6

Single-Phase Induction Motors

6.1 INTRODUCTION

Single-phase induction motors are used extensively in industrial, commercial, and domestic applications. They are used in clocks, refrigerators, freezers, fans, blowers, pumps, washing machines, machine tools, and range in size from a fraction of a horsepower to 15 hp.

Large single-phase induction motors are split-phase machines that have two separate windings physically displaced by 90 electrical degrees, and phase-splitting circuits that cause the current and associated flux of one winding to lag or lead the current and associated flux of the other winding. The net effect is the production of a rotating magnetic field that sweeps a squirrel-cage rotor, developing induction-motor action. Smaller single-phase induction motors use a much simpler device called a shading coil to provide the phase-splitting effect.

Phase-splitting circuits are also used to operate three-phase induction motors from a single-phase source. This enables larger motors to be operated in isolated areas where three-phase sources are not available. Such operation, however, must not be confused with a fault condition, called *single phasing*, where a three-phase motor operating from a three-phase source has an open circuit occur in one of the three lines. In this case, there is no phase splitter, and the resultant excessive current and vibration may damage the motor if protective devices do not function.

6.2 QUADRATURE FIELD THEORY AND INDUCTION-MOTOR ACTION

Single-phase induction motors are unique in that they cannot develop a rotating field, and hence cannot develop motor torque unless auxiliary methods are used to start the rotor turning. This is illustrated in Figure 6.1(a), where the stationary squirrel-cage rotor acts as the short-circuited secondary of a transformer. For the instant shown, the

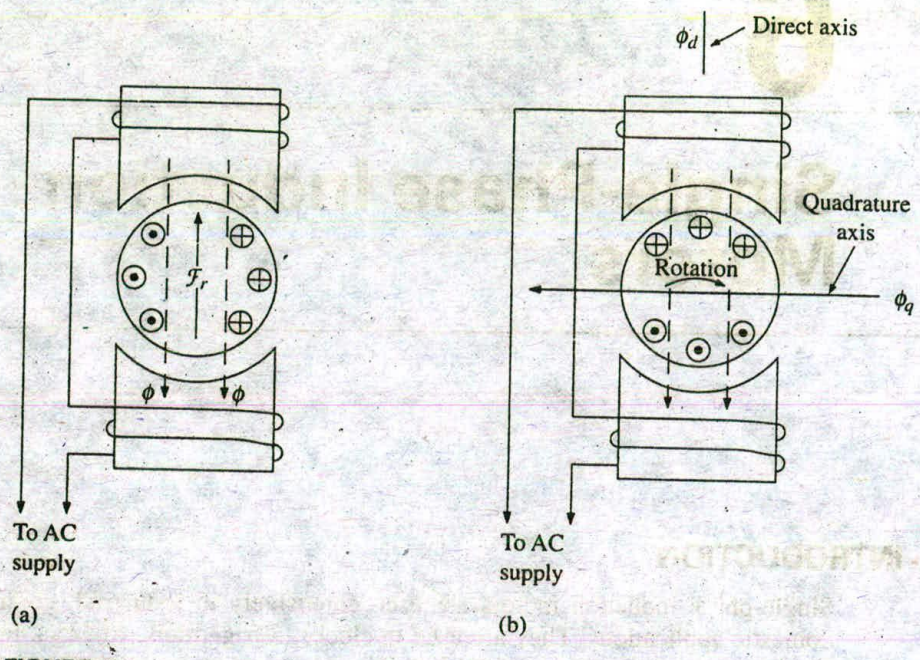


FIGURE 6.1
Instantaneous direction of current in rotor bars caused by (a) transformer action; (b) speed voltage.

stator flux, also called main-pole flux, is increasing in the downward direction, causing the rotor mmf $\mathcal{F}_r$ to be in the upward direction. Since the magnetic axis of the rotor is in line with the magnetic axis of the stator, no rotating field will be developed, and hence no motor action will be produced.

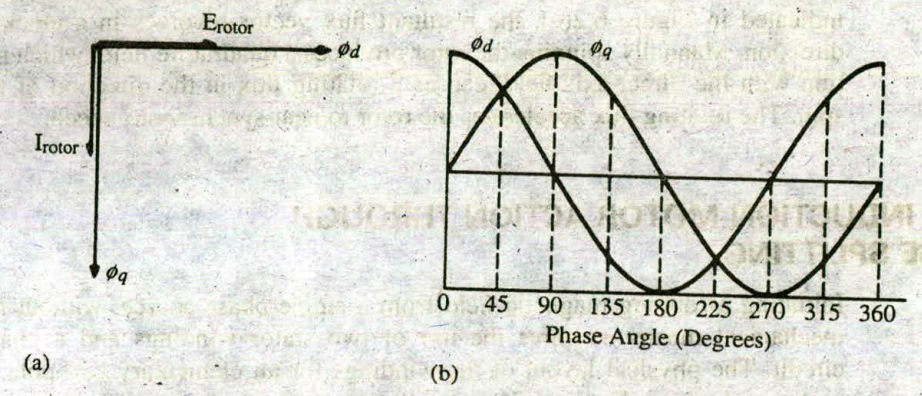
If the rotor is rotated mechanically (by hand or otherwise), as shown in Figure 6.1(b), however, a rotating field will be created, induction-motor action will occur, and the machine will accelerate to near synchronous speed. This behavior is explained by the quadrature field theory, or the double-revolving field theory **[5]**.

Quadrature Field Theory

Spinning the rotor mechanically, as shown in Figure 6.1(b), causes the rotor conductors to cut the flux of the main poles. For the instant shown the current in the rotor bars, caused by the induced speed-voltage, produces a magnetic field that is 90 electrical degrees from the centerline of the main-pole flux. The main pole flux is called the *direct-axis flux,* and the flux caused by the speed-voltage is called the *quadrature-axis flux.* The instantaneous directions of speed-voltage, associated current, and quadrature flux are determined from Lenz's law. As the rotor turns, the conductors shift their relative positions, but other conductors take their place, thus maintaining the quadrature flux. Note that *the flux produced by the stator winding alternates sinusoidally along*

the direct axis, and the induced flux in the rotor alternates sinusoidally along the quadrature axis.

Figure 6.2(a) shows the phase relationship between the main-pole flux, speed-voltage, associated rotor current, and quadrature flux. Note that the high ratio of rotor reactance to rotor resistance causes the rotor current, and hence the quadrature flux, to lag the induced voltage by almost 90°.



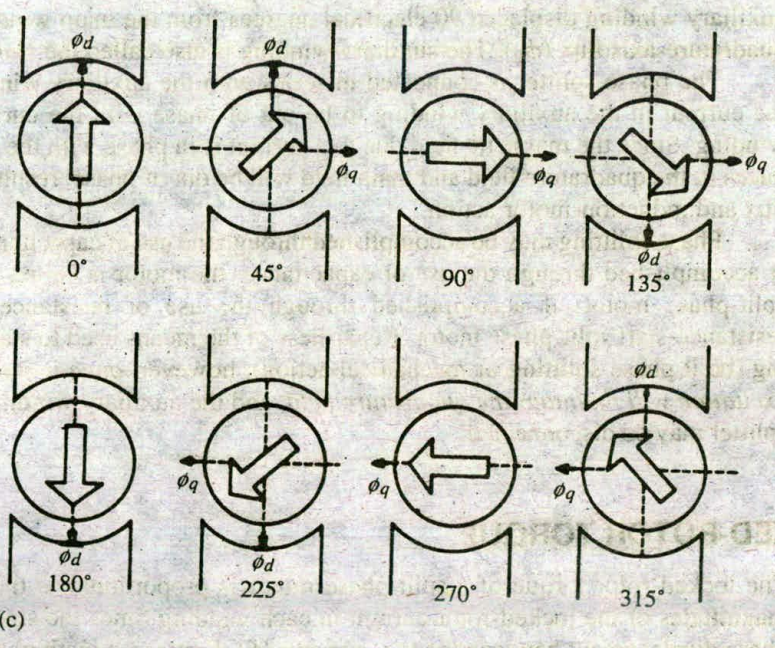


FIGURE 6.2
(a) Phase relationship between main-pole flux, speed-voltage, associated rotor current, and quadrature flux; (b) flux waves corresponding to phasor diagram in (a); (c) instantaneous directions of ϕ_d, ϕ_q, and resultant flux for corresponding phase angles in (b).

A speed-voltage is always in phase with the flux that caused it, and flux due to current is always in phase with the current that caused it. Thus, as shown in Figure 6.2(a), the quadrature flux lags the direct-axis flux by almost 90 electrical degrees. The flux waves corresponding to those in the phasor diagram are shown in Figure 6.2(b).

Figure 6.2(c) shows the instantaneous directions of the direct-axis flux, quadrature-axis flux, and resultant flux for the corresponding phase angles in Figure 6.2(b). The large arrows indicate the instantaneous direction of the resultant flux. As indicated in Figure 6.2(c), the resultant flux vector revolves in a clockwise (CW) direction. Manually spinning the rotor produces a quadrature field, which in combination with the direct-axis field, causes a rotating flux in the direction of the original spin. The rotating flux accelerates the rotor to near-synchronous speed.

6.3 INDUCTION-MOTOR ACTION THROUGH PHASE SPLITTING

Initiating a rotating magnetic field from a single-phase source, without resorting to mechanical means, requires the use of two stator windings and a phase-splitting circuit. The physical layout of the windings for an elementary two-pole split-phase motor is shown in Figure 6.3(a), and the corresponding equivalent-circuit diagram is shown in Figure 6.3(b). The main winding supplies the direct-axis flux (ϕ_d), and an auxiliary winding displaced 90 electrical degrees from the main winding supplies the quadrature-axis flux (ϕ_q). The auxiliary winding is also called the *starting winding.*

The phase splitter is connected in series with the auxiliary winding and causes the current in the auxiliary winding to be out of phase with the current in the main winding. Since the magnetic field due to a current is in phase with the current that produces it, the quadrature field and main field will be out of phase, resulting in a rotating flux and induction-motor action.

Phase splitting may be accomplished through the use of capacitance or resistance. If accomplished through the use of capacitance, the motor is called a capacitor-start split-phase motor; if accomplished through the use of resistance, it is called a resistance-start split-phase motor. Regardless of the means used to start the rotor turning (be it phase splitting or mechanical action), however, *once it starts turning, self-excitation will maintain the quadrature field,* and the auxiliary winding with its phase splitter may be disconnected.

6.4 LOCKED-ROTOR TORQUE

The locked-rotor torque of a split-phase motor is proportional to the product of the magnitudes of the locked-rotor current in each winding times the sine of the angle of phase displacement between the two currents **[6]**. Expressed mathematically,

$$T_{lr} = k_{sp} \cdot I_{mw} \cdot I_{aw} \sin \alpha \qquad \textbf{(6–1)}$$

$$\alpha = |\theta_{i,mw} - \theta_{i,aw}| \qquad \textbf{(6–2)}$$

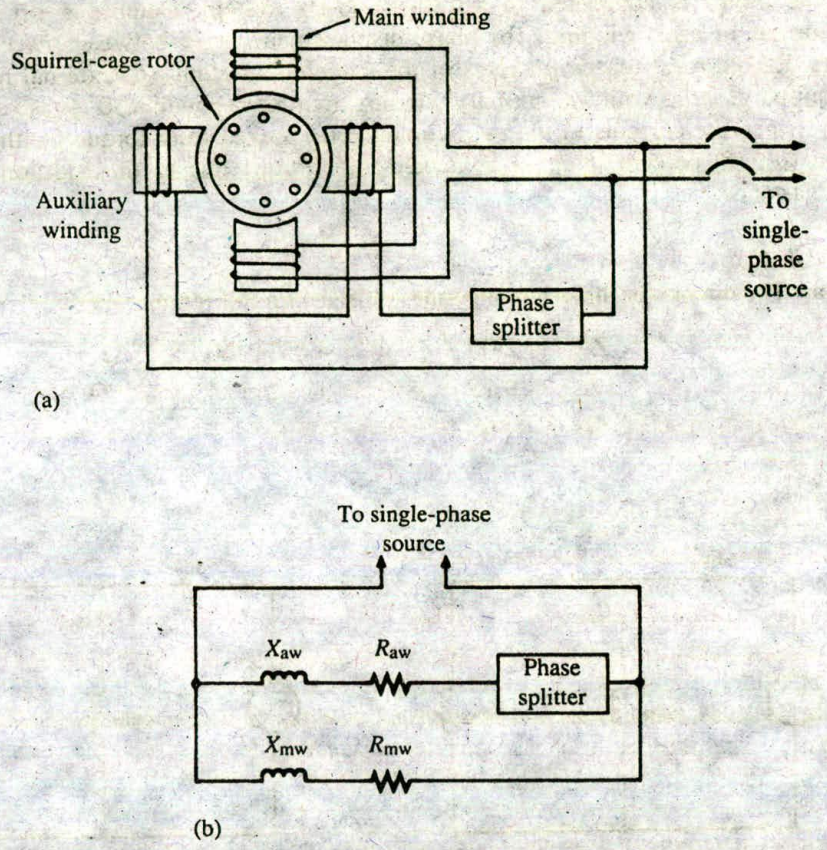


FIGURE 6.3
(a) Elementary two-pole single-phase motor with phase splitter; (b) equivalent-circuit diagram.

where:
k_{sp} = machine constant, split-phase motor
I_{aw} = current in auxiliary winding (A)
I_{mw} = current in main winding (A)
$\theta_{i,aw}$ = phase angle of current in auxiliary winding
$\theta_{i,mw}$ = phase angle of current in main winding
α = phase displacement between $\mathbf{I}_{aw}$ and $\mathbf{I}_{mw}$

EXAMPLE 6.1 The main and auxiliary windings of a hypothetical 120-V, 60-Hz, split-phase motor have the following locked-rotor parameters:

$$R_{mw} = 2.00\ \Omega \qquad X_{mw} = 3.50\ \Omega$$
$$R_{aw} = 9.15\ \Omega \qquad X_{aw} = 8.40\ \Omega$$

The motor is connected to a 120-V, 60-Hz system. Determine (a) locked-rotor current in each winding; (b) phase-displacement angle between the two currents; (c) locked-rotor torque in terms of the machine constant; (d) external resistance required in series with the auxiliary winding in order to obtain a 30° phase displacement between the currents in the two windings; (e) locked-rotor torque for the conditions in (d); (f) percent increase in locked-rotor torque due to the addition of external resistance.

Solution

(a) The circuit for the original condition is shown in Figure 6.4(a).

$$\mathbf{Z}_{mw} = 2.00 + j3.50 = 4.0311\underline{/60.2551°}\ \Omega$$
$$\mathbf{Z}_{aw} = 9.15 + j8.40 = 12.4211\underline{/42.5530°}\ \Omega$$

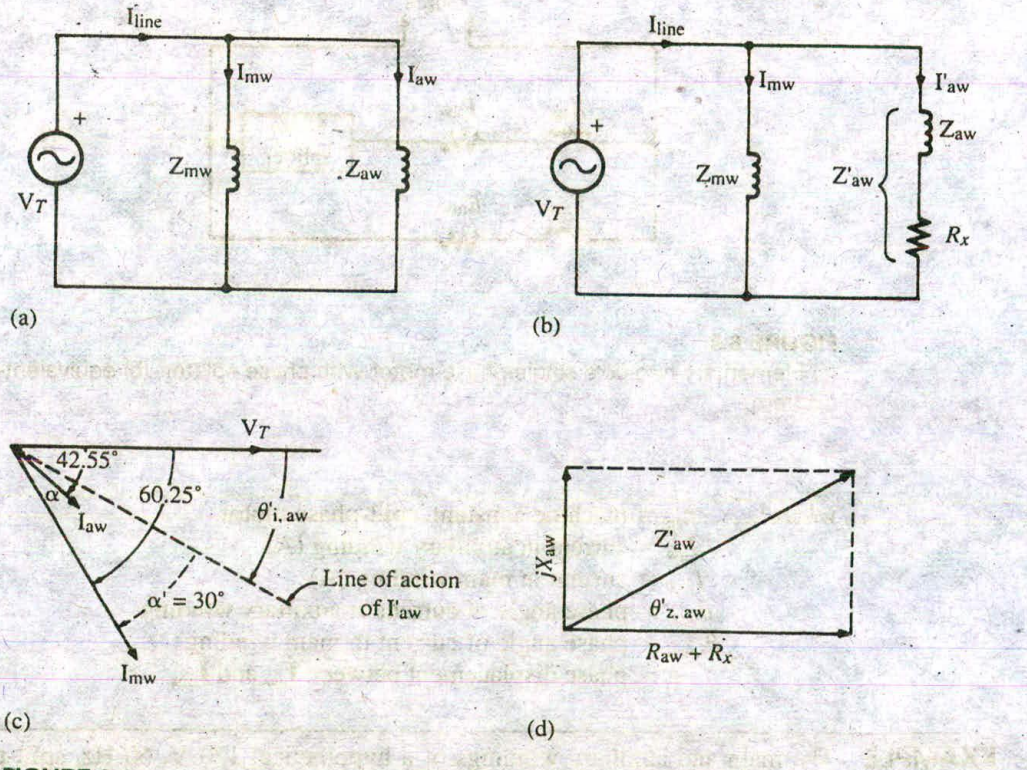


FIGURE 6.4
Diagrams for Example 6.1: (a) original circuit; (b) modified circuit; (c) phasor diagram for determining required phase angle of auxiliary current for new conditions; (d) impedance diagram for new auxiliary-circuit branch.

$$\mathbf{I}_{mw} = \frac{120\angle 0°}{4.0311\angle 60.2551°} = 29.7688\angle -60.2551° \quad \Rightarrow \quad \underline{29.8\angle -60.3°\ \text{A}}$$

$$\mathbf{I}_{aw} = \frac{120\angle 0°}{12.4211\angle 42.5530°} = 9.6610\angle -42.5530° \quad \Rightarrow \quad \underline{9.66\angle -42.6°\ \text{A}}$$

(b) $\alpha = |\theta_{i,mw} - \theta_{i,aw}| = |-60.2551 - (-42.5530)| = 17.7021° \quad \Rightarrow \quad \underline{17.7°}$

(c)
$$T_{lr} = k_{sp} \cdot I_{mw} \cdot I_{aw} \sin \alpha$$
$$T_{lr} = k_{sp} \times 29.7688 \times 9.6610 \times \sin 17.7021 = \underline{87.45k_{sp}}$$

(d) The circuit for the new condition, with a resistor in series with the auxiliary winding, is shown in Figure 6.4(b). A phasor diagram showing the respective currents for the old condition and the desired location of the new auxiliary winding current $\mathbf{I}'_{aw}$ is shown in Figure 6.4(c). From Figure 6.4(c), the required phase angle for $\mathbf{I}'_{aw}$ is

$$\theta'_{i,aw} = -60.2551° + 30° = -30.2551°$$

Applying Ohm's law to the auxiliary branch in Figure 6.4(b),

$$\mathbf{I}'_{aw} = \frac{\mathbf{V}_T}{\mathbf{Z}'_{aw}} \quad \Rightarrow \quad I'_{aw}\angle -30.2551° = \frac{V_T\angle 0°}{Z'_{aw}\angle \theta'_{z,aw}}$$
$$\theta'_{z,aw} = 30.2551°$$

From the impedance diagram for the new auxiliary-circuit branch shown in Figure 6.4(d),

$$\tan \theta'_{aw} = \frac{X_{aw}}{R_{aw} + R_x} \quad \Rightarrow \quad R_x = \frac{X_{aw}}{\tan(\theta'_{aw})} - R_{aw}$$

$$R_x = \frac{8.40}{\tan(30.2551°)} - 9.15 = 5.2508 \quad \Rightarrow \quad \underline{5.25\ \Omega}$$

(e)
$$\mathbf{I}'_{aw} = \frac{\mathbf{V}_T}{\mathbf{Z}'_{aw}} = \frac{120\angle 0°}{9.15 + 5.2508 + j8.40} = 7.1979\angle -30.2551°\ \text{A}$$
$$T_{lr} = k_{sp} \cdot I_{mw} \cdot I_{aw} \sin \alpha$$
$$T_{lr} = k_{sp} \times 29.7688 \times 7.1979 \times \sin 30° = \underline{107.1 \cdot k_{sp}}$$

(f)
$$\frac{107.1 - 87.45}{87.45} \times 100 = \underline{22.5\%\ \text{increase}}$$

Note: The added resistance in the auxiliary winding circuit decreased the auxiliary winding current, but increased the locked-rotor torque.

Graphical Analysis of Example 6.1

Because only the auxiliary winding has series-connected elements to provide phase splitting, the current in the main winding may be assumed constant, permitting Eq. (6–1) to be written as

$$T_{lr} \propto I_{aw} \sin \alpha \qquad \textbf{(6–3)}$$

Graphs of I_{aw}, α, and $(I_{aw} \cdot \sin \alpha)$ vs. the total resistance of the auxiliary winding circuit in Example 6.1, as R_x is increased from 0 Ω to 20 Ω, are shown in Figure 6.5. Note that the current in the auxiliary winding decreases with increasing resistance; angle α increases with increasing resistance; but the locked-rotor torque, represented by $(I_{aw} \sin \alpha)$, reaches a peak value with an auxiliary circuit resistance of approximately 14.2 Ω, and then falls off with increasing resistance.

For every split-phase motor there is an optimum value of auxiliary-circuit resistance that will maximize the locked-rotor torque. The phase displacement (α) for this optimum value of resistance is generally between 25° and 30°.

6.5 PRACTICAL RESISTANCE-START SPLIT-PHASE MOTORS

The circuit diagram for a general-purpose resistance-start split-phase motor is shown in Figure 6.6(a). The auxiliary winding is wound with smaller diameter wire than the main winding, causing the auxiliary winding to have a higher ratio of resistance to reactance than the main winding. The switch in the auxiliary circuit is a magnetic relay, a solid-state switch, or a centrifugally operated switch. The centrifugally operated switch shown in the circuit diagram of Figure 6.6(a) is closed when the motor is at rest, and opens when the rotor is at approximately 75 to 80 percent synchronous speed. A solid-state switch, called a triac, is shown with broken lines in Figure 6.6(a); the switch closes when starting and is set to open at approximately 75 percent synchronous speed. A magnetic relay (not shown) is closed by the high motor-starting current, and springs open when acceleration of the motor reduces the current to approximately 80 percent of the locked-rotor current. A representative phasor diagram for the motor (when starting) is shown in Figure 6.6(b). A typical torque-speed characteristic for a resistance-start split-phase motor is shown in Figure 6.6(c).

The resistance-start split-phase motor is adaptable to loads such as centrifugal pumps, oil burners, blowers, and other loads of similar characteristics that require moderate torques and constant speed. This motor offers no means for speed control from a fixed-frequency source other than that obtainable by reconnecting for different pole arrangements.

A cutaway view of a general-purpose split-phase induction motor is shown in Figure 6.7. Note the centrifugal mechanism mounted on the squirrel-cage rotor and the associated switch mounted on the end bell.

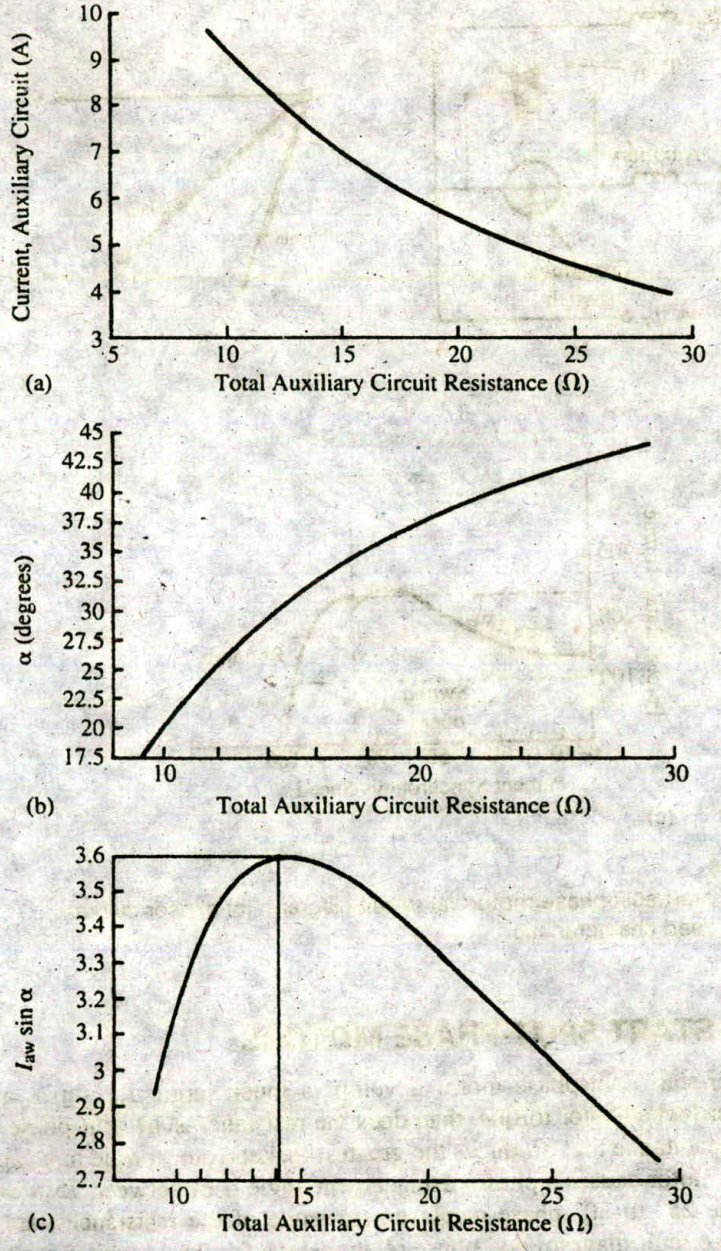


FIGURE 6.5
Graphs of auxiliary winding current, phase-displacement angle α, and locked-rotor torque represented by $I_{aw} \sin \alpha$, for the split-phase motor in Example 6.1.

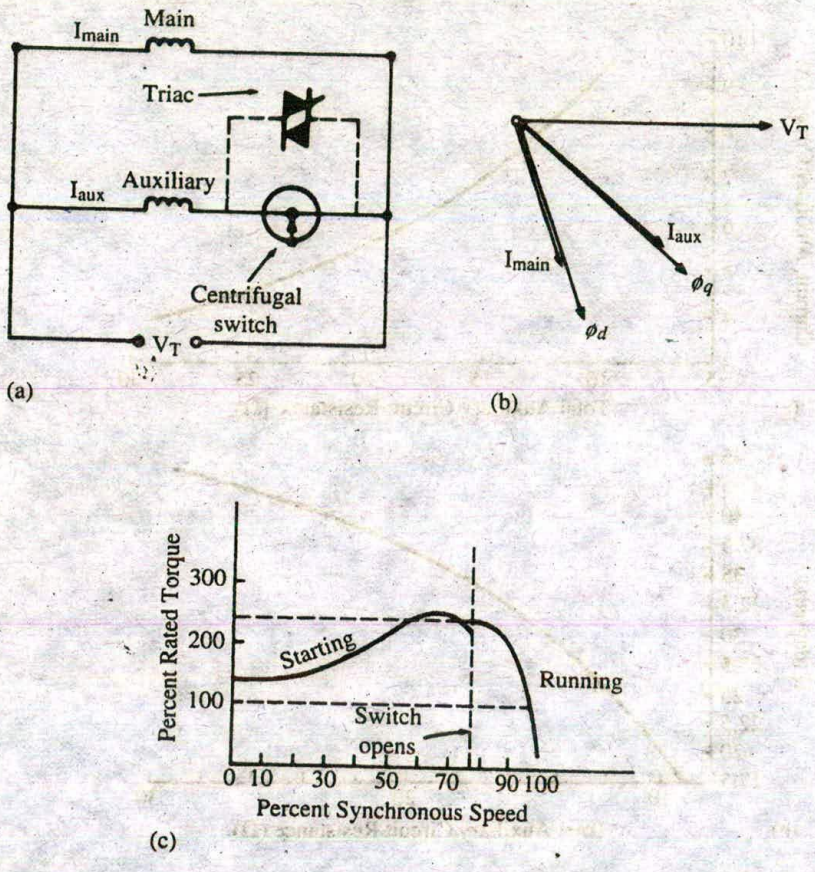


FIGURE 6.6
Resistance-start split-phase motor: (a) circuit diagram; (b) phasor diagram; (c) torque-speed characteristic.

6.6 CAPACITOR-START SPLIT-PHASE MOTORS

A capacitor-start split-phase motor develops a much larger $I_{aw} \sin \alpha$, and hence a much larger locked-rotor torque, than does the resistance-start split-phase motor. The value of capacitance that produces the greatest locked-rotor torque in a capacitor-start split-phase motor causes a phase-displacement angle α of between 75° and 88°, compared to the 25° to 30° phase-displacement angle of the resistance-start split-phase motor. The circuit diagram and phase relationships for the capacitor-start split-phase motor are shown in Figures 6.8(a) and (b), respectively.

A typical torque-speed characteristic for a capacitor-start split-phase induction motor is shown in Figure 6.8(c). The starting curve shows the motor characteristics with both auxiliary and main windings energized. The running curve shows the characteristic behavior after the auxiliary winding is disconnected. A comparison of this

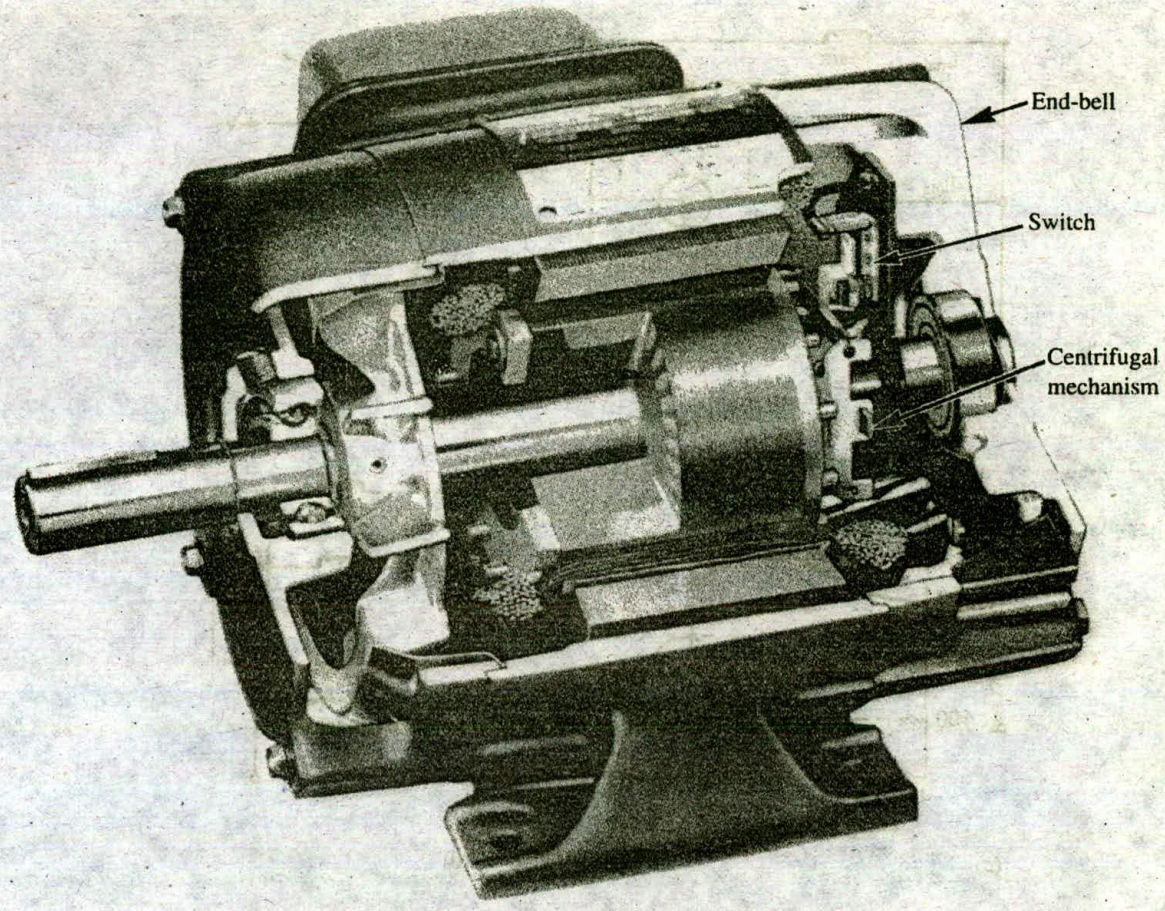


FIGURE 6.7
Cutaway view of a split-phase motor. (Courtesy Magnetek-Century Electric Company)

characteristic with that of the resistance-start split-phase motor in Figure 6.6(c) shows that the running characteristics of both machines are essentially the same. The significant difference between the two machines is the starting torque; about 130 percent rated for the resistance-start split-phase motor, and 300 percent rated for the capacitance-start split-phase motor. The high starting torque and good speed regulation of the capacitor-start motor make it well suited for applications to stokers, compressors, reciprocating pumps, and other loads of similar characteristics. This motor offers no means for speed control from a fixed-frequency source other than that obtainable by reconnecting for a different number of poles.

Note that neither the resistance-start split-phase motor nor the capacitance-start split-phase motor can attain synchronous speed. The rotating flux is dependent on

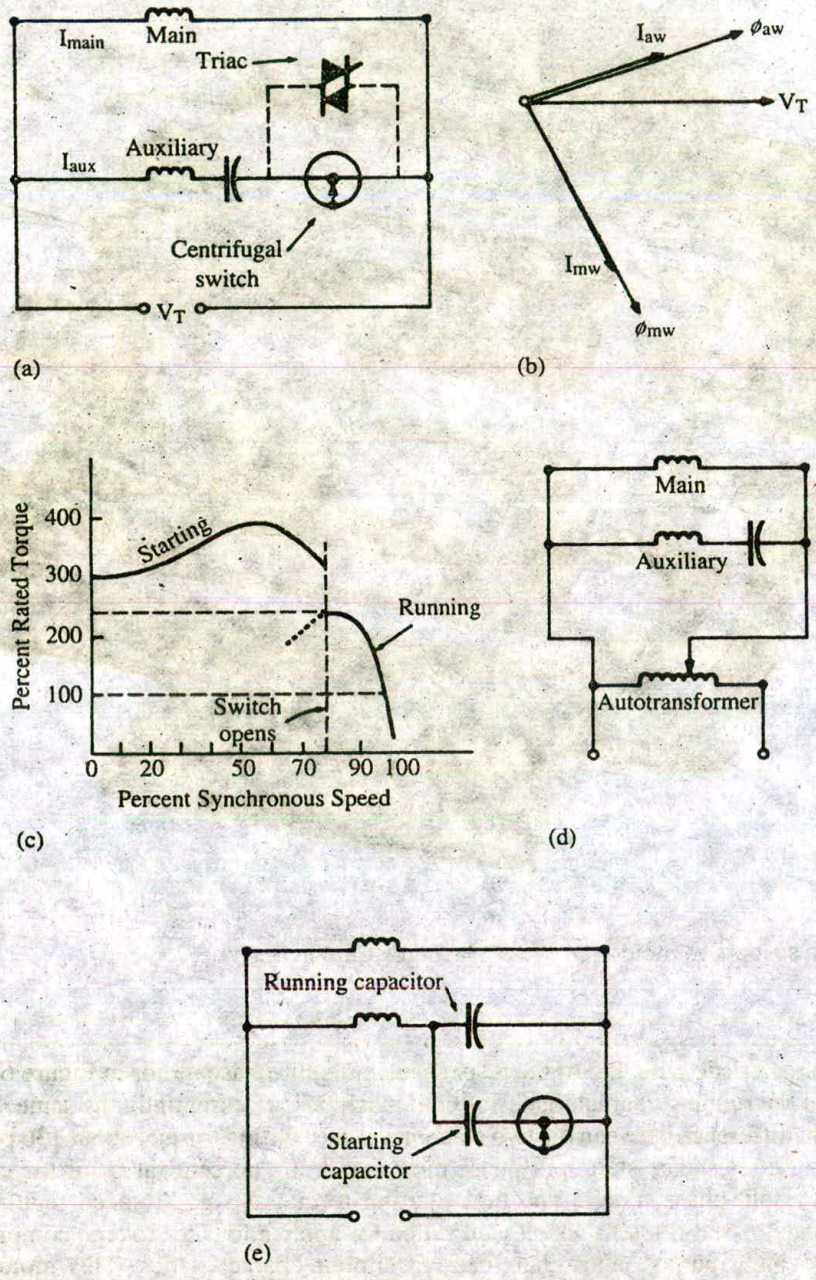


FIGURE 6.8
Capacitor motors: (a) circuit for capacitor-start motor; (b) phasor diagram corresponding to (a); (c) torque-speed characteristic for motor in (a); (d) permanent-split capacitor motor; (e) two-value capacitor motor.

induced current in the rotor to produce the quadrature field. Thus, as the rotor approaches synchronous speed, the speed-voltage induced in the rotor, the associated current in the rotor, and the quadrature flux approach zero. Hence, the accelerating torque will become zero at slightly below synchronous speed.

However, permanent-split capacitor motors and two-value capacitor motors (described below) are in effect two-phase motors, and at no load could attain synchronous speed.

Permanent-Split Capacitor Motor

A permanent-split capacitor motor utilizes a permanently connected auxiliary circuit containing a capacitor. There is no switch in the auxiliary circuit, and its operation is smoother and quieter than a capacitor-start or resistance-start motor of the same power rating. The value of capacitance for this type of motor is smaller than the one used in the capacitor-start motor and is a compromise between the best starting and best running performances. The primary field of application for a permanent-split capacitor motor is for shaft-mounted fans used in unit heaters and for ventilating fans. Its speed may be varied by a tapped or slide-wire autotransformer in the main line, as shown in Figure 6.8(d); by using an external resistor or reactor in series with the main winding or in series with both windings; or by adjusting the number of turns in the main winding through the use of taps and a selector switch or by solid-state control [8].

Two-Value Capacitor Motor

A two-value capacitor motor, having the circuit shown in Figure 6.8(e), provides a greater amount of capacitance for starting than for running. This enables a greater locked-rotor torque than is obtainable with the permanent-split capacitor motor, and the reduced capacitance when running results in improved power factor, improved efficiency, and a higher breakdown torque [8].

EXAMPLE 6.2 Using the given data for the split-phase motor windings in Example 6.1, determine (a) the capacitance required in series with the auxiliary winding in order to obtain a 90° phase displacement between the current in the main winding and the current in the auxiliary winding at locked rotor; (b) locked-rotor torque in terms of the machine constant.

Solution

(a) The winding impedances in Example 6.1 are

$$\mathbf{Z}_{mw} = 2.00 + j3.50 = 4.0311\underline{/60.2551°}\ \Omega$$

$$\mathbf{Z}_{aw} = 9.15 + j8.40 = 12.4211\underline{/42.5530°}\ \Omega$$

The circuit for the original conditions is shown in Figure 6.9(a).

$$\mathbf{I}_{mw} = \frac{120\angle 0°}{4.0311\angle 60.2551°} = \underline{29.7688\angle -60.2551° \text{ A}}$$

$$\mathbf{I}_{aw} = \frac{120\angle 0°}{12.4211\angle 42.5530°} = \underline{9.6610\angle -42.5530° \text{ A}}$$

The circuit diagram for the new condition (with a capacitor in series with the auxiliary winding) is shown in Figure 6.9(b), and a phasor diagram showing the respective

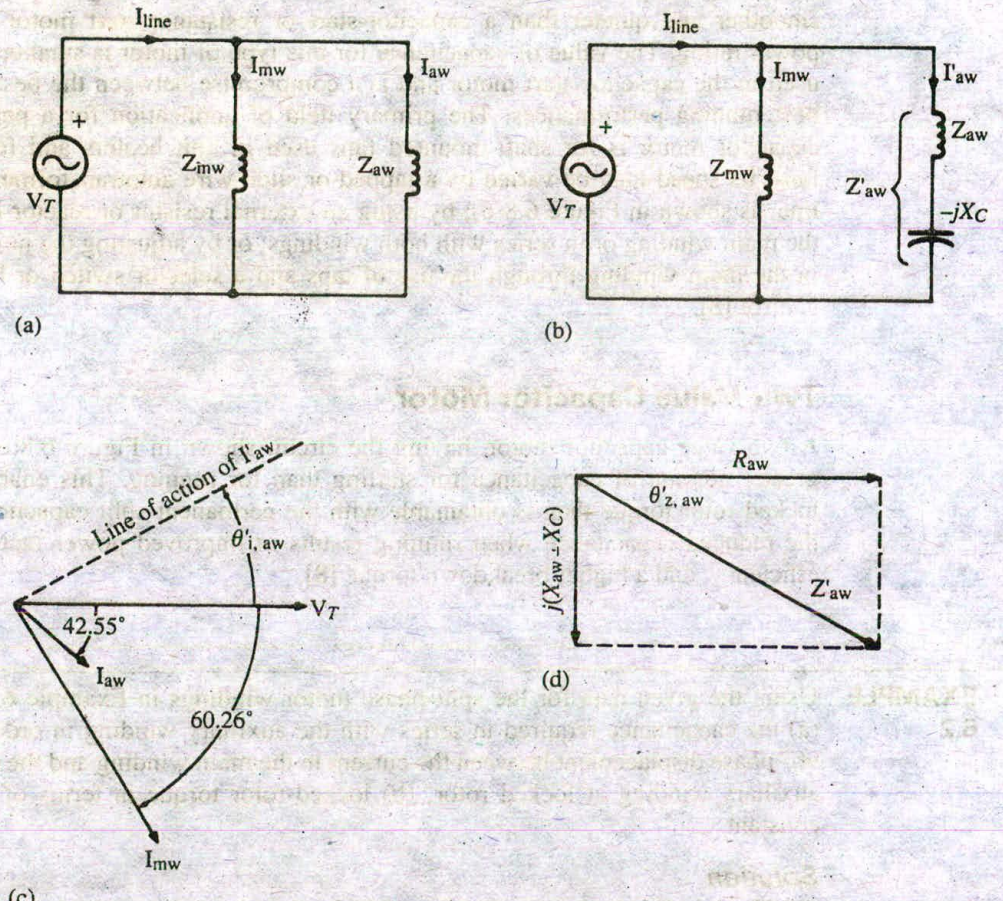


FIGURE 6.9
Diagrams for Example 6.2: (a) original circuit; (b) modified circuit; (c) phasor diagram for determining required phase angle of auxiliary current for new conditions; (d) impedance diagram for new auxiliary-circuit branch.

currents for the original condition and the desired location of the new auxiliary-winding current is shown in Figure 6.9(c). The required phase angle for $\mathbf{I}'_{aw}$ is

$$\theta'_{i,aw} = 90° - 60.26° = 29.74°$$

Applying Ohm's law to the auxiliary branch in Figure 6.9(b),

$$\mathbf{Z}'_{aw} = Z'_{aw}\underline{/\theta'_{z,aw}} = \frac{V_T\underline{/0°}}{I'_{aw}\underline{/29.74°}}$$

Thus,

$$\theta'_{z,aw} = -29.74°$$

From the impedance diagram, shown in Figure 6.9(d) for the new auxiliary circuit branch,

$$\tan(\theta'_{z,aw}) = \frac{X_{aw} - X_C}{R_{aw}} \quad \Rightarrow \quad X_C = X_{aw} - R_{aw} \cdot \tan(\theta'_{z,aw})$$

$$X_C = 8.40 - 9.15 \times \tan(-29.74°) = 13.628\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi 60 \times 13.628} = \underline{194.6\ \mu F}$$

(b)

$$\mathbf{I}'_{aw} = \frac{120\underline{/0°}}{9.15 + j8.40 - j13.628} = 11.387\underline{/29.74°}$$

$$T_{lr} = k_{sp} \cdot I_{mw} \cdot I_{aw} \sin\alpha$$

$$T_{lr} = k_{sp} \times 29.7688 \times 11.387 \times \sin 90° = \underline{338.9k_{sp}}$$

Note: The percent increase in locked-rotor torque obtained by capacitor start in Example 6.2, with respect to the locked-rotor torque obtained by resistor start in Example 6.1, is

$$\frac{338.9 - 107.1}{107.1} \times 100 = 216\%$$

Graphical Analysis of Example 6.2

Graphs of I_{aw}, α, and (I_{aw} sin α), plotted against X_C for Example 6.2 are shown in Figure 6.10. Note that the current in the auxiliary winding increases and then decreases with increasing capacitive reactance (resonance phenomena); angle α increases with increasing capacitive reactance; and the locked-rotor torque, represented by I_{aw} sin α, increases to some peak value, and then falls off with increasing capacitive reactance. Note that, for the given winding parameters, the optimum value of capacitive reactance that resulted in a phase-displacement angle of approximately 75° produced the greatest locked-rotor torque.

