14. Show that in the case of progressive longitudinal waves
Particle velocity=Wave velocity × Compression. *(Panjab)*

15. Deduce the equation of a simple harmonic wave travelling in the positive x direction in the form

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

and explain how energy is distributed in such a progressive wave. *(Delhi)*

CHAPTER 5

Velocity of Sound

5.1. Origin of Sound

Sound is produced by a vibrating body. When the gong of a bell is struck with a hammer, sound is produced. The bell is set into vibration and sound is propagated through air. These vibrations reach the ear and the ear drum is set into vibration. These vibrations are communicated to the brain. By touching the gong with the hand, one can feel the vibration of the gong. Similarly the cycle bell produces sound due to the vibrations produced by the gong. When the cycle bell is touched with hand, the vibrations are stopped and the bell does not produce sound. If a pith ball pendulum is held in contact with the edges of a vibrating gong, the pith ball moves to and fro. This shows that the gong vibrates as long as the sound is produced.

A tuning fork is set into vibration by striking one of its prongs against a rubber pad. The vibration of the prongs of the tuning fork can be seen. Similarly, vibrating strings, air columns, vibrating plates etc. produce sound.

5.2. Material Medium is a Necessity

It can be proved by means of an experiment that a material medium is a necessity for the propagation of sound waves. In the absence of a medium, no sound waves can travel.

Experiment. Take a jar and fix a bell inside it as shown. The connecting wires pass through an air tight rubber cork fixed to the neck of the jar. The electric bell is connected to a battery and a key outside the jar (Fig. 5.1). The jar is placed on the platform of an air pump. Insert the key. The bell rings and the sound is heard. When the air in the jar is removed gradually, the sound becomes fainter although the same current is passing

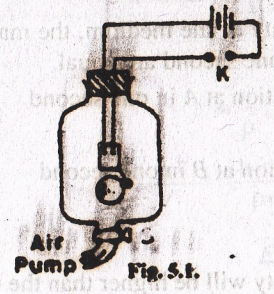


Fig. 5.1.

through the bell. After some time, when the pressure of air inside the jar is extremely low, only a very feeble sound is heard. It is not possible to create perfect vacuum with an air pump. In vacuum, no sound travels. Hence, material medium is a necessity for the propagation of sound waves.

5.3. Velocity of Longitudinal Waves in Gases

Consider a long tube of area of cross-section a. Let A and B be two cross-sections of the tube. Suppose, in the region of normal density, the medium moves from right to left along the length of the tube with a velocity U. The sound waves travel from left to right with a velocity U. In such a case, the

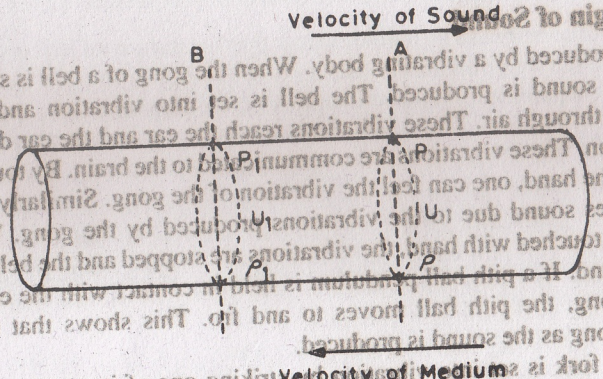


Fig. 5.2.

positions of the condensations and rarefactions in the tube with respect to the ground will remain fixed. For any small region, the pressure, velocity and density of the medium remain constant. The values of pressure, velocity and density of the medium will be different at different cross-sections of the tube.

Let A correspond to the region of normal density and B correspond to the region of condensation. At A, the pressure is P, density is ρ and velocity of the medium is U. At B, the pressure is P_1, density is ρ_1 and velocity of the medium is U_1.

As there is no change in the average density of the medium, the masses of the medium crossing the sections A and B in one second are equal.

Mass of the medium entering the cross-section at A in one second

$$= a \times U \times \rho$$

Mass of the medium leaving the cross-section at B in one second

$$= a \times U_1 \times \rho_1$$

$$\therefore \quad m = a\,U\,\rho = aU_1\,\rho_1 \qquad \ldots (1)$$

If B is at a region of condensation, its density will be higher than the normal density ρ at A.

$$\therefore \quad \rho_1 > \rho$$

and $$U_1 < U$$

The momentum per second of the medium entering at $A = mU$

The momentum per second of the medium leaving at $B = mU_1$

Change in momentum per second $= mU - mU_1$. This change in momentum per second is due to the difference in pressure $(P_1 - P)$ between A and B. Since the tube has an area of cross section a, the force applied,

$$F = (P_1 - P)\,a.$$

Also, according to Newton's second law of motion, rate of change of momentum is equal to the applied force.

$$\therefore \quad (P_1 - P)\,a = mU - mU_1$$

$$(P_1 - P)\,a = m(U - U_1)$$

$$(P_1 - P)\,a = mU\left(1 - \frac{U_1}{U}\right) \qquad \ldots (1)$$

Substituting the value of $m = a\,U\,\rho$,

and $\dfrac{U_1}{U} = \dfrac{\rho}{\rho_1}$, from equation (1), we get

$$(P_1 - P)\,a = aU^2\,\rho\left(1 - \frac{\rho}{\rho_1}\right)$$

$$(P_1 - P)\,a = U^2\,\rho\left(\frac{\rho_1 - \rho}{\rho_1}\right)$$

$$\therefore \quad U^2 = \frac{(P_1 - P)}{\left(\dfrac{\rho_1 - \rho}{\rho_1}\right)\rho} \qquad \ldots (2)$$

The bulk modulus of elasticity of the medium,

$$E = \frac{(P_1 - P)}{\left(\dfrac{V - V_1}{V}\right)}$$

But $$V \propto \frac{1}{\rho}$$

$$\therefore \quad \frac{V - V_1}{V} = \frac{\rho_1 - \rho}{\rho_1}$$

$$\therefore \quad E = \frac{(P_1 - P)}{\left(\dfrac{\rho_1 - \rho}{\rho_1}\right)} \qquad \ldots (3)$$

From equations (2) and (3)

$$U^2 \rho = E$$

$$U^2 = \frac{E}{\rho}$$

or $$U = \sqrt{\frac{E}{\rho}}$$

The formula is true only in the case of *plane waves* where the original disturbance is of very small amplitude. This requirement is fulfilled by sound waves of ordinary intensity. The relation also holds good for the velocity of simple harmonic plane waves through homogeneous isotropic media *viz.*, solids, liquids and gases.

This relation is also applicable to torsional waves in solids and transverse vibrations in strings. However, in the case of transverse vibrations of bars and plates, the formula needs modification.

5.4. Newton's Formula for Velocity of Sound

The velocity of sound in a medium—solid, liquid or gas, depends on the elasticity and density of the medium.

$$U = \sqrt{\frac{E}{\rho}}$$

where U is the velocity of sound in the medium, E the elasticity and ρ the density of the medium.

Newton also found that the elasticity of air is equal to its pressure. He assumed that the temperature of air remains constant when sound waves travel through air. The process is isothermal and Boyle's law can be applied. At the point of condensation, pressure increases and volume decreases.

Suppose, initial pressure $= P$
Initial volume $= V$
Increase in pressure $= p$
Decrease in volume $= v$
Final pressure $= (P + p)$
Final volume $= (V - v)$

$$\therefore \quad (P + p)(V - v) = PV$$

$$PV + pV - Pv - pv = PV$$

(pv is negligibly small)

$$pV = Pv \text{ or } P = \frac{pV}{v} \qquad \ldots (1)$$

From the definition of elasticity of the medium

$$E = \frac{\text{Change in pressure}}{\dfrac{\text{Change in volume}}{\text{Original volume}}}$$

$$E = \frac{P}{\dfrac{v}{V}} = \frac{pV}{v} \qquad \ldots (2)$$

Equating (1) and (2)

$$E = P$$

$\therefore$ Substituting $E = P$ in the original formula

$$U = \sqrt{\frac{P}{\rho}} \qquad \ldots (3)$$

Taking normal pressure $P = 0{\cdot}76$ m of Hg

$$= 0{\cdot}76 \times 136 \times 10^3 \times 9{\cdot}81 \ N/m^2$$

and density of air at NTP $= 1 \cdot 293 \ kg/m^3$

$$U = \sqrt{\frac{0{\cdot}76 \times 13{\cdot}6 \times 10^3 \times 9{\cdot}81}{1{\cdot}293}} \text{ cm/s } 280 \text{ m/s}$$

The experimental value determined from various experiments gives the velocity of sound at NTP

$$= 332 \text{ m/s.}$$

Therefore, there is a difference of about 52 m/s between the theoretical and the experimental values. The large difference cannot be attributed to the experimental errors. Newton was unable to explain the error in his formula and the correction was explained by a French scientist Laplace.

Laplace correction. According to Laplace, when sound waves travel through air, there is condensation and rarefaction in the particles of the medium. Where there is condensation, particles come near each other and are heated up. Where there is rarefaction, particles go apart and there is fall in temperature. Therefore, the temperature does not remain constant when sound waves travel through air or any other gas. The process is not isothermal but it is adiabatic. The total quantity of heat of the system as a whole remains constant. It neither gains nor loses any heat to the outside. During an adiabatic process

$$PV^\gamma = \text{constant}$$

Suppose, initial pressure $= P$
Initial volume $= V$
Change in pressure $= p$
Change in volume $= v$

$$PV^\gamma = (P + p)(V - v)^\gamma$$

$$PV^\gamma = P\left[1+\frac{p}{P}\right]V^\gamma\left[1-\frac{v}{V}\right]^\gamma$$

$$1 = \left[1+\frac{p}{P}\right]\left[1-\frac{v}{V}\right]^\gamma$$

$$1 = \left[1+\frac{p}{P}\right]\left[1-\frac{\gamma v}{V}\right]$$

$$1 = 1 + \frac{p}{P} - \frac{\gamma v}{V} - \frac{V}{P}\frac{p}{V}\frac{v}{}$$

Neglecting $\frac{\gamma pv}{PV}$, $\frac{p}{P} = \frac{\gamma v}{V}$

or $$\gamma P = \frac{pV}{v} \quad \ldots (1)$$

From the definition of elasticity

$$E = \frac{pV}{v} \quad \ldots (2)$$

Equating (1) and (2)

$$E = \gamma P$$

Substituting this value of $E = \gamma P$, in the equation

$$U = \sqrt{\frac{E}{\rho}}, \text{ we get}$$

$$U = \sqrt{\frac{\gamma P}{\rho}} \quad \ldots (3)$$

Here γ is the ratio between the two specific heats of air or a gas

For air $\gamma = 1{\cdot}42$

The velocity of sound in air at NTP

$$U = \sqrt{\frac{1{\cdot}42 \times 0{\cdot}76 \times 13{\cdot}6 \times 10^3 \times 9{\cdot}81}{1{\cdot}293}}$$

$$= 332{\cdot}8 \text{ m/s.}$$

This value is in agreement with the experimental value.

Hence the correct formula for the velocity of sound in air or a gas is

$$U = \sqrt{\frac{\gamma P}{\rho}}$$

5.5. Effect of Temperature

The density of air or a gas changes with the change in temperature.

At 0° C $$U_0 = \sqrt{\frac{\gamma P}{\rho_0}} \quad \ldots (1)$$

At t^0 C $$U_t = \sqrt{\frac{\gamma P}{\rho_t}} \quad \ldots (2)$$

Dividing (2) by (1)

$$\frac{U_t}{U_0} = \sqrt{\frac{\rho_0}{\rho_t}} = \sqrt{1+\frac{t}{273}} = \sqrt{\frac{T}{T_0}}$$

$$U \propto \sqrt{T}$$

Therefore, velocity of sound in air or a gas is directly proportional to the square root of the absolute temperature.

Example 5.1. *Calculate the increase in the velocity of sound in air per degree celsius rise in temperature.*

or

Prove that the velocity of sound in air increases by 0·61 m/s *for each degree celsius rise in temperature.*

$$\frac{U_t}{U_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273}} = \left(1+\frac{t}{273}\right)^{\frac{1}{2}}$$

$$U_t = U_0\left(1+\tfrac{1}{2}\times\frac{t}{273}\right) = U_0\left(1+\frac{t}{546}\right)$$

(*i*) $U_0 = 332$ metres/second, $t = 1$

$$U_t = 332\left(1+\frac{1}{546}\right)$$

$$U_t = 332 + 0{\cdot}61 = 332{\cdot}61 \text{ m/s}$$

$\therefore U_t - U_0 = 0{\cdot}61$ **m/s**

Example 5.2. *The velocity of sound in air at 14°C is 340* metres/second. *What will it be when the pressure of the gas is doubled and its temperature is raised to 157.5° C ?*

Change of pressure has no effect on the velocity of sound.

Here, $t_1 = 14°C$, $U_1 = 340$ metres/second.

$t_2 = 157{\cdot}5°C$ $U_2 = ?$

$$\frac{U_2}{U_1} = \sqrt{\frac{273+t_2}{273+t_1}}$$

$$\frac{U_2}{340} = \sqrt{\frac{273+157{\cdot}5}{273+14}} = \sqrt{\frac{430{\cdot}5}{287}}$$

$$\sqrt{\frac{3}{2}} = \sqrt{1{\cdot}5}$$

$$U_2 = 340\sqrt{1{\cdot}5} = \textbf{416·14 metres/second.}$$

Example 5.3. *The velocity of sound in air at 16°C is 340 metres/second. Find the wave-lengths in air of a note of frequency 680 at 16° C and 51 °C.*

(1) Suppose the wavelength = λ_1 at 16°C

$$U_1 = 340 \text{ metres/second}, \quad n = 680$$

$$\therefore \quad U_1 = n\lambda_1$$

or
$$\lambda_1 = \frac{U_1}{n} = \frac{340}{680}$$

$$= \textbf{0·50 metre}$$

(2) Suppose the wavelength = λ_2 at 51°C

$$t_1 = 16^0\text{C}, \quad U_1 = 340 \text{ metres/second}$$

$$t_1 = 51^0\text{C}, \quad U_2 = ?$$

$$\frac{U_2}{U_1} = \sqrt{\frac{273 + t_2}{273 + t_1}}$$

$$\frac{U_2}{340} = \sqrt{\frac{273 + 51}{273 + 16}} = \sqrt{\frac{324}{289}} = \frac{18}{17}$$

$$U_2 = \frac{340 \times 18}{17} = 360 \text{ metres/second}$$

$$\lambda_2 = \frac{U_2}{n} = \frac{360}{680} = \textbf{0·53 metre.}$$

Example 5.4. *Find the temperature at which sound travels in hydrogen with the same velocity as in oxygen at 1000°C. Assume γ to be the same for the two gases.* (Delhi)

Here, for oxygen

$$T_1 = 273 + 1000$$

$$= 1273 \text{ K}$$

$$\text{Density} = \rho_1$$

For hydrogen

$$T_2 = ?$$

$$\text{Density} = \rho_2$$

$$\frac{\rho_1}{\rho_2} = 16$$

Also
$$\frac{T_1}{\rho_1} = \frac{T_2}{\rho_2}$$

$$T_2 = \frac{T_1 \rho_2}{\rho_1}$$

or
$$T_2 = \frac{1273 \times 1}{16}$$

$$= 79{\cdot}56 \text{ K}$$

$$= -193{\cdot}44^o \text{ C}$$

5.6. Effect of Pressure

Velocity of sound in air is independent of change of pressure.

$$\text{Volume} = \frac{\text{mass}}{\text{density}} = \frac{m}{\rho}$$

$$\therefore \quad PV = \frac{Pm}{\rho} = \text{constant, if the temperature is constant.}$$

Since, the mass remains constant, $\frac{P}{\rho}$ is constant

In the formula

$$U = \sqrt{\frac{\gamma P}{\rho}}$$

$\sqrt{\frac{P}{\rho}}$ is constant.

Therefore, the velocity of sound in air or a gas is independent of changes in pressure provided the temperature remains constant.

5.7. Effect of Density of the Medium

Consider two media, of densities ρ_1 and ρ_2

$$U_1 = \sqrt{\frac{\gamma P}{\rho_1}} \quad \ldots (1)$$

and
$$U_2 = \sqrt{\frac{\gamma P}{\rho_2}} \quad \ldots (2)$$

Dividing (2) by (1)

$$\frac{U_2}{U_1} = \sqrt{\frac{\rho_1}{\rho_2}}$$

or
$$U \propto \frac{1}{\sqrt{\rho}} \quad \ldots (3)$$

Velocity of sound in air or a gas is inversely proportional to the square root of the density of the medium.

5.8. Effect of Humidity

The density of water vapour at NTP $= \frac{18}{22 \cdot 4} = 0{\cdot}8$ kg/m^3 whereas the density of dry air at NTP $= 1{\cdot}293$ kg/m^3. Therefore, water vapour has a density less than the density of dry air. Consequently the density of moist air is less than the density of dry air. As the velocity of sound is more in a medium of lesser density, velocity of sound in moist air is more than the velocity of sound in dry air.

5.9. Effect of Wind

If the wind is blowing in the direction of propagation of the wave, it will increase the velocity of sound. On the other hand, if the wind is blowing in a direction opposite to the direction of propagation of the sound waves, it will decrease the velocity of sound in air.

Example 5.5. *Prove that the velocity of sound in hydrogen is four times the velocity of sound in oxygen.*

Density of oxygen $= D_1$

Velocity of sound in oxygen $= U_1$

Density of hydrogen $= D_2$

Velocity of sound in hydrogen $= U_2$

$$\frac{D_1}{D_2} = \frac{16}{1} \text{ and } \frac{U_2}{U_1} = \sqrt{\frac{D_1}{D_2}} = \sqrt{\frac{16}{1}} = 4$$

$$\therefore \quad U_2 = 4 \times U_1.$$

5.10. Velocity of Sound in Water

According to Newton's formula, the velocity of sound in water is given by

$$U = \sqrt{\frac{E}{\rho}}$$

Volume elasticity of water $= 2{\cdot}23 \times 10^9$ N/m^2

and density of water $= 1$ g/cc

$$U = \sqrt{\frac{2{\cdot}23 \times 10^9}{10^3}} = 10^3 \text{ kg/m}^2$$

$= 1493$ metres/s.

The velocity of sound in water is approximately four times the velocity of sound in air.

Experiment. This experiment is due to Colladon and Strum who performed the experiment in a lake at Geneva in 1826.

Two boats A and B were stationed at a distance of about 13,487 metres. A large bell G was suspended in water and a hammer H was arranged as shown in Fig. 5.3. At the station B, another boat B was arranged with a microphone and an ear piece. The person sitting in A presses the handle at the point P. The

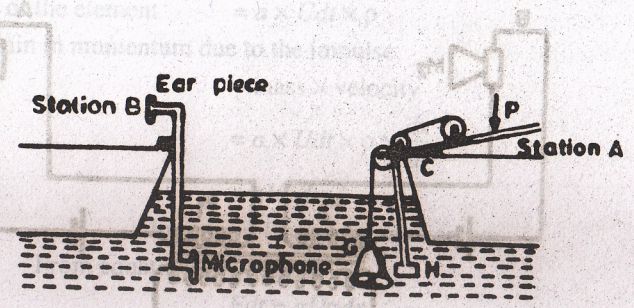


Fig. 5.3.

hammer H strikes against the gong and also a flash of light is produced at C on the boat A. The observer at B, observes the flash at C at the time the hammer H strikes against the bell G. After some time the sound reaches the microphone M and is picked up by the ear piece. The observer at B finds the time interval between the instant he sees the flash of light and the instant the sound reaches him through water. Knowing the distance and time, the velocity of sound in water can be determined.

The value of the velocity of sound in water was found to be 1435 metres per second which agrees with the theoretical result within the limits of experimental error.

5.11. Velocity of Sound in Air

In this case, two microphones M_1 and M_2 are fixed at two stations about 10 kilometres apart (Fig. 5.4). Microphone M_1 is connected to a primary coil P_1 of a transformer and a battery in the circuit. Microphone M_2 is connected to a primary coil P_2 of the transformer and a battery in the circuit. The secondary coil of the transformer (common for P_1 and P_2) is connected to a sensitive galvanometer. Light from the bulb after reflection from the mirror of the galvanometer falls on a photographic film wrapped on a rotating drum. The drum rotates with a uniform speed.

When a gun is fired at the station A, the microphone M_1 receives the sound. A change in current in the primary coil P_1 is produced and an induced current is produced in the secondary coil S. The galvanometer coil is deflected and the spot on the drum moves up as shown in Fig. 5.4. After some time, sound

reaches the microphone M_2 and the change in current in the primary coil P_2 produces an induced current in the secondary coil S. Again deflection, in the

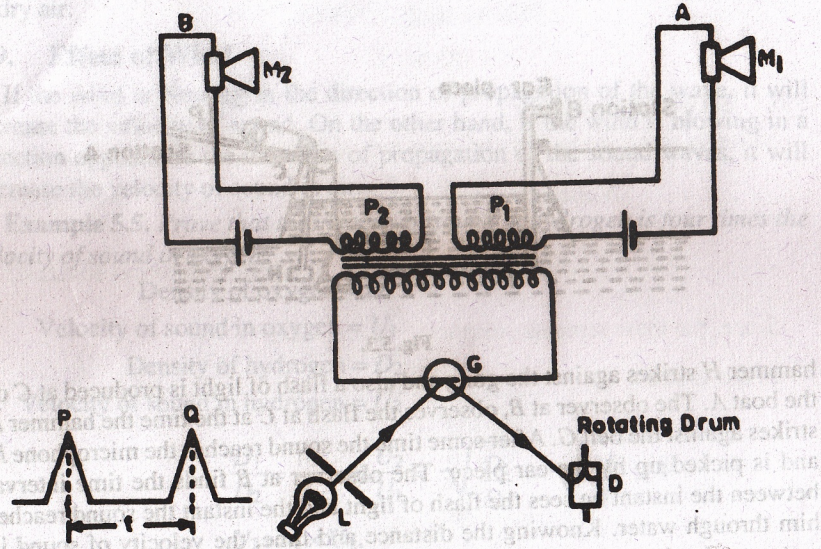


Fig. 5.4

galvanometer coil moves the spot of light on the photographic film on the rotating drum.

