CHAPTER IX

MAGNETIC MEASUREMENTS

MAGNETIC tests can be divided into two general classes: direct-current tests and alternating-current tests. Although they may be subdivided to a considerable extent, these are the two most distinctly defined classes of tests. The methods of testing magnetic specimens will therefore be dealt with under these two general headings.

Direct Current Tests. Such tests are most generally made upon solid (as distinct from laminated) materials, the alternating-current test methods being used chiefly for laminated materials.

The two most important quantities to be measured in these tests are the flux density in a specimen and the magnetizing force producing this flux density.

Magnetometer Methods. These are fundamentally the simplest of all methods of magnetic testing, and were largely used in the early work on magnetism. Magnetometers are used for the measurement of magnetic field. Often the horizontal component of such a field is measured by them. They may be applied, also, to the measurement of flux density in bar specimens of magnetic material, their advantage for this purpose being that they measure the actual (or static) value of flux density in the specimen as distinct from methods such as those using a ballistic galvanometer, which measure a *change* in flux density. The intensity of magnetization J of a specimen is measured by the magnetometer, and the corresponding flux density B is obtained from the formula

$$B = \mu_0 H + J$$

where H is the strength of the magnetic field producing this intensity of magnetization. For full details of such applications the reader should refer to the works given in Refs. (2) and (3).

Magnetometers consist essentially of a suspended magnetic needle or system of needles, the suspension itself having good torsional elastic qualities and exerting, usually, only a small torsional control upon the needle.

Referring to Fig. 9.1, let ns represent a magnetic needle, suspended at O, and of length l metres. Let n and s be its poles, of strength m webers, and suppose that a horizontal control field of strength H_x amperes per metre exists in the direction XX'. If another horizontal field, of strength H_y, having a direction at right angles to XX', is made to act upon the needle, a deflection θ is produced. The couples acting upon the needle are $H_x ml \sin \theta$ clockwise, and $H_y ml \cos \theta$ anti-clockwise. When the needle is at rest in the deflected position,

$$H_x ml \sin \theta = H_y ml \cos \theta$$

from which the strength of the field H_y is given by

$$H_y = H_x \tan \theta$$

If H_x is the horizontal component of the earth's magnetic field—as it often is—its value can be easily found, from physical tables, for any point on the earth's surface.

If the deflecting field is due to a bar magnet in the neighbourhood of the magnetometer needle, the pole strength of this magnet can be obtained as follows—

Theory. Referring to Fig. 9.2, let *ns* be the magnetometer needle of length l and pole strength m. Let *NS* be the bar magnet under

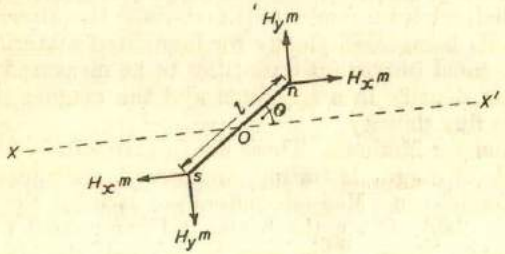


FIG. 9.1

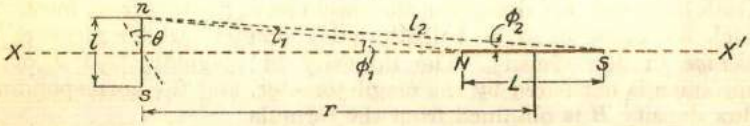


FIG. 9.2. MAGNETOMETER MEASUREMENT

test, of length L (between its poles) and pole strength m'. Suppose that both *ns* and *NS* are in the horizontal plane and are placed as shown. Then, the force upon pole n of the needle, due to pole N of the bar magnet, when resolved into the direction XX' is $\frac{mm'}{4\pi\mu_0 l_1^2} \cos \phi_1$ where l_1 and ϕ_1 are as shown. Pole N will exert the same force upon pole s of the needle, but in the opposite direction to the force on pole n. The pole S will exert forces of $\frac{mm'}{4\pi\mu_0 l_2^2} \cos \phi_2$ in direction XX' upon the two poles of the needle in the opposite directions to the forces due to N. Thus, the deflecting moment upon the needle due to the two poles of the bar magnet will be

$$\frac{mm'l}{4\pi\mu_0}\left[\frac{\cos \phi_1}{l_1^2} - \frac{\cos \phi_2}{l_2^2}\right]$$

Now, $\cos \phi_1 = \dfrac{r - \dfrac{L}{2}}{l_1}$ and $\cos \phi_2 = \dfrac{r + \dfrac{L}{2}}{l_2}$. The deflecting moment is therefore

$$\frac{mm'l}{4\pi\mu_0}\left[\frac{r - \frac{L}{2}}{l_1^3} - \frac{r + \frac{L}{2}}{l_2^3}\right]$$

If the needle is situated in a horizontal control field H in a direction perpendicular to XX', and if the deflection of the needle is θ, then we have

$$Hml \sin \theta = \frac{mm'l}{4\pi\mu_0}\left[\frac{r - \frac{L}{2}}{l_1^3} - \frac{r + \frac{L}{2}}{l_2^3}\right] \cos \theta$$

Hence,

$$m' = \frac{4\pi\mu_0 H \tan \theta}{\left[\dfrac{r - \frac{L}{2}}{l_1^3} - \dfrac{r + \frac{L}{2}}{l_2^3}\right]}$$

from which the ferric induction in the magnet or sample may be determined.

It is assumed that the length l of the needle is small compared with the distance r.

Instead of using the horizontal component of the earth's field as the controlling field, a stronger control field may be used, this being obtained by using a permanent magnet, or otherwise. The strength of such a field may be measured by comparing it with that of the earth by the *oscillation method*, which forms a simple means of measuring field strength. The magnetic needle is allowed to oscillate freely whilst situated in the field whose strength is to be measured, and the time of one complete oscillation is measured. Let this time be T_1 sec. The needle is then placed in a known magnetic field such as that of the earth, and the time of free oscillation again measured. Let this time be T_2.

Then, from the expression for the time of one complete free oscillation, namely

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

(where I is the moment of inertia of the needle about the axis of oscillation, M is the magnetic moment of the needle, and

H the strength of the magnetic field in which it oscillates), we have

$$T_1 = 2\pi\sqrt{\frac{I}{MH_1}}$$

and

$$T_2 = 2\pi\sqrt{\frac{I}{MH_2}} \quad . \quad . \quad . \quad . \quad (9.1)$$

H_1 and H_2 being the strengths of the two fields, the latter being known.

Hence

$$\frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{I}{MH_1}}}{2\pi\sqrt{\frac{I}{MH_2}}} = \sqrt{\frac{H_2}{H_1}}$$

or

$$H_1 = H_2 \cdot \frac{T_2^2}{T_1^2} \quad . \quad . \quad . \quad . \quad (9.2)$$

The magnetometer method has the advantage that it is an absolute method, but the disadvantage that it is susceptible to the influence of external magnetic fields, and also that it requires the samples under test to be in the form either of long, thin rods, or of ellipsoids, owing to the demagnetizing effect of the ends of bar-shaped samples.*

During the Second World War several accurate and sensitive magnetometers were developed in Great Britain. These took advantage of the high permeability of alloys, such as Mumetal (see p. 722) to concentrate the magnetic field. Differential arrangements could be made so that a very small movement of a coil removed it from the field to be measured. Another development in magnetometer design is based on the fact that, Mumetal becoming saturated at a relatively low flux density, it is possible to obtain high sensitivity by using this alloy in the region of saturation; a small change in the magnetic field causes a large change in permeability.

A Mumetal-wire magnetometer is described by S. Whitehead (Ref. (49)).

The Ballistic Galvanometer. Before proceeding with the description of methods of testing bar- and ring-shaped samples of magnetic material, the ballistic galvanometer and fluxmeter—instruments which are largely used in such tests—will be described.

The ballistic galvanometer is used to measure a *quantity of electricity* passed through it. This quantity, in magnetic measurements, is the result of an e.m.f. instantaneously induced in a "search coil" connected to the galvanometer terminals, when the magnetic flux

* Several forms of magnetometer are described by D. W. Dye in the *Dictionary of Applied Physics*, p. 455, amongst which is F. E Smith's magnetometer for the measurement of the intensity of the earth's magnetic field.

interlinking with the search coil is changed. Such a galvanometer is usually of the D'Arsonval type, since this type is least affected by external magnetic fields. It does not show a steady deflection when in use, owing to the transitory nature of the current passing through, but gives a "throw" which is proportional to the quantity of electricity instantaneously passed through it. This quantity—and hence the change in the flux producing it—is determined from the calibration of the galvanometer, as will be described later. The proportionality of the throw only holds if the discharge of the electricity through the galvanometer has been completed before any appreciable deflection of the moving system has taken place. For this reason the moving system of such a galvanometer must have a large moment of inertia—often obtained by the addition of weights to the moving system—compared with the restoring moment due to the suspension. This means that the galvanometer has a long period of vibration—usually from 10 to 15 seconds in practice. The damping of the galvanometer should also be small in order that the first deflection (or throw) shall be great.

For convenience in working, a galvanometer which is almost dead-beat is best, but the damping must be electromagnetic so that it may be determined from the constants of the instrument. Appreciable air damping should not be present, as this is indeterminate. A key by which the galvanometer may be short-circuited saves time in bringing the moving system to rest.

Other important points in the construction of such galvanometers are that the moving coil should be free from magnetic material, and also that the suspension strip should be carefully chosen and mounted to avoid "set." The terminals, coil, and connections within the instrument, should be of copper throughout, in order to avoid thermo-electric effects at the junctions. In the best instruments the suspension is non-conducting, the current being led into the coil by delicate spirals of very thin copper strip.

Theory. As already stated above, the quantity of electricity must be discharged through the galvanometer in a very short time, during which, the moment of inertia of the moving system being large, the movement from the zero position is negligibly small. The passage of the electricity through the instrument gives to the moving system energy which is dissipated gradually thereafter in friction and electromagnetic damping.

During the actual motion the deflecting torque is thus zero, and the equation of motion is

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = 0$$

a being the moment of inertia of the moving system, b the damping constant, c the control constant, θ the deflection in radians, and t the time in seconds.

As shown on page 277, the solution of this equation is

$$\theta = Ae^{m_1 t} + Be^{m_2 t}$$

where A and B are constants and

$$m_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad m_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The damping, and therefore b, is small so that both m_1 and m_2 are imaginary. Under these conditions, as on page 278, the solution may be written

$$\theta = e^{-\frac{bt}{2a}} F \cdot \sin\left(\frac{\sqrt{4ac - b^2}}{2a} \cdot t + \alpha\right)$$

F being a constant which may be evaluated from a knowledge of the initial conditions of the motion. Since b is small, a justifiable simplification is

$$\theta = e^{-\frac{bt}{2a}} \cdot F \cdot \sin\left(\sqrt{\frac{c}{a}}\, t + \alpha\right)$$

It should be noted that a ballistic galvanometer is sometimes used in a damped condition and then b cannot be neglected.

Initial Conditions. When $t = 0$ the deflection $\theta = 0$.

Again, if i is the current in amperes at any instant during the discharge of electricity through the instrument, the torque may be represented by Gi and hence

$$Gi = a\frac{d^2\theta}{dt^2}$$

from which $\int_0^\tau Gi\, dt = \int_0^\tau a \frac{d^2\theta}{dt^2} \cdot dt$, where τ = total time of the discharge. Since $\int_0^\tau idt$ = the quantity of electricity discharged = Q coulombs, we may write

$$G\int_0^\tau idt = GQ = a\frac{d\theta}{dt}$$

$\frac{d\theta}{dt}$ is the velocity of the moving system at the end of time τ, i.e. at the beginning of the first deflection, since τ is very small.

We may thus write (as a close approximation) when $t = 0$, $\frac{d\theta}{dt} = \frac{G}{a} \cdot Q$.

Now, differentiating the above expression for θ we have

$$\frac{d\theta}{dt} = -\frac{b}{2a} \cdot e^{\frac{b}{2a}} F \sin\left(\sqrt{\frac{c}{a}}\, t + \alpha\right) + e^{-\frac{b}{2a}t} \cdot F\sqrt{\frac{c}{a}} \cos\left(\sqrt{\frac{c}{a}}\, t + \alpha\right)$$

and when $t = 0$,

$$\frac{d\theta}{dt} = -\frac{b}{2a} \cdot e^{0} \cdot F \sin \alpha + e^{0} \cdot F\sqrt{\frac{c}{a}} \cos \alpha$$

Again, since $\theta = 0$ when $t = 0$,

$$0 = e^{0} \cdot F \sin (0 + \alpha) \text{ or, } \alpha = 0$$

$$\therefore \frac{d\theta}{dt} = F\sqrt{\frac{c}{a}} = \frac{G}{a} \cdot Q$$

or

$$F = \frac{G}{a} \cdot \sqrt{\frac{a}{c}} \cdot Q$$

Substituting in the expression for θ we have

$$\theta = \frac{G}{a} \cdot Q \cdot e^{-\frac{b}{2a}t} \sqrt{\frac{a}{c}} \sin\sqrt{\frac{c}{a}} \cdot t$$

The deflection at any time is thus proportional to Q and the motion is oscillatory, the frequency of the oscillation being

$$\frac{\omega}{2\pi} = \frac{\sqrt{\frac{c}{a}}}{2\pi}$$

The periodic time of the motion is thus

$$T' = \frac{1}{f} = 2\pi\sqrt{\frac{a}{c}}$$

The graph of deflection against time is shown in Fig. 9.3, the successive diminishing maxima corresponding to times

$$\frac{T'}{4}, \frac{3T'}{4}, \frac{5T'}{4}, \text{ etc.}$$

Substituting the values of time in the expression for θ gives

$$\theta_1 = \frac{G}{a} \cdot Q \cdot e^{-\frac{\pi}{4}\frac{b}{\sqrt{ac}}} \sqrt{\frac{a}{c}}$$

$$\theta_2 = \frac{G}{a} \cdot Q \cdot e^{-\frac{3\pi}{4}\frac{b}{\sqrt{ac}}} \sqrt{\frac{a}{c}}$$

$$\cdot = \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\theta_n = \frac{G}{a} \cdot Q \cdot e^{-\frac{\pi}{4}(2n-1)\frac{b}{\sqrt{ac}}} \sqrt{\frac{a}{c}}$$

Without damping the amplitudes would all have been

$$\theta' = \frac{G}{a} \cdot Q \cdot \sqrt{\frac{a}{c}}$$

Now $$\theta' = \frac{\theta_1}{e^{-\frac{\pi}{4}\frac{b}{\sqrt{ac}}}} = \theta_1 e^{\frac{\pi}{4}\frac{b}{\sqrt{ac}}}$$

and $$\frac{\theta_1}{\theta_2} = \frac{e^{-\frac{\pi}{4}\frac{b}{\sqrt{ac}}}}{e^{-\frac{3\pi}{4}\frac{b}{\sqrt{ac}}}} = e^{\frac{\pi}{2}\cdot\frac{b}{\sqrt{ac}}}$$

Hence $$\sqrt{\frac{\theta_1}{\theta_2}} = e^{\frac{\pi}{4}\cdot\frac{b}{\sqrt{ac}}} \text{ and } \theta' = \theta_1\sqrt{\frac{\theta_1}{\theta_2}}$$

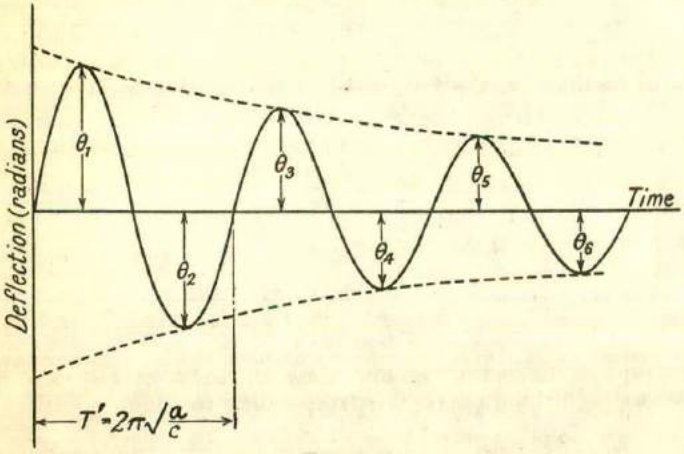


FIG. 9.3

Logarithmic Decrement. The "logarithmic decrement", λ, is a constant of the galvanometer which is proportional to the damping and depends upon the resistance of the galvanometer circuit.