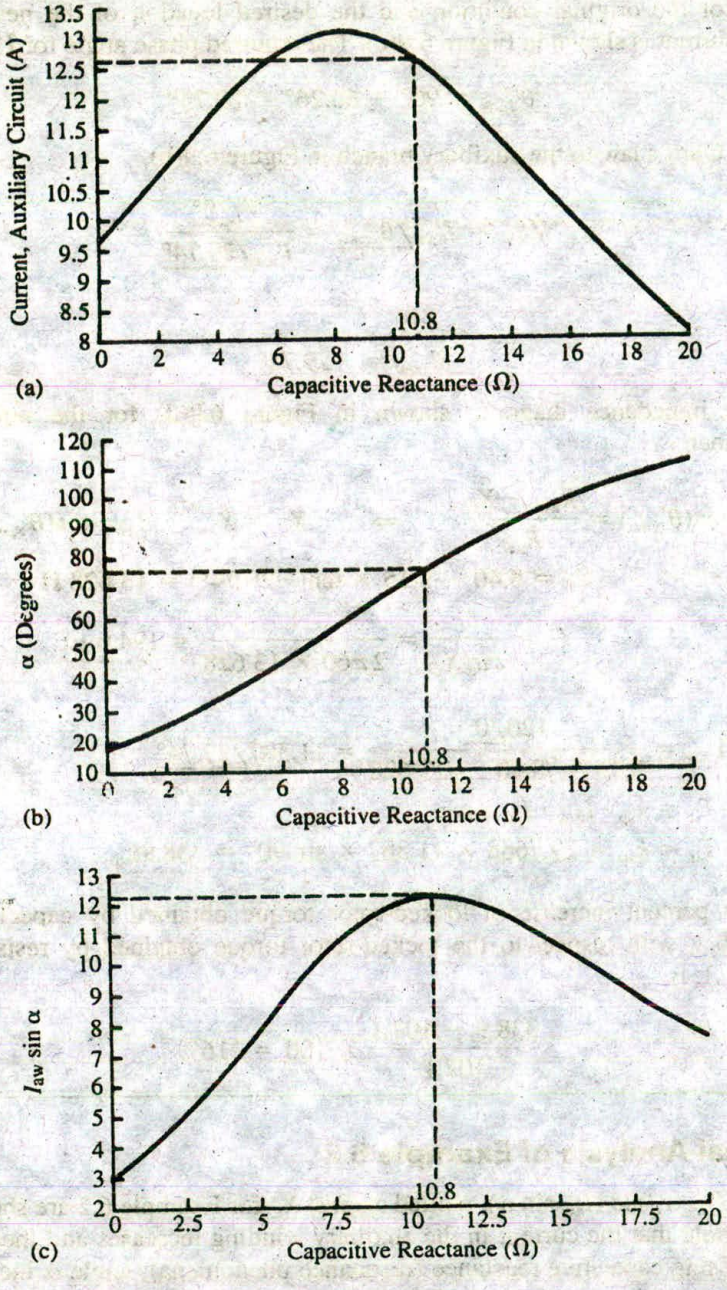


FIGURE 6.10
Graphs of auxiliary-winding current, phase-displacement angle α, and locked-rotor torque represented by $I_{aw} \sin \alpha$, for the capacitor-start motor in Example 6.2.

A comparison of the I_{aw} sin α curve for the capacitor motor in Figure 6.10(c) with that of the split-phase motor in Figure 6.5(c) shows that with the same windings, phase shifting with capacitance can produce significantly greater locked-rotor torques than can phase shifting with resistance.

6.7 REVERSING SINGLE-PHASE INDUCTION MOTORS

Reversing a single-phase induction motor is accomplished by stopping the machine, interchanging the leads to the auxiliary circuit, and then restarting. This reverses the quadrature-axis flux, causing flux rotation to be in the opposite direction.

6.8 SHADED-POLE MOTORS

The shaded-pole motor, illustrated in Figure 6.11(a), utilizes a short-circuited coil or copper ring, called a *shading coil*, to provide starting torque. The shading coil is wound around a part of the pole face and acts as the short-circuited secondary of a transformer. In accordance with Lenz's law the current in the shading coil sets up a magnetomotive force (mmf) in a direction to oppose the mmf of the field that produced it. This causes a time delay in the buildup of flux in the shading portion of the pole face. The flux reaches its maximum in the unshaded portion of the pole face before it reaches its maximum in the shaded portion, thus causing a shifting flux in the direction from the unshaded part to the shaded part. The shifting flux "sweeping" the squirrel-cage rotor develops induction-motor action. Figure 6.11(b) shows a representative torque-speed characteristic for the shaded-pole motor. A shaded-pole motor cannot be reversed unless it has two sets of shading coils with two switches, one set for each direction of rotation. The principal field of application for shaded-pole motors is in clocks, record players, small fans, and other loads with similar characteristics.

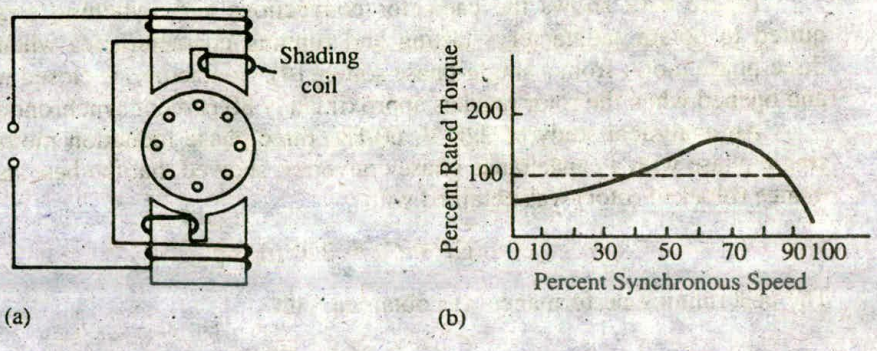


FIGURE 6.11
Shaded-pole motor: (a) construction; (b) torque-speed characteristic.

TABLE 6.1
Range of standard power ratings, single-phase motors

Motor	Power Range
Capacitor start	1 mhp to 10 hp
Resistance start	1 mhp to 10 hp
Two-value capacitor	1 mhp to 10 hp
Permanent-split capacitor	1 mhp to 1.5 hp
Shaded-pole	1 mhp to 1.5 hp

6.9 NEMA-STANDARD RATINGS FOR SINGLE-PHASE INDUCTION MOTORS

The standard frequency and voltage ratings for single-phase induction motors are

60 Hz—115 V and 230 V
50 Hz—110 V and 220 V

The ranges of NEMA-standard power ratings for single-phase induction motors are listed in Table 6.1.[1]

6.10 OPERATION OF THREE-PHASE MOTORS FROM SINGLE-PHASE LINES

In many developing countries, and in certain remote regions of the United States, and on isolated farms, the only source of power is a single-phase system. The power rating of single-phase motors, however, is generally limited to 10 hp. Hence, to fill the need for higher power requirements, three-phase motors are purchased and operated from a single-phase supply using capacitors for phase conversion.

Figure 6.12 shows the capacitor connections and switching arrangement required to obtain satisfactory starting and running characteristics when operating a three-phase motor from a single-phase source [4]. Contactor S is closed when starting, and opened when the rotor reaches approximately 80 percent synchronous speed.

An analytical study of 220-V, 60-Hz, three-phase induction motors, operating single phase with a capacitance phase converter, showed that the best starting performance (blocked rotor) was obtained with

$$C_1 + C_2 \approx 230\ \mu\text{F/hp} \tag{6–4}$$

The best running performance was obtained with

$$C_1 \approx 26.5\ \mu\text{F/hp} \tag{6–5}$$

[1]See Reference [7] for complete tables of power ratings, speed ratings, locked-rotor torque, and breakdown torque.

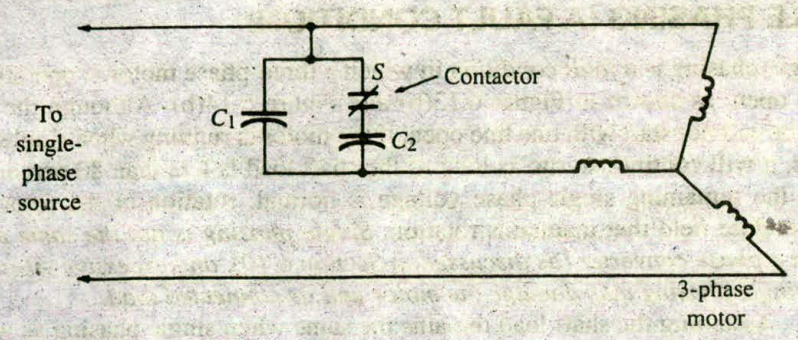


FIGURE 6.12
Capacitor connections and switching arrangement for operating a three-phase motor from a single-phase line.

Because of reduced breakdown torque and higher operating temperature, however, a three-phase motor operating single phase with a capacitance phase converter should be derated to approximately two-thirds of the three-phase rating. Thus,

$$P_{\text{rated } 1\phi} = \frac{2}{3} P_{\text{rated } 3\phi} \tag{6–6}$$

Three-phase wye-connected motors that have all six leads available, called an *open-wye motor,* provide greater flexibility for designing three-phase to single-phase conversion systems. Large open-wye motors such as a 100-hp, three-phase machine driving a rock crusher have been successfully operated from a single-phase source [2].

EXAMPLE 6.3 A large exhaust fan for a farm building on a remote farm requires a 35-hp, 1175-rpm motor. The motor is to be operated from a 220-V, 60-Hz, single-phase system. Determine (a) the NEMA-standard horsepower rating of a three-phase, design *B* motor that will provide satisfactory service when used with a capacitance phase converter; (b) the required running capacitance; (c) the additional capacitance required for starting.

Solution

(a) From Eq. (6–6),

$$P_{\text{rated } 3\phi} = 35 \times \frac{3}{2} = 52.5 \text{ hp}$$

From Table 5.1 in Chapter 5, the motor that most closely fits the requirement, without overloading, is a four-pole, 220-V, 60-hp machine.

(b) $$C_1 = 26.5\ \mu\text{F} \times 60 = \underline{1590\ \mu\text{F}}$$

(c) $$C_1 + C_2 = 230\ \mu\text{F} \times 60 = 13{,}800\ \mu\text{F}$$
$$C_2 = (13{,}800 - 1590) = \underline{12{,}210\ \mu\text{F}}$$

6.11 SINGLE PHASING (A FAULT CONDITION)

Single phasing is a *fault condition* in which a three-phase motor is operating with one line open, as shown in Figure 6.13(b) and Figure 6.14(b). Although the three-phase motor will not start with one line open, if the motor is running when single phasing occurs, it will continue to run as long as the shaft load is less than 80 percent rated load and the remaining single-phase voltage is normal; rotation of the rotor produces a quadrature field that maintains rotation. *Single phasing is not the same as operating with a phase converter (as discussed in Section 6.10), and excessive vibration caused by single phasing may damage the motor and its connected load.*

Assuming the shaft load remains the same when single phasing as when operating three phase,

$$\sqrt{3} \cdot V_{\text{line}} \cdot I_{\text{line } 3\phi} \cdot F_{P3\phi} = V_{\text{line}} \cdot I_{\text{line } 1\phi} \cdot F_{P1\phi}$$

$$I_{\text{line } 1\phi} = \sqrt{3} \cdot I_{\text{line } 3\phi} \times \frac{F_{P3\phi}}{F_{P1\phi}} \qquad \textbf{(6–7)}$$

where: $I_{\text{line } 1\phi}$ = line current when single phasing
$I_{\text{line } 3\phi}$ = line current when operating three phase
$F_{P1\phi}$ = power factor when single phasing
$F_{P3\phi}$ = power factor when operating three phase

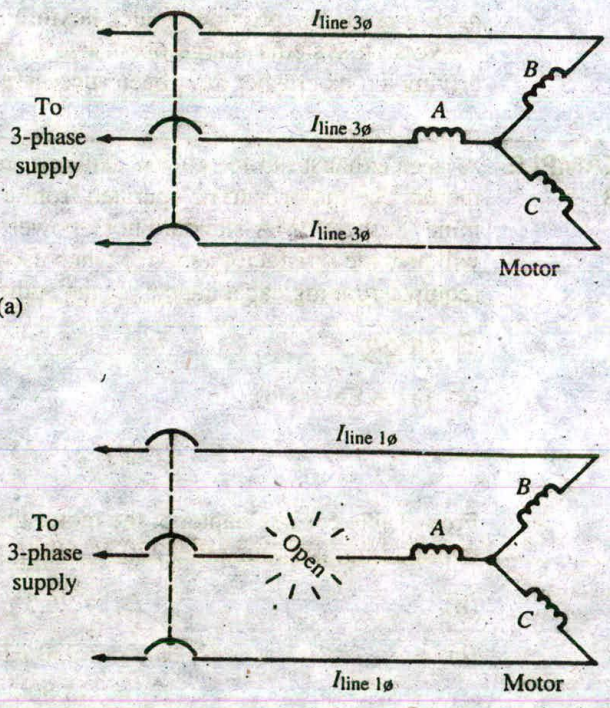


FIGURE 6.13
Wye-connected motor: (a) normal; (b) single phasing.

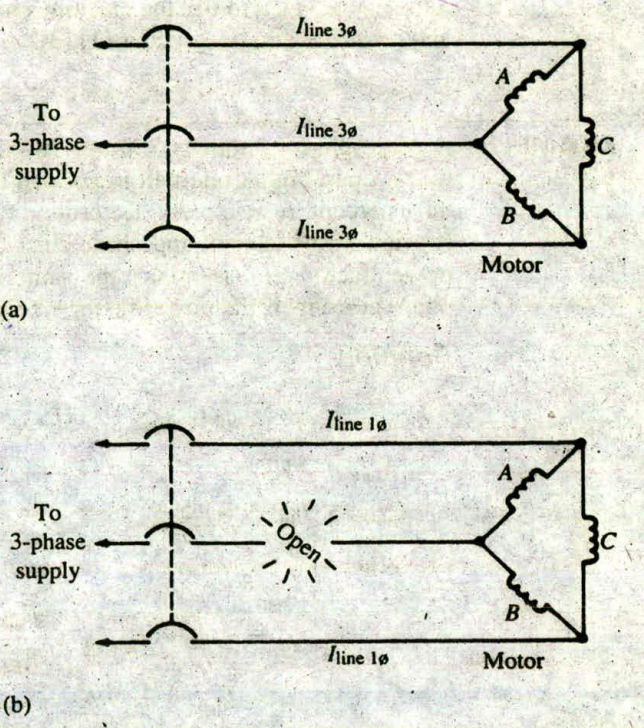


FIGURE 6.14
Delta-connected motor: (a) normal; (b) single phasing.

Thus, for the wye-connected stator in Figure 6.13 and for the delta-connected stator in Figure 6.14, when single phasing occurs, the current in the two remaining *lines* will be increased by 73 percent.[2] The distribution of current in the *stator windings* will depend on whether the windings are connected in wye or delta.

For the wye connection, shown in Figure 6.13(b), the phase current and line current are the same. Thus,

$$I_{YA} = 0$$
$$I_{YB} = I_{YC} = I_{\text{line } 1\phi} \tag{6–8}$$

For the delta connection, shown in Figure 6.14(b), the phase currents will divide in an inverse proportion to the paralleled impedances. Thus, the current distribution for the delta connection will be

$$I_{\Delta A} = I_{\Delta B} = \frac{1}{3} I_{\text{line } 1\phi}$$
$$I_{\Delta C} = \frac{2}{3} I_{\text{line } 1\phi} \tag{6–9}$$

If single phasing occurs while operating at or near rated load, the increase in phase current will cause rapid heating of the windings, and therefore protective devices,

[2]A small reduction in power factor occurs when single phasing, resulting in a somewhat higher line current.

must be provided to trip the machines from the supply lines, or severe damage to stator and rotor winding may occur **[1],[3]**.

EXAMPLE 6.4 A three-phase, 2300-V, 350-hp, 78-A, 60-Hz, four-pole, wye-connected, design *B* motor operating at one-half rated load has an efficiency and power factor of 93.6 and 84.4 percent, respectively. Determine (a) motor line current and motor phase current; (b) motor line current and phase current if one line opens; assume the efficiency and power factor are essentially the same as before the open occurred; (c) the line and phase currents if the power factor when single phasing is 82.0 percent.

Solution

(a) $P_{in} = \sqrt{3}\, V_{line} I_{line} F_P \quad \Rightarrow \quad \dfrac{(350/2)746}{0.936} = \sqrt{3}(2300) \cdot I_{line}(0.844)$

$$I_{line} = \underline{41.5\ \text{A}}$$

$$I_{phase} = I_{line} = \underline{41.5\ \text{A}}$$

(b) When single phasing,

$$I_{line\ 1\phi} = \sqrt{3} \times I_{line\ 3\phi} \times \frac{F_{P3\phi}}{F_{P1\phi}} = \sqrt{3} \times 41.5 \times \frac{0.844}{0.844} = \underline{71.9\ \text{A}}$$

$$I_{phase} = I_{line} = \underline{71.9\ \text{A}}$$

(c)
$$I_{line} = 71.9 \times \frac{0.844}{0.820} = \underline{74.0\ \text{A}}$$

$$I_{phase} = I_{line} = \underline{74.0\ \text{A}}$$

Single phasing is a fault condition that should not be allowed to continue.

SUMMARY OF EQUATIONS FOR PROBLEM SOLVING

Locked-Rotor Conditions (Single-Phase Motors)

$T_{lr} \propto I_{mw} \cdot I_{aw} \sin \alpha \qquad \alpha = |\theta_{i,mw} - \theta_{i,aw}|$ **(6–1, 6–2)**

Operating Three-Phase 220-V Motors From a Single-Phase Line

$C_1 + C_2 \approx 230\ \mu\text{F/hp}$ **(6–4)**

$C_1 \approx 26.5\ \mu\text{F/hp}$ **(6–5)**

$P_{rated\ 1\phi} = \dfrac{2}{3} P_{rated\ 3\phi}$ **(6–6)**

Single Phasing (Fault Condition)

$$I_{\text{line } 1\phi} = \sqrt{3} \cdot I_{\text{line } 3\phi} \times \frac{F_{P3\phi}}{F_{P1\phi}} \tag{6–7}$$

$$I_{YA} = 0 \qquad I_{YB} = I_{YC} = I_{\text{line } 1\phi} \tag{6–8}$$

$$I_{\Delta A} = I_{\Delta B} = \frac{1}{3} I_{\text{line } 1\phi} \qquad I_{\Delta C} = \frac{2}{3} I_{\text{line } 1\phi} \tag{6–9}$$

SPECIFIC REFERENCES KEYED TO TEXT

1. Brighton, R. S., Jr., and P. N. Ranade. Why overload relays do not always protect motors. *IEEE Trans. Industry Applications,* Vol. IA-18, No. 6, Nov./Dec. 1982.
2. Elliott, K. C., Jr. Open-wye-type phase conversion systems. *IEEE Trans. Industry and General Applications,* Vol. IGA-6, No. 2, Mar./Apr. 1970.
3. Griffith, M. S. A penetrating gaze at one open phase: Analyzing the polyphase induction motor dilemma. *IEEE Trans. Industry Applications,* Vol. IA-13, No. 6, Nov./Dec. 1977.
4. Habermann, R., Jr. Single-phase operation of a three-phase induction motor with a single-phase converter. *Trans. AIEE-PAS,* 1954, p. 833.
5. Matsch, L. W., and J. D. Morgan. *Electromagnetic and Electromechanical Machines.* Harper & Row, New York, 1986.
6. McPherson, G. *An Introduction to Electrical Machines and Transformers.* Wiley, New York, 1981.
7. National Electrical Manufacturers Association. *Motors and Generators.* Publication No. MG 1-1998, NEMA, Rosslyn, VA, 1999.
8. Vienott, C. G., and J. E. Martin. *Fractional and Sub-fractional Horsepower Electric Motors.* McGraw-Hill, New York, 1986.

REVIEW QUESTIONS

1. Explain how a resistance-start split-phase induction motor develops a rotating magnetic field.
2. Explain how a capacitance-start split-phase induction motor develops a rotating magnetic field.
3. Explain how a shaded pole motor develops a rotating magnetic field.
4. A grindstone operated by a resistance-start split-phase motor will start only when spun by hand; it will accelerate to near rated speed and then go into a recycling state: slow-down, speed-up, slow-down, speed-up, etc. Determine the cause and explain the behavior.
5. A capacitor-start motor does not run when the breaker is closed; it hums loudly and starts to smoke. If the breaker is closed while the shaft is spun by hand, however, the machine comes up to speed and operates properly. What are the three probable faults?

PROBLEMS

6–1/4 An experimental 120-V, 1/4-hp, 60-Hz, split-phase motor has the following locked-rotor parameters referred to the respective main and auxiliary windings:

$$R_{mw} = 3.94\ \Omega \qquad X_{mw} = 4.20\ \Omega$$
$$R_{aw} = 8.42\ \Omega \qquad X_{aw} = 6.28\ \Omega$$

Determine (a) the amount of external resistance required in series with the auxiliary winding in order to obtain a 30° phase displacement between the current in the main winding and the current in the auxiliary winding; (b) the locked-rotor current drawn by each winding; (c) the line current at locked rotor.

6–2/4* (a) For the machine in Problem 6–1/4, tabulate and plot $I_{aw} \sin \alpha$ vs. total auxiliary circuit resistance for 30 equal increments of R_x ranging from 0.0 Ω to 15.0 Ω. (b) Determine from the curve the value of R_x that will provide maximum starting torque.

6–3/6 Using the given data in Problem 6–1/4, determine (a) the amount of capacitance required in series with the auxiliary winding in order to obtain a phase displacement of 80.6° between the current in the main winding and the current in the auxiliary winding; (b) line current at locked rotor with capacitance in the auxiliary-winding circuit.

6–4/6* For the machine in Problem 6–3/6, tabulate and plot $I_{aw} \sin \alpha$ vs. capacitive reactance for 30 equal increments of X_C ranging from 0.0 Ω to 20 Ω. (b) Determine from the curve the value of capacitive reactance that will provide maximum starting torque. (c) Determine the value of capacitance for (b). (d) Determine the line current at locked rotor.

6–5/10 A three-phase, 50-hp, 60-Hz, 220-V, delta-connected motor is to be operated from a 220-V single-phase system using a capacitance phase converter. Determine (a) the total capacitance required for starting; (b) the running capacitance; (c) the rerated motor horsepower.

6–6/10 A centrifugal pump that requires a 20-hp input at 1185 r/min must be driven from a 220-V, 60-Hz, single-phase system. (a) Select a three-phase motor (to be used with a capacitance phase converter) that most closely suits the desired characteristics of the load. (b) Determine the required running capacitance. (c) Determine the additional capacitance required for good starting characteristics.

6–7/11 A 460-V, 25-hp, 60-Hz, six-pole, delta-connected, design *B* motor with Code letter G, operating at 75 percent rated output, has an efficiency, power factor, and slip of 94.4, 83.0, and 1.67 percent, respectively. (a) Determine motor line currents and motor phase currents. (b) Neglecting changes in power factor and slight changes in efficiency, determine the motor line currents and motor phase currents if a single-phase fault occurs.

*Solution by computer is recommended.

6–8/11 A three-phase, 4000-V, 150-hp, 890-r/min, 60-Hz, design *B*, wye-connected motor has a full-load current of 21 A and draws a locked-rotor current of 120 A. The motor is operating at 60 percent rated shaft output, and has an efficiency and power factor of 93.0 and 73.8 percent, respectively, at that load. A fault in the power supply causes an open in one of the three line leads feeding the motor. (a) Assuming the efficiency remains essentially the same, but the power factor drops to 70.1 percent, determine the motor line current and the current in each of the three motor phases. (b) If the motor is stopped, by opening its breaker, will it be able to restart with one line out? Explain.

7

Specialty Machines

7.1 INTRODUCTION

This chapter provides a brief introduction to machines with special applications. Machines, such as reluctance motors and hysteresis motors, are used for timing devices, tape recorders, turntables, and others with constant-speed requirements. They are used extensively in process industries, such as the man-made fiber industry, where many components of the process line must operate in synchronism.

Stepper motors are used in conjunction with pulse-driving circuits for precise positioning of mechanical systems. They are essential components of disk drives, printers, plotters, robots, and other applications that require accurate step-by-step positioning.

Linear induction motors are used to apply mechanical forces and to cause movement in a straight or curved line. They are used in conveyer systems, door openers, aircraft launchers, electromagnetic guns, liquid-metal pumps for nuclear reactors, high-speed rail transportation, and the like.

Universal motors have applications in low-power apparatus, such as vacuum cleaners, small power tools, and kitchen appliances.

7.2 RELUCTANCE MOTORS

The reluctance motor, also called a reluctance-synchronous motor, is an induction motor with a modified squirrel-cage rotor such as those shown in Figure 7.1. The *notches, flats,* or *barrier slots* provide equally spaced areas of high reluctance. The sections of rotor periphery between the high-reluctance areas are called salient poles; the number of salient poles must match the number of stator poles. The stator winding may be three phase, as discussed in Chapter 5, or single phase, as discussed in Chapter 6.

When the stator is energized, the rotor accelerates as a squirrel-cage induction motor. As the rotor approaches synchronous speed, the slip is very small and the rotating flux of the stator moves slowly past the salient poles of the rotor. At some critical speed, the low-reluctance paths provided by the salient poles cause them to *snap into synchronism* with the rotating flux of the stator. When this occurs, the slip is zero, induction-motor action ceases, and the rotor is pulled around by simple magnetic attraction called *reluctance torque.*

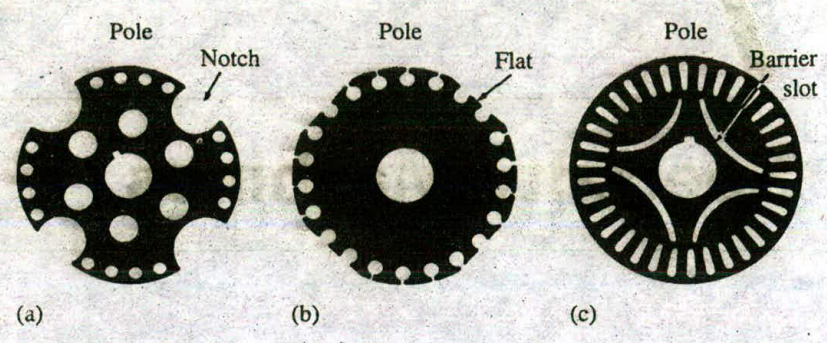


FIGURE 7.1
Types of rotor laminations used in reluctance motors. (Courtesy of Bodine Electric Company, Chicago, Illinois 60618)

Figure 7.2 shows simulated stroboscopic views of the reluctance rotor for different load conditions, as it operates in synchronism with the rotating stator flux. Assuming no shaft load and negligible rotational losses, the centerline of the rotor poles remains coincident with the centerline of the rotating poles produced by the stator, as shown in Figure 7.2(a).

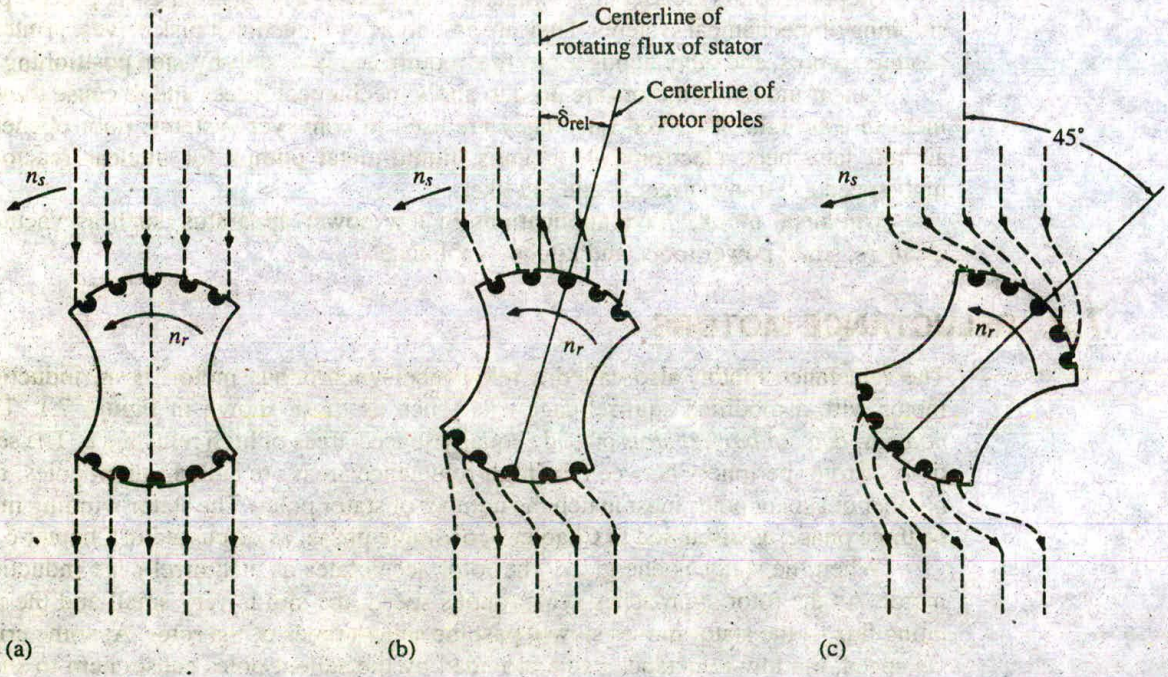


FIGURE 7.2
Simulated stroboscopic view of a reluctance motor, showing position of rotor with respect to the rotating field for (a) no load; (b) partial load; (c) maximum load.

With a step increase in shaft load, the rotor momentarily slows down, causing the salient poles of the rotor to lag the rotating poles of the stator. The angle of lag, called the *torque angle*, is shown in Figure 7.2(b). As soon as the transient slowdown for angular adjustment is completed, the rotor returns to synchronous speed at the new torque angle; the increase in reluctance torque caused by the increase in torque angle just balances the torque load on the shaft plus windage and friction. Reluctance torque increases with increasing torque angle, attaining its maximum value at $\delta_{rel} = 45°$, as shown in Figure 7.2(c).

If the shaft load is increased to a value that causes $\delta_{rel} > 45°$, the increased length of the magnetic flux path between the centerline of the rotating stator poles and the centerline of the corresponding rotor poles causes a decrease in flux and hence a decrease in magnetic attraction. In effect, the flux lines are "over-stretched," the rotor falls out of synchronism, and the machine runs as an induction motor at a slip speed determined by the amount of overload.

The average value of reluctance torque, expressed in terms of applied voltage, applied frequency, and torque angle is[1]

$$T_{rel} = K\left(\frac{V}{f}\right)^2 \times \sin(2\delta_{rel}) \qquad (7\text{–}1)$$
$$s=0$$

where: T_{rel} = average value of reluctance torque
V = applied voltage (V)
f = line frequency (Hz)
δ_{rel} = torque angle (electric degrees)

Constant K takes into account the constants of the motor, such as reluctance, number of turns of wire, and units used.

As indicated in Eq. (7–1), and as shown in Figure 7.2, each increment increase in shaft load causes additional increases in the torque angle. Maximum reluctance torque, called the pull-out torque, occurs at $\delta_{rel} = 45°$.

The constant-speed characteristic of reluctance motors, their simple construction, ruggedness, and particularly their low cost make them desirable for the more economical timing devices, turntables, and tape recorders. They are also used in automatic processes such as are found in the food processing and packaging industries. Synthetic textile industries often require that as many as 90 to 100 machines, each driving different sections of the production line, be operated in synchronism over a wide range of speeds. Three-phase reluctance motors rated at 100 hp and more are used in larger power applications where the complexity and cost of "standard" synchronous motors are not warranted.[2]

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[1]For a more detailed analysis and derivations see Reference [3].

[2]"Standard" synchronous motors rotors have electromagnets that require a direct current source. See Chapter 8 for further details.

EXAMPLE 7.1 A certain 10-hp, four-pole, 240-V, 60-Hz reluctance motor, operating under rated load conditions, has a torque angle of 30°. Determine (a) torque load on the shaft; (b) torque angle if the voltage drops to 224 V. (c) Will the rotor pull out of synchronism?

Solution

(a)

$$n_s = \frac{120 \cdot f}{P} = \frac{120 \times 60}{4} = \underline{1800 \text{ r/min}}$$

$$P_{\text{shaft}} = \frac{T \cdot n}{5252} \quad \Rightarrow \quad 10 = \frac{T \times 1800}{5252}$$

$$T_{\text{rel}} = \underline{29.18 \text{ lb-ft}}$$

(b) With the same torque load and constant frequency,

$$[V^2 \sin(2\delta_{\text{rel}})]_1 = [V^2 \sin(2\delta_{\text{rel}})]_2$$

$$240^2 \sin(2 \times 30) = 224^2 \sin(2\delta_{\text{rel}})$$

$$\delta_{\text{rel}} = \underline{41.90°}$$

(c) Because δ_{rel} is less than 45°, the rotor will not pull out of synchronism.

7.3 HYSTERESIS MOTORS

The stator of an hysteresis motor (also called a *hysteresis-synchronous motor*) is the same as that for an induction motor. The rotor, however, consists of a smooth cylinder made of very hard permanent-magnet alloy material and a nonmagnetic support, as shown in Figure 7.3.

The principle of hysteresis-motor action is explained using the elementary hysteresis motor shown in Figure 7.4. The magnets represent the stator flux, which serves to induce opposite magnetic polarity in the hardened-alloy rotor. With the magnets stationary, as shown in Figure 7.4(a), the magnetic axis of the rotor poles is coincident with the magnetic axis of the stator.

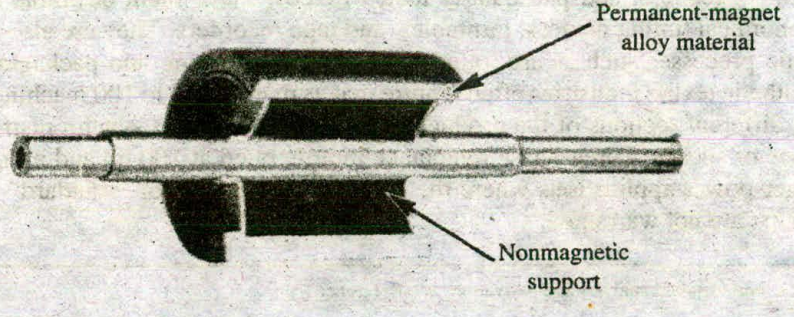


FIGURE 7.3
Rotor for hysteresis motor. (Courtesy of Bodine Electric Company, Chicago, Illinois 60618)

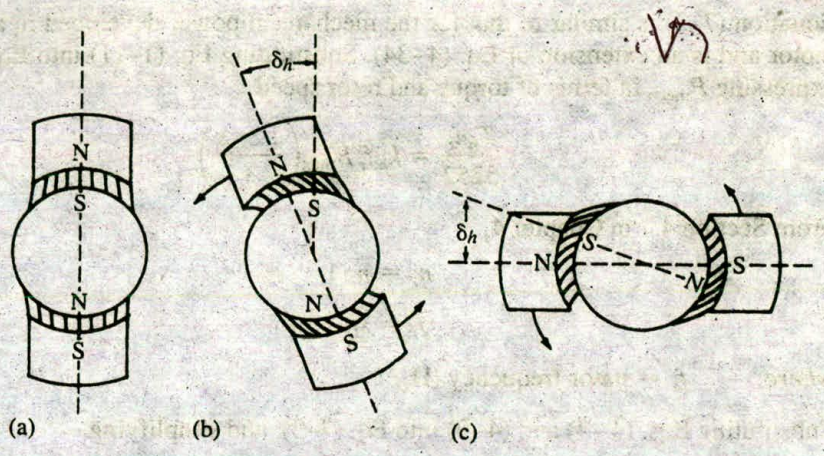


FIGURE 7.4
Hysteresis motor behavior: (a) magnets and rotor stationary; (b) and (c) rotor blocked and magnets rotating.

Spinning the "stator magnets," *with the rotor blocked,* as shown in Figures 7.4(b) and (c), provides a rotating magnetic field that exerts a torque on the induced magnetic poles of the rotor As the stator poles rotate, the induced magnetic poles in the rotor are constantly reforming in new positions, following the rotating flux. Because of hysteresis, however, the *rotor poles always lag the stator poles by angle* δ_h. The constant lag-angle results in a constant force of attraction, and hence a constant accelerating torque. Releasing the rotor and assuming no overload, the constant torque will accelerate the rotor to synchronous speed.

The energy transferred to the rotor by the rotating flux of the stator is in the form of hysteresis energy, and is expended as heat-power losses at locked rotor. From Section 1.7 of Chapter 1, the hysteresis power loss, expressed in terms of frequency and flux density, is

$$P_h = k_h f_r B_{max}^n \qquad \textbf{(1–11)}$$

where:
f_r = frequency of flux reversal in rotor (Hz)
B_{max} = maximum value of flux density in air gap (T)
P_h = heat-power loss due to hysteresis (W)
k_h = constant

With the rotor free to turn, however, hysteresis power is converted to mechanical power. Expressing the mechanical power developed in the rotor during acceleration as a function of hysteresis loss and slip,

$$P_{mech} = P_h\left(\frac{1-s}{s}\right) \qquad \textbf{(7–2)}$$

Equation (7–2) is similar to that for the mechanical power developed in an induction motor and is an extension of Eq. (4–34). Substituting Eq. (1–11) into Eq. (7–2), and expressing P_{mech} in terms of torque and rotor speed,

$$\frac{T_h n_r}{5252} = k_h f_r B_{max}^n \left(\frac{1 - s}{s}\right) \tag{7–3}$$

From Section 4.7 in Chapter 4,

$$n_r = n_s(1 - s) \tag{4–4}$$

$$f_r = sf_s \tag{4–9}$$

where: f_s = stator frequency (H_z)

Substituting Eqs. (4–4) and (4–9) into Eq. (7–3), and simplifying,

$$\frac{T_h n_s(1 - s)}{5252} = k_h \cdot s \cdot f_s B_{max}^n \left(\frac{1 - s}{s}\right)$$

$$T_h = \frac{5252 k_h f_s B_{max}^n}{n_s} \tag{7–4}$$

Expressing synchronous speed in terms of frequency, and substituting into Eq. (7–4),

$$n_s = \frac{120 \cdot f_s}{P}$$

$$T_h = \frac{5252 k_h f_s B_{max}^n}{(120 \cdot f_s)/P}$$

$$T_h = \frac{5252 k_h B_{max}^n}{120/P} \tag{7–5}$$

Equation (7–5) shows the hysteresis torque to be constant, independent of frequency and speed. In fact, *hysteresis torque depends solely on the area of the rotor's hysteresis loop.*

The hysteresis motor develops a constant hysteresis torque at all speeds from standstill up to but not including synchronous speed. At synchronous speed the rotor becomes magnetized along some random axis and pulled around by simple magnetic attraction. *The hysteresis power* P_h *is zero at synchronous speed, hysteresis torque is zero at synchronous speed, and the machine runs as a permanent-magnet synchronous motor, developing magnet torque.*

Figure 7.5 shows simulated stroboscopic views of the hysteresis rotor for different load conditions while operating at synchronous speed. Assuming no shaft load and negligible rotational losses, the centerline of induced rotor-magnets remains coincident with the centerline of the rotating poles produced by the stator, as shown in Figure 7.5(a).

With a step increase in shaft load, the rotor momentarily slows down, causing the induced rotor-magnets to lag the rotating poles of the stator by angle δ_{mag} as shown in Figure 7.5(b). As soon as the transient slowdown for angular adjustment is

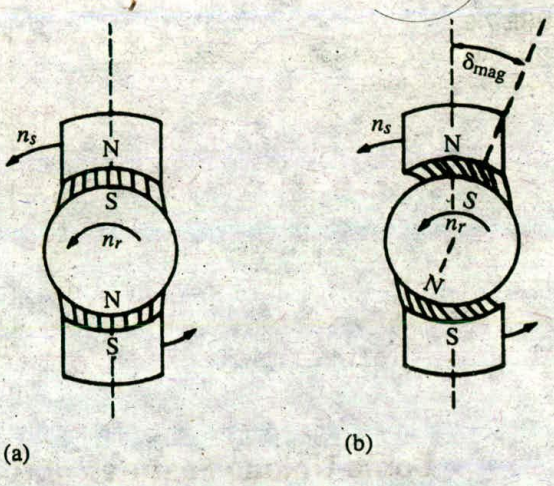


FIGURE 7.5
Simulated stroboscopic views of an hysteresis motor operating at synchronous speed: (a) no load; (b) partial load.

completed, the rotor returns to synchronous speed at the new torque angle; the increase in magnet torque caused by the increase in torque angle just balances the load torque on the shaft. *Note:* Hysteresis torque is also developed during the transient period when shaft load is increasing.

Magnet torque occurs only at synchronous speed and is proportional to the sine of the torque angle.[3] That is,

$$\underbrace{T_{\text{mag}}}_{s=0} \propto \sin(\delta_{\text{mag}}) \tag{7–6}$$

The magnet torque increases with increasing torque angle, attaining its maximum value at $\delta_{\text{mag}} = 90°$.

If the shaft load is increased to a value that causes $\delta_{\text{mag}} > 90°$, the rotor pulls out of synchronism, the magnet torque drops to zero, and although the machine develops hysteresis torque, it is not sufficient to carry the load torque that caused loss of synchronism.

A representative torque-speed curve for a hysteresis motor is shown in Figure 7.6. The dark vertical line indicates the normal range of operation while running at synchronous speed. Expressed mathematically,

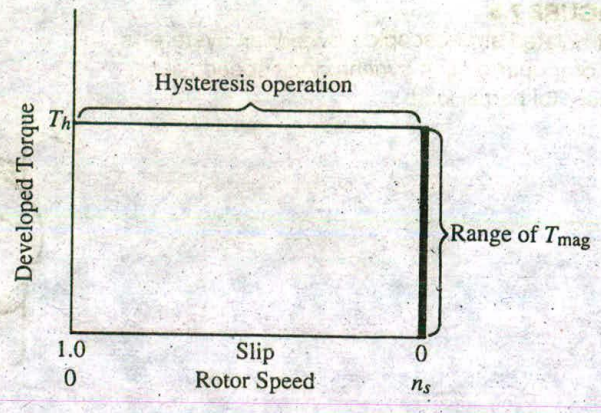
$$\underbrace{T_{\text{mag}}}_{s=0} \leq T_h \tag{7–7}$$

$$\underbrace{T_{\text{mag, max}}}_{s=0} = T_h \tag{7–8}$$

Note: Maximum magnet torque, which precedes "pull-out," is equal to the hysteresis torque.

[3]Proportionality (7–6) may be derived from Eq. (8–14) for magnet power, developed for cylindrical-rotor synchronous machines in Chapter 8.

FIGURE 7.6
Representative torque-speed characteristic of an hysteresis motor.



Some Unique Features of the Hysteresis Motor

1. The constant hysteresis torque from locked rotor to synchronous speed permits the hysteresis motor to synchronize any load that it can accelerate; no other motor can perform in this manner.
2. The smooth rotor provides quiet operation. It does not suffer from magnetic pulsations caused by slots and/or salient poles that are present in the rotors of other motors.
3. The relatively high resistance and high reactance of the hysteresis rotor limit the starting current to approximately 150 percent rated current. This contrasts significantly with the reluctance rotor, whose low reactance and low resistance results in a locked-rotor current of approximately 600 percent rated current.

Although more expensive than a reluctance motor of equivalent power, the very quiet operation of the hysteresis motor makes it particularly desirable for driving phonographs and tape recorders. Furthermore, its ability to synchronize any load that it can accelerate and its smooth synchronization without any abrupt transition make it very adaptable to drives such as those used in synthetic fiber production, where snap-synchronization may cause product damage.

Hysteresis-Reluctance Motors

An hysteresis-reluctance motor combines the torque characteristic of an hysteresis motor with that of a reluctance motor. The characteristic is obtained by cutting reluctance notches in the hysteresis ring to produce a salient pole effect. The hysteresis-reluctance motor has applications in robotics, machine tools, and textile industries **[5]**.

7.4 STEPPER MOTORS

Stepper motors, also called *stepping motors* or *step motors,* are highly accurate pulse-driven motors that change their angular position in steps, in response to input pulses from digitally controlled systems. Stepper motors are used for precise positioning of

mechanical systems and may be used without feedback. Examples of their applications include head positioning in computer disk drives, positioning of carriage, ribbon, print head, and paper feed in typewriters and printers, robots, and the like.

The step angle per input pulse depends on the construction of the stepper motor and the control system used. Steppers with a 45° step angle provide a *resolution* of 360/45 = 8 steps per revolution; steppers with a 1.8° step angle provide 360/1.8 = 200 steps per revolution; and so on. The total angle traveled by the rotor is equal to the step angle times the number of steps. Expressed in equation form,

$$\text{Resolution} = \text{steps/rev} = \frac{360°}{\beta} \tag{7–9}$$

$$\theta = \beta \times \text{steps} \tag{7–10}$$

where: β = step angle (deg/pulse)
θ = total angle traveled by rotor (deg)

The speed of a stepper motor is a function of the step angle and the *stepping frequency* (called the *pulse rate*). Thus,

$$n = \frac{\beta \times f_p}{360} \tag{7–11}$$

where: n = shaft speed (r/s)
f_p = stepping frequency (pulses/s)

EXAMPLE 7.2 A stepper motor has a 2.0° step angle. Determine (a) resolution; (b) number of steps required for the rotor to make 20.6 revolutions; (c) shaft speed if the stepping frequency is 1800 pulses/s.