As the speed of rotation of the drum is known, the time between the two deflections in the galvanometer is calculated. Suppose the distance between the two stations is d and the time taken by sound is t.

Then $$v = \frac{d}{t}$$

This gives the velocity of sound in air at room temperature.

5.12. Velocity of Sound in Isotropic Solids

In the case of solids, different types of strain are possible and hence different kinds of waves are possible in them. If one end of a thin long rod is struck with a hammer, longitudinal compressional waves are set up. In this case there is increase in length in the region of rarefaction and decrease in length in the region of condensation. As there is change in length only, it is a case of longitudinal strain involving only the Young's modulus of elasticity.

Consider a thin long rod of uniform area of cross-section a. Let one end of

the rod be struck with a hammer with a force F for a small interval of time dt.

$\therefore$ Impulse $= Fdt$ ... (1)

This impulse produces a compressional wave. The distance travelled by the wave in time $dt = Udt$.

Here U is the velocity of the wave. Let dx be the compression produced in a length Udt. The particle velocity of the element

$$= = dx / dt$$

Mass of the element $= a \times Udt \times \rho$

The gain in momentum due to the impulse

$$= \text{mass} \times \text{velocity}$$

$$= a \times Udt \times \rho \times \frac{dx}{dt}$$

$$= aU\rho dx \quad \ldots (2)$$

But, Impulse = Change in momentum

$\therefore$ From equations (1) and (2)

$$Fdt = aU\rho dx \quad \ldots (3)$$

If Y be the Young's modulus of the material,

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$Y = \frac{F/a}{\frac{adx}{aUdt}}$$

or $$Fdt = Ya\frac{dx}{U} \quad \ldots (4)$$

Comparing (3) and (4)

$$aU\rho\ dx = Ya\ \frac{dx}{U}$$

$$U^2 = \frac{Y}{\rho}$$

$$U = \sqrt{\frac{Y}{\rho}} \quad \ldots (5)$$

[**Note.** In the case of extended solids the contraction or expansion in length is not possible. In such a case, the *elongational elasticity*,

$$E = K + \frac{4}{3}\eta$$

is to be used.

$$U = \sqrt{\frac{K + \frac{4}{3}\eta}{\rho}}.$$

Here K is the bulk modulus of elasticity and η is the modulus of rigidity.]

Example 5.6. *Calculate the velocity of propagation of longitudinal sound waves through a steel rod. Y for steel* $= 2 \times 10^{11}$ N/m^2

ρ *for steel* $= 7{\cdot}6 \times 10^3$ kg/m^3.

Here $Y = 2 \times 10^{12}$ N/m^2

$\rho = 7 \cdot 6 \times 10^3$ kg/m^3

$\therefore \quad U = \sqrt{\frac{Y}{\rho}}$

$\therefore \quad U = \sqrt{\frac{2 \times 10^{11}}{7{\cdot}6 \times 10^3}}$

$U =$ **5130 m/s.**

5.13. Wave Velocity and Molecular Velocity

The velocity of propagation of sound waves through a gas is,

$$U = \sqrt{\frac{\gamma P}{\rho}} \qquad \ldots (1)$$

According to this equation the velocity of sound will depend on the pressure of the gas. Also, according to kinetic theory of gases, the pressure of the gas is,

$$P = \frac{1}{3} \rho c^2$$

Here c is the root mean square velocity of the molecules at a particular temperature.

$$\therefore \quad \frac{P}{\rho} = \frac{c^2}{3} \qquad \ldots (2)$$

Substituting the value of P/ρ in equation (1)

$$U = \sqrt{\frac{\gamma c^2}{3}}$$

or $$U = c\sqrt{\frac{\gamma}{3}} \qquad \ldots (3)$$

Also $$c = U\sqrt{\frac{3}{\gamma}} \qquad \ldots (4)$$

From equation (3), if the value of velocity of sound and the ratio of the two specific heats of the gas are known, the root mean square velocity of the molecules can be calculated.

5.14. Velocity of Sound and Frequency

The velocity of sound propagation through a medium is practically constant. It is independent of the change in frequency of sound. It has been found that the velocity of sound in a medium remains constant upto a frequency of about hundred million hertz. The velocity of sound in different media is given in Table 5.1.

TABLE 5.1

SOLIDS AT 20°C

Substances	*Velocity in* m/s
Lead	1230
Copper	3750
Aluminium	5100
Iron	5130
Granite	6000

LIQUIDS AT 25° C

Liquid	*Velocity in* m/s
Kerosene oil	1315
Mercury	1450
Water	1493
Sea water	1533

GASES AT 0° C

Gas	*Velocity in* m/s
Oxygen	317
Air	332
Nitrogen	339
Steam (100° C)	405
Hydrogen	1270

EXERCISES

1. Obtain an expression for the velocity of a plane progressive wave through a gaseous medium. *(Punjab)*
2. State and explain Newton's formula for the velocity of sound through air. Discuss Laplace correction. *(Calcutta)*
3. Discuss the effect of temperature, pressure and humidity on the velocity of sound through air. *(Osmania; Calcutta)*
4. Describe briefly an experiment to determine the velocity of sound in (*i*) water and (*ii*) air.

5. Show that the velocity of sound in isotropic solids is given by $U = \sqrt{\frac{Y}{\rho}}$.

6. Show that the velocity of sound in the case of extended solids is given by

$$U = \sqrt{\frac{3K + 4\eta}{3\rho}}.$$

7. Distinguish between wave velocity and molecular velocity and show that

$$U = c\sqrt{\frac{\gamma}{3}}.$$

8. Derive from elementary principles an expression for the velocity of sound in air (Newton's formula). Explain Laplace correction. What is the effect of temperature on the velocity of sound in air.
(Mysore; Delhi)

9. Derive the formula for the velocity of a plane progressive wave in a fluid.
(Delhi)

10. At what temperature is the velocity of sound in nitrogen gas equal to its velocity in oxygen at 20°C. The atomic weights of oxygen and nitrogen are in the ratio 16 :14. *(Delhi)*
[Hint. $T_2 \rho_1 = T_1 \rho_2$ **]** **[Ans.** 16·7° C]

11. Obtain an expression for the velocity of sound in a gas. Show how it is effected by temperature and pressure. *(Delhi)*
12. Derive the formula for the velocity of sound waves in a gas. What is Laplace's correction? Discuss the effects of temperature and pressure on the velocity of sound propagation. *(Delhi)*
13. Derive an expression for the velocity of longitudinal waves in a metallic rod and hence derive an expression for the fundamental frequency emitted by a rod of finite length. *(Delhi)*
14. Obtain an expression for the velocity of longitudinal waves in a gas. Discuss Laplace's correction. *(Delhi)*
15. Show that for every 1°C change in temperature the velocity of sound in air changes by about 0·61 m/s. *(Delhi)*
16. Derive Newton's equation for the velocity of sound waves in a material medium. Discuss its application to solids and explain what corrections need be made in the case of gases. Explain the effect of various factors that influence the velocity of sound in free atmosphere.
[Delhi (Sub.)]
17. Obtain an expression for the velocity of sound in a gas discussing in detail Newton's formula and Laplace's correction. What is the effect of temperature variation on the velocity of sound in a gas. *(Delhi)*

18. Obtain an expression for the velocity of sound in air. How does the velocity depend on humidity, temperature and pressure? What are the other factors which influence the velocity of sound ?
[Delhi (Suppl.)]
19. Derive an expression for the excess of pressure at a point in a compression wave in a fluid and hence obtain the velocity of propagation of waves.
(Bhagalpur, 1990)
20. Find an expression for the velocity of longitudinal waves through a homogeneous elastic medium. *(Gauhati, 1992)*

CHAPTER 6

Stationary Waves, Interference and Beats

6.1. Stationary Waves

When two simple harmonic waves of the same amplitude, frequency and time period travel in *opposite directions* in a straight line, the resultant wave obtained is called a stationary or a standing wave. The formation of stationary waves is due to the superposition of the two waves on the particles of the medium.

The formation of stationary waves can be represented graphically as follows :

Consider two wave trains A and B of the same amplitude, frequency and wavelength travelling in opposite directions. At an instant of time $t = 0$,

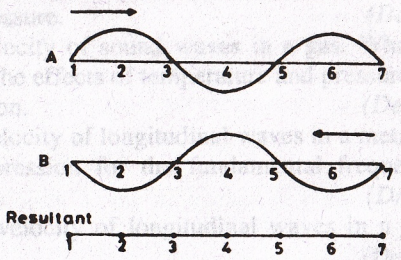


Fig. 6.1.

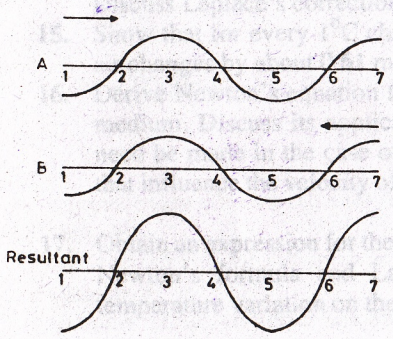


Fig. 6.2.

the waves are as shown in Fig. 6.1. The resultant displacement curve is a straight line. All the particles of the medium are at their mean positions.

At time $t = T/4$, the wave A will advance through a distance $\lambda/4$ towards right, and the wave B will advance through a distance $\lambda/4$ towards left. The resultant displacement pattern is shown in Fig. 6.2.

The particles 1, 3, 5 and 7 are at their extreme positions and particles 2, 4 and 6 are at their mean positions.

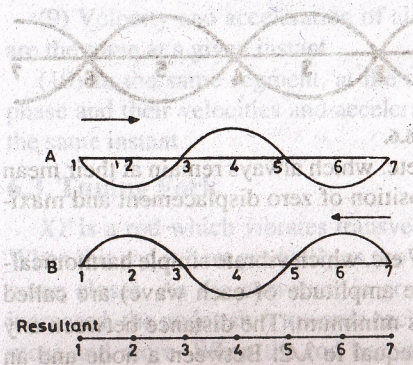


Fig. 6.3.

The resultant displacement pattern is shown in Fig. 6.4.

The particles 1, 3, 5 and 7 are at their extreme positions and 2, 4, 6 are at their mean positions.

At time $t = T$, the wave A will advance through a distance λ towards right and the wave B will advance through a distance λ towards left (with reference to zero time). The resultant displacement pattern is shown in Fig. 6.5.

All the particles are at their mean positions.

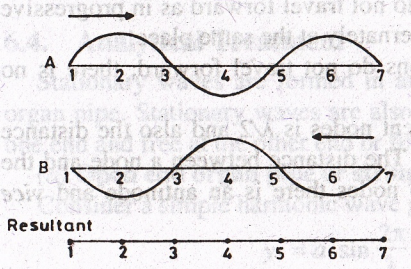


Fig. 6.5.

At time $t = T/2$, the wave A will advance through a distance $\lambda/2$ towards right and the wave B will advance through a distance $\lambda/2$ towards left (with reference to zero time).

The resultant displacement pattern is shown in Fig. 6.3.

All the particles of the medium are at their mean positions.

At time $t = 3T/4$, the wave A will advance through a distance $3\lambda/4$ towards right and the wave B will advance through a distance $3\lambda/4$ towards left (with reference to zero time).

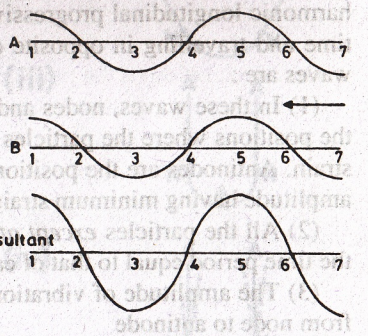


Fig. 6.4.

From the patterns discussed above, it is clear that the particles of the medium such as 2, 4, 6 etc. always remain at their mean positions. The particles such as 1, 3, 5, 7 etc. continue to vibrate simple harmonically about their mean positions with double the amplitude of each wave. It appears as though the wave pattern is stationary in space. The resultant displacement patterns at intervals of time,

$$0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4} \text{ and } T$$

are shown in Fig. 6.6.

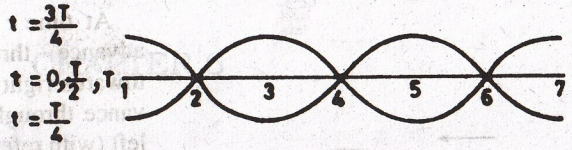


Fig. 6.6.

The positions of the particles 2, 4, 6 etc. which always remain at their mean positions are called *nodes*. Node is a position of zero displacement and maximum strain.

The positions of the particles 1, 3, 5, 7 etc. which vibrate simple harmonically with maximum amplitude (twice the amplitude of each wave) are called antinodes. At the antinodes, the strain is minimum. The distance between any two consecutive nodes or antinodes is equal to $\lambda/2$. Between a node and an antinode, the amplitude gradually increases from zero to maximum.

6.2. Properties of Stationary Longitudinal Waves

The stationary waves are formed due to the superposition of two simple harmonic longitudinal progressive waves of the same amplitude and periodic time and travelling in opposite directions. The important properties of these waves are :

(1) In these waves, nodes and antinodes are formed alternately. Nodes are the positions where the particles are at their mean positions having maximum strain. Antinodes are the positions where the particles vibrate with maximum amplitude having minimum strain.

(2) All the particles *except at the nodes*, vibrate simple harmonically with the time period equal to that of each component wave.

(3) The amplitude of vibration gradually increases from zero to maximum from node to antinode

(4) The medium is split into segments and all the particles of the same segment vibrate in phase. The particles in one segment have a phase difference of π with the particles in the neighbouring segment.

(5) Condensations and rarefactions do not travel forward as in progressive waves but they appear and disappear alternately at the same place.

(6) As condensations and rarefactions do not travel forward, there is no transference of energy.

(7) The distance between two adjacent nodes is $\lambda/2$ and also the distance between two adjacent antinodes is $\lambda/2$. The distance between a node and the adjacent antinode is $\lambda/4$. Between two nodes there is an antinode and *vice versa*.

(8) The general appearance of the wave can be represented by a sine curve but it reduces to straight line twice in each time period.

(9) Velocity and acceleration of all the particles separated by a distance λ are the same at a given instant.

(10) In the same segment, at the same instant, all the particles will be in phase and their velocities and accelerations will be maximum or minimum at the same instant.

6.3. Tuning Fork

XY is a rod which vibrates transversely with nodes at *N* and *N*. (Fig. 6.7). When this rod is bent gradually as shown in Fig. 6.7 *(ii)*, *(iii)* and *(iv)*, the nodes come closer. As the curvature is gradually increased, the nodes shift towards each other more and more. When the rod becomes U-shaped, the positions of nodes and antinodes are as shown in Fig. 6.7*(iv)*. When this U-shaped rod is attached to a metal stem, they come still closer and this arrangement is called a tuning fork. The tuning fork vibrates in three portions as shown in Fig. 6.7*(v)*. The free ends of the tuning fork behave as antinodes. The position where the stem is attached also behaves as an antinode.

A tuning fork is of great use as a source of standard frequency. It is made of an alloy of nickel and steel. A tuning fork maintains its pitch for a number of years and it can be set into vibration when one of the prongs is struck against a hard rubber pad. Tuning forks of frequencies 256, 288, 320, 341. 33, 384, 426.66, 480 and 512 are commonly manufactured.

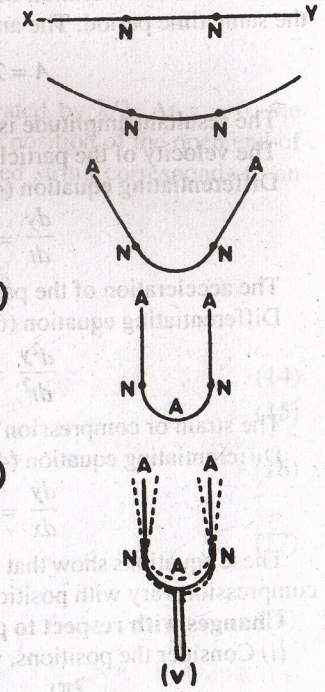


Fig. 6.7

6.4. Analytical Treatment

Stationary waves are formed in an open end organ pipe or a closed end organ pipe. Stationary waves are also formed with a stretched string fixed at one end and free at the other end or fixed at the other end.

(a) **Open end organ pipe or string free at the other end.**

Consider a simple harmonic wave given by the equation,

$$y_1 = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \textbf{Incident wave} \qquad \ldots (1)$$

The displacement of the same particle at the same instant due to the reflected wave is given by

$$y_2 = a \sin \frac{2\pi}{\lambda}(vt + x) \quad \textbf{Reflected wave.} \quad \ldots (2)$$

The resultant displacement of the wave,

$$y = y_1 + y_2$$

or
$$y = a \sin \frac{2\pi}{\lambda}(vt - x) + \sin \frac{2\pi}{\lambda}(vt + x) \quad \ldots (3)$$

$$y = 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda} \quad \ldots (4)$$

The resultant vibration of a particle represents a simple harmonic wave of the same time period. The amplitude of the resultant wave is given by

$$A = 2a \cos \frac{2\pi x}{\lambda} \quad \ldots (5)$$

The resultant amplitude is a function of x.

The velocity of the particle at any instant of time is dy/dt.

Differentiating equation (4) with respect to time

$$\frac{dy}{dt} = -\frac{4\pi av}{\lambda} \cos \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda} \quad \ldots (6)$$

The acceleration of the particle at any instant of time is d^2y/dt^2.

Differentiating equation (6) with respect to time

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 av^2}{\lambda^2} \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda} \quad \ldots (7)$$

The strain or compression at any point of the resultant vibration is dy/dx.

Differentiating equation (4) with respect to x

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \sin \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda} \quad \ldots (8)$$

These equations show that the amplitude, velocity, acceleration and strain or compression vary with position and time.

Changes with respect to position

(i) Consider the positions, where

$$\sin \frac{2\pi x}{\lambda} = 0 \text{ and } \cos \frac{2\pi x}{\lambda} = \pm 1$$

From equations (4) to (8)

Displacement $\quad y = \pm 2a \sin \frac{2\pi vt}{\lambda} \quad \ldots (9)$

Amplitude $\quad A = \pm 2a \quad \ldots (10)$

Velocity $\quad \frac{dy}{dt} = \pm \frac{4\pi av}{\lambda} \cos \frac{2\pi vt}{\lambda} \quad \ldots (11)$

Acceleration $\quad \frac{d^2y}{dt^2} = \mp \frac{8\pi^2 av^2}{\lambda^2} \sin \frac{2\pi vt}{\lambda} \quad \ldots (12)$

Strain $\quad \frac{dy}{dx} = 0. \quad \ldots (13)$

As the strain is zero, these positions correspond to antinod

$$\sin \frac{2\pi x}{\lambda} = 0$$

But $\quad \sin m\pi = 0$

where $\quad m = 0, 1, 2, 3 \ldots$ etc.

$$\therefore \quad \frac{2\pi x}{\lambda} = m \quad \text{or} \quad x = \frac{m\lambda}{2}$$

or
$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \ldots \text{ etc.}$$

Thus, the antinodes are equidistant and separated by $\lambda/2$. At $x = 0$, the position of interface is an antinode. Therefore, the position of the open end of the open end organ pipe or free end of a stretched string corresponds to an **antinode.**

(ii) Consider the positions, where

$$\sin \frac{2\pi x}{\lambda} = \pm 1, \text{ and } \cos \frac{2\pi x}{\lambda} = 0.$$

From equations (4) to (8)

Displacement $\quad y = 0 \quad \ldots (14)$

Amplitude $\quad A = 0 \quad \ldots (15)$

Velocity $\quad \frac{dy}{dt} = 0 \quad \ldots (16)$

Acceleration $\quad \frac{d^2y}{dt^2} = 0 \quad \ldots (17)$

Strain $\quad \frac{dy}{dx} = \mp \frac{4\pi a}{\lambda} \sin \frac{2\pi vt}{\lambda} \quad \ldots (18)$

These positions correspond to nodes.

$$\because \quad \cos \frac{2\pi x}{\lambda} = 0,$$

But $\quad \cos (2m+1) \frac{\pi}{2} = 0$

where $\quad m = 0, 1, 2, 3, \ldots$ etc.

$$\therefore \quad \frac{2\pi x}{\lambda} = (2m+1) \frac{\pi}{2} \quad \text{or} \quad x = \frac{(2m+1)\lambda}{4}$$

$$\therefore \quad x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots \text{ etc.}$$

Thus, the antinodes are also equidistant and separated by a distance of $\lambda/2$.

It is clear that between two consecutive antinodes there is a node and *vice versa*. Distance between a node and an antinode is $\lambda/4$.