From above we have

$$\frac{\theta_1}{\theta_2} = e^{\frac{\pi}{2}\cdot\frac{b}{\sqrt{ac}}}$$

so that $$\log_e \frac{\theta_1}{\theta_2} = \frac{\pi}{2}\frac{b}{\sqrt{ac}} = \lambda$$

Then $$\theta' = \theta_1 \, e^{\frac{\pi}{4}\frac{b}{\sqrt{ac}}} = \theta_1 \, e^{\frac{\lambda}{2}}$$

$$= \theta_1\left(1 + \frac{\lambda}{2}\right) \text{ approx.}^*$$

Obviously, from the equations for θ_1, θ_2, etc.,

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \ldots\ldots = \frac{\theta_{n-1}}{\theta_n} = e^{\frac{\pi}{2}\frac{b}{\sqrt{ac}}} = e$$

$$\therefore \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} \times \ldots\ldots \times \frac{\theta_{n-1}}{\theta_n} = (e^{\lambda})^{n-1}$$

or $$\frac{\theta_1}{\theta_n} = e^{\lambda(n-1)}$$

$$\therefore \log_e\left(\frac{\theta_1}{\theta_n}\right) = \lambda(n-1)$$

or $$\lambda = \frac{1}{n-1}\log_e\frac{\theta_1}{\theta_n}$$

From the preceding equation for the ideal undamped deflection θ', namely $\theta' = \frac{G}{a} \cdot Q\sqrt{\frac{a}{c}}$

we have $$Q = \frac{\sqrt{ac}}{G} \cdot \theta' \text{ coulombs}$$

showing that the quantity of electricity to be measured is directly proportional to the undamped deflection θ'.

Since the periodic time $T' = 2\pi\sqrt{\frac{a}{c}}$ we have, by substitution

$$Q = \frac{\sqrt{ac}}{G} \cdot \theta' = \frac{c}{G}\sqrt{\frac{a}{c}}\,\theta' = \frac{c}{G} \cdot \frac{T'}{2\pi} \cdot \theta'$$

To eliminate the quantities c and G, suppose that a direct current of I_g amperes passed through the galvanometer produces a steady deflection θ; then

$$G \cdot I_g = c \cdot \theta$$

$$\text{or } \frac{c}{G} = \frac{I_g}{\theta}$$

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* This follows since

$$e^{\frac{\lambda}{2}} = 1 + \frac{\frac{\lambda}{2}}{\underline{|1}} + \frac{\left(\frac{\lambda}{2}\right)^2}{\underline{|2}} + \frac{\left(\frac{\lambda}{2}\right)^3}{\underline{|3}} + \ldots\ldots$$

Hence, finally,

$$Q = \frac{T}{2\pi} \cdot \frac{I_g}{\theta} \cdot \theta'$$

or

$$Q = \frac{T'}{2\pi} \cdot \frac{I_g}{\theta}\left(1 + \frac{\lambda}{2}\right)\theta_1 \quad . \quad . \quad . \quad (9.3)$$

This equation may be written shortly as

$$Q = K \,.\, \theta_1$$

where

$$K = \frac{T'}{2\pi} \cdot \frac{I_g}{\theta}\left(1 + \frac{\lambda}{2}\right)$$

and must be found by calibration.

Since the value of K depends upon the damping and shunting of the galvanometer it is essential that the resistance of the galvanometer circuit during calibration shall be the same as it is when the galvanometer is being used for testing purposes.

Calibration of the Galvanometer. This may be carried out in a number of ways. Some of the methods used are—

(*a*) By Means of a Capacitor. A capacitor which has been charged to a known voltage is discharged through the galvanometer. The quantity of electricity discharged can be calculated from the known voltage and the capacitance of the capacitor. This method is not in general use owing to the difficulty of determining exactly the capacitance of the capacitor under all conditions, and also because of the fact that the damping of the galvanometer during calibration is different from that during testing.

(*b*) By Means of a Standard Solenoid. This method is most commonly used for calibration purposes. A standard solenoid consists of a long coil of wire wound on a cylinder of insulating and non-magnetic material. There may be one or more layers of wire, but the design is such that the axial length of the solenoid is large compared with its mean diameter. Usually, the axial length is at least 1 metre, while the mean diameter is of the order of 10 cm. The winding must be uniform and the number of turns per metre axial length should be such that a field strength, H, of 10,000 A/m or more is obtained at the centre of the coil when carrying its maximum allowable current. If the axial length is great compared with the mean diameter, the field strength in the neighbourhood of the centre of the core of the solenoid is uniform and is given by

$$H = \frac{NI}{L}$$

where N = No. of turns on the solenoid,

I = the current in amperes flowing in the winding,

L = the axial length in metres.

If this condition as regards axial length and mean diameter is not fulfilled, the field strength at the centre is given by

$$H = k \,.\, \frac{NI}{L} \quad . \quad . \quad . \quad . \quad (9.4)$$

where $$k = \frac{L}{2d}\log_e\left[\frac{\left(r+\frac{d}{2}\right)+\sqrt{\left(r+\frac{d}{2}\right)^2+\frac{L^2}{4}}}{\left(r-\frac{d}{2}\right)+\sqrt{\left(r-\frac{d}{2}\right)^2+\frac{L^2}{4}}}\right]$$

where d = the radial depth of the winding on the solenoid in metres,

r = mean radius of the solenoid in metres.

If the radial depth, d, is small then, as seen in Chapter I, the field strength at the centre of the solenoid is given by

$$H = \frac{NI}{L}\cos\theta$$

where θ is the angle subtended at the centre of the coil by the mean radius r at one end of the coil, i.e. $\theta = \tan^{-1}\left(r \div \frac{L}{2}\right)$

Thus
$$H = \frac{NI}{L}\frac{\frac{L}{2}}{\sqrt{r^2+\left(\frac{L}{2}\right)^2}}$$

$$= \frac{1}{2}\frac{NI}{\sqrt{r^2+\left(\frac{L}{2}\right)^2}} \quad . \quad . \quad . \quad (9.5)$$

At the centre of the solenoid is wound a small secondary coil, usually of several hundred turns of thin wire. The axial length of the secondary coil should be small, and it may either be wound over the solenoid or placed within and coaxial with it. In either case its dimensions must be accurately known.

This secondary coil is connected to the ballistic galvanometer, and a measured current is passed through the solenoid from a battery, through a reversing switch (Fig. 9.4). An e.m.f., producing a throw of the galvanometer, is induced in the secondary coil when the solenoid current is reversed. The number of flux linkages producing this throw is obtained from the known value of H at the centre of the solenoid, and from the number of turns and dimensions of the secondary coil. For example, suppose that H at the centre of the solenoid is 5,000 when a certain current is flowing in it. Suppose, also, that the secondary coil has 400 turns, and that its mean area is 15×10^{-4} sq. m.

Then, Flux threading through the secondary coil $= 4\pi \times 10^{-7} \times 5{,}000 \times 15 \times 10^{-4}$

$= 3\pi \times 10^{-6}$ Wb

No. of weber-turns $= 12\pi \times 10^{-4}$

Change in the number of weber-turns during reversal of the solenoid current $= 24\pi \times 10^{-4}$

(c) By Means of a Mutual Inductance. The solenoid and search coil provide a calculable mutual inductance, but it is a common practice to use a variable mutual inductor. This procedure has the advantage that a large range of calibration points can be obtained, and the mutual inductor is usually smaller and more convenient than the long solenoid. If a current I amperes is reversed in the primary of a mutual inductor of value M henrys the change in secondary flux linkages is

$2MI$ weber-turns

USE OF THE BALLISTIC GALVANOMETER FOR THE MEASUREMENT OF MAGNETIC FLUX. Referring to Fig. 9.5, in order to measure the flux in the ring specimen of magnetic material, corresponding to a given current I in the magnetizing winding, which is uniformly wound on the specimen, a search coil of a convenient number of turns is wound on the specimen and connected, through a resistance and calibrating coil, to a ballistic galvanometer BG as shown. The magnetizing current I is reversed, and the galvanometer throw θ observed, the change in the flux produced by the current reversal being given by $kK\theta'$, where θ' is the undamped deflection (i.e. $\theta_1\left(1+\frac{\lambda}{2}\right)$), and K is the ballistic galvanometer constant, obtained by the use of the calibrating coil, which forms the secondary coil of a standard solenoid as previously described. k is a constant depending upon the resistance of the galvanometer circuit and number of search coil turns.

Theory.

Let N = No. of turns on search coil.

ϕ = flux embraced by the search coil.

R = resistance of the ballistic galvanometer circuit.

t = the time (in seconds) taken to reverse the magnetizing current (and hence to reverse the flux ϕ).

Then, e.m.f. induced in search coil upon reversal of the flux

$$= N\frac{d\phi}{dt} \text{ volts}$$

$$\text{Average e.m.f. induced} = N \times \frac{2\phi}{t} \text{ volts}$$

Average current in the ballistic galvanometer circuit

$$= N \times \frac{2\phi}{tR} \text{ amp}$$

Quantity Q of electricity discharged through the galvanometer during t sec is

$$Q = N \times \frac{2\phi}{tR} \times t \text{ coulombs}$$

$$= 2\frac{N\phi}{R}$$

But this equals $K\theta'$.

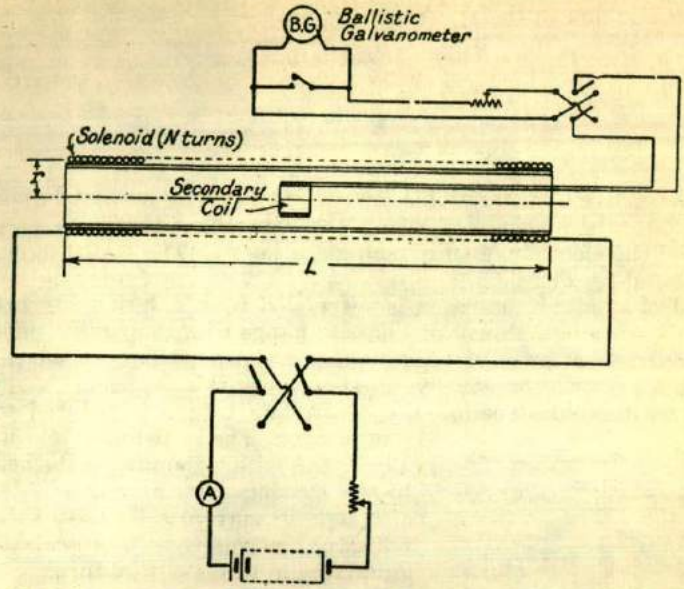


FIG. 9.4. BALLISTIC GALVANOMETER CALIBRATION BY STANDARD SOLENOID

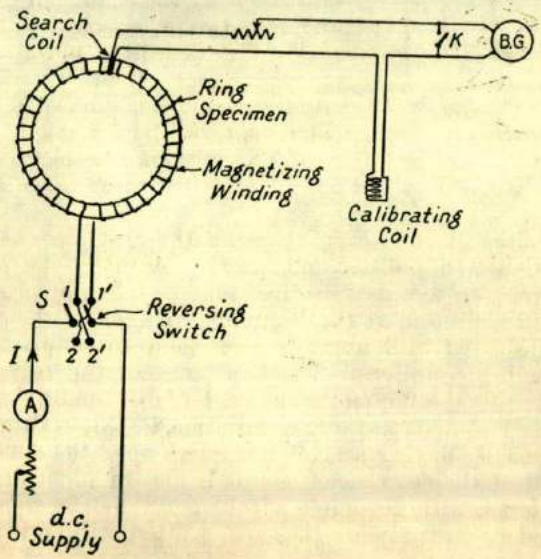


FIG. 9.5. FLUX MEASUREMENT BY BALLISTIC GALVANOMETER

Hence, the flux in the specimen is given by

$$\phi = \frac{RK\theta'}{2N} \quad . \quad . \quad . \quad . \quad (9.6)$$

so that $$k = \frac{R}{2N}$$

The Grassot Fluxmeter. This instrument is really a special type of ballistic galvanometer in which the controlling torque is very small and the electromagnetic damping is heavy. The construction is illustrated by Fig. 9.6.

A coil of small cross-section is suspended from a spring support by means of a single thread of silk, and hangs with its parallel sides in the narrow air gaps of a permanent-magnet system, as shown. Current is led into the coil by spirals of very thin, annealed silver strips. By this construction the controlling torque is reduced to a minimum. The instrument is usually fitted with a pointer (attached to the moving system) and a scale, although it may also be used as a reflecting instrument. The scale is graduated in terms of flux-turns.

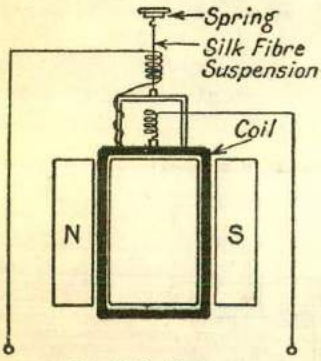


FIG. 9.6. GRASSOT FLUXMETER

The instrument is very portable, and although not so sensitive as a ballistic galvanometer, it has the great advantage that the length of time taken for the change in the flux producing the deflection need not be small. The deflection obtained, for a given change of flux interlinking with the search coil connected to the instrument, will, in a good instrument, be the same whether the time taken for the change be a fraction of a second or as much as one or two minutes.

If no controlling torque were present the instrument would remain in its deflected position indefinitely. Actually the pointer returns very slowly to zero, but readings may be taken by observing the difference in deflection at the beginning and end of the change in flux to be measured without waiting for the pointer to return to zero, the scale being uniform. The resistance of the search coil circuit connected to the fluxmeter should be fairly small, although its actual value is not important, a variation in this resistance of several ohms usually having a negligible effect upon the deflection. The inductance of the search coil circuit is also unimportant, and may be quite large, with negligible effect upon the deflection.

In some modern instruments the coil is fitted with pivots and

mounted in jewelled bearings, the current being led into the coil by fine ligaments. This form of construction is rather more robust than the suspended instruments.

Theory of the Fluxmeter. Assume that the controlling torque is negligibly small.

Let N = No. of turns on the search coil which is connected to the fluxmeter terminals,

r_s and l_s = the resistance and inductance of the search coil circuit,

R and L = the resistance and inductance of the fluxmeter,

e_s = the e.m.f. induced in the search coil at any instant,

e_f = the e.m.f. being induced at any instant in the fluxmeter coil due to its movement in the permanent-magnet field,

i = the current in the circuit at any instant.

Then $e_s = N\frac{d\phi}{dt}$, where $\frac{d\phi}{dt}$ is the rate of change of flux linking with the search coil. Also $e_f = G\frac{d\theta}{dt}$, where G is the "displacement constant" of the fluxmeter coil and is given by the product of the coil area, the flux density in the gap, and the number of turns on the coil; $\frac{d\theta}{dt}$ is the angular velocity of the fluxmeter coil. The equation of motion of the coil is therefore

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} = G \cdot i \quad . \quad . \quad . \quad . \quad (9.7)$$

The equation connecting the electrical quantities is

$$e_s = e_f + (L + l_s)\frac{di}{dt} + (r_s + R)i$$

giving, after substitution,

$$i(R + r_s) = N\frac{d\phi}{dt} - G\frac{d\theta}{dt} - (L + l_s)\frac{di}{dt}$$

Substituting for i in Equation (9.7) gives

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} = \frac{G}{(R + r_s)}\left[N\frac{d\phi}{dt} - G\frac{d\theta}{dt} - (L + l_s)\frac{di}{dt}\right]$$

Rearranging, we have

$$\frac{a(R + r_s)}{G}\frac{d^2\theta}{dt^2} + \left[\frac{b(R + r_s)}{G} + G\right]\frac{d\theta}{dt} + (L + l_s)\frac{di}{dt} = N\frac{d\phi}{dt}$$

Write $\frac{d^2\theta}{dt^2}$ as $\frac{d\omega}{dt}$, where ω = angular velocity, and integrate with respect to t.