Solution

(a) $$\text{Steps/rev} = \frac{360}{2.0} = \underline{180}$$

(b) $$\theta = \beta \times \text{Steps} \quad \Rightarrow \quad 20.6 \times 360 = 2.0 \times \text{steps}$$
$$\text{Steps} = \underline{3708}$$

(c) $$n = \frac{\beta \times f_p}{360} = \frac{2.0 \times 1800}{360} = \underline{10 \text{ r/s}}$$

7.5 VARIABLE-RELUCTANCE STEPPER MOTORS

The variable reluctance stepper motor, shown in Figure 7.7, will be used to illustrate the simple construction and general principles of operation. The toothed stator and toothed rotor are constructed from soft steel that retains very little residual magnetism. Coils wound around the stator teeth provide the magnetic attraction that establishes

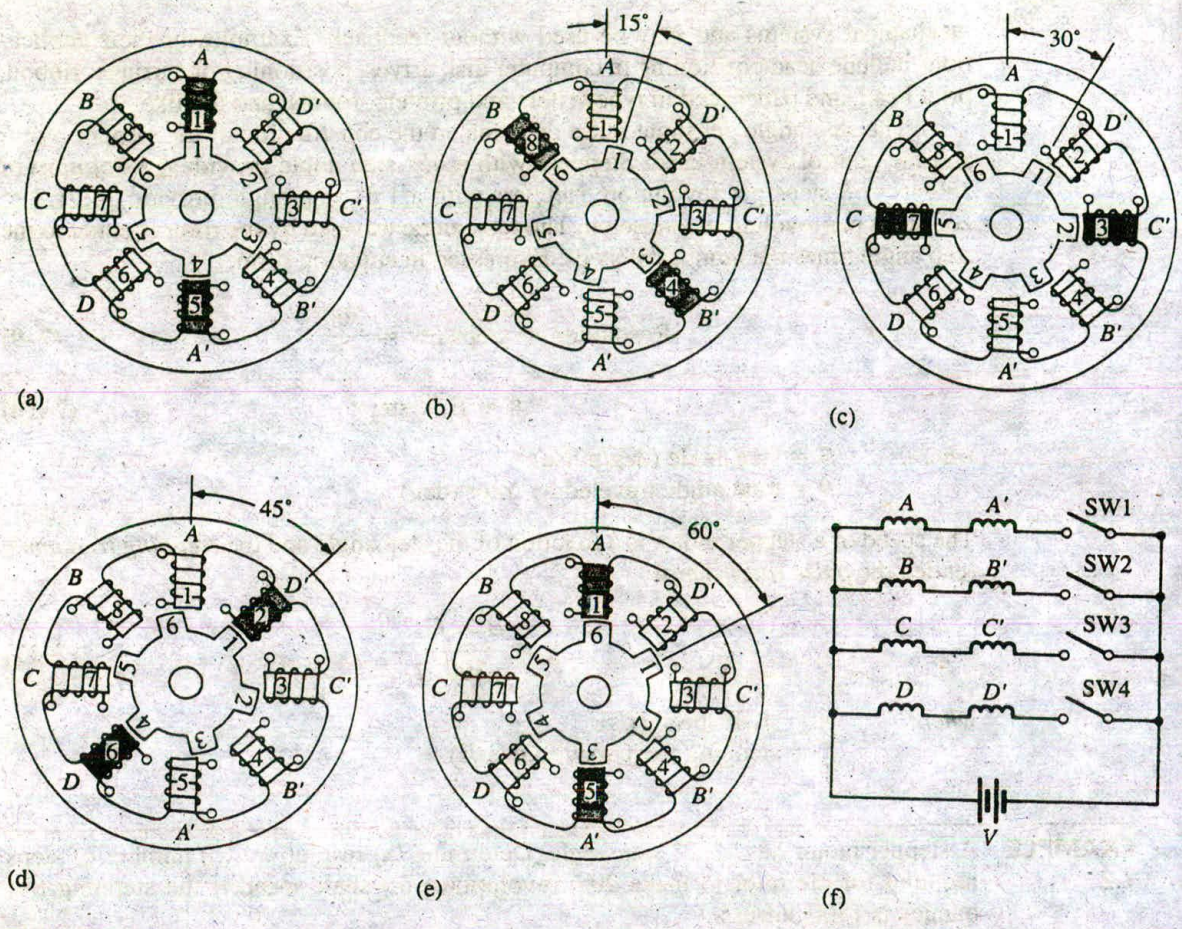


FIGURE 7.7
Variable-reluctance stepper motor showing different step positions corresponding to the switching sequence in (f). (Courtesy Superior Electric Company)

the rotor position. The motor shown in Figure 7.7 is called a *variable-reluctance stepper* because the reluctance of the magnetic circuit formed by the rotor and stator teeth varies with the angular position of the rotor. Energizing one or more stator coils causes the rotor to step forward (or backward) to a position that forms a path of least reluctance with the magnetized stator teeth.

A simple circuit arrangement for sequencing current to the stator coils is shown in Figure 7.7(f). The eight stator coils are connected in two-coil groups to form four separate circuits called *phases*. Each phase has its own independent switch. Although shown as mechanical switches in Figure 7.7(f), in actual practice, switching of phases is accomplished with solid-state control.

Figure 7.7(a) shows the position of the rotor with switch *SW*1 closed, energizing phase *A;* the rotor is in a position of minimum reluctance with rotor teeth 1 and 4 aligning with stator teeth 1 and 5, respectively. Closing switch *SW*2 and opening *SW*1 energizes phase *B,* causing rotor teeth 3 and 6 to align with stator teeth 4 and 8, respectively, as shown in Figure 7.7(b), for an angular step of 15°. Closing switch *SW*3 and opening switch *SW*2 energizes phase *C,* causing rotor teeth 2 and 5 to align with stator teeth 3 and 7, as shown in Figure 7.7(c). As each switch is closed, and the preceding one opened, the rotor moves an additional step angle of 15°. The stepping sequence, shown in Figures 7.7(a) to (e), follows the sequence of switches, repeating 1 through 4, over and over, until the desired number of revolutions or fraction of a revolution is achieved.

The direction of rotation for the 1-to-4 switching sequence, shown in Figure 7.7, resulted in clockwise (CW) stepping of the rotor. Reversing the sequence of pulses by closing the switches in the order of 4–3–2–1, will cause counterclockwise (CCW) stepping.

The relationship between step angle and the number of teeth in the rotor and the number of teeth in the stator is

$$\beta = \frac{|N_s - N_r|}{N_s \cdot N_r} \times 360 \qquad \textbf{(7–12)}$$

where: β = step angle in space degrees
N_s = number of teeth in stator core
N_r = number of teeth in rotor core

Thus, the step angle for the stepper motor in Figure 7.7 is

$$\beta = \frac{8 - 6}{8 \times 6} \times 360 = 15°$$

Half-Step Operation

Half-step operation is accomplished by using the modified pulsing sequence: *A, A & B, B, B, & C, C,* and so forth. This is illustrated in Figure 7.8 for three successive pulses. Energizing only phase *A* causes the rotor position shown in Figure 7.8(a); energizing phase *A* and phase *B* moves the rotor to the position shown in Figure 7.8(b); and energizing only phase *B* moves the rotor to the position shown in Figure 7.8(c). With each pulse the rotor moves 7.5° in a CW direction.

Microstepping

Microstepping utilizes two phases with unequal currents. For example, in Figure 7.8(b), instead of exciting phase *A* and phase *B* equally, the current in phase *A* is held constant while the current in phase *B* is increased in very small increments (pulses) until maximum current is reached. The current in phase *A* is then reduced to zero using the same very small increments. The resultant very small step angles, called

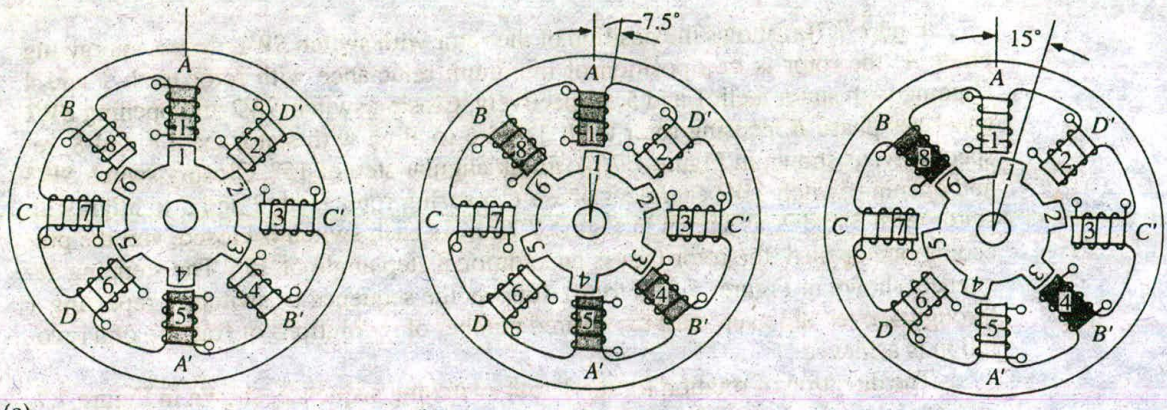


FIGURE 7.8
Variable-reluctance stepper motor sequenced for half-step operation, showing three successive pulses.

microsteps, provide smooth low-speed operation and high resolution. For example, a stepper motor with a resolution of 200 steps/rev (1.80° step angle) can with microstepping have a resolution of 20,000 steps/rev (0.0180° step angle).

Holding Torque

The holding torque, called *static torque*, is the maximum load-torque that can be applied to an *energized* stepper motor without having it slip poles.[4]

Assume that phase *A* is energized and the rotor is at rest, as indicated in Figure 7.9(a). If from this rest position the rotor is mechanically forced one 15° step clockwise, as shown in Figure 7.9(b), the resultant force of attraction on rotor teeth 1 and 4, by the respective magnetized stator teeth 1 and 5, develops a restoring torque. Releasing the rotor will cause it to flip back to the original position shown in Figure 7.9(a). Forcing the rotor 30° clockwise (two steps), while phase *A* is still energized, will result in the relative tooth positions shown in Figure 7.9(c). This is an unstable equilibrium position because rotor teeth 1 and 6 have equal forces of attraction; the net torque is zero. When released, the rotor can go in either direction: back two steps to the original position shown in Figure 7.9(a), or ahead an additional two steps to the position shown in Figure 7.9(d).

A plot of restoring torque vs. forced step displacement, called a *static-torque curve*, is shown in Figure 7.10. Note that maximum restoring torque is produced with one-step displacement. A two-step displacement has zero restoring force, but the rotor is unstable when released, and can move backwards to its original position, or move ahead for a total of four steps from its original position.

[4] The variable-reluctance stepper motor provides holding torque only while the windings are energized. Residual magnetism has no significant effect on the developed torque.

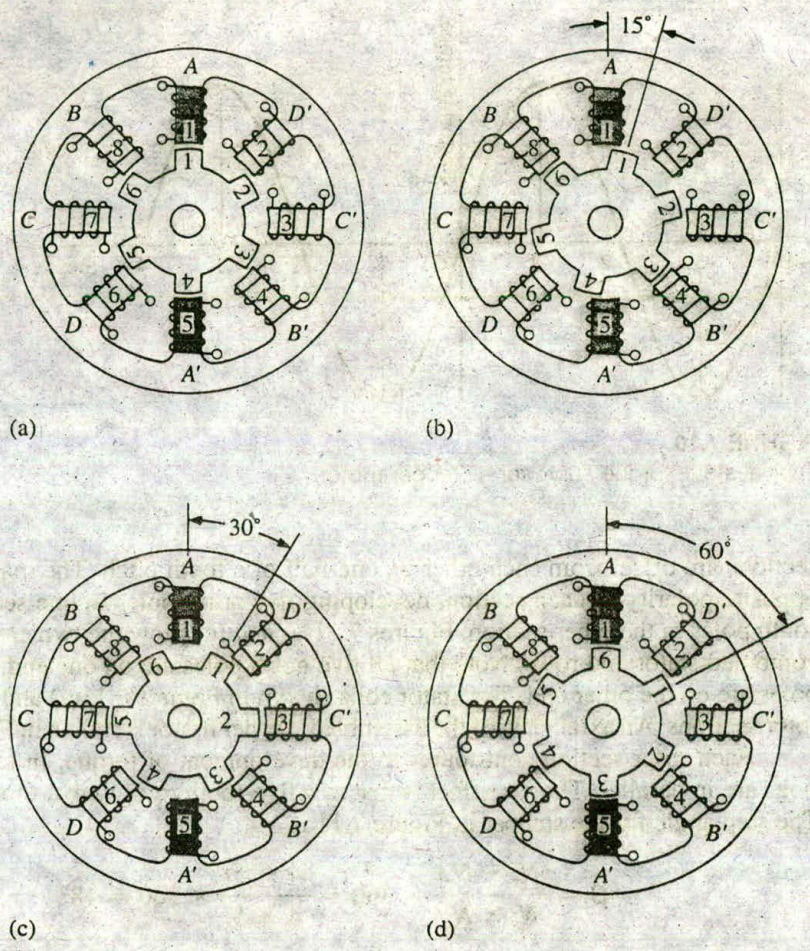


FIGURE 7.9
Rotor positions that develop restoring torque in a variable-reluctance stepper motor with phase *A* energized: (a) rotor at rest; (b) rotor forced 15° CW; (c) rotor forced 30° CW (unstable); (d) rotor flips to 60° position.

Step Accuracy

Step accuracy, expressed in percent, indicates the total error introduced by a stepper motor in a single step movement. The error is noncumulative in that it does not increase as additional steps are taken. Typical step accuracies for commercial stepper motors range from 1 to 10 percent of step size.

7.6 PERMANENT-MAGNET STEPPER MOTORS

A simplified diagram of a permanent-magnet stepper motor, shown in Figure 7.11, is used to illustrate the physical features common to this type of motor. The rotor, shown in Figure 7.11(b), has two toothed sections, separated by a permanent magnet. The two

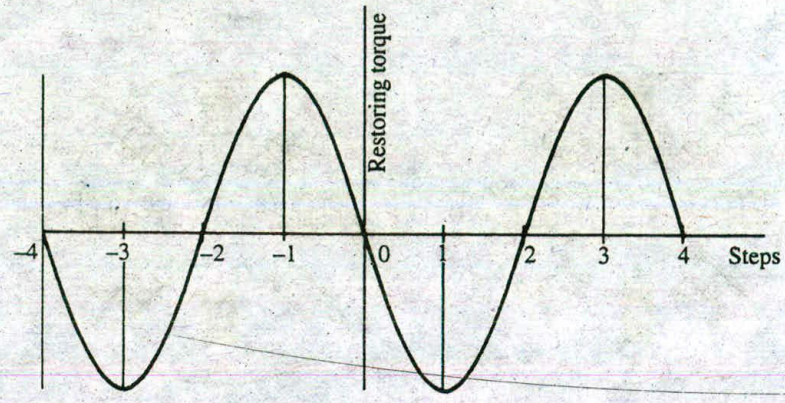


FIGURE 7.10
Typical static-torque curve for a stepper motor.

sections are offset from each other by one-half of a tooth pitch. The magnet provides opposite-polarity to each section, developing five north poles in one section and five south poles in the other section. Figures 7.11(a) and (c) show the two end views of the combined stator and rotor. Note that all five north poles are on one end, and all south poles are on the other end. The stator coils shown in Figures 7.11(a) and (c) span both rotor sections. An axial view of the assembled stepper motor is shown in Figure 7.11(d).

Each rotor section contributes to the development of torque. In effect, the sections are in parallel. The net effect is that of a five-tooth rotor with a four-tooth stator. The step angle for the stepper in Figure 7.11 is

$$\beta = \frac{|N_s - N_r|}{N_s \times N_r} \times 360 = \frac{|4 - 5|}{4 \times 5} \times 360 = 18°$$

The principle of operation of a permanent-magnet stepper motor is developed using the circuit diagram, switching table, and the corresponding rotor positions in Figure 7.12. For simplicity, only the south section of the rotor is shown. The rotor positions are keyed to the switching sequence for CW rotation; phase A is energized by $SW1$, and phase B is energized by $SW2$. Half-stepping and microstepping may also be accomplished by using methods similar to those described for variable-reluctance steppers.

7.7 STEPPER-MOTOR DRIVE CIRCUITS

Figure 7.13 shows the general structure of a drive circuit for a variable-reluctance stepper motor such as the one shown in Figure 7.7. The input pulses that control the motor are filtered in block F, and passed on to bidirectional counter C. The output bits of the counter are fed to the input of decoder D. Electronic switches $S1$, $S2$, $S3$, and $S4$ energize and de-energize the windings in accordance with the input pulse commands.

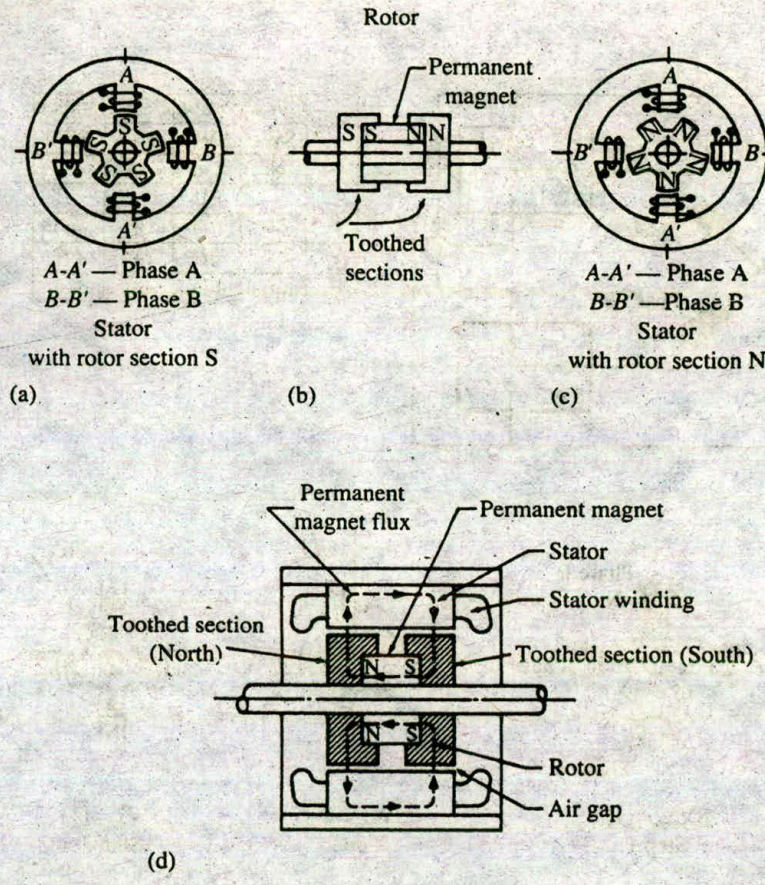
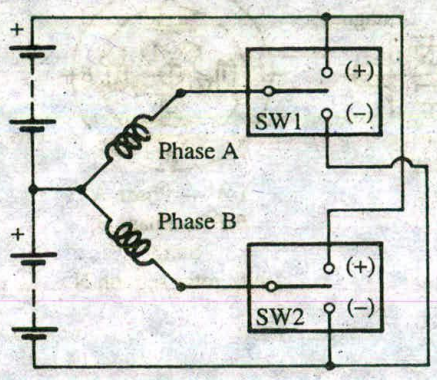


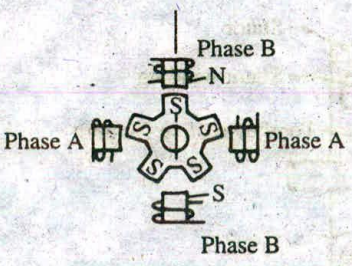
FIGURE 7.11
Permanent-magnet stepper motor: (a) stator and south section of rotor; (b) rotor; (c) stator and north section of rotor; (d) axial view of assembled motor. (Courtesy Superior Electric Company)

A typical solid-state switch is shown in Figure 7.14. Amplifier A converts the logic levels from the decoder to adequate voltage and current levels to turn transistor Q_A on and off. Resistor R_s is used to discharge the energy in the coil when Q_A is turned off.

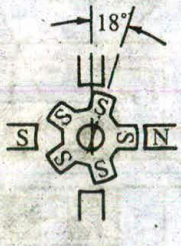
The drive-circuit structure for a permanent-magnet stepper is essentially the same as far as the filter, counter, and decoder are concerned. The interconnection of phase windings, switches, and power supply, however, is significantly different because of the flux reversal requirement. Figure 7.15 shows one example of a switching arrangement that may be used for controlling a permanent-magnet stepper motor such as the one shown in Figure 7.12. The diodes provide a path for the decaying phase-currents whenever a conducting transistor is turned off. For example, referring to Figure 7.15(b), Q_1 is on and current i_1 is supplying phase A. When Q_1 is turned off, the



Step	CW Rotation		CCW Rotation	
	SW1	SW2	SW1	SW2
1	off	(–)	off	(–)
2	(+)	off	(–)	off
3	off	(+)	off	(+)
4	(–)	off	(+)	off
1	off	(–)	off	(–)



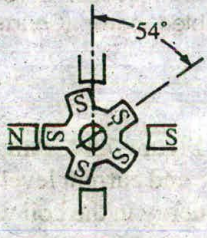
SW2 = (–)
Step 1



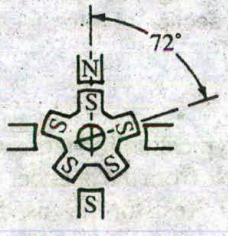
SW1 = (+)
Step 2



SW2 = (+)
Step 3



SW1 = (–)
Step 4



SW2 = (–)
Step 1

FIGURE 7.12
Circuit diagram of a permanent-magnet stepper motor with rotor positions keyed to switching sequence for CW rotation. (Courtesy Superior Electric Company)

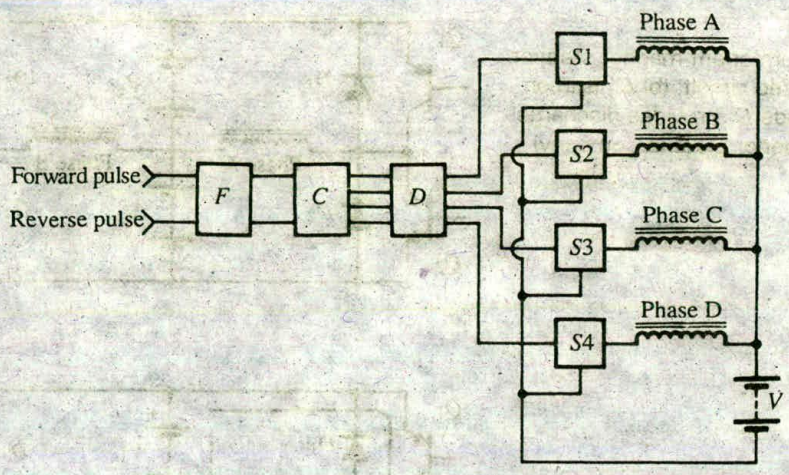


FIGURE 7.13
Drive circuit for a variable-reluctance stepper motor. (Courtesy Superior Electric Company)

inductance of phase A would prevent the immediate decay of current in it. As shown in Figure 7.15(c), however, the presence of diode D_2 provides a discharge path through an opposing V_2, quickly squelching the current. The main disadvantage of this switching arrangement is the requirement for two power supplies [7].

7.8 LINEAR INDUCTION MOTOR

The principle of linear-induction-motor (LIM) operation is the same as that for the three-phase induction motor discussed in Chapter 4 and the single-phase induction motor discussed in Chapter 6. Instead of using a rotating field to sweep a squirrel-cage rotor, however, the LIM uses a linearly moving field to sweep a conducting rail or the conductors of a ladder-type cage. This is illustrated in Figure 7.16(a), where a magnetic field, moving at velocity U_s, is shown sweeping the rungs of an aluminum ladder.

FIGURE 7.14
Solid-state switching circuit.

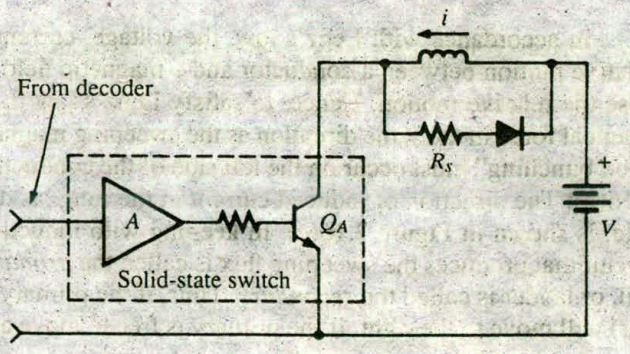
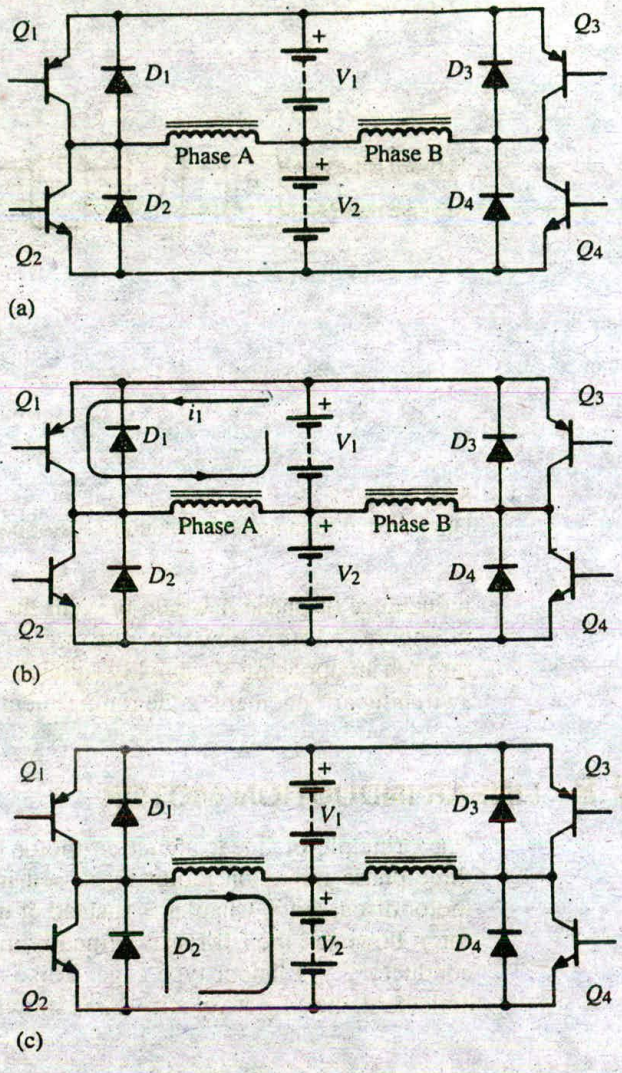
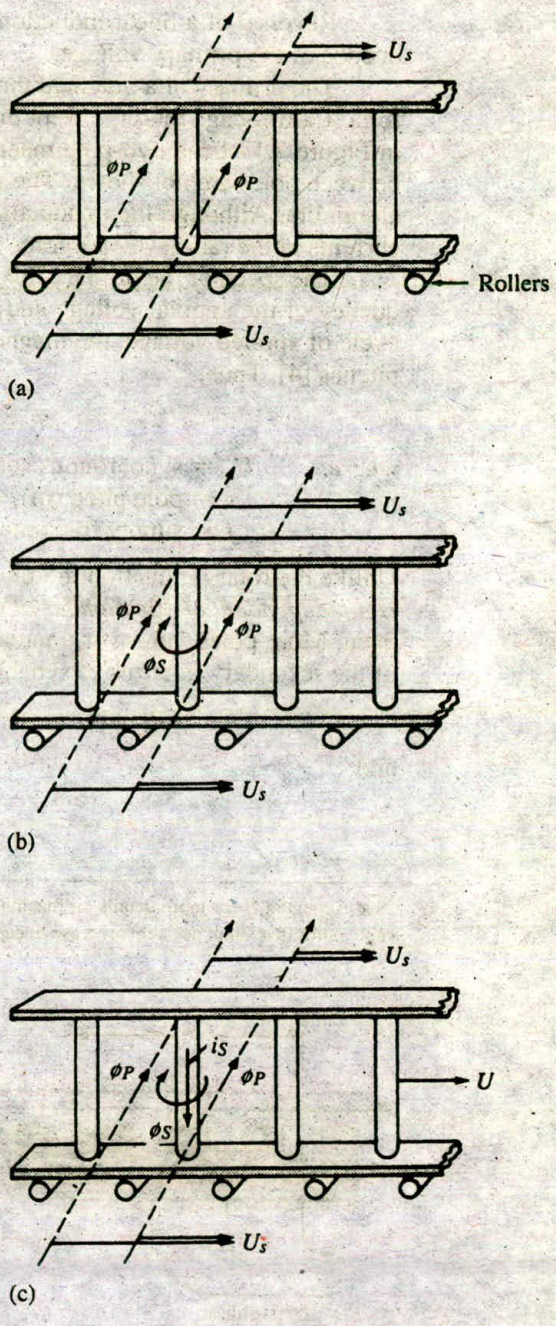


FIGURE 7.15
Drive circuit for a permanent-magnet stepper motor: (a) unenergized circuit; (b) Q_1 is in on state; (c) Q_1 off, diode D_1 provides discharge path. (Courtesy Superior Electric Company)



In accordance with Lenz's law, the voltage, current, and flux generated by the relative motion between a conductor and a magnetic field will be in a direction to oppose the relative motion. Hence, to satisfy Lenz's law the rungs must develop a mechanical force in the same direction as the sweeping magnetic field. For this to happen, "flux bunching" must occur on the left side of the conducting rung, as shown in Figure 7.16(b). The direction of induced current in the rung, as determined by the right-hand rule, is shown in Figure 7.16(c). In keeping with induction-motor nomenclature, the circuit that produces the sweeping flux is called the *primary*, and the conducting sheet, rail, or ladder is called the *secondary*. Thus, if the primary is fixed, the secondary (ladder) will move to the right. If the primary is free to move and the secondary is blocked, the primary will move to the left.

FIGURE 7.16
Diagrams of an elementary linear induction motor: (a) magnetic field sweeping rungs of ladder; (b) direction of flux around rung as determined by Lenz's law; (c) direction of current in rung as determined by right-hand rule.



Reversal of a linear induction motor is accomplished by reversing the phase sequence of the primary voltage.

The primary of a practical three-phase LIM is wound in a manner similar to that for a squirrel-cage motor, except that the windings are laid in a straight line, as shown in Figure 7.17 for a two-pole machine with one coil per phase per pole; connections between poles are not shown. The secondary is a conducting sheet or rail of copper or aluminum. Although the conducting rail has no squirrel-cage bars, the induced eddy currents in the rail develop a force in a direction to oppose the relative motion **[2],[6]**.

The speed of the primary flux, called *synchronous speed,* is a function of the frequency of the applied voltage and the span of the primary coils (pole pitch).[5] In one cycle of applied voltage, the magnetic field travels a linear distance equal to two pole pitches **[4]**. Thus,

$$U_s = 2\tau f \tag{7–13}$$

where: U_s = synchronous speed (m/s)
τ = pole pitch (m)
f = supply frequency (Hz)

Unlike the rotary induction motor, *the synchronous speed of a linear induction motor is not dependent on the number of poles,* and any number of poles may be used, odd or even. More poles, however, increase the total force developed. Expressing the speed of the secondary in terms of synchronous speed and slip,

$$U = U_s(1 - s) \tag{7–14}$$

and

$$s = \frac{U_s - U}{U_s} \tag{7–15}$$

[5]See Appendix B for more details on machine windings, including a developed view of the stator of a squirrel-cage motor that simulates a three-phase linear motor.

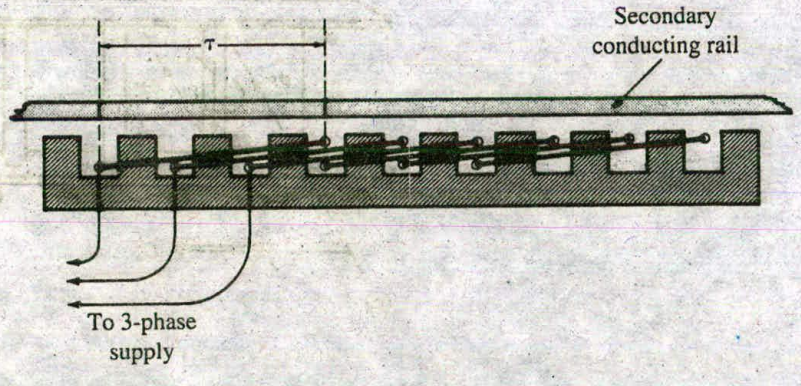


FIGURE 7.17
Linear induction motor showing conducting rail and pole pitch.

where: s = slip
U = speed of secondary (m/s)

The relatively large air gap in a LIM causes its slip at rated load to be much higher than that for a corresponding squirrel-cage induction motor.

Large three-phase LIMs have applications in traveling cranes, aircraft and missile launching, traction motors for high-speed railroads, conveyers, liquid-metal pumps, and other high-power applications where linear motion is required. Smaller linear induction motors that use a split-phase-type primary have applications in door closers and other low-power positioning requirements.

EXAMPLE 7.3 A three-phase moving-rail LIM has three poles and a pole pitch of 0.24 m. The LIM is operated from a 50-Hz system. Determine (a) the synchronous speed; (b) the rail speed assuming a slip of 16.7 percent.

Solution

(a) $$U_s = 2\tau f = 2 \times 0.24 \times 50 = \underline{24 \text{ m/s}}$$

(b) $$U = U_s(1 - s) = 24(1 - 0.167) = \underline{20.0 \text{ m/s}}$$

7.9 UNIVERSAL MOTOR

A universal motor is a relatively small *series motor* specially designed to operate at approximately the same speed and output power on either DC or single-phase AC at a frequency of 60 Hz or less, and an rms voltage that is approximately equal to the DC voltage. Such motors have NEMA-standard ratings of from 0.010 hp to 1.0 hp at a rated speed of 5000 r/min or above.

An elementary universal motor, shown in Figures 7.18(a) and (b), has its rotating part, called the *armature,* connected in series with the *series-field* winding. The rotating commutator and stationary brushes constitute a rotary switch that reverses the direction of the current in the armature coil as the coil rotates. The equivalent-circuit diagram is shown in Figure 7.18(c).

The direction of developed torque, and hence the direction of armature rotation, is independent of the polarity of the AC source. This is shown in Figures 7.18(a) and (b) for the respective alternations of the AC source. The direction of the mechanical force exerted on each conductor is determined by the flux bunching rule.

The torque developed by the universal motor is proportional to the flux density of the series field and the current in the armature conductors. That is,

$$T_D \propto B_p I_a \qquad \textbf{(7–16)}$$

where: T_D = developed torque
B_p = flux density due to current in series-field winding
I_a = armature current

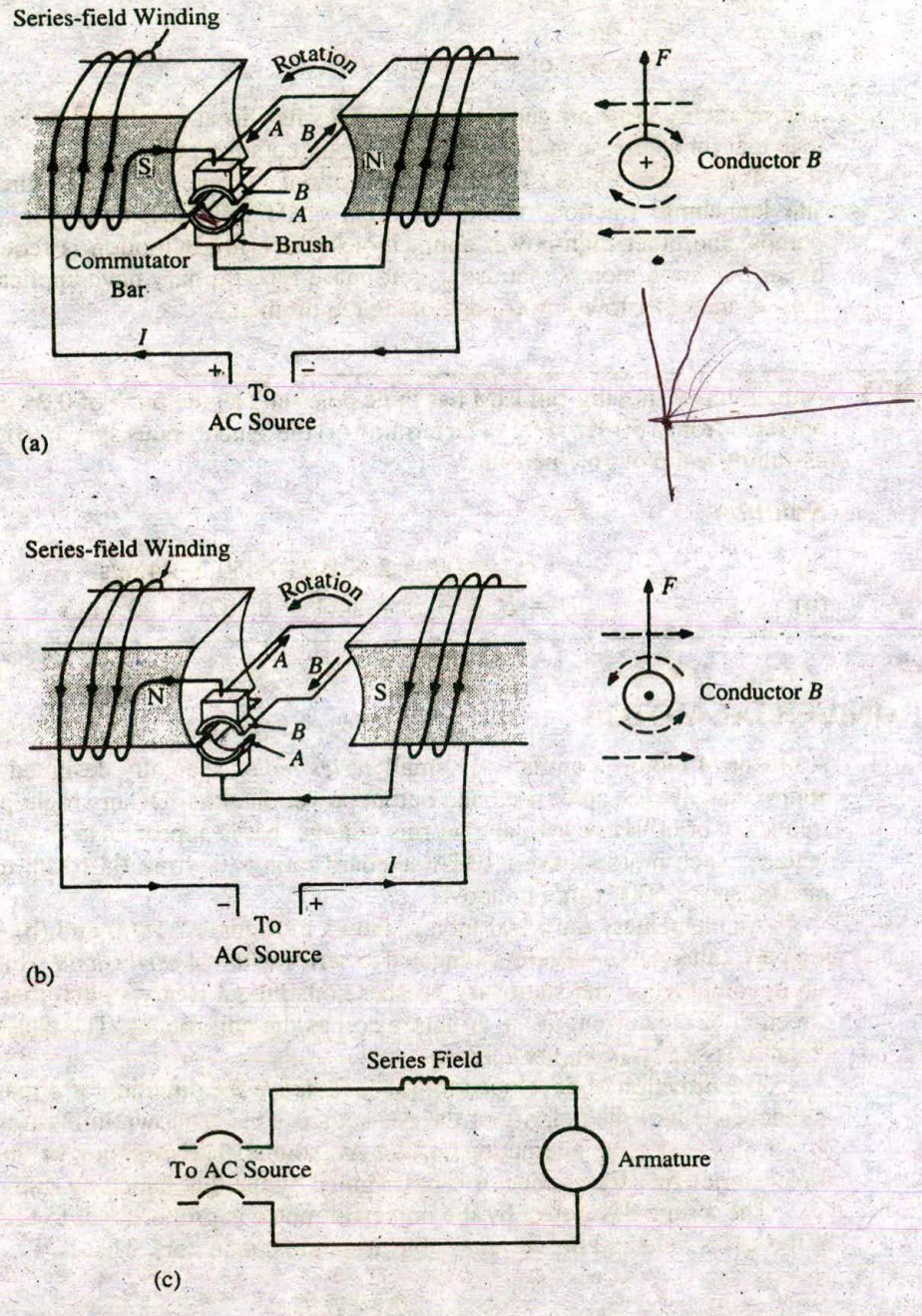


FIGURE 7.18
Elementary universal motor: sketches (a) and (b) show the same direction of rotation regardless of polarity of voltage source; (c) equivalent circuit.

As indicated in Figure 7.18(c), however, the current in the series field is the armature current. Hence, neglecting magnetic saturation effects,

$$B_p \propto I_a$$

Substituting into Eq. (7–16),

$$T_D \propto I_a^2 \tag{7–17}$$

As indicated in Eq. (7–17), the torque developed by a universal motor is approximately proportional to the square of the armature current.[6]

Universal motors can develop higher torques, can accelerate to higher speeds, and have a higher power-to-weight ratio than induction motors of the same power ratings.

Reversing the direction of rotation of a universal motor is accomplished by reversing the direction of current in the series field or in the armature, but not in both. Speed adjustment is accomplished by using an autotransformer or solid-state control to reduce the voltage applied to the motor; reducing the applied voltage reduces the armature current, which reduces the developed torque, and hence reduces the speed.

The torque and speed characteristics of the universal motor are essentially the same whether operating on AC or DC. Furthermore, because of its relatively small dimensions, no-load speeds in excess of 12,000 r/min are achieved without damage. Universal motors have applications in vacuum cleaners, portable power tools, and kitchen appliances.

Series motors, operating from a 25-Hz single-phase system are used for traction purposes on some electrified railroads [1].

[6]More detailed theory and characteristics of commutator-type motors are given in Chapters 10 and 11.

SUMMARY OF EQUATIONS FOR PROBLEM SOLVING

Reluctance Motor

$$\underbrace{T_{\text{rel}}}_{s=0} = K\left(\frac{V}{f}\right)^2 \times \sin(2\delta_{\text{rel}}) \tag{7–1}$$

Hysteresis Motor

$$T_h = \frac{5252 k_h B_{\text{max}}^n}{120/P} \tag{7–5}$$

$$\underbrace{T_{\text{mag}}}_{s=0} \propto \sin(\delta_{\text{mag}}) \tag{7–6}$$

$$\underbrace{T_{\text{mag}}}_{s=0} \leq T_h \tag{7-7}$$

$$\underbrace{T_{\text{mag, max}}}_{s=0} = T_h \tag{7-8}$$

Stepper Motors

$$\text{steps/rev} = \frac{360^\circ}{\beta} \qquad \theta = \beta \times \text{steps} \tag{7-9, 7-10}$$

$$n = \frac{\beta \times f_p}{360} \qquad \beta = \frac{|N_s - N_r|}{N_s \cdot N_r} \times 360 \tag{7-11, 7-12}$$

Linear Induction Motors

$$U_s = 2\tau f \qquad U = U_s(1 - s) \tag{7-13, 7-14}$$

$$s = \frac{U_s - U}{U_s} \tag{7-15}$$

Series Motor

$$T_D \propto I_a^2 \tag{7-17}$$

SPECIFIC REFERENCES KEYED TO TEXT

1. Jones, A. J. Amtrack's Richmond static frequency converter project. *IEEE Vehicular Technology Society News,* May 2000, pages 4–10.
2. Laithwaite, E. R. Linear electric machines—A personal view. *Proc. IEEE,* Vol. 63, No. 2, Feb. 1975.
3. Mablekos, V. E. *Electric Machine Theory for Power Engineers.* Harper & Row, New York, 1980.
4. Nasar, S. A., and I. Boldea. *Linear Motion Machines.* Wiley, New York, 1976.
5. Rahman, M. A., and A. M. Osheiba. Steady-state performance analysis of polyphase hysteresis-reluctance motors. *IEEE Trans. Industry Applications,* Vol. IA-21, No. 4, May/June 1985.
6. Say, M. G. *Alternating Current Machines.* Wiley, New York, 1978.
7. Superior Electric Company. Step motor systems. Tech. Paper A-1-A.

REVIEW QUESTIONS

1. Explain the principle of reluctance-motor operation: how it starts, accelerates, and synchronizes.
2. At what stages of reluctance-motor operation (locked rotor, acceleration, steady-state synchronous speed, loading and unloading at synchronous speed) does the machine develop (a) induction-motor torque; (b) reluctance-motor torque?
3. Explain the principle of hysteresis-motor operation: how it starts, accelerates, and synchronizes.
4. At what stages of hysteresis-motor operation (locked rotor, acceleration, steady-state synchronous speed, loading and unloading at synchronous speed) does the machine develop hysteresis torque?
5. (a) Explain the principle of stepper-motor operation. (b) How does it differ in operation from a squirrel-cage induction motor?
6. Explain how half-stepping and microstepping are accomplished.
7. Will the overall accuracy of a stepper motor be greater at 100 steps than at 10 steps? Explain.
8. Explain the principle of linear-induction-motor operation, and state how a LIM may be reversed.
9. What determines the synchronous speed of a LIM? How does this compare with the factors that determine the synchronous speed of a squirrel-cage induction motor?
10. Explain why the torque developed by a universal motor varies as the square of the armature current.
11. Using suitable diagrams, show how the direction of rotation of a universal motor is reversed.
12. How may the speed of a universal motor be adjusted?

PROBLEMS

7–1/2 A reluctance motor operating at rated voltage and frequency has a shaft load that results in a torque angle of 25°. How much additional torque load (expressed in percent) can be added to the shaft without causing loss of synchronism?

7–2/2 A reluctance motor operating at 240 V, 60 Hz, has a torque angle of 15.6°. The motor is supplied by an isolated generator. Heavy demands on the generator cause a 10 percent drop in voltage and a 5 percent drop in generator speed. Assuming a constant torque load on the motor shaft, determine the new torque angle.

7–3/3 Rated torque load for a certain hysteresis motor occurs at a torque angle of 36°. Determine (a) percent rated torque the motor can carry before pulling out of synchronism; (b) hysteresis torque developed if the motor pulls out of synchronism.

7–4/3 Determine the magnet torque developed by a hysteresis motor, in percent of locked-rotor torque, for $\delta_{mag} = 60°$.

7–5/4 A stepper motor with a step angle of 1.8° is driven at a rate of 6000 pulses/s. Determine (a) resolution; (b) rotor speed; (c) number of pulses required to rotate 46.8°.

7–6/4 Determine the pulse rate required to obtain 3600 r/min from a stepper motor with a resolution of 500 steps/rev.

7–7/4 A stepper motor that makes 500 steps/rev is operated in half-step mode. Determine (a) resolution; (b) number of steps required to turn the rotor 75.6°.

7–8/4 Determine the required resolution for a stepper motor that is to operate at a frequency of 4000 pulse/s, and travel 137.52° in 0.0191 s.

7–9/8 A three-phase, four-pole LIM with a pole pitch of 32 cm is operated from a 240-V, 60-Hz system. When loaded, the motor runs at 30.72 m/s. Determine the slip.

7–10/8 (a) Determine the required pole pitch for a two-pole, 60-Hz, 450-V LIM that will provide a synchronous speed of 40.6 m/s. (b) Assuming a 32 percent slip, determine the speed of the secondary.

8

Synchronous Motors

8.1 INTRODUCTION

Synchronous motors are used principally in large power applications because of their high operating efficiency, reliability, controllable power factor, and relatively low sensitivity to voltage dips. They are constant-speed machines with applications in mills, refineries, power plants, and the like, to drive pumps, compressors, fans, pulverizers, and other large loads, and to assist in power-factor correction. One very interesting application is the 44-MW, 10-kV, 60-Hz, 50-pole, 144 r/min synchronous motors used to drive the *Queen Elizabeth II* passenger ship. A solid-state volts/hertz drive circuit provides speed control through frequency adjustment.

Synchronous machines designed specifically for power-factor control have no external shafts and are called *synchronous condensers.* They "float" on the bus, supplying reactive power to the system. The direction of the reactive power, and hence the power factor of the system, is adjusted by changing the field excitation of the machine.

Unless otherwise specified, when discussing the behavior of individual motors and/or generators, it will be assumed that the machine is connected to a power source of unlimited capacity and zero impedance, called an *infinite bus.* The terminal voltage and frequency of the infinite bus remain constant and are unaffected by any power drawn from or supplied to the infinite bus. Large power systems in the United States and other highly industrialized countries may be considered to approximate those of the infinite bus.

8.2 CONSTRUCTION

The stator of a three-phase synchronous motor, called the *armature,* is identical to that of a three-phase induction motor, and when energized from a three-phase supply develops a rotating field in the same manner as described for induction motors.