Changes with respect to time

(*i*) Consider the instant of time, when

$$\sin \frac{2\pi vt}{\lambda} = 0 \text{ and } \cos \frac{2\pi vt}{\lambda} = \pm 1.$$

From equations (4) to (8)

Displacement $y = 0$

Amplitude $A = 2a \cos \frac{2\pi x}{\lambda}$ (independent of time)

Velocity $\frac{dy}{dt} = \pm \frac{4\pi av}{\lambda} \cos \frac{2\pi x}{\lambda}$

Acceleration $\frac{d^2y}{dt^2} = 0$

Strain $\frac{dy}{dx} = 0$

$$\therefore \quad \sin \frac{2\pi vt}{\lambda} = 0,$$

$$\sin m\pi = 0,$$

Where $m = 0, 1, 2, 3, \ldots$ etc.

$$\therefore \quad \frac{2\pi vt}{\lambda} = m\pi \quad \text{or} \quad t = \frac{m\lambda}{2v}$$

But $\frac{v}{\lambda} = n$ (frequency) $= \frac{1}{T}$

$$\therefore \quad t = \frac{mT}{2} \quad \text{or} \quad t = 0, \frac{T}{2}, T, \frac{3T}{2} \ldots \text{etc.}$$

It follows that twice in each time period the particles pass through their mean positions.

(*ii*) Consider the instant of time,

when $\sin \frac{2\pi vt}{\lambda} = \pm 1$ and $\cos \frac{2\pi vt}{\lambda} = 0.$

From equations (4) to (8)

Displacement $y = \pm 2a \cos \frac{2\pi x}{\lambda}$

Amplitude $A = 2a \cos \frac{2\pi x}{\lambda}$ (independent of time)

Velocity $\frac{dy}{dt} = 0$

Acceleration $\frac{d^2y}{dt^2} = \mp \frac{8\pi^2 av}{\lambda} \cos \frac{2\pi x}{\lambda}$

The strain or compression,

$$\frac{dy}{dx} = \mp \frac{4\pi a}{\lambda} \sin \frac{2\pi x}{\lambda}$$

$$\therefore \quad \sin \frac{2\pi vt}{\lambda} = \pm 1,$$

and $\cos \frac{2\pi vt}{\lambda} = 0$

But $\sin (2m+1)\frac{\pi}{2} = \pm 1$

Where $m = 0, 1, 2, \ldots$ etc.

$$\therefore \quad \frac{2\pi vt}{\lambda} = (2m+1)\frac{\pi}{2} \quad \text{or} \quad t = \frac{(2m+1)\lambda}{4v}$$

But $\frac{v}{\lambda} = n = \frac{1}{T}$

$$\therefore \quad t = \frac{(2m+1)T}{4} = \frac{mT}{2} + \frac{T}{4}$$

or $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4} \ldots$ etc.

At these instants, the displacement, acceleration and strain are maximum at all positions and the velocity of the particles is zero. At these instants, each particle is at its extreme position and the pattern is stationary at that instant. This instant is called the stationary instant.

(*b*) **Closed end organ pipe or string fixed at the other end**

Consider a simple harmonic wave given by the equation

$$y_1 = a \sin \frac{2\pi}{\lambda}(vt - x) \qquad \textbf{Incident wave. . . .} (1)$$

The displacement of the same particle at the same instant due to the reflected wave is given by

$$y_2 = -a \sin \frac{2\pi}{\lambda}(vt + x) \qquad \textbf{Reflected wave . . .} (2)$$

The resultant displacement

$$y = y_1 + y_2$$

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) - a \sin \frac{2\pi}{\lambda}(vt + x) \quad \ldots (3)$$

$$y = -2a \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda} \quad \ldots (4)$$

The resultant vibration of the particle represents a simple harmonic wave of the same time period. The amplitude of the resultant wave is given by

$$A = 2a \sin \frac{2\pi x}{\lambda} \quad \ldots (5)$$

The resultant amplitude is a function of x.

The velocity of the particle at any instant of time is dy/dt.

Differentiating equation (4) with respect to time,

$$\frac{dy}{dt} = \frac{4\pi av}{\lambda} \sin \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda} \quad \ldots (6)$$

The acceleration of the particle at any instant of time is d^2y/dt^2.

Differentiating equation (6) with respect to time,

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 av^2}{\lambda^2} \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda} \quad \ldots (7)$$

The strain or compression at any point of the resultant vibration is dy/dx.

Differentiating equation (4) with respect to x

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \cos \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda} \quad \ldots (8)$$

These equations show that the amplitude, velocity, acceleration and strain or compression vary with position and time.

Changes with respect to position

(1) Consider the positions, where

$$\sin \frac{2\pi x}{\lambda} = 0 \quad \text{and} \quad \cos \frac{2\pi x}{\lambda} = \pm 1.$$

From equations (4) to (8)

Displacement $y = 0 \quad \ldots (9)$

Amplitude $A = 0 \quad \ldots (10)$

Velocity $\frac{dy}{dt} = 0 \quad \ldots (11)$

Acceleration $\frac{d^2y}{dt^2} = 0 \quad \ldots (12)$

Strain $\frac{dy}{dx} = \mp \frac{4\pi a}{\lambda} \cos \frac{2\pi vt}{\lambda} \quad \ldots (13)$

From equations (9) to (13) it is clear that at these positions, displacement, amplitude, velocity and acceleration are zero but the strain is maximum. These positions correspond to **nodes.**

$$\sin \frac{2\pi x}{\lambda} = 0$$

But $\sin m\pi = 0$

where $m = 0, 1, 2, \ldots$ etc.

$$\therefore \quad \frac{2\pi x}{\lambda} = m\pi \quad \text{or} \quad x = \frac{m\lambda}{2}$$

or $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \ldots$ etc.

Thus, the nodes are equidistant and separated by $\lambda/2$. At $x = 0$, the position of interface is a node. Therefore, the position of the closed end organ pipe or fixed end of a stretched string corresponds to a **node.**

(*ii*) Consider the positions, where

$$\sin \frac{2\pi x}{\lambda} = \pm 1 \quad \text{and} \quad \cos \frac{2\pi x}{\lambda} = 0$$

From equations (4) to (8),

Displacement $y = \mp 2a \cos \frac{2\pi vt}{\lambda} \quad \ldots (14)$

Amplitude $A = \pm 2a \quad \ldots (15)$

Velocity $\frac{dy}{dt} = \pm \frac{4\pi av}{\lambda} \sin \frac{2\pi vt}{\lambda} \quad \ldots (16)$

Acceleration $\frac{d^2y}{dt^2} = \pm \frac{8\pi^2 av^2}{\lambda^2} \cos \frac{2\pi vt}{\lambda} \quad \ldots (17)$

Strain $\frac{dy}{dx} = 0 \quad \ldots (18)$

As the strain is zero, these positions correspond to antinodes.

$$\sin \frac{2\pi x}{\lambda} = \pm 1$$

Also $\sin \frac{(2m+1)\pi}{2} = \pm 1$

where $m = 0, 1, 2, \ldots$ etc.

$$\therefore \quad \frac{2\pi x}{\lambda} = \frac{(2m+1)\pi}{2}$$

or $x = \frac{(2m+1)\lambda}{4}$ or $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots$ etc.

Thus the antinodes are equidistant and separated by $\lambda/2$. The first antinode is at a distance of $\lambda/4$ from the closed end of the closed end pipe or fixed end of a stretched string.

Changes with respect to time

(*i*) At the instant of time, when

$$\sin \frac{2\pi vt}{\lambda} = 0 \text{ and } \cos \frac{2\pi vt}{\lambda} = \pm 1$$

From equations (6) to (8)

Displacement $y = \mp 2a \sin \frac{2\pi x}{\lambda}$

Amplitude $A = 2a \sin \frac{2\pi x}{\lambda}$ (independent of time)

Velocity $\frac{dy}{dt} = 0$

Acceleration $\frac{d^2y}{dt^2} = \pm \frac{8\pi^2 av^2}{\lambda^2} \sin \frac{2\pi x}{\lambda}$

Strain $\frac{dy}{dx} = \pm \frac{4\pi a}{\lambda} \cos \frac{2\pi x}{\lambda}$

These equations show that at these instants, the displacement, acceleration and strain are maximum at all positions and the velocity of the particles is zero.

Here $\cos \frac{2\pi vt}{\lambda} = \pm 1$

Also $\cos m\pi = \pm 1$

$\therefore$ $\frac{2\pi vt}{\lambda} = m\pi$

But $\frac{v}{\lambda} = \frac{1}{T}$ or $t = \frac{mT}{2}$

$t = 0, \frac{T}{2}, T, \frac{3T}{2} \ldots$ etc.

At these instants, each particle is at its extreme position and the pattern is stationary at that instant. This instant is called the stationary instant.

(*ii*) At the instant of time, when

$$\sin \frac{2\pi vt}{\lambda} = \pm 1 \quad \text{and} \quad \cos \frac{2\pi vt}{\lambda} = 0$$

From equations (4) to (8),

Displacement $y = 0$

Amplitude $A = 2a \sin \frac{2\pi x}{\lambda}$ (independent of time)

Velocity $\frac{dy}{dt} = 0$

Acceleration $\frac{d^2y}{dt^2} = 0$

Strain $\frac{dy}{dx} = 0.$

These equations show that displacement, velocity, acceleration and strain are zero.

$\because$ Here $\sin \frac{2\pi vt}{\lambda} = \pm 1$

Also $\sin \frac{(2m+1)\pi}{2} = \pm 1$

$\therefore$ $\frac{2\pi vt}{\lambda} = \frac{(2m+1)\pi}{2}$

But $\frac{v}{\lambda} = \frac{1}{T}$

$\therefore$ $t = \frac{(2m+1)T}{4}$ or $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4} \ldots$ etc.

At these instants of time, all the particles are at their mean positions. It follows that twice in each time period the particles pass through their mean positions.

6.5. Energy of a Stationary Wave

When a wave is propagated through a fluid,

$$E = -p \Big/ \frac{dy}{dx}$$

where E is the bulk modulus of elasticity, p is the exeess of pressure and $\frac{dy}{dx}$ is the volume strain.

$\therefore$ $p = -E \frac{dy}{dx}$ . . . (1)

In the case of an open end organ pipe, strain is given by

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \sin \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda}$$

Also $$v = \sqrt{\frac{E}{\rho}}$$

or $$v^2 = \frac{E}{\rho} \quad \text{or} \quad E = v^2\rho$$

Substituting the values of $\frac{dy}{dx}$ and E in equation (1)

$$\therefore \quad p = v^2\rho \frac{4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi vt}{\lambda} \quad \ldots (2)$$

When $\sin\frac{2\pi x}{\lambda} = 1$ and $\sin\frac{2\pi vt}{\lambda} = 1$,

the excess of pressure is maximum

$$p = p_0 = v^2\rho \frac{4\pi a}{\lambda} \ldots (3)$$

From equations (2) and (3),

$$\therefore p = p_0 \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi vt}{\lambda} \ldots (4)$$

Taking $$p_0 \sin\frac{2\pi x}{\lambda} = p_x$$

$$p = p_x \sin\frac{2\pi vt}{\lambda} \quad \ldots (5)$$

The velocity of the particle,

$$U = \frac{dy}{dt} = \frac{4\pi av}{\lambda} \cos\frac{2\pi x}{\lambda} \cos\frac{2\pi vt}{\lambda} \quad \ldots (6)$$

Taking $$\frac{4\pi av}{\lambda} \cos\frac{2\pi x}{\lambda} = U_x$$

$$U = U_x \cos\frac{2\pi vt}{\lambda} \quad \ldots (7)$$

Work done or the energy transfer per unit area in a small interval of time dt

$$= pU \, . \, dt$$

The total energy transfer in time T

$$= \int_0^T pU \, . \, dt$$

Rate of energy transfer

$$= \frac{\int_0^T pU \, . \, dt}{T}$$

$$= \frac{1}{T}\int_0^T \left(p_x \sin\frac{2\pi vt}{\lambda}\right) \times \left(U_x \cos\frac{2\pi vt}{\lambda}\right) dt$$

$$= \frac{p_x U_x}{T}\int_0^T \sin\frac{2\pi vt}{\lambda} \cdot \cos\frac{2\pi vt}{\lambda} \cdot dt$$

$$= \frac{p_x U_x}{2T}\int_0^T \sin\frac{4\pi vt}{\lambda} \cdot dt$$

But $$= \int_0^T \sin\frac{4\pi vt}{\lambda} \cdot dt = 0$$

$\therefore$ Rate of energy transfer = 0.

Thus in the case of stationary waves no **energy** is *transferred.*

Example 6.1. *The equation of a transverse wave on a stretched string is*

$$y = 2 \sin \pi \ (8t - 0{\cdot}005\, x - 0{\cdot}40).$$

Write down the equation of a wave that would produce a stationary wave in the string on superposition with the given wave. (Bhagalpur, 1990)

Here $$y = 2 \sin \pi \ (8t - 0{\cdot}005\, x - 0{\cdot}40)$$

$$y_1 = 2 \sin \pi \ [8t - (0{\cdot}005\, x + 0{\cdot}40)] \quad \ldots (i)$$

This is the incident wave.

The reflected wave is

$$y_2 = 2 \sin \pi \ [8t + (0{\cdot}005x + 0{\cdot}40)] \quad \ldots (ii)$$

The resultant of y_1 and y_2 will be a stationary wave.

6.6. Interference of Sound Waves

If two or more sound waves travel through the same medium, superposition of the waves takes place. The particles of the medium vibrate according to the vector sum (resultant) of the displacements due to interfering waves at that instant. As a result of the superposition, the distribution of energy in the medium is different from the distribution of energy due to the individual waves influencing the medium separately. The resultant vibration of the particles will depend upon the amplitude, time period, phase difference and direction of the interfering waves.

Consider two simple harmonic waves of the same frequency. Let a and b be the amplitudes of the two waves and phase difference ϕ. It is assumed that the two waves are plane and that they do not spread laterally. The displacement of a particle at any instant due to each wave is given by

$$y_1 = a \sin\frac{2\pi}{\lambda}(vt - x) \quad \ldots (1)$$

$$y_2 = b \sin\left[\frac{2\pi}{\lambda}(vt - x) + \phi\right] \quad \ldots (2)$$

Here v is the velocity of propagation of each wave and λ is the wave length.

The resultant displacement of the particle due to the superposition of the two waves,

$$y = y_1 + y_2$$

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) + b \sin\left[\frac{2\pi}{\lambda}(vt - x) + \phi\right]$$

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) + b \left[\sin \frac{2\pi}{\lambda}(vt - x)\cos\phi + \cos\frac{2\pi}{\lambda}(vt - x)\sin\phi\right]$$

$$y = \left[\sin \frac{2\pi}{\lambda}(vt - x)\right][a + b\cos\phi] + \left[\cos\frac{2\pi}{\lambda}(vt - x)\right][b\sin\phi]$$

Let $a + b\cos\phi = A\cos\theta$

$b\sin\phi = A\sin\theta$

$$\therefore \quad A = \sqrt{a^2 + b^2 + 2ab\cos\phi} \quad \ldots (3)$$

and $$\tan\theta = \frac{b\sin\phi}{a + b\cos\phi} \quad \ldots (4)$$

$$\therefore \quad y = \left[\sin\frac{2\pi}{\lambda}(vt - x)\right][A\cos\theta] + \left[\cos\frac{2\pi}{\lambda}(vt - x)\right][A\sin\theta]$$

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt - x) + \theta\right] \quad \ldots (5)$$

The resultant vibration has the same time period, (*i.e.*, frequency and wave length). The amplitude and phase difference are different.

(1) When $\phi = 0, 2\pi, n(2\pi) \ldots$ etc.

and $a = b$,

$A = \sqrt{a^2 + a^2 + 2a^2}$

or $A = 2a$

Intensity of sound, $I = A^2 = 4a^2$

Therefore the intensity of sound is maximum at those points, where the two waves differ in phase by $n(2\pi)$ or the path difference is $n\lambda$.

Here $n = 0, 1, 2, 3, \ldots$ etc.

(2) When $\phi = \pi, 3\pi, (2n + 1)\pi \ldots$ etc.

and $a = b$

$A = 0$

Intensity of sound $I = A^2 = 0$

Therefore, the intensity of sound is minimum at those points, where the two waves differ in phase by $(2n + 1)\pi$ or the path difference is $(2n + 1)\lambda/2$.

Here $n = 0, 1, 2, 3, \ldots$ etc.

6.7. Special Cases

(1) *When two waves of the same frequency and amplitude travel in the same direction having phase difference zero.*

Here $a = b$ and $\phi = 0$

From equation (3)

$A = 2a$

From equation (4)

$\tan\theta = 0$ or $\theta = 0$

From equation (5)

$$y = 2a\sin\frac{2\pi}{\lambda}(vt - x) \quad \ldots (6)$$

The resultant vibration has an amplitude $2a$ as shown in Fig. 6.8.

In this condensations and rarefactions are coincident.

(2) *When two waves of the same frequency and amplitude travel in the same direction having phase difference* π.

Here $a = b$

and $\phi = \pi$

From equation (3)

$A = 0$

From equation (4)

$\tan\theta = 0$ or $\theta = 0$

From equation (5)

$y = 0$

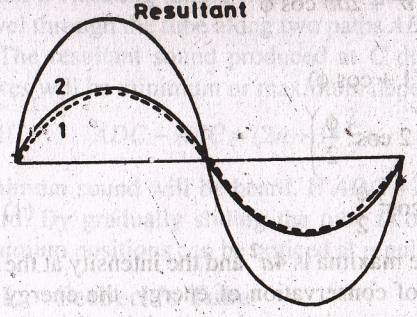


Fig. 6.8.

The resultant vibration has zero amplitude and medium remains undisturbed as shown in Fig. 6.9.

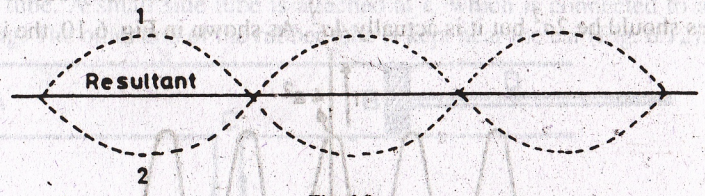


Fig. 6.9.

(3) When two waves of the same frequency and amplitude travel in opposite directions, the interference phenomenon gives rise to standing or stationary waves as discussed earlier.

6.8. Conditions for Interference of Sound Waves

1. The two sources of sound must be coherent. They must emit waves of the same frequency and amplitude.

2. The phase difference between the two sources must remain constant such that the phase difference between the two waves at any point does not change with time.

3. The two wave trains must travel in the same direction.

Condition (1) is necessary so that the positions of maximum and minimum intensity of sound are distinct. If the difference between a and b is large, the minimum intensity positions are not distinct.

Condition (2) is necessary to have sustained maxima and minima. If the phase difference between the two waves continuously changes, the maximum and minimum intensity positions are not fixed.

Condition (3) is necessary because with increase in obliquity between the waves, the resultant intensity decreases.

6.9. Energy Distribution Due to Interference of Sound Waves

From equation (3) of Article 6.6

$$A = \sqrt{a^2 + b^2 + 2ab\cos\phi}$$

When $a = b$

$$A^2 = 2a^2(1 + \cos\phi)$$

$$A^2 = 2a^2\left(2\cos^2\frac{\phi}{2}\right)$$

$$A^2 = 4a^2\cos^2\frac{\phi}{2} \qquad \ldots(1)$$

From equation (1), the intensity at the maxima is $4a^2$ and the intensity at the minima is zero. According to the law of conservation of energy, the energy cannot be destroyed. Here also the energy is not destroyed but is only redistributed from the positions of minimum intensity to those of maximum intensity. For, at the maximum intensity positions the intensity due to the two waves should be $2a^2$ but it is actually $4a^2$. As shown in Fig. 6.10, the intensity

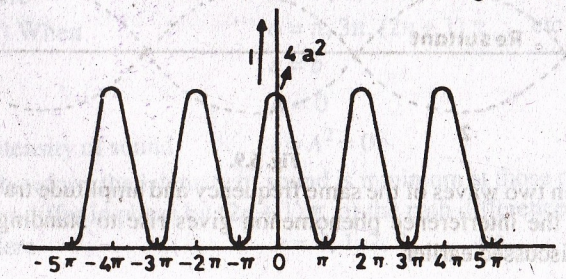


Fig. 6.10.

varies from 0 to $4a^2$ and the average is still $2a^2$. It is equal to a uniform intensity of $2a^2$ which will be present in the absence of interference phenomenon between the two waves.

Hence the formation of maxima and minima due to the interference of two sound waves is in accordance with the law of conservation of energy.

6.10. Quincke's Tube

The phenomenon of interference of sound waves can be demonstrated with a Quincke's tube. The tube D can slide inside the tube ABC as shown Fig. 6.11

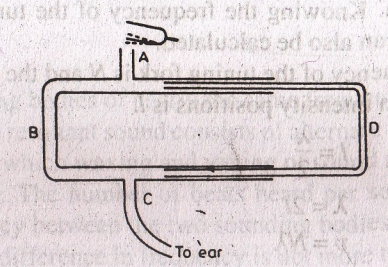


Fig. 6.11.

When a vibrating tuning fork is held near the opening at A, the sound waves travel through the tube along two paths ABC and ADC.

The resultant sound produced at C due to the interference of these two waves will be minimum or maximum depending upon the path difference.

If $ADC - ABC = (2n + 1)\frac{\lambda}{2}$

minimum sound will be heard. If $ADC - ABC = n\lambda$, maximum sound will be heard. By gradually sliding the tube D outwards or inwards maximum and minimum positions can be noticed at regular intervals.

6.11. Seebeck's Tube

The phenomenon of interference can also be demonstrated with the help of Seebeck's tube. It consists of a long glass tube open at both ends. A piston can slide in the tube. A small side tube is attached at C which is connected to a rubber tubing. The other end of the rubber tube is kept near the ear (Fig. 6.12).