Then

$$\int_0^T \frac{a(R + r_s)}{G} \cdot \frac{d\omega}{dt} \cdot dt + \int_0^T \left(\frac{b(R + r_s)}{G} + G\right)\frac{d\theta}{dt} \cdot dt + \int_0^T (L + l_s)\frac{di}{dt} \cdot dt = \int_0^T N\frac{d\phi}{dt} \cdot dt$$

where T is the total time taken for the change in flux. Thus

$$\int_{\omega_1}^{\omega_2} \frac{a(R+r_s)}{G}\,d\omega + \int_{\theta_1}^{\theta_2}\left[\frac{b(R+r_s)}{G}+G\right]d\theta + \int_{i_1}^{i_2}(L+l_s)\,di = \int_{\phi_1}^{\phi_2} N\,.\,d\phi$$

where ϕ_1 and ϕ_2 are the interlinking fluxes at the beginning and end of the change, i_1 and i_2 the corresponding currents, θ_1 and θ_2 the deflections, and ω_1 and ω_1 the angular velocities. But i_1 and i_2 are both zero as also are ω_1 and ω_2. Hence

$$\left[\frac{b(R+r_s)}{G}+G\right](\theta_2-\theta_1) = N(\phi_2-\phi_1) \quad . \quad . \quad (9.8)$$

If the total circuit resistance $(R + r_s)$ is kept small and the open-circuit damping coefficient b is small, this reduces to

$$G(\theta_2-\theta_1) = N(\phi_2-\phi_1)$$

or, if ϕ is the change in the flux and θ the change in fluxmeter deflection,

$$\phi = \frac{G}{N}\,.\,\theta \quad . \quad . \quad . \quad . \quad . \quad (9.9)$$

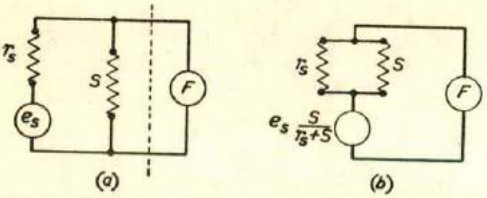


FIG. 9.7. SHUNTED FLUXMETER
(*a*) Circuit, (*b*) Equivalent Circuit

If the fluxmeter permanent-magnet field is uniform for all positions of the moving coil, G is constant, and the change in flux is directly proportional to the change in deflection.

The term $\frac{b(R+r_s)}{G}$ will cause a reading error if the resistance of the search coil is not kept small: in a typical instrument noticeable errors occur when the search-coil resistance exceeds 20 ohms.

The fluxmeter may be calibrated with the aid of a mutual inductance having a low-resistance secondary winding. If the fluxmeter is connected to the secondary and a known direct current I_p is reversed in the primary, the secondary flux linkages will be $2MI_p$. The limiting fluxmeter circuit resistance may be determined by introducing resistance until the deflection corresponding to a reversal of a given current alters.

The Shunted Fluxmeter. When a very large flux is to be measured, the number of flux linkages may be too great even when a single-turn search coil is used. The range of the fluxmeter can be extended for such measurements by the use of shunts. Fig. 9.7 (*a*) gives a diagram of the circuit when a shunt is used. r_s is the search coil resistance and S is the shunt.

When the fluxmeter is shunted, the circuit resistance may be expected to be below the limiting value, and by inspecting the solution for the unshunted instrument it is apparent that this solution is given by the equation

$$N\frac{d\phi}{dt} = G\frac{d\theta}{dt}$$

which when integrated gives

$$N(\phi_2 - \phi_1) = G(\theta_2 - \theta_1)$$

The e.m.f. induced in the search coil is therefore balanced by that induced in the fluxmeter coil due to its movement.

Applying Thevenin's theorem to the circuit in Fig. 9.7 (*a*), the circuit to the left of the dotted line may be replaced by the equivalent circuit shown in Fig. 9.7 (*b*). The search coil giving an e.m.f. e_s, together with its shunt, is replaced by an e.m.f. of value $e_s\frac{S}{r_s + S}$ together with a resistance equal to S and r_s in parallel.

As the fluxmeter reading is independent of the circuit resistance,

$$\frac{S}{r_s + S}N\frac{d\phi}{dt} = G\frac{d\theta}{dt}$$

Integrating with respect to t,

$$\frac{S}{r_s + S}N\int_0^T \frac{d\phi}{dt}\,.\,dt = G\int_0^T \frac{d\theta}{dt}\,.\,dt$$

or

$$\frac{S}{r_s + S}N\phi = G\,.\,\theta$$

i.e.

$$\phi = \frac{r_s + S}{S}\cdot\frac{G}{N}\,.\,\theta \quad . \quad . \quad . \quad . \quad (9.10)$$

Thus, when shunted, the fluxmeter reading must be multiplied by $\frac{r_s + S}{S}$.

It should be noted that it is the resistance r_s of the search coil which is important when shunting is used, and not the resistance of the fluxmeter itself.

Measurement of Leakage Factor by Means of the Fluxmeter. In dynamo-electric machinery the magnetic flux per pole which crosses the air gap—the "useful flux"—is less than the flux in the body of the pole. This is due to the fact that some lines of force—referred to as "leakage flux"—pass from the pole to the adjacent poles without crossing the air gap to the armature. The flux at the root of the pole is called the "total" flux, and the ratio $\frac{\text{Total flux}}{\text{Useful flux}}$ is the "leakage factor" of the pole.

This factor can be measured by means of a fluxmeter, a ballistic galvanometer being unsuitable on account of the high inductance of the field winding, which results in a slow rate of increase of the flux when the voltage is switched on to the field winding.

The total flux may be measured by winding two search coils on

the yoke of the machine—in the case of a direct-current machine with a stationary field—one on either side of the pole (see Fig. 9.8). As the yoke carries half of the total flux, these search coils must be connected in series so that the fluxmeter measures the flux embraced by both of them. The flux so measured will be the total flux. Another search coil, placed on the (stationary) armature in such a position that it embraces the useful flux from the pole, is then connected to the fluxmeter and the useful flux measured, the leakage factor being obtained from the two measurements.

It will usually be found that search coils of one turn only will be most suitable, in which case the fluxmeter reading gives the flux directly. In the case of a large machine it may be necessary to use shunts across the search coils, as described previously.

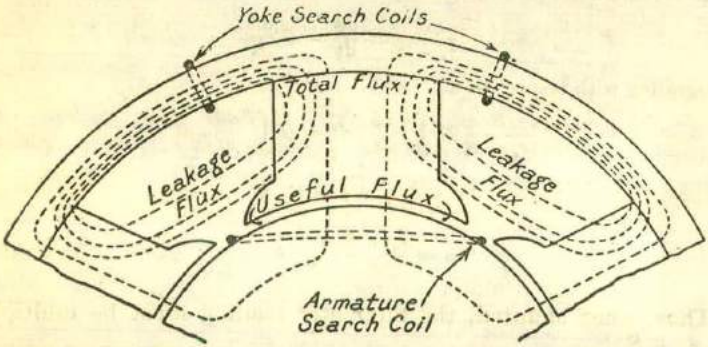


FIG. 9.8. MEASUREMENT OF LEAKAGE FACTOR

The Chattock Magnetic Potentiometer. Before proceeding with the methods of testing magnetic specimens, this device for the measurement of the magnetic potential between any two points in a magnetic field will be described. The device consists of a uniform helix of thin wire, wound on a thin strip, or rod, of some flexible insulating and non-magnetic material. This can be used, in conjunction with a ballistic galvanometer, to measure magnetic potential differences.

Let the cross-sectional area—assumed uniform—of the strip upon which the helix is wound be A sq. m, and the number of turns per metre length be n. Suppose that, when the helix is situated in a magnetic field, p_{av} is the average magnetic potential difference between the two ends. Then $p_{av} = \int H dl$, where H is the field strength, at any point within the helix, in the direction of the element of length of path dl, the integral being taken between a certain "average" point on one end of the strip and a corresponding point on the other end.

The flux in the strip at any point is $BA = \mu_0 HA$ and the flux

linkages in the element of path dl are $\mu_0 HAndl$. The total flux linkage over the whole strip is given by $\mu_0 nA\int Hdl$, the integral being taken between the ends of the strip, and provided that n is uniform over the whole strip, the flux linkage is proportional to $\int Hdl$.

In use, the winding on the strip is connected to a ballistic galvanometer and the throw on the galvanometer noted when the magnetic potential difference between the ends of the strip is changed. This change may be effected by reversing the magnetizing current giving the field, or by fixing one end of the strip at a convenient point and then moving the other end rapidly between the two points in the field under investigation. The galvanometer throw will be proportional to the total change in flux linkages through the strip, which, in accordance with the previous considerations, is proportional to $\int Hdl$.

The device can be calibrated in a long solenoid, where positioning the ends a known distance apart will give a known value of $\int Hdl$. Alternatively, and more simply, it can be formed into a closed loop encircling a coil of a known number of turns N carrying a current I amperes; then $\int Hdl = NI$.

Applications of this device are the measurement of the magnetic potential drop across a given part of a magnetic circuit, such as a joint, and the measurement of magnetic leakage. Measurements may be made upon alternating magnetic fields by using a vibration galvanometer instead of a ballistic galvanometer.

From the theory of the potentiometer it can be seen that the results are the same whether the strip upon which the helix is wound is straight or otherwise. This is a great advantage, as the use of a strip of flexible material is very convenient in investigations of this nature.

Other Methods of Measuring Magnetic Field Strength. Several other methods of exploring magnetic fields, although not in very general use, deserve mention.

(*a*) STANDARD SEARCH COIL A search coil, consisting of a single layer of (say) 50 to 100 turns of fine silk-covered wire, wound upon a short cylinder of non-magnetic material—usually marble—may be used to investigate the variation in strength, from point to point, of a magnetic field in air. The marble cylinder must be carefully turned so that its cross-sectional area is uniform throughout its axial length (about 2 or 3 cm).

The product of cross-sectional area and number of turns for the search coil may be determined to within a few parts in 10,000 by the use of a standard solenoid.

(*b*) BISMUTH SPIRAL. This method depends upon the fact that the resistance of a bismuth wire is increased when it is placed in a strong magnetic field. A curve, showing the order of this increase, is given in Fig. 9.9, the curve relating to a temperature of 20° C. A flat spiral of pure bismuth wire—about 1 mm diameter—is used

for the exploration of magnetic fields. The resistance of the spiral, when situated at a point in the magnetic field, is measured, and this resistance is compared with the resistance when the spiral is removed from the field, the temperature being the same in both cases. The field strength is obtained from a curve such as that of Fig. 9.9, when the ratio $\frac{\text{Resistance of spiral whilst in the magnetic field}}{\text{Resistance of spiral when removed from the field}}$ has been determined.

The disadvantages of this method are that it is rather insensitive —the change in resistance per weber change in field strength being

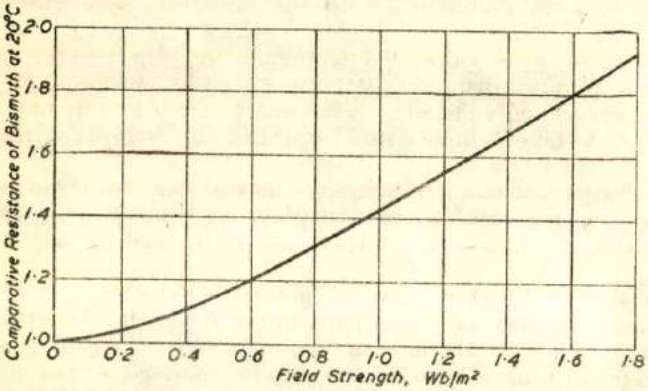


FIG. 9.9. RESISTANCE–FIELD-STRENGTH CURVE FOR BISMUTH

comparatively small—and also that the change in resistance depends very largely upon the temperature.

Its advantages are that it is very simple to use, the resistance being conveniently measured by the Wheatstone bridge method, and that, since the spiral covers a small area—of the order of 1 sq. cm—the exploration of the magnetic field can be carried out in greater detail than is possible with most other methods.

(c) MAGNETIC BALANCE AND DEFLECTING-COIL METHODS. Such methods depend upon the fact that a force is exerted upon a current-carrying conductor when it is placed in a magnetic field, the force depending upon the magnitude of the current in the conductor and upon the strength of the magnetic field.

In the magnetic balance, the force upon the conductor is balanced by weights on a pivoted beam, to one end of which the conductor is attached, the weight required for balance being used to determine the strength of the magnetic field in which the current-carrying conductor is situated.

In deflecting-coil methods a narrow coil, some 2 or 3 mm wide and 1 or 2 cm long, is used. This coil is suspended by a fine strip

suspension from a torsion head, a mirror being attached to the suspension for use in conjunction with a lamp and scale to indicate deflection. A current is passed through the coil, which is placed in the magnetic field to be measured with its plane parallel to the field. If this current is known, the field strength corresponding to a certain deflection may be determined by calibration, using a magnet of known gap flux-density.

The reader is referred to the works given in Refs. (1), (3), and (7) for fuller information regarding such methods.

Hall Effect Devices. The Hall effect is named after its discoverer, E. H. Hall, who found that when a current flows in a conductor which is perpendicular to a magnetic field, a potential difference

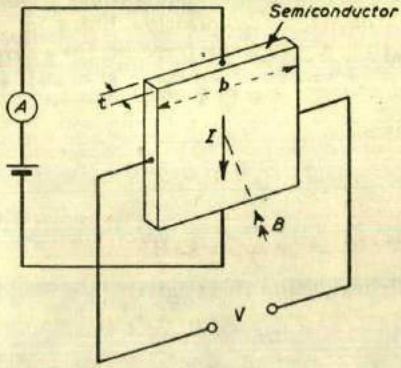


FIG. 9.10. FIELD MEASUREMENT USING HALL EFFECT

appears between the opposite edges of the conductor at right angles to the current and to the magnetic field. This e.m.f. is very small in conductors and is difficult to measure, but in some semiconductors such as germanium, the e.m.f. is sufficiently large to be measured with a sensitive moving-coil instrument. This phenomenon now forms the basis of some simple and elegant field-measuring instruments.

Consider the thin plate of semiconductor illustrated in Fig. 9.10. The current is made up from electrons and positive holes travelling in opposite directions, and since current is the rate of flow of charge, it can be expressed as

$$I = N \,.\, q \,.\, v \,.\, b \,.\, t \quad . \quad . \quad . \quad (9.11)$$

where N is the number of charge carriers per unit volume, each having charge q coulombs, v is the average velocity of the carriers in the direction of the current, and b and t are the breadth and thickness of the semiconductor plate respectively.

Now, if there is a magnetic field, of flux density B, crossing the thin plate, there will be a force on each charge of

$$F_B = q \,.\, v \,.\, B$$

and this force will deflect the carriers to one side in accordance with the left-hand rule. There is a tendency, therefore, for charge to accumulate on one side of the semiconductor, but the existence of this charge establishes an electric field which, in turn, exercises an opposing force on the carriers. Ultimately a state of equilibrium is reached when the force due to the magnetic field is equal to that due to the electric field and there is then no further accumulation. The polarity of the electric field depends upon whether conduction is due to negative electrons or positive holes of charge. If the electric field-strength is E, then the force on the charge carriers is

$$F_E = q \,.\, E$$

Hence, at equilibrium,

$$q \,.\, E + qvB = 0$$

from which

$$v = -E/B$$

Substituting for v in Equation (9.11) gives

$$E = -IB/Nqbt$$

and, since the electric field is uniform, the potential difference between opposite edges of the semiconductor is

$$V = -Eb = IB/Nqt \quad . \quad . \quad . \quad (9.12)$$

The quantity $1/Nq$ is the Hall coefficient of the material.

It can be seen from Equation (9.12) that for a given current the potential difference appearing between the edges of the material is directly proportional to the flux density of the field.

Germanium and indium antimonide have both been used in this application (Ref. (51)). The flux-measuring probe in instruments using this principle can be very small, and such instruments are very useful for the exploration of magnetic fields in confined spaces. A direct-reading meter using the principle for the measurement of flux density is manufactured by Associated Electrical Industries (Rugby) Ltd. The probes for the instrument are offered in a variety of lengths and are $\frac{1}{8}$ in. wide by $\frac{1}{32}$ in. thick. Only a small area at the top of the probe is sensitive to flux. The instrument has three ranges, 0–5,000, 0–10,000, and 0–25,000 gauss, corresponding to 0–0·5, 0–1·0, and 0–2·5 Wb/m^2 respectively.

Nuclear Magnetic Resonance. The most precise methods for the measurement of magnetic fields are based upon the phenomenon of nuclear magnetic resonance (Refs. (52) (53)).

Nearly all atomic nuclei possess spin and magnetic moments and can be looked upon as very small magnets each spinning about an axis. When an external magnetic field is superimposed on such a system the force exerted by the field on the spinning magnets causes them to precess around the direction of the field. This phenomenon is known as the "Larmor precession" and it occurs at a frequency known as the Larmor precession frequency. There is a specific constant relationship between the Larmor frequency and the flux density of the applied magnetic field for each nucleus, and this relationship is dependent upon the condition in which the nuclei are present, i.e. in elementary or molecular form.

The Larmor frequency has been determined for many substances by means of measurements made in precisely established fields, and it has been found that, for the proton in distilled water, the relationship between the magnetic flux density B, in webers per square metre, and f, the Larmor precession frequency, in cycles per second, is

$$B = 2{\cdot}348 \times 10^{-8} f \quad . \quad . \quad . \quad (9.13)$$

It can be seen from this expression that when $B = 1$ Wb/m^2 the resonance occurs at 42·58 Mc/s.