Coils wound (or mounted) on the rotor are energized from a DC source to form electromagnets that lock in synchronism with poles of opposite polarity produced by the rotating flux of the stator. The number of rotor poles is made equal to the number of stator poles.

The rotor of a two-pole 60-Hz motor, such as the cylindrical rotor shown in Figure 8.1, is constructed from a solid-steel forging so as to withstand the large centrifugal stresses inherent in high-speed operation. *Cylindrical rotors* (also called *round rotors*) cannot accelerate high-inertia loads, however, and are thus limited in application to pumps, fans, blowers, and other loads with similar low-starting-torque requirements.

Synchronous motors built for use with high-inertia low-speed loads have many poles projecting radially outward from a steel *spider,* as shown in Figure 8.2. These *salient poles* are bolted or keyed to the spider, and the spider is keyed to the shaft.

The squirrel-cage winding, called a *pole-face* winding, *amortisseur* winding, or *damper* winding, shown in Figure 8.2, is used to accelerate the rotor to near synchronous speed.[1] In those applications where high accelerating torque is required throughout the accelerating period, the rotor is designed with a double squirrel-cage winding, similar to that in a NEMA design *C* induction motor. The damper winding also helps suppress oscillations (called *hunting*) caused by torque pulsations when driving loads such as reciprocating compressors. Currents generated in the damper windings during such rotor oscillations provide a countertorque that serves to dampen the oscillations.

The *excitation windings,* also called *magnet windings, field coils,* or *field windings,* form the rotor poles. The individual field coils are connected in series or series–parallel combinations, in a manner that provides alternate north and south poles. The rotor circuit terminates at slip rings, also called collector rings, and graphite

[1]Some salient-pole designs obtain starting and accelerating torque from eddy currents and hysteresis losses in massive solid-steel pole plates that form the faces of the rotor poles. The eddy currents provide induction-motor torque, and the hysteresis effect provides hysteresis torque.

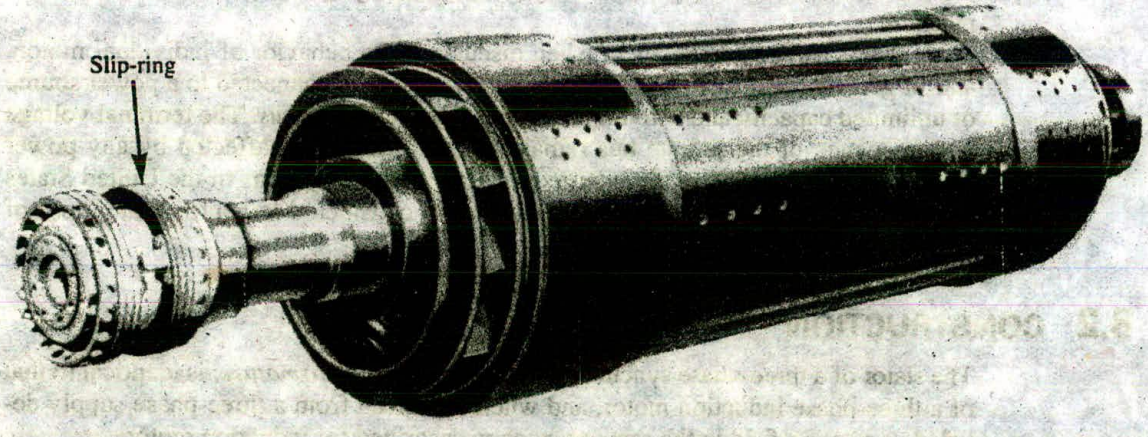


FIGURE 8.1
Two-pole cylindrical rotor for high-speed synchronous-machine applications. (Courtesy TECO Westinghouse)

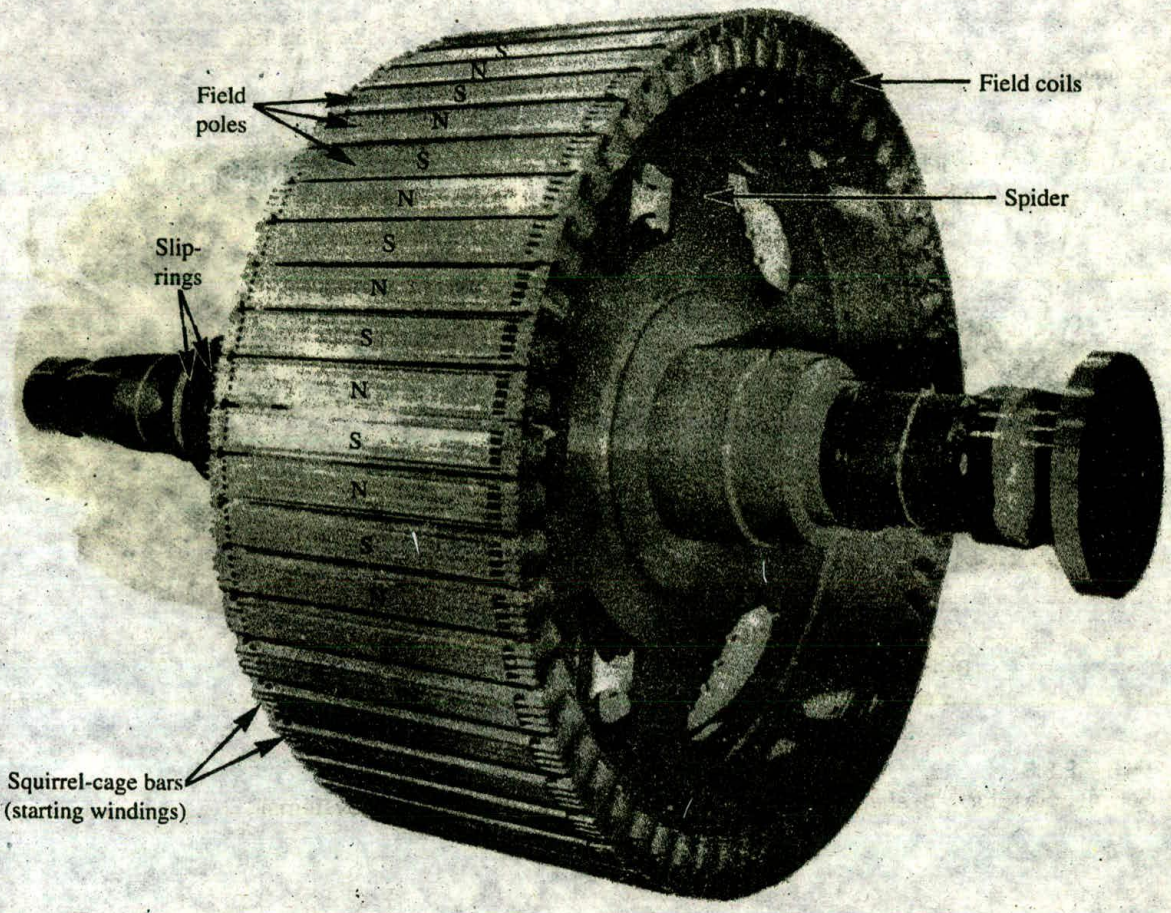


FIGURE 8.2
Salient-pole rotor for slow-speed synchronous-machine applications. (Courtesy TECO Westinghouse)

brushes pressed against the rings provide the connection between the field windings and the DC excitation source.

A salient-pole rotor with a shaft-mounted DC exciter is shown in Figure 8.3. Direct current from the exciter armature is supplied to the field windings by means of carbon brushes (not shown) riding on the commutator that connects to carbon brushes riding on the slip rings.

A salient-pole rotor equipped with a brushless excitation system is shown in Figure 8.4. Brushless excitation is provided by a small three-phase exciter armature, a three-phase rectifier, and control circuitry, all mounted on the same shaft.

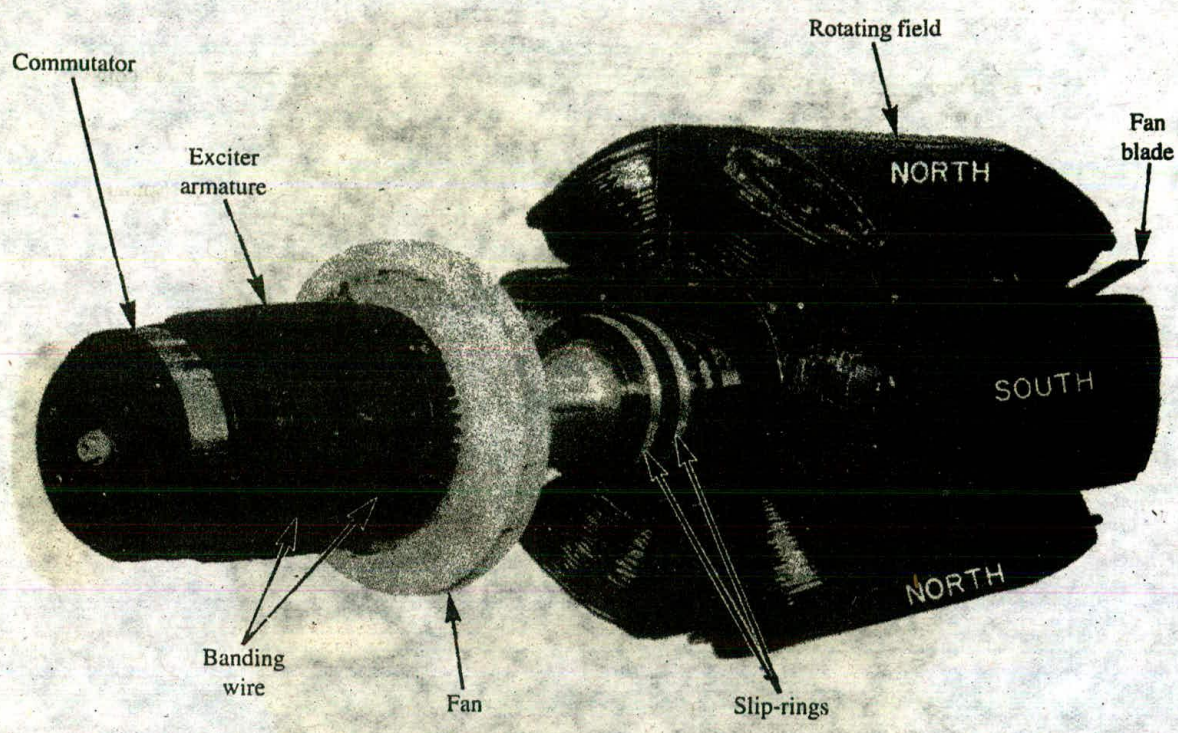


FIGURE 8.3
Salient-pole rotor with shaft-mounted DC exciter. (Courtesy GE Industrial Systems)

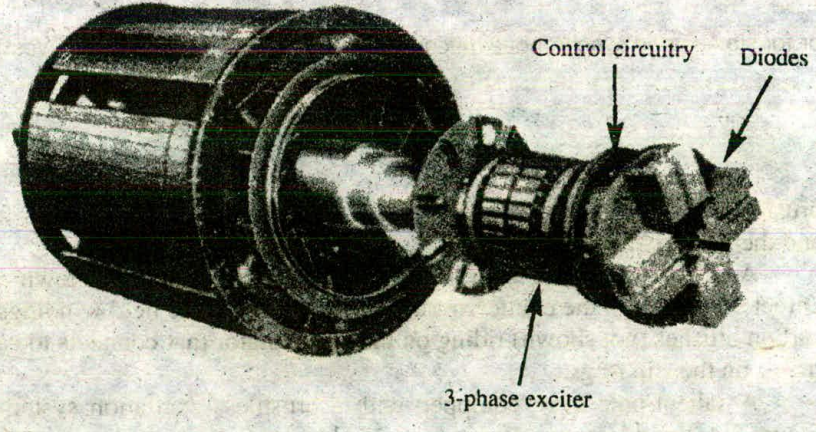


FIGURE 8.4
Salient-pole rotor with a brushless excitation system. (Courtesy Dresser Rand, Electric Machinery)

8.3 SYNCHRONOUS MOTOR STARTING

When starting, the rotor is allowed to accelerate to its maximum speed as an induction motor using its built-in damper windings.[2] At this speed the slip is very small and the rotating flux of the stator moves very slowly relative to the revolving rotor. Direct current is then applied to the magnet windings, forming alternate north and south poles that "lock" in rotational synchronism with the corresponding opposite poles of the rotating flux, the slip is zero, all induction-motor action ceases, and the machine operates at synchronous speed. The synchronous speed, as developed in Section 4.5 in Chapter 4, is expressed in terms of the stator frequency and the number of poles:

$$n_s = \frac{120 \cdot f}{P} \quad \text{r/min} \tag{4–1}$$

If the rotor magnets are energized before the machine reaches its maximum speed as an induction motor, the rotor may not synchronize and severe vibration will occur; every time a pole of the rotating stator flux passes a rotor pole, alternate attraction and repulsion occurs. Such out-of-step operation, called *pole slipping,* also causes cyclic current surges and torque pulses at slip frequency in the armature windings.

The adverse effects of frequent starting, or operating with unbalanced three-phase line voltages, are the same as for induction motors.[3]

A simplified circuit diagram for rotor and stator connections is shown in Figure 8.5. A varistor, or resistor and switch, connected across the field leads during locked rotor and acceleration, prevents a high induced emf in the field windings, and the induced current in the circuit formed by the field windings and external resistor provides additional induction-motor torque.

The circuit for a brushless excitation system is shown in Figure 8.6. A frequency-sensitive solid-state control circuit monitors the frequency of the emf induced in the motor field winding by the rotating flux of the stator. The frequency of the emf is the same as that in the squirrel-cage winding and is a function of the frequency of the applied stator voltage and the slip. That is,

$$f_r = sf_s \tag{8–1}$$

At locked rotor $s = 1.0$, causing the rotor frequency to equal the applied stator frequency. As the rotor accelerates, the slip becomes smaller, and the frequency decreases.

When the frequency-sensitive circuit observes the "high frequency" generated in the rotor coils at locked rotor, and during high-slip acceleration, it closes SCR-2 and opens SCR-1; opening SCR-1 blocks the exciter current, and closing SCR-2 connects the discharge resistor across the field windings. At near-synchronous speed, the rotor

---

[2]Although some synchronous motors are accelerated by eddy-current torque and hysteresis torque, most are accelerated by induction-motor torque through damper windings.

[3]In Chapter 5, see Section 5.13 for frequent starting, and Section 5.15 for unbalanced voltages.

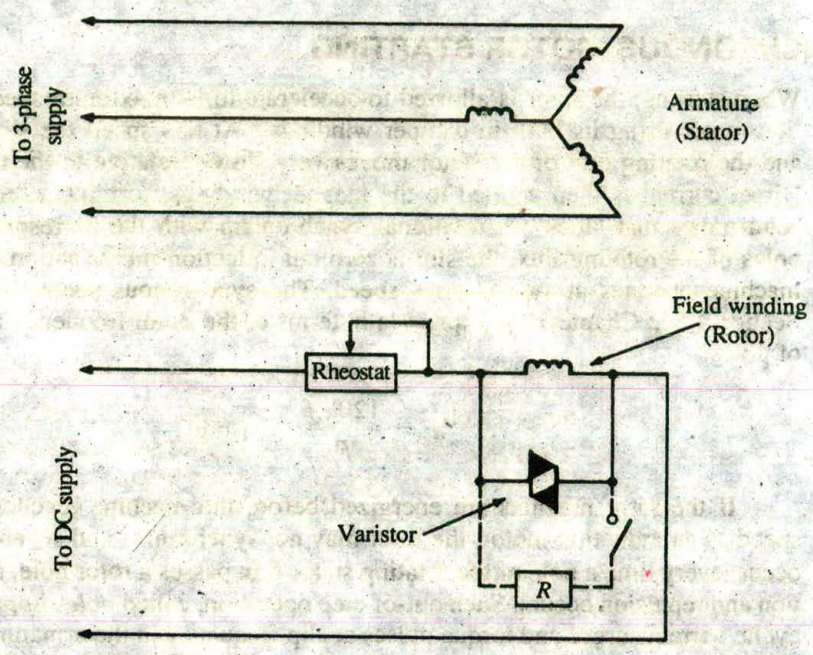


FIGURE 8.5
Simplified circuit diagrams for rotor and stator connections of a synchronous motor.

frequency is very low (approaching zero), and the control circuit opens SCR-2 and closes SCR-1; opening SCR-2 disconnects the discharge resistor, and closing SCR-1 admits current to the field windings. The solid-state switching is programmed to close SCR-1 at an instant that will ensure that the rotor poles will be facing stator poles of opposite polarity, thus preventing pole slipping.

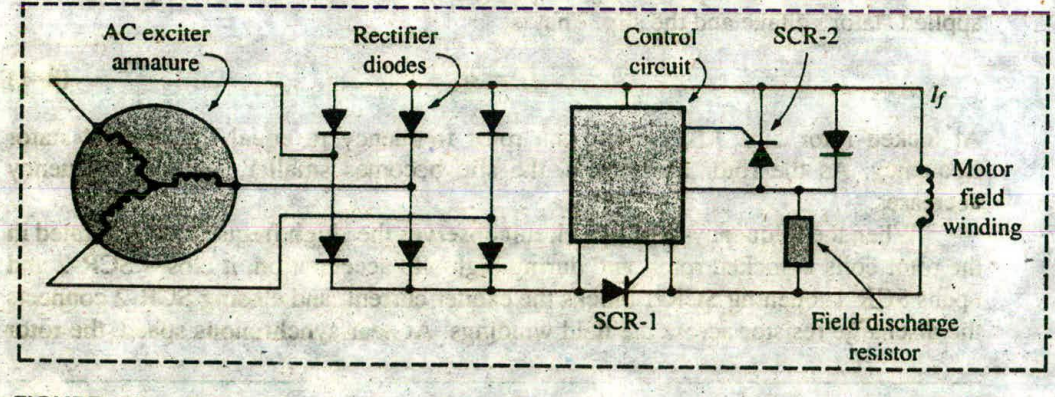


FIGURE 8.6
Circuit diagram of a brushless excitation system for a synchronous machine. (Courtesy Dresser Rand, Electric Machinery)

Synchronous motors without built-in starting components are not energized until they have been accelerated to approximately synchronous speed by an auxiliary motor or turbine. No load is connected to a synchronous motor during the acceleration period. When near synchronous speed, the magnet windings are energized, and the field current adjusted to obtain an induced stator voltage approximately equal to the supply voltage. The circuit breaker connecting the stator to the three-phase supply is closed when the induced stator voltage is in phase with the supply voltage. This procedure, called paralleling, is discussed in Section 9.6 of Chapter 9.

Reversing a Synchronous Motor

The direction of rotation of a synchronous motor is determined by its starting direction, as initiated by induction-motor action. Thus, to reverse the direction of a three-phase synchronous motor, it is necessary to first stop the motor and then reverse the phase sequence of the three-phase connections at the stator. Reversing the current to the field windings will not affect the direction of rotation.

8.4 SHAFT LOAD, POWER ANGLE, AND DEVELOPED TORQUE

Although the rotor of a synchronous motor rotates in synchronism with the rotating flux of the stator, increases in shaft load cause the rotor magnets to change their angular position with respect to the rotating flux. This displacement angle can be seen by viewing the rotor with a strobe light synchronized with the stator frequency.

Figure 8.7 shows a "strobe view" of a salient-pole motor operating at synchronous speed with no load on the shaft. The direction of rotation is counterclockwise (CCW). As the machine is loaded, the rotor changes its relative position with respect to the rotating flux of the stator, lagging behind it by angle δ. Angle δ, expressed in electrical degrees, is called the *power angle, load angle,* or *torque angle.* The broken line on the south pole in Figure 8.7 shows the position of the rotating magnets relative to the rotating flux of the stator for a representative load.

A synchronous motor operates at the same average speed for all values of load from no load to peak load. When the load on a synchronous motor is increased, the motor slows down just enough to allow the rotor to change its angular position in relation to the rotating flux of the stator, and then goes back to synchronous speed. Similarly, when the load is removed, it accelerates just enough to cause the rotor to decrease its angle of lag in relation to the rotating flux, and then goes back to synchronous speed. When the peak load that the machine can handle is exceeded, the rotor pulls out of synchronism.

The torque developed by all synchronous motors has two components: a reluctance-torque component and a magnet-torque component. The *reluctance-torque component* is due to the normal characteristic of magnetic materials in a magnetic field to align themselves so that the reluctance of the magnetic circuit is a minimum[4]

[4]See Section 7.2, Chapter 7.

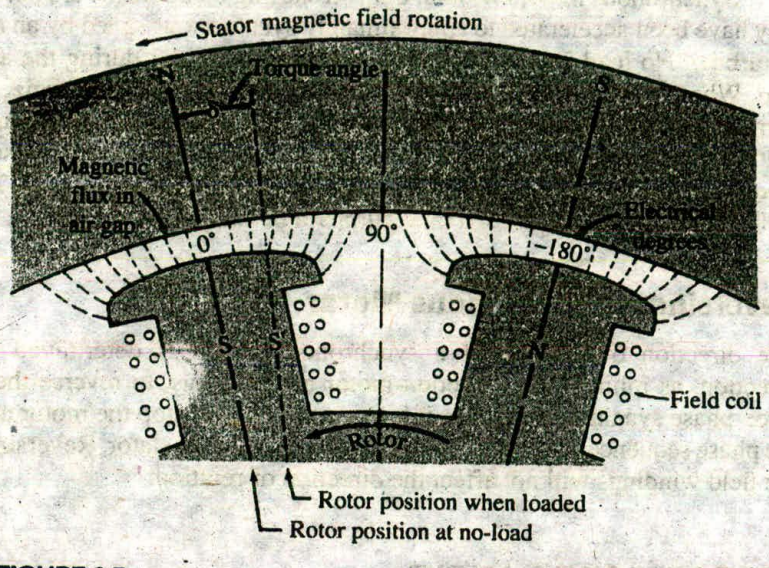


FIGURE 8.7
Strobe view of a salient-pole motor operating at synchronous speed with no load on the shaft. (Courtesy Dresser Rand, Electric Machinery)

The *magnet-torque component* is due to the magnetic attraction between the field poles (magnets) on the rotor and the corresponding opposite poles of the rotating stator flux.

The analysis of synchronous-motor behavior in this chatper is based on magnet-torque alone. This is justified for cylindrical-rotor motors because the smooth rotor surface provides an insignificant reluctance torque. It is also justified for salient-pole motors operating from 50 percent rated load to above 100 percent rated load, with power factors ranging from unity to leading; the reluctance torque for such loads is significantly smaller than the magnet torque. The performance of lightly loaded salient-pole motors is discussed in Section 8.13.

8.5 COUNTER-EMF AND ARMATURE-REACTION VOLTAGE

The air-gap flux in a synchronous motor includes a rotating field flux (Φ_f) due to DC current (I_f) in the rotating magnets, and a rotating armature flux, called *armature-reaction flux* (Φ_{ar}), caused by the three-phase armature currents in the stator windings. The magnitude and phase angle of the armature-reaction flux is a function of the magnitude and phase angle of the armature current. Both the magnet flux and the armature-reaction flux rotate in the same direction, and at synchronous speed with respect to the armature windings.

The rotating fluxes Φ_f and Φ_{ar} generate speed-voltages in the stator conductors, as visualized in Figure 8.8 for one armature phase of a three-phase synchronous

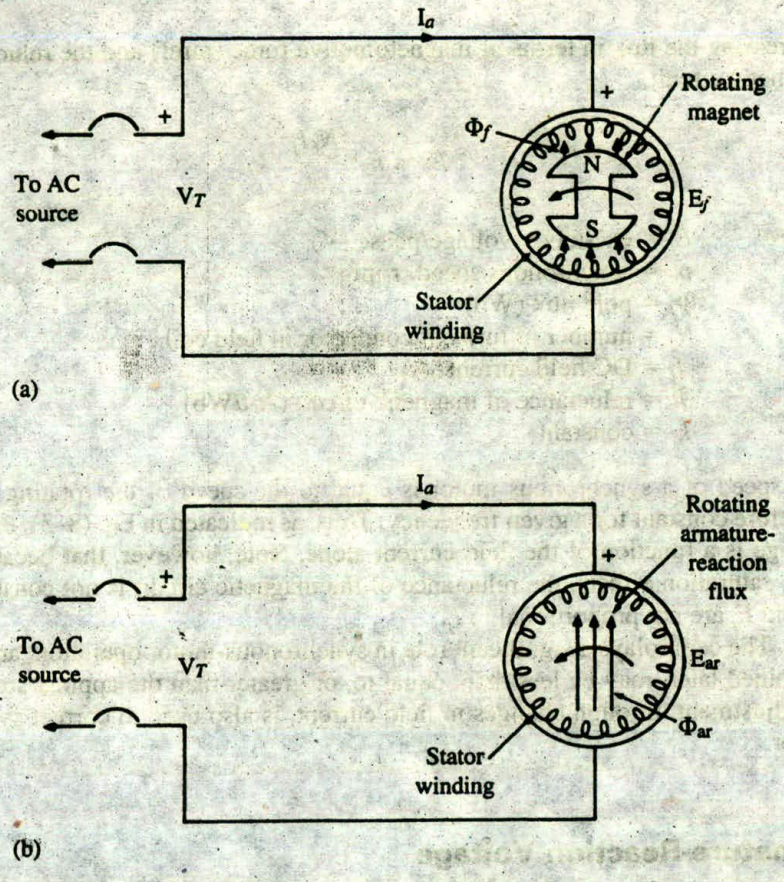


FIGURE 8.8
Separate circuits showing emfs generated by component magnetic fields for one phase of a synchronous motor: (a) due to rotating magnets: (b) due to rotating armature-reaction flux.

motor. Although shown as separate diagrams for ease of visualization, both Φ_f and Φ_{ar} rotate within the same three-phase stator winding.

Counter-emf

The rotor magnets sweeping the stator conductors, as shown in Figure 8.8(a), generate a speed-voltage, called a counter-emf (cemf) or *excitation voltage*, that acts in opposition to the applied voltage. The speed-voltage is proportional to the field flux and the speed of rotation. Expressed in equation form:

$$E_f = n_s \Phi_f k_f \qquad (8\text{–}2)$$

Expressing the flux in terms of magnetomotive force (mmf) and the reluctance of the magnetic circuit,

$$E_f = n_s \cdot \frac{N_f I_f}{\mathcal{R}} \cdot k_f \tag{8–3}$$

where: E_f = excitation voltage/phase (V)
n_s = synchronous speed (r/min)
Φ_f = pole flux (Wb)
N_f = number of turns of conductor in field coil
I_f = DC field current (A)
$\mathcal{R}$ = reluctance of magnetic circuit (A-t/Wb)
k_f = constant

The speed of a synchronous motor is equal to the speed of the rotating flux, and is therefore constant for a given frequency. Thus, as indicated in Eq. (8–3), the excitation voltage is a function of the field current alone. Note, however, that because of magnetic saturation effects, the reluctance of the magnetic circuit is not constant. Hence, Φ_f and E_f are not proportional to I_f.

The cemf plays a significant role in synchronous-motor operation, and as will be explained later, may be less than, equal to, or greater than the applied stator voltage. Its adjustment, through changes in field current, is also used to correct system power factor.

Armature-Reaction Voltage

The rotating armature-reaction flux sweeping the stator conductors, as visualized in Figure 8.8(b), generates a speed voltage called the *armature-reaction voltage*. The armature-reaction speed voltage, expressed in terms of armature-reaction flux is

$$E_{ar} = n_s \Phi_{ar} k_a \tag{8–4}$$

where: E_{ar} = armature-reaction voltage (V)
Φ_{ar} = armature-reaction flux (Wb)
n_s = synchronous speed (r/min)
k_a = constant

Neglecting the effects of magnetic saturation, the armature-reaction flux is proportional to the armature current. Hence, the armature-reaction voltage may be conveniently expressed in terms of the armature current and a fictitious *armature-reaction reactance*. Thus,

$$\mathbf{E}_{ar} = \mathbf{I}_a \cdot jX_{ar} \tag{8–5}$$

where: X_{ar} = armature-reaction reactance (Ω/phase).

8.6 EQUIVALENT-CIRCUIT MODEL AND PHASOR DIAGRAM OF A SYNCHRONOUS-MOTOR ARMATURE

The equivalent-circuit model for one armature phase of a *cylindrical rotor* synchronous motor is shown in Figure 8.9(a). All values are per phase. Applying Kirchhoff's voltage law to Figure 8.9(a),

$$\mathbf{V}_T = \mathbf{I}_a R_a + \mathbf{I}_a jX_\ell + \mathbf{I}_a jX_{\text{ar}} + \mathbf{E}_f \tag{8–6}$$

Combining reactances,

$$X_s = X_\ell + X_{\text{ar}} \tag{8–7}$$

Substituting Eq. (8–7) into Eq. (8–6)

$$\mathbf{V}_T = \mathbf{E}_f + \mathbf{I}_a(R_a + jX_s) \tag{8–8}$$

$$\mathbf{V}_T = \mathbf{E}_f + \mathbf{I}_a \mathbf{Z}_s \tag{8–9}$$

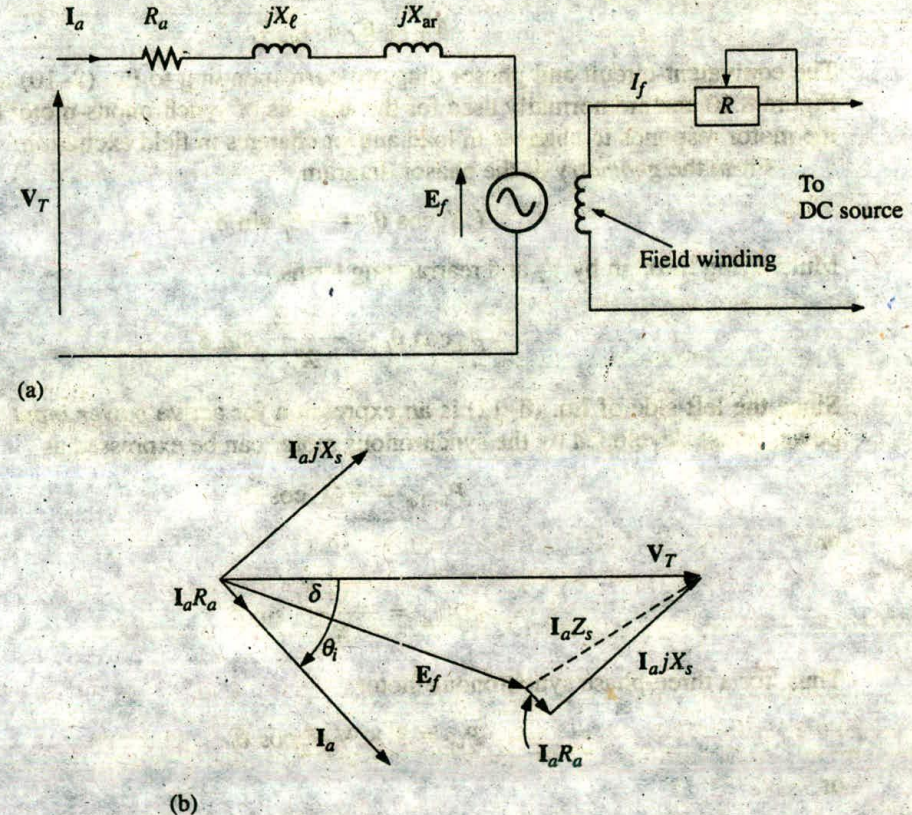


FIGURE 8.9
(a) Equivalent-circuit model for one phase of a synchronous-motor armature; (b) phasor diagram corresponding to the equivalent-circuit model in (a).

where: R_a = armature resistance (Ω/phase)
X_ℓ = armature leakage reactance (Ω/phase)
X_s = synchronous reactance (Ω/phase)
Z_s = synchronous impedance (Ω/phase)
V_T = applied voltage/phase (V)

A phasor diagram showing the component phasors and the tip-to-tail determination of $\mathbf{V}_T$ is shown in Figure 8.9(b). *The phase angle δ of the excitation voltage in Figure 8.9(b) is equal to the torque angle in Figure 8.7.* The torque angle is also called the *load angle* or *power angle.*

8.7 SYNCHRONOUS-MOTOR POWER EQUATION (MAGNET POWER)

Except for very small machines, the armature resistance of a synchronous motor is relatively insignificant compared to its synchronous reactance, enabling Eq. (8–9) to be approximated as

$$\mathbf{V}_T = \mathbf{E}_f + \mathbf{I}_a jX_s \tag{8–10}$$

The equivalent-circuit and phasor diagram corresponding to Eq. (8–10) are shown in Figure 8.10 and are normally used for the analysis of synchronous-motor behavior, as the motor responds to changes in load and/or changes in field excitation.

From the *geometry* of the phasor diagram,

$$I_a X_s \cos \theta_i = -E_f \sin \delta \tag{8–11}$$

Multiplying through by V_T and rearranging terms,

$$V_T I_a \cos \theta_i = \frac{-V_T E_f}{X_s} \sin \delta \tag{8–12}$$

Since the left side of Eq. (8–12) is an expression for active *power input,* the magnet power/phase developed by the synchronous motor can be expressed as

$$P_{\text{in},1\phi} = V_T I_a \cos \theta_i \tag{8–13}$$

or

$$P_{\text{in},1\phi} = \frac{-V_T E_f}{X_s} \sin \delta \tag{8–14}$$

Thus, for a three-phase synchronous motor,

$$P_{\text{in}} = 3 \times V_T I_a \cos \theta_i \tag{8–15}$$

or

$$P_{\text{in}} = 3 \times \frac{-V_T E_f}{X_s} \sin \delta \tag{8–16}$$

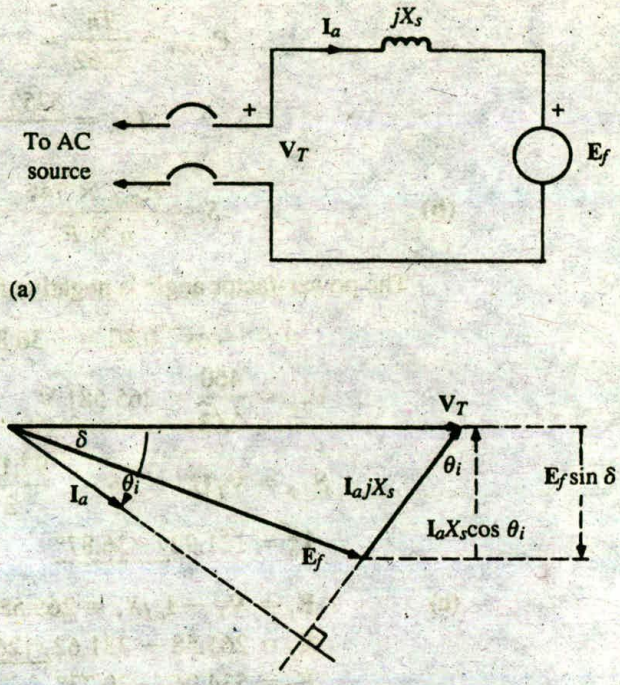


FIGURE 8.10
(a) Equivalent-circuit; (b) phasor diagram for a synchronous motor, assuming armature resistance is negligible.

Equation (8–14), called the *synchronous-machine power equation,* expresses the magnet power/phase developed by a cylindrical-rotor motor, in terms of its excitation voltage and power angle. Assuming a constant source voltage and constant frequency, Eqs. (8–13) and (8–14) may be expressed as proportionalities that are very useful in synchronous-machine analysis:

$$P \propto I_a \cos \theta_i \tag{8–17}$$

$$P \propto E_f \sin \delta \tag{8–18}$$

EXAMPLE 8.1 A 100-hp, three-phase, wye-connected, 60-Hz, 460-V, four-pole, cylindrical-rotor synchronous motor is operating at rated conditions and 80 percent power-factor leading. The efficiency, excluding field and stator losses, is 96 percent, and the synchronous reactance is 2.72 Ω/phase. Determine (a) developed torque; (b) armature current; (c) excitation voltage; (d) power angle; (e) maximum torque (also called pull-out torque).

Solution

(a)
$$n_s = \frac{120f}{P} = \frac{120 \times 60}{4} = 1800 \text{ r/min}$$

$$P_{mech} = \frac{Tn}{5252} = \quad \Rightarrow \quad T_{dev} = \frac{5252 \cdot P_{mech}}{n}$$

$$T_{dev} = \frac{5252 \times 100/0.96}{1800} = \underline{304 \text{ lb-ft}}$$

(b)
$$S = \frac{P_{shaft} \times 746}{\eta \times F_P} = \frac{100 \times 746}{0.96 \times 0.80} = 97{,}135 \text{ VA}$$

The power-factor angle is negative for a leading power factor.[5]

$$\theta = -\cos^{-1}0.80 = -36.87°$$

$$V_{1\phi} = \frac{460}{\sqrt{3}} = 265.581 \text{ V}$$

$$\mathbf{S}_{1\phi} = \mathbf{V}_T \mathbf{I}_a^* \quad \Rightarrow \quad \frac{97{,}135}{3}\underline{/-36.87°} = 265.581\underline{/0°} \times \mathbf{I}_a^*$$

$$\mathbf{I}_a^* = 121.92\underline{/-36.87°} \quad \Rightarrow \quad \underline{\mathbf{I}_a = 121.92/36.87° \text{ A}}$$

(c)
$$\mathbf{E}_f = \mathbf{V}_T - \mathbf{I}_a jX_s = 265.581\underline{/0°} - (121.92\underline{/36.87°})(2.72\underline{/90°})$$

$$\mathbf{E}_f = 265.58 - 331.62\underline{/126.87°}$$

$$\mathbf{E}_f = 534.96\underline{/-29.73°} \quad \Rightarrow \quad \underline{535/-29.7° \text{ V}}$$

(d)
$$\delta = \underline{-29.7°}$$

(e) Pull-out torque occurs at $\delta = -90°$.

$$P_{in} = 3 \cdot \frac{-V_T E_f}{X_s} \cdot \sin\delta = 3 \cdot \frac{-265.581 \times 534.96}{2.72} \cdot \sin(-90°)$$

$$P_{in} = 156{,}700 \text{ W}$$

$$P = \frac{Tn}{5252} \quad \Rightarrow \quad T_{pull\text{-}out} = \frac{5252 \cdot P}{n}$$

$$T_{pull\text{-}out} = \frac{5252 \times 156{,}700}{746 \times 1800} = \underline{613 \text{ lb-ft}}$$

8.8 EFFECT OF CHANGES IN SHAFT LOAD ON ARMATURE CURRENT, POWER ANGLE, AND POWER FACTOR

The effect that changes in shaft load have on armature current, power angle, and power factor are illustrated in Figure 8.11; the applied stator voltage, frequency, and

[5]See Sections A.5 and A.12 in Appendix A for the relationship between leading and lagging power factor and the sign of the power-factor angle.

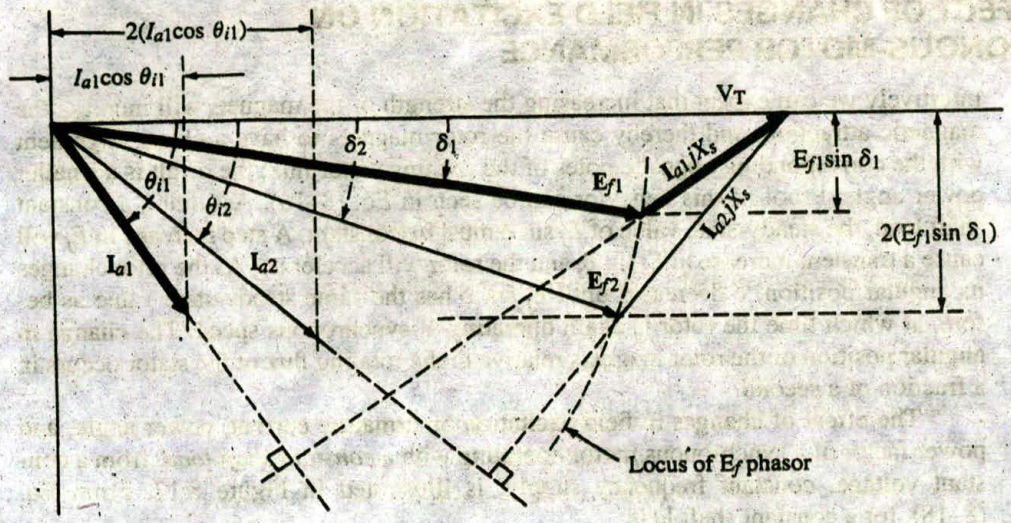


FIGURE 8.11
Phasor diagram showing effect of changes in shaft load on armature current, power angle, and power factor of a synchronous motor.

field excitation are assumed constant. The heavy lines indicate the initial load conditions, and the light lines indicate the new steady-state conditions that correspond to doubling the shaft load. *Note:* In accordance with Eqs. (8–17) and (8–18), doubling the shaft load doubles both $I_a \cos \theta_i$ and $E_f \sin \delta$. When adjusting phasor diagrams to show new steady-state conditions, the line of action of the new $\mathbf{I}_a jX_s$ phasor must be perpendicular to the new $\mathbf{I}_a$ phasor. Furthermore, as shown in Figure 8.11, if the excitation is not changed, increasing the shaft load causes the locus of the $\mathbf{E}_f$ phasor to describe a circular arc, increasing its phase angle with increasing shaft load. Note also that an increase in shaft load is also accompanied by a decrease in θ_i resulting in an increase in power factor.

As additional load is placed on the machine, the rotor continues to increase its lag relative to the rotating flux, thereby increasing both the angle of lag of the cemf phasor and the magnitude of the stator current. During all this loading, however, except for the transient conditions whereby the rotor assumes a new position in relation to the rotating flux, the average speed of the machine does not change. Finally, with increased loading, a point is reached at which a further increase in δ fails to cause a corresponding increase in motor torque, and the rotor pulls out of synchronism. The point of maximum torque occurs at a power angle of approximately 90° for a cylindrical-rotor machine, as is evidenced in Eq. (8–16). The critical value of torque that causes a synchronous motor to pull out of synchronism is called the *pull-out torque.*

8.9 EFFECT OF CHANGES IN FIELD EXCITATION ON SYNCHRONOUS-MOTOR PERFORMANCE

Intuitively we can expect that increasing the strength of the magnets will increase the magnetic attraction, and thereby cause the rotor magnets to have a closer alignment with the corresponding opposite poles of the rotating stator flux; the result is a smaller power angle. Proof of this behavior can be seen in Eq. (8–16). Assuming a constant shaft load, the steady-state value of $E_f \sin \delta$ must be constant. A step increase in E_f will cause a transient increase in $E_f \sin \delta$, and the rotor will accelerate. As the rotor changes its angular position, δ decreases until $E_f \sin \delta$ has the same steady-state value as before, at which time the rotor is again operating at synchronous speed. The change in angular position of the rotor magnets relative to the rotating flux of the stator occurs in a fraction of a second.

The effect of changes in field excitation on armature current, power angle, and power factor of a synchronous motor operating with a *constant shaft load,* from a constant voltage, constant frequency supply, is illustrated in Figure 8.12. From Eq. (8–18), for a constant shaft load,

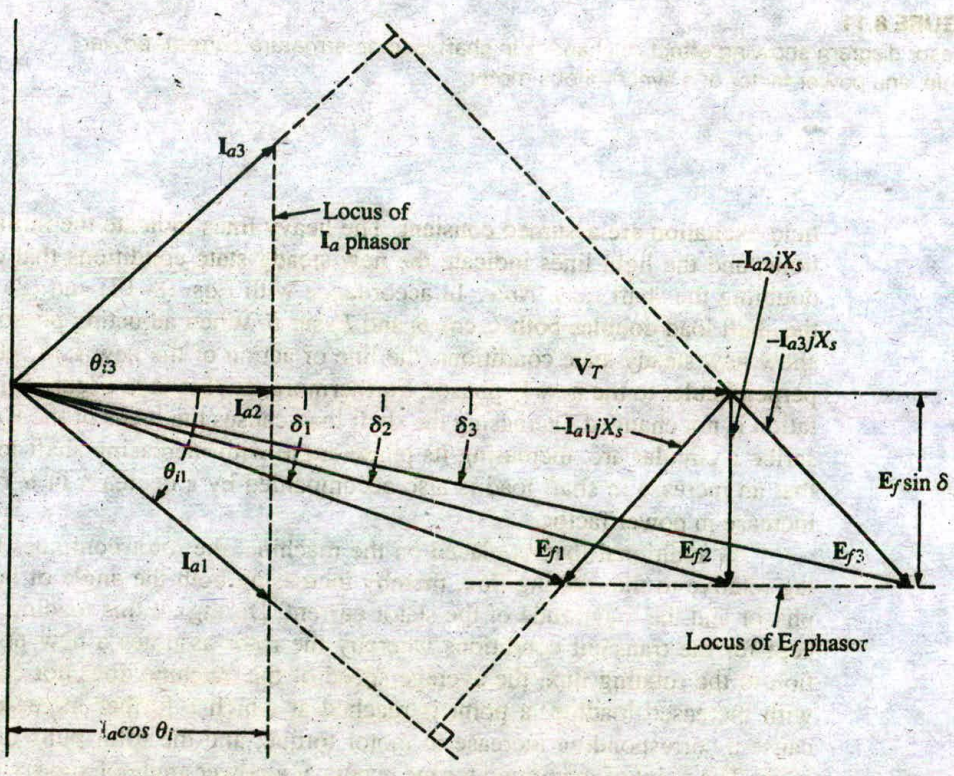


FIGURE 8.12
Phasor diagram showing the effect of changes in field excitation on armature current, power angle, and power factor of a synchronous motor.

$$E_{f1} \sin \delta_1 = E_{f2} \sin \delta_2 = E_{f3} \sin \delta_3 = E_f \sin \delta \qquad \textbf{(8–19)}$$

This is shown in Figure 8.12, where the locus of the tip of the $\mathbf{E}_f$ phasor is a straight line parallel to the $\mathbf{V}_T$ phasor. Similarly, from Eq. (8–17), for a constant shaft load,

$$I_{a1} \cos \theta_{i1} = I_{a2} \cos \theta_{i2} = I_{a3} \cos \theta_{i3} = I_a \cos \theta_i \qquad \textbf{(8–20)}$$

This is shown in Figure 8.12, where the locus of the tip of the $\mathbf{I}_a$ phasor is a line perpendicular to the $\mathbf{V}_T$ phasor.

