13. In Melde's experiment in the longitudinal mode of vibration a string vibrates in 4 loops with a load of 8 grams. How much load will be required in order that the string may vibrate in 8 loops in the transverse mode. [Ans. 8 grams] *(Delhi, 1991)*
14. Derive an expression for the frequency of transverse waves in a stretched string. *(Delhi, 1992)*
15. Discuss the production of harmonics in an open pipe. *(Delhi, 1992)*
16. Give the construction and working of a Kundt's tube. Explain, how it can be used to determine the velocity of sound in a metallic rod. *(Delhi, 1994)*
17. Discuss the characteristics of musical sound. *(Calcutta, 1993)*
18. Explain the use of Kundt's apparatus for the different acoustical determinations. *(Delhi, 1986)*
19. Derive an expression for the velocity of transverse waves in a stretched string and deduce the laws of vibrating strings. Describe how you would verify these laws with a sonometer. *[Delhi (Supple.), 1986]*
20. Discuss the vibrations of an air column in an open organ pipe. *[Delhi (Supple.), 1986]*
21. Describe and explain Melde's experiment. *[Delhi (Supple.), 1986]*
22. What is resonance? Explain through an experiment how this phenomenon is used to find the velocity of sound in air. *(Osmania, 1992)*
23. Describe the method of determining the velocity of sound by Kundt's tube. *(Gauhati, 1992)*
24. Derive an expression for the velocity of transverse waves along a string. *(Delhi, 1991)*

CHAPTER 8

Doppler Effect

8.1. Doppler Effect

It is commonly observed that the pitch of a note apparently changes when either the source or the observer are in motion relative to each other. When the source approaches the observer or when the observer approaches the source or when both approach each other the apparent pitch is higher than the actual pitch of the sound produced by the source. Similarly, when the source moves away from the observer or when the observer moves away from the source or when both move away from each other, the apparent pitch is lower than the actual pitch of the sound produced by the source.

Suppose a person is standing on a platform. The apparent pitch of the whistle of the engine increases, when the engine is approaching the person. When the engine moves away from the person, the apparent pitch of the whistle of the engine decreases. This apparent change in the pitch due to the relative motion between the source and the observer is called **Doppler effect.**

Doppler effect in sound is asymmetric. When the source moves towards the observer with a certain velocity, the apparent pitch is different to the case when the observer is moving towards the source with the same velocity. But it is not so in the case of light. Doppler effect in light is symmetric. The apparent pitch in different cases is calculated in the subsequent articles.

8.2. Observer at Rest and Source in Motion

(a) When the source moves towards the stationary observer

Suppose a source S is producing sound of pitch n and wavelength λ. The velocity of sound is v. (Fig. 8.1).

Let the source move with a velocity a towards the observer. In one second, n waves will be contained in a length $(v - a)$ and the apparent wavelength,

$$\lambda' = \frac{(v-a)}{n}$$

The apparent pitch,

$$n' = \frac{v}{\lambda'}$$

$$\therefore \quad n' = \left(\frac{v}{v-a}\right) n \qquad \ldots (1)$$

Thus, the apparent pitch of the note increases when the source moves towards a stationary observer.

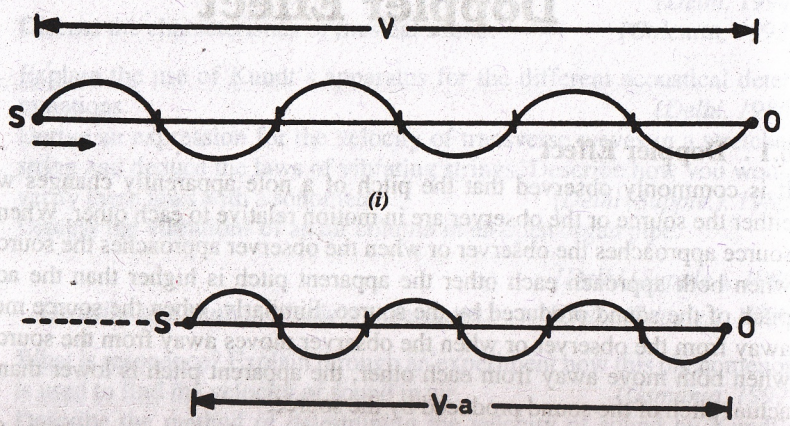


Fig. 8.1.

(b) When the source moves away from the stationary observer

Suppose a source S is producing sound of pitch n and wavelength λ. The velocity of sound is v (Fig. 8.2). Let the source move with a velocity a away from the observer. In one second, n waves will be contained in a length $(v + a)$ and the apparent wavelength,

$$\lambda' = \frac{v+a}{n}$$

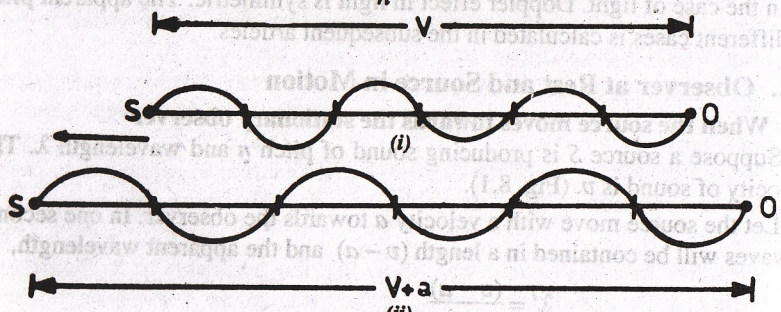


Fig. 8.2.

The apparent pitch,

$$n' = \frac{v}{\lambda'}$$

$$\therefore \quad n' = \left(\frac{v}{v+a}\right) n . \qquad \ldots (2)$$

Thus the apparent pitch of the note decreases when the source moves away from a stationary observer.

Note. For the same values of v, a and n, the values of n' in equations (1) and (2) will be different.

8.3. Source at Rest and Observer in Motion

(a) When the observer moves towards a stationary source

Suppose a source S is producing sound of pitch n and wavelength λ. The velocity of sound is v (Fig. 8.3). Let the observer move with a velocity b towards a stationary source. In this case the observer receives more number of waves in one second. The apparent wavelength remains the same. The apparent frequency

$$n' = n + \frac{b}{\lambda}$$

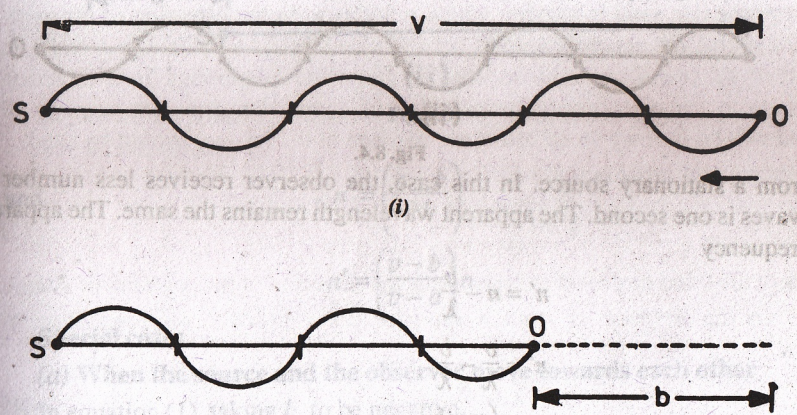


Fig. 8.3.

$$n' = \frac{v}{\lambda} + \frac{b}{\lambda}$$

$$n' = \left(\frac{v+b}{\lambda}\right)$$

But $$\lambda = \frac{v}{n}$$

$$\therefore \quad n' = \left(\frac{v+b}{v}\right) n \qquad \ldots (1)$$

Thus the apparent pitch of the note increases when the observer moves towards the stationary source.

(*b*) When the observer moves away from a stationary source

Suppose a source S is producing sound of pitch n and wavelength λ (Fig. 8.4). The velocity of sound is v. Let the observer move with a velocity b away

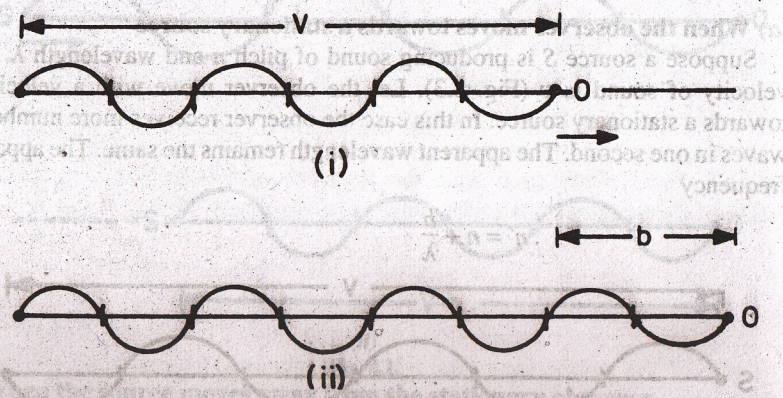


Fig. 8.4.

from a stationary source. In this case, the observer receives less number of waves is one second. The apparent wavelength remains the same. The apparent frequency

$$n' = n - \frac{b}{\lambda}$$

$$n' = \frac{v}{\lambda} - \frac{b}{\lambda}$$

$$n' = \left(\frac{v-b}{\lambda}\right)$$

But $$\lambda = \frac{v}{n}$$

$$\therefore \quad n' = \left(\frac{v-b}{v}\right) n \qquad \ldots (2)$$

Thus the apparent pitch of the note decreases when the observer moves away from a stationary source.

Note. For the values of v, b and n, the values of n' in equations (1) and (2) will be different.

8.4. When both the Source and the Observer are in Motion

When the source moves towards the observer and the observer moves away from the source. Suppose a source S is producing a sound of pitch n and wavelength λ. The velocity of sound is v (Fig. 8.5). The velocity of the source is a and the velocity of the observer is b.

Let the source move towards the observer with a velocity a and the observer move away from the source with a velocity b.

The apparent wavelength

$$\lambda' = \left(\frac{v-a}{n}\right).$$

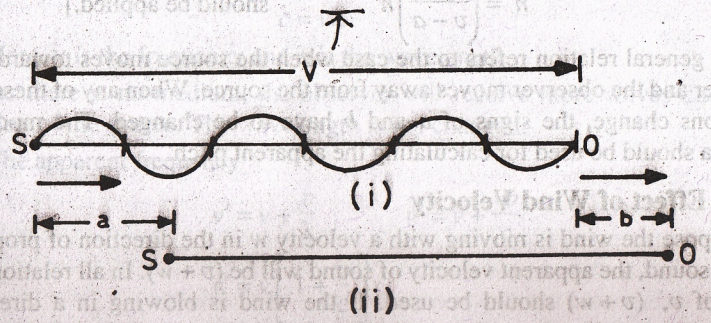


Fig. 8.5.

and $$n' = \left(\frac{v-b}{\lambda'}\right)$$

$$\therefore \quad n' = \left(\frac{v-b}{v-a}\right) n \qquad \ldots (1)$$

Special cases

(*a*) When the source and the observer move towards each other

In equation (1), taking b to be negative,

$$n' = \left[\frac{v-(-b)}{v-a}\right] n$$

$$n' = \left(\frac{v+b}{v-a}\right) n \qquad \ldots (2)$$

(*b*) When the source and observer move away from each other

In equation (1), taking a to be negative

$$n' = \left[\frac{v-b}{v-(-a)}\right] n$$

$$n' = \left(\frac{v-b}{v+a}\right)n \quad \ldots (3)$$

(c) Source moving away from the observer and the observer moving towards the source

In equation (1), taking both a and b negative,

$$n' = \left[\frac{v-(-b)}{v-(-a)}\right]n$$

$$n' = \left(\frac{v+b}{v+a}\right)n \quad \ldots (4)$$

[**Note.** While solving numerical problems, the general formula

$$n' = \left(\frac{v-b}{v-a}\right)n$$ should be applied.]

The general relation refers to the case when the source moves towards the observer and the observer moves away from the source. When any of these two directions change, the signs of a and b have to be changed. The modified formula should be used for calculating the apparent pitch.

8.5. Effect of Wind Velocity

Suppose the wind is moving with a velocity w in the direction of propagation of sound, the apparent velocity of sound will be $(v + w)$. In all relations, in place of v, $(v + w)$ should be used. If the wind is blowing in a direction opposite to the direction of propagation of sound, the apparent velocity of sound will be $(v - w)$. In all relations, in place of v, $(v - w)$ should be used.

The general relation will be

$$n' = \left[\frac{(v+w)-b}{(v+w)-a}\right]n \quad \ldots (5)$$

Here the wind direction is the same as the direction of propagation of sound.

When the direction of wind is opposite to the direction of propagation of sound,

$$n' = \left[\frac{(v-w)-b}{(v-w)-a}\right]n \quad \ldots (6)$$

8.6. Doppler Effect in Light

There is a change in the frequency of light radiation when the source or the observer move with respect to one another. This phenomenon is known as Doppler effect. It is similar to the apparent change in the pitch of sound when either the source or the observer is in motion with respect to one another.

According to the theory of relativity, a material medium is not necessary for the propagation of light waves whereas for sound waves material medium is a necessity. In sound, the Doppler effect is asymmetric *i.e.*, the apparent frequencies are different when the source is moving towards a stationary observer and when the observer is moving towards a stationary source. In the first case

$$n' = \left(\frac{v}{v-a}\right)n$$

and in the second case $$n' = \left(\frac{v+b}{v}\right)n$$

But it is not so in the case of light. In light, the Doppler effect is symmetric. The apparent frequency is the same when either the source moves towards a stationary observer or the observer moves towards a stationary source.

(*i*) Suppose, the observer is stationary and the source is moving towards the observer with a velocity v. Let the velocity of light be c.

$$c = \nu\lambda$$

Here, ν is the frequency and λ is the wavelength.

As there is no medium, the observer will receive more waves due to the motion of the source. The wavelength is not changed.

The apparent frequency

$$\nu' = \nu + \frac{v}{\lambda} \qquad \nu' = \nu + \frac{v\nu}{c}$$

$$\nu' = \nu\left[1 + \frac{v}{c}\right] \quad \ldots (1)$$

(*ii*) When the source is stationary and the observer is moving towards the source with a velocity v.

$$\nu' = \nu + \frac{v}{\lambda} \qquad \nu' = \nu + \frac{v\nu}{c}$$

$$\nu' = \nu\left[1 + \frac{v}{c}\right] \quad \ldots (2)$$

Therefore, equations (1) and (2) are similar.

(*iii*) When the source moves away from the stationary observer or the observer moves away from the stationary source with a velocity v.

$$\nu' = \nu - \frac{v}{\lambda} \qquad \nu' = \nu - \frac{v\nu}{\lambda}$$

$$\therefore \nu' = \nu\left[1 - \frac{v}{c}\right] \quad \ldots (3)$$

(*iv*) When the source and the observer move towards each other and each is moving with a velocity v.

$$\nu' = \nu\left[1 + \frac{v}{c}\right]^2 \quad \ldots (4$$

(*v*) When the source and the observer move away from each other and each is moving with a velocity v.

$$v' = v\left[1 - \frac{v}{c}\right]^2 \quad \ldots (5)$$

The phenomenon of Doppler effect is of great importance in light. It shows that there is a shift in the frequency when the source of light moves with a velocity comparable to the velocity of light. In the field of astronomy, Doppler effect has been found to be very useful.

Applications

(1) **Velocity and rotation of the sun.** By the study of Doppler shift from the light received from the western and eastern edges of the sun, it has been found that the shift is due to a velocity of about 2 km/s. Moreover, no such shift has been observed from the light received from the north and the south edges. This shows that the sun rotates about the north-south axis.

(2) **Discovery of double stars.** By the constant observation of the sky, it has been found that some of the stars that appear to be single are actually double stars and are known as spectroscopic binaries. These stars revolve about each other. When one is approaching the earth, and the other is going away from the earth, there is a shift in their spectral lines and a single spectral line is split up into two lines whose separation depends upon time and the time period is equal to the time period of revolution of the stars. By this method a number of double-star systems have been found.

(3) **Red shift.** It has been observed that some distant nebulae are moving away with a velocity greater than 20×10^3 km/s and the important spectral lines appear to shift towards the red end of the spectrum by 200 Å. This gives the idea that the universe is expanding.

(4) **Saturn's rings.** The planet Saturn has been found to be surrounded by concentric rings. With the help of Doppler effect it has been found that these rings are not solids but consist of a number of 'satellites' moving around the Saturn in these orbits. If the rings were solids, the outer edge of the ring should have greater velocity than the inner edge. But according to the principle of a satellite

$$\frac{mV^2}{R} = \frac{GMm}{R^2} \quad \text{or} \quad V^2 = \frac{GM}{R}$$

i.e., the velocity of the satellite in the inner orbit is more than that in the outer orbit. This fact has been established by the Doppler shift. Thus, the rings of the Saturn are not solids but there are a large number of satellites moving in these orbits called the rings of Saturn.

Example 8.1. *A motor car sounding a horn at a frequency of 100* hertz *moves away from a stationary observer towards a rigid flat wall with a velocity of 36* km/hr. *How many beats per second will be heard by the observer? The velocity of sound in air at room temperature = 350* m/s.

Here $V = 350$ m/s

$a = 36$ km/hr $= 10$ m/s

$n = 100$ hertz

Here the observer receives sound of apparent frequency n_1 from the moving source directly and also sound of apparent frequency n_2 from the sound waves reflected from the wall.

$$n_1 = n\left[\frac{V}{V+a}\right]$$

$$n_1 = 100\left[\frac{350}{350+10}\right]$$

$$n_1 = 97{\cdot}23 \text{ hertz}$$

$$n_2 = n\left[\frac{V}{V-a}\right]$$

$$n_2 = 100\left[\frac{350}{350-10}\right]$$

$$n_2 = 102{\cdot}9 \text{ hertz}$$

Number of beats/s

$$n_2 - n_1 = 102{\cdot}9 - 97{\cdot}23 = \mathbf{5{\cdot}67.}$$

Example 8.2. *The apparent frequency of the whistle of an engine changes in the ratio 6 : 5 as the engine passes a stationary observer. If the velocity of sound is 352* m/s, *calculate the velocity of the engine.* (Delhi, 1994)

(*i*) When the source moves towards the stationary observer,

$$n_1 = \left(\frac{v}{v-a}\right)n \quad \ldots (1)$$

(*ii*) When the source moves away from the stationary observer,

$$n_2 = \left(\frac{v}{v+\alpha}\right)n \quad \ldots (2)$$

Dividing (1) by (2)

$$\frac{n_1}{n_2} = \frac{v+a}{v-a}$$

Here $\frac{n_1}{n_2} = \frac{6}{5}$ (given)

$v = 352$ m/s

$a = ?$

$\therefore \quad \frac{6}{5} = \frac{352 + a}{352 - a}$ or $a = \mathbf{32\ m/s.}$

Hence the velocity of the engine is 32 m/s.

Example 8.3. *Two trains travelling in opposite directions at 100* km/hr *each, cross each other while one of them is whistling. If the frequency of the note is 800* Hz *find the apparent pitch as heard by an observer in the other train:*

(a) before the trains cross each other

(b) after the trains have crossed each other.

Velcocity of sound in air =340 m/s. [Delhi (Sub.), 1986]

Here, $V = 340$ m/s

$a = 100$ km/hr $= \frac{250}{9}$ m/s

$b = 100$ km/hr $= \frac{250}{9}$ m/s

$n = 800$ Hz

(*a*) In the first case, the source and the observer are moving towards each other. Therefore,

$$n' = n\left[\frac{V + b}{V - a}\right]$$

$$n' = 800\left[\frac{340 + \left(\frac{250}{9}\right)}{340 - \left(\frac{250}{9}\right)}\right]$$

$$n' = 800\left(\frac{3310}{2810}\right)$$

$$n' = \mathbf{942{\cdot}3\ Hz.}$$

(*b*) In the second case, the source and the observer are moving away from each other.

$$n' = n\left[\frac{V - b}{V + a}\right]$$

$$n' = 800\left[\frac{340 + \left(\frac{250}{9}\right)}{340 - \left(\frac{250}{9}\right)}\right]$$

$$n' = 800\left(\frac{2810}{3310}\right)$$

$$n' = \mathbf{679{\cdot}2\ Hz.}$$

Example 8.4. *An observer on a railway platform noticed that when a train passed through the station, at a speed of 72* km/hr, *the frequency of the whistle appeared to drop by 500* Hz. *Calculate the actual frequency of the note given by the whistle. Velocity of sound in air = 340* m/s. [Delhi, 1986]

Here, $V = 340$ m/s

$a = 72$ km/hr $= 20$ m/s

$n = ?$

$n_1 - n_2 = 500$

Here n_1 is the apparent pitch when the train is approaching the observer and n_2 is the apparent pitch when the train is moving away from the observer.

$\therefore \quad n_1 = n\left[\frac{V}{V - a}\right]$

and $\quad n_2 = n\left[\frac{V}{V + a}\right]$

$\therefore \quad n_1 - n_2 = n\left[\frac{V}{V - a}\right] - n\left[\frac{V}{V + a}\right]$

$$n_1 - n_2 = n\left[\left(\frac{V}{V - a}\right) - \left(\frac{V}{V + a}\right)\right]$$

$$500 = n\left[\left(\frac{340}{340 - 20}\right) - \left(\frac{340}{340 + 20}\right)\right]$$

or $\quad n = \frac{500}{\left(\frac{340}{320}\right) - \left(\frac{340}{360}\right)}$

$$n = \mathbf{4235{\cdot}3\ Hz.}$$

Example 8.5. *A person is standing on a railways platform. An engine while approaching the platform blows a whistle of pitch 660* hertz. *The speed of the engine is 72* km/hr, *velocity of sound = 350* m/s. *Calculate the apparent pitch of the whistle as heard by the person.*

Here V = 350 m/s

$a = 72$ km/hr $= 20$ m/s

$n = 660$ hertz.

The source is moving away from a stationary observer.

$$\therefore \quad n' = \left(\frac{V}{V-a}\right)n$$

$$n' = \left(\frac{350}{350-20}\right)\times 660$$

$$n' = \mathbf{700\ hertz.}$$

Example 8.6. *A person is standing on a platform. A railway engine moving away from the person with a speed of 72* km/hr *blows a whistle of pitch 740* hertz. *Calculate the apparent pitch of the whistle as heard by the person. The velocity of sound = 350* m/s.

Here $V = 350$ m/s

$a = 72$ km/hr $= 20$ m/s

$n = 740$ hertz

The source is moving away from a stationary observer.

$$\therefore \quad n' = \left(\frac{V}{V+a}\right)n$$

$$n' = \left(\frac{350}{350+20}\right)\times 740$$

$$n' = \mathbf{700\ hertz.}$$

Example 8.7. *A person is standing near a railway track and a train moving with a speed of 36* km/hr *is approaching him. The apparent pitch of the whistle as heard by the person is 700* hertz. *Calculate the actual frequency of the whistle. Velocity of sound = 350* m/s.

Here, $V = 350$ m/s

$a = 36$ km/hr $= 10$ m/s

$n' = \mathbf{700\ hertz}$

$n = ?$

The source is moving towards a stationary observer

$$\therefore \quad n' = \left(\frac{V}{V-a}\right)n$$

$$n = n'\left(\frac{V-a}{V}\right)$$

$$n = 700\left(\frac{350-10}{350}\right);$$

$$n = \mathbf{680\ hertz.}$$

Example 8.8. *Two aeroplanes A and B are approaching each other with a speed of 360* km/hr. *The frequency of the whistle emitted by A is 1000* hertz.

Calculate the apparent pitch of the whistle as heard by the passengers of aeroplane B. Velocity of sound in air = 350 m/s.

Here, $V = 350$ m/s

$a = 360$ km/hr $= 100$ m/s

$b = 360$ km/hr $= 100$ m/s

$n = 1000$ hertz

The source and the observer are moving towards each other

$$\therefore \quad n' = n\left[\frac{V+b}{V-a}\right]$$

$$\therefore \quad n' = 1000\left(\frac{350+100}{350-100}\right); \qquad n' = \mathbf{1800\ hertz.}$$

Example 8.9. *Two aeroplanes A and B are moving away from one another with a speed of 720* km/hr. *The frequency of the whistle emitted by A is 1100* hertz. *Calculate the apparent frequency of the whistle as heard by the passengers of the aeroplane B. Velocity of sound in air = 350* m/s.

Here, $V = 350$ m/s

$a = 720$ km/hr $= 200$ m/s

$b = 720$ km/hr $= 200$ m/s

$n = 1100$ hertz

The source and the observer are moving away from one another

$$\therefore \quad n' = n\left(\frac{V-b}{V+a}\right)$$

$$n' = 1100\left(\frac{350-200}{350+200}\right); \qquad n' = \mathbf{300\ hertz.}$$

Example 8.10. *Two aeroplanes A and B are approaching each other and their velocities are 108* km/hr *and 144* km/hr *respectively. The frequency of a note emitted by A as heard by the passengers in B is 1170* hertz. *Calculate the frequency of the note heard by the passengers in A. Velocity of sound = 350* m/s.