An instrument using this principle has been described by N. J. Hopkins (Ref. (53)). A proton sample of distilled water in a glass cell, 1–2 cm long by 0·5 cm inside diameter, is surrounded by a small coil and inserted in the magnetic field being measured, with the axis of the coil at right angles to the field. The coil is energized by a radio-frequency current, and when the frequency coincides with the Larmor precession frequency of the protons, changes of energy state occur in the nuclei which result in an absorption of energy from the r.f. field. The resonance is extremely sharp, and in this device, detection is aided by superimposing, by means of suitable inducing coils, a weak magnetic field varying at 50 c/s, on the field being measured, so that the resonance point is passed through 50 times a second. The absorption in energy from the oscillator is shown as a V-shaped dip on a cathode-ray oscillograph trace.

When the Larmor precession frequency is reached there is radiation from the sample, and another method of detection is to mount a second coil in a position of zero mutual inductance with the first. The second coil is connected to a receiver which detects the radiation. A very high accuracy in field measurement is possible with these techniques. It is possible to reduce the errors to the order of one part in 10^6. The fields being measured must be uniform since any non-homogeneity broadens the resonance line. A non-homogeneity, over the sample, of 1 part in 500 makes the resonance unrecognizable.

The test sample is fairly bulky, and this restricts the application of the method, but it has been found to be very useful in the exploration of the earth's magnetic field.

The Testing of Ring Specimens. Ring specimens are used, in preference to rods or strips, when accurate measurements of permeability, and hysteresis loss, up to a maximum value of field strength (H) of 2×10^4 A/m, are required. The use of such specimens has the disadvantage that they are more difficult to prepare than bar specimens, and also are more difficult to wind with the magnetizing winding which, when bar specimens are used, may be a permanently wound solenoid inside which the bar is slipped. The more reliable results obtainable with ring specimens on account of their freedom from self-demagnetizing effects may, in some cases, justify their use.

Such specimens are cut from a representative piece of the iron, the rings being machined so that their dimensions may be accurately determined. The radial thickness of such rings should be fairly small compared with their mean diameter.* If this condition is not fulfilled, most of the flux in the iron passes through the portion of the ring nearest to the inner circumference, thus causing a distribution of flux density across the cross-section of the ring which is far from uniform. The mean value of the flux density (as given by $\frac{\text{Total flux}}{\text{Cross-section}}$ under these conditions) will not correspond to the mean value of H for the cross-section of the ring, and the B-H curve obtained will be erroneous. For accurate results the ratio of outside diameter of the ring to the radial thickness should be at least 15.

If sheet material is to be tested, the ring specimen should be built up of ring punchings taken, if possible, from a number of different sheets. The punchings should be built up with the direction of rolling used in the manufacture of the sheet distributed radially to obtain a uniform distribution of reluctance round the ring. The permeability in a direction perpendicular to that of rolling is only some 75 per cent of the permeability, at the same flux density, in a direction parallel to the direction of rolling. During the shearing of the rings from the sheet, the material near the sheared edges (and for some distance inwards from the edge) is strained, the effect being to reduce its permeability. Unless the rings are to be annealed, after punching, to remove these strains, their radial width should be fairly large—say 2 or 3 cm—in order that the strained portion shall not form an appreciable percentage of the whole cross-section.

Determination of the Magnetization, or B-H, Curve. (*a*) METHOD OF REVERSALS. For this test a ballistic galvanometer is used as previously described. Before winding, the dimensions of the ring must be determined. When sheet material is being tested it may be necessary to determine the effective cross-section from the weight of the ring, taking the specific gravity as 7·8 (for soft sheet iron). This is necessary for accurate measurements on account of the air spaces between individual punchings, which cause measurements of thickness to be erroneous.

* This question is fully considered in a paper by E. Hughes (Ref. (8)).

layer of thin tape is then wound on the ring, and a search coil consisting of a few turns of thin wire, insulated by paraffined silk, is wound over the tape. The number of search-coil turns depends upon the sensitivity of the ballistic galvanometer. This number of turns must, of course, be noted. The search coil is protected by another layer of tape, over which the magnetizing winding is uniformly wound.

The connections for the test are shown in Fig. 9.5. Before commencing the test, it is essential that any residual magnetism which may be present in the specimen shall be removed by demagnetization. The short-circuiting key K of the galvanometer is left closed, and the current in the magnetizing winding is given such a value that the magnetizing force H acting upon the specimen is greater than the maximum value to be used in the test. This current is then very gradually reduced—the reversing switch S being continually thrown backwards and forwards meanwhile—in order to pass the iron specimen through as many cycles of magnetization as possible during the process. The minimum value of the current finally reached should give a magnetizing force in the specimen which is well below the smallest test value.

After demagnetization, the test is started by setting the magnetizing current at the lowest test value (such that $H = 100$ (say)). The galvanometer key K being closed, the iron specimen is then brought into a "reproducible cyclic magnetic state" by throwing the reversing switch S backwards and forwards some twenty or more times. The key K is next opened, and the flux in the specimen, corresponding to this value of H, is measured on the ballistic galvanometer as previously described. The change in flux, measured by the galvanometer, when the reversing switch S is quickly reversed, will be twice the flux in the specimen corresponding to the value of H which has been applied.

This value of H is given by $\frac{NI_1}{l}$, where N = number of turns on the magnetizing winding, I_1 = the magnetizing current, l = the length (in metres) of the mean circumference of the ring specimen.

The flux density B_1 corresponding to this value of H is obtained by dividing the value of the flux in the specimen (as measured by the ballistic galvanometer) by the cross-sectional area of the specimen.

In order to check whether the demagnetization of the specimen has been complete, and also to determine whether a reproducible cyclic state has been attained, a second measurement of flux density—for the same value of H—may be made after subjecting the specimen to a further number of reversals of magnetization. The second value of B should agree with the first. As further checks upon the measurements, a reversing switch in the ballistic galvanometer circuit (not shown in Fig. 9.5) may be used as follows: Measure the flux density for a given value of H, with this reversing switch in one position, first by throwing the reversing switch S over from terminals 11′ to 22′, and repeat the test by throwing over from 22′ to 11′, having carried out a number of

reversals of S in between the two measurements. This procedure may be repeated with the ballistic galvanometer reversing switch in its other position, four measurements of B being thus obtained. These should be very nearly equal to one another, the mean giving the value of the flux density.

The whole of this procedure is repeated for various increased values of H up to the maximum testing point, the 20 or more reversals of the magnetizing current at each value of H, before the measurement is made, being important. It should be noted also that, if the resistance of the ballistic galvanometer circuit is changed during the test, the logarithmic decrement λ must be determined for each resistance value, in order that the observed galvanometer throws may be properly corrected, the deflection used in determining the flux being given by

$$\theta' = \theta_1\left(1 + \frac{\lambda}{2}\right)$$

where θ_1 is the observed throw.

The B-H curve may be plotted from the measured values of B corresponding to the various values of H.

(*b*) THE "STEP-BY-STEP" METHOD is sometimes used. In this method there is no reversal of the magnetizing current, the procedure being as follows. The circuit shown in Fig. 9.5 is set up in the same way as for the test by the method of reversals, but the direct-current supply to the magnetizing circuit is through a potential divider having a number of tappings, as shown in Fig. 9.11. The tappings are arranged so that the magnetizing force H may be increased, in a number of suitable steps, up to the desired maximum value. The specimen, after the application of the search coil and magnetizing winding, is first demagnetized. The tapping switch S_2 is then set on tapping 1 and the switch S_1 closed, the galvanometer throw corresponding to this increase in the flux density in the specimen, from zero to some value B_1, being observed, and B_1 calculated as previously described. H_1, corresponding to this position of switch S_2, can be determined from the magnetizing current which then flows. The magnetizing force is then increased to H_2 by switching S_2 suddenly on to tapping 2, and the corresponding increase in flux density ΔB determined from the galvanometer throw observed. Then B_2—corresponding to H_2—is given by $B_1 + \Delta B$. This process is repeated for other values of H up to the maximum point, and the complete B-H curve is thus obtained without any reversal of the flux in the specimen.

Determination of the Hysteresis Loop. (*a*) STEP-BY-STEP METHOD. The determination of the hysteresis loop by this method is carried out simply by continuing the procedure just described for the determination of the B-H curve. When the point of maximum H has been reached—with S_2 (Fig. 9.11) on tapping 10—the magnetizing current is next reduced, in steps, to zero by moving switch S_2 down

through the tapping points 9, 8, 7, etc. After the reduction of the magnetizing force to zero, negative values of H are obtained by reversing the reversing switch S (Fig. 9.5) and then moving the switch S_2 (Fig. 9.11) in steps as before.

(*b*) By the Method of Reversals. This test again is carried out by means of a number of steps, but the change in flux density measured at each step is the change from the maximum value $+ B_{max}$ down to some lower value, the iron specimen being passed through the remainder of the cycle of magnetization back to the flux density $+ B_{max}$ before commencing the next step in the test, thus preserving the cyclic state.

The connections for the test are shown in Fig. 9.12. R_1, R_2, and R_4

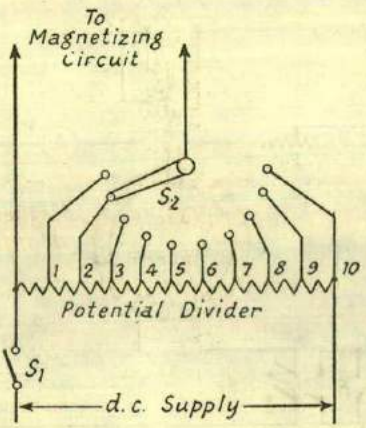


Fig. 9.11. Potential Divider for Step-by-step Method

are resistors for the adjustment of the resistances of the magnetizing and ballistic galvanometer circuits. R_3 is a variable shunting resistor which is connected across the magnetizing winding by moving over the switch S_2, thus reducing the current in this winding from its maximum value down to any desired value—depending upon the value of R_3.

The procedure is as follows—

The value of H_{max} required to produce the value of B_{max} to be used during the test is obtained from the previously determined B-H curve of the specimen. The resistances R_2 and R_4 are then adjusted so that the magnetizing current is such that this value of H is obtained when switch S_2 is in the "off" position. (H is, of course, given by $H = \frac{NI}{l}$, where I is the magnetizing current, N the number of turns on the magnetizing winding, and l the length of

magnetic path, or mean circumference, of the specimen.) The resistance R_1 is adjusted so that a convenient deflection of the galvanometer is obtained when the maximum value of the magnetizing current is reversed. R_3 is adjusted to such a value that a suitable reduction of the current in the magnetizing winding is obtained when this resistance is switched in circuit.

Switch RS_2 is then placed on contacts 11′ and the short-circuiting key K opened. The magnetization of the specimen—since the

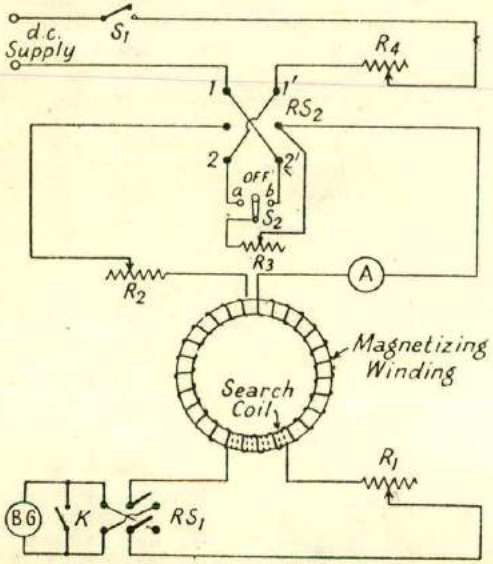


Fig. 9.12. Connections for Method of Reversals

maximum magnetizing current is now flowing—corresponds to point A on the loop (Fig. 9.13).

The next step is to throw switch S_2 quickly over from the "off" position to contact b, thus shunting the magnetizing winding by R_3 and reducing the magnetizing force to H_c (say). The corresponding reduction in flux density, $-\Delta B$, is obtained from the galvanometer throw, and hence the point C on the loop is obtained.

The key K is now closed, and switch RS_2 reversed on to contacts 22′. Switch S_2 is then opened and RS_2 moved back again to contacts 11′. This procedure passes the specimen through the cycle of magnetization and back to point A again, ready for the next step in the test. The section AD of the loop is obtained by continuation of this procedure.

To obtain the section DEF of the loop, with K closed and S_2 in

the "off" position, place RS_2 on contacts 11'. Then place S_2 on contact a, open the key K, and rapidly reverse RS_2 on to contacts 22', observing the corresponding galvanometer throw. From this throw the change in flux $\Delta B'$ (Fig. 9.13) may be obtained, since the switching operations described cause H to be changed from $+ H_{max}$ to $- H_x$ (say). To bring the magnetization of the specimen back to point A, close key K, open S_2 and reverse RS_2 on to contacts 11'.

By continuing this process, other points on the section *DEF* of the loop are obtained. The section *FGLA* of the loop may be

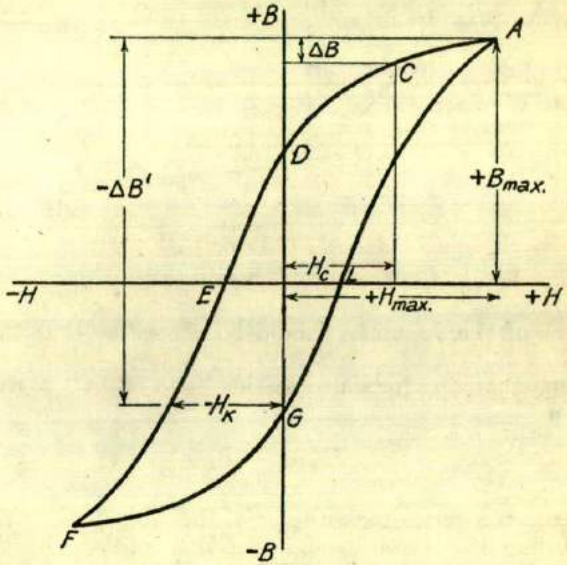


FIG. 9.13. HYSTERESIS LOOP

obtained by drawing in the reverse of *ADEF*, since the two halves are identical.

By measuring the area of the hysteresis loop so obtained—by means of a planimeter—and expressing this area in *B-H* units of area, the hysteresis loss for the material may be obtained, since

Hysteresis loss per cycle per cubic metre, in joules
= Area of loop in *B-H* units

The Testing of Specimens in the Form of Rods or Strips. It is obviously much easier to prepare a specimen in the form of a machined rod than to prepare a ring specimen as previously described. Such specimens suffer, however, from the disadvantage of "*self-demagnetization.*" When a rod is magnetized electromagnetically, poles are produced at its ends, and these poles produce, inside

the rod, a magnetizing force from the north pole to the south which is in opposition to the applied magnetizing force, thus rendering the true value of H acting on the rod a somewhat uncertain quantity. For accurate results, therefore, if the methods of measurement using a ballistic galvanometer as described above are used, this demagnetizing effect must be corrected for, or, since the effect is least when the ratio of diameter to length of the rod is small, the

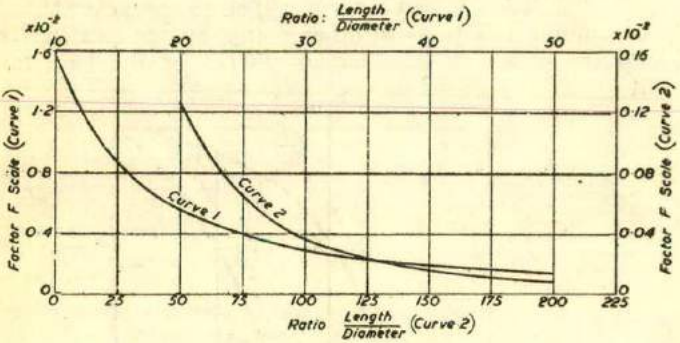


FIG. 9.14. CURVES OF DEMAGNETIZING FACTORS

dimensions of the specimen should be chosen so that the effect is negligible.

The demagnetizing force due to this "end effect" is given by the expression

$$H_d = \frac{F}{\mu_0} B_f \quad . \quad . \quad . \quad . \quad (9.14)$$

where B_f is the ferric induction, i.e. the flux density due to the magnetization of the iron itself, and F is a constant which depends upon the relative dimensions of the rod. The value of F for various ratios of length to diameter for cylindrical rods may be determined from a curve such as that given in Fig. 9.14, which has been plotted from values given by Du Bois (Ref. (5)) and by Thompson and Moss (Ref. (9)).