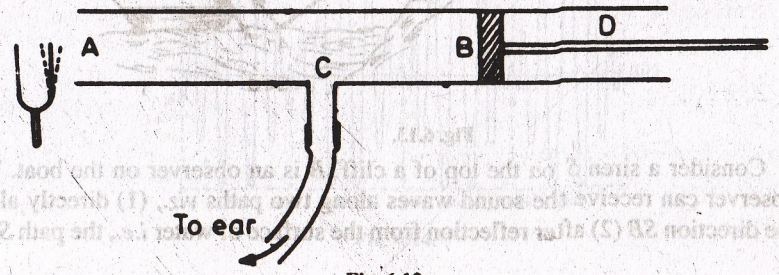


Fig. 6.12.

When a tuning fork is allowed to vibrate near the open end A, the ear perceives the resultant effect of two waves, the direct wave and the reflected wave from B. If the piston is gradually moved outwards, alternately maximum and minimum intensity of sound is heard. When the path difference between the direct and the reflected waves, is $(2n + 1)\,\lambda/2$ due to interference of sound waves, the intensity of sound is minimum. When the path difference is $n\lambda$, maximum intensity of sound is heard.

It means that, when $BC = (2n + 1)\,\lambda/4$, minimum intensity of sound is heard. When $BC = n\lambda/2$, maximum intensity of sound is heard.

This arrangement is suitable to determine the wavelength of sound waves in air or a gas medium. Knowing the frequency of the tuning fork, velocity of sound in a medium can also be calculated.

Suppose the frequency of the tuning fork is N and the distance between two successive maximum intensity positions is l.

Then, $$l = \frac{\lambda}{2}$$

or $$\lambda = 2l$$

$\therefore$ $$v = N\lambda$$

$$v = 2Nl.$$

6.12. Silence Zones

When a siren is operated from the top of a cliff, the sound energy is received by an observer (a man standing on a boat or a ship). These sirens are primarily meant as a warning that there is a cliff or a big rock near the boat or the ship. However, a person on the boat does not receive the sound from the siren at all the positions.

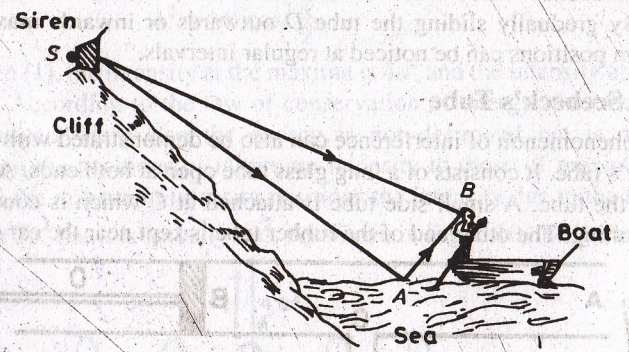


Fig. 6.13.

Consider a siren S on the top of a cliff. B is an observer on the boat. The observer can receive the sound waves along two paths *viz.*, (1) directly along the direction SB (2) after reflection from the surface of water *i.e.*, the path SAB.

These two waves reaching B will interfere. If the path difference $(SA + AB - SB)$ is an odd multiple of $\lambda/2$, the two waves produce destructive interference and no sound will be heard at B. This place will be a zone of silence. If the path difference is an even multiple of $\lambda/2$ (integral multiples of λ), the two waves reinforce with one another and maximum intensity of sound will be heard.

As the boat approaches the cliff, the path difference continuously changes. At positions where the path difference is equal to odd multiples of half the wavelength of sound, no sound will be heard. These positions are called **silence zones.**

6.13. Beats

When two sounding bodies of nearly the same frequency and amplitude are sounded together, the resultant sound consists of alternate maxima and minima. This phenomenon in which waxing and vaning of sound at regular intervals is heard is called *beats*. The number of beats heard per second is equal to the difference in frequency between the two sounding bodies. However, beats are heard only when the difference in frequency is not more than ten.

Graphical method. The formation of beats between two notes can be easily understood by the graphical method. In Fig. 6.14 (*i*), the graph represents the displacement curve for a note of frequency 20 hertz. Fig. 6.14 (*ii*) represents the displacement curve for a note of frequency 16 hertz.

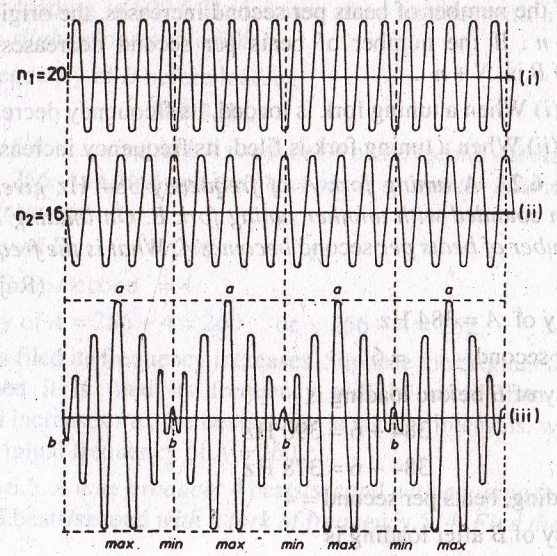


Fig. 6.14.

Fig. 6.14 (*iii*) represents the resultant displacement curve when the notes 1 and 2 are sounded simultaneously. From the resultant displacement curve it is clear that in one second there are four positions of maximum intensity of sound and four positions of minimum intensity of sound. The maxima are represented by *'a'* and minima are represented by *'b'*. The number of maxima or minima per second is four. This is also equal to the difference in frequency between the two notes.

Suppose the frequency of one tuing fork $A = 256$ and a second tuning fork $B = 260$. When the two tuning forks are sounded together, after $\frac{1}{8}$ th of a second, A completes 32 vibrations and B completes 32·5 vibrations. Minimum sound is produced. After $\frac{1}{4}$ th of a second, A completes 64 vibrations and B completes 65 vibrations. Maximum sound is produced. The process repeats and it is found that in one second, four maxima and four minima are produced. Hence, in this case, four beats are heard in one second.

The frequency of a tuning fork can be determined by the method of beats.

Take the tuning fork A of known frequency. The tuning fork B is of unknown frequency. The two tuning forks have nearly the same frequency. Find the number of beats produced per second. Let the frequency of A be N and the number of beats produced per second be n. Therefore, the frequency of B will be $N \pm n$.

To find the correct value of B *viz.*, $(N + n)$ or $(N - n)$, the tuning fork B is loaded with a little wax. When a tuning fork is loaded with wax, its frequency decreases. If the number of beats per second increases, the original frequency of B is $N - n$. If the number of beats per second decreases, the original frequency of B is $N + n$.

Note. (*i*) When a tuning fork is loaded, its frequency decreases.

(*ii*) When a tuning fork is filed, its frequency increases.

Example 6.2. *A tuning fork A of frequency 384* Hz *gives 6* beats *per* second *when sounded with another tuning fork B. On loading B with a little wax, the number of beats per* second *becomes 4. What is the frequency of B?*

(Rajasthan, 1986)

Frequency of $A = 384$ Hz

Beats per second $= 6$

Frequency of B before loading is

Either $384 + 6 = 390$ Hz

or $384 - 6 = 378$ Hz

After loading, beats per second = 4

Frequency of B after loading is

Either $384 + 4 = 388$ Hz

or $384 - 4 = 380$ Hz.

Since, after loading the frequency decreases, frequency of B before loading is **390 Hz.**

Example 6.3. *When two tuning forks were sounded together, 20* beats *were produced in 8* seconds. *After loading one of the tuning forks with a little wax, they produce 32* beats *in 8* seconds. *If the unloaded fork had a frequency of 512* Hz, *calculate the frequency of the other.* [Delhi (Supple), 1986]

Frequency of A $= 512$ Hz

Frequency of B $= ?$

Before loading beats per second

$$= \frac{20}{8} = 2{\cdot}5$$

Frequency of B before loading is

Either $512 + 2{\cdot}5 = 514{\cdot}5$

or $512 - 2{\cdot}5 = 509{\cdot}5$

After loading, beats per second

$$= \frac{32}{8} = 4$$

Frequency of B after loading is

Either $512 + 4 = 516$

or $512 - 4 = 508$

Since, after loading the frequency of a tuning fork is lowered, the frequency of B, before loading is 509·5 Hz because after loading with a *little* wax, its frequency is *slightly* lowered to 508.

Hence frequency of B before loading

= 509·5 Hz.

Example 6.4. *A tuning fork A produces 4* beats/second *with a tuning frok B of frequency 256. A is filed and the beats occur at shorter intervals. What was its original frequency ?*

Frequency of B $= 256$

Beats/second $= 4$

Frequency of $A = 256 + 4 = 260$ or $256 - 4 = 252$.

When A is filed its frequency increases. Suppose the original frequency of A is 260. When it is filed its frequency will be more than 260 and the beats/second increases *i.e.*, the beats occur at shorter intervals, which is given. Hence the original frequency of $A = 260$.

Example 6.5. *A note produces 4* beats/second *with a tuning fork of frequency 512 and 6* beats/second *with a fork of frequency 514. Find the frequency of the note.*

In the first case

Frequency of the tuning fork = 512

Beats/second = 4

∴ Possible frequencies of the note are 512 + 4 = 516

or 512 − 4 = **508**

In the second case

Frequency of the tuning fork = 514

Beats/second = 6

Possible frequencies of the note are 514 + 6 = 520

or 514 − 6 = **508**

Hence the frequency of the note is 508.

Example 6.6. *Two tuning forks A and B give 5* beats/second. *The frequency of A=512. When B is filed, 5* beats/second *are again produced. Find the frequency of B before and after filing.*

Frequency of A = 512

Beats per second = 5

Frequency of B before filing is either

512 + 5 = 517

or 512 − 5 = 507

Since, after filing 5 beats/second are again produced, frequency of B after filing is either

512 + 5 = 517

or 512 − 5 = 507

Let us consider the frequency of B before filing as 517. After filing the frequency increases. Therefore, after filing the frequency of B cannot be equal to either 517 or 507. Therefore, frequency of B cannot be equal to 517.

Consider the frequency of B before filing 507

After filing its frequency can be equal to 517

Therefore, this is the possible value.

Hence, frequency of B before filing = **507**

" " " after filing = **517.**

Example 6.7. *Calculate the velocity of sound in a gas in which two waves of lengths 1* metre *and 1·01* metres *produce 10* beats *in 3* seconds.

Number of beats in 3 s = 10

∴ Beats/second $= \frac{10}{3}$

Here, $\lambda_1 = 100$ cm

$\lambda_2 = 1{\cdot}01$ metres = 101 cm

Suppose the velocity = V

Frequency of the first wave $= n_1$

Frequency of the second wave $= n_2$

$$\therefore \quad n_1 - n_2 = \text{beat/s} = \frac{10}{3}$$

$$n_1 - n_2 = \frac{10}{3}$$

But $n_1 = \frac{V}{\lambda_1} = \frac{V}{100}$ and $n_2 = \frac{V}{\lambda_2} = \frac{V}{101}$

Substituting the values of n_1 and n_2

$$\frac{V}{100} - \frac{V}{101} = \frac{10}{3} \quad \text{or} \quad = \frac{V}{10100} = \frac{10}{3}$$

$$V = \frac{101000}{3} = 33666{\cdot}67 \text{ cm/s}$$

$$= \mathbf{336{\cdot}67 \text{ m/s.}}$$

6.14. Analytical Treatment of Beats

Consider two wave trains of frequencies n_1 and n_2 where $(n_1 - n_2)$ is small. Let a and b be the amplitudes of the waves respectively. For the sake of simplicity, it is assumed that the two waves are in phase at any point in the medium at $t = 0$. The displacements y_1 and y_2 due to each wave are given by

$$y_1 = a \sin \omega_1 t$$

$$y_2 = b \sin \omega_2 t$$

Here $\omega_1 = 2\pi n_1$

and, $\omega_2 = 2\pi n_2$

$$\therefore \quad y_1 = a \sin 2\pi n_1 t \qquad \ldots (1)$$

and $y_2 = b \sin 2\pi n_2 t \qquad \ldots (2)$

The resultant displacement is given by

$$y = y_1 + y_2$$

$$y = a \sin 2\pi n_1 t + b \sin 2\pi n_2 t$$

$$y = a \sin 2\pi n_1 t + b \sin 2\pi [n_1 - (n_1 - n_2)] t$$

$$y = a \sin 2\pi n_1 t + b [\sin 2\pi n_1 t \cos 2\pi (n_1 - n_2) t$$

$$- \cos 2\pi n_1 t \sin 2\pi (n_1 - n_2) t] \qquad \ldots (3)$$

$$y = \sin 2\pi n_1 t\,[a + b\cos 2\pi (n_1 - n_2)\,t] - \cos(2\pi n_1 t)\,[b \sin 2\pi (n_1 - n_2)\,t]$$

Take $a + b\cos 2\pi (n_1 - n_2)\,t = A\cos\theta$

and $b \sin 2\pi (n_1 - n_2)\,t = A\sin\theta$

$\therefore$ $y = A\sin(2\pi n_1 t - \theta)$

Here $$\tan\theta = \frac{b\sin 2\pi (n_1 - n_2)\,t}{a + b\cos 2\pi (n_1 - n_2)\,t} \quad \ldots (4)$$

and $$A = \sqrt{a^2 + b^2 + 2ab\cos 2\pi (n_1 - n_2)\,t} \quad \ldots (5)$$

From equation (4), it is evident that the phase angle θ changes with respect to time. Similarly from equation (5), the amplitude of the resultant vibration also changes with time.

(1) When $2\pi (n_1 - n_2)\,t = 2K\pi$

where $K = 0, 1, 2, 3 \ldots$ etc.

the resultant amplitude,

$$A = \sqrt{a^2 + b^2 + 2ab}$$

$$A = (a + b) \quad \ldots (6)$$

The resultant amplitude is maximum, when

$$t = \frac{K}{(n_1 - n_2)}$$

i.e., at time instants, $0, \frac{1}{(n_1 - n_2)}, \frac{2}{(n_1 - n_2)}$ etc., the amplitude of the resultant is maximum. As the intensity of sound is directly proportional to the square of the amplitude, the maximum intensity of sound will be heard at the instants,

$$0, \frac{1}{(n_1 - n_2)}, \frac{2}{(n_1 - n_2)}, \frac{3}{(n_1 - n_2)} \ldots \text{etc.}$$

(2) When $2\pi (n_1 - n_2)\,t = (2K + 1)\,\pi$

where $K = 0, 1, 2, 3,$ etc.

the resultant amplitude

$$A = \sqrt{a^2 + b^2 - 2ab}$$

$$A = (a - b)$$

The resultant amplitude is minimum when,

$$t = \frac{(2K + 1)}{2(n_1 - n_2)}$$

i.e., at time instants,

$$\frac{1}{2(n_1 - n_2)}, \frac{3}{2(n_1 - n_2)}, \frac{5}{2(n_1 - n_2)} \ldots \text{etc.}$$

the amplitude of the resultant is minimum.

Hence minimum intensity of sound will be heard at the instants,

$$\frac{1}{2(n_1 - n_2)}, \frac{3}{2(n_1 - n_2)}, \frac{5}{2(n_1 - n_2)} \ldots \text{etc.}$$

Thus, the maxima and minima occur alternately after equal intervals of time $\frac{1}{2(n_1 - n_2)}$. The time interval between two successive maxima or between two successive minima is $\frac{1}{(n_1 - n_2)}$.

The number of beats produced per second $= (n_1 - n_2)$.

If the amplitudes of the two wave trains are equal, the maximum amplitude of the resultant $= 2a$ and the minimum amplitude $= 0$.

In this case, the positions of minima will be of zero intensity.

6.15. Combination Tones

By mathematical analysis it can be shown that, when two strong notes of frequencies P and Q are sounded together, a series of tones of other frequencies are produced along with the original notes. These tones are called *combination tones*. Combination tones are different from the harmonics. The strongest of the combination tones is the **First Difference Tone** of frequency $(P - Q)$ or $(Q - P)$. The difference tones can be infinite in number, *viz.*, $P - 2Q$, $P - 3Q$, $P - 4Q \ldots$ etc. It is also possible that two difference tones of frequencies $(P - 2Q)$ and $(P - 4Q)$ can theoretically produce a difference tone of frequency $(P - 2Q) - (P - 4Q) = 2Q$.

In a similar manner one can expect summation tones of frequencies $(P + Q)$, $(P + 2Q)$, $(2P + Q)$ etc. In fact, an infinite number of combination tones, difference or summation type, are theoretically possible. Usually it is the first difference tone that is easily audible. The first difference tone is weaker than the two primaries. However, the second, third etc., difference tones and the summation tones are extremely weak in intensity.

6.16. Helmholtz Resonator

In various acoustical investigations, Helmholtz resonators are commonly used. Helmholtz particularly used them in the study of the quality of musical notes and hence these are called helmholtz resonators.

The principle of a resonator is that the air contained in a hollow body with a narrow neck, say a bottle, vibrates with a certain time period. If a tuning fork of the same frequency as that of the natural frequency of a resonator is held near its mouth, the sound intensity increases and resonant vibrations are heard. Two

common forms of Helmholtz resonators are shown in Fig. 6.15. Both the resonators are provided with a wide and a narrow mouth. The source of sound is kept at the wide end, and the ear is kept near the narrow end.

The natural frequency of resonator *A* is fixed because of its fixed volume of air. In the case of *B*, the volume of air enclosed can be altered by sliding the outer cylinder. The natural frequency can be varied as desired.

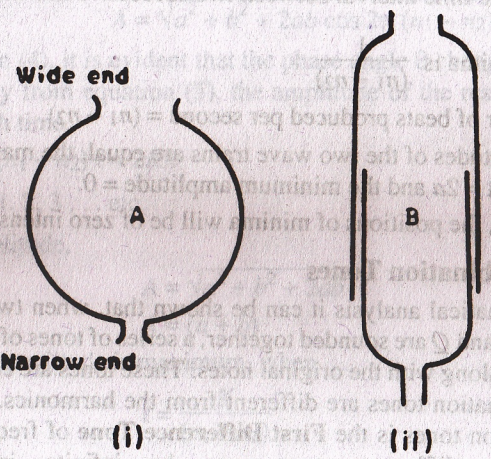


Fig. 6.15.

6.17. Theory of Resonator

Consider a flask containing air and of narrow cross section area *a*. Let the volume of the flask be *V* and the mass of air contained in the neck be *m*. The pressure inside the flask is *P* and outside pressure is P_0 (Fig. 6.16).

For equilibrium,

$$P = P_0 + \frac{mg}{a} \qquad \ldots (1)$$

When the air in the flask is in resonance with a particular frequency, the air contained in the neck moves up and down and acts like a piston on the large mass of air in the neck. Let, any instant, the mass of air in the neck move down through a distance *x*. If the compression is adiabatic, the new pressure P_1 in the vessel will be,

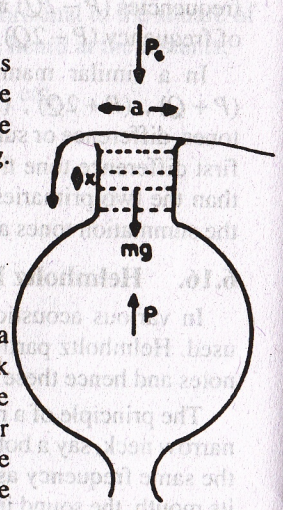


Fig. 6.16.

$$P_1 (V - ax)^\gamma = PV^\gamma \qquad \ldots (2)$$

$$\therefore \quad P_1 = P\left[\frac{V}{V - ax}\right]^\gamma$$

$$P_1 = P\left[1 + \frac{ax}{V - ax}\right]^\gamma$$

or $$P_1 = P\left[1 + \frac{\gamma a x}{V - a x}\right] \quad \text{(approx.)}$$

$$\therefore \quad P_1 - P = \frac{P\gamma ax}{V - ax} \qquad \ldots (3)$$

The net downward force *F* on the air in the resonator,

$$F = \left[\left(P_0 + \frac{mg}{x}\right) - P_1\right] a$$

$$F = [P - P_1]\, a$$

Substituting the value of $(P - P_1)$ from equation (3)

$$F = -\left(\frac{P\gamma ax}{V - ax}\right) a \quad \text{or} \quad F = -\frac{P\gamma a^2 x}{V - ax}$$

As *ax* is extremely small as compared to *V*,

$$\therefore \quad F = -\frac{P\gamma a^2 x}{V}$$

But F = Mass × Acceleration

$$\therefore \quad -\frac{P\gamma a^2 x}{V} = \text{Mass} \times \text{Acceleration}$$

$$\therefore \quad \text{Acceleration} = \left(\frac{-P\gamma a^2}{mV}\right) x$$

$$\therefore \quad \frac{\text{Acceleration}}{x} = -\left(\frac{P\gamma a^2}{mV}\right)$$

$$\therefore \quad \frac{\text{Acceleration}}{\text{Displacement}} = -\left(\frac{P\gamma a^2}{mV}\right) \qquad \ldots (4)$$

This represents simple harmonic motion.

∴ Time period,

$$T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$T = 2\pi\sqrt{\frac{mV}{P\gamma a^2}}$$

The frequency of oscillation,

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{P\gamma a^2}{mV}} \quad \ldots (5)$$

The velocity of sound in air,

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \text{or} \quad P\gamma = v^2\rho$$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{v^2\rho a^2}{mV}}$$

But mass of air, $m = al\rho$

Here l is the length of the neck.

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{v^2\rho a^2}{al\rho V}}$$

$$f = \frac{v}{2\pi}\sqrt{\frac{a}{lV}} \quad \ldots (6)$$

$$\therefore \quad f^2 V = \frac{v^2 a}{4\pi^2 l} \quad \ldots (7)$$

As a, l and v remain constant,

$$f^2 V = \text{constant}$$

$$\therefore \quad f \propto \frac{1}{\sqrt{V}}.$$

Thus, the frequency of the resonator is inversely proportional to the square root of its volume.