(Delhi, 1992)

Here, $V = 350$ m/s

$a = 108$ km/hr $= 30$ m/s

$b = 144$ km /hr $= 40$ m/s

$n' = 1170$ hertz

$n = ?$

The source and the observer are approaching each other.

$$\therefore \quad n' = n\left(\frac{V+b}{V-a}\right)$$

$$1170 = n\left(\frac{350+40}{350-30}\right)$$

$$n = \frac{1170 \times 320}{390};$$

$$n = \mathbf{960\ hertz.}$$

Example 8.11. *Two aeroplanes pass each other in opposite directions and one of them is blowing a whistle of frequency 540* hertz. *Calculate the frequencies of the notes heard in the other aeroplane (i) before and (ii) after they have passed each other. Velocity of either of the aeroplanes is 540* km-hr^{-1} *and the velocity of sound = 350* ms^{-1}. (Delhi, 1991)

Here, $V = 350$ m/s

$a = 540$ km/hr $= 150$ m/s

$b = 540$ km/hr $= 150$ m/s

$n = 540$ hertz

(*i*) In the first case, the source and the observer are moving towards each other. Therefore,

$$n' = n\left[\frac{V+b}{V-a}\right]$$

$$n' = 540\left[\frac{350+150}{350-150}\right];$$

$$n' = \mathbf{1350\ hertz.}$$

(*ii*) In the second case, the source and the observer are moving away from each other.

$$n' = n\left[\frac{V-b}{V+a}\right]$$

$$n' = 540\left[\frac{350-150}{350+150}\right];$$

$$n' = \mathbf{216\ hertz.}$$

Example 8.12. *An observer on a railway platform observed that as a train passed through the station at 90* km/hr, *the frequency of the whistle appeared to drop by 400* hertz. *Find the frequency of the whistle. Velocity of sound in air = 350* m/s. (Bombay, 1991)

Here, $V = 350$ m/s

$a = 90$ km/hr $= 25$ m/s

$n = ?$

$n_1 - n_2 = 400$

Here n_1 is the apparent pitch when the train is approaching the observer and n_2 is the apparent pitch when the train is moving away from the observer.

$$\therefore \quad n_1 = n\left[\frac{V}{V-a}\right] \text{ and } n_2 = n\left[\frac{V}{V+a}\right]$$

$$\therefore \quad n_1 - n_2 = n\left[\frac{V}{V-a}\right] - n\left[\frac{V}{V+a}\right]$$

$$n_1 - n_2 = n\left[\left(\frac{V}{V-a}\right) - \left(\frac{V}{V+a}\right)\right]$$

$$\therefore \quad 400 = n\left[\left(\frac{350}{350-25}\right) - \left(\frac{350}{350+25}\right)\right]$$

$$n = \frac{400}{\left(\frac{350}{325}\right) - \left(\frac{350}{375}\right)}$$

$$n = \mathbf{2785{\cdot}7\ hertz.}$$

Example 8.13. *The frequency of the horn of a car is observed to drop from 272* hertz *to 256* hertz *as the car passes a stationary observer. What is the speed of the car. Velocity of sound in air = 346·5* m/s.

Here, $v = 346{\cdot}5$ m/s

$$\frac{n_1}{n_2} = \frac{272}{256} = \frac{17}{16}$$

$a = ?$

(*i*) When the car approaches a stationary observer, the apparent pitch

$$n_1 = \left(\frac{v}{v-a}\right)n \qquad \ldots (1)$$

(*ii*) When the car moves away from the stationary observer, the apparent pitch

$$n_2 = \left(\frac{v}{v+a}\right)n \qquad \ldots (2)$$

Dividing (*i*) by (*ii*)

$$\frac{n_1}{n_2} = \frac{v+a}{v-a}$$

or

$$\frac{17}{16} = \frac{346{\cdot}5+a}{346{\cdot}5-a}$$

$$a = \mathbf{10{\cdot}5\ m/s}$$

The speed of the car is 10·5 m/s.

Example 8.14. *Two observers A and B carry identical sound sources of frequency 500* hertz. *If the observer A is stationary and B moves away from A at a speed of 3·6* km/hr, *how many beats/s are heard by A and B respectively. Velocity of sound in air =350* m/s.

(1) In the first case, consider A as source and B as observer. It means the source is at rest and the observer is moving away from it. The apparent pitch

$$n_1 = \left(\frac{v-b}{v}\right)n$$

Here $v = 350$ m/s

$b = 3{\cdot}6$ km/hr $= 1$ m/s

$n = 500$

$$n_1 = \left(\frac{350-1}{350}\right)500$$

$$n_1 = \frac{349 \times 500}{350}$$

Number of beats /s heard by $B = n - n_1$

$$= 500 - \frac{500 \times 349}{350} = \frac{500}{350} = 1{\cdot}43.$$

(2) In the second case, consider B as a source and A as an observer. It means the source is moving away from a stationary observer. The apparent pitch,

$$n_2 = \left(\frac{v}{v+a}\right)n$$

Here $v = 350$ m/s

$a = 1$ m/s

$n = 500$ hertz

$$\therefore \quad n_2 = \left(\frac{350}{350+1}\right)500$$

$$n_2 = \left(\frac{350}{351}\right)500$$

Number of beats/s heard by A

$$= n - n_2$$

$$= 500 - \left(\frac{500 \times 350}{351}\right)$$

$$= \frac{500}{351} = 1{\cdot}424.$$

Example 8.15. *A motor car fitted with two sounding horns which have a difference in frequency by 320 vibrations per second is speeding at the rate of 36* km/hr *towards a stationary person. Calculate the difference in the frequencies of the notes heard by him. Velocity of sound in air = 330* m/s.

(Madras, 1984)

Let n_1 and n_2 be the two frequencies

$$n_1 - n_2 = 320$$

$$n'_1 = \left(\frac{v}{v-a}\right)n_1 \qquad \ldots (1)$$

$$n'_2 = \left(\frac{v}{v-a}\right)n_2 \qquad \ldots (2)$$

Subtracting (2) from (1)

$$(n'_1 - n'_2) = \left(\frac{v}{v-a}\right)(n_1 - n_2)$$

Here $a = 36$ km/hr $= 10$ m/s

and $v = 330$ m/s

$$\therefore \quad n'_1 - n'_2 = \left(\frac{330}{330-10}\right)(320)$$

$$\therefore \quad n'_1 - n'_2 = \mathbf{330\ vibrations/s.}$$

Example 8.16. *At each of two stations A and B, a siren is sounding with a constant frequency of 250* cycles/s. *A cyclist from A proceeds straight towards B with a velocity of 12* km/hr *and hears 5* beats/s. *Calculate the velocity of sound.*

(Panjab, 1993)

(1) In the first case, the cyclist is moving away from the source A.

$$\therefore \quad n_1 = \left(\frac{v-b}{v}\right)n$$

Here $b = 12$ km/hr $= \frac{10}{3}$ m/s

$n = 250$

$$n_1 = \left(\frac{v-\frac{10}{3}}{v}\right)250 \qquad \ldots (1)$$

(2) In the second case, the cyclist is moving towards the source B.

$$\therefore \quad n_2 = \left(\frac{v+b}{v}\right)n$$

$$\therefore \quad n_2 = \left(\frac{v - \frac{10}{3}}{v}\right) 250 \qquad \ldots (2)$$

Substracting (1) from (2)

$$n_2 - n_1 = \frac{250}{v} \times \frac{20}{3}$$

Here $n_2 - n_1 = 5$

$$\therefore \quad v = \frac{250 \times 20}{5 \times 3}$$

$$v = 333{\cdot}33 \text{ m/s}.$$

Example 8.17. *The driver of a car moving towards a factory with a velocity of 30* m/s *sounds the horn with a frequency of 240* Hz. *Find the apparent frequency of sound heard by the watchman of the factory.*

Velocity of sound in air = 350 m/s. (Osmania, 1992)

Here, $v = 350$ m/s

$a = 30$ m/s

$n = 240$ Hz

The source is moving towards a stationary observer,

$$n' = \left(\frac{v}{v - a}\right) n$$

$$n' = \left(\frac{350}{350 - 30}\right) 240$$

$$n' = 262{\cdot}5 \text{ Hz}.$$

Example 8.18. *Two express trains travelling at 100* km/hr *are meeting each other while one of them is blowing the whistle. If the frequency of the note is 800, find the apparent pitch as heard by an observer in the other train after they passed each other. Velocity of sound = 340* m/s. (IAS, 1988)

Here, $V = 340$ m/s

$$a = 100 \text{ km/hr} = \frac{10 \times 10^3}{3600} = 27{\cdot}78 \text{ m/s}$$

$$b = 100 \text{ km/hr} = \frac{100 \times 10^3}{3600} = 27{\cdot}78 \text{ m/s}$$

$n = 800$ Hz

$n' = ?$

The source and the observer are moving away from each other.

$$\therefore \quad n' = n\left[\frac{V - b}{V + a}\right]$$

$$= 800\left[\frac{340 - 27{\cdot}78}{340 + 27{\cdot}78}\right]$$

$$= 800\left[\frac{312{\cdot}22}{367{\cdot}78}\right]$$

$$= 679{\cdot}15 \text{ Hz}.$$

EXERCISES

1. Explain Doppler's principle. Calculate the apparent pitch of a note due to the motion of the source and the listener. *(Calcutta; Behrampur)*
2. Discuss Doppler effect in sound and obtain an expression for the apparent frequency of the note when the source and the listener are (*i*) moving towards each other and (*ii*) moving away from each other.
3. Explain Doppler's effect. A source produces a note of frequency n and is moving towards a stationary observer with a uniform speed a. Show that the apparent pitch is
$$n' = n\left[\frac{V}{V - a}\right].$$
4. An observer on a railway platform observed a train passing through the station at a speed a. Show that the frequency of the whistle changes by
$$nV\left[\frac{2a}{V^2 - a^2}\right].$$
5. Explain Doppler effect. Explain the apparent frequency of a note when both the source and the listener are in motion, the medium being at rest. *(Delhi, 1984)*
6. Explain Doppler effect. Find an expression for the ratio of the apparent frequency to the real frequency of the notes emitted when there is relative movement between the source and the listener. *(Madras, 1984)*
7. Explain Doppler principle. A source of sound produces a note of frequency n and is moving towards a stationary observer with a speed a. Show that the apparent pitch is given by
$$n' = n\left[\frac{V}{V - a}\right]$$
where V is the velocity of sound in air. *[Delhi, 1986]*
8. State and explain Doppler effect. Derive an expression for the change in frequency of a note when both the source of sound and the observer are in motion. *[Delhi (Sub.), 1986]*

9. Explain Doppler effect. Obtain an expression for the frequency of a note heard by an observer, when both the source and the observer are in motion towards each other. *[Delhi (Supple.), 1986]*
10. A person is standing on a railway platform. An engine while approaching the platform blows a whistle of pitch 640 hertz. The speed of the engine is 108 km/hr. Velocity of sound is 350 m/s. Calculate the apparent pitch of the whistle as heard by the person. **(Ans.** 700 hertz)
11. A person is standing on a platform. A railway engine moving away from the person with a speed of 90 km/hr blows a whistle of pitch 730 hertz. Calculate the apparent pitch of the whistle as heard by the person. Velocity of sound in air = 340 m/s. **(Ans.** 680 hertz)
12. A person is standing near a railway track and a train moving with a speed of 72 km/hr is approaching him. The apparent pitch of the whistle as heard by the person is 680 hertz. Calculate the actual frequency of the whistle. Velocity of sound in air = 340 m/s. **(Ans.** 640 hertz)
13. Two aeroplanes *A* and *B* are approaching each other, with a speed of 540 km/hr. The frequency of the whistle emitted by *A* is 2000 hertz. Calculate the apparent pitch of the whistle as heard by the passengers of aeroplane *B*. Velocity of sound in air = 350 m/s. **(Ans.** 500 hertz)
14. Two aeroplanes *A* and *B* are moving away from one another with a speed of 540 km/hr. The frequency of the whistle emitted by *A* is 1000 hertz. Calculate the apparent pitch of the whistle as heard by the passengers of aeroplane *B*. Velocity of sound in air = 350 m/s.

(Ans. 400 hertz)

15. Two aeroplanes *A* and *B* are approaching each other and their velocities are 540 km/hr and 720 km/hr respectively. The frequency of the note emitted by *A* as heard by the passengers in *B* is 1100 hertz. Calculate the frequency of the note as heard by the passengers in *A*. Velocity of sound in air =350 m/s. **(Ans.** 400 hertz)
16. Two aeroplanes pass each other in opposite directions and one of them is blowing a whistle of frequency 900 hertz. Calculate the frequencies of the notes heard in the other aeroplane (*i*) before and (*ii*) after they have crossed each other. Velocity of either of the aeroplanes is 720 km/hr. Velocity of sound in air =350 m/s. **[Ans.** (*i*) 3300 hertz (*ii*) 245·4 hertz]
17. An observer on a railway platform observed that as a train passed through the station at a speed of 72 km/hr, the frequency of the whistle appeared to drop by 500 hertz. Calculate the frequency. Velocity of sound in air = 340 m/s. **(Ans.** 4235·3 hertz)
18. What is Doppler Effect? Derive an expression for the apparent frequency received by a stationary observer when the source of sound is in motion. *[Osmania, 1992]*

CHAPTER 9

Reflection, Refraction and Diffraction

9.1. Reflection of Sound

In every day life we come across many acoustic phenomena, *e.g.*, echoes, roaring of thunder, reverberation in buildings etc. All these phenomena are based on the reflection of sound waves. Reflection of sound plays an important role in many acoustic applications *e.g.*, speaking tubes, horns and trumpets, sounding boards etc. The stationary waves produced in stretched strings and the resonant vibrations produced in open and closed end organ pipes are also based on the principle of reflection of sound.

9.2. Reflection of a Plane Wave at a Plane Surface

Let *XY* be the plane reflecting surface and *AMB* the incident plane wavefront. All the particles on *AB* will be vibrating in phase. Let *i* be the angle of incidence (Fig. 9.1).

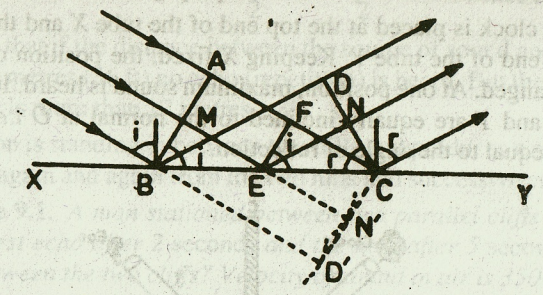


Fig. 9.1.

In the time the disturbance at *A* reaches *C*, the secondary waves from the point *B* must have travelled a distance *BD* equal to *AC*. With the point *B* as centre and radius equal to *AC* construct a sphere. From the point *C* draw tangents *CD* and *C'D*. Then *BD* = *BD'*

In the Δs BAC and BDC, BC is common

$$BD = AC$$

and $$\angle BAC = \angle BDC = 90°$$

The two triangles are congruent.

$$\therefore \quad \angle ABC = i = \angle BCD = r$$

$$i = r.$$

Thus the angle of incidence is equal to the angle of reflection. Hence, CD forms the reflected plane wavefront. It can be shown that all the points on CD form the reflected plane wavefront. In the time the disturbance from F reaches the point C, the secondary waves from E must have travelled a distance $EN = FC$. With E as centre and radius FC draw a sphere and draw tangents CN and CN' to the sphere. It can be shown that the triangles EFC and ENC are congruent.

$$AC = AF + FC$$

But $$AF = ME$$

and $$FC = EN$$

$$\therefore \quad AC = ME + EN.$$

Thus, all the secondary waves from the different points on AB reach the corresponding points on CD at the same time. Therefore, CD forms the reflected plane wavefront and also the angle of incidence is equal to the angle of reflection.

9.3. Experimental Demonstration of Reflection of Sound

In Fig. 9.2, AB is a plane reflecting surface *i.e.*, a polished metal plate, X and Y are two long tubes hinged at O. The two tubes can be moved in a vertical plane. C is an obstacle which prevents the propagation of sound through it.

An alarm clock is placed at the top end of the tube X and the ear is placed near the top end of the tube Y. Keeping X fixed, the position of the tube Y is gradually changed. At one position, maximum sound is heard. It is noticed that the tubes X and Y are equally inclined to the normal at O *i.e.*, the angle of incidence is equal to the angle of reflection.

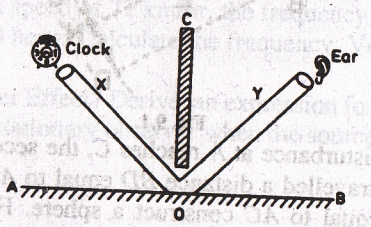


Fig. 9.2.

9.4. Whispering Galleries

S is the position of a speaker and L is the position of an observer (Fig. 9.3). Even a very low intensity sound made by S (*i.e.*, a whisper) can be heard by the observer at L when he keeps his ear close to the wall. This is due to the multiple reflection of the sound waves from the curved wall of the gallery.

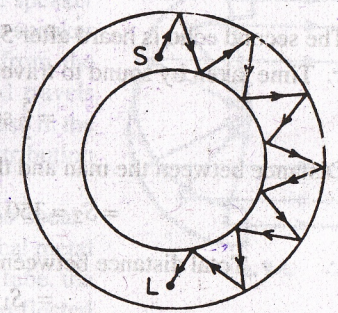


Fig. 9.3.

In **Salt Lake City,** the walls of a hall are elliptical in shape. Even if a small pin is dropped at one of its foci, the sound can be heard by the listener at the second focus.

9.5. Echo

The repetition of the sound produced due to reflection by a distant extended surface like a cliff, hill, well, building, etc. is called an *echo*. The effect of sound on the human ear remains for $\frac{1}{10}$ of a second. If the sound is reflected back in a time less than $\frac{1}{10}$th of a second, no echo is heard. Suppose, a person produces sound and this sound is reflected from an obstacle at a distance x. The time taken by the sound to travel to the obstacle and back is t.

$$t = \frac{2x}{v}$$

Here $$v = 340 \text{ metres/ second}$$

$$x = \frac{vt}{2} = \frac{340 \times 0{\cdot}1}{2} = 17 \text{ metres.}$$

It means that if the distance between the source of sour d and the obstacle is less than 17 metres (56 ft) no echo (repetition) is heard. But the echo is heard if the distance is more than 17 metres.

If a person is standing in between two hills and produces a sound, the sound is reflected again and again from the two hills and successive echoes are heard.

Example 9.1. *A man stationed between two parallel cliffs fires a gun. He hears the first echo after* 2 seconds *and the next after* 5 seconds. *What is the distance between the two cliffs? Velocity of sound in air is 350* m/s.

The first echo is heard after 2 seconds

$\therefore$ Time taken by sound to travel from the man to the first cliff

$$= \frac{2}{2} = 1 \text{ s}$$

$$v = 350 \text{ m/s}$$

Distance between the man and the first cliff $S_1 = 350 \times 1$

$= 350$ m

The second echo is heard after 5 seconds.

∴ Time taken by sound to travel from the man to the second cliff

$= \frac{5}{2}$ seconds

Distance between the man and the second cliff

$= S_2 = 350 \times \frac{5}{2} = 875$ m

∴ Total distance between the two cliffs

$= S_1 + S_2 = 350 + 875$

$= \mathbf{1225\ m.}$

Example 9.2. *A man standing between two parallel cliffs fires a rifle. He hears one echo after* $1\frac{1}{2}$ *s, another after* $2\frac{1}{2}$ *s, and a third one after 4 s. If the distance between the two cliffs is 700* m *what is the velocity of sound and what is the distance of the man from one of the cliffs.*

The third echo is heard after the sound has been reflected from the first and the second cliffs.

∴ Total distance between the two cliffs

$= 700$ m

Total distance travelled by sound

$= 2 \times 700 = 1400$ m

Time = 4 s

∴ Velocity of sound $= v = \frac{S}{t} = \frac{1400}{4} = 350$ m/s

The first echo is heard after $1\frac{1}{2}$ s $= \frac{3}{2}$ s

∴ Time taken by the sound to travel from the man to the first cliff

$= \frac{3}{4}$ s

Distance between the man and the first cliff

$S_1 = 350 \times \frac{3}{4}$

$= \mathbf{262{\cdot}5\ m}$

9.6. Applications of Reflection of Sound

(1) The principle of reflection of sound is used in an auditorium. A large plane

sounding board (cylindrical or parabolic) is placed behind the speaker (Fig. 9.4). The speaker stands at the focus of the board. When the speaker speaks, the sound waves get reflected from the sounding board and the reflected sound travels towards the audience. This enables increase in the intensity of sound and also uniform distribution of sound.

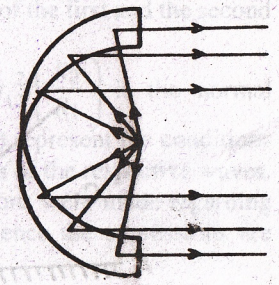


Fig. 9.4.

(2) Speaking tube or megaphone. The speaking tube or megaphone consists of a conical metal horn (Fig. 9.5). At the narrow end of the tube, the speaker speaks. The sound waves get reflected from the side of the tube and are propagated practically along the axis of the tube. This provides higher intensity of sound in a particular direction. Here, the

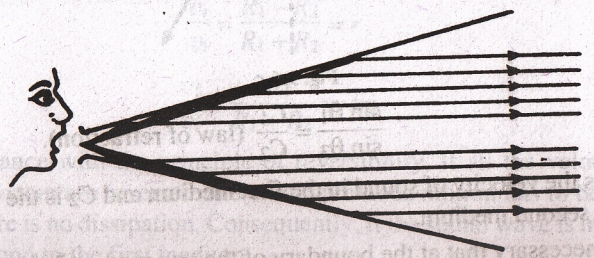


Fig. 9.5.

sound energy is not spread in all directions but it is concentrated in a desired direction.

(3) Ear trumpet. An ear trumpet is a useful device for persons who are hard of hearing. The sound waves that enter the wide end of the trumpet suffer multiple reflections. The sound entering the ear has large amplitude and thus higher intensity. This enables the person to hear even low intensity sounds.

9.7. Phase Change on Reflection

According to the laws of reflection and refraction, the velocity in each medium is independent of the direction. Let AB be a surface separating two media.

Here θ_1, θ_2, and θ_3 are the angles of incidence, reflection and refraction respectively (Fig. 9.6).

Also $$\frac{\sin\theta_1}{C_1} = \frac{\sin\theta_2}{C_1} = \frac{\sin\theta_3}{C_2} \quad \ldots(1)$$

∴ $\theta_1 = \theta_2$ (law of reflection)

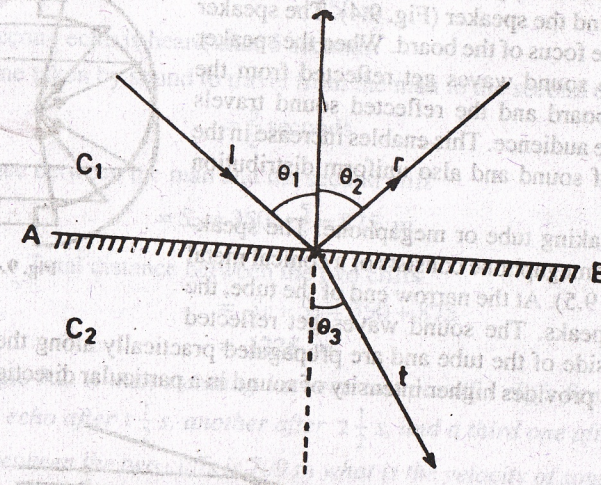


Fig. 9.6.

and $$\frac{\sin\theta_1}{\sin\theta_3} = \frac{C_1}{C_2}$$ (law of refraction).

Here C_1 is the velocity of sound in the first medium and C_2 is the velocity of sound in the second medium.

It is also necessary that at the boundary of the two media (*i*) the component of particle velocity v, normal to the surface must be continuous and (*ii*) the pressure variation ∂P must be continuous. Accordingly, considering *i*, *r* and *t* as the incident, reflected and transmitted waves, to satisfy the first condition,

$$v_i \cos\theta_1 + v_r \cos\theta_1 = v_t \cos\theta_2 \quad \ldots (2)$$

To satisfy the second condition

$$\partial P_i + \partial P_r = \partial P_t \quad \ldots (3)$$

But $$\partial P = \pm C\rho v$$

$$\therefore \quad C_1\rho_1 v_i - C_1\rho_1 v_r = C_2\rho_2 v_t \quad \ldots (4)$$

The negative sign for the second term indicates the reversed direction of the reflected wave.

Eliminating v_t from equations (2) and (4)

$$\frac{v_r}{v_i} = -\left[\frac{C_2\rho_2\cos\theta_1 - C_1\rho_1\cos\theta_2}{C_2\rho_2\cos\theta_1 + C_1\rho_1\cos\theta_2}\right] = r \quad \ldots (5)$$

Similarly by eliminating v_r,

$$\frac{v_t}{v_i} = \frac{2C_1\rho_1\cos\theta_1}{C_2\rho_2\cos\theta_1 + C_1\rho_1\cos\theta_2} = t_{12} \quad \ldots (6)$$

Take $C_1\rho_1 = R_1$ and $C_2\rho_2 = R_2$

where R_1 and R_2 represent the **radiation resistances** of the first and the second medium respectively.

Here r^2 is the normal reflection coefficient and $t_{12}^2\left(\frac{R_2}{R_1}\right)$ is the normal transmission coefficient. The expressions (5) and (6) represent the conditions at the boundary and also for the corresponding points of the respective waves. While deducing the above expressions no assumptions were made regarding the wavelength and phase of the incident wave. Hence the expressions are applicable to waves of any type.

Normal incidence. In the case of normal incidence,

$$\theta_1 = \theta_2 = 0$$

$$\therefore \quad \cos\theta_1 = \cos\theta_2 = 1$$

Substituting these values in equations (5) and (6),

$$\frac{v_r}{v_i} = \frac{R_1 - R_2}{R_1 + R_2} = r \quad \ldots (7)$$

and $$\frac{v_t}{v_i} = \frac{2R_1}{R_1 + R_2} = t_{12} \quad \ldots (8)$$

In accordance with the principle of reversibility, if all the velocities in a dynamical system are reversed, the whole previous motion has to be obtained provided there is no dissipation. Consequently, if the sound wave is transmitted from the second to the first medium,

$$t_{21} = \frac{2R_2}{R_1 + R_2}$$

and $$t_{12}\, t_{21} + r^2 = 1 \quad \ldots (9)$$

Equation (9) is independent of any assumption as regards change of phase at reflection.