For an ellipsoid or very long rod, the value of the coefficient F may be calculated from the expression

$$F = \left[\frac{1}{2k} \log_e \frac{1+k}{1-k} - 1\right]\left(\frac{1}{k^2} - 1\right)$$

where $k = \sqrt{\left(1 - \frac{a^2}{b^2}\right)}$

a = the minor axis of the ellipsoid,
b = the major axis of the ellipsoid.

To obtain the true value of the magnetizing force H acting on a bar specimen, H_d must be subtracted from the value of H calculated from the ampere-turns per metre length of the magnetizing winding. From the curves given it can be seen that the ratio $\frac{\text{Length}}{\text{Diameter}}$ of the specimen must be of the order of 25 or more for the effect to have a negligible influence upon the value of H.

On account of this demagnetizing effect the value of H is often measured by means of search coils wound on thin strips of glass and placed with the glass lying flat on the bar specimen (Refs. (2), (3)). The flux density in the air at the surface of the specimen (which is

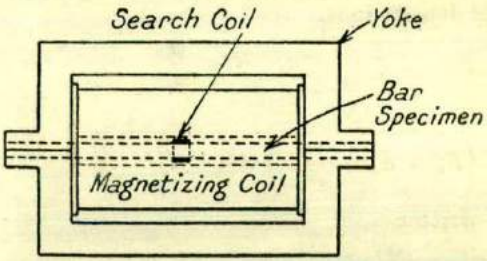


FIG. 9.15. BAR AND YOKE METHOD

the same as H in the specimen) is measured by this means instead of relying upon calculated values and corrections.

Bar and Yoke Methods. Such methods are commonly used for the testing of bar specimens. They combine the advantages of both ring and bar specimens, the demagnetizing effect being largely eliminated by the use of heavy-section yokes, while the advantages of the bar specimen, as regards preparation and ease of application of the magnetizing force, are retained.

There are a number of such methods which are all essentially modifications of the original yoke method due to Hopkinson.

A search coil is wound upon the bar specimen at its centre, and the bar is then clamped between the two halves of a massive iron yoke, whose reluctance is small compared with that of the specimen, as shown in Fig. 9.15. The magnetizing winding is fixed inside the yoke, as shown, the specimen fitting inside it.

Let N = No. of turns on the magnetizing winding

I = current in the magnetizing winding

l = length of specimen between the two halves of the yoke

A = cross-section of the specimen

$\mu_s = \mu_{sr}\mu_0$ = permeability of the specimen when the magnetizing current is I, μ_{sr} being the relative permeability of the specimen

S_y = the reluctance of the yoke

S_g = the reluctance of the two joints between specimen and yoke

ϕ = the total flux in the magnetic circuit

Then
$$\phi = \frac{NI}{S_y + S_g + \dfrac{l}{A\mu_s}} \quad . \quad . \quad . \quad (9.15)$$

Now, if H is the actual magnetizing force acting on the specimen, and B is the flux density in it,

$$B = \mu_s \,.\, H = \frac{\phi}{A}$$

Hence, $H = \dfrac{NI}{\left(S_y + S_g + \dfrac{l}{A\mu_s}\right) A\mu_s} = \dfrac{NI}{S_y A\mu_s + S_g A\mu_s + l}$

This may be written

$$H = \frac{NI}{(1+m)l}, \text{ where } m = \frac{A\mu_s}{l}(S_y + S_g) \quad . \quad (9.16)$$

The quantity m is made small by carefully fitting the specimen into the yoke, and by making the yokes of very heavy section, thus reducing both S_g and S_y to small quantities. In preparing the specimen its dimensions must be very carefully adjusted so that it exactly fits the holes in the yoke to be used. The length of the specimen generally used is about 20 or 25 cm and the diameter (if of rod form) about 1 cm.

If m is small,

$$H = \frac{NI}{l}(1 - m) \text{ approx.} \quad . \quad . \quad (9.17)$$

which means that the actual value of H in the specimen differs from the value calculated from the magnetizing ampere-turns and length of specimen by the amount

$$\frac{NI}{l} \,.\, m$$

The flux density may be measured by ballistic galvanometer in the usual way.

Permeameters. There are many different types of permeameters, which are pieces of apparatus constructed for the determination of the B-H curve of a sample in a conveniently simple manner. The

specimens are either in bar form or consist of bundles of strips of sheet magnetic material. When strip material is being tested, the strips should be cut from differing directions in the sheet so that they represent an average sample. This is not applicable to grain-orientated magnetic materials, which have anisotropic magnetic properties. If a permeameter is to give accurate results it is essential that the flux distribution should be uniform through the part of the specimen on which measurements of B and H are being taken.

Only a small number of permeameters can be described in the space available here, but many other forms are described in the works given in Refs. (1), (3), (10), (11), (12), and (14).

EWING DOUBLE-BAR METHOD. In this permeameter two exactly similar bar specimens of the material under test are used, with two pairs of magnetizing coils, one pair of the latter being exactly half

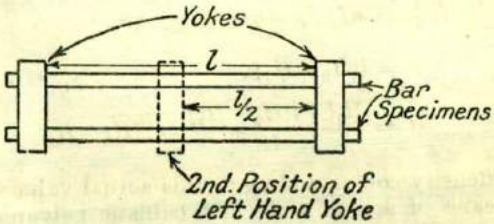


FIG. 9.16. EWING DOUBLE-BAR METHOD

the length of the other pair. The number of turns, per centimetre axial length, is the same for both pairs of coil. Two yokes of annealed soft iron, with holes to receive the ends of the bar specimens—the fit being tight—are used. The arrangement of bars and yokes is shown in Fig. 9.16.

The object of this arrangement is the elimination of the reluctance of the yokes and air gaps—assumed to be the same, for a given flux density, in all positions of the yokes—by making two tests, one with a length of specimen l and the other with a length $\frac{l}{2}$. The difference in m.m.f. required to produce the same flux density in the two cases is that required for a length of l in each specimen.

Let n = No. of turns per metre length for both pairs of magnetizing coils.

I_1 = the current in the coils when the specimen length is l.

I_2 = the current in the coils for length $\frac{l}{2}$.

H_1 = the apparent magnetizing force for length l.

H_2 = " " " " $\frac{l}{2}$.

Let a = the m.m.f. required for the yokes and air gaps in each case.

B = the flux density in the specimen (the same in each case).

Then
$$H_1 = \frac{nI_1 l}{l} = nI_1$$

$$H_2 = \frac{nI_2 \frac{l}{2}}{\frac{l}{2}} = nI_2$$

Thus, if H be the true magnetizing force in the iron for a flux density B,

$$Hl = nI_1 l - a = H_1 l - a$$

and
$$H\frac{l}{2} = nI_2 \cdot \frac{l}{2} - a = H_2 \frac{l}{2} - a$$

Hence
$$a = l(H_2 - H_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.18)$$

and
$$H = \frac{H_1 l - l(H_2 - H_1)}{l} = 2H_1 - H_2 \quad . \quad . \quad (9.19)$$

The flux density corresponding to this actual value of H is measured by means of search coils and ballistic galvanometer in the ordinary way. The complete test is carried out by first obtaining and plotting a B-H curve for the specimens with a length l—the apparent values of H being plotted. The specimens are then demagnetized, and a second B-H curve is obtained, and plotted, with a specimen length of $l/2$, the two curves being plotted on the same axes. The true B-H curve is obtained from these two, the true value of H, corresponding to any value of B, being obtained from the expression above.

The disadvantages of the method are that the joint- and yoke-reluctances are not quite the same for two different positions of the yokes. The test requires, also, two carefully prepared specimens, and is somewhat lengthy in operation.

Illiovici Permeameter. The arrangement is shown in Fig. 9.17. There are two magnetizing windings connected in parallel, one uniformly distributed on the specimen, and the other—which acts as a compensating winding—on the yoke. Each has a variable resistance in series with it, the winding on the specimen having, also, an ammeter in series with it. The operation of the permeameter depends upon the fact that when the magnetomotive forces of the two magnetizing windings—on the yoke and specimen—have been adjusted until they are just sufficient to drive the existing flux through the reluctance of that part of the circuit upon which they are wound, there will be no magnetic potential difference across

either of these parts. The magnetic potentiometer is used to indicate when this condition has been attained. Thus, when the magnetic potential drop across the specimen is zero, the m.m.f. in its magnetizing winding is just sufficient to drive the existing flux through its own reluctance. The true value of H in the specimen is then given by the m.m.f. then existing in this winding divided by the length of the specimen between the two arms of the yoke. The effect of yoke reluctance is thus eliminated. The flux density in

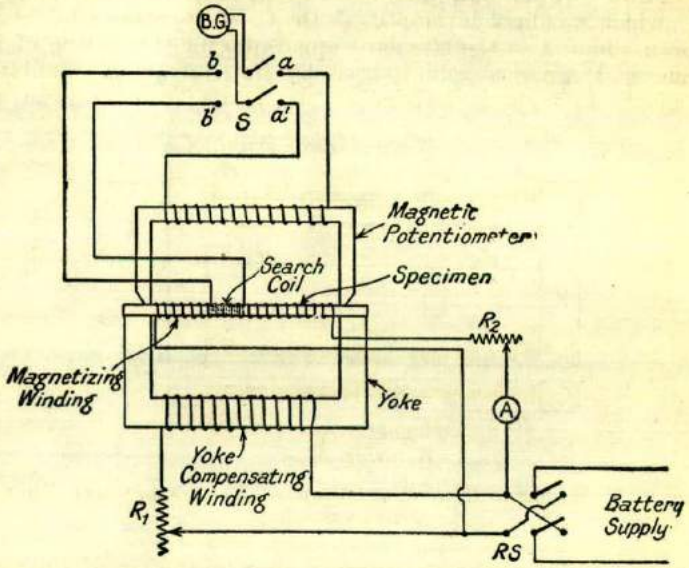


FIG. 9.17. ILLIOVICI PERMEAMETER

the specimen is measured by a search coil and ballistic galvanometer in the usual way.

In carrying out a test the specimen is placed on the yoke with the search coil and magnetizing winding over it, the magnetic potentiometer being then clamped in position and the specimen demagnetized.

The current in the magnetizing winding of the test specimen is then adjusted to give the value of H required, and the throw-over switch S is placed on contacts aa'. Resistance R_1 is then adjusted until no galvanometer throw is observed when the reversing switch RS is reversed. The value of H in the specimen is then given by

$$H = \frac{NI}{l}$$

where N is the number of turns on the magnetizing coil on the specimen, l the length (in metres) of the specimen between the arms of the yoke, and I is the current indicated by the ammeter. The switch S is now thrown on to contacts bb' and the flux density in the specimen corresponding to this value of H is measured by observing the galvanometer throw when RS is reversed.

Tests up to $H = 35{,}000$ A/m can be made with this apparatus, its principal advantages being its simplicity and its independence of the reluctance of the yoke.

BURROWS DOUBLE-BAR AND YOKE PERMEAMETER. This permeameter, which was first developed by Dr. C. W. Burrows (Ref. (12)), has been adopted as the standard apparatus for the testing of bar specimens in America, and is used by the Bureau of Standards. The effect of magnetic leakage at the joints between yoke and

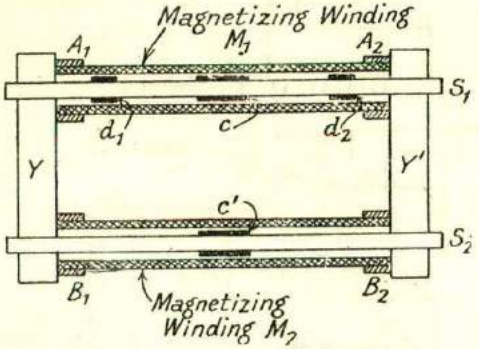


FIG. 9.18. MAGNETIC CIRCUIT OF BURROWS DOUBLE-BAR AND YOKE PERMEAMETER

specimen is eliminated in this permeameter by the use of a number of compensating coils which apply compensating m.m.f.s at different parts of the magnetic circuit, these m.m.f.s being just sufficient to drive the flux through the reluctance of the part upon which the coils are placed.

Fig. 9.18 shows the arrangement of the magnetic circuit and coils. S_1 is the bar specimen to be tested, S_2 being a bar of similar dimensions to S_1. These bars are surrounded by magnetizing windings M_1 and M_2, which are uniformly wound along the lengths of the bars. A_1, A_2, B_1, and B_2 are compensating coils for the elimination of leakage effects at the joints between the two bars and the massive yokes YY' into which the bars fit; c and c' are two exactly similar search coils wound at the centres of the two bars, while d_1 and d_2 are two similar search coils wound in the positions shown, on the test bar, and each having exactly half the number of turns of search coil c. Coils d_1 and d_2 are connected permanently in series, as are also the

four coils A_1, A_2, B_1, B_2. The dimensions of the coils M_1, M_2 are such that H in the specimen is approximately 10^4 times the current in the windings, the maximum value of H for which the apparatus is used being about 40,000 A/m. The dimensions of the bar specimens are approximately 30 cm long and 1 cm in diameter.

The coils A_1, A_2, B_1, B_2 are supplied from a separate battery supply, coil M_1 from another separate supply, and M_2 from another. In carrying out the test it is necessary, first of all, to ensure that, for a given value of the current in M_1—i.e. of H in the test bar—the flux threading through all four search coils c, c', d_1, and d_2, is the same, the current in the compensating coils, and in M_2, being adjusted until this condition is fulfilled. If the flux threading coils d_1 and d_2 is the same as that threading c and c', there can be no appreciable leakage of flux through the air in the neighbourhood of the joints. This means that the m.m.f. for the joints is being supplied by the compensating coils, and that the m.m.f. in coil M_1 is used up merely in driving the flux through the bar specimen S_1 inside it. Thus H in the specimen is given by $\frac{NI}{l}$, where

N = No. of turns on coil M_1
I = current (in amperes) in M_1
l = length of specimen in metres

It may, in some cases, be necessary to make corrections for the fact that the magnetizing solenoids are not infinitely long, but such corrections are usually negligible.

The procedure for obtaining equal flux threading all four search coils is as follows.

First, the specimen having been demagnetized, the current in the magnetizing coil M_1 is set at the required testing value. Search coils c and c' are then connected in series, but in opposition, to a ballistic galvanometer. The currents in coils M_1 and M_2 are then simultaneously reversed. A throw will be observed on the galvanometer. The current in M_2 is adjusted until no throw is obtained when the two currents are reversed. Since search coils c and c' have equal numbers of turns, this means that equal fluxes are now threading through them. Next, search coil c is connected in series with, but in opposition to, coils d_1 and d_2, and then to the ballistic galvanometer. The current in the compensating coils A_1, A_2, B_1, B_2 is then adjusted until no galvanometer throw is obtained upon simultaneous reversal of the currents in these coils and in M_1 and M_2. Then, since d_1 and d_2 together have the same number of turns as coil c, the flux threading all three coils is the same.

The flux density corresponding to the value of H in coil M_1 (obtained as described above) can be measured by connecting coil c alone to the ballistic galvanometer, and noting the throw when the

currents in the two magnetizing coils and the compensating coils are simultaneously reversed.

Fig. 9.19 gives a diagram of connections showing how the switching may be arranged for convenience in carrying out the test as described above.*

FAHY SIMPLEX PERMEAMETER. This permeameter, which has been used by the Bureau of Standards, and is being used quite com-

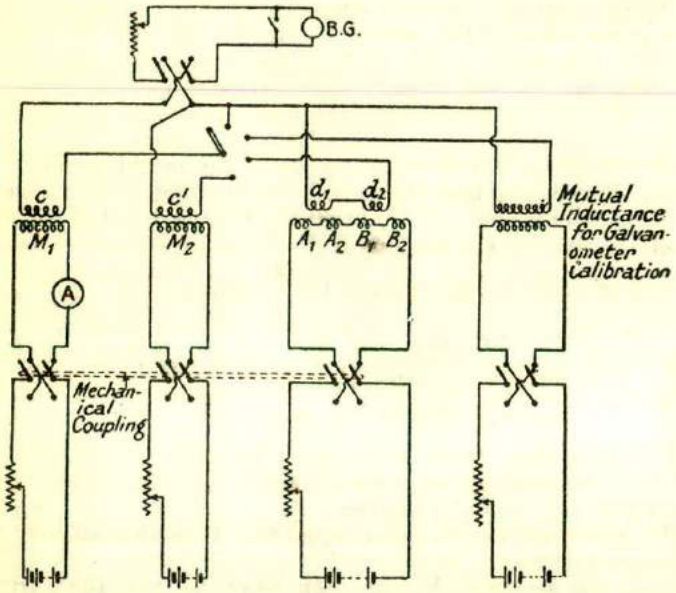


FIG. 9.19. ELECTRICAL CIRCUITS OF BURROWS DOUBLE-BAR AND YOKE PERMEAMETER

monly in other countries for the routine testing of bar specimens, has the advantage of simplicity both in construction and operation, while its accuracy is of the same order as that of the Burrows apparatus.