Note that increasing the excitation from $\mathbf{E}_{f1}$ to $\mathbf{E}_{f3}$ in Figure 8.12 caused the angle of the current phasor (and hence the power factor) to go from lagging to leading. The value of field excitation that results in unity power factor is called *normal excitation.* Excitation greater than normal is called overexcitation, and excitation less than normal is called underexcitation. Furthermore, as indicated in Figure 8.12, when operating in the overexcited mode, $|\mathbf{E}_f| > |\mathbf{V}_T|$.

8.10 V CURVES

Curves of armature current vs. field current or armature current vs. excitation voltage are called V curves, and are shown in Figure 8.13 for representative synchronous-motor loads. The curves are related to the phasor diagram in Figure 8.12, and illustrate the effect of different values of field excitation on armature current and power factor for specific shaft loads. Note that increases in shaft load require increases in field excitation in order to maintain unity power factor.

The locus of the left endpoints of the V curves in Figure 8.13 represents the *stability limit* ($\delta = -90°$). Any reduction in excitation below the stability limit for a particular load will cause the rotor to pull out of synchronism.

The constant-load V curves shown in Figure 8.13 may be plotted from laboratory data (I_a vs. I_f) as I_f is varied, or graphically by plotting $|\mathbf{I}_a|$ vs. $|\mathbf{E}_f|$ from a family of phasor diagrams such as that shown in Figure 8.12, or from the following mathematical expression for the V curves[6]:

$$I_a = \frac{1}{X_s} \cdot \left[E_f^2 + V_T^2 - 2\sqrt{E_f^2 V_T^2 - X_s^2 P_{\text{in},1\phi}^2}\right]^{0.5} \qquad \textbf{(8–21)}$$

Equation (8–21) is based on the geometry of the phasor diagram and assumes R_a is negligible. *Note:* If the developed torque is less than the shaft load plus windage and friction, instability will occur, and the expression under the square root sign will be negative.

The family of V curves shown in Figure 8.13 represents computer plots of Eq. (8–21), assuming a three-phase, 40-hp, 220-V, 60-Hz synchronous motor with a synchronous reactance of 1.27 Ω/phase.

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[6]See Reference [1] for the derivation of Eq. (8–21) and the general nature of V curves.

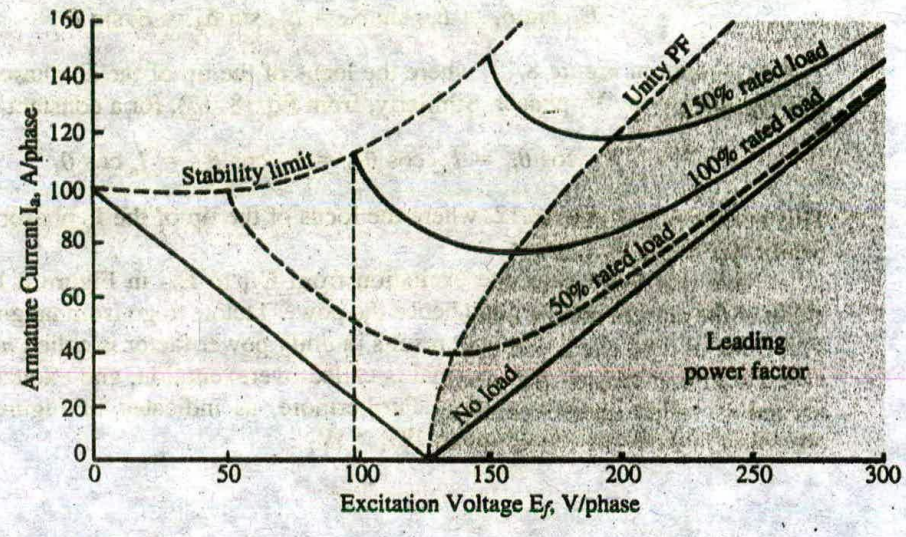


FIGURE 8.13
Family of representative V curves for a synchronous motor.

EXAMPLE 8.2 Referring to the V curve for 100 percent load in Figure 8.13, determine (a) the minimum value of excitation that will maintain synchronism. (b) Repeat (a) using Eq. (8–16). (c) Repeat (a) using Eq. (8–21). (d) Determine the power angle if the field excitation voltage is increased to 175 percent of the stability limit determined in (c).

Solution

(a) Estimation of minimum excitation voltage from the graph is

$$E_f \approx \underline{98 \text{ V}}$$

(b) From Eq. (8–16), and neglecting losses,

$$P_{\text{in}} = 3 \times \frac{-V_T E_f}{X_s} \sin \delta \quad \Rightarrow \quad E_f = \frac{-P_{\text{in}} \cdot X_s}{3 \cdot V_T \sin \delta}$$

$$E_f = \frac{-40 \times 746 \times 1.27}{3 \times 220/\sqrt{3} \times \sin(-90°)} \approx \underline{99 \text{ V}}$$

(c) From Eq. (8–21), the value of E_f that causes the expression under the square-root sign to equal zero is the minimum excitation that will maintain synchronism. Thus for,

$$E_f^2 V_T^2 - X_s^2 P_{\text{in.1}\phi}^2 = 0$$

$$E_f = \frac{X_s P_{\text{in.1}\phi}}{V_T} = \frac{1.27 \times 40 \times 746/3}{220/\sqrt{3}} \approx \underline{99 \text{ V}}$$

(d) From Eq. (8–18),

$$1.75E_{f1} \sin \delta_2 = E_{f1} \sin(-90°)$$
$$\delta_2 = \underline{-35°}$$

8.11 SYNCHRONOUS-MOTOR LOSSES AND EFFICIENCY

A power-flow diagram that illustrates the flow of power through a synchronous motor, from stator and rotor input to shaft output, is shown in Figure 8.14. As indicated in the power-flow diagram, the total power loss for the motor is

$$P_{\text{loss}} = P_{\text{scl}} + P_{\text{core}} + P_{\text{fcl}} + P_{f,w} + P_{\text{stray}} \quad \text{W} \tag{8–22}$$

where:
P_{scl} = stator-conductor loss
P_{fcl} = field-conductor loss
P_{core} = core loss
$P_{f,w}$ = friction and windage loss
P_{stray} = stray power loss

Except for the transient conditions that occur when the field current is increased or decreased (magnetic energy stored or released), the total energy input to the field coils is constant and all of it is converted to I^2R losses in the copper conductors.

Efficiency

The overall efficiency of a synchronous motor is given by

$$\eta = \frac{P_{\text{shaft}}}{P_{\text{in}} + P_{\text{field}}} = \frac{P_{\text{shaft}}}{P_{\text{shaft}} + P_{\text{loss}}} \tag{8–23}$$

The nameplates of synchronous motors and manufacturers' specification sheets generally provide only the overall efficiency for rated load conditions. Hence, only the total losses may be determined. The separation of losses into the components listed in Eq. (8–22) requires a very involved test procedure [2]. If the armature copper losses and field copper losses can be calculated, however, they should be subtracted from the total input power to obtain a closer approximation of the mechanical power developed.

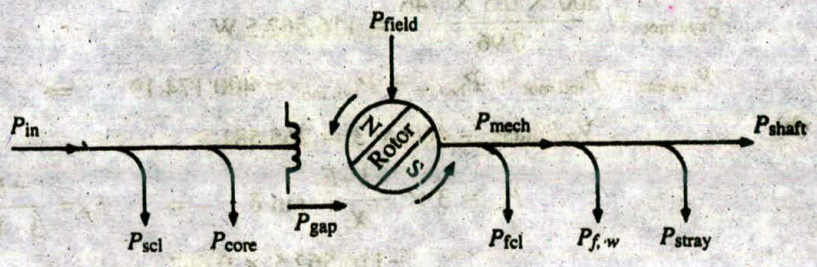


FIGURE 8.14
Power-flow diagram for a synchronous motor.

8.12 USING SYNCHRONOUS MOTORS TO IMPROVE THE SYSTEM POWER FACTOR

In industrial applications where both synchronous motors and induction motors are used, the synchronous motor is usually operated at a leading power factor to compensate for the lagging power factor of the induction motors. In those cases where a synchronous motor is operated without load, merely for the purpose of improving the power factor of a system, the machine is called a *synchronous condenser*. Machines specifically designed for this application are built without external shafts.

EXAMPLE 8.3 A three-phase, 60-Hz, 460-V system supplies the following loads:

1. A six-pole, 60-Hz, 400-hp, three-phase, wye-connected induction motor, operating at three-quarters rated load with an efficiency of 95.8 percent and a power factor of 89.1 percent.
2. A 50-kW, delta-connected, three-phase resistance heater.
3. A 300-hp, 60-Hz, four-pole, wye-connected, cylindrical-rotor synchronous motor, operating at one-half rated load, with a torque angle of $-16.4°$.

Neglecting copper losses, the synchronous motor is operating at 96 percent efficiency, and its synchronous reactance is 0.667 Ω/phase. Determine (a) system active power; (b) power factor of the synchronous motor; (c) system power factor; (d) percent change in synchronous motor field current required to adjust the system power factor to unity (neglect saturation effects); (e) power angle of the synchronous motor for the conditions in (d).

Solution

The circuit diagram is shown in Figure 8.15(a) and the power diagram in Figure 8.15(b).

(a)
$$P_{\text{ind mot}} = \frac{400 \times 0.75 \times 746}{0.958} = 233{,}611.7 \text{ W}$$
$$P_{\text{heater}} = 50{,}000 \text{ W}$$
$$P_{\text{syn mot}} = \frac{300 \times 0.5 \times 746}{0.96} = 116{,}562.5 \text{ W}$$
$$P_{\text{system}} = P_{\text{ind mot}} + P_{\text{heater}} + P_{\text{syn mot}} = 400{,}174.19 \quad \Rightarrow \quad \underline{400.2 \text{ kW}}$$

(b)
$$V_T/\text{phase} = 460/\sqrt{3} = 265.581 \text{ V}$$
$$P_{\text{in}} = 3 \times \frac{-V_T E_f}{X_s} \sin\delta \quad \Rightarrow \quad E_f = \frac{-P_{\text{in}} \cdot X_s}{3 \cdot V_T \sin\delta}$$
$$E_f = \frac{-116{,}562.5 \times 0.667}{3 \times 265.581 \times \sin(-16.4°)} = 345.614 \text{ V}$$
$$\mathbf{E}_f = \underline{345.614/-16.4° \text{ V}}$$

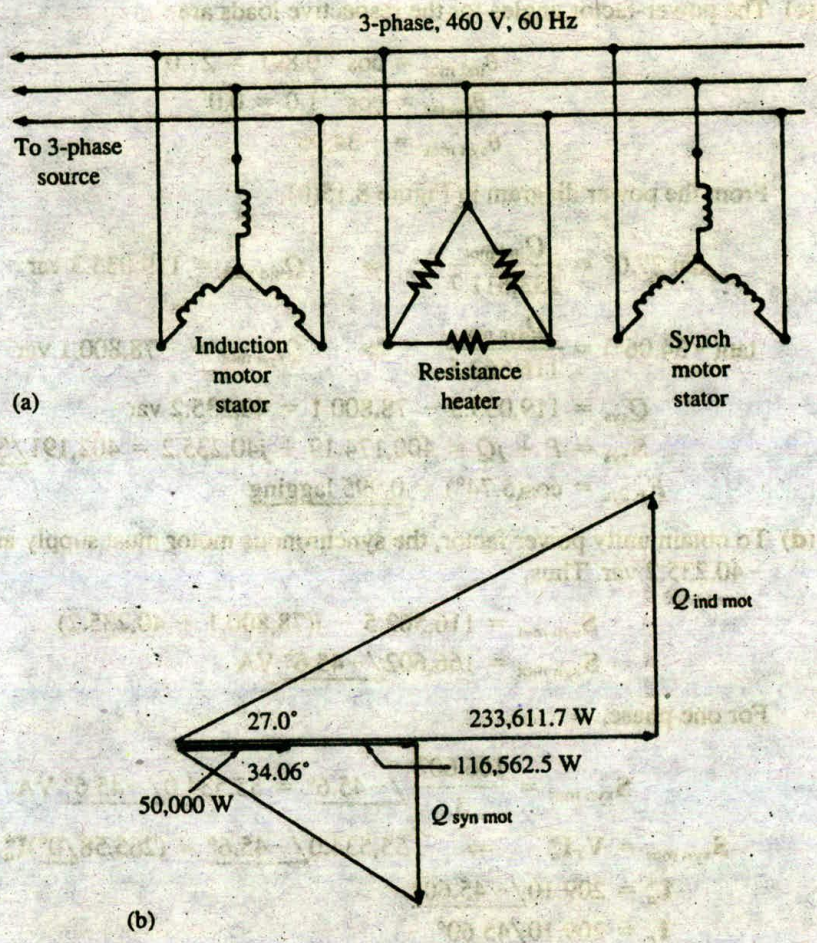


FIGURE 8.15
(a) Circuit diagram for Example 8.3; (b) power diagram.

$$\mathbf{E}_f = \mathbf{V}_T - \mathbf{I}_a jX_s$$

$$345.614\angle -16.4° = 265.581\angle 0° - \mathbf{I}_a \times 0.667\angle 90°$$

$$117.79\angle -55.94° = -\mathbf{I}_a \times 0.667\angle 90°$$

$$\mathbf{I}_a = 176.6\angle 34.06° \text{ A}$$

$$\theta = (\theta_v - \theta_i) = (0 - 34.06) = -34.06°$$

$$F_P = \cos(-34.06°) = \underline{0.828 \text{ leading}}^{7}$$

[7] See Sections A.5 and A.12 in Appendix A for the relationship between the leading and lagging power factor and the sign of the power-factor angle.

(c) The power-factor angles for the respective loads are

$$\theta_{\text{ind mot}} = \cos^{-1}0.891 = 27.0°$$
$$\theta_{\text{heater}} = \cos^{-1}1.0 = 0.0°$$
$$\theta_{\text{syn mot}} = -34.06°$$

From the power diagram in Figure 8.15(b),

$$\tan 27.0° = \frac{Q_{\text{ind mot}}}{233{,}611.7} \quad \Rightarrow \quad Q_{\text{ind mot}} = 119{,}035.3 \text{ var}$$

$$\tan(-34.06°) = \frac{Q_{\text{syn mot}}}{116{,}562.5} \quad \Rightarrow \quad Q_{\text{syn mot}} = -78{,}800.1 \text{ var}$$

$$Q_{\text{sys}} = 119{,}035.3 - 78{,}800.1 = 40{,}235.2 \text{ var}$$
$$\mathbf{S}_{\text{sys}} = P + jQ = 400{,}174.19 + j40{,}235.2 = 402{,}191\underline{/5.74°} \text{ VA}$$
$$F_{P,\text{sys}} = \cos(5.74°) = \underline{0.995 \text{ lagging}}$$

(d) To obtain unity power factor, the synchronous motor must supply an additional −40,235.2 var. Thus,

$$\mathbf{S}_{\text{syn mot}} = 116{,}562.5 - j(78{,}800.1 + 40{,}235.2)$$
$$\mathbf{S}_{\text{syn mot}} = 166{,}602\underline{/-45.6°} \text{ VA}$$

For one phase,

$$\mathbf{S}_{\text{syn mot}} = \frac{166{,}602}{3}\underline{/-45.6°} = 55{,}534.0\underline{/-45.6°} \text{ VA}$$

$$\mathbf{S}_{\text{syn mot}} = \mathbf{V}_T\mathbf{I}_a^* \quad \Rightarrow \quad 55{,}534.0\underline{/-45.6°} = (265.58\underline{/0°})\mathbf{I}_a^*$$
$$\mathbf{I}_a^* = 209.10\underline{/-45.60°}$$
$$\mathbf{I}_a = 209.10\underline{/45.60°}$$
$$\mathbf{E}_f = \mathbf{V}_T - \mathbf{I}_a jX_s = 265.581\underline{/0°} - (209.10\underline{/45.60°})(0.667\underline{/90°})$$
$$\mathbf{E}_f = 265.581 + 99.65 - j97.58 = \underline{378.04/-14.96° \text{ V}}$$

Neglecting magnetic saturation, $E_f \propto \Phi_f \propto I_f$

$$\Delta E_f = \frac{378.04 - 345.614}{345.614} \times 100 = \underline{9.38\%}$$

(e) $$\delta = \underline{-14.96°}$$

8.13 SALIENT-POLE MOTOR

The rotor poles and associated interpolar spaces in the magnetic circuit of a salient-pole motor (such as that previously shown in Figure 8.7) present a cyclic variation of reluctance to the armature-reaction flux sweeping the stator conductors: low reluc-

tance along each pole axis and high reluctance along each interpolar space. The net effect on the armature winding is the same as would be produced by two component waves of armature-reaction flux, 90° apart, sweeping the stator conductors. The component of armature-reaction flux acting along the axis of each field pole is called the *direct-axis* component, and the component of armature-reaction flux acting in the interpolar space between alternate north and south poles is called the *quadrature-axis* component. These component fluxes generate voltage drops in the armature that may be expressed in terms of armature current and fictitious armature-reaction reactances. The associated power equation, expressed in terms of these reactances, includes a magnet component and a reluctance component[8]:

$$P_{\text{salient},1\phi} = \underbrace{\frac{-V_T E_f}{X_d} \sin \delta}_{\text{magnet power}} - \underbrace{V_T^2 \left[\frac{X_d - X_q}{2X_d X_q}\right] \sin 2\delta}_{\text{reluctance power}} \qquad \textbf{(8–24)}$$

where: X_d = direct-axis synchronous reactance (Ω/phase)
X_q = quadrature-axis synchronous reactance (Ω/phase)

Normal Operation

Within its normal operating range (half-rated load to rated load), and normal excitation, the excitation voltage (E_f) in Eq. (8–24) is high enough to cause $P_{\text{mag}} >> P_{\text{rel}}$. Thus, for normal operation,

$$P_{\text{salient},1\phi} \approx \frac{-V_T E_f}{X_d} \sin \delta \qquad \textbf{(8–25)}$$

Note: If the interpolar space between north and south poles is reduced to zero, the rotor becomes a cylindrical rotor, $X_d = X_q = X_s$, the reluctance component of power = 0, and Eq. (8–24) reduces to Eq. (8–25).

Because the reluctance component of power is independent of the excitation voltage, low values of excitation will significantly reduce the magnet component without affecting the reluctance component. Thus, if the motor is unloaded, or very lightly loaded, and the field circuit is opened, the excitation voltage will drop to a very small residual value. This will cause the magnet power in Eq. (8–24) to drop to an insignificant value, and the motor will operate as a *reluctance-synchronous motor*. The reluctance-torque contribution of a salient-pole motor makes it more stable at low excitation and/or sudden overloads than the cylindrical-rotor machine.

Plots of reluctance torque and magnet torque vs. δ are shown superimposed on a strobe view of the rotor in Figure 8.16. Curve *A* represents reluctance torque, curve *B* represents magnet torque, and curve *C* is the algebraic summation of curve *A* and curve *B*. The broken sections indicate regions of unstable operation for the component torques and for the resultant torque.

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[8]See References [3] and [4] for in-depth derivations.

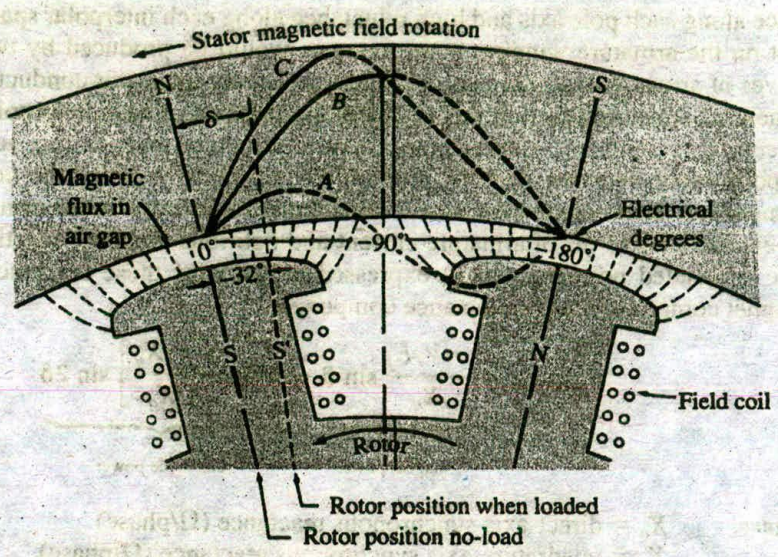


FIGURE 8.16
Plots of reluctance torque, magnet torque, and total torque vs. power angle shown superimposed on a strobe view of the rotor. (Courtesy Dresser Rand, Electric Machinery)

As indicated in Eq. (8–24), and shown in Figure 8.16, the maximum value of reluctance torque occurs when the shaft load is sufficient to cause the rotor to lag its no-load position by 45° ($\delta = -45°$). The magnet torque has its maximum value at $\delta = -90°$. The net torque for a salient-pole motor, due to combined reluctance and magnet contributions, has its maximum value, called the pull-out torque, at $\delta \approx -70°$. Rated torque for the representative motor shown in Figure 8.16 occurs at $\delta \approx -32°$.

EXAMPLE 8.4 A three-phase, 200-hp, 60-Hz, 2300-V, 900 r/min, salient-pole synchronous motor has direct-axis and quadrature-axis reactances equal to 36.66 Ω/phase and 23.33 Ω/phase, respectively. Neglecting losses, determine (a) the developed torque if the field current is adjusted so that the excitation voltage is equal to two times the applied stator voltage, and the power angle is −18°; (b) the developed torque in percent of rated torque, if the load is increased until maximum reluctance torque occurs.

Solution

(a) $$P_{\text{mag.}1\phi} = \frac{-V_T E_f}{X_d} \sin\delta = \frac{-(2300/\sqrt{3})(2 \times 2300/\sqrt{3})}{36.66} \sin(-18°)$$

$$P_{\text{mag.}1\phi} = 29{,}727.2 \text{ W}$$

$$P_{\text{rel},1\phi} = -V_T^2\left[\frac{X_d - X_q}{2X_dX_q}\right]\sin 2\delta = -(2300/\sqrt{3})^2\left[\frac{36.66 - 23.33}{2 \times 36.66 \times 23.33}\right]\sin(-36°)$$

$$P_{\text{rel},1\phi} = 8076.9 \text{ W}$$

$$P_{\text{salient},3\phi} = 3(P_{\text{mag},1\phi} + P_{\text{rel},1\phi}) = 3(29{,}727.2 + 8076.9) = 113{,}412.3 \text{ W}$$

$$P_{\text{salient},3\phi} = \frac{113{,}412.3}{746} = 152.0 \text{ hp}$$

$$P = \frac{Tn}{5252} \quad \Rightarrow \quad T = \frac{5252 \cdot P}{n}$$

$$T = \frac{5252 \times 152.0}{900} = \underline{887 \text{ lb-ft}}$$

(b) The reluctance torque has its maximum value of $\delta = -45°$.

$$P_{\text{mag},1\phi} = \frac{-V_T \cdot E_f \sin\delta}{X_d}$$

$$P_{\text{mag},1\phi} = \frac{-(2300/\sqrt{3})(2 \times 2300/\sqrt{3})}{36.66}\sin(-45°) = 68{,}023.1 \text{ W}$$

$$P_{\text{rel},1\phi} = -V_T^2 \cdot \left[\frac{X_d - X_q}{2X_dX_q}\right]\sin 2\delta$$

$$P_{\text{rel},1\phi} = -(2300/\sqrt{3})^2\left[\frac{36.66 - 23.33}{2 \times 36.66 \times 23.33}\right]\sin(-90°) = 13{,}741.3 \text{ W}$$

$$P_{\text{salient},3\phi} = 3(68{,}023.1 + 13{,}741.3) = 245{,}293.2 \text{ W}$$

$$P_{\text{salient},3\phi} = \frac{245{,}293.2}{746} = 328.8 \text{ hp}$$

Because the speed of a synchronous motor is constant,

$$\text{Percent rated torque} = \frac{328.8}{200} \times 100 = \underline{164\%}$$

This is an excessive overload, and if the machine is operated at this load for more than several minutes, rapid overheating of armature and field windings will occur.

8.14 PULL-IN TORQUE AND MOMENT OF INERTIA

The pull-in torque is the torque required to pull the rotor into synchronism with the rotating field of the stator. The amortisseur winding accelerates the rotor to its maximum speed as an induction motor, at which time application of current to the rotor magnets develops additional torque to pull the rotor into synchronism. If the load inertia is too high, however, the motor may not develop enough torque to synchronize or may take excessive time. In either event, the resultant pole slipping will cause high armature

TABLE 8.1
Synchronous-motor torques in percent of rated torque[a] (minimum values)

Speed (rpm)	hp	Power Factor	Torques		
			Locked Rotor	Pull-in (Based on Normal Wk^2 of Load)[b,c]	Pull-out[c]
500 to 1800	200 and below	1.0	100	100	150
	150 and below	0.8	100	100	175
	250 to 1000	1.0	60	60	150
	200 to 1000	0.8	60	60	175
	1250 and larger	1.0	40	60	150
		0.8	40	60	175
450 and below	All ratings	1.0	40	30	150
		0.8	40	30	200

[a] Reproduced by permission of the National Electrical Manufacturers Association from *NEMA Standards Publication MG 1-1998, Motors and Generators*, Copyright 1999 by NEMA, Washington, DC.
[b] Values of normal Wk^2 of load are given in *Motors and Generators* 1-21.12.
[c] With rated excitation current applied.

current pulses and violent torque pulses that may damage the winding. To ensure quick synchronization when the field current is applied, the moment of inertia (Wk^2) of the connected load must not exceed the normal values recommended for the specific horsepower and speed rating of the motor.

Table 8.1 lists the minimum values of locked-rotor torque, pull-in torque, and pull-out torque for NEMA-standard *salient-pole* synchronous motors for unity and 0.8 *leading* power factors.[9] The motors must be capable of delivering the minimum pull-out torque for at least one minute. The normal values of Wk^2, on which the pull-in torque values in Table 8.1 are based, are given in extensive NEMA tables **[5]**.

8.15 SPEED CONTROL OF SYNCHRONOUS MOTORS

The speed of a synchronous motor is directly proportional to the stator frequency and inversely proportional to the number of poles:

$$n_r = n_s = \frac{120f}{P}$$

Because of the nature of its construction, however, pole changing of the rotor is not practical. Hence, the only way to change the speed of a synchronous motor is to change the frequency of the applied voltage. Furthermore, as discussed in Section 5.8

[9] Corresponding values of torque for cylindrical-rotor synchronous motors are subject to individual negotiation between manufacturer and user.

in Chapter 5, to avoid overheating of the stator winding, the ratio of volts/hertz must be kept constant. This may be accomplished by means of adjustable frequency generators with volts/hertz regulators, or by using solid-state devices exclusively. With solid-state control, the fixed frequency AC supply is converted to adjustable voltage DC, and then inverted to adjustable frequency AC, or by using a cycloconverter (see Section 13.16).

8.16 DYNAMIC BRAKING

Synchronous motors may be decelerated rapidly by converting the energy stored in the moving parts to electrical energy and dissipating it as heat in resistors. To do this, the motor armature is disconnected from the three-phase supply lines and connected across a three-phase resistor bank. The field is maintained at full strength by the exciter. The motor acts as a generator and feeds current to the resistor bank, dissipating energy at a rate equal to $3 \times I^2R$. Resistance R is the resistance of each armature phase and series-connected external resistor.

SUMMARY OF EQUATIONS FOR PROBLEM SOLVING

$$n_r = n_s = \frac{120 \cdot f}{P} \tag{4–1}$$

$$E_f = n_s \Phi_f k_f \tag{8–2}$$

$$\mathbf{V}_T = \mathbf{I}_a\mathbf{R}_a + \mathbf{I}_a jX_\ell + \mathbf{I}_a jX_{ar} + \mathbf{E}_f \tag{8–6}$$

$$\mathbf{V}_T = \mathbf{E}_f + \mathbf{I}_a jX_s \tag{8–10}$$

$$P_{in,1\phi} = V_T I_a \cos\theta_i \tag{8–13}$$

$$P_{in,1\phi} = \frac{-V_T E_f}{X_s} \sin\delta \tag{8–14}$$

$$I_a = \frac{1}{X_s} \cdot \left[E_f^2 + V_T^2 - 2\sqrt{E_f^2 V_T^2 - X_s^2 P_{in,1\phi}^2}\right]^{0.5} \tag{8–21}$$

$$P_{salient,1\phi} = \underbrace{\frac{-V_T E_f}{X_d} \sin\delta}_{\text{magnet power}} - \underbrace{V_T^2\left[\frac{X_d - X_q}{2X_dX_q}\right] \sin 2\delta}_{\text{reluctance power}} \tag{8–24}$$

$$P = \frac{Tn}{5252}$$

SPECIFIC REFERENCES KEYED TO TEXT

1. Bewley, L. V. *Alternating Current Machinery.* Macmillan, New York, 1949.
2. Institute of Electrical and Electronics Engineers. *Test Procedures for Synchronous Machines.* IEEE STD 115-1983, IEEE, New York, 1995.
3. Mablekos, V. E. *Electric Machine Theory for Power Engineers.* Harper & Row, New York, 1980.
4. McPherson, G. *An Introduction to Electrical Machines and Transformers.* Wiley, New York, 1981.
5. National Electrical Manufacturers Association. *Motors and Generators.* Publication No. MG 1-1998, NEMA, Rosslyn, VA, 1999.

GENERAL REFERENCE

Smeaton, R. W. *Motor Application and Maintenance Handbook,* 2nd ed. McGraw-Hill, New York, 1987

REVIEW QUESTIONS

1. Discuss two designs of salient-pole rotors that provide starting and accelerating torque.
2. How does an amortisseur winding reduce hunting caused by pulsating loads?
3. State how a synchronous motor is started, stopped, and reversed.
4. How can the speed of a synchronous motor be adjusted?
5. What are the two components of synchronous-motor torque? What are they due to?
6. Explain why it is possible for the cemf of a synchronous motor to be greater than the applied voltage.
7. What is meant by the *torque angle* of a synchronous motor? What factors affect the magnitude of this angle?
8. What effect does increasing the excitation of a fully loaded synchronous motor, from its stability limit to its maximum excitation, have on its armature current? Explain.
9. What effect does increasing the excitation of a fully loaded synchronous motor, from its stability limit to its maximum excitation, have on its pull-out torque? Explain.
10. What effect does increasing the excitation of a fully loaded synchronous motor, from its stability limit to its maximum excitation, have on its power factor? Explain.
11. Will increasing the shaft load cause the speed of a synchronous motor to operate at a new lower steady-state speed? Explain.
12. Using a phasor diagram, explain how a synchronous motor adjusts its input power to accommodate an increase in shaft load.

13. Differentiate between pull-in torque, pull-out torque, and locked-rotor torque.
14. What effect does load inertia have on pull-in torque, pull-out torque, and locked-rotor torque?
15. Explain the dynamic braking process for synchronous motors.

PROBLEMS

8–1/3 Determine the speed of a 40-pole synchronous motor operating from a three-phase, 50-Hz, 4600-V system.

8–2/3 Determine the frequency required to operate a 16-pole, 480-V synchronous motor at 225 r/min.

8–3/3 A three-phase 50-hp, 2300-V, 60-Hz synchronous motor is operating at 90 r/min. Determine the number of poles in the rotor.

8–4/7 A two-pole, three-phase, 1000-hp, 2300-V, wye-connected synchronous motor operating at rated power, rated voltage, and rated frequency has a power factor of 0.80 leading, an efficiency of 96.5 percent, and a power angle of $-23°$. The synchronous reactance is 4.65 Ω/phase. Determine (a) line current; (b) cemf/phase.

8–5/7 A 4000-hp, 13,200 V, 60-Hz, two-pole, three-phase, wye-connected cylindrical-rotor synchronous motor, operating at rated load and 0.84 power-factor leading, has an efficiency (excluding field losses and armature resistance losses) of 96.5 percent. The synchronous reactance per phase is 49.33 Ω. Sketch the phasor diagram and determine (a) rated torque; (b) armature current; (c) excitation voltage; (d) power angle; (e) pull-out torque.

8–6/7 A 2500-hp, 6600-V, 60-Hz, 3600 r/min, three-phase, wye-connected synchronous motor is operating at a power angle of $-28.5°$ and a cemf of 4500 V/phase. The synchronous reactance is 15.8 Ω/phase. Determine (a) the developed torque; (b) the shaft-power out, assuming a mechanical efficiency of 94.3 percent; (c) power factor; (d) pull-out torque.

8–7/7 A 200-hp, 460-V, four-pole, 60-Hz, three-phase, wye-connected synchronous motor, operating at rated load, has an efficiency of 94 percent and a power factor of 80 percent leading. Assuming the synchronous reactance is 1.16 Ω/phase, determine (a) the excitation voltage and power angle; (b) the pull-out torque; (c) the pull-out torque if the excitation voltage is cut in half.

8–8/8 The phasor diagram for a synchronous motor operating at 0.5 rated load is shown in Figure 8.17. Using the same applied voltage phasor, construct a new phasor diagram that shows the new conditions, assuming the shaft load is increased to its rated value. Show all construction lines.

8–9/9 Given the phasor diagram for the synchronous motor shown in Figure 8.18, neglecting magnetic saturation, and showing all construction lines, construct a new phasor diagram for each of the following specified conditions: (a) field current unchanged, shaft load doubled; (b) shaft load unchanged, field current doubled; (c) shaft load doubled, field current doubled.

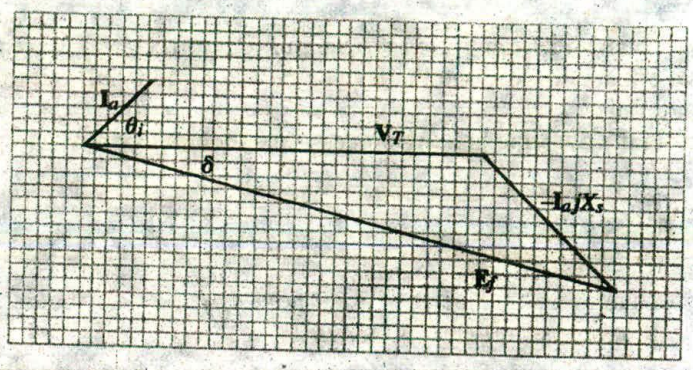


FIGURE 8.17
Phasor diagram for Problem 8–8/8.

8–10/9 Given the phasor diagram for the synchronous motor shown in Figure 8.18, neglecting magnetic saturation, and showing all construction lines, construct a new phasor diagram for each of the following specified conditions: (a) field current doubled, shaft load unchanged; (b) shaft load doubled, field current cut in half; (c) shaft load cut in half, field current halved.

8–11/10* (a) A 15,000-hp, 13,200-V, 60-Hz, 1800 r/min, wye-connected, cylindrical-rotor synchronous motor has a synchronous reactance of 14.95 Ω/phase. Plot a family of V curves corresponding to rated load, 75 percent rated load, and 50 percent rated load. (b) Sketch the stability limit and unity power-factor locus. (c) Approximate from the curves the minimum excitation voltage that will just maintain synchronism for each load condition.

8–12/10 A 250-hp, 575-V, three-phase, 60-Hz, two-pole, wye-connected synchronous motor operating at one-half rated load has the V curve shown in

*Solution by computer is recommended.

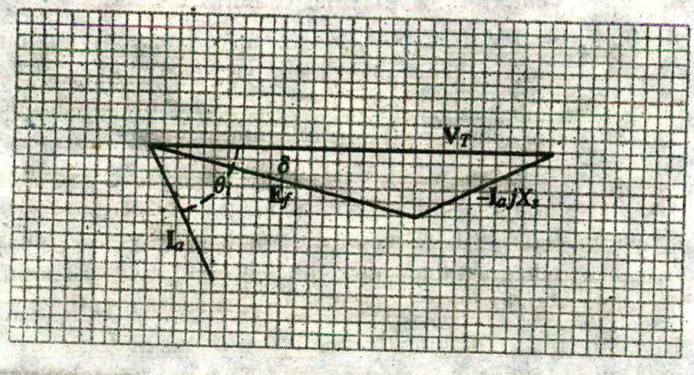


FIGURE 8.18
Phasor diagram for Problems 8–9/9 and 8–10/9.

Figure 8.19. (a) Approximate (from the curve) the phase values of excitation voltage and armature current for operation at unity power factor. (b) Approximate the stability limit. (c) Using the nameplate voltage and data obtained in (a), make a scale drawing of the corresponding phasor diagram. (d) Using the phasor diagram, determine the power angle and the synchronous reactance drop. (e) Determine the synchronous reactance.

8–13/10 The V curve shown in Figure 8.20 is for a 460-V, 350-hp, 60-Hz, 1800 r/min, cylindrical-rotor synchronous motor. The machine is operating at less than rated load, with an excitation voltage of 500 V/phase. The synchronous reactance is 0.65 Ω/phase. Sketch the corresponding phasor diagram to scale, and determine: (a) power angle; (b) active and reactive components of input power; (c) power factor; (d) developed torque.

8–14/12 A three-phase, 60-Hz, 460-V system supplies an induction motor that draws 50 kVA at 0.84 power factor, a 30-kW incandescent lighting load, and a 125-hp, six-pole, wye-connected synchronous motor, with a synchronous impedance of 1.45 Ω/phase, that draws 80 kW. Determine (a) the synchronous-motor power factor required to operate the system at unity power factor; (b) the excitation voltage required for the conditions in (a); (c) the synchronous-motor power angle for the conditions in (a).

8–15/12 Repeat Problem 8–14/12, assuming the system is to be operated at 0.80 leading power factor.

8–16/12 A 460-V, three-phase, 60-Hz system supplies power to a 10-kVA, delta-connected resistor load; a 50-kVA, 0.86 power factor induction motor, and

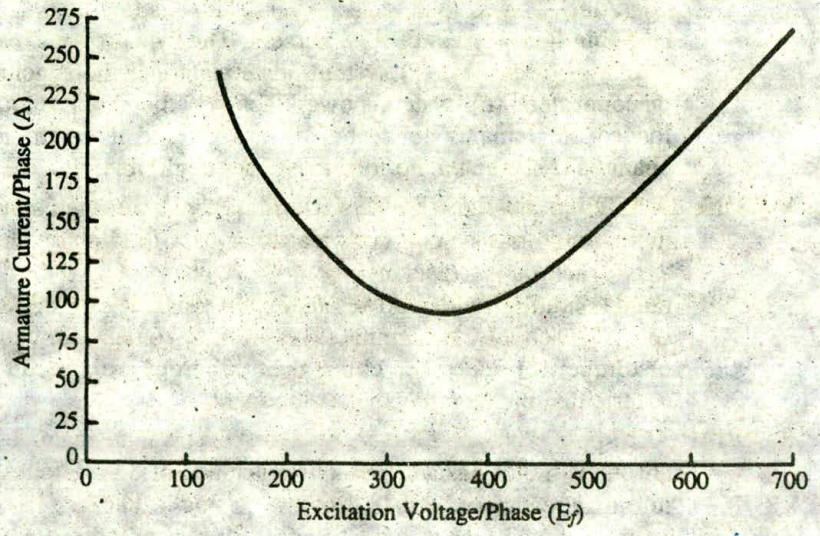


FIGURE 8.19
V curve for Problem 8–12/10.

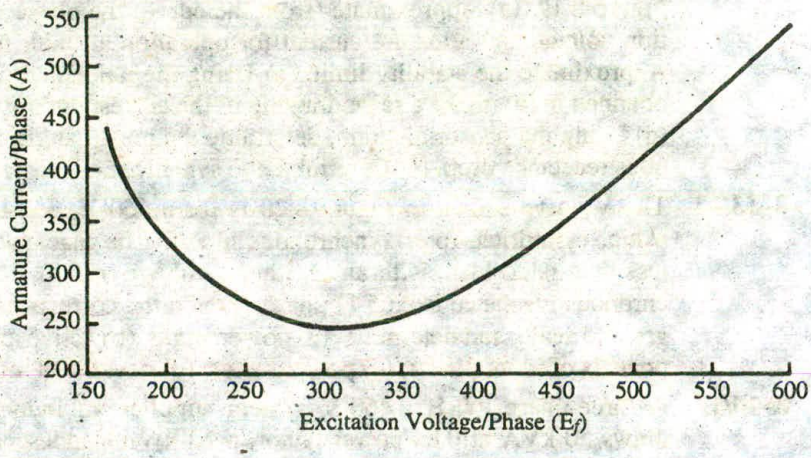


FIGURE 8.20.
V curve for Problem 8–13/10.

a wye-connected synchronous motor that is operating at 0.62 power factor leading, and has a synchronous impedance of 6.05 Ω/phase. If the system is operating at unity power factor, determine (a) active and reactive components of synchronous motor power; (b) synchronous motor current; (c) excitation line voltage.

8–17/12 A three-phase, 60-Hz, 575-V system, operating at 1.5 MW and 0.92 power factor lagging, includes a 500-hp, 575-V, 1200 r/min synchronous motor operating at rated load, 0.84 power-factor leading, and an efficiency (discounting field losses) of 96.2 percent. The synchronous reactance is 0.567 Ω/phase. Determine (a) system power factor if the synchronous motor is disconnected; (b) system power factor if the synchronous motor is unloaded but remains connected to the bus and its excitation remains unchanged. Assume the no-load losses are negligible.

8–18/13 A 3500-hp, 60-Hz, 4000-V, 450 r/min, wye-connected synchronous motor has a direct-axis synchronous reactance of 5.76 Ω/phase, and a quadrature-axis synchronous reactance of 4.80 Ω/phase. The motor is operating at rated conditions with a power angle and efficiency of −34.6° and 97.1 percent, respectively. Determine (a) excitation voltage; (b) magnet component of torque; (c) reluctance component of torque; (d) percent of total power contributed by the reluctance component.

8–19/14 A 20-pole, 700-hp, 2300-V, 60-Hz, 0.8 power-factor wye-connected synchronous motor has a direct-axis reactance of 8.91 Ω/phase and a quadrature-axis reactance of 6.48 Ω/phase. Using Table 8.1, determine (a) minimum expected locked-rotor torque; (b) minimum expected pull-in torque, assuming normal Wk^2; (c) minimum expected pull-out torque, assuming rated field current is applied.

9

Synchronous Generators (Alternators)

9.1 INTRODUCTION

Synchronous generators, also called *alternators* or *AC generators*, are the principal sources of electrical power throughout the world, and range in size from a fraction of a kVA to 1500 MVA.

The bulk of electric power is generated with high-speed steam turbines driving cylindrical rotor machines, and low-speed hydraulic turbines driving salient-pole machines. Steam for steam turbines is obtained from fossil fuels, nuclear fuel, and in some cases from geothermal energy. Water for hydraulic turbines is obtained from dammed rivers or pumped storage (water pumped into high-elevation reservoirs for later use).

Diesel-driven generators and gas-turbine-driven generators are used for large and small isolated loads, for utility standby power to handle peak demand, and for applications in remote pumping stations, ships, drilling rigs, and the like.

The construction of synchronous generators is essentially the same as for synchronous motors described in Chapter 8. It is interesting to note that synchronous generators driven by hydraulic turbines and connected to large power systems are sometimes operated as synchronous motors during periods of least usage; through a switching arrangement, the synchronous generator is driven as a synchronous motor in the reverse direction, pumping water back from a lower reservoir to a higher reservoir. Such storing of water for later use is called *pumped storage*.

This chapter starts with a simple transition from synchronous-motor action (developed in Chapter 8) to synchronous-generator action. This is followed by the general procedure for safe and efficient paralleling (and shutdown) of synchronous generators with other machines or with an "infinite bus." The theory of load transfer between machines, power-factor control, and the avoidance of potential operational problems are stressed. Voltage regulation, determination of parameters, and generator efficiency complete the chapter.

9.2 MOTOR-TO-GENERATOR TRANSITION

Figure 9.1(a) shows an infinite bus connected to a synchronous motor whose shaft is mechanically coupled to a water pump and a steam turbine. The design of the turbine is such that when steam hits the turbine blades, the torque exerted by the turbine will be in the same direction as the synchronous-motor torque. The equivalent circuit for one phase is shown in Figure 9.1(b).

With the turbine valve closed, so that no steam enters the turbine, the synchronous motor will drive the pump and the turbine at synchronous speed.[1] The phasor diagram for synchronous-motor action is shown in Figure 9.2(a) for a cylindrical-rotor machine. The corresponding phasor equation is

$$\mathbf{V}_T = \mathbf{E}_f + \mathbf{I}_a jX_s \qquad \textbf{(9–1)}$$

where: $\mathbf{V}_T$ = terminal voltage/phase (V)
$\mathbf{E}_f$ = excitation voltage/phase (V)
$\mathbf{I}_a$ = armature current/phase (A)
X_s = synchronous reactance/phase (Ω)

[1] A vacuum is maintained on the turbine to prevent windage from overheating the turbine blades.