Also incident energy must be equal to the sum of the reflected and the transmitted energy.

$$\therefore \quad E_i = E_r + E_t$$

But $$E \propto C\rho v^2$$

$$\therefore \quad C_1\rho_1 v_i^2 = C_1\rho_1 v_r^2 + C_2\rho_2 v_t^2$$

$$\therefore \quad R_1 v_i^2 = R_1 v_r^2 + R_2 v_t^2$$

$$\therefore \quad v_i^2 = v_r^2 + \left(\frac{R_2}{R_1}\right) v_t^2$$

Dividing by v_i^2

$$1 = \frac{v_r^2}{v_i^2} + \left(\frac{R_2}{R_1}\right)\frac{v_t^2}{v_i^2}$$

$$1 = r^2 + \left(\frac{R_2}{R_1}\right)t_{12}^2$$

or $$t_{12}^2\left(\frac{R_2}{R_1}\right) + r^2 = 1 \qquad \ldots (10)$$

Special cases

(1) From equation (7), if $R_1 = R_2$

$$\frac{v_r}{v_i} = 0$$

It means that there is no reflected wave and the whole of the energy is transmitted to the second medium.

(2) If $R_2 > R_1$ (*i.e.*, wave travelling from air to water)

i.e. $C_2\rho_2 > C_1\rho_1$

$\frac{v_r}{v_i}$ is $-$ ve

$\therefore$ v_r is $-$ ve

It means the particle velocity of the reflected wave is in reversed phase. However, the quantity, $\frac{v_r}{C} = S_r$, the phase of the condensation in the reflected wave remains unaltered because both v_r and C are reversed in sign.

(3) If $R_2 < R_1$ (*i.e.* wave travelling from water to air)

i.e. $C_2\rho_2 < C_1\rho_1$

$\frac{v_r}{v_i}$ is $+$ ve

It means that the phase of the particle velocity remains unchanged. However, the phase of the condensation is *reversed.*

(4) From equations (7) and (8),

$$\frac{S_r}{S_i} = -\left(\frac{v_r/C}{v_i/C}\right) = -\left(\frac{v_r}{v_i}\right) = \left(\frac{R_2 - R_1}{R_1 + R_2}\right)$$

$$\frac{S_r}{S_i} = \frac{R_2 - R_1}{R_1 + R_2} \qquad \ldots (11)$$

Also $$\frac{S_t}{S_i} = \frac{2R_1}{R_1 + R_2} \qquad \ldots (12)$$

This means that the phase of the transmitted wave remains unchanged in all cases.

(5) If the velocity of sound C_1 in the first medium is less than the velocity of sound C_2 in the second medium, there will be a critical angle of incidence θ_1, when θ_2 is equal to 90°. In this case

$$\sin\theta_1 = \frac{C_1}{C_2}$$

For an angle of incidence greater than θ_1, there will be total internal reflection. Since the velocity of sound in water is greater than the velocity of sound in air, total internal reflection will occur only when the waves are incident from air on the water surface. Here, it must be remembered that total internal reflection in sound occurs only when the incident wave is travelling from rarer to denser medium and not from denser to rarer medium.

9.8. Transmission of Sound from Air to Water

From equations (7) and (8) of Article 9.7,

$$\frac{v_r}{v_i} = \frac{R_1 - R_2}{R_1 + R_2} = r \qquad \ldots (1)$$

and $$\frac{v_t}{v_i} = \frac{2R_1}{R_1 + R_2} = t_{12} \qquad \ldots (2)$$

From equations (11) and (12) of Article 9.7,

$$\frac{S_r}{S_i} = \frac{R_2 - R_1}{R_1 + R_2} \qquad \ldots (3)$$

$$\frac{S_t}{S_i} = \frac{2R_1}{R_1 + R_2} \qquad \ldots (4)$$

The above equations are applicable for normal incidence.

For air $R_1 = \rho_1 C_1 = 0{\cdot}001293 \times 33200$

$= 40$ CGS units (app.)

and water $R_2 = \rho_2 C_2 = 1 \times 15 \times 10^4$ CGS units

Substituting these values in equation (3)

$$\frac{S_r}{S_i} = 0{\cdot}9994.$$

It indicates that the incident wave is totally internally reflected almost completely. The phase of the condensation is not changed but the particle velocity is reversed (*i.e.*, nodal condition).

To conclude, in general, in all cases of sound transmission, from gaseous to a solid or a liquid medium or *vice versa*, *i.e.*, when the radiation resistance R_1 and R_2 are widely different, there is almost complete total internal reflection.

9.9. Refraction of a Plane Wavefront at a Plane Surface

Let XY represent the surface separating the media 1 and 2. v_1 and v_2 are the velocities of sound in the two media (Fig. 9.7). APB is the incident plane wavefront. By the time the disturbance at B reaches C, the secondary waves from A must have travelled a distance $AD = V_2 t$ where t is the time taken by the waves to travel the distance BC.

$$\therefore \quad BC = v_1 . t.$$

and

$$AD = v_2 . t.$$

With A as centre and radius $AD\ (= v_2 . t)$ draw a sphere. Draw the tangent CD to the sphere from the point C. Then, CD represents the refracted plane wavefront. To prove that CD is the common wavefront, it is enough to show that in the time the disturbance travels from B to C or A to D, the disturbance at

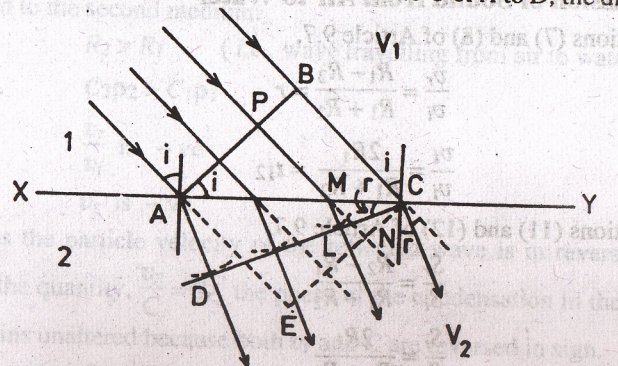


Fig. 9.7

P reaches L. With the point M as centre draw a sphere such that CD happens to be the tangent to the sphere. From the Δs ACD and MCL.

$$\frac{AD}{ML} = \frac{AC}{MC} \qquad \ldots (1)$$

Similarly from the Δs ACE and MCN

$$\frac{AE}{ML} = \frac{AC}{MC} \qquad \ldots (2)$$

From equations (1) and (2)

$$\frac{AD}{ML} = \frac{AE}{MN}$$

or

$$\frac{AE}{AD} = \frac{BC}{AD} = \frac{v_1 t}{v_2 t} = \frac{MN}{ML}$$

or

$$\frac{BC}{AD} = \frac{MN}{ML} = \frac{v_1}{v_2} \qquad \ldots (3)$$

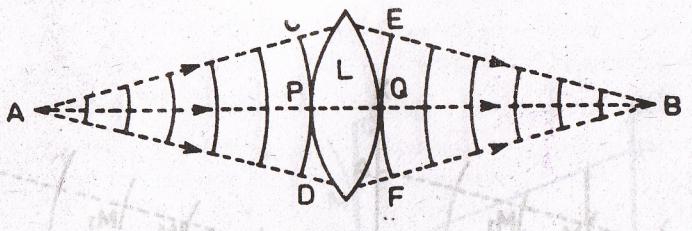


Fig. 9.8.

Hence, if AD is the radius of the secondary wavefront for the point A then ML is the radius of the secondary wavefront for the point M.

Let i and r be the angles of incidence and refraction respectively. From the Δs ABC and ACD

$$\frac{\sin i}{\sin r} = \frac{BC}{AC} \Big/ \frac{AD}{AC}$$

or

$$\frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2}$$

Thus, the bending or refraction of sound waves will depend upon the velocities of sound in the two media.

9.10. Experimental Demonstration of Refraction of Sound

The principle of refraction of sound waves can be demonstrated with a simple experiment. A rubber balloon L in the shape of a convex lens is taken and it is filled with carbon dioxide (Fig. 9.8). The density of carbon dioxide is more than air. Therefore, velocity of sound in carbon dioxide is less than in air. A source of sound is kept at A. The incident spherical wavefront CPD, after refraction through the lens L, gives rise to the refracted spherical wavefront EQF. The sound converges to the point B just similar to light waves. B is the sound image corresponding to the source A. The ear placed at B receives sound of maximum intensity.

9.11. Diffraction of Sound

It is a matter of common experience that the sound is heard behind an obstacle placed in the path of sound. This bending of sound round an obstacle is called diffraction of sound. In the case of light, even for a very small obstacle, a shadow is observed and it is difficult to observe the illumination in the geometrical shadow. Both sound and light are propagated as waves in the medium and are diffracted.

9.12. Explanation of Diffraction of Sound

Each progressive wave, according to Huygens wave theory, produces

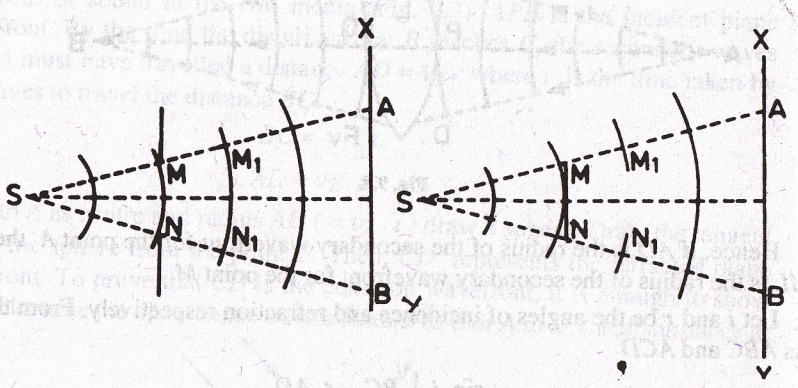


Fig. 9.9.

secondary waves, the envelope of which forms the secondary wavefront. In Fig. 9.9 *(i)*, *S* is a source of sound and *MN* is an aperture. *XY* is a screen placed in the path of sound. *AB* is the portion of the screen where sound energy is received and above *A* and below *B*, it is the region of the geometrical shadow. Considering *MN* as the primary wavefront, according to Huygens construction, if the secondary wavefronts drawn one would expect encroachment of sound in the geometrical shadow. Thus, the shadows formed by obstacles are not sharp. **This bending of sound waves round the edges of an obstacle or the encroachment of sound within the region of the geometrical shadow is called diffraction of sound.** Similarly if an opaque obstacle *MN* is placed in the path of sound [Fig.9.9 *(ii)*], the back portion of the obstacle should also receive sound waves and there should be sound intensity in the geometrical shadow region *AB* also.

Augustin Jean Fresnel in 1815, combined in a striking manner, Huygens wavelets with the principle of interference and could satisfactorily explain the bending of light and sound waves round obstacles.

9.13. Fresnel's Assumptions

According to Fresnel, the resultant effect at an external point due to a wavefront will depend on the factors discussed below.

In Fig. 9.10, *S* is a source of sound and *MN* is an aperture. *XY* is the screen and *SO* is perpendicular to *XY*. *MCN* is the incident spherical wavefront due to the point source *S*. To obtain the resultant effect at a point *P* on the screen, Fresnel assumed that: (1) a wavefront can be divided into a large number of strips or zones called Fresnel's zones of small area and the resultant effect at any point will depend on the combined effect of all the secondary waves

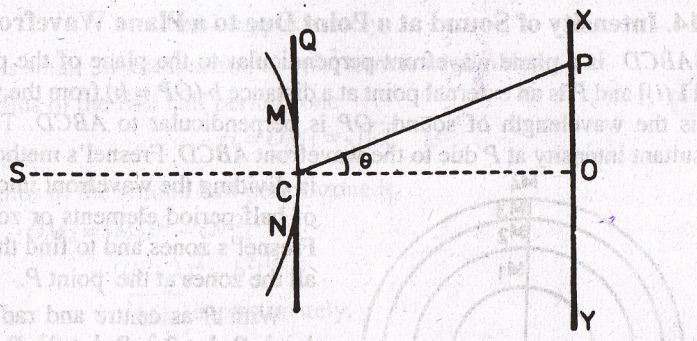


Fig. 9.10.

emanating from the various zones; (2) the effect at a point due to any particular zone will depend on the distance of the point from the zone; (3) the effect at *P* will also depend on the obliquity of the point with reference to the zone under consideration, *e.g.*, due to the part of the wavefront at *C*, the effect will be maximum at *O* and decreases with increasing obliquity. It is maximum in a direction radially outwards from *C* and it decreases in the opposite direction. The effect at a point due to the obliquity factor is proportional to $(1 + \cos\theta)$ where $\angle PCO = \theta$. Considering an elementary wavefront at *C*, the effect is maximum at *O* because $\theta = 0$ and $\cos\theta = 1$. Similarly in a direction tangential

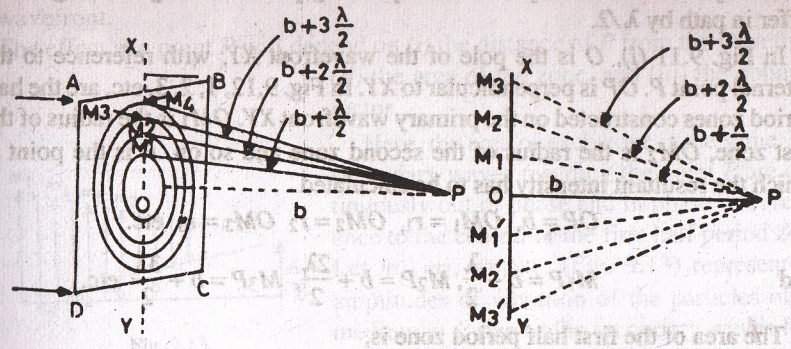


Fig. 9.11.

to the primary wavefront at *C* (along *CQ*) the resultant effect is one-half of that along *CO* because $\theta = 90°$ and $\cos 90 = 0$. In the direction *CS* the resultant effect is zero since $\theta = 180°$ and $\cos 180 = -1$ and $1 + \cos 180 = 1 - 1 = 0$. This property of the secondary waves eliminates one of the difficulties experienced with the simpler form of Huygens principle *viz.*, that if the secondary waves

spread out in all directions from each point on the primary wavefront, they should give a wave travelling forward as well as backward. As the amplitude at the rear of the wave is zero there will evidently be no back wave.

9.14. Intensity of Sound at a Point Due to a Plane Wavefront

ABCD is a plane wavefront perpendicular to the plane of the paper [Fig. 9.11 *(i)*] and *P* is an external point at a distance *b (OP = b)* from the wavefront. λ is the wavelength of sound. *OP* is perpendicular to *ABCD*. To find the resultant intensity at *P* due to the wavefront *ABCD*, Fresnel's method consists in dividing the wavefront into a number of half period elements or zones called Fresnel's zones and to find the effect of all the zones at the point *P*.

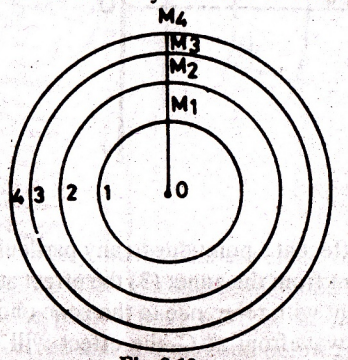


Fig. 9.12.

With *P* as centre and radii equal to $b+\lambda/2, b+2\lambda/2, b+3\lambda/2$ etc., construct spheres which will cut out circular areas of radii OM_1, OM_2, OM_3, etc., on the wavefront. These circular zones are called half period zones or half period elements. Each zone differs from its neighbour by a phase difference of π or a path difference of $\lambda/2$. Thus, the secondary waves starting from the points *O* and M_1 and reaching *P* will have a phase difference of π or a path difference of $\lambda/2$. A Fresnel half period zone with respect to an external point *P* is a thin annular zone (or a thin rectangular strip) of the primary wavefront in which the secondary waves from any two corresponding points of neighbouring zones differ in path by $\lambda/2$.

In Fig. 9.11 *(i)*, *O* is the pole of the wavefront *XY*, with reference to the external point *P*. *OP* is perpendicular to *XY*. In Fig. 9.12, 1, 2, 3, etc. are the half period zones constructed on the primary wavefront *XY*. OM_1 is the radius of the first zone, OM_2 is the radius of the second zone and so on. *P* is the point at which the resultant intensity has to be calculated.

$$OP = b,\ OM_1 = r_1,\ OM_2 = r_2\ OM_3 = r_3 \text{ etc.}$$

and
$$M_1P = b + \frac{\lambda}{2},\ M_2P = b + \frac{2\lambda}{2},\ M_3P = b + \frac{3\lambda}{2} \text{ etc.}$$

The area of the first half period zone is,

$$\pi\, OM_1{}^2 = \pi\,[M_1P^2 - OP^2]$$

$$= \pi\left[\left(b + \frac{\lambda}{2}\right)^2 - b^2\right]$$

$$= \pi\left[b\lambda + \frac{\lambda^2}{4}\right]$$

$$= \pi b\lambda \text{ appoximately.} \quad \ldots (1)$$

(As λ is small , in comparison to b, λ^2 term is negligible).

The radius of the first half period zone

$$r_1 = OM_1 = \sqrt{b\lambda} \quad \ldots (2)$$

The radius of the second half period zone is

$$OM_2 = [M_2P^2 - OP^2]^{1/2}$$

$$= [(b+\lambda)^2 - b^2]^{1/2}$$

$$= \sqrt{2b\lambda} \text{ approximately.}$$

The area of the second half period zone

$$\pi = [\ OM_2^2 - OM_1^2\ ]$$

$$= \pi\ [\ 2b\lambda - b\lambda\ ] = \pi b\lambda.$$

Thus, the area of each half period zone is equal to $\pi b\lambda$. Also, the radii of the 1st, 2nd, 3rd etc. half period zones are $\sqrt{1.b\lambda}$, $\sqrt{2b\lambda}$, , $\sqrt{3b\lambda}$ etc. Therefore, the radii are proportional to the square roots of the natural numbers. However, it should be remembered that the areas of the zones are not constant but are dependent on *(i)* λ, the wavelength of sound and *(ii)* *b*, the distance of the point from the wavefront. The area of the zone increases with increase in the wavelength of sound and with increase in the distance of the point *P* from the wavefront.

The effect at a point *P* will depend on *(i)* the distance of *P* from the wavefront, *(ii)* the area of the zone, and *(iii)* the obliquity factor.

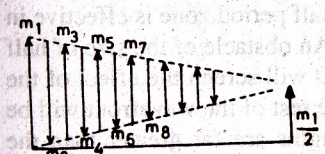


Fig. 9.13.

Here, the area of each zone is the same. The secondary waves reaching the point *P* are continuously out of phase and in phase with reference to the central or the first half period zone. Let m_1, m_2, m_3 etc. (Fig. 9.13) represent the amplitudes of vibration of the particles of the medium at *P* due to the secondary waves from the 1st, 2nd, 3rd etc. half period zones. As we consider the zones outwards from *O*, the obliquity increases and hence the quantities m_1, m_2, m_3 etc. are of continuously decreasing order. Thus, m_1 is slightly greater than m_2, m_2 is slightly greater than m_3 and so on. Due to the phase difference of π between any two consecutive zones if the displacements of the particles due to odd numbered zones is in the positive direction, then due to the even numbered zones the displacement will be in the negative direction

at the same instant. As the amplitudes are of gradually decreasing magnitude, the amplitude of vibration at P due to any zone can be approximately taken as the mean of the amplitudes due to the zones preceding and succeeding it.

e.g. $$m_2 = \frac{m_1 + m_3}{2}$$

The resultant amplitude at P at any instant is given by

$$A = m_1 - m_2 + m_3 - m_4 ..... + m_n \text{ if } n \text{ is odd.}$$

(If n is even, the last quantity is $-m_n$).

$$\therefore \quad A = \frac{m_1}{2} + \left[\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right] + \left[\frac{m_3}{2} - m_4 + \frac{m_5}{2}\right] + ... + \frac{m_n}{2}$$

But $$m_2 = \frac{m_1}{2} + \frac{m_3}{2} \text{ and } m_4 = \frac{m_3}{2} + \frac{m_5}{2}$$

$$\therefore \quad A = \frac{m_1}{2} + \frac{m_n}{2} \ldots \text{ if } n \text{ in odd}$$

and $$A = \frac{m_1}{2} + \frac{m_{n-1}}{2} - m_n \ldots \text{ if } n \text{ is even.}$$

If the whole wavefront $ABCD$ is unobstructed, the number of half period zones that can be constructed with reference to the point P is infinite *i.e.* $n \to \infty$. As the amplitudes are of gradually diminishing order, m_n and m_{n-1} tend to zero.

$\therefore$ The resultant amplitude at P due to the whole wavefront $= A = \frac{m_1}{2}$. The intensity of sound at a point is proportional to the square of the amplitude.

$$\therefore \quad I \alpha \frac{m_1^2}{4}.$$

Thus, the intensity at P is only one-fourth of that due to the first half period zone alone. Here, only half the area of the first half period zone is effective in providing the intensity of sound at the point P. An obstacle of the size of half the area of the first half period zone placed at O will screen the effect of the whole wavefront and the intensity at P due to the rest of the wavefront will be zero. In the case of sound waves, the wavelengths are far greater than the wavelength of light and hence the area of the first half period zone for a plane wavefront of sound is very large. *If the effect of sound at a point beyond an obstacle is to be shadowed, an obstacle of very large size has to be used to get no sound effect.*

EXERCISES

1. Discuss the reflection of a plane wavefront of sound at a plane surface.

2. Describe an experiment to demonstrate the reflection of sound.

3. What is an echo ? How is it produced? Explain, why an echo cannot be heard if the distance between the source of sound and the obstacle is less than 17 metres. Also explain the formation of successive echoes.

4. Describe briefly some practical applications of reflection of sound.

5. Discuss the phase change on reflection when (*i*) sound waves travel from air to water and (*ii*) from water to air.

6. Discuss the refraction of sound waves at the interface of two media and explain, how most of the sound energy is totally internally reflected when sound waves are travelling from rarer to denser medium.

7. Discuss the refraction of a plane wavefront of sound at a plane surface.

8. Describe an experiment to demonstrate refraction of sound.

9. What do you understand by diffraction of sound?

10. Discuss the intensity of sound at a point due to a progressive plane wavefront.

11. Distinguish between the diffraction of sound with diffraction of light.

12. Explain, why the diffraction effects are easily noticed in the case of sound whereas in the case of light even an obstacle of small size produces a geometrical shadow.

13. Deduce the laws of reflection with the help Huygen's theory of secondary wavelets. *(Rajasthan, 1985)*

14. Write short notes on :

(*i*) Reflection and Refraction of sound waves. *(Delhi, 1993)*

(*ii*) Diffraction of sound.

(*iii*) Echo.

(*iv*) Whispring Galleries.

CHAPTER 10

Practical Applications

10.1. Siren

It consists of a wind chest. The upper disc D has holes near its circumference and is just above the disc A of the wind chest. The disc A has a number of holes equal to the number of holes in the disc. The holes in D are slanting and are just in opposite direction to the holes in the fixed disc A (Fig. 10.1). The disc D rotates about the spindle and the number of rotations made by it, is indicated by the counters C_1 and C_2.

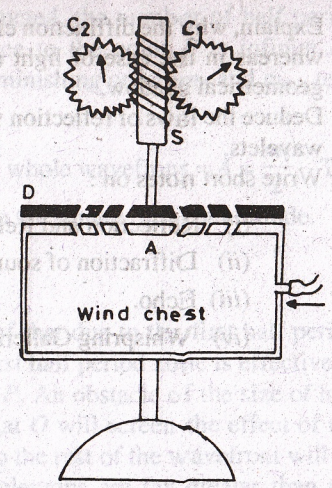


Fig. 10.1.

Working. The wind is pushed into the wind chest and each puff of air that escapes from the holes, rotates the upper disc D. When the holes in the disc D are just above the holes in A, air escapes. Suppose the upper disc has h holes and it rotates n times second. The frequency of the sound produced by the air escaping out of the holes $= N = n \times h$.

Determination of the frequency of a note. The pressure of wind entering into the wind chest is adjusted such that the frequency of the sound produced by the siren is nearly equal to that of the given note. This can be done by increasing the speed of rotation of the disc D slowly. When the frequencies are nearly equal, count the number of beats produced per second. Let the number of beats produced per second be x.

The frequency of the note $= N + x$.

To avoid the counting of beats, the speed of rotation should be adjusted such that the two notes have the same pitch. In that case the frequency of the note $= N$.

Example 10.1. *The disc of a siren having a circle of 40* holes *rotates uniformly 500 times in 1* min *24* s. *Find the frequency of the note emitted and its wavelength in air, if the velocity of sound in air is 34000* cm/s.