An outline of its construction is given in Fig. 9.20. Two iron clamping posts clamp the specimen (of cross-section $\frac{3}{4}$ in. by $1\frac{3}{4}$ in.) against a laminated steel yoke. The latter carries the magnetizing winding, and the specimen is surrounded by a search coil for the measurement of the flux density in it. The magnetizing force H acting on the specimen is measured, like the flux density, by a ballistic galvanometer, utilizing an air-cored search coil of several

* Fuller descriptions of the apparatus, testing methods, and the application of corrections, will be found in the works given in Refs. (1), (2), (3), (12).

thousand turns located between the two clamping posts as shown. The values of H so measured are corrected by calibration of the H search coil, utilizing a specimen of known magnetic characteristics in place of the test specimen.

THE NATIONAL PHYSICAL LABORATORY FORM OF PERMEAMETER. This is one of the most accurate forms of permeameter and is comparatively easy to use in practice. The arrangement of the permeameter is shown schematically in Fig. 9.21. The specimen S

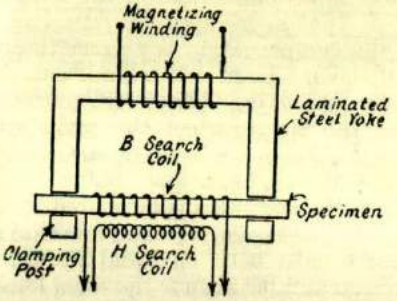


FIG. 9.20. FAHY SIMPLEX PERMEAMETER

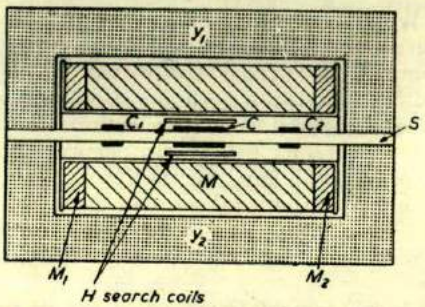


FIG. 9.21. NATIONAL PHYSICAL LABORATORY PERMEAMETER

may consist of a bundle of strips of 0·014 in. thick sheet iron, each strip being 25 cm long by 7 cm wide. The specimen is clamped between two similar rectangular iron yokes Y_1 and Y_2 which are in parallel in the magnetic circuit. The cross-sectional area of these yokes is large so that they present a low-reluctance path to the flux in the specimen. The yokes are often built with sheet stampings. The main magnetizing winding M is wound on a bobbin which surrounds the specimen, and it is provided at each end with compensating windings M_1 and M_2, whose purpose is to supply the m.m.f.s necessary to overcome the reluctance of the joints. The flux density in the specimen is measured ballistically in the usual

way by means of the small search coil C surrounding the mid-section of the specimen. Two other coils C_1 and C_2, each have half the number of turns in C and are used to check the uniformity in flux in the specimen in a manner similar to that described for the Burrows permeameter. The compensation is carried out by connecting C_1 and C_2 in series, and in opposition to C, to the ballistic galvanometer. The main magnetizing current is adjusted to give the required value of H, and the currents in the compensating windings are then adjusted until there is no indication on the ballistic galvanometer when the main and compensating currents are simultaneously reversed. When the compensation is correct the flux density over the mid-section of the specimen will be uniform. This flux density can be measured by connecting C, by itself, to the ballistic galvanometer and noting the throw when the main and compensating currents are reversed Compensation is not necessary with specimens of low relative permeability, i.e. below 1,000.

The value of H in the specimen is measured in this permeameter by means of H coils. These coils are wound on thin rigid flat cards of insulating material, usually between 1 and 2 mm thick, of the same width as the specimen and between 5 and 8 cm long. The coils have a large number of turns of very fine copper wire, the number of turns being chosen to give adequate sensitivity with the ballistic galvanometer. When tests at low H values are being carried out, thousands of turns, wound in several layers, are necessary. The windings are usually impregnated with wax or varnish, to give them rigidity. Two of these coils are normally used, one placed each side of the specimen, and if the flux density is uniform in the specimen, the flux through these coils will be proportional to the H in the specimen. The total flux linkages in the H coil in a field of strength H is $\mu_0 NAH$, where N is the total number of turns and A the mean area of the turns. It is impossible to measure the product NA from the dimensions of the coil, and the H coils are usually calibrated in a long solenoid.

The actual measurement of H is carried out in the permeameter by connecting the H coils to the ballistic galvanometer after the B test, and reversing the main and compensating currents.

This permeameter will give accurate results for H values from 10 ampere-turns per metre up to very high values in the region of 300,000 ampere-turns per metre. The field strength can be nearly doubled by mounting a short specimen between massive soft-iron pole pieces. Rod specimens of circular cross-section may also be tested by the addition of shaped pole pieces.

Drysdale Plug Permeameter. This permeameter, devised by Dr. C. V. Drysdale, is for the testing of a large mass of magnetic material. A special drill is used to cut a cylindrical hole in the mass of iron to be tested, the drill being so formed that it leaves a thin rod or pin of the metal (about 0·1 in. diameter) standing in the centre

of the hole. A rose cutter at the upper end of the drill cuts a conical hole at the surface of the iron, so that, after drilling, a section through the hole is as shown in Fig. 9.22 (*a*). A split conical plug, having a small magnetizing coil and search coil fixed to its lower end, is then pressed into the hole so that it grips the centre pin tightly and also makes good contact with the side of the conical hole in the iron mass (Fig. 9.22(*b*)). The whole then forms a bar and yoke permeameter on a small scale, the centre pin of material being the bar specimen and the mass of the material, together with the conical plug, forming the yoke. Two holes in the plug allow leads from the magnetizing coil and search coil to be brought out.

A measured current is passed through the magnetizing coil from a battery through a reversing switch, H being obtained from the

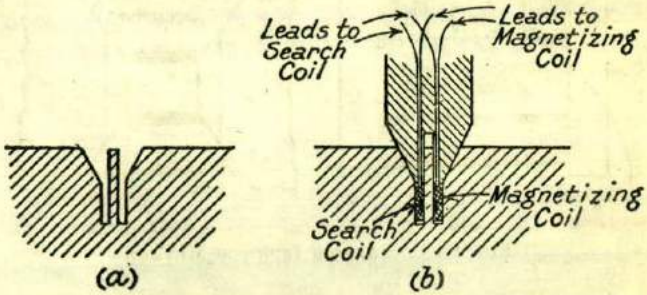


FIG. 9.22. DRYSDALE PLUG PERMEAMETER

characteristics of the magnetizing coil. The corresponding flux density is measured by the search coil and ballistic galvanometer, or fluxmeter, in the usual way.

Magnetic Testing with Intense Fields. The methods of testing so far described have been suitable for testing with values of the magnetizing force H up to 50,000 A/m in most cases. For tests at greater field strengths than these special methods have to be used.

Various "isthmus" methods, most of which are modifications of the original isthmus method due to Ewing, are used for this purpose, the name being derived from the fact that the specimen, in such methods, forms a narrow "isthmus" between the two specially-shaped poles of an electromagnet.

EWING'S ISTHMUS METHOD. Ewing carried out tests as far as $H = 2 \times 10^6$ A/m, and B about 4·5 Wb/m^2, by the use of the apparatus illustrated in Fig. 9.23.

The electromagnet carries magnetizing windings as shown. The pole pieces are conical and have a cylindrical seating, so that the specimen may be rotated through 180° during the test. The specimen itself is turned down to a bobbin shape, having a cylindrical

portion in the middle and conical ends which abut against faces on the pole pieces, the latter forming a continuation of the conical ends of the specimen. The whole of the flux in the pole pieces is thus forced to pass through a specimen of very much smaller section, thus giving a very high flux density in the specimen. The flux density obtainable depends upon the area of cross-section of the isthmus, or cylindrical part, of the specimen.

Two search coils, having equal known numbers of turns, and of known cross-sections, are fitted on the cylindrical portion of the specimen, as shown in Fig. 9.23. The inner coil—for the measurement of B—fits the specimen closely, while the outer coil—for the

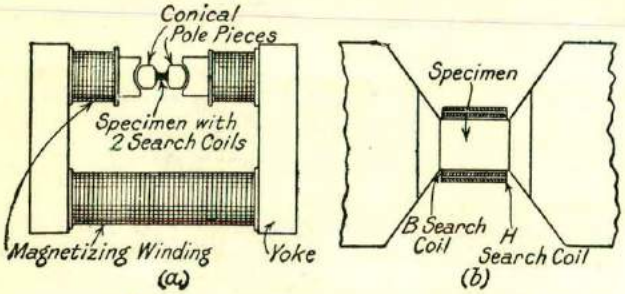


FIG. 9.23. EWING'S ISTHMUS METHOD

measurement of H—is separated from the inner coil by a small annular space.

In making a test, a magnetizing current is passed through the winding on the electromagnet, the inner search coil being connected to a ballistic galvanometer. The specimen—with its search coils—is then quickly rotated through 180°, this being equivalent, as regards interlinking flux, to a reversal of the magnetizing current. The galvanometer throw is observed and the flux density obtained therefrom. The outer search coil encloses some air flux as well as the flux in the specimen, and the difference between the flux which it encloses and that enclosed by the inner search coil is measured. This difference in flux, when divided by the difference in cross-section of the two search coils, gives the flux density in the air surrounding the specimen, and this is equal to μ_0 times the magnetizing force H in the specimen. The difference in the flux enclosed by the two search coils may be measured by connecting the two coils in series—opposing one another—and to the ballistic galvanometer, when the throw produced by a rotation of the specimen through 180° gives a measure of the flux between the coils. Otherwise, the total flux enclosed by the outer coil may be measured and the difference in enclosed flux obtained by subtraction. The inner search coil must of necessity enclose some air flux as well as the flux

in the specimen. This air flux is taken into account, using μ_0 times the measured value of H as the air flux density. The small cross-section of the specimens used in such tests renders this correction necessary in most cases.

The slope of the conical pole and specimen end pieces for maximum concentration of flux is about 60° (between the cone side and axis), whilst the angle for the most uniform field within the specimen is about 39°. A compromise is usually made in practice.

GUMLICH'S METHOD. This method is the same in principle as the method described above, but employs an improved method

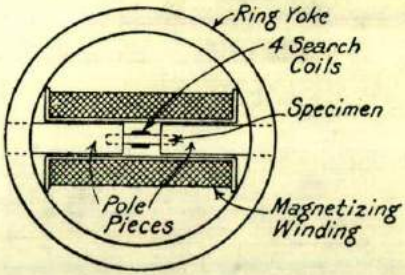


FIG. 9.24. GUMLICH'S METHOD

of determining H in the specimen. The arrangement is shown in Fig. 9.24.

A laminated ring yoke is used with two diametrically opposite pole pieces PP'. These are 25 mm in diameter, and have a central air gap of 12 mm. The pole pieces are bored centrally to take a specimen of diameter 6 mm. A magnetizing coil M surrounds the pole pieces and specimen, and upon the latter are wound four search coils in four layers. The radial spaces between the layers—and hence between the search coils—are known, and the coils have equal numbers of turns.

The flux density in the specimen is measured by connecting the inner coil—of known cross-section—to a ballistic galvanometer, and reversing the magnetizing current. Then, by connecting the search coils in pairs, opposing one another, to the ballistic galvanometer, and reversing the magnetizing current in each case, the air flux density in inter-coil spaces can be measured. These are plotted vertically on a graph, taking radial distances from the surface of the specimen as abscissae. By extrapolation the flux density at the surface of the specimen—and hence H in the specimen—can be obtained. A correction can be made also, from this graph, for the air flux enclosed by the inner search coil.

The method can be used up to $H = 6 \times 10^5$ A/m, and can be adapted to tests upon sheet material by using pole pieces with rectangular

holes. Other methods of carrying out tests at high inductions are described in the publications given in Refs. (1), (2), (3), (13).

The Testing of Feebly Magnetic Materials. For measurements upon material which is distinctly magnetic, but whose permeability is low (of the order of unity), the circuit shown in Fig. 9.25 may be used. The magnetizing winding has a search coil at its centre, this search coil being fixed so that the specimen can be slipped inside it or withdrawn without disturbing the search coil itself. The search

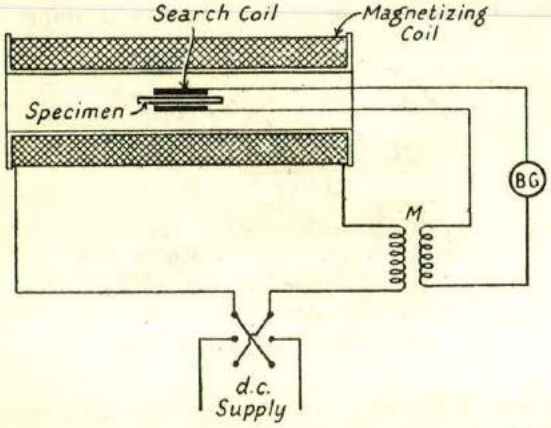


FIG. 9.25. APPARATUS FOR TESTING FEEBLY MAGNETIC MATERIALS

coil is connected, through a ballistic galvanometer, to the secondary of a mutual inductor, the primary of which is connected in the magnetizing circuit.

Since the material under test is only feebly magnetic, it is assumed that there is no demagnetization due to end effect. The procedure in testing is as follows.

Current is passed through the magnetizing coil to give the required field strength H, which is given simply by $H = \frac{NI}{l}$ (NI being the number of ampere-turns on the magnetizing coil and l its axial length), since the demagnetizing effect of the ends of the specimen is negligible. Before the specimen is placed inside the search coil the mutual inductance is adjusted to give no galvanometer throw when the magnetizing current is reversed. The specimen is then inserted, and the galvanometer throw upon reversal of the current observed. The flux density obtained from this throw will be that due to the ferric induction in the specimen. Calling this flux density B_f, we have for the total flux density, when the field strength is H, $B = B_f + \mu_0 H$, and for the relative permeability $\mu_r = \frac{B_f + \mu_0 H}{\mu_0 H}$

The Curie balance, and various modifications of it (Refs. (3), (15)), may be used for the measurement of the susceptibility of materials such as ores and rock specimens. A simplified drawing of the arrangement is shown in Fig. 9.26.

The specimen is placed in a glass tube which is suspended from a light arm which also carries a balance weight and a copper vane, the latter moving in the gap of a permanent magnet for damping purposes. The moving system also carries a mirror and another balance weight, as shown. A permanent magnet of ring shape is mounted on an arm which may be rotated so that the air gap of the magnet may be moved towards or away from the tube containing the specimen.

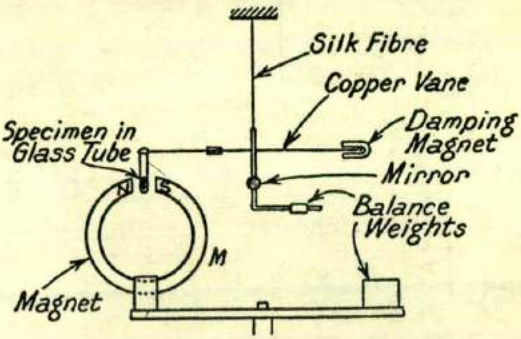


FIG. 9.26. THE CURIE BALANCE

In making the test a measurement is first made upon the glass tube alone. The magnet M is rotated until maximum deflection of the moving system is obtained, this deflection θ' being observed. The specimen is then placed in the tube and the maximum deflection θ_1 of the moving system with different positions of M again observed. A third measurement is then made with the specimen replaced by some material whose susceptibility is known.

Let the deflection now be θ_2. Then

$$\frac{\kappa_x}{\kappa_s} = \frac{m_s(\theta_1 - \theta')}{m_x(\theta_2 - \theta')} \qquad (9.20)$$

where κ_x and κ_s are the susceptibilities, and m_x and m_s the masses, of the unknown and standard materials respectively.

It should be noted that if θ' is in the opposite direction to θ_1 and θ_2 it must be treated as negative.

W. Sucksmith and R. R. Pearce (Ref. (43)) have described a method, using a balance, for the measurement of magnetic susceptibilities in a controlled atmosphere, or *in vacuo* at temperatures up to 1,500° C.