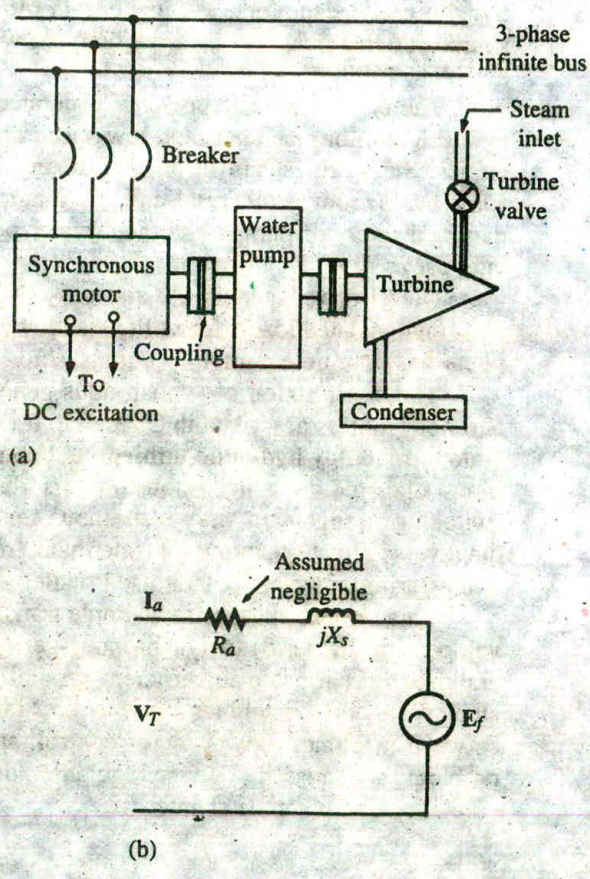


FIGURE 9.1
(a) Synchronous motor mechanically coupled to a water pump and a steam turbine; (b) equivalent circuit for one phase of the synchronous-motor armature.

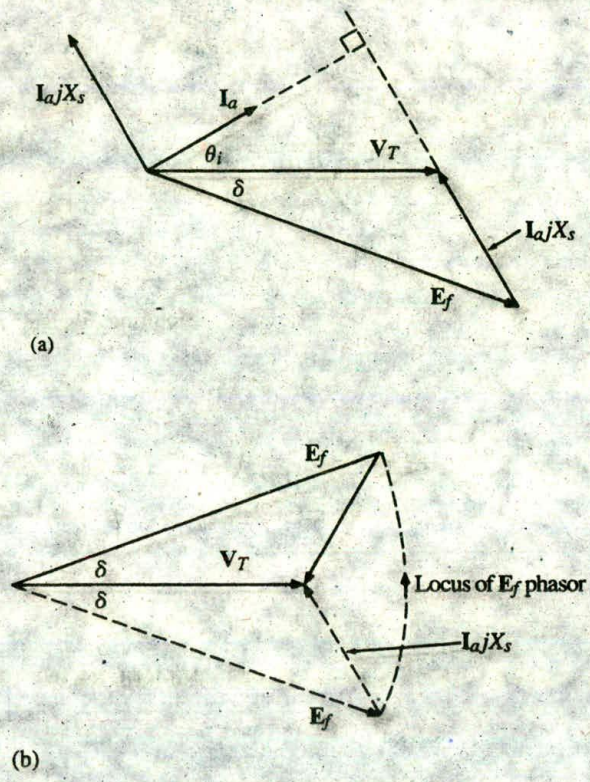


FIGURE 9.2
(a) Phasor diagram for one phase of a synchronous machine acting as a motor; (b) phasor diagram showing transition from motor action to generator action.

Figure 9.2(b) illustrates how admitting steam to the turbine affects the power angle of a synchronous machine. Admitting a small amount of steam to the turbine by partially opening the turbine valve causes the turbine to take part of the pump load, resulting in a decrease in the synchronous-motor power angle.

If the flow of steam is increased to a value that causes the turbine to carry the entire pump load, the synchronous motor will operate at no load and its power angle will be zero. Further increases in steam will cause the power angle to increase in the positive direction. Since the field current was not changed, the locus of the excitation voltage describes a circular arc. Except for transient changes in speed as the power angle in Figure 9.2(b) changes, the machine continues to run at synchronous speed.

The synchronous machine absorbs power and thus provides motor action when δ is negative, operates at no load when δ is zero, and (as will be shown) provides generator action, delivering power to the three-phase bus when δ is positive.

Strobe views of a two-pole rotor, and the corresponding phasor diagrams for motoring, for no load, and for generating are shown in Figure 9.3. When the synchronous machine is operating as a motor (δ negative), as shown in Figure 9.3(a), it is driven by the bus voltage, and the excitation voltage is a cemf (voltage drop). *However, when operating as a generator (δ positive),* as shown in Figure 9.3(c), *the excitation voltage becomes the source voltage, and the active component of current* ($I_a \cos \theta_i$) *is reversed.*

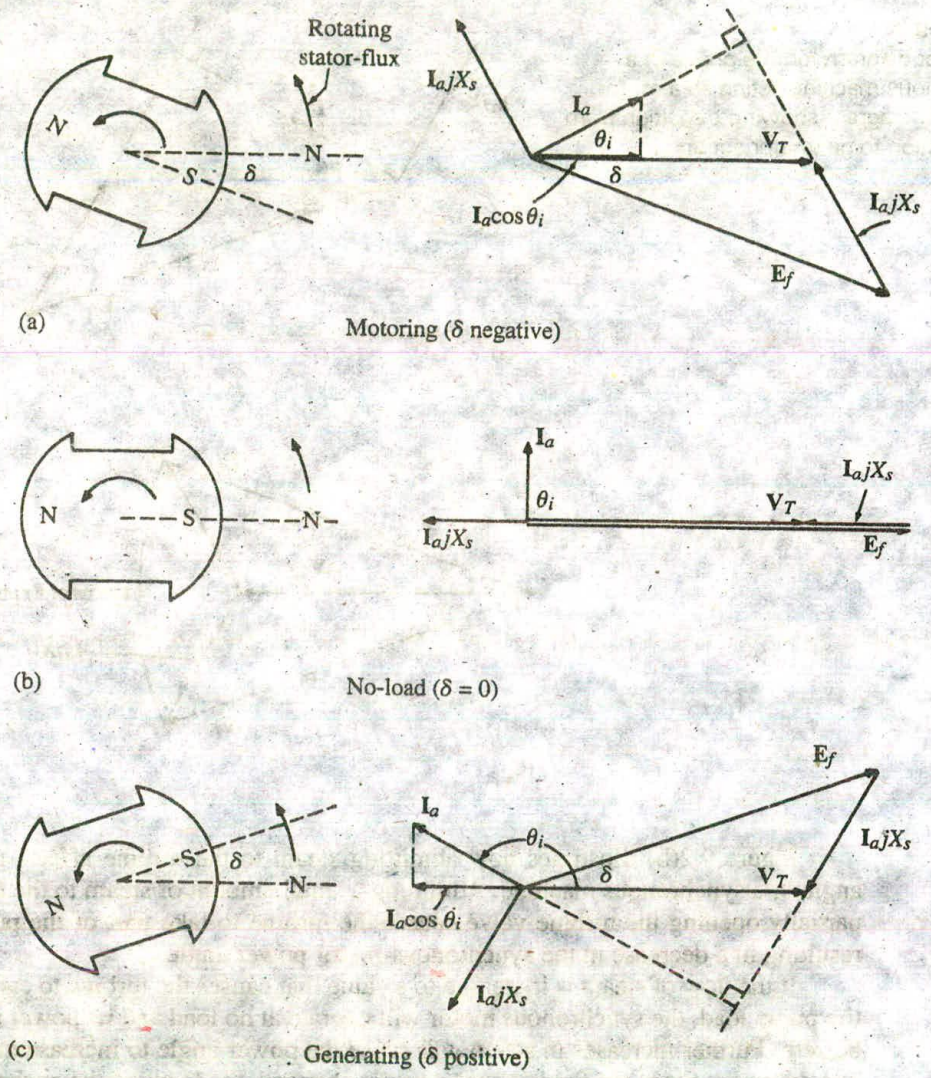


FIGURE 9.3
Strobe views of a two-pole rotor and the corresponding phasor diagrams for (a) motoring, (b) no load, and (c) generating.

Although the phasor diagram shown in Figure 9.3(c) is correct for a synchronous generator, it is not used in that form when analyzing generator behavior because one would have to contend with a negative power factor.[2] The reason for this awkward situation is that the changes in the phasor diagrams, as the machine went from motor ac-

[2]The power factor would have a negative value because $\theta_v = 0, \theta_i > 90°$: $\theta = (\theta_v - \theta_i)$ and $F_P = \cos\theta$.

tion in Figure 9.3(a) to generator action in Figure 9.3(c), were based on an assumed direction of *electric power in* for both motor and generator action.

Phasor Diagram of a Synchronous Generator

When thinking in terms of generator operation and *power out,* as shown in Figure 9.4(a), the excitation voltage $\mathbf{E}_f$ becomes the source voltage and the bus voltage $\mathbf{V}_T$ becomes a voltage drop. Thus, in terms of Kirchhoff's voltage law,

$$\mathbf{E}_f = \mathbf{V}_T + \mathbf{I}_a jX_s$$

Rearranging terms,

$$\mathbf{V}_T = \mathbf{E}_f - \mathbf{I}_a jX_s \qquad \textbf{(9–1a)}$$

where: $\mathbf{V}_T$ = terminal voltage/phase (V)
$\mathbf{E}_f$ = excitation voltage/phase (V)
$\mathbf{I}_a$ = armature current/phase (A)
X_s = synchronous reactance/phase (Ω)

As previously developed in Section 8.5 of Chapter 8, the excitation voltage is related to the pole flux and field current in the following manner:

$$E_f = n_s \Phi_f k_f \qquad \textbf{(9–2)}$$

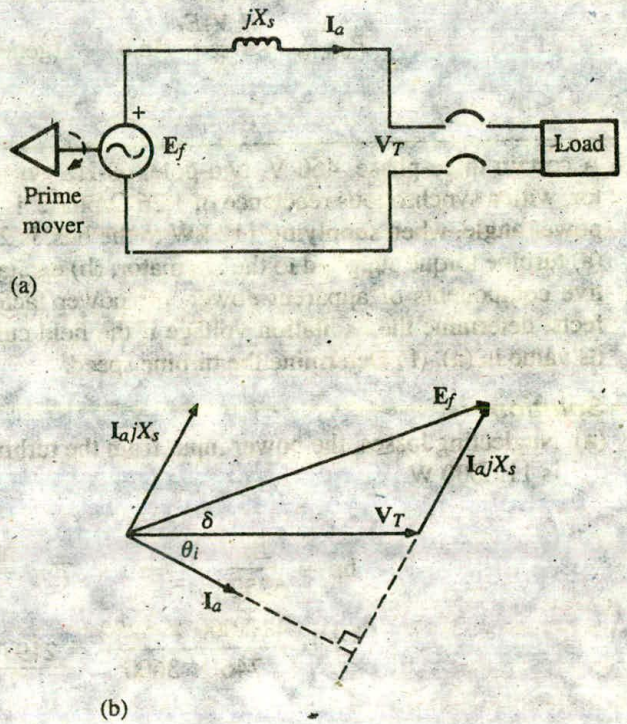


FIGURE 9.4
(a) Equivalent circuit; (b) preferred phasor diagram for generator action.

$$E_f = n_s \cdot \frac{N_f I_f}{\mathcal{R}} \cdot k_f \qquad (9\text{–}3)$$

where:
n_s = synchronous speed (r/min)
Φ_f = flux/magnet pole (Wb)
k_f = constant
N_f = turns of wire/pole
I_f = DC field current (A)
$\mathcal{R}$ = reluctance of magnetic circuit (A-t/Wb)

Note that the difference between Eq. (8–10) for motor action and Eq. (9–1) for generator action is that the armature current is reversed. The phasor diagram corresponding to Eq. (9–1) for generator action is shown in Figure 9.4(b).

9.3 SYNCHRONOUS-GENERATOR POWER EQUATION

The equations developed in Section 8.7 for electric power input to a synchronous machine acting as a motor apply equally well to the synchronous machine acting as a generator. The only difference is the *direction* of electric power: *in* for a motor and *out* for a generator. Thus,

$$P_{\text{in},1\phi} = \frac{-V_T E_f}{X_s} \sin\delta \qquad \text{(motor action)} \qquad (8\text{–}14)$$

$$P_{\text{out},1\phi} = \frac{+V_T E_f}{X_s} \sin\delta \qquad \text{(generator action)} \qquad (9\text{–}4)$$

EXAMPLE 9.1 A certain three-phase, 460-V, two-pole, 60-Hz, wye-connected synchronous alternator, with a synchronous reactance of 1.26 Ω/phase, is connected to an infinite bus. The power angle, when supplying 112 kW to the bus, is 25°. Neglecting losses, determine (a) turbine torque supplied to the alternator; (b) excitation voltage; (c) active and reactive components of apparent power; (d) power factor. (e) Neglecting saturation effects, determine the excitation voltage if the field current is reduced to 85 percent of its value in (a). (f) Determine the turbine speed.

Solution

(a) Neglecting losses, the power input from the turbine to the synchronous machine is 112,000 W.

$$\text{hp} = \frac{Tn}{5252} \quad \Rightarrow \quad T = \frac{\text{hp} \times 5252}{n}$$

$$T = \frac{112{,}000 \times 5252}{746 \times 3600} = \underline{219.0 \text{ lb-ft}}$$

(b)
$$V_T = \frac{460}{\sqrt{3}} = 265.581 \text{ V/phase}$$

$$P_{\text{out}} = 3 \times \frac{V_T E_f}{X_s} \sin\delta \quad \Rightarrow \quad E_f = \frac{P_{\text{out}} \cdot X_s}{3 \cdot V_T \cdot \sin\delta}$$

$$E_f = \frac{112{,}000 \times 1.26}{3 \times 265.581 \times \sin(25°)} = \underline{419.1 \text{ V/phase}}$$

(c) Using Eq. (9–1),

$$\mathbf{V}_T = \mathbf{E}_f - \mathbf{I}_a jX_s \quad \Rightarrow \quad 265.581\angle 0° = 419.1\angle 25° - \mathbf{I}_a \times 1.26\angle 90°$$

$$114.253 + j177.119 = \mathbf{I}_a \times 1.26\angle 90°$$

$$\mathbf{I}_a = 167.279\angle -32.824° \text{ A}$$

$$\mathbf{S} = 3 \times \mathbf{V}_T \mathbf{I}_a^* = 3 \times 265.581\angle 0° \times 167.279\angle 32.824°$$

$$\mathbf{S} = 133{,}278.3\angle 32.824° = 111{,}990 + j72{,}245 \text{ VA}$$

$$P = 112 \text{ kW}$$

$$Q = 72.2 \text{ kvar lagging}$$

(d)
$$F_P = \cos(32.824°) = \underline{0.84 \text{ lagging}}$$

(e) From Eq. (9–3), and neglecting saturation effects, $E_f \propto I_f$. Thus,

$$E_f = 0.85 \times 419.1 = \underline{356.2 \text{ V/phase}}$$

(f)
$$n_s = \frac{120f}{P} = \frac{120 \times 60}{2} = \underline{3600 \text{ r/min}}$$

Salient-Pole Generator

The equation for *electric power in,* previously discussed for salient-pole motors in Section 8.13, also applies to salient-pole generators. Thus, in terms of *electric power out,*

$$P_{\text{out},1\phi} = \frac{V_T E_f}{X_d} \sin\delta + V_T^2 \left[\frac{X_d - X_q}{2X_d X_q}\right] \sin 2\delta \qquad \textbf{(9–5)}$$

A plot of Eq. (9–5) for electric *power out* is shown in Figure 9.5 for a representative salient-pole synchronous machine connected to an infinite bus. Reading from left to right, the plot shows a smooth transition from motor action to generator action as load torque on the motor shaft is removed and turbine torque is applied. Although the curve is plotted from −180° to +180°, loading beyond the maximum values will cause the machine to pull out of synchronism, whether it is operating as a motor or as a generator. Maximum power is called *pull-out power.*

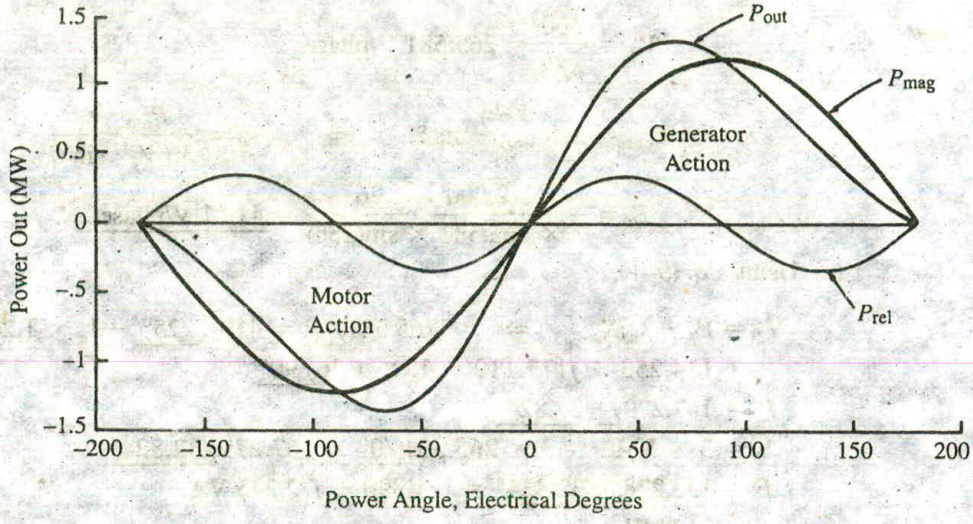


FIGURE 9.5
Plot of *electric power out* for a representative synchronous machine, showing a smooth transition from motor action to generator action.

9.4 GENERATOR LOADING AND COUNTERTORQUE

When load is connected to a generator, the interaction of the armature current with the pole flux develops a torque in opposition to the driving torque of the prime mover.[3] This opposing torque, called *countertorque,* causes the machine to slow down. As the machine decelerates, its speed-governor responds to admit more energy to the prime mover. When the energy input to the prime mover is equal to the load on the generator plus losses, the system is in equilibrium and the machine operates at the new lower speed. If the prime mover has a constant-speed governor (isochronous), however, the machine will initially drop in speed, and as sufficient energy is admitted to the prime mover, the speed will return to its initial no-load value.

9.5 LOAD, POWER FACTOR, AND THE PRIME MOVER

The voltage at the terminals of a generator is affected considerably by the power factor of the load. Hence, the rated voltage of an alternator, as indicated on the nameplate, is always given for rated kVA at a specific power factor and field current.

The multitude of different loads in distribution systems—resistive, inductive, and capacitive—generally results in an average system power factor of approximately 80 percent lagging. Hence, the terminal voltage for rated kVA at 80 percent power

[3]See Section 1.11, Chapter 1.

factor is generally indicated on the nameplate of most general-service alternators. If an alternator is designed to supply only a unity power-factor load, however, the generator voltage indicated on the nameplate will be the value for rated kVA at unity power factor.

The energy supplied to an alternator by its prime mover must be adequate to carry the load that the generator is called on to deliver. Although the generator can deliver the same kVA at different power factors, the prime mover supplies only active power. Hence, if the kVA demand from the generator is always at 80 percent power factor or less, the maximum active power that the prime mover will be required to supply will be equal to the rated kVA of the machine multiplied by a power factor of 0.80 or less. Under such circumstances, it is not necessary to install a prime mover large enough to supply rated kVA at unity power factor.

9.6 PARALLELING SYNCHRONOUS GENERATORS

Two or more generators operating in parallel provide economy of operation and flexibility for scheduling routine maintenance. As increases in oncoming load approaches the rated load of the machine or machines already on the bus, additional machines may be paralleled to share the load. Similarly, as the bus load decreases, one or more machines are taken off the line to allow the remaining units to operate at higher efficiencies. A simplified one-line diagram[4] for a two-generator system designed for parallel operation is shown in Figure 9.6. Generator *A* is connected to the bus and is supplying

[4]Each line from generator to breaker to load represents the three line leads of a three-phase generator. Three slash marks indicate three conductors/line (IEEE STD 315-1975).

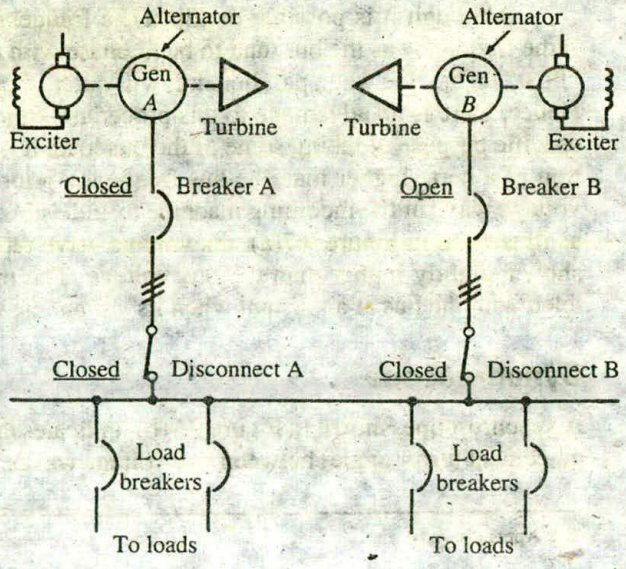


FIGURE 9.6
Simplified one-line diagram for a two-generator system.

the entire load. Generator *B* is the incoming machine; its breaker is open. The disconnect switch is closed prior to starting the prime mover.

Machines that are to be paralleled must have the same phase sequence.[5] Any attempt to parallel an alternator with another already on the bus, but with opposite phase sequence, may not only black out the plant, but may also cause serious damage to the machines and associated equipment. Most modern plants have automatic synchronizers that correctly synchronize a generator with the bus. Knowledge of manual synchronization is necessary for nonautomatic plants, however, and for plants with defects in the automatic synchronizing equipment.

Procedure for Paralleling

The voltage of the incoming machine should be adjusted to obtain a value approximately equal to the bus voltage. Although a few volts higher is preferred, a few volts lower will not adversely affect the operation. The frequency of the incoming machine should be adjusted to be a fraction of a hertz higher than the bus frequency. Then assuming that the bus-disconnect switch is closed, the circuit breaker should be closed at the instant the voltage wave of the incoming machine is in phase with the bus voltage. The no-load speed (and hence the no-load frequency) of a synchronous generator is changed by remote control from the generator panel. A control switch on the generator panel actuates a servomotor that raises or lowers the no-load speed setting of the governor.

The frequency of the incoming machine should be a fraction of hertz higher than the bus frequency to ensure its taking load at the instant it is paralleled. If its frequency is slightly lower than the bus frequency, it will be motorized at the instant the circuit breaker is closed. Alternators in parallel are locked in rotational synchronism. Hence, an incoming machine with a frequency lower than the bus frequency will be driven as a motor by the other alternators on the bus.

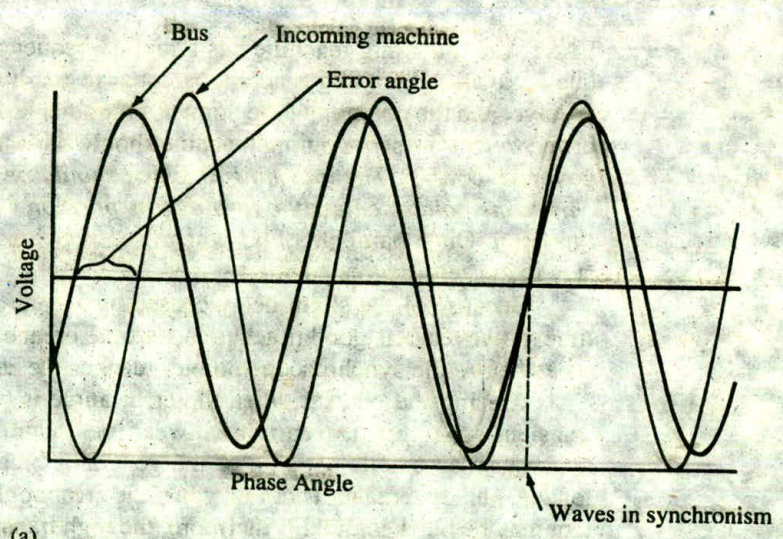
Although it is possible to adjust the frequency of the incoming machine to the same frequency as the bus, and to be in phase with the bus, the incoming machine will "float" on the line when paralleled. It will not be driven as a motor, nor will it take load. Hence, there is no advantage to this procedure. Since an alternator is paralleled for the specific purpose of taking some of the bus load, it is advantageous to adjust it to a fraction of a hertz higher than the bus frequency prior to paralleling. This will cause the voltage wave of the incoming machine to slide slowly past the voltage wave of the bus, as illustrated in Figure 9.7(a); the voltage wave representing the incoming machine is shown slightly higher than the bus voltage. The incoming machine should be paralleled with the bus at an instant when its voltage wave coincides with that of the bus.

Synchroscope

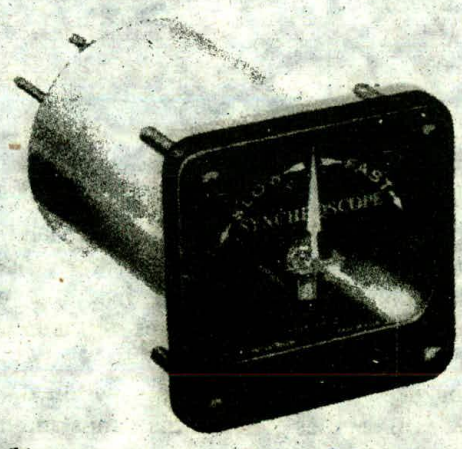
A synchroscope, shown in Figure 9.7(b), indicates the instantaneous angle of phase displacement (error angle) between two voltage waves. If the voltage waves have different

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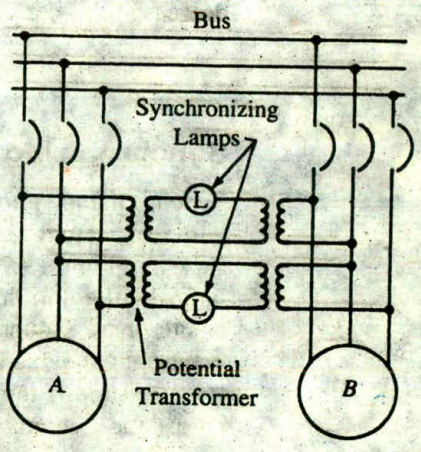
[5] See Appendix A.10.



(a)



(b)



(c)

FIGURE 9.7
(a) Voltage waves of bus and incoming generator, with incoming machine at a slightly higher voltage and a slightly higher frequency; (b) synchroscope (Courtesy GE Industrial Systems); (c) circuit showing synchronizing lamp connections.

frequencies, the angle between the two waves, called the *error angle*, will always be changing, causing the pointer to revolve. When properly connected, a synchroscope indicates the electrical speed of the incoming machine with respect to the bus. If the synchroscope revolves in the direction marked SLOW, the incoming machine has a lower frequency than that of the bus. If the synchroscope revolves in the direction marked

FAST, the incoming machine has a higher frequency than that of the bus. If the frequency of the incoming machine is equal to the bus frequency, the pointer will not revolve, and the position of the pointer will indicate the error angle. If this happens, the prime mover of the incoming machine should have its speed increased by adjusting its governor control. *The incoming machine should be paralleled at the instant the synchroscope pointer enters the zero-degree position while revolving slowly in the FAST direction.* Once paralleled, the synchroscope pointer will no longer revolve, but will stay in the zero-degree position.

Because of the slight delay caused by human reaction time and breaker closing time, however, it is good practice to start the breaker closing operation one or two degrees *before* the synchroscope pointer reaches the zero-degree position.

Closing the breaker when the error angle is greater than 10° will cause a high transient synchronizing current between the incoming generator and the bus, which may cause automatic tripping of the generator breaker. If the bus is already heavily loaded, and the breaker is closed when the error angle approaches 180°, the entire system may be blacked out. Furthermore, the high transient current associated with an extreme out-of-phase attempt at synchronization causes excessive oscillating torques on both the stator and rotor windings that may result in immediate damage, or at best, a decrease in its service life.

Synchronizing Lamps

Synchronizing lamps are used to indicate the phase sequence of the incoming machine with respect to the bus and as a check on the synchroscope.[6] The lamp connections are shown in Figure 9.7(c). If both generator and bus have the same phase sequence, both lamps will go bright and dark in unison. If, however, the incoming machine is of opposite phase sequence with respect to the bus, the lamps will alternate repeatedly—one bright and one dark.

Assuming the phase sequence is correct, both lamps will be dark when the synchroscope is at approximately zero degrees. *Note:* If the lamps and the synchroscope are in conflict, and the lamps go bright and dark in unison, the lamps are correct.

Synchronizing with lamps has the disadvantage of a dark period that may extend over 15° or more of error angle. Hence, when this method is used, the circuit breaker should be closed midway through the dark period. Another disadvantage is that it does not indicate whether the incoming machine is fast or slow.

A functional diagram for a two-generator system supplying a common bus load is shown in Figure 9.8; it illustrates the basic instrumentation required for effective operation. A plant with only one generator, synchronized with an infinite bus, would omit panel 3, but would provide protective relaying on a substitute panel.

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[6]Phase sequence may also be checked with a small handheld phase-sequence indicator or with a small three-phase motor [5].

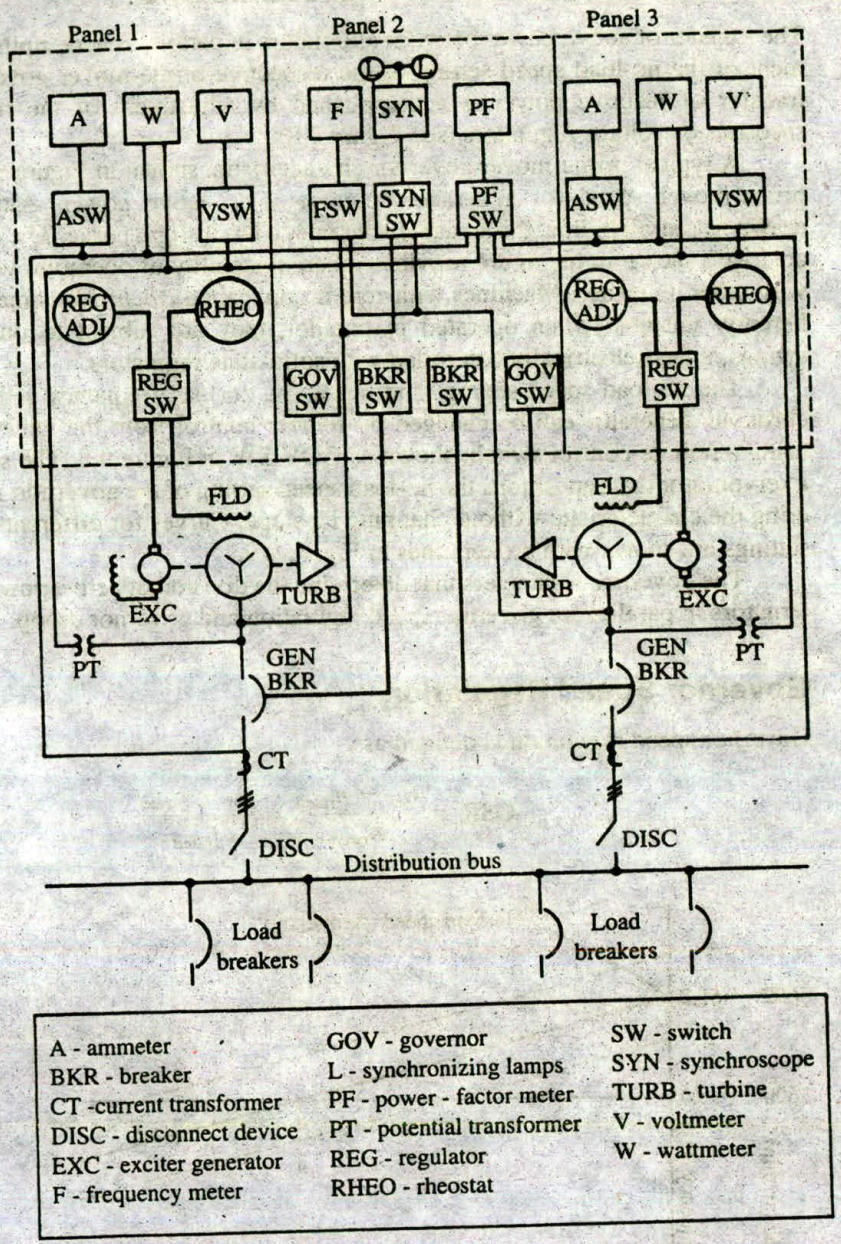


FIGURE 9.8
Functional diagram for a two-generator system supplying a common bus load.

9.7 PRIME-MOVER GOVERNOR CHARACTERISTICS

The transfer of active power between alternators in parallel is accomplished by adjustment of the no-load speed setting of the respective prime-mover governors, and the transfer of reactive power is accomplished by adjustment of the respective field rheostats or voltage regulators (see Figure 9.8).

A typical prime-mover governor characteristic, shown in Figure 9.9, is a plot of prime-mover speed (or generator frequency) vs. active power. Although usually drawn as a straight line, the actual characteristic has a slight curve. The drooping characteristic shown in the figure provides inherent stability of operation when paralleled with other machines. Machines with zero droop, called *isochronous* machines, are inherently unstable when operated in parallel; they are subject to unexpected load swings, unless electrically controlled with solid-state regulators.

The no-load speed setting (and hence the no-load frequency setting) of a synchronous generator can be changed by remote control from the generator panel by using a remote-control switch, shown as GOV SW in Figure 9.8. The switch actuates a servomotor that repositions the no-load speed setting of the governor, raising or lowering the characteristic without changing its slope. Curves for different no-load speed settings are shown with broken lines in Figure 9.9.

The governor parameters that determine the division of active power between alternators in parallel are governor speed regulation and governor droop.

Governor Speed Regulation

Governor speed regulation is defined as

$$\text{GSR} = \frac{n_{\text{nl}} - n_{\text{rated}}}{n_{\text{rated}}} = \frac{f_{\text{nl}} - f_{\text{rated}}}{f_{\text{rated}}} \tag{9–6}$$

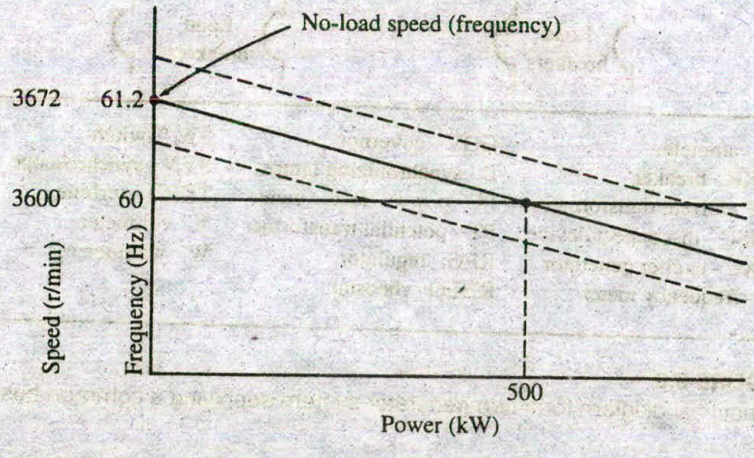


FIGURE 9.9
Typical prime-mover governor characteristics.

where: n_{rated} = rated speed (r/min)
n_{nl} = no-load speed (r/min)
f_{rated} = rated frequency (Hz)
f_{nl} = no-load frequency (Hz)

Governor Droop

Governor droop (GD) or droop rate is defined as the ratio of the change in frequency to the corresponding change in active power:

$$\text{GD} = \frac{\Delta f}{\Delta P} = \frac{f_{nl} - f_{rated}}{P_{rated}} \tag{9–7}$$

The governor droop may be expressed in hertz/watt, hertz/kilowatt, or hertz/megawatt, as appropriate. The droop of a properly functioning governor is constant for a given droop setting. It is independent of the no-load speed setting and is unaffected by parallel operation.[7]

[7]Wear or other defects in the governing mechanism can affect the droop.

EXAMPLE 9.2 Referring to Figure 9.9, the governor characteristic drawn with a solid line represents the rated operating conditions for a 500-kW, 460-V, 60 Hz, three-phase, two-pole synchronous generator. Determine (a) speed regulation; (b) governor droop.

Solution

(a)
$$\text{GSR} = \frac{f_{nl} - f_{rated}}{f_{rated}} = \frac{61.2 - 60}{60} = \underline{0.02}$$

(b)
$$\text{GD} = \frac{\Delta_f}{\Delta_P} = \frac{61.2 - 60}{500} = 0.00240 \text{ Hz/kW} \quad \text{or} \quad \underline{2.4 \text{ Hz/MW}}$$

9.8 DIVISION OF ACTIVE POWER BETWEEN ALTERNATORS IN PARALLEL

The division of oncoming bus load between alternators in parallel is determined inherently by the droops of the respective governors, and cannot be readily changed by the operator. The shifting of bus load between machines to provide a desirable distribution is easily accomplished, however, by changing the no-load speed settings of the respective governors. *To shift some load from one generator to another requires an energy shift between prime movers.* The generator that is to take a greater share of the existing bus load must have more energy admitted to its prime mover; this is accomplished by raising the no-load speed setting of its governor. Similarly, the turbine generator that is to lose some load must have less energy admitted to its prime mover; this is accomplished by lowering the no-load speed setting of its governor.

Machines With Identical Governor Characteristics

Figure 9.10 illustrates graphically the transfer of load between two turbine generators with identical droop characteristics. Machine *B* carries a load of 150 kW at 60 Hz. Machine *A* has just been paralleled and is carrying no load. The initial conditions are indicated by subscript 1 and the solid lines. To transfer some load from machine *B* to machine *A* without permanently changing the frequency of the system requires adjustment of both governors.

Assume for the sake of argument that one-third of the load on machine *B* is to be transferred to machine *A*; that is, machine *B* is to carry 100 kW and machine *A* is to carry 50 kW. The governor control switch of machine *A* is turned to RAISE and held there until one-half the amount of load that is to be transferred is shifted, namely, 25 kW. This adjustment raises the no-load speed setting, and hence raises the entire characteristic of machine *A* without changing its droop. Since 25 kW was removed from machine *B*, it also increases in speed. Its characteristic, however, has not been shifted. The system is now operating at some higher frequency f_2, and the characteristic of machine *B* intersects the new and higher frequency line at 125 kW, as denoted by subscript 2. To complete the transfer of load, the governor control switch of machine *B* is moved to LOWER and held there until the remaining 25 kW is transferred. The characteristic of machine *B* is thereby lowered, and the frequency of the system comes back to its original 60-Hz value. This is indicated by subscript 3, where machine *A* now takes 50 kW and machine *B* takes 100 kW. Throughout the transition period the slope of the curves was not changed. The entire characteristic of each machine was moved either up or down.

During the transition period, when more energy was admitted to the prime mover of machine *A*, the energy balance of the system was disturbed; more energy was

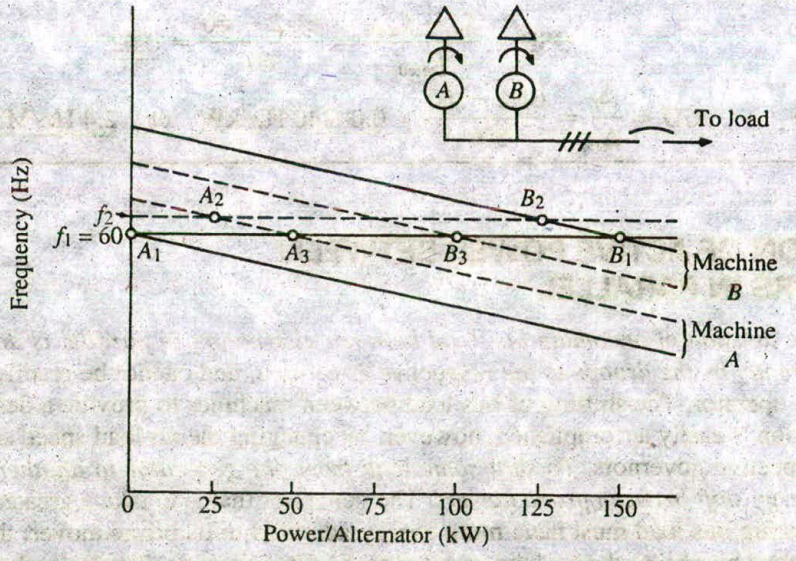


FIGURE 9.10
Graphical illustration of load transfer between machines with identical governor characteristics.

fed into the turbine generators than was going out. The total input was greater than the output, resulting in an increase in system speed and frequency. Prime-mover *A* increased its speed because of the direct increase in energy input brought about by raising the no-load speed setting of its governor. Prime-mover *B* increased its speed because of a reduction in its load. Except for the brief interval when the power angles are changing, both machines maintain identical speeds. Machines in parallel cannot run at different electrical speeds; if one increases in speed, so must the other. Like two engines driving a bull gear through identical pinions and flexible couplings, both machines are locked in rotational synchronism; but the engine with the greater fuel input will do more work than the other.

In actual practice, the division of active power between machines is accomplished with very little transient change in frequency, by making very small governor adjustments (alternately on each machine) until the desired transfer is accomplished.

If the governor control switches are located on a common synchronizing panel, both controls may be operated simultaneously, holding one on RAISE and the other on LOWER until the desired transfer is accomplished. This may be a little tricky, however.

Machines with identical governor droops (droop rate) will divide all increases in bus load and all decreases in bus load equally between them, regardless of the power ratings of the individual machines and the number of machines in parallel. For example, assume that the system indicated by subscript 3 in Figure 9.10 has an additional 50 kW added to the bus load. The system will operate at a new lower frequency, and the additional load will be divided equally between both machines. This is shown in Figure 9.11, where at 60 Hz, generator *A* is supplying 50 kW and generator *B* is supplying 100 kW. The additional load caused the frequency of the system to drop to a value that resulted in generator *A* delivering 75 kW and generator *B* delivering 125 kW. The new distribution is indicated by subscript 4 and the intersection of the new frequency line with the governor characteristics.

Machines With Dissimilar Governor Characteristics

Synchronous-generator sets with dissimilar governor characteristics do not divide increases or decreases in bus load equally. *The machine with the least droop assumes a greater portion of the change in bus load.* This is shown in Figure 9.12, where three paralleled generators, each with a different governor droop, are taking equal shares of the bus load at frequency f_1. The broken lines show the effect that different governor droops have on the distribution of oncoming bus load; the new lower frequency f_2 caused by the oncoming bus load causes the machine with the least droop to take a greater share of the oncoming load. Thus, if generators with different power ratings are to be operated in parallel, the respective governor droops should be adjusted so that load distribution between generators will be in proportion to their respective power ratings.

Machines With Isochronous Governors

The extreme case of dissimilar governor droops is illustrated in Figure 9.13. Generator *A* has a drooping governor characteristic and generator *B* is isochronous. The loads on machines *A* and *B* are 40 kW and 120 kW, respectively, and are indicated in Figure 9.13 by subscript 1. To transfer 60 kW from generator *B* to generator *A*, the governor

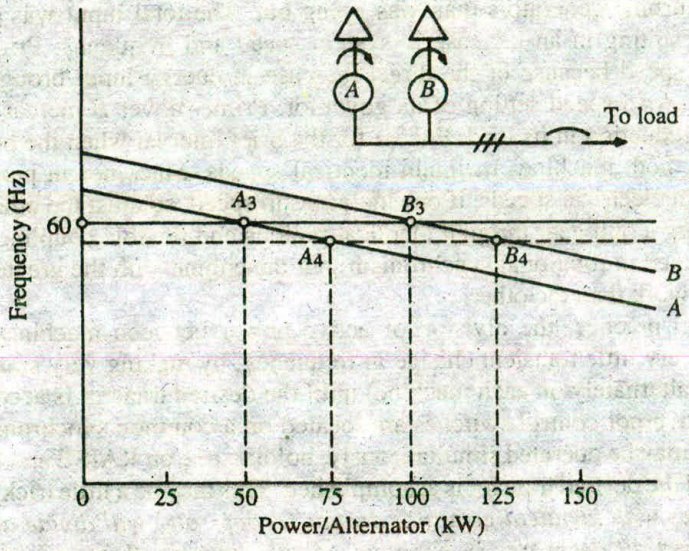


FIGURE 9.11
Effect of oncoming load on frequency and load division, for paralleled machines that have identical governor droops.

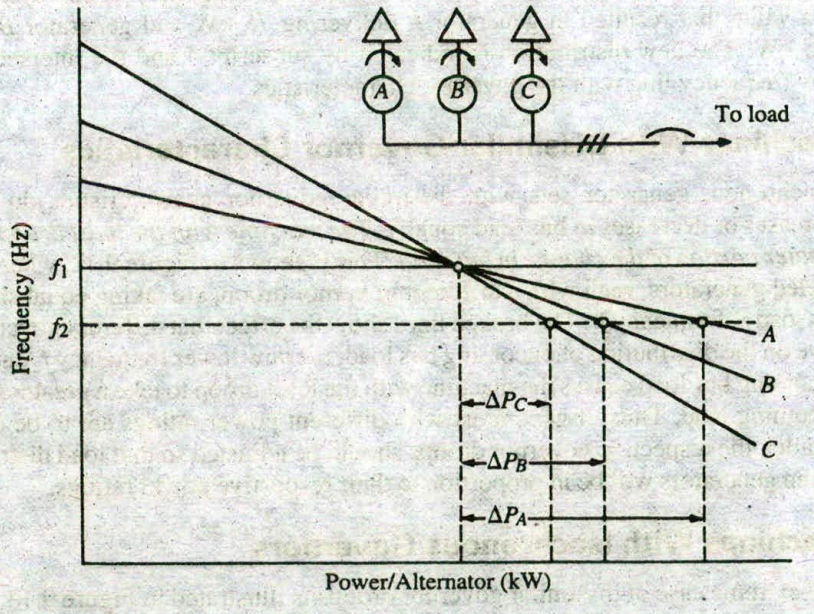


FIGURE 9.12
Effect of oncoming load on frequency and load division, for paralleled machines that have different governor droops.