Here $h = 40$, $V = 34000$ cm/s

Number of rotations $= 500$, Time $= 1$ min 24 s $= 84$ s

$\therefore$ Number of rotations /s $n = \dfrac{500}{84}$

$\therefore$ $N = n \times h = 40 \times \dfrac{500}{84} = 238{\cdot}1$

But $V = N\lambda$

$\therefore$ $\lambda = \dfrac{V}{N} = \dfrac{34000}{238{\cdot}1} = \mathbf{14{\cdot}28\ cm.}$

Example 10.2. *The disc of a given siren has 32 holes. What must be the speed of rotation per minute of the siren disc so that the note emitted by the siren may be in unison with that of a tuning fork which makes 256 vibrations per second.*

Suppose, the number of rotations /minute $= x$

$\therefore$ Number of rotations/s $= \dfrac{x}{60}$

Here $n = \dfrac{x}{60}$

$h = 32$, $N = 256$

$N = n \times h$

$256 = \dfrac{x}{60} \times 32$

or $\mathbf{x = 480\ rotations/minute.}$

Example 10.3. *The disc of a siren having 60* holes *makes 420* revolutions *per minute. Find the wavelength of its sound in air at 0* °C *if the velocity of sound at 0* °C *is 332* m/s. (IAS, 1989)

Number of holes, $h = 60$

Number of revolutions $= 420$ per minute

$n = \dfrac{420}{60}$

$= 7$ per second

$v = nh$

$v = 7 \times 60$

$= \mathbf{420\ Hz}$

$V = 332$ m/s

$$\lambda = \frac{V}{\nu}$$

$$= \frac{332}{420}$$

0·79 m.

10.2. Falling Plate Method

To the prong of a tuning fork a fine style is attached. The style touches a smoked paper fixed on a vertical plate. The vertical plate is fixed at the top. The tuning fork is set into vibration so that its prong having the style vibrates horizontally. The plate is allowed to fall under the action of gravity. The style traces a wave on the smoked paper as shown in Fig. 10.2.

The wave traces near the lower edge of the plate are narrow and the wavelength increases with increase in distance from the lower end. Take a point A on the smoked paper and mark points B and C such that there are n complete waves in AB and also n complete waves in BC. Measure the distances AB and BC.

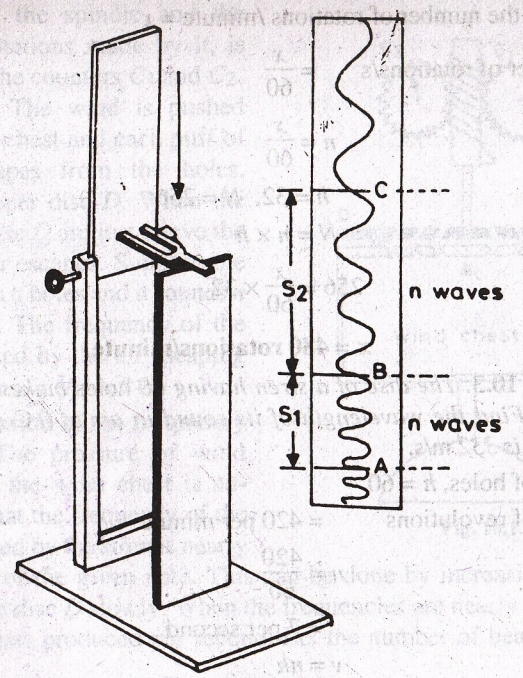


Fig. 10.2.

If u is the initial velocity of the plate at A and the time taken from A to B or B to C is t, then

$$S_1 = ut + \frac{1}{2}gt^2 \quad \ldots (1)$$

$$(S_1 + S_2) = u(2t) + \frac{1}{2}g(2t)^2 \quad \ldots (2)$$

Multiplying (1) by 2

$$2S_1 = 2ut + gt^2 \quad \ldots (3)$$

Subtracting (3) from (2)

$$S_2 - S_1 = gt^2$$

$$t = \sqrt{\frac{S_2 - S_1}{g}} \quad \ldots (4)$$

The number of waves in AB or $BC = n$

Frequency of the tuning fork $= \frac{n}{t}$

$$\therefore \quad N = \frac{n}{t} = n\sqrt{\frac{g}{S_2 - S_1}} \quad \ldots (5)$$

Hence, the frequency of the tuning fork can be determined. The only possible error is due to the decrease in frequency of the fork when the style is attached to it. Therefore, the style should be light.

10.3. Determination of Frequency of a Tuning Fork by Revolving Drum Method

[Chronographic Method]

The apparatus consists of a revolving drum D which rotates at a uniform speed. T is a tuning fork having a fine style attached to one of its prongs. A smoked paper is wrapped on the surface of the drum. The tip of the style just touches the smoked paper (Fig.10.3). AB is a second's pendulum and C is a cup containing mercury. When the pendulum is set into vibration, the bob B just touches the surface of mercury twice in each vibration. The pendulum works as an automatic make and break arrangement in the primary circuit (Fig. 10.3). The secondary circuit is also shown in Fig. 10.3. Whenever B makes contact with mercury, an induced *emf* is produced across S, S.

A spark is produced at the tip of the style and a mark is made on the smoked paper. These marks will be obtained on the smoked paper at equal intervals of time of one second.

The tuning fork and the pendulum are set into vibration. The style marks a wavy pattern on the smoked paper of the rotating drum. The marks are also obtained on the smoked paper. The smoked paper is removed and the number of complete waves between two consecutive marks is counted. The average

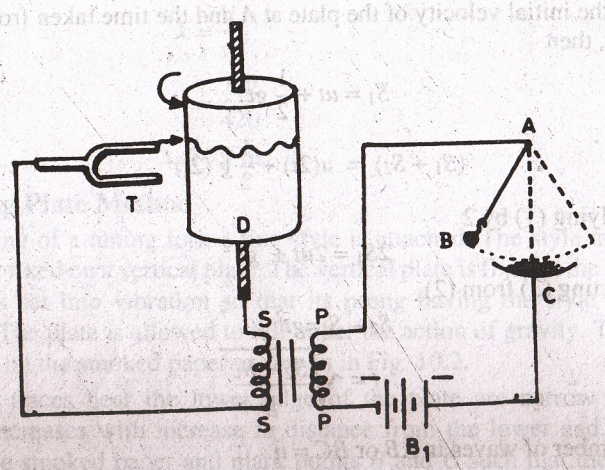


Fig.10.3.

number of waves between two marks gives the value of the frequency of the tuning fork.

Here the frequency of the tuning fork is determined with the style attached to one of its prongs. The experimental value is less than the actual frequency of the tuning fork. To determine this correction, a tuning fork of known frequency is taken with a style fixed to one of its prongs. The experiment is performed and the experimental value is found as discussed earlier. The difference between the actual frequency and the experimental value gives the value of the correction to be applied.

Suppose the actual frequency of the tuning fork is x, and the experimental value is y. The correction to be applied $= +(x - y)$.

To determine the unknown value of the frequency of any tuning fork, the correction $(x - y)$ is added to the experimental value.

10.4. Determination of Frequency of a Tuning Fork by Stroboscopic Method

The tuning fork whose frequency is to be determined has two thin aluminium foils A and B attached to its prongs. The size of the foils is such that the window (slit) in one of the foils, opens when the prongs vibrate outward and it closes when the prongs vibrate inward. The window opens only once in each vibration of the tuning fork (Fig. 10.4).

D is a white circular disc having dots marked near the edge. These dots are equally spaced. The tuning fork F is placed in such a position that the dot lies in front of the window when it opens. The dots can be observed with the help of a telescope T. The disc D is rotated at a uniform speed with the help of an electric motor (Fig. 10.4).

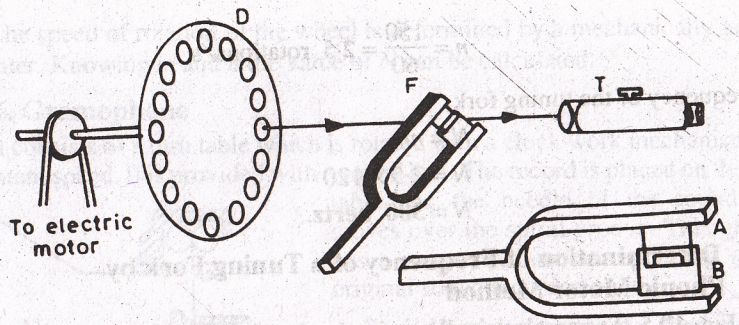


Fig. 10.4.

The tuning fork is set into vibration and the speed of rotation of the disc is gradually increased. At a particular minimum speed of the disc the dot *appears stationary* in the field of veiw of the telescope. This means that during one complete vibration of the tuning fork, the position of one dot is exactly replaced by the next dot.

Let n be the minimum number of rotations of the disc per second when the dot appears stationary. Suppose the number of dots on the disc $= m$.

Time taken for one complete rotation of the disc$=1/n$.

Time interval for one dot to be exactly replaced by the next dot,

$$t = \frac{1}{nm} \qquad \ldots (1)$$

This time t also corresponds to the time taken by the tuning fork to complete one vibration.

Suppose the frequency of the tuning fork is N

$$\therefore \quad t = \frac{1}{N} \qquad \ldots (2)$$

From equations (1) and (2)

$$\frac{1}{N} = \frac{1}{nm}$$

$$\therefore \quad N = nm.$$

If the speed of rotation of the disc is made double, the dot will again appear stationary. Similarly at the speeds $3n$, $4n$, etc., the dot will appear stationary.

Example 10.4. *A stroboscopic disc marked with 120 dots is rotated and the minimum speed adjusted till the dot appears stationary when looked through a slit attached to the prongs of a tuning fork. If the minimum speed of rotation of the disc is 150* rotations per minute, *calculate the frequency of the tuning fork.* (Delhi, 1984)

Here $m = 120$

$$n = \frac{150}{60} = 2{\cdot}5 \text{ rotations/s}$$

Frequency of the tuning fork,

$$N = nm$$

$$N = 2{\cdot}5 \times 120$$

$$N = \mathbf{300 \text{ hertz.}}$$

10.5. Determination of Frequency of a Tuning Fork by Phonic Motor Method

In Fig. 10.5, F is an electrically maintained tuning fork whose frequency is to be determined. N is a soft iron toothed wheel that can rotate about a horizontal axis. M_1 and M_2 are two electromagnets having their axes passing through the centre of the wheel. The electrical arrangement is shown in Fig. 10.5.

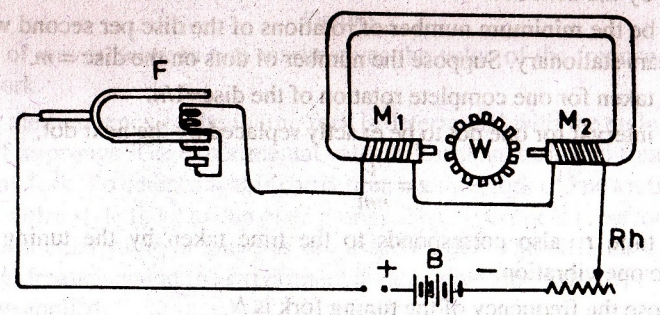


Fig. 10.5.

The tuning fork F is set into vibration. Initially the wheel W is rotated by hand. When only two diametrically opposite teeth of the wheel just pass the electromagnets at the instant of excitation, the teeth get impluses and move forward. The rotor will just advance through one tooth during one full vibration of the fork. The wheel will get an impluse during each excitation and will continue to rotate. The speed of rotation will depend on the frequency of the tuning fork. If the wheel is moving faster, the force of attraction between the tooth and the magnet, retards its motion. If the wheel is moving slower, the force of attraction accelerates it.

Suppose the wheel makes n rotations in one second and the number of teeth on the wheel is m.

Number of impluses received per second = $m \times n$.

If the frequency of the tuning fork is N, then

$$N = \mathbf{m \times n}$$

The speed of rotation of the wheel is determined by a mechanically geared counter. Knowing m and n the value of N can be calculated.

10.6. Gramophone

It consists of a turn table which is rotated with a clock work mechanism at a constant speed. It is provided with a sound box. The record is placed on the turn table and the needle of the sound box moves over the spiral groove. This groove has a wavy structure correponding to the original sound recorded on it.

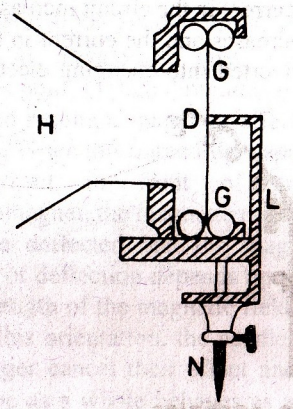


Fig. 10.6.

Sound box. It consists of a diaphragm D and a horn H (Fig. 10.6). The diaphragm is held in position with the help of the rubber gaskets G, G. A lever L is fixed to the middle of the diaphragm. The lever is attached to the needle N. The needle N vibrates and these vibrations are magnfied with a lever system and the diaphragm vibrates accordingly. These vibrations of the diaphragm produce sound.

Recording. To prepare a gramophone record, a circular wax disc with a spiral groove on it is taken. The speaker speakes before the diaphragm and a sharp needle called a style vibrates along the path of the rotating wax record. A transverse wave is formed.

To prepare a large number of records, this wax disc is coated with graphite and electroplated uniformly with copper. The wax coating is removed by gently heating it. The copper disc remains with the grooves projecting outwards. This parent record is pressed against a plastic or shellac disc and the grooves are formed on it. These records prepared from the parent record are called sister records. Thus a large number of records can be obtained.

Sound reproduction. The record is kept on the turn table of the gramophone and the needle of the sound box is gently placed at the starting point of the groove, of the rotating record. When the needle moves along the groove, it vibrates and forces the diaphragm to vibrate accordingly. Thus, the original recorded sound is reproduced. In a modern sound box, a pick up arrangement is used.

10.7. Microphone and Loud Speaker

Telephones are used to send and receive messages through electrical signals between two stations. As the electrical signals can flow round a closed circuit, two line wires are required. At both the ends of the line, a mouthpiece and an ear piece are provided. The mouthpiece works on the principle of microphone and the ear piece works as a small loud speaker.

The microphone consists of a small ebonite box. Between the diaphragm *D* and the thin metal disc *P*, the space is filled with carbon granules [Fig. 10.7 (*a*)]. The lead wires are taken from *P* and *D*. When a person speaks before the microphone, the diaphragm *D* vibrates. Due to the vibration of the diaphragm, the pressure on the carbon granules continuously changes. Consequently the resistance offered by the carbon granules in the circuit changes. When they are closely packed, the resistance decreases and the current in the circuit increases. When they are loosely packed, the resistance increases and the current in the circuit decreases. Thus the sound energy is converted into changing electric current.

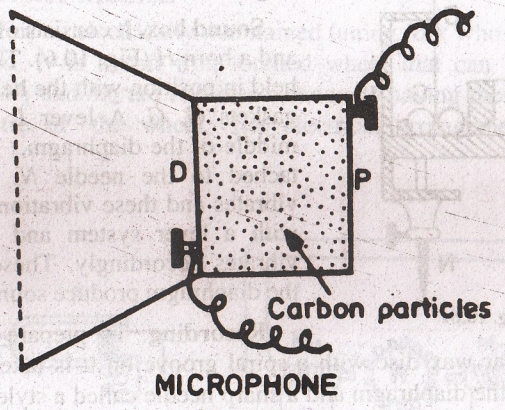


Fig. 10.7 *(a)*

The loudspeaker consists of a diaphragm *D* and an electromagnet [Fig. 10.7(*b*)]. When the changing current flows through the coil of the electromagnet, the magnetic flux changes. Due to the change in the magnetic flux, the diaphragm is set into vibration. Here electrical energy is converted into sound energy. The original sound is reproduced.

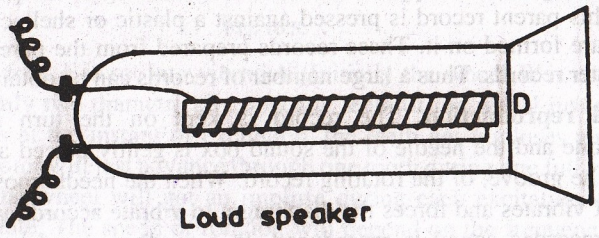


Fig. 10.7 *(b)*

10.8. Tape Recording

The most commonly used tape recorders are based on the principle of magnetism and are called magnetic tape recorders. Magnetic tapes of the form

usually employed for tape recording and reproduction behave similar to an assembly of magnetic compass needles.

One side (dull side) of the tape is coated with a paste of ferric oxide (Fe_2O_3) containing needle shaped iron oxide crystals. These crystals can be imagined to be similar to magnetic compass needles. Each needle behaves as a tiny magnet (magnetic dipole) and has south and north polarity.

In an unrecorded tape, the needles point in such directions so that the resultant magnetic effect is zero. When this unrecorded tape is moved in front of an electromagnet, the magnetic needles are deflected and the magnitude of deflection depends upon the strength of the magnetic field. After this orientation, the needles no longer cancel their effect and the tape as a whole behaves as a magnet, magnetised to different extents at different positions of the tape. The specific property of these iron oxide crystals is that they can retain their oriented positions for a very long time. Once recorded, the tape can serve as a permanent record unless it is erased.

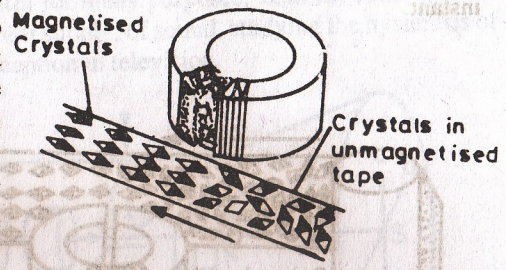


Fig. 10.8

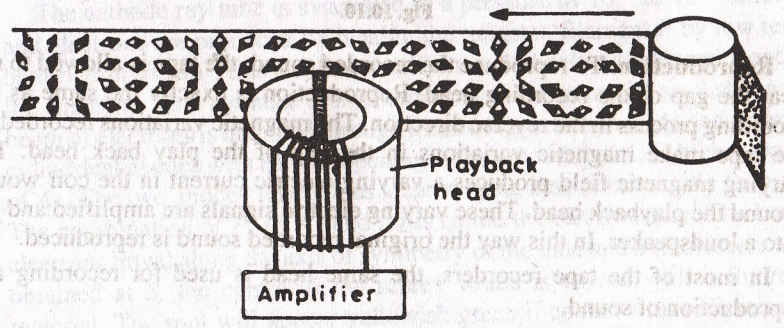


Fig. 10.9.

Recording. A magnetic tape has two sides. The dull side is coated with ferric oxide containing needle shaped crystals. The other side is shining. The tape is usually made of *cellulose acetate* or *poly vinyl chloride (PVC).* The tape is thin so that maximum length can be wound on a reel. The material of the tape must have flexibility but it should not stretch. The stretching of the tape can distort the recording. Sometimes Mylar, a kind of terylene is also used for the manufacture of the tape.

When a person speaks before a microphone, the sound vibrations are converted into varying electrical impulses. These electrical variations are fed into the amplifier. The amplified signal is fed to an iron ring having a very small air gap. This gap is not visible to the naked eye.

The dull side of the tape faces the gap and it is run, close to the gap. Varying electrical impulses produce corresponding variations in the magnetic field strength in the gap. The magnetic needles get oriented to different directions. The magnitude of orientation at any point depends on the field strength at that instant.

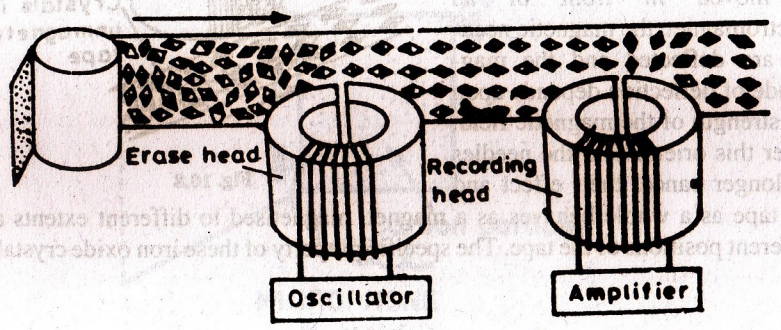


Fig. 10.10.

Reproduction. To reproduce the recorded sound, the tape is allowed to run near the gap of the recording head. Reproduction is exactly the same as the recording process in the reverse direction. The magnetic variations recorded on the tape make magnetic variations in the gap of the play back head. The varying magnetic field produces a varying electric current in the coil wound around the playback head. These varying electric signals are amplified and fed into a loudspeaker. In this way the original recorded sound is reproduced.

In most of the tape recorders, the same head is used for recording and reproduction of sound.

When the sound is to be recorded on a tape, it is initially passed in front of an erasing head, so that the previous record is erased

Erasing the tape. Whenever new recording has to be made on a tape, it is essential that the previous recording is completely erased. If it is not done, the two records will overlap and both the patterns will be heard.

The erasing of a tape is done by passing it before an erasing head. An erasing head consists of an electromagnet (An iron ring with a small air gap) which is fed with a strong current from an oscillator of frequency 50 kilohertz. This rapidly varying strong electric current completely obliterates the previous

recorded pattern and the tape is ready for new recording.

10.9. Cathode Ray Oscillograph

A cathode ray oscillograph consists of a thermionic vacuum tube fitted with an electron gun to produce electrons. The shape and the internal components of the cathode ray tube are shown in Fig.10.11. The electrons traverse along the axis of symmetry of the tube and, in the absence of any deflecting electric or magnetic field, strike the centre of the screen and produce a fluorescent spot. The beam can be deflected by suitable electric or magnetic fields. The cathode ray oscillograph is a device useful for many purposes, such as, observing the wave form of an alternating current supply, of sound, studying the hysteresis of ferro-magnetic materials and reception in television.

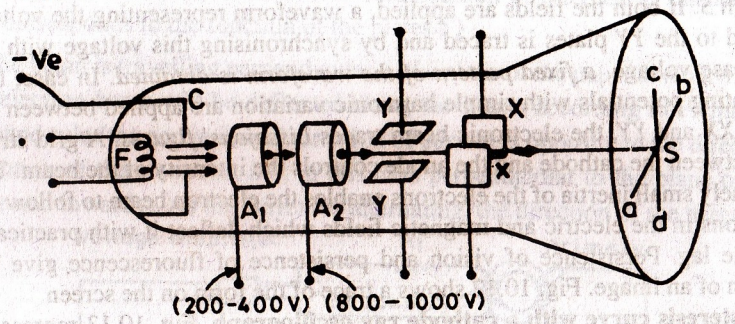


Fig. 10.11.

The cathode ray tube is evacuated to a pressure of 10^{-5} to 10^{-6} mm of Hg and electrons are produced by heating the tungsten filament F by low tension battery (not shown). C is a cylinder called the Wehnelt cylinder and it encloses the filament. This cylinder is maintained at a –ve potential so that a narrow beam of electrons is obtained through a narrow aperture in C. A_1 and A_2 are two cylindrical anodes with narrow openings. The potential of A_2 is higher than that of A_1. The two anodes function as a converging lens (electrostatic focussing). The combination of F, C, A_1 and A_2 is called the *electron gun*. The beam of electrons travel along the axis of symmetry of the tube and a fluorescent spot is obtained at S, the centre of the screen which is coated with a fluorescent material. The spot will appear yellowish green if coated with willemite (zinc ortho-silicate), blue with calcium or cadmium tungstate and white with zinc sulphide or zinc-cadmium sulphate.

The beam may be deflected horizontally or vertically by applying suitable electrostatic fields between the deflecting plates XX and YY respectively. The deflection of the beam is also possible by suitable magnetic fields produced by coils outside the tube.

The deflecting plates XX are connected to a *'time-base'* voltage of known frequency of alternation. This voltage causes the spot to sweep across the

screen (along the *X*-axis) at a uniform speed and then to quickly flash back to the starting point. The *PD* of which the waveform is to be investigated is applied between the plates *YY*.

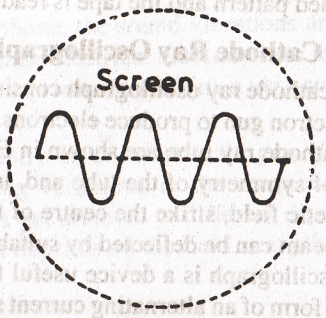


Fig. 10.12.

If the voltage is applied only to the *XX* plates the spot sweeps a horizontal line passing through *S* and if the voltage is applied only to the *YY* plates the spot moves along a vertical line passing through *S*. If both the fields are applied, a waveform representing the voltage applied to the *YY* plates is traced and by synchronising this voltage with the time base voltage, *a fixed pattern of the waveform is obtained*. In case, two alternating potentials with simple harmonic variation are applied between the plates *XX* and *YY*, the electronic beam traces *Lissajous' figures*. A grid structure between the cathode and the anode controls the intensity of the beam. The extremely small inertia of the electrons enables the electron beam to follow the variations in the electric and magnetic fields which deflect it with practically no time lag. Persistence of vision and persistence of fluorescence give the illusion of an image. Fig. 10.12 shows a trace of the form on the screen.

Hysteresis curve with a cathode ray oscillograph. Fig. 10.13 represents the diagram of connections to study the phenomenon of hysteresis in a ferromagnetic bar. The bar *PQ* (say of soft iron) is supported horizontally near the cathode ray tube so that the length of the bar is parallel to the plates *YY* as shown. A solenoid would on this bar is connected to an alternating voltage from the secondary of a step down transformer and the plates *XX* are connected as shown. The strength of the current in the circuit is adjusted with the help of two variable non-inductive resistances R_1 and R_2. *S* is the screen. The axis of the tube is perpendicular to the plane of the paper.

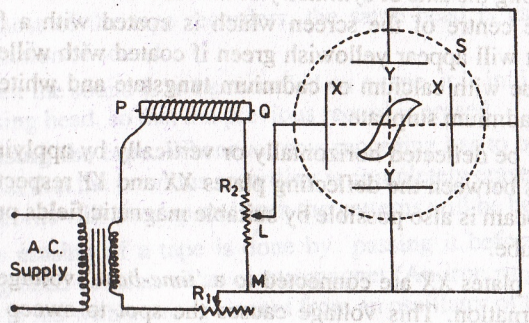


Fig. 10.13.