Example 6.8. *A Helmholtz resonator has a volume of 1* litre. *The radius of the neck is 1* cm *and the length of the neck is 0·2* cm. *Calculate the frequency at resonance. Take the velocity of sound at room temperature=350* m/s.

Here $V = 1000\ \text{cm}^3 = 10^{-3}\ \text{m}^3$

$l = 0{\cdot}2\ \text{cm} = 2 \times 10^{-3}\ \text{m}$

$r = 1\ \text{cm} = 10^{-2}\ \text{m}$

$a = \pi r^2 = 3{\cdot}14 \times 10^{-4}\ \text{m}^2$

$v = 350\ \text{m/s}$

The resonant frequency,

$$f = \frac{v}{2\pi}\sqrt{\frac{a}{lV}} \quad \text{or} \quad f = \frac{350}{2 \times 3{\cdot}14}\sqrt{\frac{3{\cdot}14 \times 10^{-4}}{2 \times 10^{-3} \times 10^{-3}}}$$

$f = 698{\cdot}4$ **hertz.**

6.18. Dependence of the Frequency of Resonator on the Size and Shape of the Mouth

The frequency of a Helmholtz resonator also depends upon the size and shape of the mouth. The time taken by the excess of pressure inside to drive out air depends upon the size of the mouth of the resonator. If air can escape more easily (wider mouth), the time period is short *i.e.*, the frequency is high and *vice versa.* The property of the mouth to allow the air to escape less easily or more easily is denoted by the term acoustic conductivity of the mouth. This term acoustic conductivity depends upon the area of cross section (a) and the length (l) of the mouth.

The acoustic conductivity,

$$k = \frac{a}{l} \quad \ldots (1)$$

The frequency of the resonator,

$$f = \frac{v}{2\pi}\sqrt{\frac{a}{lV}} = \frac{v}{2\pi}\sqrt{\frac{k}{V}} \quad \ldots (2)$$

For a circular aperture in a thin wall, l is very small. Raleigh showed that in such cases, the acoustic conductivity k is approximately equal to the diameter.

$$\therefore \quad k = \frac{a}{l} = 2r$$

or

$$f = \frac{v}{2\pi}\sqrt{\frac{2r}{V}} \quad \ldots (3)$$

Example 6.9. *A small hole is drilled in a spherical resonator of redius* 5×10^{-2} m. *The frequency at resonance is 300* hertz. *(a) Calculate the radius of the hole. Velocity of sound in air at room temperature=350* m/s. *(b) Also calculate the resonant frequency if three additional holes of the same size are drilled in the resonator.*

Here $v = 350\ \text{m/s}$

$V = \frac{4}{3}\pi (5 \times 10^{-2})^3\ \text{m}^3$

$f = 300$ hertz

$r = ?$

(*a*) $f = \frac{v}{2\pi}\sqrt{\frac{2r}{V}}$

or $$r = \frac{2\pi^2 f^2 V}{v^2}$$

or $$r = \frac{2 \times (3{\cdot}14)^2 \times (300)^2 \times 1{\cdot}33 \times 3{\cdot}14\,(5 \times 10^{-2})^3}{(350)^2}$$

$r = 0{\cdot}007561$ m.

(b) For a total number of 4 holes of the same size, the total area of cross section is 4 times the original area. But the effective length and the volume of air in the resonator remain the same.

$$\therefore \quad f' = \frac{v}{2\pi}\sqrt{\frac{4a}{lV}}$$

$$f' = 2\left[\frac{v}{2\pi}\sqrt{\frac{a}{lV}}\right]$$

$$f' = 2\left[\frac{v}{2\pi}\sqrt{\frac{2r}{V}}\right]$$

$$f' = 2f$$

$$f' = 2 \times 300$$

$$f' = \mathbf{600\ hertz.}$$

Example 6.10. *A direct radiating loud speaker of radius 10* cm *is mounted in a rigid walled cabinet of inside dimensions* $40 \times 50 \times 60$ cm^3 *and wall thickness 2* cm. *Calculate the resonance frequency of the cabinet, considering that the cabinet works as a Helmholtz resonator. Velocity of sound in air at room temperature=350* m/s.

Here $t = 2$ cm $= 0{\cdot}02$ m

$r = 10$ cm $= 0{\cdot}1$ m

$a = \pi r^2 = 3{\cdot}14 \times (0{\cdot}1)^2$

or $a = 0{\cdot}0314$ m^2

Effective length $l = \left(t + \frac{\pi r}{2}\right)$

$= 0{\cdot}02 + 0{\cdot}157 = 0{\cdot}177$ m

$V = 40 \times 50 \times 60$ cm^3

or $V = 0{\cdot}12$ m^3

$v = 350$ m/s

$f = ?$

$$f = \frac{v}{2\pi}\sqrt{\frac{a}{lV}}$$

$$= \frac{350}{2 \times 3{\cdot}14}\sqrt{\frac{0{\cdot}0314}{0{\cdot}177 \times 0{\cdot}12}}$$

$$= \mathbf{67{\cdot}76\ hertz.}$$

EXERCISES

1. Explain the formation of stationary waves when two wave trains of exactly the same amplitude and wavelength are travelling along the same string in opposite directions. Calculate the expression for the distance between two consecutive nodes. *[Behrampur (Hons.), 1992]*
2. Explain clearly the distinction between progressive waves and longitudinal waves.
3. Give briefly the theory of formation of stationary waves and discuss the characteristics of this wave motion. *(Calcutta)*
4. Discuss analytically the formation of beats and show that the number of beats per second is equal to the difference in frequency of the two notes. *(Calcutta)*
5. Discuss analytically the formation of stationary waves and explain how the characteristics change *(i)* with distance and *(ii)* with time.
6. Give a general account of distribution of energy in a stationary wave.
7. Explain fully the equation
$$y = 2a \cos\frac{2\pi x}{\lambda} \sin\frac{2\pi vt}{\lambda}.$$
8. Explain the principle of superposition. Use it to obtain the formation of stationary waves in an elastic medium. Discuss the property of the medium at the node and antinode positions. Show that in the case of a stationary wave, no energy is transferred across any section of the medium. *(Delhi, 1993)*
9. Describe an experiment to demonstrate interference of sound waves. *(Delhi)*
10. Discuss analytically the formation of beats. *(Delhi)*
11. Two closed organ pipes 51 cm and 52 cm long produce three beats per second when they are simultaneously sounding their fundamental notes. Calculate the velocity of sound ignoring end correction. *(Delhi, 1993)* **[Ans.** 318·24 m/s]
12. What are stationary waves and how are they formed ? State their important properties. *(Lucknow; Delhi)*

13. What are beats? Discuss analytically the formation of beats.

14. A tuning fork of unknown frequency gives 5 beats per second when sounded together with another tuning fork of frequency 384 hertz. The fork is now loaded with wax and three beats per second are heard. Calculate the unknown frequency. (**Ans**.389)

15. Discuss the theory of the formation of stationary waves. State their important properties.

16. What are stationary waves and how are they formed?

17. What is interference of sound? How are beats produced? *(Calcutta)*

18. What are beats? How are they produced? Derive an expression for the frequency of beats.

19. Describe an experiment to demonstrate the formation of beats.

20. Describe Helmholtz resonator and obtain an expression for the natural frequency of such a resonator. Indicate its use in the analysis of sound. *[Madras; Behrampur (Hons.)]*

21. Calculate the velocity of sound in a gas in which two waves of lengths 50 cm and 50 · 5 cm produce 6 beats/s. [**Ans**. 303 m/s] *(Calcutta)*

22. Discuss the theory of Helmholtz resonator and give its uses. *(Calcutta)*

23. Two organ pipes, one closed and the other open are found to have their first over tones of the same frequency. Find the ratio of their lengths. [**Ans.** (3 : 4) *Calcutta)*

24. Discuss how sound waves are reflected at the open and closed ends of a tube producing stationary waves in air. How are their properties studied analytically? *(Delhi)*

25. What are stationary waves? Investigate theoretically the formation of stationary waves and discuss their properties with regard to changes of displacement and pressure at different points. *[Delhi (Supple), 1986]*

26. What are beats? Differentiate between the phenomenon of beats and interference. Show that the number of beats per second is equal to the difference between the frequencies of the two sources sounded together. *[Delhi (Supple), 1986]*

27. What are beats? Explain graphically their formation. *(Rajasthan, 1985)*

28. Discuss the formation of beats. *(Kanpur, 1985)*

29. Deduce the equation for a stationary wave and it derive from it its characteristic properties. Compare these properties with those of a progressive wave. (Kanpur, 1985)

30. What are beats and how are they formed? *[Delhi (Add Physics), 1986]*

31. Explain the formation of stationary waves and discuss their characteristics. How do they differ from progressive waves? *(Calcutta, 1992)*

32. Describe a Helmohtz's resonator. How can the frequency of sound be measured with it. *(Calcutta, 1992)*

33. Explain analytically the formation of beats when two sources of nearly equal frequencies are sounded together. *(Gauhati, 1992)*

34. Explain the principle of superposition. Use it to obtain the formation of stationary waves in an elastic medium. Discuss the property of the medium at the node and antinode positions *(Delhi, 1993)*



CHAPTER 7

Vibrations in Strings and Air Columns

7.1. Velocity of Transverse Waves Along a Stretched String

First Method

Consider a string PQ stretched by a tension T. Let the string be plucked at the centre O and left free (Fig. 7.1). The string vibrates transversely. These vibrations are simple harmonic in nature. The force tending to bring any element of the string back to the equilibrium position is the component of tension acting at right angles to PQ.

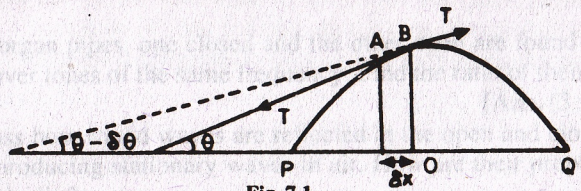


Fig. 7.1.

Let the undisplaced position of the string correspond to the X-axis and let the displacement be along the Y-axis. Consider a small element AB of length δx. The tangents drawn at A and B make angles θ and $\theta - \delta\theta$ with the X-axis (Fig. 7.1). The tension at A is resolved into two rectangular components. The downward component of tension at $A = T \sin \theta$.

As θ is small $\quad \sin \theta = \tan \theta$

$\therefore$ Downward component of tension at $A = T \tan \theta$.

But $\quad \tan \theta = \dfrac{dy}{dx}$ at A (*i.e.*, slope at A)

$\therefore$ Downward component of tension at A

$$= T \frac{dy}{dx} \qquad \ldots (1)$$

The rate of change of slope with respect to the length of the element,

$$= \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

The change in slope for a distance δx

$$= \frac{d^2y}{dx^2} \cdot \delta x$$

The slope at the point B

$$\tan (\theta - \delta\theta) = \frac{dy}{dx} - \frac{d^2y}{dx^2} \cdot \delta x$$

The upward component of tension acting at $B = T \sin (\theta - \delta\theta)$

As θ is small, $\sin (\theta - \delta\theta) = \tan (\theta - \delta\theta)$

$\therefore$ Upward component of tension at B

$$= T \tan (\theta - \delta\theta)$$

$$= T\left[\frac{dy}{dx} - \frac{d^2y}{dx^2} \cdot \delta x\right] \qquad \ldots (2)$$

From equations (1) and (2), resultant downward tension,

$$F = T\frac{dy}{dx} - T\left(\frac{dy}{dx} - \frac{d^2y}{dx^2} \cdot \delta x\right)$$

$$F = T\frac{d^2y}{dx^2}\delta x \qquad \ldots (3)$$

Let m be the mass of the string per unit length.

Mass of the element $= m\,\delta x$

Acceleration of the element in the direction of y-axis

$$= \frac{d^2y}{dt^2}$$

Force acting on the element = mass × acceleration

$$F = m\,\delta x \frac{d^2y}{dt^2} \qquad \ldots (4)$$

From equations (3) and (4),

$$m\,\delta x \frac{d^2y}{dt^2} = T\frac{d^2y}{dx^2}\delta x$$

$$\frac{d^2y}{dt^2} = \frac{T}{m}\frac{d^2y}{dx^2} \qquad \ldots (5)$$

This is similar to the differential equation of wave motion,

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \quad \ldots (6)$$

Here v is the velocity of propagation of the wave.

Comparing equations (5) and (6)

$$v^2 = \frac{T}{m}$$

$$v \sqrt{\frac{T}{m}} \quad \ldots (7)$$

Also $v = n\lambda$

In case the string of length l vibrates in p segments, length of each segment $= \frac{l}{p}$ and each segment corresponds to half wavelength

$$\therefore \quad \frac{l}{p} = \frac{\lambda}{2}$$

$$\lambda = \frac{2l}{p}$$

$$\therefore \quad v = \frac{n(2l)}{p} \quad \ldots (8)$$

Substituting this value of v in equation (7)

$$\frac{n(2l)}{p} = \sqrt{\frac{T}{m}}$$

$$n = \frac{p}{2l} \sqrt{\frac{T}{m}} \quad \ldots (9)$$

In case the string vibrates in one segment

$$p = 1,$$

and $$n = \frac{1}{2l} \sqrt{\frac{T}{m}} \quad \ldots (10)$$

7.2. Alternative Method

Consider the portion $PABQ$ of a string in which a transverse wave is travelling from left to right with a velocity v. The string is also drawn towards the left with the same velocity so that the wave is stationary (Fig. 7.2).

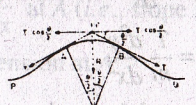


Fig. 7.2.

Let AB be a small element of the string. The centre of curvature of the element AB is O. The angle $AOB = \theta$. It is assumed that the curvature of AB is small and hence θ is small. Let T be the tension at A and B. The directions of these tensions are tangential to the element at A and B.

Resolving the tension at A into two rectangular components,

$T \sin \frac{\theta}{2}$ and $T \cos \frac{\theta}{2}$ perpendicular and parallel to the string in the undisplaced portion.

Similarly the components at B will be $T \sin \frac{\theta}{2}$ and $T \cos \frac{\theta}{2}$ perpendicular and parallel to the string in the undisplaced portion.

The parallel components are equal and opposite. The perpendicular components are along CO.

The resultant tension along $CO = 2T \sin \frac{\theta}{2}$

As θ is small, $\sin \frac{\theta}{2} = \frac{\theta}{2}$

$\therefore$ Resultant tension $= 2T \frac{\theta}{2} = T\theta \quad \ldots (1)$

For equilibrium position, this resultant tension provides the necessary centripetal force,

$$= \frac{(m\,\delta x)\, v^2}{R} \quad \ldots (2)$$

$$\therefore \quad \frac{m\,\delta x\, v^2}{R} = T\theta$$

But $$\theta = \frac{\delta x}{R}$$

$$\therefore \quad \frac{m \,.\, \delta x \,.\, v^2}{R} = \frac{T \,.\, \delta x}{R} \quad \text{or} \quad v^2 = \frac{T}{m}$$

$$v = \sqrt{\frac{T}{m}} \quad \ldots (3)$$

Here m is the per unit length of the string.

Also $v = n\lambda$

In case the string of length l vibrates in p segments, length of each segment $= \frac{l}{p}$ and each segment corresponds to half wave length.

$$\therefore \quad \frac{l}{p} = \frac{\lambda}{2} \quad \text{or} \quad \lambda = \frac{2l}{p}$$

$$\therefore \quad v = \frac{n\,(2l)}{p} \qquad \ldots (4)$$

Substituting this value of v in equation (3)

$$\frac{n\,(2l)}{p} = \sqrt{\frac{T}{m}}$$

$$n = \frac{p}{2l}\sqrt{\frac{T}{m}} \qquad \ldots (5)$$

In case the string vibrates in one segment

$$p = 1$$

and

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} \qquad \ldots (6)$$

7.3. Laws of Transverse Vibration of Strings

There are three laws of transverse vibration of strings :

(1) The fundamental frequency is inversely proportional to the length of the string

$$n \propto \frac{1}{l}$$

(2) The fundamental frequency is directly proportional to the square root of the stretching force or tension

$$n \propto \sqrt{T}$$

(3) The fundamental frequency is inversely proportional to the square root of the mass per unit length

$$n \propto \frac{1}{\sqrt{m}}$$

Combining the above three laws,

$$n \propto \frac{1}{l}\sqrt{\frac{T}{m}} \quad \text{or} \quad n = \frac{k}{l}\sqrt{\frac{T}{m}}$$

The value of the constant $k = \frac{1}{2}$

$$\therefore \quad n = \frac{1}{2l}\sqrt{\frac{T}{m}} \qquad \ldots (1)$$

If D is the diameter of the wire and d is the density of the material of the wire, then

$$m = \frac{\pi D^2}{4} \times 1 \times d$$

$$n = \frac{1}{2l}\sqrt{\frac{4T}{\pi D^2 d}}$$

$$n = \frac{1}{lD}\sqrt{\frac{T}{\pi d}} \qquad \ldots (2)$$

7.4. Verification of the Laws of Transverse Vibration of Strings

(1) $n \propto \frac{1}{l}$ or nl = constant

Take a sonomter and a tuning fork of frequency 256 (Fig. 7.3). Keeping a load of 1 kg find the position of the two bridges when the wire is in unsion with

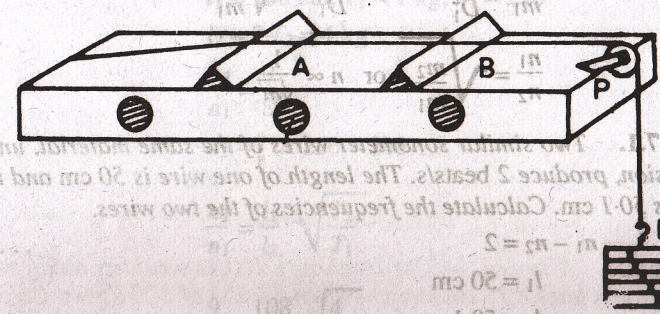


Fig. 7.3.

the tuning fork. This can be found by using a small paper rider on the sonometer wire. Suppose the length = l_1. Now, using a tuning fork of frequency 512, find the reasonating length, keeping tension the same. Let this length be l_2. It is found that l_2 is half l_1. Also $n_1 l_1 = n_2 l_2$.

$$n \propto \frac{1}{l} \quad \text{or} \quad nl = \text{constant}$$

(2) $n \propto \sqrt{T}$

For a tuning fork of frequency 256, find the resonating length l_1 for a load of 1 kg. If will be found that when the load is increased to 4 kg the tuning fork of frequency 512 is in resonance with the same length of the wire l_1. Here

$$\frac{n_2}{n_1} \sqrt{\frac{T_2}{T_1}}, \quad \text{or } n \propto \sqrt{T}$$

Also $n^2 \propto T$ or $T \propto n^2$

(3) $n \propto \frac{1}{\sqrt{m}}$

Take two wires of the same material and of diameters in the ratio of 1 : 2.

Stretch the first wire with a tension of 1 kg wt. Find the resonating length with a tuning fork of frequency 512. Now stretch the second wire with the same tension of 1 kg and find the resonating length with a tuning fork of frequency 256. It is found that the two lengths are equal.

Here $\frac{n_1}{n_2} = \frac{D_2}{D_1}$

$$m_1 = \frac{\pi D_1^2 \times d}{4} \quad \text{and} \quad m_2 = \frac{\pi D_2^2 \times d}{4}$$

But $\frac{m_2}{m_1} = \frac{D_2^2}{D_1^2}$ or $\frac{D_2}{D_1} = \sqrt{\frac{m_2}{m_1}}$

$\therefore$ $\frac{n_1}{n_2} = \sqrt{\frac{m_2}{m_1}}$ or $n \propto \frac{1}{\sqrt{m}}$.

Example 7.1. *Two similar sonometer wires of the same material, under the same tension, produce* 2 beats/s. *The length of one wire is 50* cm *and that of the other is 50·1* cm. *Calculate the frequencies of the two wires.*

Here $n_1 - n_2 = 2$ ... (1)

$l_1 = 50$ cm

$l_2 = 50{\cdot}1$ cm

As the tensions and material of the wires are the same,

$n_1 l_1 = n_2 l_2$

or $\frac{n_1}{n_2} = \frac{l_2}{l_1}$

$\frac{n_1}{n_2} = \frac{50{\cdot}1}{50}$

or $n_1 = \left(\frac{50{\cdot}1}{50}\right) n_2$ ... (2)

Substituting this value of n_1 in equation (1),

$\left(\frac{50{\cdot}1}{50}\right) n_2 - n_2 = 2$

$n_2 =$ **1000 hertz**

$\therefore$ $n_1 = n_2 + 2$

$n_1 = 1000 + 2$

$n_1 =$ **1002 hertz.**

Example 7.2. *Two strings each of length 60* cm *are stretched, one by a force of 4* kg wt *and the other by a force of 9* kg wt. *What is the interval between the two notes that are produced?*

$$\frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

The interval between the two notes is the fifth.

Example 7.3. *A wire of length* 108 cm *produces a fundamental note of frequency 256 when stretched by a weight of 1* kg. *By how much its length should be increased so that its pitch is raised by a major tone if it is now stretched by a weight of 4* kg.