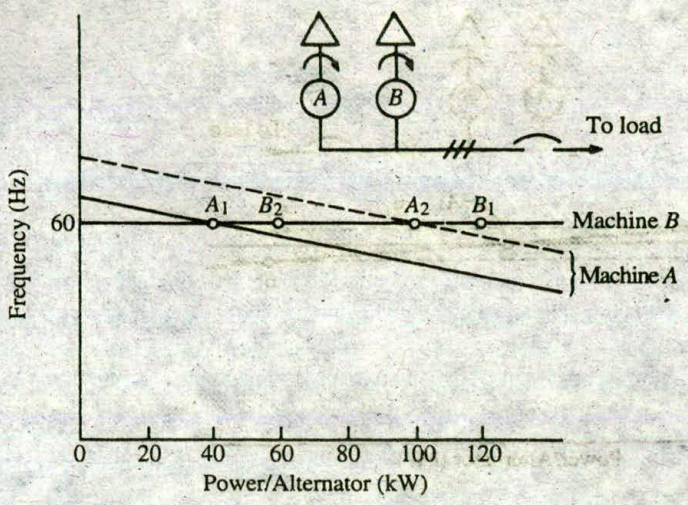


FIGURE 9.13
Effect of governor adjustment in a constant frequency system.

control switch of generator *A* should be turned to RAISE and held there until the entire transfer is completed. The governor control switch of generator *B* should not be used, since adjustment of its governor would affect the frequency of the system. The transfer of load must be made by adjusting only the governor control of the machine with droop. The final conditions are indicated by subscript 2.

For a given no-load speed setting of its governor, *an alternator with a drooping characteristic can carry only one value of load at a given frequency.* An isochronous machine, however, can carry any value of load from no load to rated load at the given frequency. Thus, if an isochronous machine is in parallel with a droop machine, any increase or decrease in bus load will be absorbed by the isochronous machine.

Inherent Instability of Two or More Isochronous Machines in Parallel

When two or more alternators with isochronous governors are in parallel, their operation is very unstable unless controlled by special-purpose electronic governors. The slightest change in frequency may cause a considerable interchange of load between machines. This is illustrated in Figure 9.14. Since no two governors are identical in the true sense of the word, it may be assumed that generator *A* has zero droop and generator *B* has a very minute droop.

The initial conditions are illustrated by subscript 1, where both generators are shown as having equal shares of the bus load. If a slight decrease in system speed occurs, due perhaps to very slight irregularities in the isochronous governing mechanism, a considerable portion of the bus load will be thrown on generator *B*, and generator *A* will then be lightly loaded or driven as a motor; the new condition is shown by subscript 2. Likewise, a slight increase in speed would shift the load in the other direction, as indicated by subscript 3.

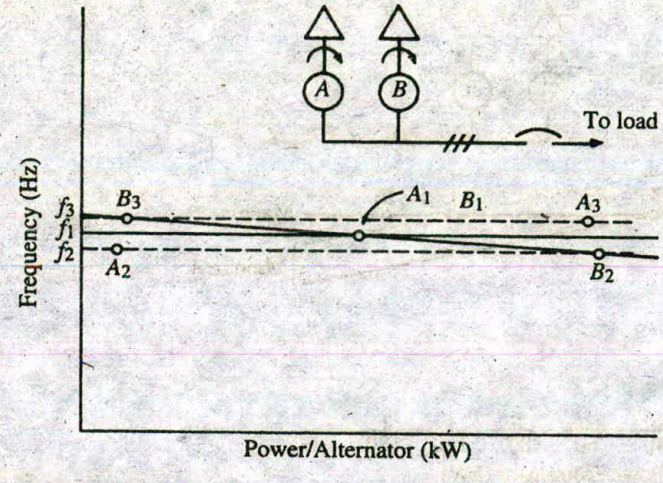


FIGURE 9.14
Graphic illustration of the inherent instability of isochronous machines in parallel.

The parallel operation of isochronous machines requires fast-acting electronic governors. Mechanical governors do not respond fast enough to avoid wild load swings [1].

9.9 MOTORING OF ALTERNATORS

When the energy input to the prime mover of an alternator is insufficient to drive it at its synchronous speed, it will not only lose its load but will be driven by the other alternators on the bus at the synchronous speed of the system. Such "motoring" is automatically detected by a reverse-power relay, also called a *power-directional relay,* which trips the circuit breaker of the affected machine. Magnetic damping action on a rotating disk provides a time delay (adjustable) to prevent a transient load swing from tripping the generator breaker. Sustained motoring will trip the breaker, however, after expiration of the time delay. The only other indication of motoring is the backward deflection of the wattmeter pointer. If observed early enough, the operator may correct the condition by admitting more energy to the prime mover of the motorized machine. Sustained motoring is undesirable, because it results in a useless expenditure of energy. Furthermore, in the case of steam turbines, windage losses during motorization can overheat the turbine blades.

9.10 GENERAL PROCEDURE FOR SAFE SHUTDOWN OF AC GENERATORS IN PARALLEL WITH OTHER MACHINES

For safe shutdown of AC generators in parallel with other machines, these steps should be followed:

1. Using governor and field controls, gradually reduce the active and reactive power of the generator to essentially zero (see Section 9.12).

2. When the wattmeter indicates zero, or reverses (motoring), trip the generator breaker.
3. Switch the automatic voltage regulator to manual.
4. Reduce the voltage to its minimum value and then trip the exciter breaker.
5. Trip the turbine.

Note: In some large steam-turbine-generator applications (1000 MVA and above) the manufacturer may recommend a different sequence for shutdown to provide an increased margin against potential overspeed [4].

9.11 CHARACTERISTIC TRIANGLE AS A TOOL FOR SOLVING LOAD DISTRIBUTION PROBLEMS BETWEEN ALTERNATORS IN PARALLEL

The solution of problems involving load distribution between generators may be accomplished in a straightforward manner by defining a *characteristic triangle* for each machine, and then using similar triangles to determine a solution.

Referring to Figure 9.15, the characteristic triangle is formed by the governor characteristic and the rated frequency line, with the intersection occurring at rated frequency and rated active power. The characteristic triangle is fixed for a given governor droop, and it does not change with changes in load or changes in the no-load speed setting of the governor.

An increase in load causes a decrease in frequency, as shown in Figure 9.15, and the intersection of the new frequency line with the governor characteristic establishes

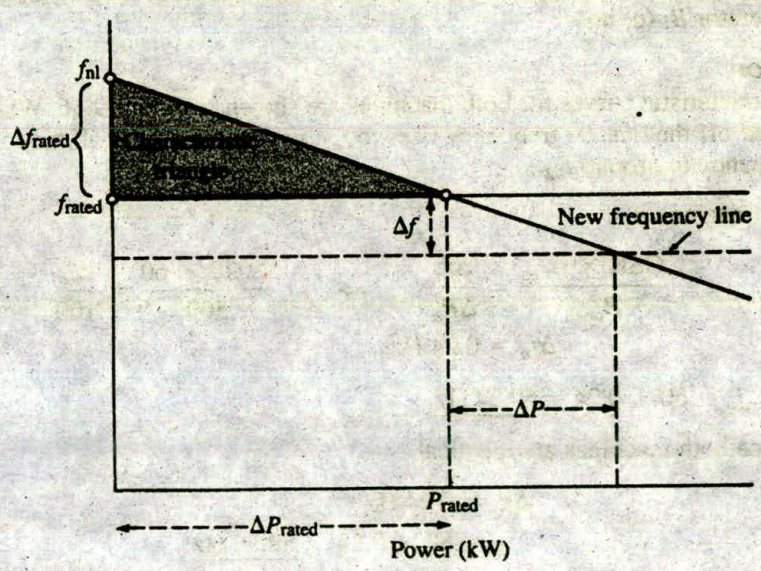


FIGURE 9.15
Characteristic triangle formed by the governor characteristic and the rated frequency line.

a new triangle similar to the characteristic triangle. Thus, from the geometry of similar triangles, and Eq. (9–7)

$$\text{GD} = \frac{f_{\text{nl}} - f_{\text{rated}}}{P_{\text{rated}}} = \frac{\Delta f}{\Delta P} \tag{9–8}$$

From Eq. (9–6),

$$f_{\text{nl}} - f_{\text{rated}} = \text{GSR} \times f_{\text{rated}}$$

Substituting into Eq. (9–8),

$$\text{GD} = \frac{\text{GSR} \times f_{\text{rated}}}{P_{\text{rated}}} = \frac{\Delta f}{\Delta P} \tag{9–9}$$

where: GD = governor droop
GSR = governor speed regulation
Δf = change in frequency due to change in load
ΔP = change in load

Since the governor droop is unaffected by increases or decreases in load, Eq. (9–9) holds true for all operating frequencies and for all no-load speed settings of the governor.

EXAMPLE 9.3 Two generators, *A* and *B*, are in parallel and sharing a total bus load of 300 kW at 60 Hz. Both generators are rated at 460 V, 500 kW, 60 Hz, and have 2.0 percent speed regulation. Generator *A* is carrying 100 kW, and generator *B* is carrying 200 kW. Assuming generator *A* is tripped off the bus, determine the frequency of (a) generator *A*; (b) generator *B*; (c) bus.

Solution

The characteristic curves for both machines are shown in Figure 9.16. With generator *A* tripped off the bus, its frequency rises to f_{a2}; generator *B* takes all the load, causing its frequency to drop to f_{b2}.

(a) For machine *A*,

$$\frac{\text{GSR} \times f_{\text{rated}}}{P_{\text{rated}}} = \frac{\Delta f_a}{\Delta P_a} \quad \Rightarrow \quad \frac{0.020 \times 60}{500} = \frac{\Delta f_a}{100}$$

$$\Delta f_a = 0.24 \text{ Hz}$$

Thus, $f_a = 60 + 0.24 = \underline{60.24 \text{ Hz.}}$

(b) Since both machines are identical,

$$\Delta f_b = 0.24 \text{ Hz}$$

$$f_b = 60 - 0.24 = \underline{59.76 \text{ Hz}}$$

(c) The bus frequency is the frequency of generator *B*.

$$f_{\text{bus}} = \underline{59.76 \text{ Hz}}$$

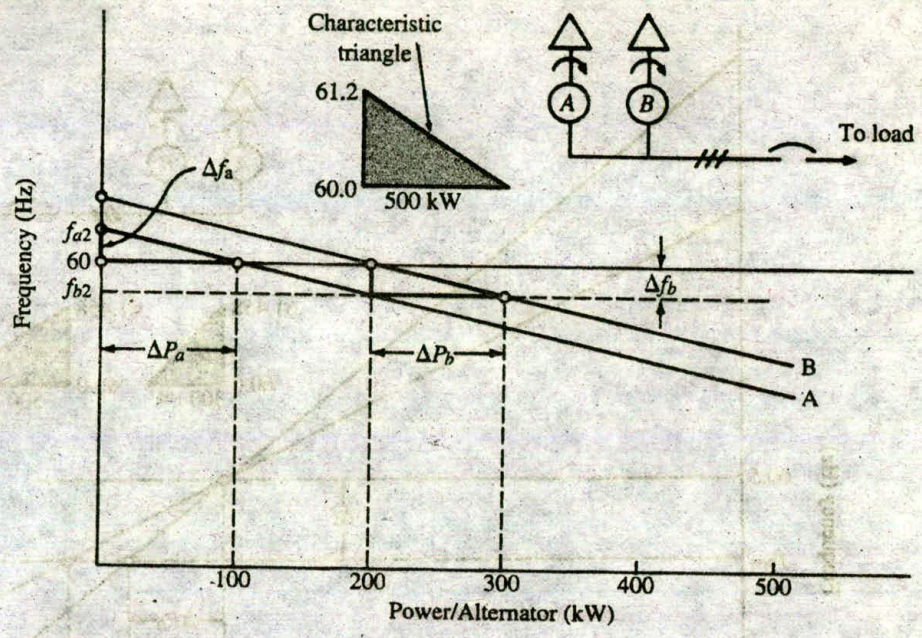


FIGURE 9.16
Governor characteristics for Example 9.3.

EXAMPLE 9.4 A 500-kW, 60-Hz, 2300-V, six-pole alternator *A* is paralleled with a 300-kW, 60-Hz, 2300-V, four-pole machine *B*. Both machines have a speed regulation of 2.43 percent. The machines are carrying equal shares of a 400-kW bus load at a frequency of 60.5 Hz. If the bus load increases to a total of 500 kW, determine (a) operating frequency; (b) load carried by each machine.

Solution

The characteristic curves are shown in Figure 9.17.

(a)

$$\frac{\text{GSR} \times f_{\text{rated}}}{P_{\text{rated}}} = \frac{\Delta f}{\Delta P}$$

Machine *A* **Machine *B***

$$\frac{0.0243 \times 60}{500} = \frac{\Delta f}{\Delta P_a} \qquad \frac{0.0243 \times 60}{300} = \frac{\Delta f}{\Delta P_b}$$

$$\Delta P_a = 342.936\Delta f \qquad \Delta P_b = 205.761\,\Delta f$$

$$\Delta P_a + \Delta P_b = [342.936 + 205.761]\Delta f$$

$$500 - 400 = 548.697\Delta f$$

$$\Delta f = 0.182 \text{ Hz}$$

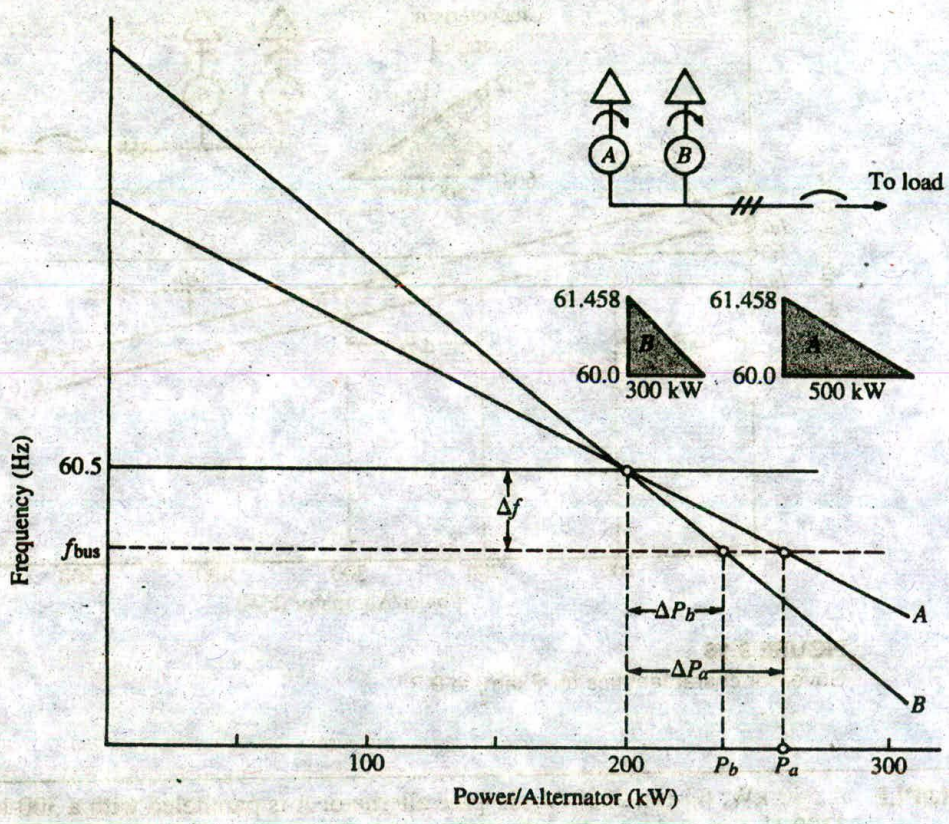


FIGURE 9.17
Governor characteristics for Example 9.4.

Referring to Figure 9.17,

$$f_{bus} = 60.5 - 0.182 = \underline{60.318 \text{ Hz}}$$

(b)

$$\Delta P_a = 342.936\Delta f = 342.936 \times 0.182 = 62.414 \text{ kW}$$
$$\Delta P_b = 100 - 62.414 = 37.586 \text{ kW}$$
$$P_a = 200 + 62.41 = \underline{262.41 \text{ kW}}$$
$$P_b = 200 + 37.59 = \underline{237.59 \text{ kW}}$$

EXAMPLE 9.5 A 1000-kW, 60-Hz synchronous alternator *A* is in parallel with a 600-kW, 60-Hz alternator *B*. Both machines have identical governor droops of 0.0008 Hz/kW. Machine *A* takes two-thirds of a 900-kW bus load at 60.2 Hz. If an additional 720-kW load is connected to the bus, determine (a) bus frequency; (b) load on each machine.

Solution

(a) Since both machines have identical governor droops, they will take equal shares of the additional bus load. Thus,

$$\Delta P_a = \Delta P_b = \frac{720}{2} = 360 \text{ kW}$$

$$\text{GD} = \frac{\Delta f}{\Delta P_a} \quad \Rightarrow \quad 0.0008 = \frac{\Delta f}{360}$$

$$\Delta f = 0.288 \text{ Hz}$$

$$f_{\text{bus}} = 60.2 - 0.288 = \underline{59.91 \text{ Hz}}$$

(b)

$$P_a = \frac{2}{3} \times 900 + 360 = \underline{960 \text{ kW}}$$

$$P_b = \frac{1}{3} \times 900 + 360 = \underline{660 \text{ kW}}$$

Note: Machine *B* is operating at a 10 percent overload.

EXAMPLE 9.6 Three diesel-driven 60-Hz alternators *A*, *C*, and *B* are in parallel and taking equal shares of a 210-kW, 60-Hz bus load. The ratings of the alternators are 500 kW, 200 kW, and 300 kW, respectively. The circuit is shown in Figure 9.18(a), and the governor characteristics are shown in Figure 9.18(b). If a three-phase, 440-kW resistance load and a three-phase, 60-Hz induction motor that draws 200 kVA at 0.80 power factor are added to the bus, determine (a) the system kilowatts; (b) the system frequency; (c) the kilowatt loads carried by each machine.

Solution

(a) The increase in bus load is

$$\Delta P_{\text{bus}} = 440 + 0.8 \times 200 = 600 \text{ kW}$$

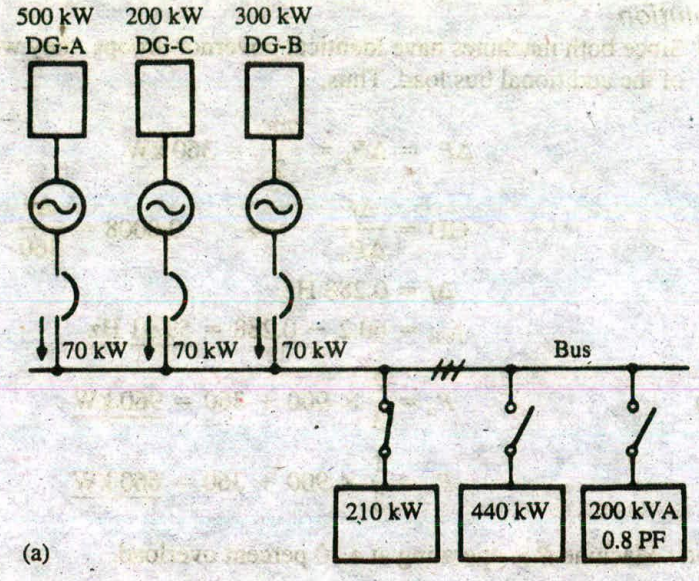
$$P_{\text{syst}} = 210 + 600 = \underline{810 \text{ kW}}$$

(b) The respective governor droops obtained from the initial loadings shown in Figure 9.18(b) are

$$\text{GD}_a = \frac{\Delta f_a}{\Delta P_a} = \frac{60.2 - 60}{70} = 0.002857 \text{ Hz/kW}$$

$$\text{GD}_b = \frac{\Delta f_b}{\Delta P_b} = \frac{60.4 - 60}{70} = 0.005714 \text{ Hz/kW}$$

$$\text{GD}_c = \frac{\Delta f_c}{\Delta P_c} = \frac{60.6 - 60}{70} = 0.008571 \text{ Hz/kW}$$



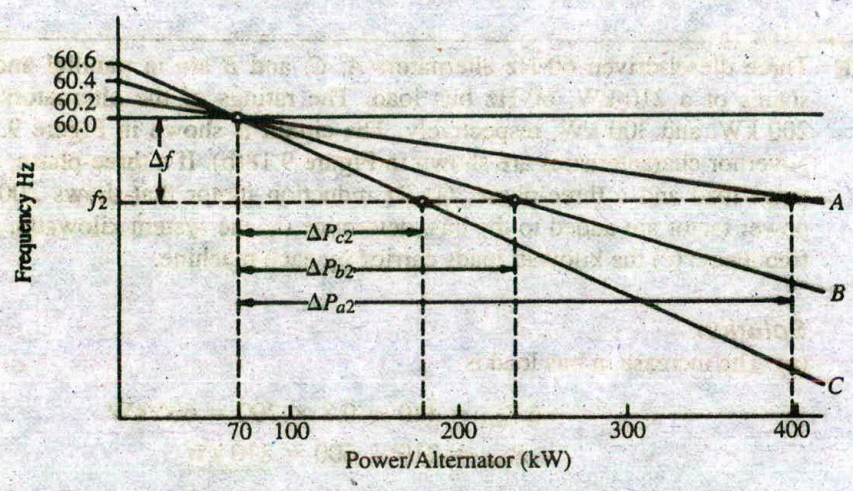


FIGURE 9.18
Governor characteristics for Example 9.6.

The increased load causes the bus frequency to drop to f_2. Referring to Figure 9.18 using similar triangles, and noting that Δf is the same for all machines,

$$\frac{\Delta f}{\Delta P_{a2}} = 0.002857 \qquad \frac{\Delta f}{\Delta P_{b2}} = 0.005714 \qquad \frac{\Delta f}{\Delta P_{c2}} = 0.008571$$

$$\Delta P_{a2} = 350\Delta f \qquad \Delta P_{b2} = 175\Delta f \qquad \Delta P_{c2} = 116.6667\,\Delta f$$

$$\Delta P_{a2} + \Delta P_{b2} + \Delta P_{c2} = 600$$
$$(350 + 175 + 116.6667)\Delta f = 600 \quad \Rightarrow \quad \Delta f = 0.9351 \text{ Hz}$$
$$f_2 = 60 - \Delta f = 60 - 0.9351 = \underline{59.06 \text{ Hz}}$$

(c)
$$P_{a2} = 70 + \Delta P_{a2} = 70 + 350 \times 0.9351 = \underline{397.3 \text{ kW}}$$
$$P_{b2} = 70 + \Delta P_{b2} = 70 + 175 \times 0.9351 = \underline{233.6 \text{ kW}}$$
$$P_{c2} = 70 + \Delta P_{c2} = 70 + 116.6667 \times 0.9351 = \underline{179.1 \text{ kW}}$$

9.12 DIVISION OF REACTIVE POWER BETWEEN ALTERNATORS IN PARALLEL

Adjustment of the no-load speed setting of the respective governors to balance the active power between paralleled alternators does not balance the reactive power. Balancing reactive power between machines requires adjustment of the respective field currents.

The effect of active power transfer on reactive power loading will be discussed for the two identical paralleled generators shown in Figure 9.19; generator 1 is carrying the entire bus load, and generator 2 is floating on the bus. The phasor diagrams corresponding to the initial load conditions are shown with *solid lines* in Figures 9.20(a) and (b). Current $\mathbf{I}_{a1}$ supplied by generator 1 is the bus current. The drawings in Figure 9.20 are constructed to scale.

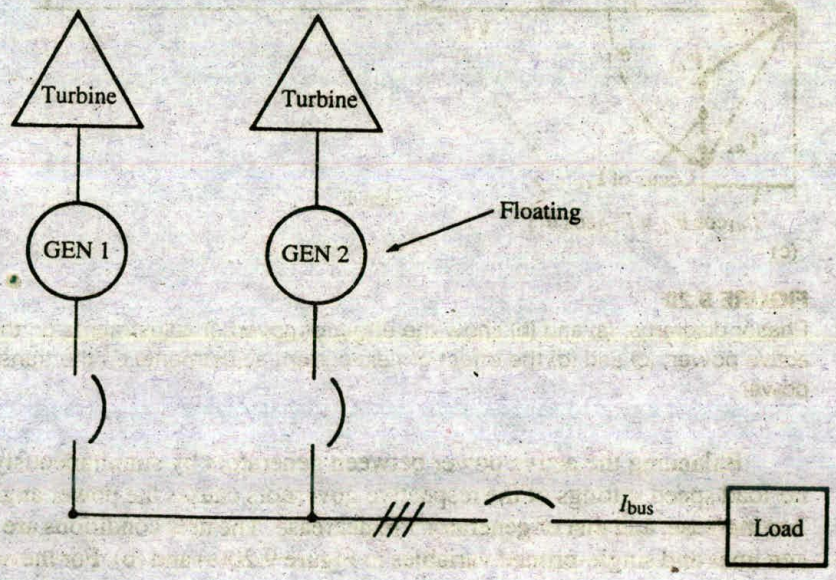


FIGURE 9.19
Two generators in parallel, one floating on the bus.

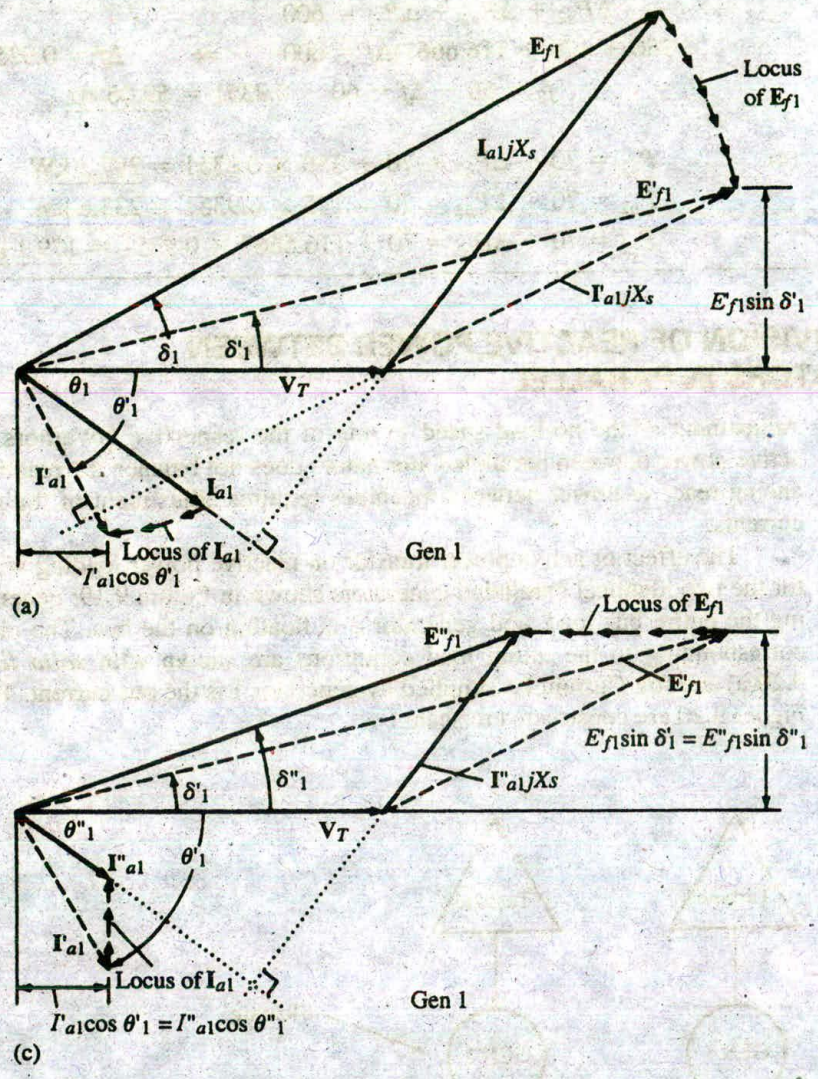
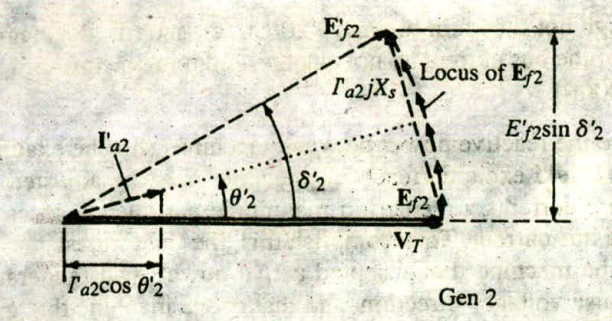


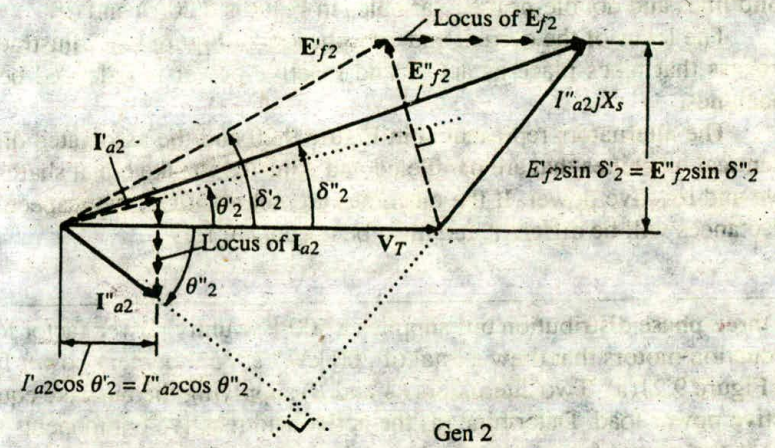
FIGURE 9.20
Phasor diagrams: (a) and (b) show the effect of governor adjustments on the transfer of active power; (c) and (d) the effect of field-current adjustments on the transfer of reactive power.

Balancing the active power between generators by simultaneously adjusting the no-load speed settings of the respective governors causes the power angle of generator 2 to increase, and that of generator 1 to decrease. The new conditions are shown by broken lines and single-primed variables in Figure 9.20(a) and (b). For the new conditions,

$$\frac{V_T E'_{f1}}{X_{s1}} \sin\delta'_1 = \frac{V_T E'_{f2}}{X_{s2}} \sin\delta'_2$$



(b)



(d)

FIGURE 9.20 ***(Continued)***

Since the machines are assumed to be identical, $X_{s1} = X_{s2}$, and since they are in parallel V_T is the same for both machines. Hence,

$$E'_{f1} \sin \delta'_1 = E'_{f2} \sin \delta'_2$$

The equivalence is shown in Figures 9.20(a) and (b). Furthermore, with both machines supplying equal active power to the load,

$$V_T I'_{a1} \cos \theta'_1 = V_T I'_{a2} \cos \theta'_2$$
$$I'_{a1} \cos \theta'_1 = I'_{a2} \cos \theta'_2$$

This equivalence is also shown on the drawings in Figures 9.20(a) and (b). *Note:*

1. The armature current $\mathbf{I}'_{a1}$ in generator 1 is lagging, but the armature current $\mathbf{I}'_{a2}$ in generator 2 is leading, indicating a difference in power factor, and hence a difference in reactive power distribution between machines.

2. Although not shown in Figures 9.20(b), (c), and (d), the magnitude and phase angle of the bus current do not change, and remain equal to $\mathbf{I}_{a1}$ [shown in Figure 9.20(a)].

To balance the reactive power between machines (for the example illustrated in Figure 9.20), the field excitation of each machine must be adjusted so that their respective armature currents are equal in magnitude and their respective phase angles equal that of the bus current. To accomplish this, the field rheostat (or voltage regulator control) of the machine that accepted active power load (generator 2) should be turned in the "raise voltage" direction, and that of the machine that gave up some active power load (generator 1) should be turned in the "lower voltage" direction, until the power-factor meter (or var meter) indicates the same for both machines.

The phasor diagrams corresponding to the final steady-state condition for balanced active power and balanced reactive power between machines is shown with solid lines and double-primed variables in Figures 9.20(c) and (d).

The locus of the current and emf phasors in Figure 9.20 illustrates the transition process that takes place as active and reactive power is balanced between identical machines.

The alternators represented in Figure 9.20 and the associated discussion are for identical machines that are paralleled and adjusted to take equal shares of the total active and reactive power. If the machines are not identical, the respective synchronous reactances will be different and must be accounted for.

EXAMPLE 9.7 A three-phase distribution bus supplies a 500-kW unity power-factor load, and several induction motors that draw a total of 200 kVA at 85.2 percent power factor, as shown in Figure 9.21(a). Two alternators (A and B) supplying the bus take equal shares of the active power load. Determine (a) the active and reactive components of the bus load. (b) If the power factor of generator A is 0.94 lagging, determine the reactive power supplied by each machine.

Solution

(a)

$$P_{\text{bus}} = 500 + 200 \times 0.852 = \underline{670.40 \text{ kW}}$$

$$\theta_{\text{motors}} = \cos^{-1}(0.852) = 31.57°$$

$$Q_{\text{bus}} = Q_{\text{motors}} = 200 \times \sin(31.57) = \underline{104.7 \text{ kvar}}$$

(b)

$$P_a = \frac{670.40}{2} = 335.2 \text{ kW} \qquad \theta_a = \cos^{-1}(0.94) = 19.95°$$

From the power diagram shown in Figure 9.21(b),

$$\tan(19.95°) = \frac{Q_a}{335.2} \quad \Rightarrow \quad Q_a = \underline{121.7 \text{ kvar}}$$

$$Q_b = Q_{\text{bus}} - Q_a = 104.7 - 121.7 = \underline{-17.0 \text{ kvar}}$$

The negative sign indicates a leading power factor.

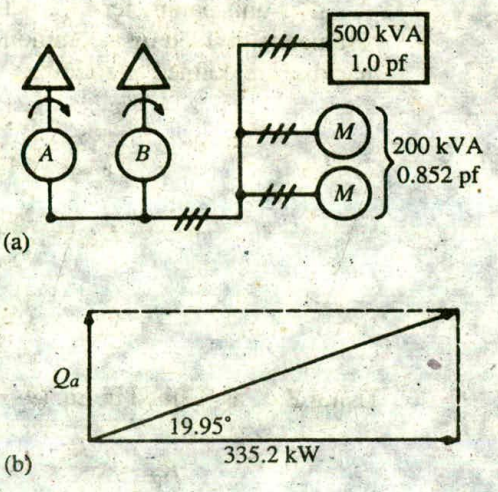


FIGURE 9.21
(a) One-line diagram and (b) power diagram for Example 9.7.

9.13 ACCIDENTAL LOSS OF FIELD EXCITATION

Accidental loss of field excitation supplied to a synchronous machine in parallel with other machines will cause the affected unit to pull out of synchronism. The resultant reduction in its load will cause its prime mover to accelerate and drive the alternator as an induction generator.[8] The effect will be rapid overheating of the rotor forging by eddy currents, with possible damage to rotor wedges that hold the rotor end turns in place. If protective equipment, such as a loss-of-field relay, does not respond to trip the generator breaker, and the field excitation cannot be immediately restored, the generator breaker must be tripped by the operator.

Furthermore, the large amount of reactive power drawn by the affected machine as it operates asynchronously (as an induction generator) will cause overheating of the stator windings. On complete loss of excitation, the rotor may overheat in as little as 10 seconds [2].

9.14 PER-UNIT VALUES OF SYNCHRONOUS MACHINE PARAMETERS

Manufacturers of electrical machinery generally publish synchronous machine parameters as per-unit impedance instead of actual impedance values. Per-unit values are useful in machine design in that a wide range of machine sizes have approximately the same values. This makes it easy to detect gross errors in design calculations, and provides a convenient means of comparing the relative performance of machines without regard to machine size.

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[8]See Section 5.18 on induction generators.

Per-unit parameters are defined as a ratio of the actual value of the respective parameters divided by a common base impedance. The base values for a synchronous machine are defined as follows:

S_{base} = rated apparent power/phase (VA)

V_{base} = rated voltage/phase (V)

$$I_{base} = \frac{S_{base}}{V_{base}} \quad \text{A} \tag{9–10}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}} \quad \Omega \tag{9–11}$$

Using Z_{base} as defined in Eq. (9–11), per-unit parameters are

$$R_{pu} = \frac{R_a}{Z_{base}} \qquad X_{pu} = \frac{X_s}{Z_{base}} \qquad \mathbf{Z}_{pu} = R_{pu} + jX_{pu} \tag{9–12}$$

EXAMPLE 9.8 Determine the per-unit impedance of a three-phase, wye-connected, 100-kVA, 480-V, 60-Hz synchronous generator whose synchronous impedance is $(0.0800 + j2.300)\ \Omega$.

Solution

$$S_{base} = \frac{100{,}000}{3}\ \text{VA} \qquad V_{base} = \frac{480}{\sqrt{3}}\ \text{V}$$

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(480/\sqrt{3})^2}{100{,}000/3} = 2.304\ \Omega$$

$$R_{pu} = \frac{R_a}{Z_{base}} = \frac{0.0800}{2.304} = \underline{0.0347} \qquad X_{pu} = \frac{X_s}{Z_{base}} = \frac{2.300}{2.304} = \underline{0.9983}$$

$$\mathbf{Z}_{pu} = R_{pu} + jX_{pu} = 0.0347 + j0.9983 = \underline{0.9989 \angle 88.01°}$$

9.15 VOLTAGE REGULATION

The voltage regulation of an alternator is the percentage change in terminal voltage from no load to rated load, with respect to rated voltage, with the field current held constant. The voltages are all phase voltages or all line voltages.

$$\text{VR} = \frac{V_{nl} - V_{rated}}{V_{rated}} \times 100 \tag{9–13}$$

where: VR = voltage regulation (%)

V_{rated} = voltage indicated on nameplate of machine (V)

V_{nl} = no-load voltage (open-circuit voltage) (V)

Note 1. *V_{nl} is the voltage measured after tripping the generator breaker while the generator is operating at rated load and rated voltage.*

Note 2. V_{nl} *is not the excitation voltage.* Substituting E_f for V_{nl} in Eq. (9–13) results in gross errors and should not be used as an approximate value for V_{nl} because E_f does not account for the effect of magnetic saturation.

Note 3. The voltage regulation will be positive for unity power-factor and lagging power-factor loads (usual operating conditions), but for leading power-factor loads the regulation may be negative.

The voltage regulation of an alternator is a figure of merit used for comparison with other machines. Machines with high values of voltage regulation have relatively large values of synchronous reactance and suffer large drops in output voltage with lagging power-factor loads. Thus, the voltage regulation of a machine is an indicator of the change in field current required to maintain system voltage, when going from no load to rated load at some specific power factor. Furthermore, when operating in parallel, alternators with wide differences in voltage regulation will not divide the reactive power load in proportion to their apparent power ratings.

The no-load voltage for a specific power factor may be determined by operating the machine at rated load conditions, then removing the load and observing the no-load voltage. Since this is generally not a practical method, especially for very large machines, other methods have been developed to determine the no-load voltage without having to load and unload the machine. The IEEE standard for determining the no-load voltage, as given in Reference [6], is an exact but complicated and time-consuming process.

The Approximate Method

A graphical method that provides a reasonable approximation of the no-load voltage may be obtained by reference to the actual magnetization curve, and a linearized magnetization curve drawn from the origin through the rated voltage point on the magnetization curve, as shown in Figure 9.22. See References [3],[8] for a more detailed analysis.

Defining points on the field-current axis of the magnetization curve,

$$I_{f2} = I_{f1} = \Delta I_f$$

where:
I_{f1} = field current required to produce rated armature voltage at no load
I_{f2} = field current required to obtain a value of V_{nl} that will result in rated voltage when rated load is applied
ΔI_f = increment increase in field current required to compensate for the synchronous reactance drop and magnetic saturation
ΔV = voltage drop due to magnetic saturation

To determine the no-load voltage using the approximate method:

1. Enter the value of E_f calculated from Eq. (9–1) on the linearized magnetization curve (point *a*) in Figure 9.22.
2. Project vertically from the linearized curve to the actual magnetization curve (point *b*). Then project horizontally to the voltage axis and read V_{nl}.

EXAMPLE 9.9 A hypothetical three-phase, two-pole, 60-Hz, 1000-kVA, 4.8-kV, wye-connected synchronous generator has a synchronous reactance of 13.80 Ω/phase. The machine is operating at rated kVA, rated voltage, and a power factor of 90.0 percent lagging.

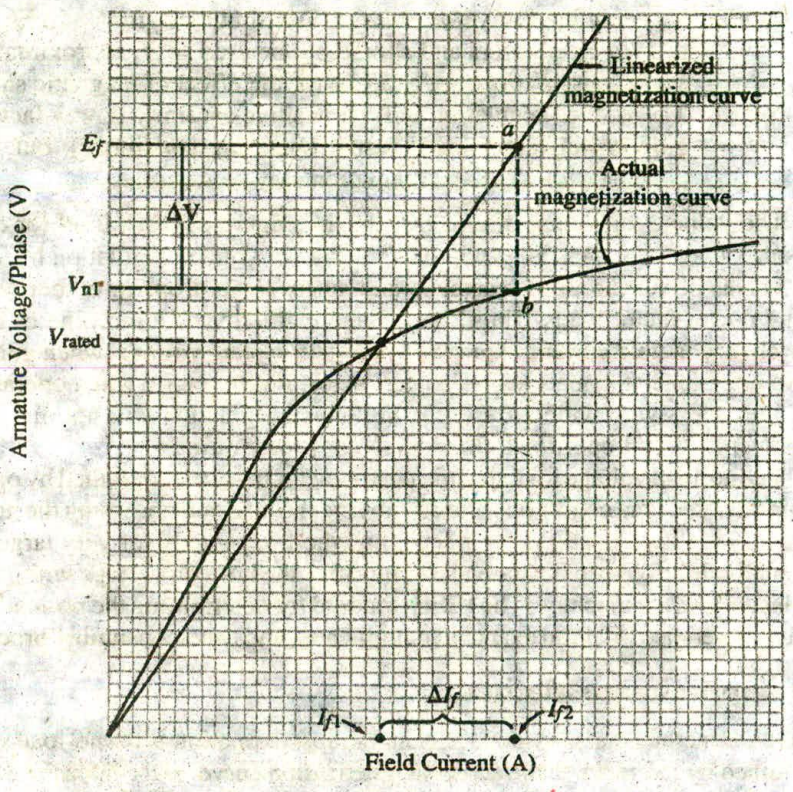


FIGURE 9.22
Magnetization curve and its linear approximation for a representative synchronous generator.

Determine (a) excitation voltage; (b) power angle; (c) no-load voltage, assuming the field current is not changed; (d) voltage regulation; (e) no-load voltage if the field current is reduced to 80 percent of its value at rated load.

Solution

$$V_T = \frac{4800}{\sqrt{3}} = 2771.28 \text{ V} \qquad \theta = \cos^{-1}(0.900) = 25.84°$$

$$\mathbf{S} = \mathbf{V}_T \mathbf{I}_a^* \quad \Rightarrow \quad \frac{1000 \times 10^3}{3} \angle 25.84° = (2771.28 \angle 0°) \cdot \mathbf{I}_a^*$$

$$\mathbf{I}_a^* = 120.28 \angle 25.84° \quad \Rightarrow \quad \mathbf{I}_a = 120.28 \angle -25.84°$$

(a) The circuit diagram for one phase is shown in Figure 9.23(a).

$$\mathbf{E}_f = \mathbf{V}_T + \mathbf{I}_a jX_s = 2771.28 \angle 0° + (120.28 \angle -25.84°)(13.80 \angle 90°)$$
$$\mathbf{E}_f = 3494.8 + j1493.9 = 3800.7 \angle 23.1° \text{ V}$$
$$E_f = \underline{3801 \text{ V}}$$

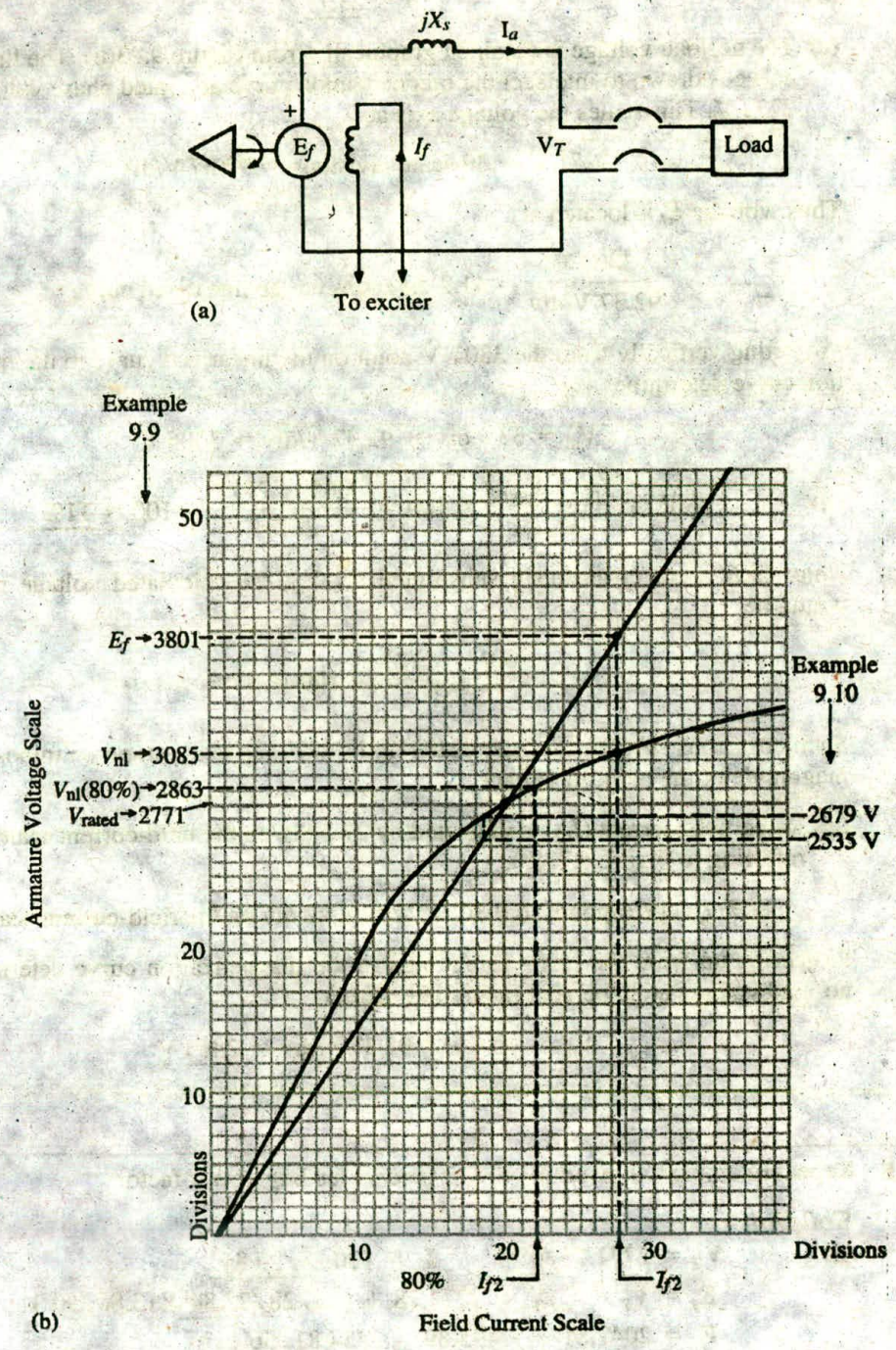


FIGURE 9.23
Equivalent circuit and magnetization curve for Examples 9.9 and 9.10.