The potential difference applied between the plates *XX* at any instant is equall to the potential difference between the points *L* and *M*. This potential difference will in turn depend on the strength of the current in the secondary circuit and hence is proportional to the magnetising field (*H*). The displacement of the beam in the direction of the *X*-axis is proportional to *H* at any instant.

The plates YY are connected. When the iron bar is magnetised by the current passing through the solenoid, there is a magnetic field inside the tube parallel to the direction of the *X* axis (the position of the rod *PQ* outside the tube corresponds to the function of the plates *YY*).

The beam is travelling perpendicular to the plane of the paper and outwards. The electron beam will be deflected along the *Y*-axis according to Fleming's left hand rule (the conventional direction of current is opposite to the direction of motion of the electrons, *i.e.*, in this case perpendicular to the plane of the paper and inwards). As the current is alternating, the direction of the magnetic

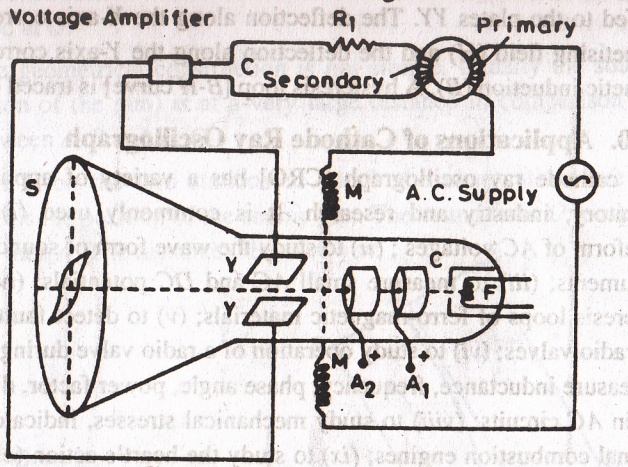


Fig. 10.14. B-H Curve with CRO.

field also changes and hence the beam is deflected upwards or downwards depending on the magnetic polarity of *PQ* at that instant. The vertical shift of the beam corresponds to the intensity of magnetisation (*I*) of the specimen.

The force acting on an electron moving in an electric field is *Xe* where *X* is the intensity of the field and *e* is the charge on the electron. Due to the magnetic field the force is *He v* where *H* is the intensity of the magnetic field and *v* is the velocity of the electron. These two forces *Xe* and *Hev* must be of the same order to obtain satisfactory results. Due to persistence of vision a hysteresis

loop (*I-H* curve) is traced on the screen, the shift along the *X*-axis represents *H* and the shift along the *Y*-axis represents *I*.

Hysteresis curve for a specimen given in the form of a ring using a cathode ray oscillograph. To trace the hysteresis curve for a specimen given in the form of a ring, the circuit shown in Fig. 10.14 is employed. *R* is the ring specimen. The alternating current supply is connected to the primary in series with the magnetic deflection coils (*MM*) which provide the time-base voltage. These coils are placed outside the tube. The direction of the magnetic field is along the plane of the paper and perpendicular to the axis of the tube. Due to these coils the beam is deflected along the *X*-axis (Fleming's left hand rule). Due to magnetic induction an alternating voltage is produced in the secondary and this voltage is connected to a condenser *C* through a high resistance R_1. The condenser helps to keep the induced *emf* in phase with the magnetising current. The voltage across the condenser is amplified with a valve amplifier and fed to the plates *YY*. The deflection along the *X*-axis corresponds to the magnetising field (*H*) and the deflection along the *Y*-axis corresponds to the magnetic induction (*B*). A hysteresis loop [*B-H* curve] is traced on the screen.

10.10. Applications of Cathode Ray Oscillograph

A cathode ray oscillograph [CRO] has a variety of applications in the laboratory, industry and research. It is commonly used (*i*) to study the waveform of *AC* voltages ; (*ii*) to study the wave form of sound from musical instruments; (*iii*) to measure small *AC* and *DC* potentials; (*iv*) to study the hysteresis loops of ferro-magnetic materials; (*v*) to detect faults in radio sets and radio valves; (*vi*) to study operation of a radio valve during working; (*vii*) to measure inductance, frequency, phase angle, power factor, dielectric losses etc. in *AC* circuits; (*viii*) to study mechanical stresses, indicator diagrams of internal combustion engines; (*ix*) to study the heart's action (electro-cardiography), and (*x*) in Television receivers and Radar. A cathode ray oscillograph is very widely used in television receivers and radar equipments. Very small intervals of time of the order of micro-seconds can be measured with a cathode ray oscillograph.

10.11. Sound Ranging

The position of an enemy gun can be located by observing the time of the reception of sound at three different positions. This technique is called sound ranging.

Let *A*, *B* and *C* be three positions of observation. Suppose the gun is fired from some unknown position *O* (Fig.10.15). Microphones are fitted at each position. The sound received by each microphone is converted into an electrical

impluse and fed into oscilloscopes separately at some common control position. As the distances of *A*, *B* and *C* are different from *O*, sound will be received at different instants. From the signals obtained on the oscillograph screen, it is possible to find the difference in time intervals between various signals. Let these time intervals be (*i*) t_1 between *A* and *B*, and (*ii*) t_2 between *A* and *C*.

Suppose *v* is the velocity of sound in air. It means sound reaches *B*, t_1 seconds later than *A*. The distance of *B* from the source is vt_1 more than the distance of *A* from the source of sound. Similarly the distance of *C* is vt_2 more than the distance of *A* from the source.

With *B* as centre draw a circle of radius vt_1 and with *C* as centre draw a circle of radius vt_2 (Fig.10.15).

An imaginary arc is drawn touching the circles and also passing through *A*. The points *D*, *A* and *E* lie on this arc. The centre of the arc is found by geometrical construction. Here *O* is the centre of the circular arc. The enemy gun is situated at *O*.

The above geometrical construction is not simple. Usually the source of sound (position of the gun) is at a very large distance in comparison to the distances between *A*, *B* and *B*, *C*.

Let *A* and *B* be the two stations at which the observations are made (Fig.10.16). *O* is the position of the enemy gun. The sound from *O* reaches *B*, *t* seconds later than *A*. The distance $OB - OA = vt$.

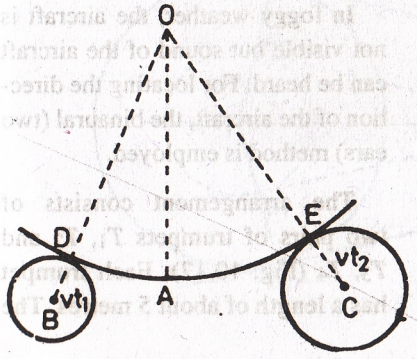


Fig. 10.15.

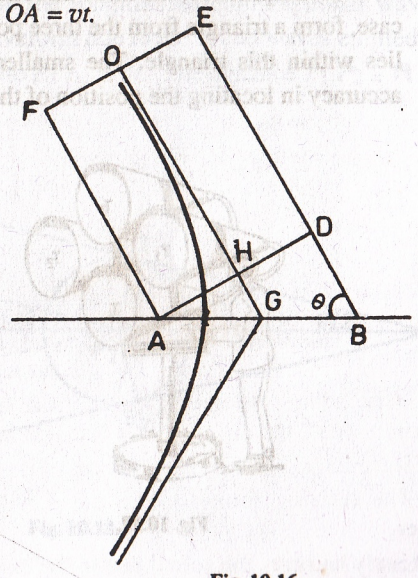


Fig. 10.16.

The point O lies on a hyperbola drawn with A and B as foci. If O is at a large distance from G, (the centre of AB) and EOF and AD are perpendiculars on OG, then O lies on the asymtote OG such that

$$OB - OA = EB - FA$$
$$= BD$$

or $$BD = vt$$

Take $$AB = l$$

and $$\angle ABD = \angle \theta = \angle OGA$$

$$\cos\theta = \frac{BD}{AB} = \frac{vt}{l}$$

$$\theta = \cos^{-1}\left[\frac{vt}{l}\right]$$

The value of θ can be calculated as v, t and l are known. Here, θ gives the direction of the source of sound with respect to the positions A and B.

Similarly the angles θ corresponding to the positions of B and C, and also for C and A are found. The point of intersection of these three directions gives the position of the gun.

In actual practice, the three directions may not intersect at one point. In that case, form a triangle from the three points of intersection. The source of sound lies within this triangle. The smaller the triangle, the higher will be the accuracy in locating the position of the gun.

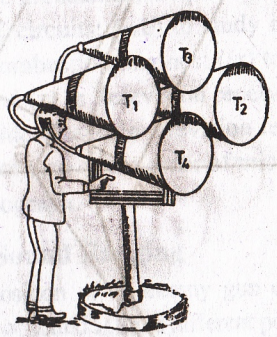


Fig. 10.17.

10.12. Locating the Direction of Aircraft

In foggy weather, the aircraft is not visible but sound of the aircraft can be heard. For locating the direction of the aircraft, the binaural (two ears) method is employed.

The arrangement consists of two pairs of trumpets T_1, T_2 and T_3, T_4 (Fig. 10.17). Each trumpet has a length of about 5 metres. The

four trumpets are arranged in the form of a cross and mounted on a stand. The distance between the trumpets T_1 and T_2 or between T_3 and T_4 is about 4 meters. Two rubber tubes are fitted at the narrow ends of the trumpets T_1 and T_2 and these tubes are fixed in the ears of one of the listeners. This listener controls the horizontal motion. Similarly two rubber tubes from T_3 and T_4 are fitted in the ears of the second listener who controls the vertical motion.

Initially the trumpets T_1 and T_2 are rotated in a horizontal plane. This is done to locate the position where the sound in the two ears is of equal intensity. Now, the trumpets T_3 and T_4 are also brought in the directions of T_1 and T_2. This fixes the vertical plane.

If the sound is heard in the two ears by the second listener, the sound may not be of equal intensity. The trumpets T_3 and T_4 are now rotated in a vertical plane so as to get the position of equal intensity of sound in the two ears of the second listener. Now, the trumpets T_1 and T_2 are brought in the direction of T_3 and T_4. Now all the four trumpets point in the direction of the aircraft.

10.13. Wavefront at Supersonic Speeds: Flight of the Bullet

Any body moving with a speed higher than that of sound is said to be moving with supersonic speed. In this case, the nature of the wavefront is conical.

Suppose a bullet is moving at supersonic speed V. The velocity of sound is v. At any instant, let P_1 be the position of the bullet. After time t, the bullet reaches the point P_2. In this time the longitudinal compressional waves of sound travel a distance vt. The wavefornt due to the sound waves emitted at P_1 will be a sphere of radius vt. Here $P_1A_1 = vt$. The distance $P_1P_2 = Vt$. Here $P_1P_2 > P_1A_1$ (Fig. 10.18).

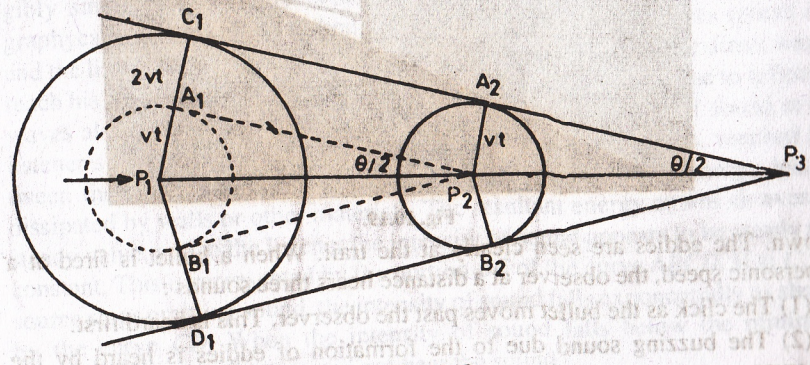


Fig. 10.18.

After a time $2t$ the bullet reaches the point P_3. From P_2 to P_3 the time taken is t. The sound waves emitted at P_1 will reach C_1 and D_1 and those emitted at P_2 will reach A_2 and B_2. $P_3\ A_2\ C_1$ and $P_3\ B_2\ D_1$ are tangents to the wavefronts at P_2 and P_3. Thus, the wavefront is conical with the apex always at the position of the bullet. The cone angle is constant and depends upon the speed of sound and the speed of the bullet.

In Fig. 10.18, $\Delta P_3 P_1 C_1$ and $P_3 P_2 A_2$ are similar

$$\sin \frac{\theta}{2} = \frac{P_2 A_2}{P_3 P_2} = \frac{P_1 C_1}{P_3 P_1}$$

or $$\sin \frac{\theta}{2} = \frac{vt}{Vt} = \frac{2vt}{2Vt}$$

or $$\sin \frac{\theta}{2} = \left(\frac{v}{V}\right)$$

or $$\sin \frac{\theta}{2} = \sin^{-1}\left(\frac{v}{V}\right)$$

or $$\theta = 2\left(\sin^{-1}\frac{v}{V}\right) \quad \ldots (1)$$

From equation (1), it is clear that the cone angle θ decreases with the increase in the speed of the bullet.

In Fig. 10.19 the picture of the flight of the bullet at supersonic speed is

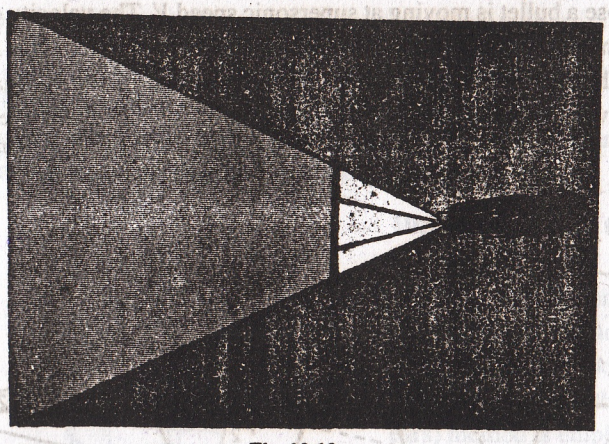


Fig. 10.19.

shown. The eddies are seen clearly at the trail. When a bullet is fired at a supersonic speed, the observer at a distance hears three sounds :

(1) The click as the bullet moves past the observer. This is heard first.

(2) The buzzing sound due to the formation of eddies is heard by the observer.

(3) Lastly he hears the sound due to the explosion of the gun powder.

10.14. Acoustics

The branch of Physics that deals with the process of generation, reception and propagation of sound is called acoustics. This branch in fact covers many fields and is closely related to various branches of engineering. Some of the important fields of acoustics are (*i*) design of acoustical instruments (*ii*) electro-acoutics *viz.*, the branch relating to the methods of sound production and recording (microphones, amplifiers, loudspeakers etc.) (*iii*) architectural acoustics dealing with the design and construction of buildings, operas, music halls, recording rooms in radio and television broadcasting stations. (In general architectural acoustics deals with the behaviour of sound waves in a closed space) and, (*iv*) musical acoustics deals with the design of musical instruments.

10.15. Reverberation

It is observed that for a listener in a room or an auditorium, whenever a sound pulse is produced, he receives directly compressional sound waves from the source, as well as sound waves from the walls, ceiling and other materials present in the room. The waves received by the listener are : (*i*) direct waves and (*ii*) reflected waves due to multiple reflections at the various surfaces. The quality of the note received by the listener will be the combined effect of these two sets of waves. There is also a time gap between the direct wave received by the listener and the waves received by successive reflection. Due to this, the sound persists for sometime even after the source has stopped. This persistance of sound is termed as *reverberation*. The time gap between the initial direct note and the reflected note upto the minimum audibility level is called *reverberation time*. The reverberation time will depend on the size of the room or the auditorium, the nature of the reflecting material on the wall and the ceiling and the area of the reflecting surfaces.

In a good auditorium it is necessary to keep the reverberation time negligibly small. The intensity of sound as received by the listener, is shown graphycally in Fig. 10.20. When a source emits sound, the waves spread out and the listener is aware of the commencement of sound when the direct waves reach his ears. Subsequently the listener receives sound energy due to reflected waves also. If the note is continuously sounded, the intensity of sound at the listener's ears gradually increases. After sometime a balance is reached between the energy emitted per second by the source and the energy lost or dissipated by walls or other materials. The resultant energy attains an average steady value, and to the listener the intensity of sound appears to be steady and constant. This is represented by the portion BC of the curve $ABCD$. If at C, the source stops emitting sound, the intensity of sound falls exponentially as shown by the curve CD. When the intensity of sound falls below the minimum audibility level, the listener will not hear the sound.

When a series of notes are produced in an auditorium (say speech or music) each note will give rise to its own intensity curve with respect to time. The curves for these notes are shown in Fig. 10.21.

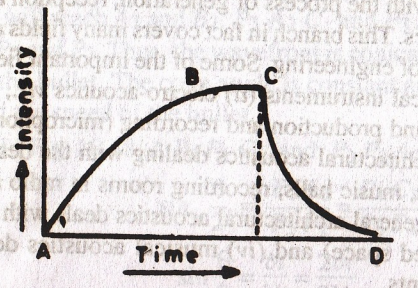


Fig. 10.20.

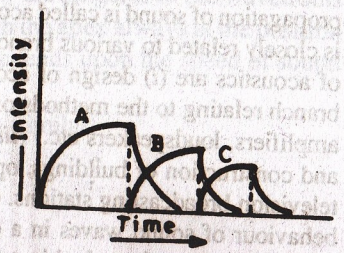


Fig.10.21.

For clear audibility of speech or music, it is necessary that (*i*) each separate note should give sufficient intensity of sound in every part of the auditorium, and (*ii*) each note should die down rapidly before the maximum average intensity due to the next note is heard by the listener. (Fig. 10.21). This is particularly important with speech. In the case of music *comparatively more* reverberation can be tolerated.

10.16 Sabine's Reverberation Formula

Sabine developed the reverberation formula to express the rise and fall of sound in an auditorium. The main assumptions are :

(1) The average energy per unit volume is uniform. It is represented as σ.

(2) The energy is not lost in the auditorium. The energy lost is only due to the absorption of the material of the walls and ceiling and also due to the escape through the windows and ventilators. Both these factors are included in the term '**absorption**' of energy.

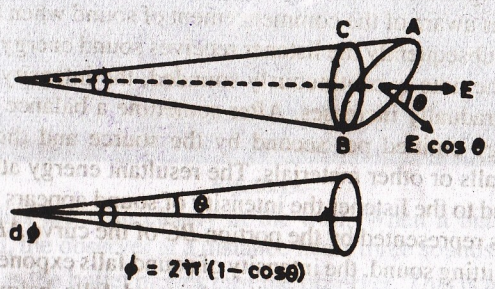


Fig. 10.22.

Suppose a source is producing sound continuously. This sound energy is propagated in all directions. Let σ be the energy contained in a unit volume. The energy that is contained in a solid angle $d\phi$

$$= \frac{\sigma \cdot d\phi}{4\pi}$$

Let this energy by incident on a unit surface area of the wall at an angle θ. If the velocity of sound is v, then the toal energy falling per second on a unit surface area of the wall.

$$= \left(\frac{\sigma \cdot d\phi}{4\pi}\right) (\cos\theta) \cdot v$$

The total energy falling per second within a hemisphere

$$= \frac{\sigma v}{4\pi} \cdot \int \cos\theta \cdot d\phi$$

But $$\phi = 2\pi (1 - \cos\theta)$$

or $$d\phi = 2\pi \sin\theta \cdot d\theta$$

Substituting this value of $d\phi$,

The total energy falling per second within a hemisphere

$$= \frac{\sigma v}{4\pi} \int_0^{\pi/2} 2\pi \sin\theta \cdot \cos\theta \cdot d\theta$$

$$= \frac{\sigma v}{2} \left[-\frac{\cos^2\theta}{2} \right]_0^{\pi/2}$$

$$= \frac{\sigma v}{4}.$$

Suppose α is the absorption coefficient of the walls that refers to the fraction of the incident energy not reflected from the walls. The amount of energy absorbed per second per unit area $= \frac{\alpha\sigma v}{4}$. If A is the area of the walls and the other absorbing materials including ceiling, windows and ventilators etc., the amount of energy absorbed per second $= \frac{A\alpha\sigma v}{4}$.

Let V be the volume of the auditorium, the total energy = $V\sigma$. The rate of increase of energy $= \frac{d}{dt}(V\sigma)$

$$= V\frac{d\sigma}{dt} \qquad \ldots (1)$$

Suppose, the source supplies energy at the rate of Q units per second.

Then, the rate of increase of energy $= Q - \frac{A\alpha\sigma v}{4}$ ... (2)

Equating (1) and (2)

$$V \cdot \frac{d\sigma}{dt} = Q - \frac{A\alpha\sigma v}{4} \quad \text{...(3)}$$

Take $\frac{A\alpha v}{4} = K$

$$\frac{K}{V} = \beta \text{ and } B = \frac{Q}{K} = \frac{4Q}{A\alpha v}.$$

From equation (3)

$$V \cdot \frac{d\sigma}{dt} = Q - K\sigma$$

$$\frac{d\sigma}{dt} = \frac{Q}{V} - \frac{K}{V} \cdot \sigma \quad \text{...(4)}$$

The general solution of this equation is

$$\sigma = B + be^{-\beta t} \quad \text{...(5)}$$

When $t = 0, \quad \sigma = 0$

$\therefore$ From equation (5)

$$0 = B + b$$

or

$$b = -B$$

$$\sigma = B - Be^{-\beta t}$$

$$\sigma = [B - e^{-\beta t}]$$

Substituting the values of B and β

$$\sigma = \frac{4Q}{A\alpha v}\left[1 - \frac{e^{-A\alpha v t}}{4v}\right] \quad \text{...(6)}$$

Equation (6) represents the rise of average sound energy per unit time from the time the source commences to produce sound.

The maximum value of average energy per unit volume

$$\sigma_{max} = \frac{4Q}{A\alpha v} \quad \text{...(7)}$$

Similarly, after the source ceases to emit sound, the decay of the average energy per unit volume is given by

$$\sigma = \frac{4Q}{A\alpha v} e^{-\frac{A\alpha v}{4V} \cdot t} \quad \text{...(8)}$$

$$\sigma = \sigma_{max}\, e^{\frac{-A\alpha v t}{4v}} \quad \text{...(9)}$$

The factor $\frac{A\alpha v}{4V}$ gives the reverberation time in the auditorium. If σ_0 represents the minimum audible intensity after a time t_1, then from equation (9)

$$\sigma_0 = \sigma_{max}\, e^{-\frac{A\alpha v}{4V} \cdot t_1} \quad \text{...(10)}$$

Here t_1 is the time interval between the cutting off the sound and the time at which intensity falls below the minimum audible level.

From equation (10)

$$\sigma_{max} = \sigma_0\, e^{+\frac{A\alpha v}{4V} \cdot t_1}$$

Taking logarithms

$$\log_e\left(\frac{\sigma_{max}}{\sigma_0}\right) = \frac{A\alpha v}{4V} \cdot t_1 \quad \text{...(11)}$$

Here α and σ_0 change with the frequency of sound.

For calculating the reverberation time, a standard steady intensity is required. Sabine took the value of $\frac{\sigma_{max}}{\sigma_0} = 10^6$.

From equation (11)

$$\log_e (10^6) = \frac{A\alpha v}{4V} \cdot t_1$$

$$2{\cdot}303 \times 6 = \frac{A\alpha v}{4V} \cdot t_1$$

Taking velocity of sound approximately at room temperature as 350 m/s,

$$2{\cdot}303 \times 6 = \frac{A\alpha \times 350}{4V} \cdot t_1$$

or

$$t_1 = \frac{2{\cdot}303 \times 24V}{350A\alpha}$$

$$t_1 = \frac{0{\cdot}158V}{A\alpha} \quad \text{...(12)}$$

In general

$$t_1 = \frac{0{\cdot}158V}{\Sigma A\alpha} \quad \text{...(13)}$$

In equation (12), the quantities are represented in M.K.S. units

[**Note.** In F.P.S. units, taking $v = 1120$ ft/s]

$$t_1 = \frac{0{\cdot}05V}{A\alpha}$$

Equation (12) represents the Sabine's reverberation time formula.

According to equation (12), the reverberation time is (*i*) directly proportional to the volume of the auditorium ; (*ii*) inversely proportional to the area of the celilings, area of the walls etc., and (*iii*) inversely proportional to the total absorption plus transmission through open surfaces. It has been experimentally found that the reverberation time of 1·03 seconds is most suitable for all rooms having approximately a volume of less than 350 cubic metres.

To decrease the reverberation time, the walls of the auditorium are usually covered with material having large absorption coefficients. The area of the surfaces of the walls is also increased in good cinema halls to decrease the reverberation time.