The Testing of Permanent Magnets. The details of the method of testing permanent magnets, when manufactured, depend upon the shape of the magnet, this varying greatly according to the purpose for which the magnet is to be used. Owing to the self-demagnetizing force, it is necessary, in testing such magnets, to measure H by means of search coils, of small cross-section, laid flat against the surface of the magnet as previously described. These search coils give the value of the air flux density at the surface of the magnet at different points, and thus measure H for these points. The flux density B is measured by search coils in the ordinary way.*

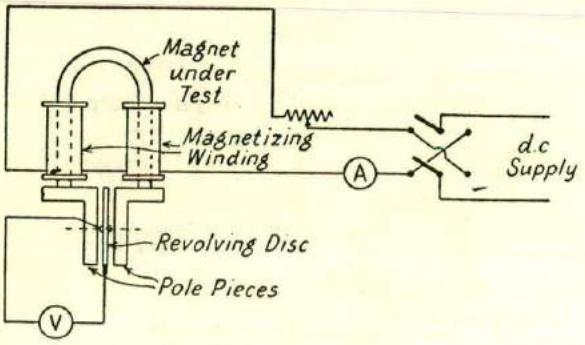


FIG. 9.27. BETTERIDGE APPARATUS

BETTERIDGE APPARATUS FOR MAGNET TESTING (Ref. (17)). The principle of this apparatus for the commercial testing of permanent magnets may be understood from Fig. 9.27. The magnet to be tested is placed with its straight ends inside two magnetizing coils and so that its ends press against two pole pieces, in the air gap between which a thin iron disc, plated with copper, is mounted. This disc is mounted in ball bearings and the clearances between it and the pole pieces are small. It is driven round at constant speed by a small motor, and has two small brushes making contact with its spindle and its rim. When a current flows in the magnetizing windings, flux crosses the gap between the pole pieces, and the flux density in the gap will be proportional to that in the magnet. An e.m.f. will be induced in the revolving disc, this e.m.f. being proportional to the gap flux density and being measured by the millivoltmeter V.

Thus, the voltmeter reading gives a measure of the flux density in the specimen, while H is obtained by the usual formula from the

* Details of tests upon different shapes of magnets, upon magnet steels, and of tests for the determination of temperature effects upon magnets, will be found in the works given in Refs. (3), (13), (14), and (16).

constants of the magnetizing coils and from the current in these coils.

Some leakage exists between the two arms of the magnet, but with proper design of the apparatus its effect upon the results is small.

British Standard Specification No. 406—1931 gives a complete specification for this apparatus, and also for another apparatus of the moving-coil pattern which uses a fluxmeter of special design.

Magnetic Testing with Alternating Current. When iron is subjected to an alternating magnetic field a loss of power occurs due to hysteresis effects in the iron. Power is absorbed, also, due to eddy currents which are set up in the iron, since it is electrically conducting material and the flux threading through it is changing. Such eddy currents will be discussed more fully in Chapter XIV.

Although the hysteresis loss per cycle in iron may be determined from the hysteresis loop obtained in a d.c. test, this loss may be somewhat different under the actual alternating magnetization conditions with which it will be used in practice. Also, eddy current losses can only be measured by the use of alternating current. For these reasons, inspection tests upon sheet steel which is to be used in the manufacture of transformers and other a.c. apparatus are very commonly a.c. tests.

Separation of Iron Losses. It is often sufficient, in acceptance tests of sheet material, to measure the total loss in the steel at the standard frequency (50 c/s) and with a maximum flux density of about 1 weber per square metre. The separation of the losses into their two components—i.e. hysteresis loss and eddy current loss—involves a rather more lengthy test.

Hysteresis loss, as already seen, is given by the expression

$$W_h = k \,.\, f \,.\, B_{max}^{1.6}$$

where W_h is the loss in watts per cubic metre of material, f being the number of cycles of magnetization per second (i.e. the frequency), B_{max} the maximum flux density, and k a constant for any given material. This law holds approximately for values of B_{max} between 0·1 and 1·2 Wb/m².

Eddy current loss, provided the sheets are sufficiently thin for "skin effect" (see Chapter XIV) to be negligible, is given by the expression

$$W_e = k' K_f^2 f^2 t^2 B_{max}^2$$

where W_e is the loss in watts per cubic metre, f the frequency, t the thickness of the sheet, and B_{max} the maximum flux density. K_f is the "form factor" of the alternating wave of flux and depends upon the shape of this wave.

Thus, if the form factor remains constant throughout a test, and the maximum flux density B_{max} is kept constant, the total power

loss being measured at different frequencies, then this total loss may be written

$$W_t = Mf + Nf^2$$

where $M = kB_{max}^{1.6}$

and $N = k'K_f^2 \cdot t^2 \cdot B_{max}^2$

both M and N being constant for this test.

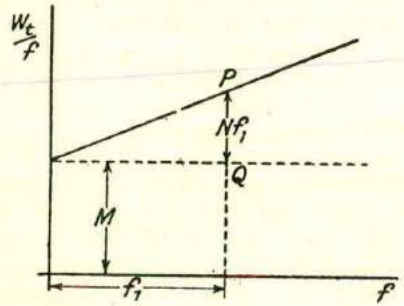


FIG. 9.28. SEPARATION OF IRON LOSSES

These constants may be determined—and the total loss thus split up into its two components, for any given frequency—by plotting $\frac{W_t}{f}$ against f as shown in Fig. 9.28.

Then, $\frac{W_t}{f} = M + Nf$, so that the intercept on the vertical axis gives M; and N can be obtained from the slope of the graph. M is the hysteresis loss per cycle. The eddy-current loss for any frequency f_1 is given by the intercept PQ, as shown in Fig. 9.28. In this way the two components of the total loss, for any given form factor and maximum flux density, can be separated.

Again, if the frequency and B_{max} are kept constant and the form factor is varied, the total loss being measured for various values of form factor, then we have

$$W_t = C + DK_f^2 \quad . \quad . \quad . \quad (9.21)$$

If W_t is now plotted vertically against values of K_f^2 horizontally the constants C and D may be obtained. The intercept upon the vertical axis (Fig. 9.29) gives C, and the slope of the line gives D.

Thompson and Walmsley (Ref. (20)) have described a method of measuring and of separating the iron losses of a transformer, using thermionic valve rectification to obtain the form factor of the voltage wave. The effects of variation of both form factor of the applied voltage and of temperature are considered in this paper.

Methods of Measurement. WATTMETER METHOD. This is perhaps the commonest method of measuring the total loss in sheet steel with alternating current. The sheet material to be tested is arranged in the form of a "magnetic square," of which there are several forms, Epstein being the originator of the arrangement. In this square there are four bundles of strips of the sheet material. These are bound with tape to form four cores to fit inside four magnetizing coils, the individual strips being insulated from one another by thin tissue paper. The ends of the cores, projecting beyond the magnetizing windings, are, in the Epstein apparatus, interleaved and clamped at the corners.

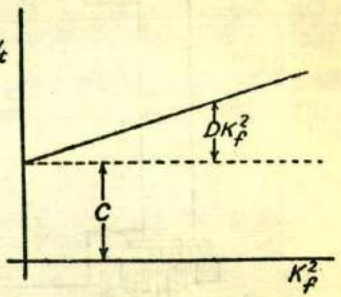


FIG. 9.29. SEPARATION OF IRON LOSSES

LLOYD–FISHER MAGNETIC SQUARE. Fig. 9.30 shows a magnetic square as used by Lloyd and Fisher and now in use at the National Physical Laboratory and Bureau of Standards. The strips of material—cut half in the direction of rolling of the sheet during manufacture and half perpendicular to this direction—are about 25 cm long and 5 or 6 cm wide. They are built up into four bundles and assembled to form a complete magnetic circuit with the aid of bent corner pieces and clamps, as shown in the figure. These corner pieces should be of the same material as the strips, or at least of material having similar magnetic properties. The overlap at the corners should be only a few millimetres, and a correction may be applied, if necessary, to account for the fact that the cross-section of the magnetic circuit is doubled at the overlapping points.

A very small number of strips is shown in the figure for the sake of clearness. The bundles of strips are placed inside four similar magnetizing coils of heavy wire, connected in series to form the primary winding. Each of these coils has, underneath it, two single-layer coils of thin wire and having equal numbers of turns. These secondary coils are connected in series in groups of four —one on each core—to form two separate and similar secondary windings.

The primary winding is connected either directly, or through a transformer having a variable secondary, to an alternator having a waveform which is as nearly as possible sinusoidal. In this way regulation of the magnetizing current by means of resistance, with consequent alteration of waveform, is avoided. If the total loss in the sample is to be measured, the connections are as shown in Fig. 9.31, the wattmeter voltage coil being supplied from one secondary winding and an electrostatic voltmeter from the other.

The iron specimen must be weighed and its cross-section determined before assembly. The magnetizing current is adjusted to give the value of B_{max} required, the frequency of the supply being

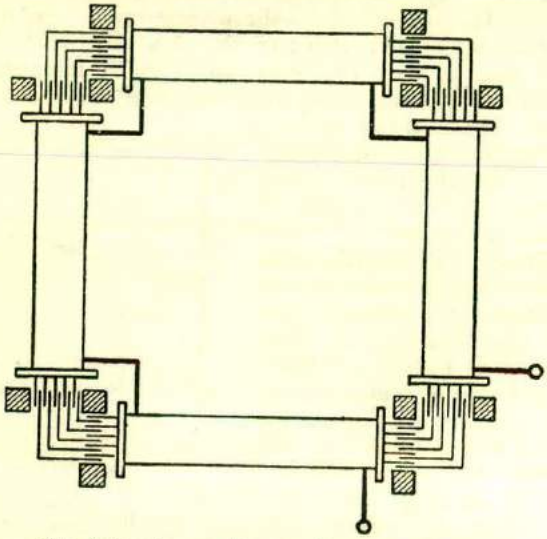


FIG. 9.30. LLOYD–FISHER MAGNETIC SQUARE

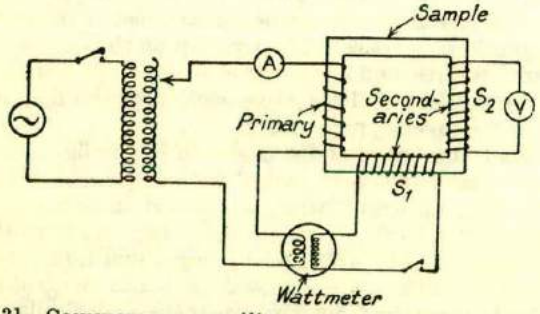


FIG. 9.31. CONNECTIONS FOR WATTMETER METHOD OF IRON-LOSS MEASUREMENT

previously adjusted to the correct value. The wattmeter and voltmeter readings are observed.

Theory of the Method. Then, the voltage induced in secondary winding S_2, whose r.m.s. value is measured by the voltmeter, is given by the expression

$$E = 4K_f \cdot B_{max} A \cdot f \cdot N_2 \text{ volts} \qquad (9.22)$$

where K_f is the form factor, A the cross-section of the specimen, in which the maximum flux density is B_{max}, f the frequency, and N_2 the number of turns on secondary winding S_2.

Hence the value of B_{max} may be obtained—

$$B_{max} = \frac{E}{4 K_f . A . f . N_2} \quad . \quad . \quad . \quad (9.23)$$

It may be necessary to correct this expression, especially at high values of B_{max}, for the fact that the coil S_2 encloses some air flux as well as the flux in the sample, since the cross-sectional area of the coil must be greater than that of the sample itself.

Let A_c = the cross-sectional area of the coil.

A_s = the cross-sectional area of the sample.

H_{max} = the maximum magnetizing force (equal to that in the air space within the coil).

B_{max} = the actual maximum flux density in the sample

Then the total flux within the coil is

$$B_{max} A_s + \mu_0 H_{max} (A_c - A_s) = B'_{max} A_s$$

where B'_{max} is the apparent maximum flux density in the sample. Thus,

$$E = 4K_f . f N_2 [B_{max} A_s + \mu_0 H_{max} (A_c - A_s)]$$
$$= 4K_f . f . N_2 . A_s B'_{max} \text{ volts}$$

where $$B'_{max} = B_{max} + \mu_0 H_{max} \left(\frac{A_c - A_s}{A_s} \right)$$

H_{max} may be determined from the permeability curve of the sample.

As regards the power loss in the iron—

Let W_i = total iron loss.

W = wattmeter reading.

V = voltage applied to wattmeter voltage coil.

E = voltmeter reading

= voltage induced in coil S_1, since S_1 and S_2 have equal numbers of turns.

r_p = resistance of wattmeter voltage coil.

r_c = resistance of coil S_1

i_p = current in the voltage-coil circuit.

Then $$E = i_p (r_p + r_c)$$
$$V = i_p r_p$$

Hence, the power loss in the iron, together with the copper loss in the winding S_1 and in the wattmeter pressure coil

$$= W \cdot \frac{E}{V} = W \frac{i_p (r_p + r_c)}{i_p r_p} = W \frac{(r_p + r_c)}{r_p}$$

$$= W \left(1 + \frac{r_c}{r_p}\right)$$

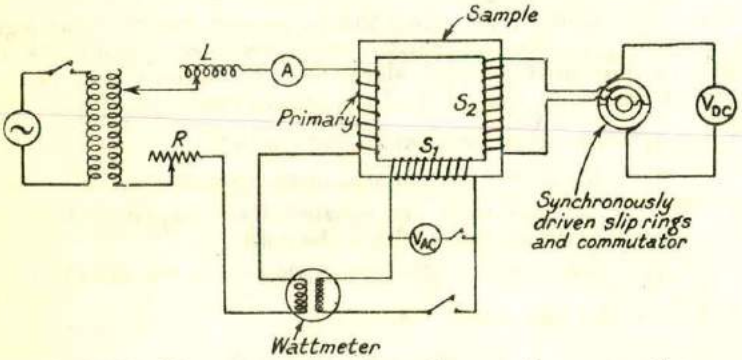


FIG. 9.32. MEASUREMENT OF POWER LOSS AT DIFFERENT FORM FACTORS

Again, the copper losses in r_p and r_c are

$$i_p^2 (r_p + r_c) = \left(\frac{E}{r_p + r_c}\right)^2 (r_p + r_c) = \frac{E^2}{r_p + r_c}$$

Therefore,
$$W_t = W \left(1 + \frac{r_c}{r_p}\right) - \frac{E^2}{r_p + r_c} \quad . \quad . \quad (9.24)$$

As mentioned previously, the hysteresis and eddy current components of the loss can be graphically determined from the results of power measurements such as the above, at different frequencies.

Fig. 9.32 gives the connections for the measurement of power loss at different form factors, the form factor being obtained from measurements of r.m.s. and mean voltages induced in the two windings S_1 and S_2, which have equal numbers of turns. Variation of form factor is obtained by adjustment of R and L—variable resistance and inductance—and of the number of turns on the secondary of the supply transformer. The voltmeter V_{AC} measures the r.m.s. voltage, and voltmeter V_{DC}, in conjunction with a synchronously-driven two-part commutator and slip rings, measures the mean, or average, voltage.

It is now a common practice to use a permanent-magnet moving-coil instrument in conjunction with a full-wave copper-oxide or

selenium rectifier bridge, in place of the synchronous rectifying commutator. This is quite satisfactory for most industrial measurements. Commercial rectifier instruments are usually calibrated in r.m.s. values for an assumed form factor of 1·11, and in this case the reading should be divided by 1·11 to obtain the mean volts.

It should be noted that the value of B_{max} in the specimen is normally derived from the reading of the rectifier voltmeter. Suppose we have any periodic non-sinusoidal flux wave-form whose maximum value is Φ_{max}. The instantaneous e.m.f. induced in the search coil S_2 is

$$e = N_2 \frac{d\phi}{dt}$$

The average e.m.f. for any time interval $t_2 - t_1$ is

$$E_{av} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} N_2 \frac{d\phi}{dt} \cdot dt$$

$$= \frac{1}{t_2 - t_1} N_2(\Phi_2 - \Phi_1)$$

where Φ_2 and Φ_1 are the fluxes corresponding to t_2 and t_1. The zeros of e.m.f. occur at the peak values of the flux wave, and in the half-period between them, the flux changes from $+\Phi_{max}$ to $-\Phi_{max}$. The time interval in this case is $\frac{1}{2f}$, where f is the fundamental frequency.

Hence

$$E_{av} = \frac{2\Phi_{max}N_2}{\frac{1}{2f}} = 4N_2 f \Phi_{max}$$

This result holds for full-wave rectification.

An essential condition, in the methods of loss separation previously discussed, was that B_{max} in the specimen should be constant. When the form factor separation method is being used, the reading on the mean voltmeter must be kept constant, and with the frequency variation method the quotient of its reading and frequency must be constant.

It was stated earlier that the r.m.s. voltmeter should be an electrostatic instrument, but the voltage normally developed in the search coils is relatively low for this type of instrument, and a suspended spotlight-reflecting instrument is usually necessary. An excellent alternative is to use a thermal voltmeter consisting of a 5 or 10 mA vacuo-thermo-junction with a series resistor in the heater circuit, and a sensitive millivoltmeter connected to the thermocouple.