$l_1 = 108$ cm, $T_1 = 1$ kg, $n_1 = 256$

$l_2 = ?$ $T_2 = 4$ kg

$\frac{n_2}{n_1} = \frac{9}{8}$

or $n_2 = \frac{9}{8} \times 256 = 288$

$\frac{n_2}{n_1} = \frac{l_1}{l_2}\sqrt{\frac{T_2}{T_1}}$

$\frac{9}{8} = \frac{108}{l_2}\sqrt{\frac{4}{1}}$

$l_2 = 192$ cm

The increase in length = 192 – 108 = **84 cm.**

Example 7.4. *A wire of specific gravity 7, one metre long and 1* mm *in diameter is stretched by a weight of 11* kg. *Calculate the pitch of the fundamental note.*

Here $d = 7$, $l = 100$ cm

$D = 1$ mm $= \frac{1}{10}$ cm

$T = 11$ kg $= 11 \times 1000 \times 980$ dynes

$n = ?$

$$n = \frac{1}{lD}\sqrt{\frac{T}{\pi d}}$$

$$n = \frac{1}{100 \times \frac{1}{10}}\sqrt{\frac{11 \times 1000 \times 980 \times 7}{22 \times 7}}$$

$= 70{\cdot}$

Example 7.5. *A string when stretched by a weight of 4* kg *gives a note of frequency 256. What weight will produce an octave of this note?*

Here, $n_1 = 256$, $n_2 = 2 \times 256 = 512$

$T_1 = 4$ kg wt ; $T_2 = ?$

$\frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}}$ or $\frac{512}{256} = \sqrt{\frac{T_2}{T_1}}$

or $\sqrt{\frac{T_2}{4}} = 2,\ T_2 = \mathbf{16\ kg\ wt.}$

Example 7.6. *Calculate the frequency of the fundamental note of a string, 1* metre *long and weighing 2* grams *when stretched by a weight of 400* kg.

Here, $l = 1$ metre

Total mass $= 2\ = 0{\cdot}002$ kg

$$m = \frac{0{\cdot}002}{1} = 0{\cdot}002 \text{ kg/m}$$

$$T = 400 \text{ kg} = 400 \times 9{\cdot}8 \text{ newtons}$$

$$n = \frac{l}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2\times 1}\sqrt{\frac{400\times 9{\cdot}8}{0{\cdot}002}}$$

$$n = \tfrac{1}{2}\sqrt{400\times 4900} = \mathbf{700.}$$

Example 7.7. *A tuning fork of frequency 160 is sounded along with a sonometer wire of length 25* cm *stretched to a tension of 1·25* kg wt. *Calculate the number of beats/s.*

(Mass per unit length = 0·025 g/cm*).*

Here $l = 25$ cm, $m = 0{\cdot}025$ g/cm

$$T = 1{\cdot}25 \text{ kg wt} = 1{\cdot}25 \times 1000 \times 980 \text{ dynes}$$

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2\times 25}\sqrt{\frac{1{\cdot}25\times 1000\times 980}{0{\cdot}025}}$$

$$n = \frac{7000}{50} = 140$$

Frequency of the tuning fork $N = 160$

$\therefore$ Beats/s $= N - n = 160 - 140 = \mathbf{20.}$

Example 7.8. *A wire gives a fundamental note of 256* cycles *per second when it is under a tension of 10* kg wt.

(i) Under what tension will the string emit a frequency of 512 cycles per second?

(ii) How would you make the wire emit a note of 768 cycles per second keeping the tension at 10 kg wt.

(i) Here, $n_1 = 256,\ T_1 = 10$ kg wt.

$n_2 = 512,\ T_2 = ?$

$$\frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{512}{256} = \sqrt{\frac{T_2}{10}} \quad \text{or} \quad T_2 = \mathbf{40\ kg\ wt.}$$

(ii) Here, in the first case

$n_1 = 256$

Length of the wire $= l_1$

In the second case, $n_2 = 768$

Length of the wire $= l_2$

$$\frac{n_2}{n_1} = \frac{l_1}{l_2} \text{ or } \frac{768}{256} = \frac{l_1}{l_2} \text{ or } l_2 = \frac{l_1}{3}$$

Therefore, by changing the length to $\frac{1}{3}$ of the length in the first case the wire will emit a note of frequency 768.

Example 7.9. *A flexible string of length 0·99* metre *and mass one* gram *is stretched by a tension of T* newtons. *The string vibrates in three segments with a frequency of 500* hertz. *Calculate the tension.*

$$n = \frac{p}{2l}\sqrt{\frac{T}{m}}$$

or $$n^2 = \frac{p^2 T}{4l^2 m}$$

or $$T = \frac{4n^2 l^2 m}{p^2}$$

Here $n = 500$ hertz

$l = 0{\cdot}99$ m

$$m = \frac{10^{-3}}{0{\cdot}99} \text{ kg/m}$$

$p = 3$

$$\therefore \quad T = \frac{4\times(500)^2\times(0{\cdot}99)^2\times 10^{-3}}{9\times 0{\cdot}99}$$

$T = \mathbf{110\ newtons.}$

Example 7.10. *A flexible string of length 0·88* m *is stretched by a force of 55* newtons. *The mass of the string is 1* gram. *Calculate the frequency of vibration of the string if it vibrates in 5 segments.*

$$n = \frac{p}{2l}\sqrt{\frac{T}{m}}$$

Here $p = 5$

$T = 55$ newtons

$l = 0{\cdot}88$ m

$$m = \frac{10^{-3}}{0{\cdot}88} \text{ kg/m}$$

$$n = \frac{5}{2\times 0{\cdot}88}\sqrt{\frac{55\times 0{\cdot}88}{10^{-3}}}$$

$n = \mathbf{625\ hertz.}$

7.5. Melde's Experiment

In Melde's Experiment, one end of the string is connected to the prong of an electrically maintained tuning fork. The other end is connected to the scale pan. The string passes over a smooth frictionless pulley. The distance between the tuning fork and the pulley can be adjusted. There are two modes of vibration (*i*) transverse mode of vibration and (*ii*) longitudinal mode of vibration (Fig. 7.4).

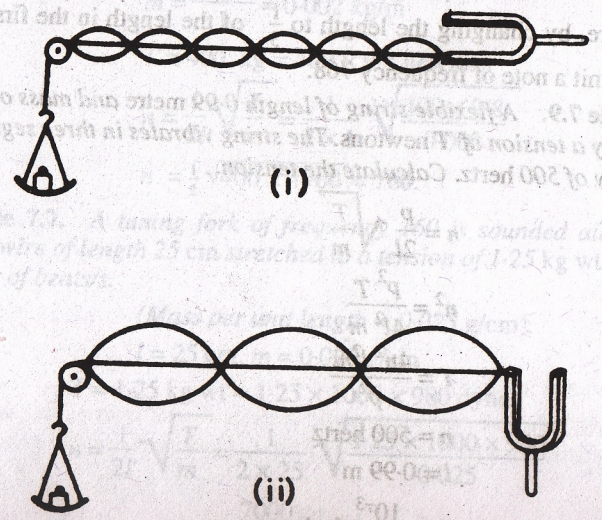


Fig. 7.4.

Transverse mode of vibration. The tuning fork vibrates at right angles to the length of the string [Fig. 7.4 (*i*)]. In this case the frequency of vibration of the string, is equal to the frequency of the tuning fork. Suppose N is the frequency of the tuning fork and the string of length l vibrates in p_1 segments.

$$\therefore \quad N = \frac{p_1}{2l}\sqrt{\frac{T}{m}} \qquad \ldots (1)$$

Longitudinal mode of vibration. The tuning fork vibrates along the direction of the length of the string [Fig. 7.4 (*ii*)]. In this case for one complete vibration of the tuning fork the string completes half vibration. Suppose the frequency of the tuning fork is N. Therefore, the frequency of vibration of the string is $N/2$. If the string of length l vibrates in p_2 segments, the frequency of vibration of the string,

$$\frac{N}{2} = \frac{p_2}{2l}\cdot\sqrt{\frac{T}{m}} \qquad \ldots (2)$$

Special Case

(*i*) Suppose the string vibrates in p_1 segments in the transverse mode, then for the same tuning fork and the same tension, the string will vibrate in half the number of segments in longitudinal mode of vibration.

From equations (1) and (2),

$$p_2 = \frac{p_1}{2}$$

(*ii*) From equation (1), for the transverse mode of vibration

$$N = \frac{p_1}{2l}\sqrt{\frac{T}{m}}$$

$$N^2 = \frac{p_1^2 T}{4l^2 m}$$

or $$Tp_1^2 = 4N^2 l^2 m = \text{constant}$$

Similarly from equation (2), for longitudinal mode of vibration

$$\frac{N}{2} = \frac{p_2}{2l}\sqrt{\frac{T}{m}}$$

$$Tp_2^2 = N^2 l^2 m = \textbf{constant}$$

or in general $$Tp^2 = \textbf{constant.}$$

Example 7.11. *Find the tension needed to produce stationary waves with four loops in a string one metre long and 0·5* gram *in weight, fixed to a tuning fork of frequency 200* hertz, *when the prongs of the fork are vibrating perpendicular to the string.* (Delhi, 1992)

Here, $$N = \frac{p_1}{2l}\sqrt{\frac{T}{m}} \quad \text{or} \quad T = \frac{4l^2N^2m}{p_1^2}$$

Here $l = 100$ cm $= 1$ metre

$p_1 = 4$

$N = 200$ hertz

$$m = \frac{0{\cdot}5}{100} = 0{\cdot}005 \text{ g/cm} = 5 \times 10^{-4} \text{ kg/m}$$

$$\therefore \quad T = \frac{4 \times (1)^2 \times (200)^2 \times 5 \times 10^{-4}}{(4)^2}$$

$$T = \textbf{5 newtons.}$$

Example 7.12. *In Melde's experiment, when a string is stretched by a piece of glass it vibrates with 7 loops. When the glass piece is completely immersed in water the string vibrates in 9 loops. What is the specific gravity of glass?* (Punjab, 1993)

Here $$T_1 p_1^2 = T_2 p_2^2$$

$$T_1 \times (7)^2 = T_2 (9)^2$$

$$\frac{T_1}{T_2} = \frac{81}{49}$$

$$\frac{T_2}{T_1} = \frac{49}{81}$$

$$1 - \frac{T_2}{T_1} = 1 - \frac{49}{81} = \frac{32}{81}$$

$$\frac{T_1 - T_2}{T_1} = \frac{32}{81}$$

or
$$\frac{T_1}{T_1 - T_2} = \frac{81}{32} = 2{\cdot}531$$

Therefore the specific gravity of glass = **2·531.**

Example 7.13. *A flexible string of length 0·99* m *and mass 1* gram *is stretched by a tension T newtons. The string vibrates in three segments with a frequency of 500* Hz. *Calculate the tension.* (Delhi, 1992)

Here $l = 0{\cdot}99$ m

$M = 10^{-3}$ kg

$$m = \frac{M}{l} = \frac{10^{-3}}{0{\cdot}99} \text{ kg/m}$$

$n = 500$ Hz

$p = 3$

$$n = \frac{p}{2l}\sqrt{\frac{T}{m}}$$

$$T = \frac{4n^2 l^2 m}{p^2}$$

$$T = \frac{4 \times (500)^2 \times (0{\cdot}99)^2 \times 10^{-3}}{9 \times 0{\cdot}99}$$

$T = \mathbf{110\ N.}$

Example 7.14. *Two wires of the same material and diameter are subjected to the same tension. They produce 3 beats per second when sounded together. Calculate their frequencies if their lengths are 42·5* cm *and 43* cm *respectively* (Gauhati, 1992)

Here, $l_1 = 42{\cdot}5$ cm

$l_2 = 43$ cm

$$n_1 = \frac{1}{2l_1}\sqrt{\frac{T}{m}}$$

$$n_2 = \frac{1}{2l_2}\sqrt{\frac{T}{m}}$$

Here T and m are the same

$$\frac{n_1}{n_2} = \frac{l_2}{l_1} = \frac{43}{42{\cdot}5}$$

$$n_1 = \left(\frac{43}{42{\cdot}5}\right) n_2 \quad \ldots (i)$$

Also $n_1 - n_2 = 3$

$$\left(\frac{43}{42{\cdot}5}\right) n_2 - n_2 = 3$$

$n_2 = \mathbf{255}$

$n_1 = 255 + 3 = \mathbf{258.}$

Example 7.15. *Two strings of the same material and of the same cross section are suspended on a sonometer. One is loaded with 12* kg *and the other with 3* kg. *The first string is tuned to the second harmonic of the second string. If the first string is 100* cm *in length, what is the length of the second string?* (Bhagalpur, 1990)

Here $n_1 = \frac{1}{2l_1}\sqrt{\frac{T_1}{m}}$

$$n_2 = \frac{1}{2l_2}\sqrt{\frac{T_2}{m}}$$

$$\frac{n_1}{n_2} = \frac{l_2}{l_1}\sqrt{\frac{T_1}{T_2}}$$

Here $n_1 = 3\, n_2$

$$\frac{T_1}{T_2} = \frac{12}{3} = 4$$

$$\therefore \quad 3 = \frac{l_2}{l_1}\sqrt{4}$$

$l_2 = 1{\cdot}5\ l_1$

Here $l_1 = 1$ m

$l_2 = \mathbf{1{\cdot}5\ m.}$

Example 7.16. *An addition of 24* kg *to the tension of a stretched string changed its frequency to three times the original frequency. What was the original tension?* (IAS, 1984)

Frequency, $f_1 = 2\pi\sqrt{\frac{T}{m}} \quad \ldots (i)$

$$f_2 = 2\pi\sqrt{\frac{T+24}{m}} \quad \ldots (ii)$$

$$\frac{f_2}{f_1} = \sqrt{\frac{T+24}{T}}$$

$$(3)^2 = \frac{T+24}{24}$$

$$T = \mathbf{3\ kg\ wt}$$

$$= 3 \times 9{\cdot}8\ \text{N}$$

$$= \mathbf{29{\cdot}4\ N.}$$

Example 7.17. *A movable bridge divides a sonometer wire into two parts, which differ in length by 1* cm *and produce 4 beats per second when sounded together. If the whole length is 100* cm, *find the frequencies of the parts.*

(IAS, 1989)

Here $l_1 = 49{\cdot}5$ cm

$l_2 = 50{\cdot}5$ cm

As $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

$n\,l$ = constant

$n_1 l_1 = n_2 l_2$

$49{\cdot}5\, n_1 = 50{\cdot}5\, n_2.$ ... *(i)*

$$n_2 = \left(\frac{49{\cdot}5}{50{\cdot}5}\right) n_1 \quad \text{... } (ii)$$

Also $n_1 - n_2 = 4$ (given)

$$n_1 - \left(\frac{49{\cdot}5}{50{\cdot}5}\right) n_1 = 4$$

$$n_1 = \mathbf{202} \quad \text{... } (iii)$$

$202 - n_2 = 4$

$n_2 = \mathbf{198}$ ... *(iv)*

7.6. Vibrations of Air Columns

Stationary longitudinal waves are produced in wind instruments which are provided with a column of air called the resonator. The vibrations are caused by the mouth piece and the mouth piece is different for different instruments. The mouth piece acts as a source and supplies the necessary energy to maintain the vibrations in the column of air. Depending upon the nature of excitation, the organ pipes can be classified into two categories :

(i) **Flute or Flue Pipes** and *(ii)* **Reed Pipes.**

Both these categories may have many varieties which differ in shape and material. The tubes which are normally used have one end open and the other end either open or closed. The nature of the reflected wave will be different in the open end and closed end organ pipes. In the case of the closed end organ pipe the reflection takes place at a rigid wall while in the case of the open end pipe the reflection takes place at a yielding wall. In the case of an open end organ pipe the ends have always antinodes. In the case of a closed end organ pipe, the open end has an antinode and the closed end has always a node.

These sounding bodies *viz.*, the organ pipes can vibrate in a number of ways, giving notes of different pitch. The resultant sound is a complex note. The lowest vibrating frequency of the instrument is called the *fundamental frequency.*

The rest of the notes are the integral multiples of the fundamental frequency and are called *harmonics* or overtones.

Suppose the fundamental frequency is n, then the overtones are $2n$, $3n$, $4n$, etc.

7.7. Resonance

Free vibration. When a vibrating body is set to vibrate it begins to vibrate with its natural frequency. In the case of a simple pendulum

$$n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

and in the case of a stretched string

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Such vibrations are called free vibrations.

Forced vibrations. A vibrating body can set or force another body into vibration even if their natural frequencies are different. In such a case, a body which is forced to vibrate, does not vibrate for a long time. Its vibration dies off quickly.

Resonance. It is a phenomenon of vibratory motion in which a body set into vibration with its natural frequency by another body vibrating with the same frequency.

7.8. Velocity of Sound in Air by Resonance Method

A resonance tube apparatus is taken. It is properly levelled and the reservoir R is filled with water. A tuning fork of known frequency is struck gently on a rubber pad and brought near the top end of the tube (Fig. 7.5).

With the help of a reservoir R and a pinch cock, the position of the water level is changed. The position of maximum intensity of sound is found. This corresponds to a position of resonance. The particles of the air column in the tube vibrate with the same frequency as that of the tuning fork.

When the prong of the tuning fork moves from a to b, it sends a wave of

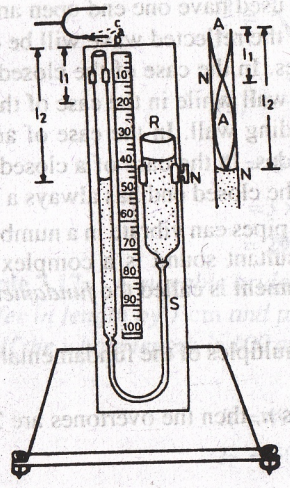


Fig .7.5.

compression. When the prong moves from b to a, a wave of compression comes back after reflection from the water surface. Similarly when the prong moves from a to c, it sends a wave of rarefaction through the air column in the tube. When the prong moves from c to a, the wave of rarefaction is reflected back through the air column. It means, that when the tuning fork completes one-fourth of the vibration, the sound wave moves from the top end to the level of water in the tube. Therefore, the length of the air column.

$$l_1 = \frac{\lambda}{4}$$

End correction. In practice l_1 is less than $\lambda/4$ because the reflection of sound does not take place just at the end of the tube but a small distance above the open end. The end correction $x = 0{\cdot}3\ D$ where D is the diameter of the tube.

Actually, the open end of the tube behaves as an antinode and the closed end behaves as a node. The distance between an adjacent node and an antinode is $\lambda/4$.

Suppose, the frequency of the tuning fork is n. Let the first resonance occur at l_1 and the second resonance at a length l_2 of the air column.

Then
$$l_1 + x = \frac{\lambda}{4} \quad \ldots (1)$$

$$l_2 + x = \frac{3\lambda}{4} \quad \ldots (2)$$

Subtracting (1) from (2)

$$l_2 - l_1 = \frac{\lambda}{2}$$

$$V_t = n\lambda = 2n\,(l_2 - l_1) \quad \ldots (3)$$

Also velocity of sound at 0° C $= (V_t - 0{\cdot}61t)$ m/s . . . (4)

Multiply equation (1) by 3

$$3l_1 + 3x = \frac{3\lambda}{4} \quad \ldots (5)$$

Equating (2) and (5)

$$3l_1 + 3x = l_2 + x$$

$$x = \left(\frac{l_2 - 3l_1}{2}\right)$$

Note . The frequency of a given tuning fork can also be determined using resonance tube apparatus.

Here $V_t = 2n\,(l_2 - l_1)$

But $V_t = V_0 + 0{\cdot}61\,t$ m/s

Take V_0 to be 332 metres/ second and find V_t

$$n = \frac{V_t}{2(l_2 - l_1)}$$

As, V_t, l_2 and l_1 are known, n can be calculated.

Example 7.18. *Resonance was observed with an air column when its length was 16* cm *and again when it was 50* cm. *If the frequency of the tuning fork is 512, calculate (i) the velocity of sound and (ii) end correction.*

Here $l_1 = 16$ cm, $l_2 = 50$ cm , $n = 512$

$$V_t = 2n\,(l_2 - l_1)$$
$$= 2 \times 512(50 - 16)$$
$$= \mathbf{34816\ cm/s}$$
$$n = \frac{l_2 - 3l_1}{2}$$
$$= \frac{50 - 48}{2} = \mathbf{1\ cm}.$$

Example 7.19. *If the lengths of the frist and second resonating air columns are 16·5* cm *and 51·5* cm *respectively with a tuning frok of frequency 512* Hz, *calculate the velocity of sound in air.* (Osmania, 1992)

Here $l_1 = 16{\cdot}5$ cm

$l_2 = 51{\cdot}5$ cm

$l_2 - l_1 = 51{\cdot}5 - 16{\cdot}5 = 35$ cm

$= 0{\cdot}35$ m

$n = 512$ Hz

$V = 2n\,(l_2 - l_1)$

$= 2 \times 512 \times 0{\cdot}35$

$= \mathbf{358{\cdot}4\ m/s.}$

7.9. Organ Pipe

It is a wind instrument. It consists of the mouthpiece M, a slot A and a lip L (Fig. 7.6). If the other end of the pipe is closed it is called a closed end organ pipe. If the other end is open it is called an open end organ pipe. Air is blown through the mouthpiece M and is ejected out through the slot A. The lip L vibrates. The air column acts as a resonator. The air column resonates with a particular fundamental frequency depending on its length. The open end behaves as an antinode and the closed end as a node.