Example 10.5. *A hall of volume 5500* metre^3 *is found to have a reverberation time of 2·3* s. *The sound absorbing surface of the hall has an area of 750* m^2. *Calculate the average absorption coefficient.* (Bhagalpur, 1990)

Here, $V = 5500\ m^3$

$t = 2{\cdot}3s$

$A = 750\ m^2$

Absorption coefficient $= \alpha$

Reverberation time,

$$t = \frac{0{\cdot}158\ V}{A\alpha}$$

$$\alpha = \frac{0{\cdot}158V}{At}$$

$$= \frac{0{\cdot}158 \times 5500}{750 \times 2{\cdot}3}$$

$$= \mathbf{0{\cdot}504.}$$

10.17. Determination of Absorption Coefficient

A source of frequency 512 hertz is taken. The time of reverberation in a hall is determined using a chronograph, (*i*) without the absorbing materials in the hall, and (*ii*) with the absorbing materials in the hall. Let these two times be t_1 and t_2 respectively. According to Sabine's formula

$$t_1 = \frac{0{\cdot}158V}{\Sigma\alpha_1 A_1} \quad \ldots(1)$$

and

$$t_2 = \frac{0{\cdot}158V}{\Sigma\alpha_1 A_1 + \alpha_2 A_2} \quad \ldots(2)$$

Here α_2 is the absorption coefficient of the material of area A_2.

$$\therefore \quad \frac{1}{t_1} = \frac{\Sigma\alpha_1 A_1}{0{\cdot}158V} \quad \ldots(3)$$

$$\frac{1}{t_2} = \frac{(\Sigma\alpha_1 A_1) + \alpha_2 A_2}{0{\cdot}158V} \quad \ldots(4)$$

$$\therefore \quad \frac{1}{t_2} - \frac{1}{t_1} = \frac{\alpha_2 A_2}{0{\cdot}158V}$$

$$\therefore \quad \alpha_2 = \frac{0{\cdot}158V}{A_2}\left[\frac{t_1 - t_2}{t_1 t_2}\right] \quad \ldots(5)$$

From equation (5), knowing the values of t_1, t_2, A_2 and V, the value of α_2 can be calculated.

The absorption coefficients of some common materials calculated with a source of frequency 512 hertz are given in Table 10.1.

The absorption coefficient of a given material is different at different frequencies. It is generally higher at higher frequencies.

TABLE 10.1.

No.	*Material*	*Absorption Coefficient*
1.	Marble	0.01
2.	Common Plaster	0.03
3.	Glass	0.027
4.	Concrete	0.17
5.	Cork	0.23
6.	Asbestos	0.26
7.	Carpet	0.30
8.	Acoustic Plaster	0.30
9.	Acoustic Felt	0.45
10.	Fibre Board	0.50
11.	Heavy Curtains	0.50
12.	Hair or Felt	0.58
13.	Fibre Glass	0.75
14.	Perforated Cellulose Fibre Tiles	0.85

Example 10.6. *A room has dimensions* $6 \times 4 \times 5$ metres. *Calculate (i) the mean free path of the sound waves in the room, (ii) the number of reflections made per second by the sound wave with the walls of the room. Velocity of sound in air = 350* m/s.

(*i*) The mean free path of the sound waves is defined as the average distance travelled by a sound wave through air between any two consecutive encounters with the walls of the room.

Mean free path,

$$L = \frac{4\,[\text{Volume of the room}]}{\text{Total surface area}}$$

Here volume of the room $= 6 \times 4 \times 5 = 120\ m^3$

Total surface area $= 2[6 \times 4 + 4 \times 5 + 6 \times 5]$

$= 148\ m^2$

$$\therefore \quad L = \frac{4 \times 120}{148}$$

or $L = 3{\cdot}243$ **m**

(ii) The number of reflections made per second

$$N = \frac{\text{Velocity of sound}}{\text{Mean free path}}$$

$$N = \frac{350}{3{\cdot}243}$$

$N = 107{\cdot}9$.

Example 10.7. *The volume of a room is 600 m^3. The wall area of the room is 220 m^2, the floor area is 120 m^2 and the ceiling area is 120 m^2. The average sound absorption coefficient, (i) for the walls is 0·03; (ii) for the ceiling is 0·80, and (iii) for the floor is 0·06. Calculate the average sound absorption coefficient and the reverberation time.*

The average sound absorption coefficient,

$$\alpha = \frac{\Sigma \alpha A}{\Sigma A}$$

or

$$\alpha = \frac{\alpha_1 A_1 + \alpha_2 A_2 + \alpha_3 A_3}{A_1 + A_2 + A_3}$$

Here $A_1 = 220\ m^2, \quad \alpha_1 = 0{\cdot}03$

$A_2 = 120\ m^2, \quad \alpha_2 = 0{\cdot}80$

$A_3 = 120\ m^2, \quad \alpha_3 = 0{\cdot}06$

$$\therefore \quad \alpha = \frac{0{\cdot}03 \times 220 + 0{\cdot}8 \times 120 + 0{\cdot}06 \times 120}{220 + 120 + 120}$$

$$= \frac{109{\cdot}8}{460}$$

$= 0{\cdot}2389 = 0{\cdot}24$ (approx.)

The total sound absorption of the room

$= 0{\cdot}24 \times 460$

$= 110{\cdot}40$ **metric sabines.**

Reverberation time,

$$t = \frac{0{\cdot}158V}{\alpha \Sigma A} = \frac{0{\cdot}158 \times 600}{110{\cdot}4}$$

$= 0{\cdot}8588$ s

$= 0{\cdot}86\ s$ **(approx).**

10.18. Acoustic Intensity

Acoustic intensity of a sound wave is defined as the average power transmitted per unit area in the direction of propagation of the wave.

Instantaneous power per unit area is equal to the product of instantaneous pressure (P) and instantaneous particle velocity (v). Average power per unit area measures the acoustic intensity.

$\therefore$ Acoustic intensity,

$$I = \frac{1}{T}\int_0^T Pv\,dt$$

Here $$P = -\rho C^2 \left(\frac{du}{dx}\right)$$

But $$u = A\cos(\omega t - kx)$$

and $$v = \frac{du}{dt} = -\omega A \sin(\omega t - kx)$$

$$\frac{du}{dx} = -Ak\sin(\omega t - kx)$$

But $$kC = \omega$$

$\therefore$ $$P = -\rho CA\omega \sin(\omega t - kx)$$

Here C is the velocity of sound and ρ is the density of the medium.

$$\therefore I = \frac{1}{T}\int_0^T \left[-\rho CA\,\omega \sin(\omega t - kx)\right]\left[-\omega A \sin(\omega t - kx)\right] dt$$

$$I = \frac{\rho C\,\omega^2 A^2}{T}\int_0^T \sin(\omega t - kx)^2\,dt$$

$$= \frac{\rho C\,\omega^2 A^2}{T}\int_0^T (\cos^2 kx \sin^2 \omega t + \sin^2 kx \cos^2 \omega t - \tfrac{1}{2}\sin 2\omega t \sin 2kx)\,dt$$

$$= \frac{\rho C\omega^2 A^2}{T}\left(\frac{T}{2}\right)$$

$$I = \tfrac{1}{2}\rho C\omega^2 A^2 \qquad \ldots (1)$$

Since $$P = -\rho CA\omega \sin(\omega t - kx)$$

$$P_{max} = -\rho CA\omega$$

Root mean square value of pressure,

$$P_{RMS} = \frac{P_{max}}{\sqrt{2}}$$

From equation (1)

$$\therefore \quad I = \frac{\rho^2 C^2 A^2 \omega^2}{2\rho C}$$

$$I = \frac{(P_{max})^2}{2\rho C}$$

$$I = \frac{\left(\frac{P_{max}}{\sqrt{2}}\right)^2}{\rho C}$$

or $$I = \frac{P^2_{RMS}}{\rho C}$$

Example 10.8. *Calculate the amplitude of the displacement wave, given that for a frequency of 400* hertz, *the feeblest sound that could be heard corresponds to a pressure amplitude of* 8×10^{-5} newton/m^2. *The density of air is 1·29* kg/m^3 *and velocity of sound in air is 345* m/s.

Here $$A = \frac{P_{max}}{\rho C \omega}$$

Here $P_{max} = 8 \times 10^{-5}$ newton/m^2

$\rho = 1{\cdot}29$ kg/m^3

$C = 345$ m/s

$\omega = 2\pi f = 2\pi \times 400$

$= 800\,\pi$ radians/s

$$\therefore \quad A = \frac{8 \times 10^{-5}}{1{\cdot}29 \times 345 \times 800\,\pi}$$

$$A = 7{\cdot}15 \times 10^{-11} \text{ m.}$$

Note. It may be noted that this amplitude is of the order of molecular dimensions.

Example 10.9. *Compare the acoustic intensities in air and in water for the same acoustic pressure.*

Acoustic intensity,

$$I = \frac{P^2 RMS}{\rho C}$$

$$\frac{I_{air}}{I_{water}} = \frac{\left(\frac{P^2_{RMS}}{\rho C}\right)_{air}}{\left(\frac{P^2_{RMS}}{\rho C}\right)_{water}}$$

But $$(P_{RMS})_{air} = (P_{RMS})_{water}$$

$$\therefore \quad \frac{I_{air}}{I_{water}} = \frac{(\rho C)_{water}}{(\rho C)_{air}}$$

ρ for air $= 1{\cdot}29$ kg/m^3

C for air $= 350$ m/s

ρ for water $= 1000$ kg/m^3

C for water $= 1470$ m/s

$$\therefore \quad \frac{I_{air}}{I_{water}} = \frac{1000 \times 1470}{1{\cdot}29 \times 350}$$

$$= 3256.$$

This shows that for the same acoustic pressure, the acoustic intensity in air is 3256 times that in water.

Example 10.10. *Compare the acoustic intensities in air and in water for the same frequency and amplitude.*

$$\frac{I_{air}}{I_{water}} = \frac{\frac{1}{2}(\rho C A^2 \omega^2)_{air}}{\frac{1}{2}(\rho C A^2 \omega^2)_{water}}$$

Here $$(A\omega)_{air} = (A\omega)_{water}$$

$$\therefore \quad \frac{I_{air}}{I_{water}} = \frac{(\rho C)_{air}}{(\rho C)_{water}}$$

ρ for air $= 1{\cdot}29$ kg/m^3

ρ for water $= 1000$ kg/m^3

C for air $= 350$ m/s

C for water $= 1470$ m/s

$$\therefore \quad \frac{I_{air}}{I_{water}} = \frac{1{\cdot}29 \times 350}{1000 \times 1470}$$

$$= \frac{1}{3256}$$

or $$I_{water} = 3256\ I_{air}.$$

This shows that for the same frequency and amplitude, the acoustic intensity in water is 3256 times that in air.

Example 10.11. *A plane acoustic wave in air has an intensity of 21* W/m^2. *This wave strikes a wall at right angles to its surface. The area of the wall is 50* m^2. *Calculate the force exerted on the wall. Velocity of the wave in air is 350* m/s.

Acoustic intensity is the power per unit area.

$$I = \frac{\text{Power}}{\text{Area}} = \left(\frac{\text{Force}}{\text{Area}}\right) \times \text{Velocity}$$

or $I = PC$ watts/m^2

$$\therefore \quad P = \frac{I}{C}$$

Here $I = 21$ watts/m^2

and $C = 350$ m/s

$$\therefore \quad P = \frac{21}{350}$$

or $P = 0{\cdot}06$ newton/m^2

Area = 50 m^2

$\therefore$ Force = Pressure × Area

$= 0{\cdot}06 \times 50$

= 3 newtons.

10.19. Acoustic Measurements

In acoustic measurements, it is necessary to use the logarithmic scale for measuring acoustic intensity, acoustic power and acoustic pressure. The logarithmic scale is called the decibel scale, abbreviated as db. The decibel scale always refers to the quantity to be measured logarithmically to some standard reference. The decibel is a dimensionless quantity because it refers to the ratio of two similar quantities.

The decibel value is equal to 10 times the logarithm of the ratio of the quantities measured, to the base 10.

1 bel = 10 decibels = 10 db.

(1) Acoustic intensity level

Acoustic intensity level is written as

$$IL = 10 \log \left(\frac{I}{I_0}\right) \text{ db reference to } I_0 \text{ watts/m}^2$$

The standard acoustic intensity reference,

$$I_0 = 10^{-12} \text{ watts/m}^2$$

$$\therefore \quad IL = 10 \log \left(\frac{I}{10^{-12}}\right) \text{ db reference } 10^{-12} \text{ watt/m}^2$$

$$= 10 [\log I + \log 10^{12}] \text{ db}$$

$$= [10 \log I + \log 120] \text{ db}$$

(2) Acoustic pressure level

Acoustic pressure level is written as

$$PL = 10 \log_{10} \left(\frac{P}{P_0}\right)^2 \text{ db}$$

reference to a pressure of P_0 newtons/m^2.

$$\therefore \quad PL = 20 \log_{10} \left(\frac{P}{P_0}\right) \text{ db}$$

The standard acoustic pressure reference,

$$P_0 = 2 \times 10^{-5} \text{ newton/m}^2$$

$$\therefore \quad PL = 20 \log \left(\frac{P}{2 \times 10^{-5}}\right) \text{ db}$$

reference to a pressure of 2×10^{-5} newton/m^2

$$\therefore \quad PL = 20 [\log P + \log 5 \times 10^4]$$

$$= [20 \log P + 20 \log 5 \times 10^4]$$

$$= [20 \log P + 20 \times 4{\cdot}6990]$$

$$= [20 \log P + 94] \text{ db} \quad \text{(approx)}.$$

Example 10.12. *The sound from a drill gives a noise level of 90* decibels *at a point at a short distance from it. What is the noise level at this point if four such drills are working simultaneously at the same distance from the point?*

Here, $\frac{I_2}{I_1} = 4$

and $IL_1 = 90$ db

$$IL = 10 \log \left(\frac{I}{I_0}\right) \text{ db}$$

In the first case $IL_1 = 10 \log \left(\frac{I_1}{I_0}\right)$

In the second case $IL_2 = 10 \log \left(\frac{I_2}{I_0}\right)$

Increase in noise level $IL_2 - IL_1 = 10\left[\log\left(\frac{I_2}{I_0}\right) - \log\left(\frac{I_1}{I_0}\right)\right]$

$$= 10 \log\left(\frac{I_2}{I_1}\right)$$

But $\frac{I_2}{I_1} = 4$

$\therefore$ $IL_2 - IL_1 = 10 \log_{10} 4$

$= 10 \times 0{\cdot}6021 = 6{\cdot}021$ db

$\therefore$ $IL_2 = IL_1 + 6{\cdot}021 = 90 + 6{\cdot}021 = \mathbf{96{\cdot}021\ db.}$

Example 10.13. *Calculate the increase in the acoustic intensity level when the sound is doubled.*

$$IL = 10 \log\left(\frac{I}{I_0}\right) \text{ db}$$

In the first case

$$IL_1 = 10 \log\left(\frac{I_1}{I_0}\right)$$

In the second case

$$IL_2 = 10 \log\left(\frac{I_2}{I_0}\right)$$

Increase in acoustic intensity level,

$$IL_2 - IL_1 = 10\left[\log\left(\frac{I_2}{I_0}\right) - \log\left(\frac{I_1}{I_0}\right)\right]$$

$$= 10 \log\left(\frac{I_2}{I_1}\right)$$

But $\frac{I_2}{I_1} = 2$

$\therefore$ $IL_2 - IL_1 = 10 \log_{10} 2 = 10 \times 0{\cdot}3010 = \mathbf{3{\cdot}01\ db.}$

Example 10.14. *Calculate the increase in sound pressure level when the sound pressure is doubled.*

$$PL = 10 \log\left(\frac{P}{P_0}\right)^2$$

$$= 20 \log\left(\frac{P}{P_0}\right) \text{ db}$$

In the first case

$$PL_1 = 20 \log\left(\frac{P_1}{P_0}\right) \text{ db}$$

In the second case

$$PL_2 = 20 \log\left(\frac{P_2}{P_0}\right) \text{ db}$$

Increase in sound pressure level,

$$PL_2 - PL_1 = 20\left[\log\left(\frac{P_2}{P_0}\right) - \log\left(\frac{P_1}{P_0}\right)\right]$$

$$= 20 \log\left(\frac{P_2}{P_1}\right)$$

But $\frac{P_2}{P_1} = 2$

$\therefore$ $PL_2 - PL_1 = 20 \log_{10} 2 = 20 \times 0{\cdot}3010 = \mathbf{6{\cdot}02\ db.}$

Example 10.15. *Calculate (i) the acoustic intensity, (ii) the acoustic pressure of a plane acoustic wave in air of intensity level of 100* decibels *reference to* 10^{-12} W/m^2

(1) $IL = 10 \log\left(\frac{I}{I_0}\right)$ db

Here $I_0 = 10^{-12}$ watt/m^2

$\therefore$ $IL = 10 \log\left(\frac{I}{10^{-12}}\right)$ db

reference to 10^{-12} watt/m^2

$= 10\ [\log I + \log 10^{12}]$ db

$= 10\ [\log I + 12]$ db

$IL = [10 \log I + 120]$ db ... (1)

But $IL = 100$ db ... (2)

From (1) and (2)

$\therefore$ $100 = 10 \log I + 120$

or $10 \log I = -20$

$\log I = -2$

$I = \mathbf{10^{-2}\ watt/m^2}$.

(2) Acoustic pressure

Acoustic intensity, $I = \frac{P^2}{\rho C}$

or $P = \sqrt{I \rho C}$

Here $I = 10^{-2}$ watt/m^2

Density of air, $\rho = 1{\cdot}29$ kg/m^3

Velocity of sound in air,

$C = 350$ m/s

$\therefore$ $P = \sqrt{10^{-2} \times 1{\cdot}29 \times 350}$

or $P = \mathbf{2{\cdot}124}$ **newtons/m^2.**

Example 10.16. *Calculate the acoustic pressure of a plane acoustic wave of pressure level of 100* decibels *reference to a standard acoustic pressure level of* 2×10^{-5} newton/m^2.

Here $PL = 10 \log \left(\frac{P}{P_0}\right)^2$ db

reference to a pressure P_0 newton/m^2

$PL = 20 \log \left(\frac{P}{P_0}\right)$ db

Here $P_0 = 2 \times 10^{-5}$ newton/m^2

$\therefore$ $PL = 2 \log \left(\frac{P}{2 \times 10^{-5}}\right)$ db

reference to a pressure of 2×10^{-5} newton/m^2

$PL = 20\ [\log P + \log 5 \times 10^4]$ db

$PL = 20\ [\log P + 4{\cdot}6990]$ db

or $PL = [20 \log P + 94]$ db

But $PL = 100$ db

$\therefore$ $100 = 20 \log P + 94$

$20 \log P = 6$

$\log P = 0{\cdot}3$

$P = \mathbf{1{\cdot}995}$ **newtons / m^2**

Example 10.17. *Calculate the acoustic intensity level in each case, at a distance of 10* metres *from a source which radiates energy at the rate of 3·14* watts. *Using reference intensities of (1) 100* watts/m^2, *(2) 1* watt/m^2, *(3)* 10^{-12} watts/m^2.

Power radiated $= W = 3{\cdot}14$ watts

Distance $= 10$ m

Intensity $I = \frac{W}{\text{Area}}$

$= \frac{3{\cdot}14}{4\pi r^2}$

$I = \frac{1}{4r^2}$

or $I = \frac{1}{400} = 0{\cdot}0025$ watt/m^2

(1) $IL = 10 \log \left(\frac{I}{I_0}\right)$ db

Here $I_0 = 100$ watts/m^2

$I = 0{\cdot}0025$ watt/m^2

$\therefore$ $IL = 10 \log \left(\frac{0{\cdot}0025}{100}\right)$

$IL = \mathbf{-46{\cdot}021}$ **db reference to 100 watts/m^2**

(2) Here $I_0 = 1$ watt/m^2

$I = 0{\cdot}0025$ watt/m^2

$\therefore$ $IL = 10 \log \left(\frac{0{\cdot}0025}{1}\right)$ db

$IL = 10 \log (25 \times 10^{-4})$ db

$IL = \mathbf{-26{\cdot}021}$ **db reference to 1 watt/m^2**

(3) Here, $I_0 = 10^{-12}$ watt/m^2

$I = 0{\cdot}0025$ watt/m^2

$IL = 10 \log \left(\frac{I}{I_0}\right)$ db

$= 10 \log \left(\frac{0{\cdot}0025}{10^{-12}}\right)$ db

$= 10 \log (25 \times 10^8)$ db

$IL = \mathbf{93{\cdot}979}$ **db reference to 10^{-12} watt/m^2.**

Example 10.18. *An air-conditioning unit operates at a sound intensity level of 75* db. *If it is operated in a room with an existing sound intensity level of 70* db, *what will be the resultant intensity level.*

In the first case,

$IL_1 = 10 \log \left(\frac{I_1}{I_0}\right)$ db

But $IL_1 = 75$ db

$$\therefore \quad 75 = 10 \log \left(\frac{I_1}{I_0}\right)$$

or $$\text{antilog } 7{\cdot}5 = \left(\frac{I_1}{I_0}\right)$$

or $$I_1 = I_0 \ (\text{antilog } 7{\cdot}5)$$

$$I_1 = 6{\cdot}99 \times 10^7 \ I_0 \text{ watts/m}^2 \qquad \ldots (1)$$

In the second case,

$$IL_2 = 10 \log \left(\frac{I_2}{I_0}\right) \text{ db}$$

But $$IL_2 = 70 \text{ db}$$

$$\therefore \quad 70 = 10 \log \left(\frac{I_2}{I_0}\right)$$

or $$I_2 = I_0 \ (\text{antilog } 7)$$

$$I_2 = 10^7 \ I_0 \text{ watts/m}^2 \qquad \ldots (2)$$

The resultant sound intensity,

$$I = I_1 + I_2$$

$$I = 6{\cdot}99 \times 10^7 \ I_0 + 10^7 \ I_0$$

$$I = (7{\cdot}99 \times 10^7) \ I_0 \text{ watts/m}^2$$

The resultant intensity level

$$IL = 10 \log \left(\frac{I}{I_0}\right) \text{ db}$$

$$IL = 10 \log \left(\frac{7{\cdot}99 \times 10^7 \ I_0}{I_0}\right) \text{db}$$

$$IL = 10 \log 7{\cdot}99 \times 10^7$$

or $$IL = \mathbf{79{\cdot}025 \text{ db}}$$

The resultant intensity level is **79·025 decibles.**

Example 10.19. *Two sources of sound A and B emit sound waves of different frequencies. The two sound pressure levels as recorded at a place C are 80* db *and 75* db *respectively. Calculate the resultant sound pressure level at C due to the combined effect.*

In the first case, $$PL_1 = 20 \log \left(\frac{P_1}{P_0}\right) = 80 \text{ db}$$

or $$\log \left(\frac{P_1}{P_0}\right) = 4$$

$$P_1 = P_0 \text{ (antilog 4)}$$

$$P_1 = 10^4 \ P_0 \text{ newtons/m}^2 \qquad \ldots (1)$$

In the second case,

$$PL_2 = 20 \log \left(\frac{P_2}{P_0}\right) = 75 \text{ db}$$

or $$\log \left(\frac{P_2}{P_0}\right) = 3{\cdot}75$$

$$P_2 = P_0 \text{ (antilog } 3{\cdot}75)$$

$$= 5{\cdot}6 \times 10^3 \ P_0 \text{ newtons/m}^2 \qquad \ldots (2)$$

The total sound pressure at *C* due to the combined effect,

$$P = P_1 + P_2$$

$$= (10^4 + 5{\cdot}6 \times 10^3) \ P_0$$

$$P = 15{\cdot}6 \times 10^3 \ P_0 \text{ newtons/m}^2 \qquad \ldots (3)$$

or $$\frac{P}{P_0} = 15{\cdot}6 \times 10^3 \qquad \ldots (4)$$

The combined sound pressure level,

$$PL = 20 \log \left(\frac{P}{P_0}\right) \text{ db}$$

$$= 20 \log (15{\cdot}6 \times 10^3) \text{ db}$$

$$= 20 \ (4{\cdot}195) \text{ db}$$

$$= \mathbf{83{\cdot}90 \text{ db.}}$$

Thus the total sound pressure level at *C* is **83·90 decibles.**

Example 10.20. *A light source 1* kWh *is radiating energy uniformly. Determine the intensity of the electric field at a distance of one meter from the source.* (IAS, 1988)

$$E = 1 \text{ kWh}$$

It means energy radiated by the source in one hour = 1 kWh

Energy radiated in one second $= 1 \text{ kWs}$

$$= 10^3 \text{ J/s}$$

$$R = 1 \text{ m}$$

Intensity,

$$I = \frac{E}{4\pi R^2}$$

$$= \frac{10^3}{4\pi \times 1}$$

79·57 watts/m^2 (or J/m^2 -s).

Example 10.21. *A source of sound emits energy in all di ections at a rate of 0·638 J/s. Find the intensity level at a distance of 11·5 m reckoned above a datum of $1·2 \times 10^{4}$ W/cm^2.* (Bhagalpur, 1990)

Here power radiated $= 0·638$ J/s

$P = 0·638$ watts

Distance $r = 11·5$ m

Intensity, $$I = \frac{P}{\text{Area}}$$

$$= \frac{0·638}{4\pi r^2}$$

$$= \frac{0·638}{4\pi (11·5)^2}$$

$$= 3·84 \times 10^{-4} \text{ watts/m}^2$$

Also $I_0 = 1·2 \times 10^{4}$ watts /cm^2

$= 1·2 \times 10^{8}$ watts/m^2

Intensity level $$IL = 10 \log \left(\frac{I}{I_o}\right)$$

$$IL = 10 \log \left(\frac{3·84 \times 10^{-4}}{1·2 \times 10^{8}}\right) \text{db}$$

$$= 10 \log (3·2 \times 10^{-12}) \text{ db}$$

$$= \mathbf{-114·95 \text{ db.}}$$

Example 10.22. *The noise from an aeroplane engine 100* m *from an observer is 40* db *in intensity. What will be intensity when the aeroplane flies overhead at an altitude of 2* km? (IAS)

$$I_1 = \frac{P}{4\pi R_1^2}$$

$$I_2 = \frac{P}{4\pi R_2^2}$$

$$\frac{I_2}{I_1} = \frac{R_1^2}{R_2^2}$$

Here $R_1 = 100$ m, $R_2 = 2000$ m

$$\frac{I_2}{I_1} = \frac{(100)^2}{(2000)^2} = \frac{1}{400}$$

$$\left(\frac{I_1}{I_2}\right) = 400$$

$$IL_1 = 10 \log \left(\frac{I_1}{I_0}\right)$$

$$IL_2 = 10 \log \left(\frac{I_2}{I_0}\right)$$

Decrease in intensity level

$$IL_1 - IL_2 = 10 \left[\log \left(\frac{I_1}{I_0}\right) - \log \left(\frac{I_2}{I_0}\right)\right]$$

$$= 10 \log \left(\frac{I_1}{I_0}\right)$$

$$= 10 \log_{10} (400)$$

$$= 10 \times 2·6021$$

$$= 26·021 \text{ db}$$

$$IL_2 = IL_1 - 26·021$$

$$= 40 - 26·021$$

$$= \mathbf{13·979 \text{ db.}}$$

10.20. Factors Affecting the Acoustics of Buildings

Reverberation is one of the important single factors that affects the acoustics of a room or a hall. Besides reverberations there are also other factors like loudness, focussing, echelon effect, extraneous noise, resonance etc.