(b) $$\delta = \underline{23.1^\circ}$$

(c) The no-load voltage is obtained graphically from Figure 9.23(b). The linearized curve is drawn to intersect the magnetization curve at a rated phase voltage of 2771 V. This scales the voltage axis at

$$2271 \text{ V} \div 30 \text{ scale divisions} = 92.37 \text{ V/div}$$

Thus, voltage E_f is located at

$$\frac{3801 \text{ V}}{92.37 \text{ V/div}} = 41.2 \text{ divisions on the linearized curve}$$

Projecting vertically from the 3801-V point on the linearized curve to the magnetization curve determines V_{nl}.

$$V_{nl} = 33.4 \text{ div} \times 92.37 \text{ V/div} \approx \underline{3085 \text{ V}}$$

(d) $$\text{VR} = \frac{V_{nl} - V_{rated}}{V_{rated}} \times 100 = \frac{3085 - 2771}{2771} \times 100 \approx \underline{11\%}$$

Note: If E_f were erroneously substituted for V_{nl}, the calculated voltage regulation would be

$$\frac{3801 - 2771}{2771} \times 100 = 37\%$$

Such erroneous substitution ignores the limits that magnetic saturation imposes on the magnitude of the induced voltage.

(e) Rated field current (I_{f2}) is indicated by 27.5 div on the field-current scale. Thus, referring to Figure 9.23(b),

$$80\% \, I_{f2} \text{ corresponds to } 0.80 \times 27.5 = 22 \text{ div on the field-current scale}$$

Projecting up from the field-current axis to the magnetization curve determines the no-load voltage at 80 percent rated field current.

$$V_{nl} = 31 \text{ div} \times 92.37 \text{ V/div} \approx \underline{2863 \text{ V}}$$

EXAMPLE 9.10 Repeat Example 9.9 assuming a 90.0 percent leading power factor.

Solution

(a) $$\mathbf{V}_T = 2771.28\underline{/0^\circ} \text{ V} \qquad \mathbf{I}_a = 120.28\underline{/25.84^\circ}$$

$$\mathbf{E}_f = \mathbf{V}_T + \mathbf{I}_a jX_s = 2771.28 + (120.28\underline{/25.84^\circ})(13.80\underline{/90^\circ})$$

$$\mathbf{E}_f = 2047.81 + j1493.89 = 2534.81\underline{/36.11^\circ}$$

$$E_f = \underline{2535 \text{ V}}$$

(b) $\delta = \underline{36.1°}$

(c) Referring to Figure 9.23(b), voltage E_f is located at

$$\frac{2535}{92.37} \approx 27.4 \text{ div on the linearized curve (shown with dotted lines)}$$

Projecting vertically from the 2535 point on the linearized curve to the magnetization curve determines the no-load voltage. Thus,

$$V_{nl} = 29 \times 92.37 \approx \underline{2679 \text{ V}}$$

(d) $$\text{VR} = \frac{V_{nl} - V_{rated}}{V_{rated}} \times 100 = \frac{2679 - 2771}{2771} \times 100 \approx \underline{-3.32\%}$$

Note: The leading power-factor condition resulted in a negative voltage regulation.

9.16 DETERMINATION OF SYNCHRONOUS MACHINE PARAMETERS

The synchronous reactance depends on the degree of magnetic saturation, and will have different values for different loads and different field currents. The value of synchronous reactance supplied by the manufacturer for voltage-regulation calculations, and for analysis involving rated conditions, *is based on a field current that provides rated open-circuit voltage and an associated short-circuit current* **[6],[9]**.

In those cases where synchronous machine parameters are not readily available from the manufacturer, they can be *approximated* from a DC test, an open-circuit test, and a short-circuit test.

DC Test

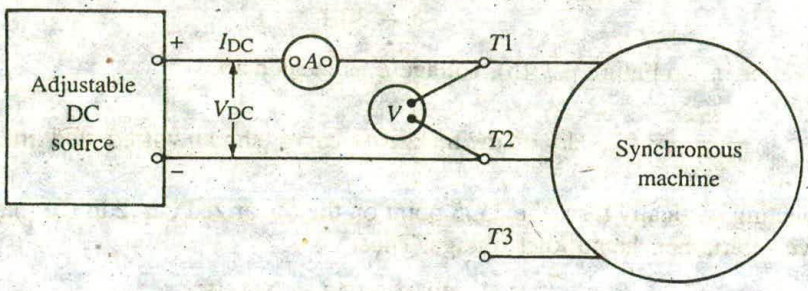
The purpose of the DC test is to determine R_a. This is accomplished by connecting any two stator leads to a low-voltage DC source, as shown in Figure 9.24(a). The rheostat is adjusted to provide readable values of current and voltage, and the resistance between the two stator leads is determined from Ohm's law. Thus, from Figures 9.24(a) and (b) for a wye connection,

$$R_{DC} = \frac{V_{DC}}{I_{DC}} \tag{9–14}$$

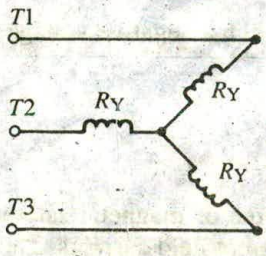
$$R_Y = \frac{R_{DC}}{2} \tag{9–15}$$

If the stator is delta connected as shown in Figure 9.24(c),

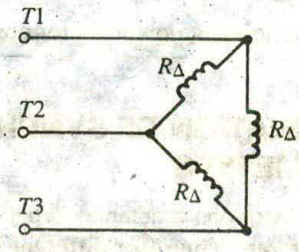
$$R_{DC} = \frac{R_\Delta \cdot 2R_\Delta}{R_\Delta + 2R_\Delta} \quad \Rightarrow \quad R_\Delta = 1.5R_{DC} \tag{9–16}$$



(a)



(b)



(c)

FIGURE 9.24
(a) DC test circuit for determining the equivalent resistance of a synchronous machine; (b) wye-connected stator; (c) delta-connected stator.

Note: To account for skin effect, the values of resistance obtained by Eqs. (9–15) and (9–16) should be multiplied by a skin-effect factor; although the value varies with different machines, a factor of 1.2 is a fair approximation. Thus,

$$R_a \approx 1.2 \times R_Y \quad \text{or} \quad R_a \approx 1.2 \times R_\Delta \tag{9–16a}$$

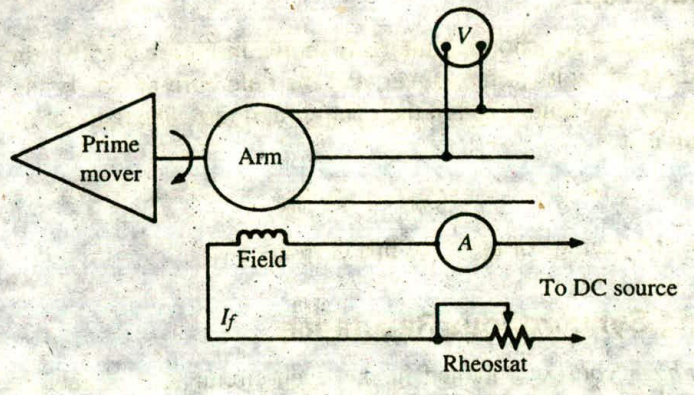
as appropriate.

Open-Circuit Test

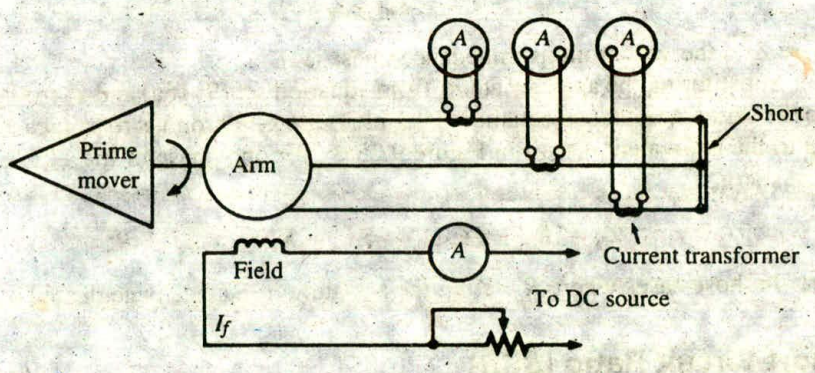
In the open-circuit test, the machine is driven at rated speed by its prime mover, and the field current is adjusted to obtain rated voltage at no load. The circuit is shown in Figure 9.25(a). The data set obtained from the open-circuit test is

$$I_{f,OC} \qquad V_{OC,line}$$

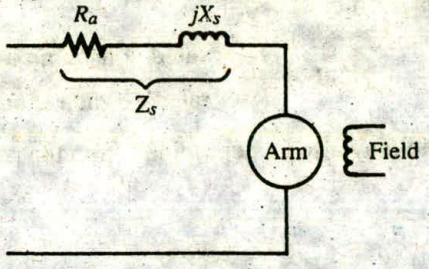
where: $I_{f,OC}$ = field current that produces $V_{OC,line}$ (A)
$V_{OC,line}$ = open-circuit line voltage (V)



(a)



(b)



(c)

FIGURE 9.25
(a) Connections for open-circuit test; (b) connections for short-circuit test; (c) equivalent circuit for one phase.

Short-Circuit Test

With the three stator leads short circuited, as shown in Figure 9.25(b), the machine is driven at rated speed by its prime mover, and the field current slowly increased from its lowest value to the value it had in the open-circuit test. The data set obtained from the short-circuit test is

$$I_{f,\text{OC}} \qquad I_{\text{SC,line}}$$

where $I_{\text{SC,line}}$ is the average of the three line currents (A).

Determining Synchronous Reactance

The ratio of *phase voltage* obtained in the open-circuit test, to the *phase current* obtained in the short-circuit test, determines the synchronous impedance/phase.

$$Z_s = \frac{V_{\text{OC,phase}}}{I_{\text{SC,phase}}} \qquad \Omega \tag{9–17}$$

where Z_s is the synchronous impedance/phase (Ω).

Substituting the appropriate R_a from equation set (9–16a) and Z_s from Eq. (9–17) into the series-impedance equation determines the synchronous reactance. Thus, referring to the equivalent circuit in Figure 9.25(c), for one phase of the synchronous machine stator,

$$\mathbf{Z}_s = R_a + jX_s \qquad \Rightarrow \qquad X_s = \sqrt{Z_s^2 - R_a^2} \tag{9–18}$$

Note: In those cases where $R_a \ll Z_s$, the armature resistance is neglected and $X_s \approx Z_s$.

Short-Circuit Ratio (SCR)

The short-circuit ratio of a synchronous generator is the ratio of field current required to obtain rated voltage at no load, to the field current required to obtain rated current when the armature is short circuited:

$$\text{SCR} = \frac{I_f \text{ for rated volts at no load}}{I_f \text{ for rated } I_a \text{ when shorted}} \tag{9–19}$$

The short-circuit ratio may also be determined from the reciprocal of the *per-unit* synchronous reactance:

$$\text{SCR} = \frac{1}{X_{\text{pu}}} \tag{9–20}$$

The short-circuit ratio is a measure of a machine's sensitivity to changes in loading. A machine with a high SCR is larger physically, weighs more, and costs more, but has a smaller voltage regulation than machines with lower SCRs. Machines with lower SCRs have larger synchronous reactance, and therefore require a voltage-regulating system that provides rapid and large changes in field current for small changes in load.

EXAMPLE 9.11 The short-circuit, open-circuit, and DC test data for a three-phase, wye-connected, 50-kVA, 240-V, 60-Hz synchronous alternator are

$$V_{OC,line} = 240.0 \text{ V} \qquad I_{SC,line} = 115.65 \text{ A}$$
$$V_{DC} = 10.35 \text{ V} \qquad I_{DC} = 52.80 \text{ A}$$

Determine (a) equivalent armature resistance; (b) synchronous reactance; (c) short-circuit ratio.

Solution

(a) $$R_{DC} = \frac{V_{DC}}{I_{DC}} = \frac{10.35}{52.8} = 0.1960\ \Omega \qquad R_Y = \frac{R_{DC}}{2} = \frac{0.1960}{2} = 0.098\ \Omega$$

$$R_a \approx 1.2 \times R_Y = 1.2 \times 0.098 = \underline{0.1176\ \Omega}$$

(b) $$Z_s = \frac{V_{OC,phase}}{I_{SC,phase}} = \frac{240/\sqrt{3}}{115.65} = 1.1981\ \Omega$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{(1.1981)^2 - (0.1176)^2} = \underline{1.1923\ \Omega}$$

(c) $$S_{base} = \frac{50{,}000}{3} \qquad V_{base} = \frac{240}{\sqrt{3}}$$

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(240/\sqrt{3})^2}{50{,}000/3} = 1.1520\ \Omega$$

$$X_{pu} = \frac{X_s}{Z_{base}} = \frac{1.1923}{1.1520} = 1.035$$

$$\text{SCR} = \frac{1}{X_{pu}} = \frac{1}{1.035} = \underline{0.966}$$

9.17 LOSSES, EFFICIENCY, AND COOLING OF AC GENERATORS

Losses in synchronous generators are the same as those presented for synchronous motors in Section 8.11. A power-flow diagram that illustrates the flow of power through a synchronous generator is shown in Figure 9.26. As indicated in the power-flow diagram, the total loss for the synchronous generator is

$$P_{loss} = P_{stray} + P_{f,w} + P_{core} + P_{fcl} + P_{scl} \qquad \textbf{(9–21)}$$

The overall efficiency of a synchronous generator is given by

$$\eta = \frac{P_{out}}{P_{in} + P_{field}} = \frac{P_{out}}{P_{out} + P_{losses}} \qquad \textbf{(9–22)}$$

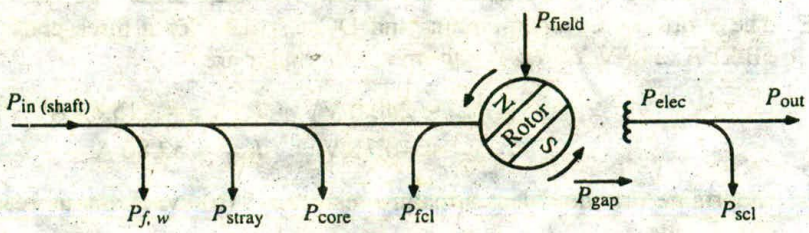


FIGURE 9.26
Power-flow diagram for a synchronous generator.

The field-power requirements for a synchronous generator vary from 1 kW for an alternator with a 25-kW output to 50 kW for a machine with a 7-MW output **[11]**. Since all power input to the field windings is a heat-power loss, many different cooling methods have evolved.

Cooling of Alternator Windings

The heat generated by the I^2R losses in the copper, and the hysteresis and eddy-current losses in the iron, are removed by ventilation. The simplest method, frequently used on small machines, uses an open housing and a fan mounted on the shaft to circulate ambient air through the windings. When the ambient temperature is relatively high, however, or the air is contaminated with moisture and dust, a totally enclosed air recirculating and cooling system is installed, as shown in Figure 9.27. The cooler, also called a heat exchanger, consists of a bank of finned tubes over which the hot air passes. Cold water circulating through the tubes absorbs the heat. The heat liberated from the windings is transferred to the air, which then passes through the cooler. A fan circulates the same air through the machine, absorbing heat from the windings and transferring it to the cooling water through the heat exchanger.

Hydrogen is often used as the cooling agent for large electrical machines. It has a thermal conductivity approximately seven times that of air, and because of its low density requires only one-tenth the amount of energy to blow it through the machine. The very considerable reduction in windage loss, coupled with the smaller physical dimensions of the hydrogen-cooled machine, more than offsets the extra cost of explosion-proof housings, complex shaft seals, and hydrogen-control equipment. The windings of some large alternators are cooled by circulating liquids through hollow tubular conductors that carry both the coolant and the current. The direct contact of coolant with the conductors results in excellent heat removal.

The new thrust in synchronous generator design is for highly efficient superconducting generators that use field coils wound with superconducting material. Prototypes of superconducting AC generators have been constructed and tested by several manufacturers **[7],[10]**.

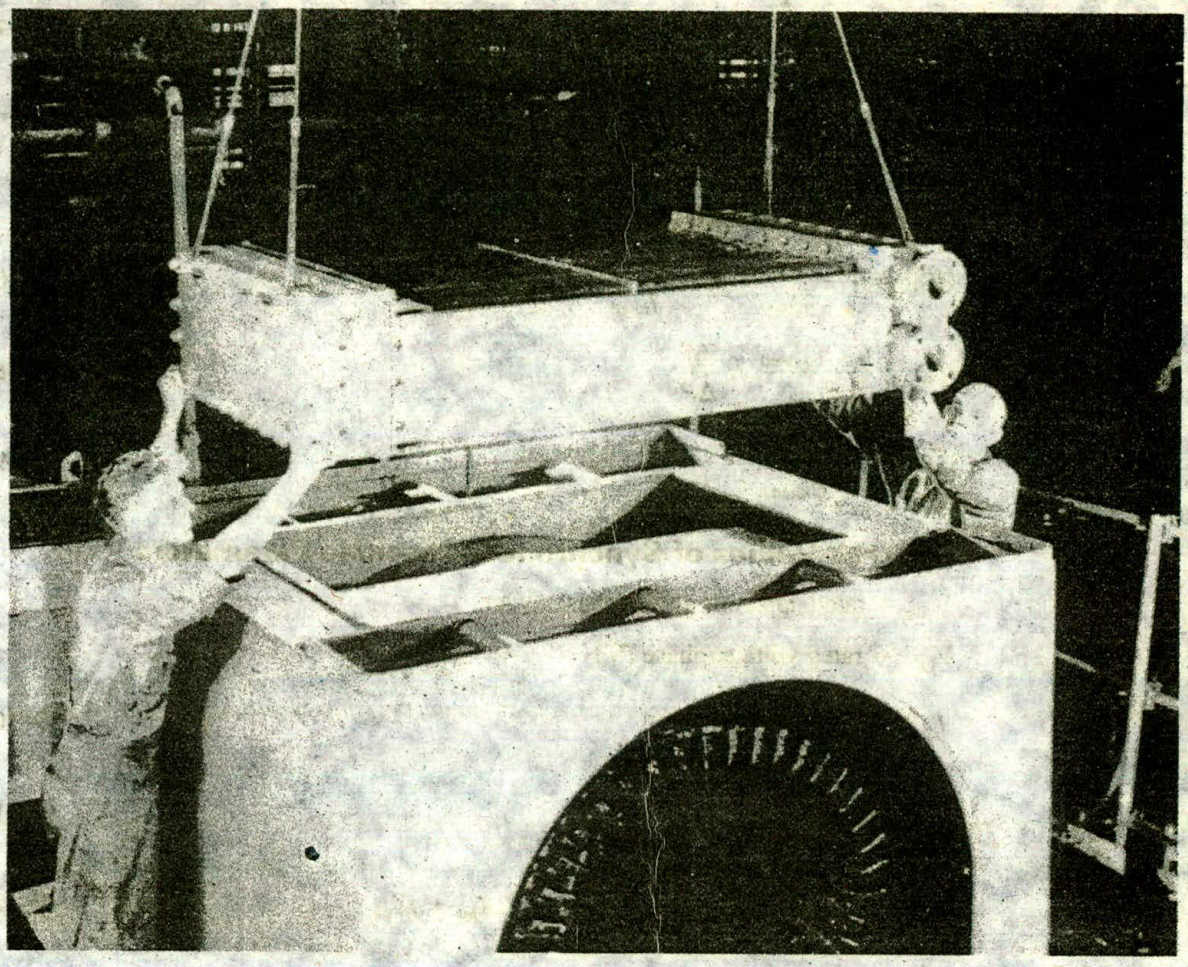


FIGURE 9.27
Installing a heat exchanger on top of the stator of a synchronous generator. (Courtesy GE Industrial Systems)

SUMMARY OF EQUATIONS FOR PROBLEM SOLVING

$$f = \frac{P \cdot n_s}{120} \quad \text{Hz} \tag{4–1}$$

$$E_f = n_s \Phi_f k_f \tag{9–2}$$

$$\mathbf{V}_T = \mathbf{E}_f - \mathbf{I}_a j X_s \tag{9–1a}$$

$$P_{\text{in},1\phi} = \frac{-V_T E_f}{X_s} \sin \delta \quad \text{(motor action)} \tag{8–14}$$

$$P_{\text{out.1}\phi} = \frac{+V_T E_f}{X_s} \sin\delta \qquad \text{(generator action)} \tag{9–4}$$

$$P_{\text{out.1}\phi} = \frac{V_T E_f}{X_d} \sin\delta + V_T^2 \left[\frac{X_d - X_q}{2X_d X_q}\right] \sin 2\delta \qquad \text{W} \tag{9–5}$$

Governor Characteristics

$$\text{GSR} = \frac{n_{\text{nl}} - n_{\text{rated}}}{n_{\text{rated}}} = \frac{f_{\text{nl}} - f_{\text{rated}}}{f_{\text{rated}}} \tag{9–6}$$

$$\text{GD} = \frac{f_{\text{nl}} - f_{\text{rated}}}{P_{\text{rated}}} = \frac{\Delta f}{\Delta P} \tag{9–8}$$

$$\text{GD} = \frac{\text{GSR} \times f_{\text{rated}}}{P_{\text{rated}}} = \frac{\Delta f}{\Delta P} \tag{9–9}$$

Per-Unit Values of Synchronous Machine Parameters

S_{base} = rated apparent power/phase (VA)

V_{base} = rated voltage/phase (V)

$$I_{\text{base}} = \frac{S_{\text{base}}}{V_{\text{base}}} \qquad \text{A} \tag{9–10}$$

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} = \frac{V_{\text{base}}^2}{S_{\text{base}}} \qquad \Omega \tag{9–11}$$

$$R_{\text{pu}} = \frac{R_a}{Z_{\text{base}}} \qquad X_{\text{pu}} = \frac{X_s}{Z_{\text{base}}} \qquad \mathbf{Z}_{\text{pu}} = R_{\text{pu}} + jX_{\text{pu}} \tag{9–12}$$

Voltage Regulation

$$\text{VR} = \frac{V_{\text{nl}} - V_{\text{rated}}}{V_{\text{rated}}} \times 100 \qquad \text{percent} \tag{9–13}$$

Determination of Parameters

$$R_{\text{DC}} = \frac{V_{\text{DC}}}{I_{\text{DC}}} \qquad \text{R}_{\text{Y}} = \frac{\text{R}_{\text{DC}}}{2} \qquad R_{\Delta} = 1.5R_{\text{DC}} \qquad \Omega \tag{9–14, 9–15, 9–16}$$

$$Z_s = \frac{V_{\text{OC.phase}}}{I_{\text{SC.phase}}} \qquad X_s = \sqrt{Z_s^2 - R_a^2} \qquad \Omega \tag{9–17, 9–18}$$

$$\text{SCR} = \frac{I_f \text{ for rated volts at no load}}{I_f \text{ for rated } I_a \text{ when shorted}} = \frac{1}{X_{\text{pu}}} \tag{9–20}$$

Losses and Efficiency

$$P_{loss} = P_{stray} + P_{f,w} + P_{core} + P_{fcl} + P_{scl} \quad \text{W} \tag{9–21}$$

$$\eta = \frac{P_{out}}{P_{in} + P_{field}} = \frac{P_{out}}{P_{out} + P_{losses}} \tag{9–22}$$

SPECIFIC REFERENCES KEYED TO TEXT

1. Daley, J. M. Design considerations for operating on-site generators in parallel with utility service. *IEEE Trans. Industry Applications,* Vol. IA-21, No. 1, Jan./Feb. 1985, pp. 69–80.
2. Darron, H. G., J. L. Koepfinger, J. R. Mather, and P. A. Rusche. The influence of generator loss of excitation on bulk power system reliability. *IEEE Trans. Power Apparatus and Systems,* Vol. PAS-94, No. 5, Sept./Oct. 1975, pp. 1473–1483.
3. Del Toro, V. *Electric Machines and Power Systems,* Sec. 5–4. Prentice Hall, Upper Saddle River, NJ, 1985.
4. General Electric Co. *Generator Instructions,* GEK-75511B, 1985.
5. Hubert, C. I. *Preventive Maintenance of Electrical Equipment.* Prentice Hall, Upper Saddle River, NJ, 2002.
6. Institute of Electrical and Electronic Engineers. *Test Procedures for Synchronous Machines,* IEEE Standard 115-1995, IEEE, New York, 1995.
7. Kumagai, M., T. Tanaka, K. Ito, Y. Watanabe, K. Sato, and Y. Gocho. Development of superconducting AC generator. *IEEE Trans. Energy Conversion,* Vol. EC-1, No. 4, Dec. 1986, pp. 122–129.
8. Mablekos, Van E. *Electric Machine Theory for Power Engineers.* Harper & Row, New York, 1980.
9. McPherson, G. *An Introduction to Electrical Machines and Transformers.* Wiley, New York, 1981.
10. Nasar, S. A. *Handbook of Electrical Machines,* McGraw-Hill, New York, 1987.
11. National Electrical Manufacturers Association. *Motors and Generators,* Publication No. MG-1-1998. NEMA, Rosslyn, VA, 1999.

REVIEW QUESTIONS

1. Using a two-pole synchronous machine and a corresponding phasor diagram, explain the motor-to-generator transition.
2. What would cause a synchronous generator to fail to build up voltage?
3. Assume that a turbine generator is operating alone at rated load when an accidental "dead-short" occurs at its three-phase output terminals. Will the machine come to a quick stop, slow down, or overspeed? Explain.
4. Explain why an alternator develops a countertorque when a load is connected to its armature.

5. Why is the rated voltage of an alternator, as indicated on the nameplate, always given for rated kVA at a specific power factor and frequency?
6. A 450-V, three-phase, 600-kW, 60-Hz generator *A* is to be paralleled with another identical machine *B*. Machine *B* is on the bus and carrying a load of 300 kVA at 60 Hz, 450 V, and 0.80 power-factor lagging. State the procedure you would follow in order to parallel machine *A* with machine *B*. Include in your statements the apparatus operated, meters observed, and the specific values of voltage and frequency for the incoming machine. Assume the prime mover has been started.
7. Explain why alternators in parallel cannot operate at different frequencies.
8. If the pointer of a synchroscope stops revolving and remains at the 30° position, what does it signify? What must be done to enable correct paralleling?
9. Explain the inherent dangers of attempting to parallel alternators when the phase angle between the two is relatively large. What is the maximum allowable error angle?
10. What methods can be used to determine the phase sequence of an incoming alternator? How is opposite phase sequence indicated?
11. How is the transfer of active power accomplished between alternators in parallel?
12. A 500-kVA, three-phase, 60-Hz, 450-V alternator is supplying a bus load of 400 kW at 0.90 power factor. State in outline form the procedure to be followed in order to have each machine take an equal share of the active power load. Mention the apparatus adjusted and the instruments observed as well as the values of voltage, frequency, and power of each machine during the transition and when the change is completed.
13. Two identical alternators with slightly drooping governor characteristics share an equal portion of the bus load at 60 Hz. (a) Sketch representative governor droop characteristics of each machine on one set of coordinate axis and draw the 60-Hz line. (b) With dotted lines appropriately drawn and lettered to indicate the transition steps, show and explain how a portion of the load of one machine may be transferred to the other machine. The bus must operate at 60 Hz when the transfer is completed.
14. Given two alternators with drooping governor characteristics, explain why admitting more energy to the prime mover of one machine results in an increase in the frequency of both machines.
15. Assume two alternators *A* and *B* are in parallel and carrying identical loads. Machine *A* has 2 percent speed regulation, and machine *B* has 3 percent speed regulation. Sketch the characteristic curves and indicate (a) the relative load division when the bus load increases; (b) the relative load division when the bus load decreases.
16. Explain the behavior of two alternators in parallel, one with governor droop and the other adjusted for isochronous operation, when the load on the bus increases and when it decreases.
17. Explain why two alternators with isochronous governors are inherently unstable when operating in parallel.
18. What is motoring? Is it harmful? How can it be detected and corrected?

19. Using phasor diagrams, explain why the transfer of active power between alternators in parallel causes a change in the division of reactive power between them.
20. How can the transfer of reactive power be accomplished between alternators in parallel? What instruments are used to observe this transfer?
21. Explain why adjustment of the field excitation of alternators in parallel affects the power factor of the machines.
22. Explain with the aid of a phasor diagram how it is possible for one alternator to operate at a leading power factor and another in parallel with it to operate at a lagging power factor when the bus power factor is unity.
23. Explain with the aid of a phasor diagram how it is possible for the individual power factors of two paralleled alternators and the power factor of the bus to be all lagging and all different.
24. (a) State in outline form the correct procedure for paralleling a 2000-V, 50-Hz, 1000-kVA alternator with another on the bus. Assume both machines have identical characteristics. Include the approximate values of the voltage and frequency of the incoming machine just prior to synchronizing. (b) Assuming the bus load is 800 kW at 0.8 power-factor lagging, what adjustments must be made to each machine in order that both machines will divide equally the active power of the bus? (c) What adjustments must be made to each machine in order that both machines will divide equally the reactive power of the bus?
25. What are the losses in an AC generator? How is the heat dissipated?

PROBLEMS

9–1/2 Determine the speed required to obtain a frequency of 50 Hz from a four-pole alternator.

9–2/2 A certain three-phase alternator operating at no load has an induced emf of 2460 V at 60 Hz. Determine the voltage and frequency if the pole flux and rotor speed are each increased by 10 percent.

9–3/2 An alternator operating at no load has a generated emf of 346.4 V/phase and a frequency of 60 Hz. If the pole flux is decreased by 15 percent and the speed is increased by 6.8 percent, determine (a) induced voltage; (b) frequency.

9–4/3 A 2400-V, 60-Hz, three-phase, six-pole, wye-connected synchronous generator is connected to an infinite bus and is supplying 350 kW at a power angle of 28.2°. The stator has a synchronous reactance of 12.2 Ω/phase. Neglecting losses, determine (a) input torque to the alternator; (b) excitation voltage per phase; (c) armature current; (d) active and reactive components of power; (e) power factor.

9–5/3 A four-pole, 600-V synchronous generator is connected to an infinite bus that supplies a 60-Hz, 600-V, 2000-kVA, 80.4 percent power-factor load. The generator is driven by a steam turbine that delivers 1955 lb-ft of torque

to the generator shaft, causing a power angle of 36.4°. The synchronous reactance is 1.06 Ω/phase. Neglecting losses, determine (a) mechanical power input to the generator rotor; (b) excitation voltage per phase; (c) armature current; (d) active and reactive components of apparent power delivered to the bus; (e) generator power factor.

9–6/3 A six-pole, 75-kVA, 340-V, 60-Hz, wye-connected diesel generator is supplying a bus load of 54.5 kVA at 78.9 percent power-factor lagging, 220 V, and 60 Hz. The armature has a synchronous impedance of $0.18 + j0.92$ Ω/phase. Determine (a) armature current; (b) excitation voltage per phase; (c) power angle; (d) shaft torque supplied by the diesel engine (neglect losses).

9–7/11 The ratings of two turbine generators *A* and *B* are 300-kW, 3.5 percent speed regulation and 600-kW, 2.5 percent speed regulation, respectively. The machines are operating in parallel at 60 Hz, 480 V. Machine *A* is supplying 260 kW, and machine *B* is supplying 590 kW. (a) Sketch a one-line diagram for the system. (b) On the same coordinate axes sketch the approximate governor-droop characteristics of both machines and label the curves. (c) Opening of a distribution breaker results in a remaining bus load of 150 kW. Determine the new frequency and the active power load carried by each machine.

9–8/11 Two 600-kW, 60-Hz, diesel-driven synchronous generators *A* and *B* have governor speed regulations of 2.0 and 5.0 percent, respectively. Both machines are in parallel and supplying equal shares of a 1000-kW bus load at 57 Hz. (a) Sketch the approximate governor characteristics for both machines on one set of coordinate axes, and indicate the operating frequency. Label both curves. (b) On the same diagram, approximate a new operating condition that assumes the load on the bus decreases to a total of 400 kW. (c) Determine the new frequency and the new load distribution for the conditions in (b).

9–9/11 A 700-kW, 60-Hz generator *A*, with 2.0 percent speed regulation, is operating in parallel with generator *B* of equal kilowatt rating, but whose speed regulation is 6.0 percent. The total bus load of 1000 kVA at 60 Hz, 2400 V, and 80.6 percent power-factor lagging is divided equally between the two machines. If a 200-kVA, 60.0 percent power-factor load is disconnected from the bus, determine (a) the new operating frequency; (b) the active power load on each machine.

9–10/11 Three 600-kW, 60-Hz, 480-V synchronous machines are in parallel, each supplying the following loads: generator *A*, 200 kW; generator *B*, 100 kW; and generator *C*, 300 kW. The speed regulations for machines *A*, *B*, and *C* are 2.0, 2.0, and 3.0 percent, respectively. The system frequency is 60 Hz. If the system load increases to 2000 kVA at 70.0 percent power-factor lagging, determine the new operating frequency and active power load carried by each machine.

9–11/11 Three 25-Hz turbine generators *A*, *B*, and *C* are connected in parallel and operating at 25 Hz, 2400 V. Generator *A* is rated at 600 kW and has a speed regulation of 2.0 percent. Generator *B* is rated at 500 kW and has a speed

regulation of 1.5 percent. Generator *C* has a rating of 1000 kW and has a speed regulation of 4.0 percent. The loads on machines *A*, *B*, and *C* are 200 kW, 300 kW, and 400 kW, respectively. If an additional 800 kW is connected to the bus, determine (a) the new frequency; (b) the load on each generator.

9–12/12 Two synchronous machines *A* and *B* are in parallel and taking equal shares of the bus load at 480 V, 60 Hz. The power factor of the bus is 76.6 percent, and the power angle of each machine is 20.0°. (a) Using a straight edge and protractor draw (on graph paper) separate phasor diagrams for each machine. (b) Assuming the governor controls of both machines are adjusted so that machine *A* takes 25 percent of the bus load, construct the new conditions on the phasor diagrams. Show all construction lines and determine the new power factor of each machine.

9–13/12 A three-phase turbine generator in parallel with an infinite bus has a power angle of 27°, a power factor of 94.2 percent, and a line voltage of 600 V. (a) Using a straight edge, protractor, and graph paper, draw the phasor diagram to scale. (b) Assume the turbine is tripped, the generator excitation is not changed, and a mechanical load equal to 60 percent of the load it delivered when acting as a generator is applied to the shaft. Show the changes on the phasor diagram and determine the new power factor and the new power angle.

9–14/12 Two identical 480-V, 60-Hz diesel generators DG 1 and DG 2 are in parallel and are taking equal shares of the total bus load of 3000 kW at 80.2 percent power-factor lagging. (a) Sketch separate phasor diagrams showing the original conditions for each machine. (b) The governor controls of both machines are manipulated so that DG 1 takes two-thirds of the bus load. Assume the bus voltage, frequency, and field excitation do not change. Using ruler and compass, construct to scale, on the diagrams in (a), the new conditions for the respective machines. Show all construction lines, and using a protractor, determine the new power angle and new power factor of each machine.

9–15/12 A turbine generator TG 1 is paralleled with an infinite bus and is carrying 100 kW at 60.2 percent power-factor lagging. (a) Sketch the phasor diagram for the given conditions. (b) Assume the governor of TG 1 is adjusted so as to double the energy input to its prime mover. Using dotted lines, construct the new conditions on the phasor diagram in (a) and determine the new power angle. (c) Assume the conditions in (b) are further modified by a 25 percent increase in the pole flux of TG 1. Using broken lines and the same phasor diagram construct the new conditions and determine the corresponding power angle.

9–16/12 Two 450-V, three-phase, 60-Hz, 600-kW synchronous machines *A* and *B* are operating in parallel and share equally the active and reactive power of the bus. The bus load is 1000 kVA, operating at a power factor of 80.4 percent lagging. If the field excitation of alternator *A* is adjusted to cause its power

factor to increase to 85.0 percent lagging, at what power factor will the other machine operate?

9–17/12 Two alternators *A* and *B* are in parallel supplying a 560-kVA, 480-V, 60-Hz bus load at a power factor of 82.8 percent lagging. Machine *A* is carrying 75 percent of the active power load at a power factor of 92.4 percent lagging. Determine the active and reactive power components supplied by each machine.

9–18/12 Two alternators *A* and *B* are in parallel and supplying a 600-V, 60-Hz bus load consisting of a 270-kVA unity power-factor load, a group of induction motors totaling 420 kVA at 89.4 percent power factor, and a synchronous motor that draws 300 kVA at 92.3 percent power-factor leading. If machine *A* is carrying 60 percent of the total active power load at a power factor of 70.4 percent lagging, determine the active and reactive components supplied by machine *B*.

9–19/12 Three identical 1000-kW diesel generating sets operating in parallel take equal shares of a 1500-kW bus load at 450 V, 60 Hz, and 80.0 percent power-factor lagging. The governor speed regulation of the three machines are adjusted to 1.0, 2.0, and 3.0 percent for prime movers *A*, *B*, and *C*, respectively. (a) Sketch representative governor droops of the three machines on one set of coordinate axes, and label the curves. (b) Calculate the new operating frequency and active power load on each generator if the total bus load is increased to 1850 kW. (c) If the system power factor for the new conditions is 95.0 percent lagging, machine *A* has a power factor of 90.0 percent lagging, and machine *B* has a power factor of 60.0 percent lagging, what is the power factor of machine *C*?

9–20/14 Determine the per-unit resistance and per-unit synchronous reactance of a 37.5-kVA, 480-V, 60-Hz, wye-connected alternator whose synchronous impedance is $1.47 + j7.68\ \Omega$/phase.

9–21/14 A 5000-kVA, 13,800-V, 60-Hz, wye-connected alternator has a synchronous reactance of 55.2 Ω/phase. Its resistance is negligible. Determine the per-unit impedance.

9–22/15 A 250-kVA, 480-V, three-phase, four-pole, 60-Hz synchronous generator with a synchronous reactance of 0.99 Ω/phase is operating at rated conditions and 83.2 percent power-factor lagging, and has the magnetization curve shown in Figure 9.28. Determine (a) excitation voltage; (b) power angle; (c) open-circuit phase voltage; (d) voltage regulation; (e) no-load voltage if the field current is reduced to 60 percent of its value at rated load.

9–23/15 Repeat Problem 9–22/15 assuming that the power factor is 83.2 percent leading.

9–24/15 A 1000-kVA, 4800-V, 60-Hz, 3600-r/min, wye-connected alternator with a synchronous reactance of 14.2 Ω/phase is supplying 600 kVA at 4800 V to a bus load operating at a power factor of 95.2 percent lagging. Assume the

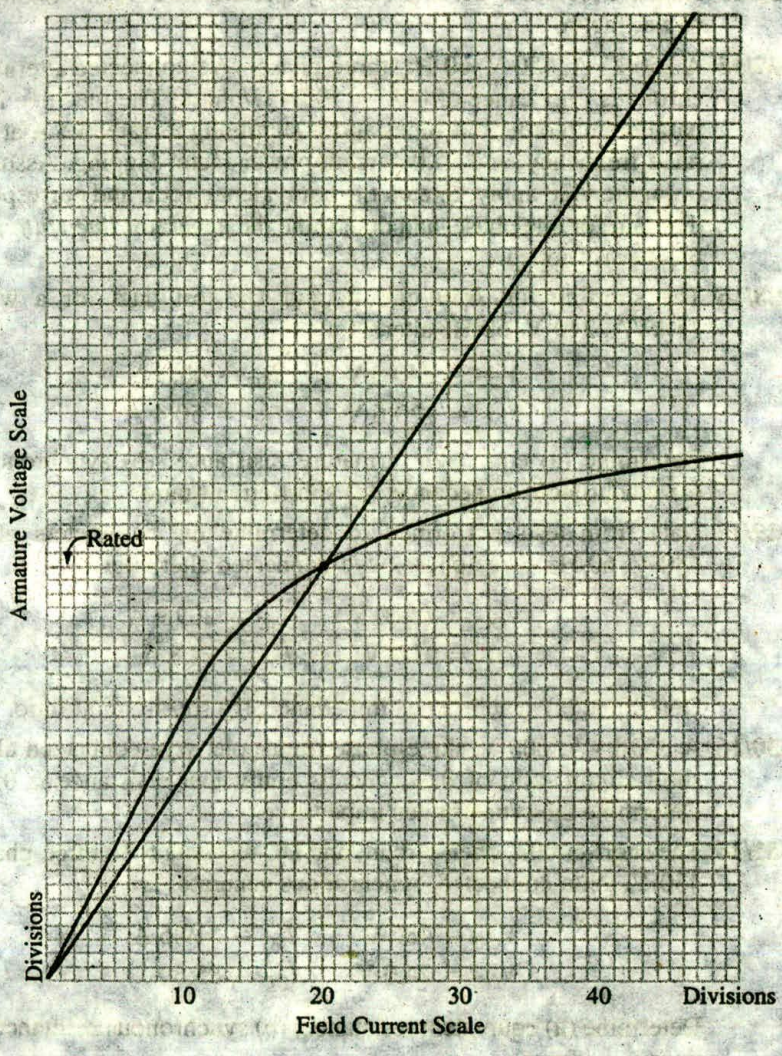


FIGURE 9.28
Magnetization curve for Problems 9–22, 9–23, 9–24, 9–25, 9–26, and 9–27.

curves in Figure 9.28 apply. Determine the open-circuit *line voltage* of the generator if a sudden short circuit tripped the generator breaker.

9–25/15 A 150-kVA, 240-V, delta-connected, 60-Hz synchronous generator has a synchronous impedance of $0.094 + j0.320\ \Omega$/phase, and is operating at rated conditions and 75.2 percent power-factor lagging. Assume the curves in Figure 9.28 apply. Determine the voltage regulation.

9–26/15 Repeat Problem 9–25/15 assuming operation is at rated conditions and unity power factor.

9–27/15 A 250-kVA, 450-V, 60-Hz, three-phase, wye-connected alternator has a synchronous impedance of $0.05 + j0.24\ \Omega$/phase. The machine is operating in parallel with other machines and is supplying its rated kVA at rated voltage, rated frequency, and 85.0 percent power-factor lagging. Assume the curves in Figure 9.28 apply. Sketch the equivalent circuit and corresponding phasor diagram for one phase, and determine the generator *line voltage* if the circuit breaker is tripped.

9–28/16 The short-circuit, open-circuit, and DC test data for a wye-connected, 25-kVA, 240-V, 60-Hz alternator are

$$V_{OC} = 240.0\ \text{V} \qquad I_{SC} = 60.2\ \text{A}$$
$$V_{DC} = 120.6\ \text{V} \qquad I_{DC} = 50.4\ \text{A}$$

Determine (a) equivalent armature resistance; (b) synchronous reactance; (c) synchronous impedance; (d) short-circuit ratio.

9–29/16 Data from tests performed to determine the parameters of a 200-kVA, 480-V, 60-Hz, three-phase, wye-connected alternator are

$$V_{OC} = 480.0\ \text{V} \qquad I_{SC} = 209.9\ \text{A}$$
$$V_{DC} = 91.9\ \text{V} \qquad I_{DC} = 72.8\ \text{A}$$

Determine (a) synchronous impedance; (b) short-circuit ratio.

9–30/16 A 350-kVA, 600-V, three-phase, four-pole, wye-connected alternator has a short-circuit ratio of 0.87, and its equivalent resistance is $0.644\ \Omega$/phase. Determine the synchronous impedance.

9–31/16 The short-circuit, open-circuit, and DC test data for a three-phase, 125-kVA, 480-V, delta-connected, synchronous generator are

$$V_{OC} = 480\ \text{V} \qquad I_{SC} = 519.6\ \text{A}$$
$$V_{DC} = 24.0\ \text{V} \qquad I_{DC} = 85.6\ \text{A}$$

Determine (a) equivalent resistance; (b) synchronous reactance; (c) synchronous impedance.