7.10. Closed End Organ Pipe

Suppose the length of the pipe is l. For the fundamental frequency of vibration of the air column

$$l = \frac{\lambda_1}{4} \qquad [\text{Fig. 7.7 } (i)]$$

Frequency

$$n_1 = \frac{V}{\lambda_1} = \frac{V}{4l} \quad \ldots (1)$$

For the first overtone

$$l = \frac{3\lambda_2}{4} \qquad [\text{Fig. 7. } (ii)]$$

Frequency

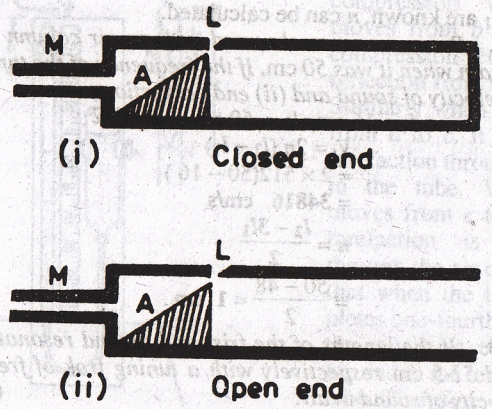


Fig.7.6.

$$n_2 = \frac{V}{\lambda_2} = \frac{3V}{4l}$$
$$= 3n_1 \quad \ldots (2)$$

For the second overtone

$$l = \frac{5\lambda_3}{4} \quad \text{[Fig. 7.7 (iii)]}$$

Frequency $n_3 = \frac{V}{\lambda_3} = \frac{5V}{4l} = 5n_1 \quad \ldots (3)$

Therefore, in the case of a closed end organ pipe, only the odd harmonics are produced and the even harmonics are missing. The fundamental frequency = $V/4l$.

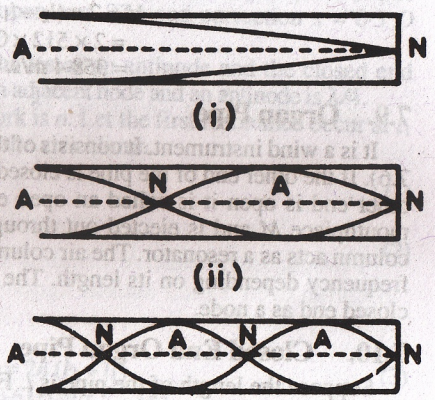


Fig.7.7.

7.11. Open End Organ Pipe

In an open end pipe antinodes are present at both the ends.

Suppose the length of the pipe is l.

For the fundamental frequency of vibration of the air column

$$l = \frac{\lambda_1}{2} \quad \text{[Fig. 7.8 (i)]}$$

$$n_1 = \frac{V}{\lambda_1} = \frac{V}{2l} \quad \ldots (1)$$

For the first overtone

$$l = \lambda_2 \quad \text{[Fig. 7.8 (ii)]}$$

Frequency

$$n_2 = \frac{V}{\lambda_2} = \frac{V}{l}$$
$$= 2n_1 \quad \ldots (2)$$

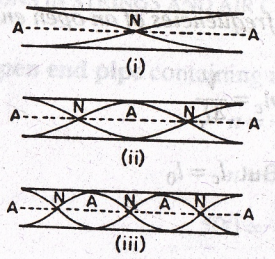


Fig. 7.8.

For the second overtone

$$l = \frac{3\lambda_3}{2} \quad \text{[Fig. 7.8 (iii)]}$$

Frequecy

$$n_2 = \frac{V}{\lambda_3} = \frac{3V}{2l} = 3n_1 \quad \ldots (3)$$

Therefore the harmonics having frequencies n_1, $2n_1$, $3n_1$ . . . *etc. are produced. Hence all the harmonics are present.*

Example 7.20. *An open pipe 20* cm *long resonates, when filled with oxygen, with a given tuning fork. Find the length of the open pipe filled with hydrogen which will be in resonance with the same tuning fork. Velocity of sound in oxygen is 320* m/s *and velocity of sound in hydrogen is 1280* m/s.

Here, $V_0 = 320$ m/s

$V_H = 1280$ m/s

$l_o = 20$ cm

$l_H = ?$

$$n = \frac{V_0}{2l_0} = \frac{V_H}{2l_H}$$

or $$l_H = \left[\frac{V_H}{V_0}\right] l_0$$

$$l_H = \left[\frac{1280}{320}\right] \times 20$$

$$l_H = \mathbf{80\ cm.}$$

Example 7.21. *Compare the lengths of an open end and closed end pipes which emit the same fundamental note.*

For a closed pipe

$$n_c = \frac{V}{4lc} \quad \ldots (1)$$

For an open pipe

$$n_0 = \frac{V}{2l_o} \quad \ldots (2)$$

Here $n_o = n_c$

$\therefore$ $$\frac{V}{4l_c} = \frac{V}{2l_o} \text{ or } \frac{l_o}{l_c} = 2.$$

Example 7.22. *Compare the fundamental frequencies of an open end and closed end pipes of the same length.*

$$n_o = \frac{V}{2l_0}, \qquad n_c = \frac{V}{4l_c}$$

$$\frac{n_0}{n_c} = \frac{2l_c}{l_0} \qquad \text{But } l_c = l_0$$

$$\frac{n_0}{n_c} = 2.$$

Example 7.23. *Two open organ pipes of lengths 50 cm and 50·5 cm produce 3 beats/second. Calculate the velocity of sound in air.*

Here, $l_1 = 50$ cm, $l_2 = 50{\cdot}5$ cm

Suppose the velocity of sound $= V$

$$n_1 = \frac{V}{2l_1} = \frac{V}{100}$$

$$n_2 = \frac{V}{2l_2} = \frac{V}{2 \times 50{\cdot}5} = \frac{V}{101}$$

Beats/second $= 3$

$$n_1 - n_2 = 3$$

$$\therefore \quad \frac{V}{100} - \frac{V}{101} = 3 \quad \text{or} \quad \frac{V}{10100} = 3$$

$$V = \mathbf{30300\ cm/\ second.}$$

Example 7.24. *Two tuning forks A and B give 6 beats/second. A resounds with a closed column of air 15 cm long and B with an open column 30·5 cm long. Calculate their frequencies.*

Suppose, the frequency of $A = n_1$

$$n_1 = \frac{V}{4l_1} = \frac{V}{4 \times 15} = \frac{V}{60}$$

Frequency of $B = n_2$

$$n_2 = \frac{V}{2l_2} = \frac{V}{2 \times 30{\cdot}5} = \frac{V}{61}$$

$\therefore$ Beats/second $= 6$ or $n_1 - n_2 = 6$

$$\frac{V}{60} - \frac{V}{61} = 6 \quad \text{or} \quad \frac{V}{3660} = 6$$

or $V = \mathbf{21960\ cm/second.}$

Example 7.25. *An organ pipe filled with air has a fundamental frequency of 500 Hz The first harmonic of another organ pipe closed at one end and filled with carbon dioxide has the same frequency as that of the first harmonic of the open organ pipe. Calculate the length of each pipe. (Velocity of sound in air=330 m/s, velocity of sound in CO_2 = 264 m/s).*

(1) **Open end pipe** containing air

$$n = \frac{V}{2l}$$

Here $n = 500, \quad V = 330$ m/s

$$500 = \frac{330}{2l} \quad \text{or} \quad l = \mathbf{0{\cdot}33\ metre.}$$

(2) **Closed end pipe** containing carbon dioxide

$$n = \frac{V}{4l}$$

Here $n = 500, \quad V = 264$ m/s

$$500 = \frac{264}{4} \quad \text{or } l = \mathbf{0{\cdot}132\ metre.}$$

7.12. Vibrations in Rods

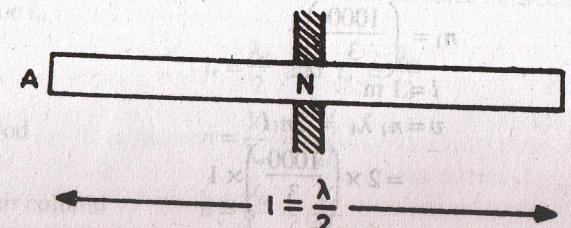


Fig. 7.9

Sound waves travel through solids, liquids and gases. The velocity of a sound wave in an isotropic medium is given as

$$u = \sqrt{\frac{E}{\rho}}$$

The elasticity of a solid is higher as compared to liquids and gases.

It has been found that velocity of sound through iron is higher than the velocity of sound through water or air.

Consider a metal rod rigidly clamped at the mid point (Fig. 7.9). If one end of the rod is gently rubbed with a resined cloth, longitudinal stationary vibrations are set up in the rod. At the clamped position, there is a node (compression). At the free ends, there are antinodes (rarefactions). The distance between the two ends of the rod is equal to half the wavelength of sound through the rod.

Here $l = \frac{\lambda}{2}$ or $\lambda = 2l$

The longitudinal vibrations in rods has been used in experiments for determining the velocity of sound through iron, glass, wood, brass, copper etc.

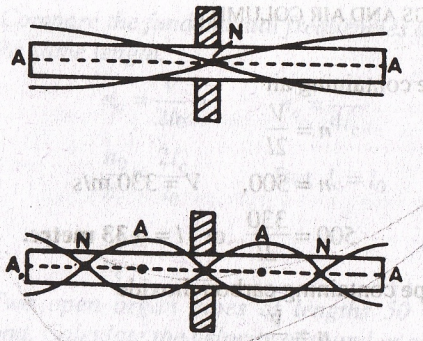


Fig. 7.9 A

Example 7.26. *The frequency of a note next higher to the fundamental, as given by a rod of an alloy, 1 m long and clamped at its midpoint is 1000. If the density fo the alloy be $7{\cdot}5 \times 10^3$ kg/m^3, calculate the value of the Young's modulus for it.* (Calicut, 1992)

Here, $n_2 = 3n_1$

$n_2 = 1000$

$$n_1 = \left(\frac{1000}{3}\right)$$

$l = 1$ m

$$v = n_1 \lambda_1 = 2n_1 l$$

$$= 2 \times \left(\frac{1000}{3}\right) \times 1$$

$$= \left(\frac{2000}{3}\right) \text{ m/s}$$

and $\rho = 7{\cdot}5 \times 10^3$ kg/m^3

Also $$v = \left[\frac{E}{\rho}\right]^{\frac{1}{2}}$$

$$E = \rho v^2$$

$$E = 7{\cdot}5 \times 10^3 \times \left[\frac{2000}{3}\right]^2$$

$$E = \mathbf{3{\cdot}3 \times 10^9 \ N/m^2}$$

7.13. Kundt's Tube

It consists of a glass tube. A cork B is attached to a wooden handle and can be moved inside the tube. The rod of the required material for which the velocity of sound is to be determined is clamped in the middle at C. Th front end of the rod has an aluminium disc A attached to it. The tube contains air at

room temperature and a little lycopodium powder is put along the length of the tube (Fig. 7.10).

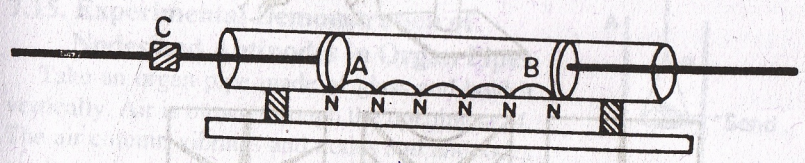


Fig. 7.10.

Velocity of sound in solids. When the rod is rubbed with a resined cloth gently, longitudinal vibrations are set up in the rod. The free ends of the rod behave as antinodes and the clamped point C acts as a node. These vibrations are transferred to the air column and the air column vibrates. The cork B is adjusted so that the air column resonates with the vibrations of the rod. At this stage, the lycopodium powder sets itself in the form of heaps at regular intervals of distance.

Let the length of the rod be l_r and the mean distance between two consecutive heaps be l_a.

$$\therefore \quad l_r = \frac{\lambda_r}{2} \text{ and } l_a = \frac{\lambda_a}{2}$$

For the rod $$n = \frac{V_r}{\lambda_r} = \frac{V_r}{2l_r} \quad \ldots (1)$$

For the air column $$n = \frac{V_a}{\lambda_a} = \frac{V_a}{2l_a} \quad \ldots (2)$$

From (1) and (2)

$$V_r = V_a \left[\frac{l_r}{l_a}\right] \quad \ldots (3)$$

Velocity of sound in gases. Here the experiment is performed first with air and then with the required gas, using the same rod.

Suppose, the distance between two consecutive heaps in the air column = l_a and the mean distance between two consecutive heaps in the gas column = l_g. From equation (3)

$$V_r = V_a \left(\frac{l_r}{l_a}\right) \quad \ldots (4)$$

and $$V_r = V_a \left(\frac{l_r}{l_a}\right) \quad \ldots (5)$$

From (4) and (5)

$$V_g = V_a \left[\frac{l_g}{l_a}\right] \quad \ldots (6)$$

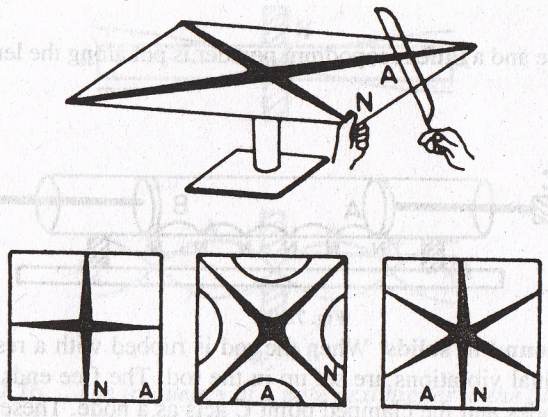


Fig . 7. 11.

7.14. Vibrations in Plates (Chaldni's Figures)

The vibrations in a plate can be studied by the following experiment.

Fix a glass plate on a stand [Fig. 7.11 (*i*)]. Sprinkle sand on the plate to form a uniform thin layer. Hold the plate at one corner and bow the middle of one side. It is noticed that the particles of sand arrange themselves into a symmetrical pattern as shown in Fig. 7.11 (*i*).

By holding the plate firmly at other points and bowing at different positions, different symmetrical patterns are obtained. In Fig. 7.11 (*ii*), the point *N* refers to the positions where the plate is firmly held and the point *A* refers to the positions where the plate is bowed. The position *N* corresponds to the node and the position *A* corresponds to the antinode. These patterns are called Chaldni's Figures.

Each pattern of the Chaldni's Figures, corresponds to a particular mode of vibration. However, these modes are not harmonically related in frequency.

Example 7.27. *A brass rod of length 3* metres *is clamped in the middle. Calculate the frequency of the fundamental note emitted by it, when it vibrates longitudinally. Density of brass* $= 8 \cdot 3$ g/cm^3 *and Young's modulus of brass* $= 10.76 \times 10^{11}$ dynes/cm^2.

Here $$v = \sqrt{\frac{Y}{p}} \quad \text{and} \quad n = \frac{v}{2l}$$

$$\therefore \quad n = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$$

Here $l = 3$ metres

$$\rho = 8 \cdot 3 \text{ g/cm}^3 = 8300 \text{ kg/m}^3$$

$$Y = 10 \cdot 76 \times 10^{11} \text{ dyns/cm}^2 = 10 \cdot 76 \times 10^{10} \text{ newtons/m}^2$$

$$\therefore \quad n = \frac{1}{2 \times 3}\sqrt{\frac{10 \cdot 76 \times 10^{10}}{8300}}$$

$$n = \mathbf{600 \text{ hertz.}}$$

7.15. Experimental Demonstration of Nodes and Antinodes in Organ Pipes

Take an organ pipe made of glass and hold it vertically. Air is blown through the opening at *M*. The air column vibrates and nodes and antinodes are formed.

Take a scale pan *P* having a small quantity of sand of fine grain (Fig. 7.12). Lower the scale pan, gradually inside the pipe. It is observed that at some positions, the sand particles remain completely at rest while at some other positions the sand particles vibrate. The positions where the sand particles are at rest, correspond to nodes. The positions where the sand particles vibrate correspond to positions of antinodes.

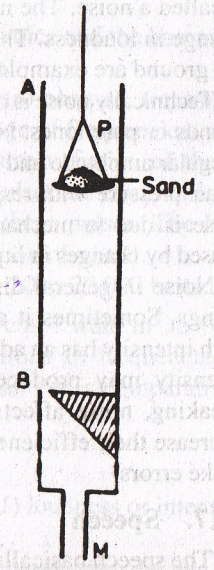


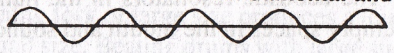
Fig. 7.12.

Initially air is blown with low pressure so that the air column vibrates with its fundamental frequency. In this case the open end at *A* will be an antinode and the closed end at *B* will be a node.

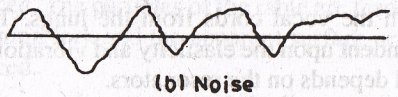
7.16. Musical Sound and Noise

Musical sound. The sound that produces a pleasing effect or sensation on the ear is called a musical sound, the musical sound succeeds at regular intervals in quick succession and without any sudden change in loudness. The sounds produced by plucking a string of an instrument and by a harmonium are musical sounds [Fig. 7.13 (*a*)].

In general, music is a complex and highly subjective mental sensation. Music is due to combination or succession of different notes produces by various sounding bodies *e.g.*, vibrating strings, stretched membranes and air columns. Musical sounds have simple harmonic structure. Their wave forms are regular. Musical notes consist of the fundamental and its harmonics.



(a) Musical Sound



(b) Noise

Fig. 7.13.

Noise. The sound that produces jarring effect or displeasing effect on the ear is called a noise. The noise succeeds at irregular intervals and there is sudden change in loudness. The sounds produced by the gun and by a plate falling on the ground are examples of noise [Fig. 7.13 (*b*)].

Technically noise is defined as the result of the combination of single frequency sounds or pure tones. Noise has a continuous frequency spectrum but possesses irregular amplitude and waveform. Airborne noise is mainly due to the variations in air pressure with respect to the mean atmospheric pressure. Structural-borne noise is due to mechanical vibrations in elastic bodies. Liquid-borne noise is caused by changes in liquid pressure about the mean static pressure.

Noise in general disturbs the normal work, sleep or recreation of human beings. Sometimes it also produces strain, irritation and headache. Noise of high intensity has an adverse cumulative effect on the human ear. Noise of high intensity may produce temporary or permanent deafness. Psychologically speaking, noise affects in an adverse way the output of workers and will decrease their efficiency. Persons working in noisy surroundings are liable to make errors.

7.17. Speech

The speech basically refers to the structure of language and its main characteristics are loudness, pitch, timbre and interpretive aspect. If a speech is well recognized and understood it is said to be intelligible. Speech sounds are complex audible acoustic waves which provide the listners with a number of clues for understanding.

An intelligible speech depends upon the acoustic power delivered during the speech, the characteristics of speech, sensitiveness to hearing and noises in the surroundings.

7.18. Human Voice

The human voice is a natural source of sound. Human voice has four main parts :

(1) **Power generator.** This comprises of diaphragm, lungs, bronchi, trachea and muscles.

(2) **Vibrator.** It is called larynx.

(3) **Resonators.** The acoustic resonators in the human system are nose, mouth, throat, other empty spaces in the mouth and sounding board *e.g.*, head, chest etc.

(4) **Articulators.** These are lips, tongue, teeth etc.

The loudness of sound in a human voice primarily depends upon the stream of air forced through the vocal cords from the lungs. The frequency of the human voice is dependent upon the elasticity and vibrations of the vocal cords. The quality of sound depends on the resonators.

7.19. Human Ear

Human ear is a natural sound receiver. The human voice and the human ear together form a fundamental and natural sound system.

The hearing mechanism is a highly sensitive electro-acoustic transducer. The human ear responds to sound waves of a wide range of frequencies, wave forms and intensity. It communicates acoustic pressure variation of the ear drum into pulses in the auditory nerve system. These pulses are communicated to the brain which in turn indentifies and interprets these pulses. The brain converts these pulses into aural sensations *viz.*, perception of sound.

The human ear responds to frequencies in the range 20 to 20,000 hertz. The range of sound intensity over which the ear is sensitive is 1 watt/ m^2 to 10^{-12} watt/m^2. The human ear is more sensitive to variations in frequencies as compared to variations in sound intensities. The human ear is comparatively more sensitive to sound of low intensity.

7.20. Characteristics of Musical Sound

There are three characteristics of musical sounds : (1) loudness or intensity, (2) pitch and (3) quality or timbre.

1. **Loudness or Intensity**. The amount of sound energy crossing per unit area around a point in one second is known as intensity of sound. Loudness depends upon intensity and also upon the sensitiveness of the ear. Loudness and intensity are related to each other by the relation

$$L \propto \log I$$

where L represents the sensations of loudness and I, the intensity of sound.

Loudness or intensity depends upon the following factors :

(*i*) **Amplitude.** Loudness is directly proportional to the square of the amplitude of the sounding body. The amplitude of sound produced by men is large and hence loud sound is produced. The amplitude of sound produced by ladies or children is small, therefore, the sound produced is feeble. Mosquito also produces a wave of small amplitude, therefore, the sound produced by a mosquito is also feeble.

(*ii*) **Surface area.** Loudness is directly proporional to the surface area of the sounding body. A tuning fork of large size produces a loud sound as compared to a tuning fork of small size. Beating drums with large surfaces produce a loud sound as compared to the beating drums with small surface area. A tuning fork ordinarily produces a feeble sound. When its stem is pressed against a table, a loud sound is produced. The particles of the table are forced to vibrate with the frequencty of the tuning fork and the apparent surface area increases, hence a loud sound is produced.

(*iii*) **Distance between the source and the listener.** The intensity of sound is inversely proportional to the square of the distance between the source and

the listener, provided the source produces sound waves in all directions. Therefore, the sound becomes feeble and feeble with increase in distance between the listener and the source.

(*iv*) **Density of the medium.** The greater the density of the medium, the louder is the sound. When the density of the medium is decreased, the sound becomes feeble.

(*v*) **Motion of air.** If air is blowing in the direction of propagation of the sound waves, loudness increases. If air is blowing in a direction opposite to the direction of propagation of the sound waves, loudness decreases.

2. **Pitch.** It is a sensation that depends upon the frequency. A shrill sound is produced by a source of high frequency whereas the pitch is lower if the frequency is lower. Pitch does not depend upon loudness or quality. The voice produced by ladies and children has high pitch because the frequency is high. The voice of an old man has low pitch and is hoarse because the frequency of sound is low. The frequency of the sound produced by a mosquito is of high pitch due to high frequency. The pitch of sound changes due to Droppler's Principle when either the source or the observer or both are in motion.