(1) **Loudness.** The speech of a person in a hall can be heard by an audience consisting of about 1000 persons. However, to ensure uniform distribution of sound intensity in the hall, electrically amplified loud speakers are used. These speakers are kept at different places in the auditorium and are located generally at a height higher than the speaker's head. Amplifiers, however, make the low frequency tones more prominent and hence the amplification has to be kept low. The presence of low artificial ceilings improves the audibility in general.

(2) **Focussing** The presence of cylindrical or spherical surfaces on the walls or the ceiling gives rise to undesirable focussing.

In Fig. 10.23 the observer at O receives sound from the speaker along the direct path SO. The observer also receives the sound waves after reflection from the ceiling. Thus the intensity of sound at O is comparatively higher than other positions in the auditorium. It may also happen that the direct and the reflected waves are in opposite phase. This results in minimum intensity of sound at O. Further, the direct and the reflected waves may form a stationary

wave pattern. This causes uneven distribution of sound intensity.

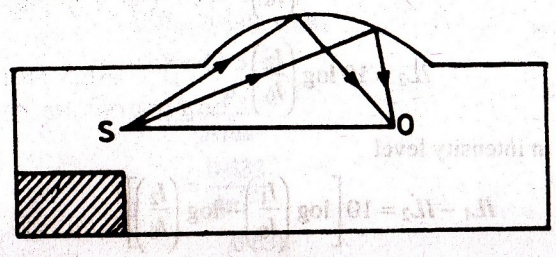


Fig. 10.23.

(3) **Echelon effect.** If there is regular structure similar to a flight of stairs or a set of railings in the hall, the sound produced in front of such a structure may produce a *musical note* due to regular successive echoes of sound reaching the observer. Such an effect is called *echelon effect.* If the frequency of this note is within the audible range, the listener will hear only this note prominently. To avoid echelon effect, the stair cases are covered with carpets to avoid reflection of sound.

(4) **Extraneous noise.** The extraneous noise may be due to (1) sound received from outside the room (2) the sound produced by fans etc., inside the auditorium. The external sound cannot be completely eliminated but can be minimised by using double or triple windows and doors. Proper attention must also be paid to maximum permissible speed of fans and the rate of air circulation in the room. The air conditioning pipes should be covered with cork and insulated acoustically from the main building.

(5) **Resonance.** The acoustics of a building may also be affected by resonance. If there is resonance for any audio frequency note, the intensity of the note will be entirely different from the intensity desired. In halls of large size, the resonance frequency is much below the audible limit and harmful effects due to resonance will not be present.

10.21. Sound Distribution in an Auditorium

The design of an auditorium requires smooth decay and growth of sound. In an auditorium, the sound must be distributed or diffused over the whole area. To ensure these factors, acoustic treatment is given *viz.*, scattering effect of objects, irregularities on the wall surfaces, fixing absorptive material on the walls etc. In Fig. 10.24, the first reflection of sound waves at different positions of the ceiling is shown. It is clear from the figure that the reflected sound is distributed evenly in the whole auditorium *viz.*, the main floor and the balcony. This design enables an even distribution of sound intensity.

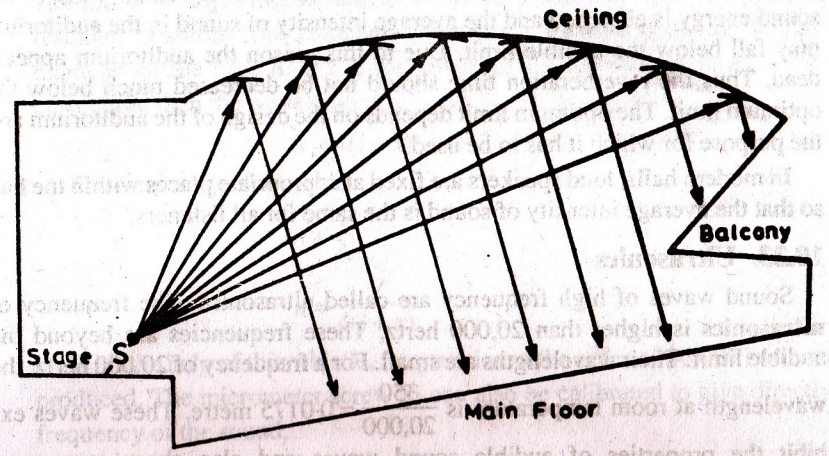


Fig. 10.24.

10.22. Requisites for Good Acoustics

The reverberation of sound in an auditorium is due to multiple reflections taking place at various surfaces present within the auditorium. The acoustics of an auditorium can be improved by using the surfaces with high absorption coefficient. This will reduce the reverberation time below the optimum value. This can be achieved as follows :

(1) By hanging heavy curtains.

(2) By hanging pictures and maps.

(3) By having a few open windows.

(4) By having good audience. Each person is equivalent to about 0.50 m^2 area of an open window.

(5) The curved walls and corners bounded by two walls should be avoided. This is done to avoid (*i*) concentration of sound and (*ii*) dead spaces.

(6) Upholstered seats should be provided so that the absorption is approximately the same with or without the audience.

(7) The walls and the ceiling should be covered with the materials having high absorption coefficient *i.e.*, with perforated card boards, felt, asbestos, fibre glass etc.

(8) The walls should be engraved and made rough with decorative materials to increase the absorption.

Reverberation Time–Optimum Value

It is important that the reverberation time should not be very much lower than the optimum value. If the time of reverberation is very small, most of the sound energy is absorbed and the average intensity of sound in the auditorium may fall below the audible limit. Due to this reason the auditorium appears dead. Thus the reverberation time should not be decreased much below the optimum limit. The optimum limit depends on the design of the auditorium and the purpose for which it has to be used.

In modern halls, loud speakers are fixed at appropriate places within the hall so that the average intensity of sound is the same for all listeners.

10.23. Ultrasonics

Sound waves of high frequency are called ultrasonics. The frequency of **ultrasonics** is higher than 20,000 hertz. These frequencies are beyond the audible limit. Their wavelengths are small. For a frequency of 20,000 hertz, the wavelength at room temperature is $\frac{350}{20{,}000} = 0{\cdot}0175$ metre. These waves exhibit the properties of audible sound waves and also show some new phenomena. The sound waves of frequency lower than the audible limit are called **infra-sonics.** Supersonics refers to the **velocities** higher than the velocity of sound *i.e.*, more than 1200 km/hour.

Ultrasonic waves have a large number of practical applications. Ultrasonic waves can be produced by the following methods : (*i*) Galton whistle, (*ii*) Magnetostriction oscillator and (*iii*) Piezo-electric oscillator.

10.24. Production of Ultrasonic Waves

1. **Galton whistle.** Galton whistle works on the principle of organ pipe. It consists of a closed end air column *A* whose length can be adjusted with the help of a movable piston. The piston *P* can be moved to the desired position with the help of a screw S_1. The open end of the pipe *A* is fitted with a lip *L*. The position of the pipe *C* can be adjusted with the help of the screw S_2. The gap between the ends of *A* and *C* can be adjusted with the help of the screw S_2. (Fig. 10.25).

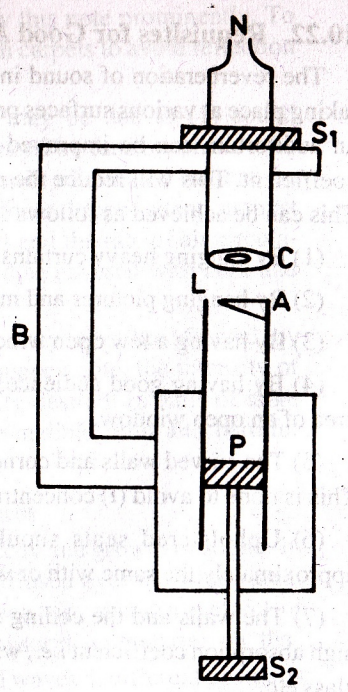


Fig. 10.25.

An air blast is blown through the nozzle *N* at the top. The blast of air coming out of *C* strikes against the lip *L* and the column of air in the pipe is set into vibration. By adjusting the length of the air column in *A*, it is brought to the resonant position. The resonant frequency will depend on the length and diameter of the pipe *A*. If l is the length of the air column in *A*, x the end correction, then the wavelength

$$\lambda = 4(l + x)$$

The frequency of sound is,

$$n = \frac{V}{\lambda}$$

$$n = \frac{V}{4(l + x)}$$

With the help of this whistle, frequencies of the order of 30,000 hertz can be produced. The micrometer screw S_1 can also be calibrated to give directly the frequency of the sound.

2. **Magnetostriction oscillator.** Ultrasonic waves can be produced by using the principle of magnetrostriction. According to this principle, if a ferromagnetic material in the form of a bar (like iron or nickel), is subjected to an alternating magnetic field, the bar expands and contracts in length alternately. The frequency of contraction or expansion is twice the frequency of the alternating magnetic field. The alternating magnetic field is produced with the help of an oscillatory circuit. Due to the longitudinal contraction and expansion of the bar, longitudinal compressional waves are produced in the medium surrounding the bar. Frequencies ranging from a few hundred hertz to about 300,000 hertz can be produced with this arrangement.

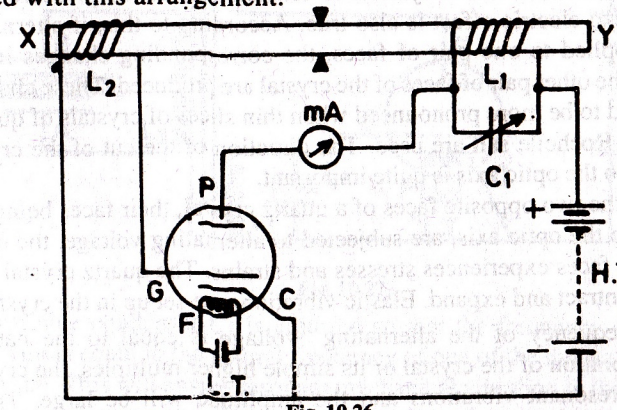


Fig. 10.26.

The experimental arrangement is shown in Fig. 10.26. XY is a bar of ferromagnetic material say of iron or nickel. The bar is clamped in the middle. L_1 and L_2 are two coils surrounding the bar PQ. L_1 and C_1 are connected in parallel and the combination is connected in the plate circuit. L_2 is connected between the grid and the cathode. The milliammeter is connected in the plate circuit. The values of L_1 and C_1 determine the frequency of the oscillatory circuit.

Initially the bar is magnetised by passing direct current. The condenser C_1 is adjusted so that the frequency of the oscillatory circuit is the same as the natural frequency of longitudinal vibrations of the bar. In this case, the oscillations are maintained by the coupling effect between the coils L_1 and L_2. Any change in the plate current brings about a change in magnetisation and consequently the length of the rod changes. This gives rise to the change in flux in the coil L_2 in the grid circuit. An induced e.m.f. is produced in the coil L_2 and this e.m.f. is amplified by the triode valve and reacts back on the coil L_1. In this way, the oscillations are maintained and the amplitude of oscillations will be large.

At the start, the milliammeter shows alternation when the oscillations are set up. The condenser C_1 is adjusted to the position where these alternations are maximum. In this position, the vibrations are most intense.

The frequency of the ultrasonic waves produced by this method will depend on the length, density and elasticity of the material of the bar.

3. **Piezo-electric oscillator.** In 1880, the Curie brothers, discovered that certain asymmetric crystals exhibit *Piezo-electric effect.* In this effect, if one pair of opposite faces of a crystal is subjected to pressure, the other pair of opposite faces develop opposite electric charges. The sign of the charges changes when the faces are subjected to tension instead of pressure. The converse of Piezo-electric effect is also true. According to this, if alternating voltages are applied to one pair of faces, the corresponding changes in the dimensions of the other pair of faces of the crystal are produced. These changes have been found to be more pronounced when thin slices of crystals of quartz, tourmaline and Rochelle salt are used. The direction of the cut of the crystal with reference to the optic axis is quite important.

Thus, when the two opposite faces of a quartz crystal, their faces being cut perpendicular to the optic axis, are subjected to alternating voltage, the other pair of opposite faces experiences stresses and strains. The quartz crystal will continuously contract and expand. Elastic vibrations are set up in the crystal.

When the frequency of the alternating voltage is equal to the natural frequency of vibration of the crystal or its simple higher multiples, the crystal is thrown into resonant vibrations and the amplitude will be large. These vibrations are longitudinal in nature. The frequency of vibration is

$$n = \frac{p}{2l}\sqrt{\frac{Y}{\rho}} \quad \ldots (1)$$

Here $p = 1, 2, 3, \ldots$ etc.

Y is the elasticity and ρ is the density of the crystal.

The velocity of longitudinal waves in the crystal,

$$v = \sqrt{\frac{Y}{\rho}} \quad \ldots (2)$$

The value of $v = 5{\cdot}5 \times 10^3$ m/s for quartz.

For a crystal of length 0·05 m, the frequency for the first mode of vibration will be

$$\frac{1 \times 5{\cdot}5 \times 10^3}{2 \times 0{\cdot}05} = 5{\cdot}5 \times 10^4 \text{ hertz.}$$

The other modes of frequency are simple integral multiples of $5{\cdot}5 \times 10^4$ hertz

For experimental arrangement, the circuit diagram is as shown in Fig. 10.27. Q is a thin slice of quartz crystal cut with its opposite faces perpendicular to the optic axis. The crystal is placed between two metal plates A and B. The plates A and B are connected to the coil L_3. Coils L_1, L_2 and L_3 are inductively coupled. Coil L_2 is connected in the plate circuit. The tank circuit L_1 and C_1 is connected between the grid and the cathode.

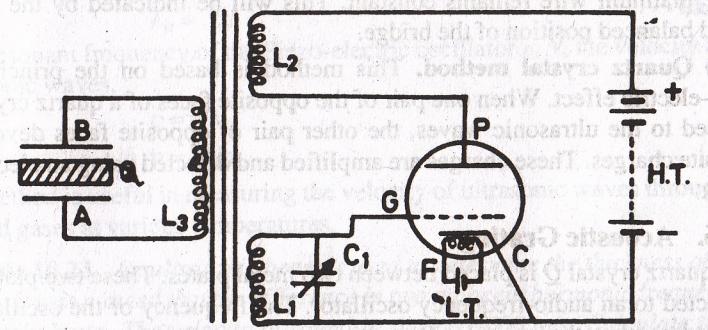


Fig. 10.27.

The variable condenser C_1 is adjusted so that the frequency of the oscillatory circuit is equal to the natural frequency of one of the modes of vibration of the crystal. This will ensure resonant mechanical vibration in the crystal due to the linear expansion and contraction. Ultrasonic frequencies as high as 5×10^8 hertz can be obtained with this arrangement.

10.25. Detection of Ultrasonic Waves

Ultrasonic waves propagated through a medium can be detected in a number of ways. Some of the methods employed are as follows :

(1) **Kundt's tube method.** Ultrasonic waves can be detected with the help of Kundt's tube. At the nodes, lycopodium powder collects in the form of heaps. The average distance between two adjacent heaps is equal to half the wavelength. This method cannot be used if the wavelength of ultrasonic waves is very small *i.e.*, less than a few millimeters. In the case of a liquid medium, instead of lycopodium powder, powdered coke is used to detect the position of nodes.

(2) **Sensitive flame method.** A narrow sensitive flame is moved along the medium. At the position of the antinode, the flame is steady. At the position of the node the flame flickers because there is change in pressure. In this way positions of nodes and antinodes can be found in a medium. The average distance between two adjacent nodes is equal to half the wavelength. If the value of the frequency of the ultrasonic wave is known, the velocity of the ultrasonic wave through the medium can be calculated.

(3) **Thermal detectors.** This is the most commonly used method of detection of ultrasonic waves. In this method, a fine platinum wire is used. This wire is moved through the medium. At the position of nodes, due to alternate compressions and rarefactions, adiabatic changes in temperature take place. The resistance of the platinum wire changes with respect to time. This can be detected with the help of Callendar and Griffith's bridge arrangement. At the position of the antinodes, the temperature remains constant and the resistance of the platinum wire remains constant. This will be indicated by the undisturbed balanced position of the bridge.

(4) **Quartz crystal method.** This method is based on the principle of Piezo-electric effect. When one pair of the opposite faces of a quartz crystal is exposed to the ultrasonic waves, the other pair of opposite faces developed opposite charges. These charges are amplified and detected using an electronic circuit.

10.26. Acoustic Grating

A quartz crystal Q is placed between two metal plates. These two plates are connected to an audio-frequency oscillator. The frequency of the oscillator is adjusted so that the crystal vibrates in resonance. Due to the longitudinal compression and expansion of the crystal, ultrasonic waves are produced. These waves are reflected by the reflector R (Fig. 10.28). Due to the superposition of the direct and the reflected waves, longitudinal stationary waves are produced in the medium. These waves give rise to fixed positions of nodal and antinodal planes. At the nodal planes, the density is maximum and at the antinodal planes, the density is minimum. This arrangement behaves like a diffraction grating and is called acoustic grating.

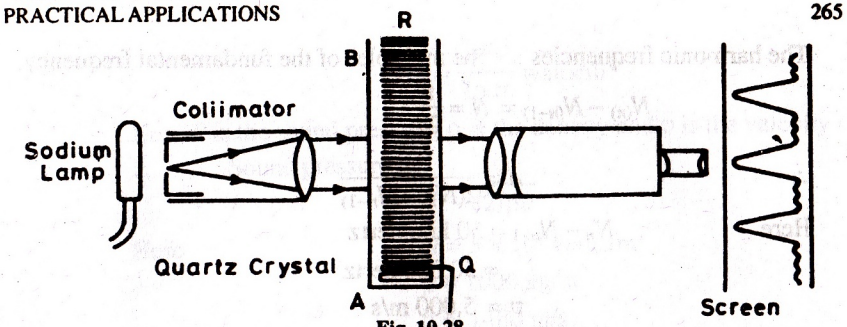


Fig. 10.28.

This arrangement is suitable for the measurement of wavelength and velocity of ultrasonic waves in a medium. In Fig. 10.28, the acoustic grating is mounted on the prism table of a spectrometer. A parallel beam of sodium light from the collimator is normally incident on the acoustic grating. The diffracted beam of light is viewed through the telescope. The diffraction pattern consists of a central maximum and principal maxima on either side. The directions of the principal maxima are given by the expression

$$d \sin \theta_n = n\lambda \quad \ldots (1)$$

Here n=1, 2, 3, etc. and θ_n is the angle of diffraction for the nth order. λ is the wavelength of sodium light and d is the distance between two adjacent nodal or antinodal planes. Knowing n, θ_n and λ, the value of d can be calculated from equation (1). If λ_a is the wavelength of the ultrasonic waves through the medium,

$$d = \frac{\lambda_a}{2}$$

or

$$\lambda_a = 2d \quad \ldots (2)$$

If the resonant frequency of the Piezo-electric oscillator is N, the velocity of the ultrasonic waves,

$$v = N\lambda_a$$

$$v = 2Nd \quad \ldots (3)$$

This method is useful in measuring the velocity of ultrasonic waves through liquids and gases at various temperatures.

Example 10.23. *An ultrasonic beam is used to determine the thickness of a steel plate. It was noticed that the difference in two adjacent harmonic frequencies is 50* kilo hertz. *The velocity of sound in steel is 5000* m/s. *Calculate the thickness of the steel plate.*

The velocity of ultrasonic waves,

$$v = 2N\,d$$

Here N is the fundamental frequency and d is the thickness.

$$\therefore \quad N = \frac{v}{2d}$$

The harmonic frequencies are the multiples of the fundamental frequency.

$$N_{(n)} - N_{(n-1)} = N = \frac{v}{2d}$$

$$\therefore \quad d = \frac{v}{2(N_n - N_{n-1})}$$

Here $N_n - N_{n-1} = 50$ kilo hertz

$= 50{,}000$ hertz

$v = 5{,}000$ m/s

$$\therefore \quad d = \frac{5{,}000}{2 \times 50{,}000}$$

$d = \mathbf{0{\cdot}05\ m.}$

Hence the thickness of the steel plate is **0·05 metre.**

Example 10.24. *A quartz crystal of thickness 0·001* m *is vibrating at resonance. Calculate the fundamental frequency. Given, Y for quartz* $= 7.9 \times 10^{10}$ newtons/m^2 *and* ρ *for quartz=2650* kg/m^3.

For longitudinal vibrations,

$$v = \sqrt{\frac{Y}{\rho}}$$

Here $Y = 7{\cdot}9 \times 10^{10}$ newtons/ m^3

and $\rho = 2650$ kg/m^3

$$\therefore \quad v = \sqrt{\frac{7{\cdot}9 \times 10^{10}}{2650}}$$

$v = 5460$ m/s

For the fundamental mode of vibration,

$$\text{thickness} = \frac{\lambda}{2}$$

$$\therefore \quad \lambda = 2 \times \text{thickness} = 2 \times 0{\cdot}001 = 0{\cdot}002 \text{ m}$$

Frequency, $n = \dfrac{v}{\lambda}$

$$n = \frac{5460}{0{\cdot}002} = 2730{,}000 \text{ hertz} = \mathbf{2730\ kilo\text{-}hertz.}$$

Example 10.25. *A quartz crystal of thickness 0·001* m *radiates ultrasonic waves into water of frequency 20* kilo-hertz *and intensity* 5×10^4 watts/m^2. *Calculate (i) maximum acceleration and (ii) maximum displacement. Velocity of sound in water=1480* m/s *and density of water=1000* kg/m^3. *Density of the crystal=2650* kg/m^3.

Sound intensity,

$$I = \frac{P^2}{2\rho v} \text{ watts/m}^2$$

Here P is the sound pressure, ρ is the density and v is the velocity of sound.

$\therefore$ Sound pressure

$$P = \sqrt{2I\rho v}$$

Here $I = 5 \times 10^4$ watts/m^2

$\rho = 1000$ kg/m^3

$v = 1480$ m/s

$$\therefore \quad P = \sqrt{2 \times 5 \times 10^4 \times 1000 \times 1480}$$

$= \mathbf{3{\cdot}85 \times 10^5\ newtons/m^2}$

Force = Press × Area = Mass × Acceleration

$\therefore \quad PA = Ma$

Here M = mass of the crystal

$=$ volume × density $= A \times t \times D$

$\therefore \quad PA = (A\,t\,D)\,a$

(i) Maximum acceleration,

$$a = \frac{P}{t\,D}$$

$$a = \frac{3{\cdot}85 \times 10^5}{0{\cdot}001 \times 2650}$$

or $a = 1{\cdot}45 \times 10^5$ **m/s^2**

(ii) Maximum displacement,

$$y = \frac{a}{\omega^2}$$

$$= \frac{1{\cdot}45 \times 10^5}{(2\pi f)}$$

Here $f = 20$ kilo-hertz $= 20{,}000$ hertz

$$\therefore \quad y = \frac{1{\cdot}45 \times 10^5}{(2\pi \times 20{,}000^2)} = \mathbf{9{\cdot}4 \times 10^{-6}\ m.}$$

Example 10.26. *A piezo-electric X-cut quartz plate has a thickness of 1·5* mm. *If the velocity of propagation of longitudinal sound waves along the X* direction is 5760 m/s, *calculate the fundamental frequency of the crystal.* **(IAS)**

Here, thickness = 1·5 mm = $1{\cdot}5 \times 10^{-3}$ m

$v = 5760$ m/s

Thickness, $t = \dfrac{\lambda}{2}$

(9) **Medical applications.** Ultrasonic waves have a large number of applications in the field of medicine. Some of the important applications are as follows :

(*a*) *Neuralgic pain.* Ultrasonic waves are useful for relieving neuralgic and rheumatic pains. The affected portion of the body is exposed to ultrasonic waves. The waves produce a soothing massage action and relieves pain.

(*b*) *Arthritis.* Ultrasonic waves are used to relieve pain due to arthritis. Here a small metal head, vibrating with a frequency of more than 10^6 hertz is moved over the skin of the patient. These vibrations after passing through the tissues, produce a deep massage action. The patient is relieved of the pain.

(*c*) *Contracted fingers.* Ultrasonic waves are used to restore the contracted fingers. They are also used to loosen up the scar tissues in various parts of the human body.

(*d*) *Broken teeth.* Ultrasonic waves are used by dentists for the proper extraction of broken teeth.

(*e*) *Bloodless surgery.* Ultrasonic waves are used in bloodless surgery. Here the ultrasonic waves are focussed on a sharp instrument and the tissues are destroyed without any loss of blood.

American doctors have used such instruments for conducting bloodless brain operations.

(10) **Sterilization.** Ultrasonic waves can destroy unicellular organisms. Bacteria perish under the action of ultrasonic waves. Ultrasonic waves are used in the sterilization of water and milk.

(11) **Enemy of lower life.** When some lower animals like rats, frogs, fish, etc., are exposed to ultrasonic waves, they become lame.

Ultrasonic waves are having more and more practical applications in all fields. Active research work is still in progress to study the effect of ultrasonic waves in mechanical, biological, chemical, physical and industrial fields.

10.28. Fourier Theorem

Fourier theorem deals with the summation of a number of simple harmonic vibrations, in which the vibrations are in the same straight line. The theorem also helps in the synthesis and analysis of complex forms of vibrations. This theorem was formulated by **J.B.T. Fourier** in 1828.

The theorem states that any single valued periodic function can be expressed as a sum of a number of simple harmonic terms which are multiples of the given function. The theorem is generally referred in relation to the study of transverse vibration of strings. However, the theorem has a wider scope.

Fourier's theorem deals primarily with the synthesis of a complex periodic vibration from simple harmonic terms and it also gives a method to analyse a complex vibration into its component vibrations.

Conditions. The theorem has the following two provisions.

(1) The displacement must be a single valued function and continuous. This condition is satisfied in all cases of mechanical vibrations because a single particle cannot actually have two different displacements simultaneously.

(2) The displacement must always have a finite value. This is true in the case of sound.

10.29 Fourier Series

The Fourier theorem can be expressed by the series

$$y = f(\omega t) = A_0 + A_1 \cos(\omega t + \alpha_1) + A_2 \cos(2\omega t + \alpha_2) + \ldots\ldots + A_m \cos(m\omega t + \alpha_m)$$

This can be written in the form

$$y = f(\omega t) = A_0 + \sum_{m=1}^{m=\infty} A_m \cos(m\omega t + \alpha_m) \quad \ldots (1)$$

Here y represents the displacement of the complex periodic vibrations of frequency $\left(\frac{\omega}{2\pi}\right)$. $A_1, A_2, \ldots, A_m$ are the amplitudes of the components of the simple harmonic vibrations and α_1, $\alpha_2, \ldots \alpha_m$, ..., represent their respective initial phases.

It may be mentioned that sometimes it is convenient to represent y as a sum of sine and cosine series in the form

$$y = f(\omega t) = \sum_{m=0}^{m=\infty} [A_m \sin m\omega t + B_m \cos m\omega t] \quad \ldots (2)$$

Here $A_0 = 0$

$$\therefore \quad y = f(\omega t) = B_0 + \sum_{m=0}^{m=\infty} A_m \sin m\omega t + \sum_{m=0}^{m=\infty} B_m \cos m\omega t \quad \ldots (3)$$

10.30. Evaluation of Fourier Coefficients

The method of finding the amplitudes of Fourier coefficients (B_0, A_m and B_m) for all values of m is called Fourier analysis. In equation (3) the value of B_0 can be obtained by integrating this equation with respect to time t over a complete time period T

$$\therefore \quad \int_0^T y\,dt = \int_0^T B_0\,dt + \int_0^T \left[\sum_{m=1}^{m=\infty} A_m \sin m\omega t\right] dt + \int_0^T \left[\sum_{m=1}^{m=\infty} B_m \cos m\omega t\right] dt \quad \ldots (4)$$

Here $$\int_0^T = \left[\sum_{m=1}^{m=\infty} A_m \sin m\omega t\right] dt = 0$$

and $$\int_0^T \left[\sum_{m=1}^{m=\infty} B_m \cos m\,\omega t\right] dt = 0$$

$$\therefore \quad \int_0^T y\,dt = B_0\,T$$

or $$B_0 = \frac{1}{T}\int_0^T y\,dt \qquad \ldots (5)$$

To find the value of A_m, of the sine series, multiply all the terms of equation (3) by sin $m\omega t$ and integrate the terms for a complete cycle 0 to T. Then

$$A_m = \frac{2}{T}\int_0^T y \sin(m\omega t)\,dt \qquad \ldots (6)$$

Similarly the values of B_m of the cosine series can be obtained by multiplying all the terms of equation (3) by cos $m\omega t$ and integrating for a complete cycle 0 to T.

Thus we have

$$B_m = \frac{2}{T}\int_0^T y \cos(m\omega t)\,dt \qquad \ldots (7)$$

Results

$$y = f(\omega t) = B_0 + \sum_{m=1}^{m=\infty} A_m \sin m\omega t + \sum_{m=1}^{m=\infty} B_m \cos m\,\omega t$$

$$B_0 = \frac{1}{T}\int_0^T y\,dt$$

$$A_m = \frac{2}{T}\int_0^T y \sin(m\omega t)\,dt \qquad \ldots (8)$$

$$B_m = \frac{2}{T}\int_0^T y \cos(m\omega t)\,dt$$

10.31. Square Wave

The analysis of a square wave according to Fourier theorem is an illustrative example. Let $y = f(\omega t) = 0$ at $t = 0$ and $t = \frac{T}{2}$. Let y be equal to $+a$ for $0 < t < \frac{T}{2}$ (Fig. 10.29). This function has a discontinuity at $t = 0$ and at $t = \frac{T}{2}$. Consequently it does not satisfy the assumption that it is smooth everywhere. Therefore, a perfect representation cannot be reasonably expected from the Fourier series.

There is a **sharp overshoot** at $t = 0$ and at $t = \frac{T}{2}$ for every partial sum of the series. Addition of more and more terms makes the overshoot sharper but the displacement (y) does not become zero.

Here $f(\omega t) = 0$ for $t = 0$,

and $f(\omega t) = +a$ for $0 < t < \frac{T}{2}$

and $f(\omega t) = 0$ for $t = \frac{T}{2}$

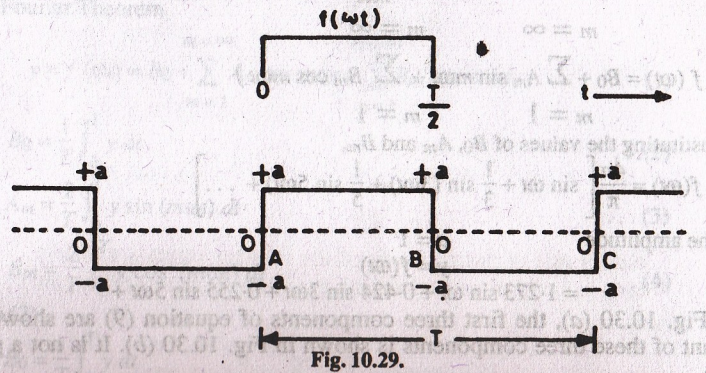


Fig. 10.29.

and $f(\omega t) = -a$ for $\frac{T}{2} < t < T$

Using the equations in (8), it is easy to obtain the values of

$B_0 = 0$

$B_m = 0$ for all values of m

$A_m = 0$ for $m = 2, 4, 6$ (even integers)

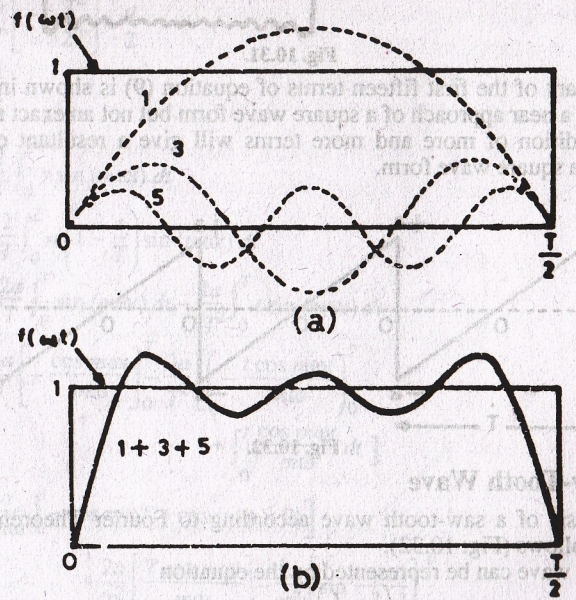


Fig. 10.30.

and $$A_m = \frac{4a}{m\pi} \text{ for } m = 1, 3, 5 \qquad \text{(odd integers)}$$

$$y = f(\omega t) = B_0 + \sum_{m=1}^{m=\infty} A_m \sin m\omega t + \sum_{m=1}^{m=\infty} B_m \cos m\omega t$$

Substituting the values of B_0, A_m and B_m,

$$y = f(\omega t) = \frac{4a}{\pi}\left[\sin \omega t + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin 5\omega t + \ldots\right]$$

If the amplitude $a = 1$

$$y = f(\omega t)$$

$$= 1{\cdot}273 \sin \omega t + 0{\cdot}424 \sin 3\omega t + 0{\cdot}255 \sin 5\omega t + \quad \ldots (9)$$

In Fig. 10.30 (*a*), the first three components of equation (9) are shown. The resultant of these three components is shown in Fig. 10.30 (*b*). It is not a perfect square.

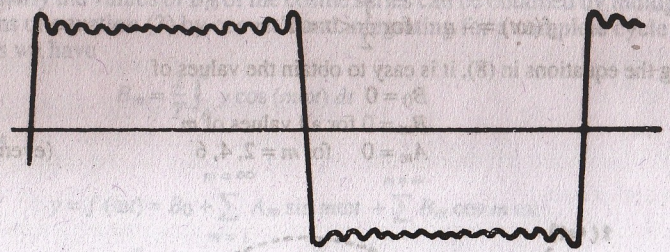


Fig. 10.31.

The resultant of the first fifteen terms of equation (9) is shown in Fig. 10.31. This figure is a near approach of a square wave form but not an exact square form. However, addition of more and more terms will give a resultant curve nearly approaching a square wave form.

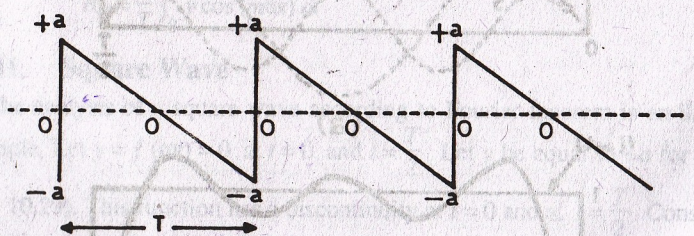


Fig. 10.32.

10.32. Saw-Tooth Wave

The analysis of a saw-tooth wave according to Fourier Theorem can be illustrated as follows (Fig. 10.32).

Saw-tooth wave can be represented by the equation

$$y = f(\omega t) = a\left(1 - \frac{t}{T}\right) \text{ for } 0 < t < T$$

Here amplitude $= a$

$y = f(\omega t) = a$ at $t = 0$

and $y = f(\omega t) = 0$ at $t = T$

After this the curve is repeated.

According to Fourier Theorem

$$y = f(\omega t) = B_0 + \sum_{m=1}^{m=\infty} A_m \sin m\omega t + \sum_{m=1}^{m=\infty} B_m \cos m\omega t \quad \ldots (1)$$

$$B_0 = \frac{1}{T}\int_0^T y\, dt \quad \ldots (2)$$

$$A_m = \frac{2}{T}\int_0^T y \sin(m\omega t)\, dt \quad \ldots (3)$$

and $$B_m = \frac{2}{T}\int_0^T y \cos(m\omega t)\, dt \quad \ldots (4)$$

From equation (2)

$$B_0 = \frac{1}{T}\int_0^T y\, dt$$

$$B_0 = \frac{1}{T}\int_0^T a\left(1 - \frac{t}{T}\right) dt$$

$$B_0 = \frac{a}{T}\left[t - \frac{t^2}{2T}\right]_0^T$$

$$B_0 = \frac{a}{T}\left[T - \frac{T^2}{2T}\right] = \frac{a}{2}$$

$$\therefore \quad B_0 = \frac{a}{2} \quad \ldots (5)$$

From equation (3)

$$A_m = \frac{2}{T}\int_0^T y \sin(m\omega t)\, dt$$

$$= \frac{2}{T}\int_0^T a\left(1 - \frac{t}{T}\right)\sin(m\omega t)\, dt$$

$$= \frac{2a}{T}\int_0^T \sin(m\omega t)\, dt - \frac{2a}{T^2}\int_0^T t \sin(m\omega t)\, dt$$

$$A_m = \frac{2a}{T}\left[-\frac{\cos m\omega t}{m\omega}\right]_0^T - \frac{2a}{T^2}\left[\left(-\frac{t\cos m\omega t}{m\omega}\right)_0^T + \int_0^T \frac{\cos m\omega t}{m\omega}\, dt\right]$$

$$A_m = \frac{2a}{Tm\omega}\left[-\cos m\omega T + \cos(m\omega \times 0)\right] + \frac{2a}{T^2}\left[\frac{T\cos m\omega t}{m\omega}\right] - \frac{2a}{T^2}\left[\sin\frac{m\omega t}{m^2\omega^2}\right]_0^T$$

$$-\cos(m\omega T) + \cos 0 = 0$$

and $$\left[\sin(m\omega t)\right]_0^T = 0 \qquad \left[\because T = \frac{2\pi}{\omega}\right]$$

$$\therefore \quad A_m = \frac{2a}{T}\left[\frac{\cos 2\pi m}{m\omega}\right] = \frac{2a}{Tm\omega} = \frac{a}{m\pi} \qquad \ldots (6)$$

From equation (4)

$$B_m = \frac{2}{T}\int_0^T y \cos(m\omega t)\, dt$$

$$B_m = \frac{2}{T}\int_0^T a\left(1 - \frac{t}{T}\right)\cos(m\omega t)\, dt$$

$$B_m = 0$$

Hence all the cosine terms of the Fourier series are absent.

In a saw-tooth wave, equation (1) of Fourier series can be written in the form

$$y = f(\omega t) = \frac{a}{2} + \sum_{m=1}^{m=\infty} \frac{a}{m\pi} \sin m\omega t \qquad \ldots (7)$$

This expression in the expanded form can be written as

$$y = \frac{a}{2} + \frac{a}{\pi}\left(\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \right.$$
$$\left. + \ldots + \frac{1}{m}\sin m\omega t \ldots\right) \qquad \ldots (8)$$

The addition of successive terms in the series given in equation (8) results in the saw-tooth waveform. Fig. 10,33 shows the resultant wave pattern for the addition of two terms, three terms, 10 terms and 30 terms respectively. Taking only the first term, will give only a sine wave.

10.33. Wave Velocity and Group Velocity

The equation for a plane progressive wave is given by

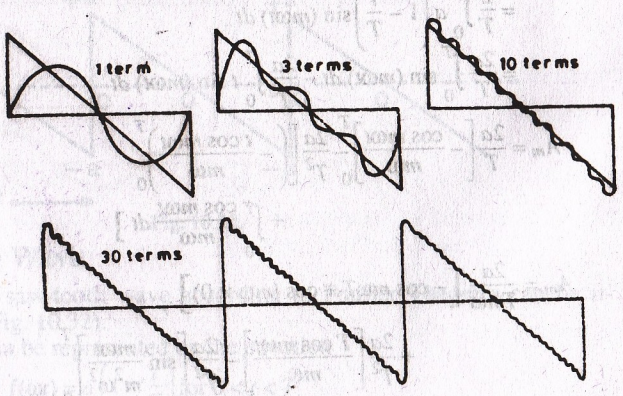


Fig. 10.33.

$$y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$y = a\sin(\omega t - kx)$$

where $$\omega = \frac{2\pi}{T} = 2\pi n$$

and $$k = \frac{2\pi}{\lambda}$$

Here ω is called the angular frequency of the wave and k is called the wave vector.

Also

$$\frac{\omega}{k} = \frac{2\pi n}{2\pi/\lambda} = n\lambda = v_p$$

Here v_p is called the **phase velocity** or the **wave velocity**. In other words, $v_p = \frac{\omega}{k}$ is the velocity with which a plane progressive wavefront travels forward. It has a constant phase = $(\omega t - kx)$

$$\omega t - kx = \text{constant}$$

Differentiating this equation with respect to t and x,

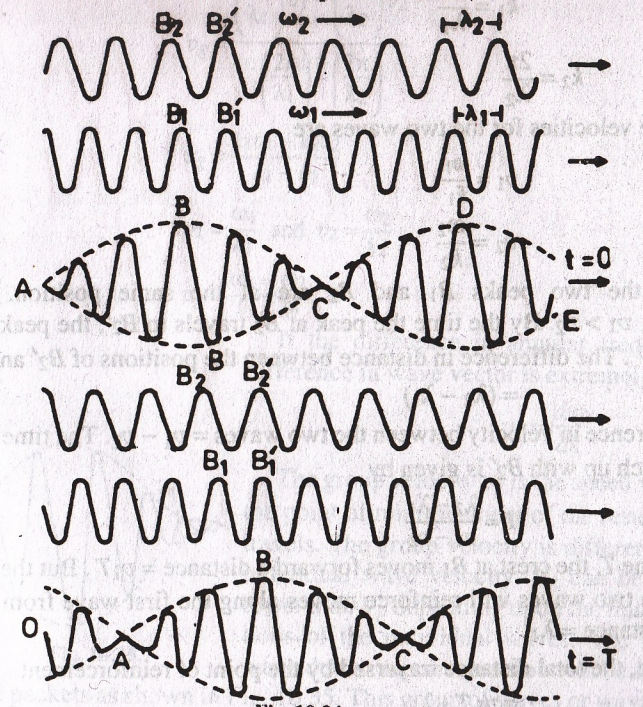


Fig. 10.34.

$$\omega - k\frac{dx}{dt} = 0$$

or $$\frac{dx}{dt} = \frac{\omega}{k} = v_p$$

Group velocity. Consider a group of progressive waves with angular frequencies in the range ω and $\omega + \Delta\omega$ and wave vectors in the range k and $k + \Delta k$ superimposing on one another. The resultant wave pattern consists of points of maxima amplitude and points of minimum amplitude. In between any two consecutive maximam and minima, the amplitude gradually decreases. Between any two consecutive minima, say A and C, there is a position of maximum amplitude midway between A and C, *i.e.*, at B. The dotted loop represents a group of waves. This group of waves is called a packet and with time the packet moves forward in the medium with a velocity, called the *group velocity.* (Fig. 10.34).

Consider two plane progressive waves travelling in the positive direction of the X-axis. The equations of the two waves at any instant are,

$$y_1 = a_1 \sin(\omega_1 t - k_1 x) \quad \ldots(1)$$

$$y_2 = a_2 \sin(\omega_2 t - k_2 x) \quad \ldots(2)$$

Here $$k_1 = \frac{2\pi}{\lambda_1} \quad \ldots(3)$$

and $$k_2 = \frac{2\pi}{\lambda_2} \quad \ldots(4)$$

The wave velocities for the two waves are

$$v_1 = \frac{\omega_1}{k_1}$$

and $$v_2 = \frac{\omega_2}{k_2}$$

Initially the two peaks B_1 and B_2 are at the same position. Here $\lambda_2 > \lambda_1$ and $v_1 > v_2$. By the time the peak at B_2 travels to B_2', the peak at B_1 travels to B_1'. The difference in distance between the positions of B_2' and B_1'

$$= (\lambda_2 - \lambda_1)$$

The difference in velocity between the two waves $= v_1 - v_2$. The time taken by B_1' to catch up with B_2' is given by

$$T = \frac{\lambda_2 - \lambda_1}{v_1 - v_2} \quad \ldots(5)$$

In this time T, the crest at B_1 moves forward a distance $= v_1 T$. But the point at which the two waves will reinforce moves along the first wave from B_1 to B_1' *i.e.*, a distance $= \lambda_1$.

Therefore, the total distance traversed by the point of reinforcement

$$= v_1 T + \lambda_1$$

$$\therefore \text{ Group velocity, } v_g = \frac{v_1 T + \lambda_1}{T} \quad \ldots(6)$$

or $$v_g = v_1 + \frac{\lambda_1}{T}$$

or $$v_g = v_1 + \left[\frac{\lambda_1 (v_1 - v_2)}{\lambda_2 - \lambda_1}\right]$$

$$v_g = \frac{v_1 \lambda_2 - v_1 \lambda_1 + v_1 \lambda_1 - v_2 \lambda_1}{(\lambda_2 - \lambda_1)}$$

or $$v_g = \frac{v_1 \lambda_2 - v_2 \lambda_1}{(\lambda_2 - \lambda_1)}$$

Dividing the numerator and denominator by $\lambda_1 \lambda_2$,

$$v_g = \frac{\frac{v_1}{\lambda_1} - \frac{v_2}{\lambda_2}}{\frac{1}{\lambda_1} - \frac{1}{\lambda_2}}$$

Multiplying the numerator and denominator by 2π

$$v_g = \frac{\left(\frac{2\pi}{\lambda_1}\right) v_1 - \left(\frac{2\pi}{\lambda_2}\right) v_2}{\left(\frac{2\pi}{\lambda_1}\right) - \left(\frac{2\pi}{\lambda_2}\right)}$$

or $$v_g = \frac{k_1 v_1 - k_2 v_2}{k_1 - k_2} \quad \ldots(7)$$

$$v_1 = \frac{\omega_1}{k_1} \text{ and } v_2 = \frac{\omega_2}{k_2}$$

$$\therefore \quad v_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} \quad \ldots(8)$$

If the difference in angular frequency and difference in wave vector is extremely small,

$$v_g = \frac{d\omega}{dk} \quad \ldots(9)$$

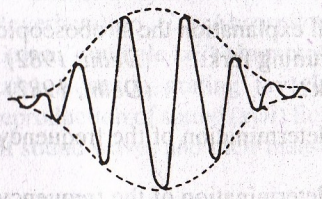


Fig. 10.35.

The group velocity v_g is the speed with which the point of reinforcement of the resultant wave travels. The group velocity is different from the individual wave velocity. V_g can be higher or lower than V_p depending upon the relative directions of the individual waves. The modulated wave due to superimposition will be in the form of wave packets as shown in Fig. 10.35. This group of waves or wave packet in each dotted envelope travels with the group velocity and not with wave velocity.

EXERCISES

1. Describe the working of a siren. Explain how it can be used to determine the frequency of a note.
2. Describe the falling plate method for determining the frequency of a tuning fork.
3. Discuss the revolving drum method (chronographic method) for determining the frequency of a tuning fork.
4. Discuss the stroboscopic method for determining the frequency of a tuning fork.
5. Explain in detail the method of determining the frequency of a tuning fork by using a phonic motor.
6. Explain briefly the principles involved in recording and reproduction of sound in a magnetic tape recorder. *(Nagpur, 1984)*
7. Give the principle, construction and working of a cathode ray oscillograph. Explain briefly the applications of a cathode ray oscillograph.
8. Explain clearly how sound ranging is done.
9. Discuss the nature of wavefront formed during the flight of a bullet at supersonic speed.
10. What is reverberation and on what factors does it depend?
11. Derive Sabine's reverberation formula and explain its significance. *(Punjab, 1983)*
12. What are ultrasonics? Describe a method for their production. *[Behrampur (Hons.), 1982]*
13. Discuss briefly the applications of ultrasonics. *(Behrampur, 1982)*
14. How are ultrasonics detected? Explain the principle and working of a thermal detector.
15. Discuss briefly the applications of ultrasonics in (*i*) depth finding and (*ii*) signalling.
16. Discuss briefly the applications of ultrasonics in medicine and surgery.
17. Describe with experimental details and full explanation the stroboscopic method of determining the frequency of a tuning fork. *(Delhi, 1982)*
18. What is a phonic wheel? How does it work? *(Delhi, 1982)*
19. Describe the falling plate method for the determination of the frequency of a tuning fork giving the relevant theory.
20. Discuss the phonic wheel method for the determination of the frequency of a tuning fork.
21. State the Fourier theorem and obtain expressions for Fourier coefficients.
22. Discuss the application of Fourier theorem for the analysis of a square wave.

23. Apply Fourier series in analysing the saw-tooth wave form.
24. Distinguish between wave velocity and group velocity and obtain expressions for their values.
25. Enumerate the features that an auditorium should have for good acoustics. How can these features be incorporated in the design of an auditorium? What remedial measures would you suggest for the following defects in an existing auditorium?
    (*i*) Uneven distribution of sound.
    (*ii*) Unintelligible audibility of speech. *(Nagpur, 1984)*
26. Explain the difference between phase velocity and group velocity. *(Lucknow, 1984)*
27. Discuss briefly the applications of ultrasonics. *[Delhi (Addl Physics)*
28. Describe the stroboscopic method for finding the frequency of a tuning fork. *(Delhi, 1986)*
29. What are ultrasonic waves? How are they produced and to what uses are they put? *(Delhi, 1986)*
30. Describe in detail the falling plate method for determining the frequency of a tuning fork. *[Delhi (Supple), 1986]*
31. Explain what causes reverberation in a hall and how it can be reduced? *(Kanpur, 1985)*
32. What are ultrasonics? How are these produced and detected? *(Kanpur, 1985)*
33. Describe with a diagram, the working of an electrically maintained tuning fork. *(Gauhati, 1992)*
34. Define intensity and loudness of sound. Also define bell and phon. *(Bhagalpur, 1993)*
35. Write short notes on :

(*i*) Gramophone (*ii*) Tape recording (*iii*) Microphone (*iv*) Loud Speaker (*v*) Cathode ray oscillograph (*vi*) Sound ranging (*vii*) Flight of a bullet (*viii*) Reverberation (*ix*) Absorption coefficient (*x*) Acoustics of buildings (*xi*) Galton's whistle (*xii*) Magnetostriction oscillator (*xiii*) Piezo-electric oscillator (*xiv*) Acoustic grating (*xv*) Ultrasonics and its uses (*xvi*) Recording and reproduction of sound (*xvii*) Echo and its use in depth ranging (*xviii*) Frequency of sound waves and their measurement (*xix*) Sabine's formula.