3. **Quality or Timbre.** It depends on the presence of overtones. The quality of sound enables us to distinguish between two sounds having the same loudness and pitch. A sounding body produces waves of frequency $2n$, $3n$, $4n$ etc. where n is the fundamental frequency. Nature has provided different overtones in the voice of different persons. Due to the quality of sound, one can recognise his friend from his voice without seeing him.

Suppose a person is calling you. The loudness and pitch will tell us whether it is a voice from a man, a lady or a child. The quality will further help us, to find the particular person producing the sound, man, woman or child. That is why quality plays a very important part. Otherwise, voices produced by all men would have been similar.

The roaring of a lion has high amplitude but low frequency. Therefore, the roaring of a lion can be heard even at far away places, but it is not a shrill sound.

The humming of a mosquito or a bee has low amplitude but *high frequency*. The sound is not heard when the mosquito is even a few metres away from the ear but it is a shrill sound.

7.21. Intensity of Sound

The intensity of sound is defined as the average rate of transfer of energy per unit area, the area being perpendicular to the direction of propagation of sound. Determination of the intensity of sound is important in practical acoustics.

Amount of energy transfer per unit area per second

$$I = 2\pi^2 \rho n^2 a^2 v \quad \ldots (1)$$

Velocity of sound,

$$v = \sqrt{\frac{E}{\rho}} \quad \text{and} \quad E = -\frac{p}{dV/V}$$

dV is the change in volume, V the original volume and p is the excess of pressure

$$v = \sqrt{\frac{-p}{\left(\frac{dV}{V}\right)\rho}}$$

Taking $\frac{dV}{V} = \frac{dy}{dx}$ and simplifying

$$p = -v^2 \rho \frac{dy}{dx} \quad \ldots (2)$$

A simple harmonic wave is represented by the equation

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x)$$

Substituting this value of $\frac{dy}{dx}$ in equation (2)

$$p = \frac{2\pi a v^2 \rho}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \quad \ldots (3)$$

The maximum excess of pressure

$$p_{max} = \frac{2\pi a v^2 \rho}{\lambda} \quad \ldots (4)$$

and

$$p = p_{max} \cos \frac{2\pi}{\lambda}(vt - x) \ldots (5)$$

$\therefore$

$$p_{max} = 2\pi a \rho v \left(\frac{v}{\lambda}\right)$$

$$p_{max} = 2\pi a \rho v n \quad \ldots (6)$$

From equations (1) and (6)

$$I = 2\pi^2 \rho n^2 a^2 v$$

$$I = \frac{(2\pi \alpha \rho v n)^2}{2\rho v}$$

$$I = \frac{p_{max}^2}{2\rho v} \quad \ldots (7)$$

Equation (7) shows that the intensity of sound varies directly as the square of the excess of pressure. Therefore, in acoustics it is important to measure the excess of pressure to obtain the value of intensity of sound. For ordinary conversation $p_{max} = 0{\cdot}1$ newton/m^2.

In the case of ordinary conversation, the sound output per square metre is $1{\cdot}13 \times 10^{-5}$ watt. Human ear is an extremely sensitive organ and can detect sound intensity lower than this value. It has been found that the ear can detect intensities as low as 10^{-12} watt/m^2.

7.22. Measurement of Intensity of Sound—Decibel and Phon

The intensity of sound is defined as the quantity of energy propagating through a unit area per unit time, the direction of propagation being perpendicular to the area. The unit of intensity in the CGS system is ergs/cm^2 -s. and in SI system it is joules/m^2-s. The amount of power transmitted per unit area is measured in watts/m^2. A convenient unit is micro-watts/m^2.

The loudness of sound is just an aural sensation and it is a physiological phenomenon rather than a physical phenomenon. The intensity of sound refers to the external or the objective measurement and the loudness refers to the internal or subjective aspect. Intensity of sound is a definite physical quantity and loudness is merely a degree of sensation. Loudness of sound increases with the intensity of sound according to Weber-Fecher law in physiology. According to this law, the loudness produced is proportional to the logarithm of intensity. This law does not hold good near the lower and the upper limits of audibility. According to this law,

$$S \propto \log I$$

or

$$S = K \log I$$

where K is a constant

or

$$\frac{dS}{dI} = \frac{K}{I}$$

The quantity dS/dI is called the sensitiveness of the ear. The sensitiveness of the ear decreases with increase in the intensity of sound.

In the case of all practical measurements, it is the relative intensity that is important and not the absolute value. Hence the intensity of sound is often measured as its ratio to a standard intensity I_0. The intensity level is equal to

$$\frac{I}{I_o}$$

The standard intensity taken is 0·01 watt/m^2.

Suppose a person speaks in a normal conversational tone, he emits energy at the rate 10^{-5} joule/s. The mouth aperture is about 10^{-3} m^2 while speaking. If he speaks into a short tube the whole of the sound energy spreads along the length of the tube and the intensity of sound is 10^{-2} watt/ m^2. This value of sound

intensity is the standard intensity I_0. The person hearing at the other end of the tube gets the feeling of standard intensity. If a person shouts into the tube as loud as he can, the intensity will be 100 I_0. When the intensity is 100 I_0; upto 1000 I the listener feels pain.

The faintest sound that can be heard depends also upon the frequency of the note. The average person's threshold of audibility is about 10^{-10} I_0 for a frequency of 400 hertz. Thus the range of hearing for a human ear is from 10^{-10} I_0 upto 100 I_0. Hence the human ear has a dynamic range of 10^{12} in intensity.

7.23. Bel

Whenever the intensity of sound increases by a factor of 10, the increase in intensity is said to be 1 bel. Therefore, the dynamic range of audibility of the human ear is 12 **bels** or 120 decibels. When the intensity increases by a factor $10^{0{\cdot}1}$, the increase in intensity is 0·1 bel or 1 decibel.

Suppose the loudness is S for intensity I and S_0 for intensity I_0

$$\therefore \quad S = K \log_{10} I \qquad \ldots(1)$$

$$S_0 = K \log_{10} I_0 \qquad \ldots(2)$$

The intensity level L is the difference in loudness.

$$L = S - S_0$$

$$L = K \log_{10} I - K \log_{10} Io$$

$$L = K \log_{10} \left(\frac{I}{I_o}\right)$$

When the value of $K = 1$,

$$L = \log_{10} \left(\frac{I}{I_o}\right)$$

Here the unit of L is bel

If L is measured in decibels,

$$L = 10 \log_{10} \left(\frac{I}{I_o}\right) \text{ decibels}$$

Suppose the intensity level changes by 1 decibel, then

$$1 = 10 \log_{10} \left(\frac{I}{I_o}\right)$$

or

$$\frac{I}{I_o} = 1{\cdot}26$$

It means that the intensity level alters by 1 decibel when the intensity of sound changes by 26%.

The lowest change in intensity level that can be detected by the human ear

is 1 decibel.

Example 7.28. *Calculate the change in intensity level when the intensity of sound increases 100 times its original intensity.*

Here, Initial intensity $= I_0$

Final intensity $= I$

$$\frac{I}{I_0} = 100$$

Increase in intensity level $= L$

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

$$L = 10 \log_{10} 100 = \mathbf{20\ decibels.}$$

Example 7.29. *Calculate change in intensity level when the intensity of sound increases by 10^6 times its original intensity.*

Here Initial intensity $= I_0$

Final intensity $= I$

$$\frac{I}{I_o} = 10^6$$

Increase in intensity level, $L = 10 \log_{10}\left(\frac{I}{I_0}\right)$

$$L = 10 \log_{10} (10^6)$$

$$L = \mathbf{60\ decibels.}$$

TABLE 7.1
Intensity Levels of Different Sounds

Source of Sound	*Intensity Level in decibels*
Threshold of hearing	0
Rustle of leaves	10
Whisper	15—20
Normal conversation	60—65
Heavy traffic	70—80
Roaring of a lion (at a distance of 6 m)	90
Thunder	100—110
Painful Sounds	130 and above

7.24. Phon

The intensity levels given in the above table refer to the loudness in decibels with the assumption that the threshold of audibility is the same, irrespective of the pitch of the sound. However, the sensitivity of the ear and the threshold of audibility vary over wide ranges of frequency and intensity. Hence the intensity level will be different at different frequencies even for the same value of I_o. For measuring the intensity level, a different unit called the *phon* is used. The measure of loudness in phons of any sound is equal to the intensity level in decibels of an equally loud pure tone of frequency 1000 hertz. Thus, the phon scale and the decibel scale agree for a frequency of 1000 hertz but the two values differ at other frequencies.

Suppose the intensity level of a note of frequency 480 hertz is to be determined. A standard source of frequency 1000 hertz is sounded and the intensity of the standard source is adjusted so that it is equal to the loudness of the given note of frequency 480. The intensity level of the standard source in decibels is numerically equal to the loudness of the given source in phons.

Suppose a note of frequency 3000 hertz and intensity level 70 decibels gives the same loudness as a standard source of frequency 1000 hertz at intensity level 67 decibels. The intensity level of the note of frequency 3000 hertz is *67 phons.*

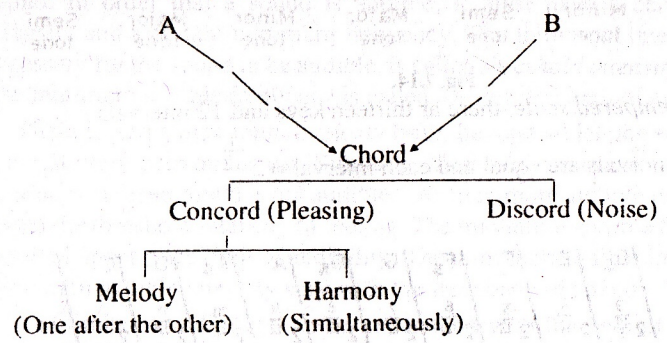
7.25. Musical Scale

Chord. When two or more notes are sounded together, the combined note produced is called chord. If the combined note produces a pleasing effect on the ear, it is called *concord*. If the combined note produces a displeasing effect on the ear, it is called *discord.*

Harmony. When two or more notes are sounded simultaneously, the combined note, producing a pleasing effect on the ear, is called *harmony.*

In English music, where a number of instruments play simultaneously, harmony is produced.

Melody. When two or more notes are sounded one after the other, the combined note producing pleasing effect on the ear, is called melody. In Indian folk music, melody is produced.



Interval. The ratio between the frequencies of two notes is called a musical interval. It is equal to 1 or more than 1. It is never less than 1.

Unison. Here the musical interval =1

$$\frac{N_2}{N_1} = 1 \quad \text{or} \quad N_2 = N_1$$

The two notes are said to be in unison, when their frequencies are equal.

Octave. Here the musical interval = 2

$$\frac{N_2}{N_1} = 2 \quad \text{or} \quad N_2 = 2N_1$$

Here N_2 is the octave of N_1

Major tone. Here the musical interval $= \frac{9}{8}$

Minor tone. Here the musical interval $= \frac{10}{9}$

Semi tone. Here the musical interval $= \frac{16}{15}$

Fifth tone. Here the musical interval $= \frac{3}{2}$.

Diatonic musical scale. This scale has eight keys and seven intervals (Fig. 7.14).

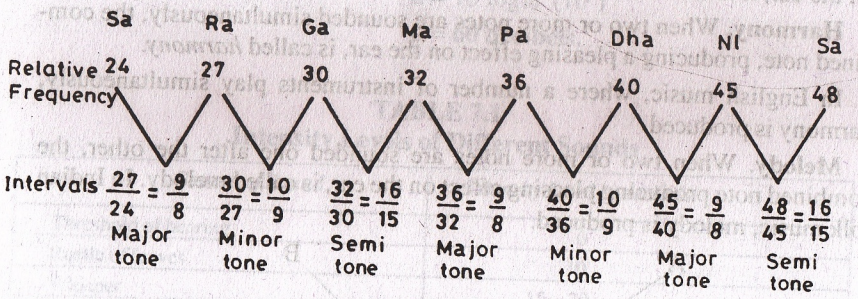


Fig. 7.14.

In an *equally tempered scale,* there ar thirteen keys and 12 intervals (Fig. 7.15). The intervals are equal and each interval $= 2^{\frac{1}{12}}$

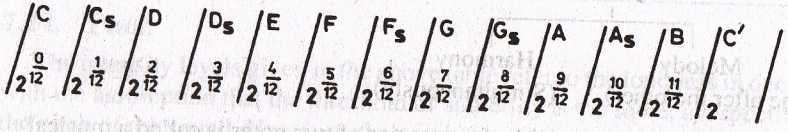


Fig. 7.15.

In this scale five notes have been introduced. They are C_s, D_s, F_s, G_s and A_s. The main advantage of this scale is that the interval is the same between the consecutive notes and a singer can conveniently use any key as his fundamental.

Suppose $C = 256$

$$D = 256 \times \frac{9}{8} = 288$$

$$E = 288 \times \frac{10}{9} = 320$$

$$F = 320 \times \frac{16}{15} = \frac{1024}{3} = 341.33$$

$$G = \frac{1024}{3} \times \frac{9}{8} = 384$$

$$A = 384 \times \frac{10}{9} = \frac{1280}{3} = 426{\cdot}66$$

$$B = \frac{1280}{3} \times \frac{9}{8} = 480$$

$$C' = 480 \times \frac{16}{15} = 512.$$

Example 7.30. *The frequency of the key note C = 512. Find the frequency of the note G.*

$C = 512$

$G = 512 \times \frac{36}{24} = 768$ (Because if $C = 24$, $G = 36$).

7.26. Limits of Audibility

The limits of audibility of sound depend upon the *intensity* and *frequency* of sound. In order that a sound is audible, it must have a certain minimum intensity and a certain minimum frequency. The minimum intensity of sound necessary for the sound to be audible, is called *threshold intensity* of audibility. The minimum audible frequency is called *lower pitch limit* of audibility.

There is also a maximum intensity limit, beyond which the sound produces a sensation of pain on the ear. Similarly, there is a maximum frequency limit, beyond which the sound is not audible. The maximum audible intensity limit is called the threshold intensity of feeling. The maximum audible frequency limit is called upper pitch limit of audibility. The minimum audible intensity and the maximum audible intensity vary with the frequency of sound.

The limits of audibility can be understood with the help of an audiogram drawn by Wegel (Fig. 7.16). In an audiogram, the frequency is plotted along the x-axis on a logarithmic scale and the intensity (pressure in newton/m^2) is

plotted along the y-axis on a logarithmic scale. In the figure curve 1 represents the threshold of audibility and curve 2 represents the threshold of feeling. These two curves when extrapolated, enclose an area on the audiogram. This enclosed area is called the auditory sensation area. This audiogram represents the auditory sensation area for a normal human ear. If the pitch and the intensity of any sound are beyond the limits set by the auditory sensation area, the sound is not heard. The sound wave may be felt by the ear as pressure but not as an audible sound.

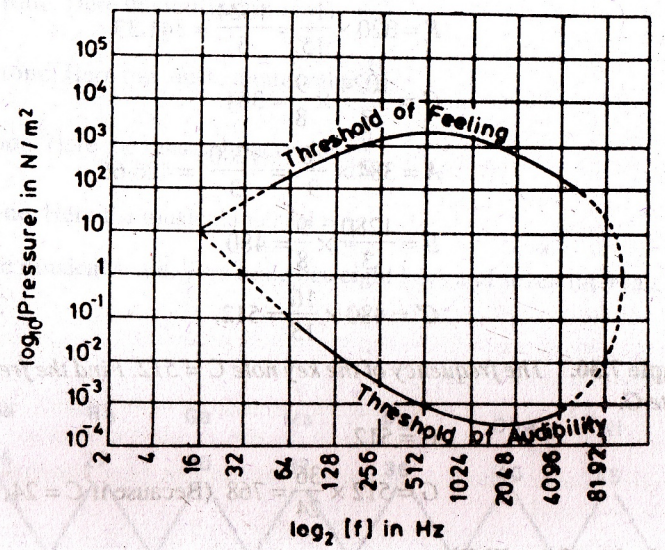


Fig. 7.16

The human ear is sensitive for frequency in the range 500 hertz to 7000 hertz. This range of frequencies represents the range of ordinary speech. The peak sensitiveness of the ear ranges from 2000 to 2500 hertz. Beyond a frequency of 7000 hertz the human ear is insensitive. The frequency range used in music extends from 40 to 4000 hertz.

The threshold intensity increases both at high and low frequency levels. The increase is more at the low frequency level. The intensity that causes painful sensation in the ear is maximum at a frequency of about 800 hertz. However, at this frequency, the threshold of audibility is low. At a frequency of 1000 hertz, the ratio between the two pressure amplitudes is 10^7 : 1. The corresponding intensity ratio is 10^{14} : 1. This ratio is enormously high.

The lowest audible frequency is 30 hertz whereas the highest is 20,000 hertz. The audible frequency range for the ear between 30 hertz and 20,000 hertz constitutes a range of eleven octaves. However, in the case of the human eye, the range of frequency in the visible region ($3{\cdot}75 \times 10^{14}$ hertz to $7{\cdot}5 \times 10^{14}$ hertz) constitutes only one octave. Out of the eleven audible octaves, seven are available in music.

EXERCISES

1. Derive an expression for the velocity of a simple harmonic wave in a stretched string.
2. Show that the fundamental frequency of vibration of a stretched string is given by
$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}.$$ *(Mysore, 1991)*
3. Discuss the laws of transverse vibration of a stretched string and explain how the diameters of two wires can be compared using a sonometer. *(Madras, 1984; Calcutta, 1984)*
4. Describe Melde's experiment and explain how the laws of vibration of strings can be verified with this experiment.
5. Show that the frequency of vibration of a string, in the case of transverse mode of vibration of a string is double the frequency for the longitudinal mode of vibration provided it vibrates with the same number of segments.
6. Discuss the harmonics in the case of closed end and open end organ pipes. *(Calcutta, 1984)*
7. Show that for the same length of the organ pipe, the fundamental frequency in the case of an open end organ pipe is double the fundamental frequency of a closed end organ pipe.
8. Describe Kundt's tube. Explain how it can be used to determine the velocity of sound in *(i)* solids and *(ii)* in gases. *(Calcutta, 1984)*
9. Describe an experiment to demonstrate the formation of nodes and antinodes in an organ pipe.
10. Derive an expression for the velocity of transverse waves in a string under tension and hence calculate the expression for the fundamental frequency of vibration. *(Delhi, 1993; Madras, 1984)*
11 State the laws of vibration of strings and describe experiments to verify the law concerning *(i)* length *(ii)* tension and *(iii)* linear density. *(Delhi, 1993)*
12. Describe Melde's experiment. Show that the frequency of vibration of a string in the longitudinal mode is half that in the transverse mode of vibration. *(Mysore, 1991; Delhi, 1991, 1994)*

13. In Melde's experiment in the longitudinal mode of vibration a string vibrates in 4 loops with a load of 8 grams. How much load will be required in order that the string may vibrate in 8 loops in the transverse mode. [Ans. 8 grams] *(Delhi, 1991)*
14. Derive an expression for the frequency of transverse waves in a stretched string. *(Delhi, 1992)*
15. Discuss the production of harmonics in an open pipe. *(Delhi, 1992)*
16. Give the construction and working of a Kundt's tube. Explain, how it can be used to determine the velocity of sound in a metallic rod. *(Delhi, 1994)*
17. Discuss the characteristics of musical sound. *(Calcutta, 1993)*
18. Explain the use of Kundt's apparatus for the different acoustical determinations. *(Delhi, 1986)*
19. Derive an expression for the velocity of transverse waves in a stretched string and deduce the laws of vibrating strings. Describe how you would verify these laws with a sonometer. *[Delhi (Supple.), 1986]*
20. Discuss the vibrations of an air column in an open organ pipe. *[Delhi (Supple.), 1986]*
21. Describe and explain Melde's experiment. *[Delhi (Supple.), 1986]*
22. What is resonance? Explain through an experiment how this phenomenon is used to find the velocity of sound in air. *(Osmania, 1992)*
23. Describe the method of determining the velocity of sound by Kundt's tube. *(Gauhati, 1992)*
24. Derive an expression for the velocity of transverse waves along a string. *(Delhi, 1991)*

CHAPTER 8

Doppler Effect

8.1. Doppler Effect

It is commonly observed that the pitch of a note apparently changes when either the source or the observer are in motion relative to each other. When the source approaches the observer or when the observer approaches the source or when both approach each other the apparent pitch is higher than the actual pitch of the sound produced by the source. Similarly, when the source moves away from the observer or when the observer moves away from the source or when both move away from each other, the apparent pitch is lower than the actual pitch of the sound produced by the source.

Suppose a person is standing on a platform. The apparent pitch of the whistle of the engine increases, when the engine is approaching the person. When the engine moves away from the person, the apparent pitch of the whistle of the engine decreases. This apparent change in the pitch due to the relative motion between the source and the observer is called **Doppler effect.**

Doppler effect in sound is asymmetric. When the source moves towards the observer with a certain velocity, the apparent pitch is different to the case when the observer is moving towards the source with the same velocity. But it is not so in the case of light. Doppler effect in light is symmetric. The apparent pitch in different cases is calculated in the subsequent articles.

8.2. Observer at Rest and Source in Motion

(a) When the source moves towards the stationary observer

Suppose a source S is producing sound of pitch n and wavelength λ. The velocity of sound is v. (Fig. 8.1).

Let the source move with a velocity a towards the observer. In one second, n waves will be contained in a length $(v - a)$ and the apparent wavelength,

$$\lambda' = \frac{(v-a)}{n}$$