The power dissipated in the instruments connected to the search

windings, if significant, must be deducted from the wattmeter reading. It is impracticable to use a moving-iron or dynamometer instrument for the r.m.s. voltmeter because of their large power consumption, which can be greater than the power loss in the specimen. In industrial work, where robustness and reliability are paramount considerations, moving-iron instruments are sometimes used in conjunction with negative-feedback amplifiers having unity gain. These devices are, in effect, impedance convertors changing the relatively low impedance of the moving-iron instrument into an effective impedance of many megohms, thus eliminating the loading effect on the square (Ref. (54)).

SINGLE-STRIP TESTING. Electrical sheet steel, as normally supplied, is guaranteed to be within a certain maximum loss figure. It is therefore of considerable assistance to a manufacturer if the loss in individual sheets can be measured immediately after manufacture and the sheets graded accordingly. A variety of apparatus has been developed for this purpose, the most recent being that due to McFarlane, Milne and Darby (Ref. (56)). This apparatus can handle sheets of 8 ft minimum length, the sheet being held in a large low-loss yoke during the test. The sheet is weighed before insertion into the yoke, and automatic adjustment is effected in the circuit so that the specific loss in watts per pound is displayed on a wattmeter. The equipment can handle between 300 and 400 sheets per hour. The paper, Ref. (56), also includes a useful list of references on single-strip testing.

A.C. BRIDGE METHODS. The a.c. bridge network can be adapted to the measurement of iron loss and effective permeability of magnetic samples. Such methods are very useful when the available samples are small, and when the test is to be carried out at commercial or audio frequencies, and with fairly low values of flux density. Fig. 9.33 shows the connections of the Maxwell bridge as applied to iron loss and permeability measurements.

R_1, R_2, and R_3 are resistors, the latter being variable. In series with R_3, in the same arm, is a variable inductor L of resistance r. It may be necessary to connect R_3 in series with the winding on the sample if the resistance of the latter is small. The specimen, in ring form, is wound with a winding whose inductance is L_s and effective resistance R_s, this effective resistance containing an iron-loss component. R_w is the actual resistance of the winding on the ring. G is a vibration galvanometer or telephone, and A an ammeter. The supply is from an alternator having a pure sine waveform.

Balance of the bridge is obtained by adjustment of L and R_3.

Theory. At balance

$$I_1 R_1 = I_2 R_2$$

and

$$I_1 (R_s + j\omega L_s) = I_2 [(r + R_3) + j\omega L]$$

where $\omega = 2\pi \times$ frequency.

Then $$\frac{I_1}{I_2} = \frac{R_2}{R_1}.$$

$$I_1 R_s = I_2 (r + R_3).$$

and $$I_1 L_s = I_2 L$$

from which $$\frac{R_2}{R_1} = \frac{r + R_3}{R_s}$$

or $$R_s = (r + R_3) \frac{R_1}{R_2}$$

and $$L_s = L \frac{R_1}{R_2}$$

The iron loss in the sample is given by $R_s I_1^2 - R_w I_1^2$.

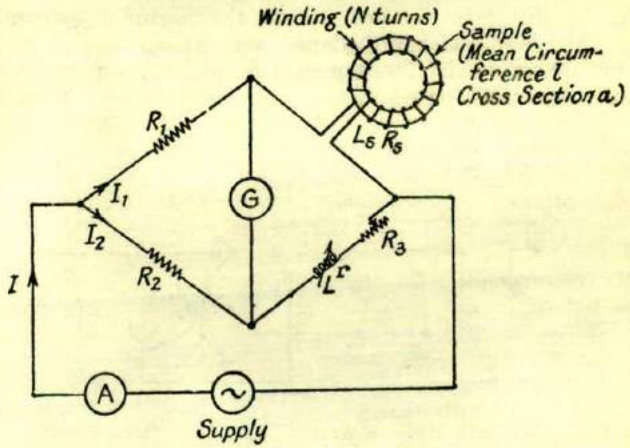


FIG. 9.33 MAXWELL BRIDGE FOR IRON-LOSS MEASUREMENTS

Now, the current $I = I_1 + I_2$ (I_1 and I_2 being in phase)

Thus $$I = I_1 + \frac{R_1}{R_2} I_1 = I_1 \left(\frac{R_1 + R_2}{R_2} \right)$$

or $$I_1 = \frac{R_2}{R_1 + R_2} \cdot I$$

Hence, the iron loss W_t is given by

$$W_t = I^2 \left(\frac{R_2}{R_1 + R_2} \right)^2 (R_s - R_w) \quad . \quad . \quad (9.25)$$

If N = No. of turns on the specimen,
l = length of mean circumference of specimen (in metres)
a = cross-section of specimen in square metres,
μ_r = relative permeability of specimen

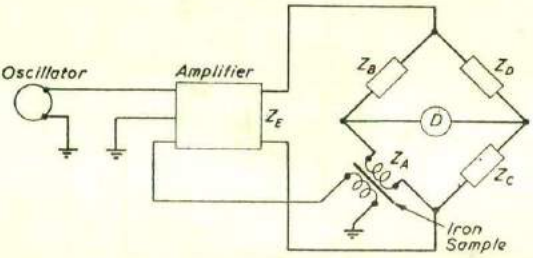
then the inductance is

$$L_s = \frac{NI_1}{l} \times \frac{N}{I_1} = \frac{N^2 a}{l} \mu_0 \mu_r \text{ henry}$$

$$a\mu_0\mu_r$$

from which μ_r may be calculated when L_s has been measured.

One of the difficulties normally encountered in the use of a.c. bridges for iron testing is due to the fact that the circuit containing the iron-cored coil consists of a linear impedance in series with a non-linear impedance (the iron-cored coil) and a sine-wave generator. This causes distortion of the flux wave-form since there is a voltage drop in the linear impedance due to the harmonic currents. In addition, the non-linear impedance acts as an energy convertor absorbing power at the fundamental frequency and supplying the harmonic power dissipated in the linear impedance. Normally, to



(*Proc. Inst. Elect. Engrs*)

FIG. 9.34. A.C. BRIDGE WITH FLUX WAVE-FORM CONTROL

obtain the true iron loss, a correction has to be made by subtracting the harmonic power $R\,(I_3^2 + I_5^2 + I_7^2 + \ldots)$, dissipated in the resistive part R of the linear impedance, from the measured fundamental power in the iron core.

J. McFarlane and M. J. Harris (Ref. (57)) have described how this difficulty may be overcome by a feedback-amplifier technique in which the flux waveform in the iron core is made to be a sine wave. The circuit used is shown schematically in Fig. 9.34. The bridge supply is obtained from an amplifier which is driven by a sine-wave oscillator, and negative feedback is provided by the voltage induced by the flux waveform in a winding on the test specimen. Provided that approximately 40 dB of negative feedback is used, the flux waveform approximates very closely to a sine wave, and in these circumstances measurement of the core impedance at fundamental frequency enables the true iron loss to be determined. Using this technique, accurate loss measurements can be made directly at flux densities up to 1·3 Wb/m², and the authors (Ref. (57)) show

how the technique can be extended to 1·7 Wb/m² by a slightly modified experimental procedure.

CAMPBELL BRIDGE METHOD. Fig. 9.35 shows the connections of a bridge method due to Campbell (Ref. (19)), this method being one of the best known for the purpose of iron-loss measurement by a bridge network.

The ring specimen carries two windings, a primary and a secondary, having N_1 and N_2 turns respectively, and having the same cross-sectional area. M is a variable mutual inductor connected as shown. R_1 and R_2 are variable resistors, and A is an ammeter. G is a vibration galvanometer or telephone according to the supply frequency. The waveform of the supply voltage should be sinusoidal, and R_1 should be made sufficiently large to ensure that the

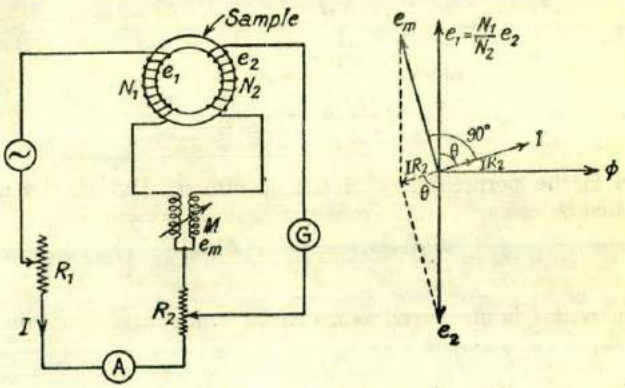


FIG. 9.35. CAMPBELL BRIDGE METHOD

current waveform is also sinusoidal. M and R_2 are adjusted until the galvanometer G shows no deflection or until the telephone gives minimum sound. Under balance conditions the vector diagram is as shown in the figure: ϕ is the flux in the sample, and I the current in the primary winding, leading the flux by a small angle on account of iron loss; e_1, e_2, and e_m are the *induced* voltages in the primary winding, the secondary winding, and the secondary of the mutual inductor M respectively. Balance is obtained when the vector sum of e_m and e_2 is equal, and opposite, to the vector IR_2 representing the voltage drop in resistor R_2.

From this vector diagram (since $e_m = \omega MI$ and $e_2 = \omega mI$, m being the mutual inductance between the primary and secondary windings on the specimen), we have,

$$e_m = e_2 \text{ (very nearly)}$$

or

$$m = M$$

and also the iron loss,

$$W_t = e_1 I \cos\theta = e_1 I \cdot \frac{IR_2}{e_2} = I^2 R_2 \frac{e_1}{e_2}$$

Now $e_1 = \frac{N_1}{N_2} e_2$; therefore the iron loss in the specimen is given by

$$W_t = \frac{N_1}{N_2} R_2 \cdot I^2 \quad . \quad . \quad . \quad (9.26)$$

Let a be the cross-section of the specimen, l its mean circumference, and a' the cross-section of the windings on the specimen.

Then, flux per ampere flowing in the primary winding, linking the two windings

$$= \frac{N_1 I}{\left[\frac{l}{a\mu + (a' - a)\mu_0}\right] I}$$

or

$$\phi = \frac{N_1[a\mu + (a' - a)\mu_0]}{l}$$

where μ is the permeability of the specimen. Hence, the mutual inductance is

$$m = \frac{N_1 N_2[a\mu + (a' - a)\mu_0]}{l} = M \quad . \quad . \quad (9.27)$$

If the current I is measured as an r.m.s. value, and is of sinusoidal waveform,

$$H_{max} \text{ in the specimen} = \frac{N_1 \cdot I_{max}}{l}$$

$$= \frac{N_1 I\sqrt{2}}{l}$$

If, also, B_{max} is the maximum flux density in the specimen, the flux threading the secondary, per ampere in the primary, is

$$\frac{aB_{max} + (a' - a)\mu_0 H_{max}}{\sqrt{2}I}$$

or

$$m = \frac{N_2[aB_{max} + (a' - a)\mu_0 H_{max}]}{\sqrt{2}I} \quad . \quad . \quad (9.28)$$

For the description of other bridge methods of iron-loss measurement, see Ref. (3).

Measurements on Iron Powders. T. H. Oddie (Ref. (44)) has described researches on the permeability and magnetic loss coefficients of carbonyl- and hydrogen-reduced iron powders, at frequencies from 200 c/s to 40 Mc/s. The method used, over the range 200 to

100,000 c/s, employs the resonance bridge circuit shown in Fig. 9.36. The powders under test were moulded in the form of (*a*) cylindrical cores with an axial hole or (*b*) toroidal cores of rectangular cross-section.

The resonance bridge network is used in combination with a beat-frequency type of oscillator-analyser. The apparent resistance and inductance of the coil containing the test core are obtained from

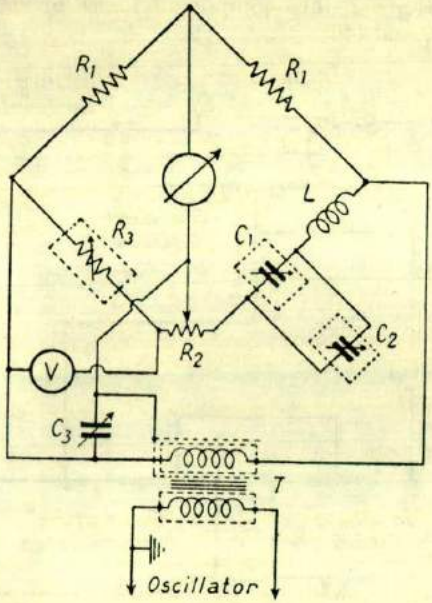


FIG. 9.36

C_1 = air capacitor, C_2 = mica capacitor, C_3 = trimmer capacitor, L = test coil
R_1 = ratio arms, R_2 = fine adjustment slide wire, R_3 = resistance box,
T = input transformer, V = valve voltmeter.

the settings of the standard capacitor, of the variable resistors and the voltage drop across R_2 for balance. These apparent values for the coil have to be corrected for losses in the mica standard capacitor, the self-capacitance of the coil (found from the apparent increase of inductance with frequency) and the small inductance due to the air path in the magnetic circuit in parallel with the core.

The corrected value of inductance is

$$L_m = 4{\cdot}61 \times 10^{-7} \cdot \mu_r \cdot N^2 \cdot h \log_{10} \frac{d_1}{d_2}$$

where N = number of turns on the coil, d_1 and d_2 are external and internal diameters of the core, h is the length of the core in metres.

The relative permeability of the core μ_r can be found from this expression. The maximum flux density is

$$B_m = \frac{2{\cdot}83 \ . I \ . L_m}{Nh(d_1 - d_2)}$$

where I is the r.m.s. current (in amperes) in the windings.

The magnetic losses in the core are found from analysis of the measured resistance R into component losses by methods given in Refs. (44), (45), and (46).

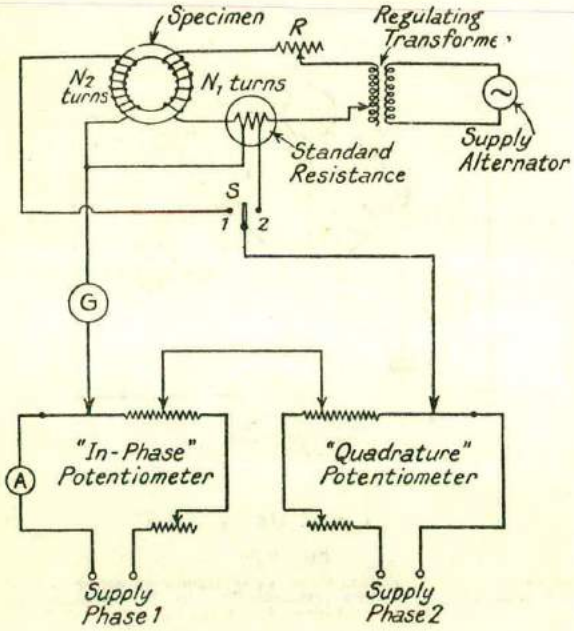


FIG. 9.37. IRON-LOSS MEASUREMENT BY A.C. POTENTIOMETER

Measurement of Iron Loss by A.C. Potentiometer. The a.c. potentiometer, which has already been described, may be used for the measurement of the power loss in samples of iron at low flux densities, and forms a very satisfactory method. The connections are given in Fig. 9.37, in which the potentiometer is assumed to be of the Tinsley-Gall pattern. This potentiometer is very suitable for such measurements, since it measures the magnetizing- and iron-loss components of the exciting current separately.

The sample—of ring form—carries two windings. The primary winding has N_1 turns and the secondary N_2 turns, the supply to the former being through a regulating transformer, from an alternator

which also supplies current to the two potentiometer slide-wire circuits. The alternator should have a sinusoidal voltage waveform, in which case the current waveform will be approximately sinusoidal if the flux density in the sample is low. A variable resistor R and a standard resistor are connected in series with the primary winding, the latter being for the purpose of measuring the current in the primary winding by measuring the magnitude and phase of the voltage drop across it. G is a vibration galvanometer.

The flux density B_{max} in the sample is obtained from the expression

$$B_{max} = \frac{E}{4K_f \cdot A \cdot f \cdot N_2}$$

where E is the e.m f. (r.m.s. value) induced in the secondary winding, A is the cross-section of the sample and f the supply frequency. If the form factor K_f is 1·11—i.e. if the supply voltage wave is sinusoidal—B_{max} is given by $\frac{E}{4{\cdot}44AfN_2}$.

The voltage E is measured by moving the switch S on to contact 1, setting the "quadrature" potentiometer at zero, and adjusting the "in-phase" potentiometer until the vibration galvanometer shows no deflection, it being assumed that the necessary standardizing adjustments of the potentiometer have first of all been made as described in Chapter VIII. The setting of the "in-phase" potentiometer for balance then gives the value of the voltage E directly. In order to satisfy this condition it is usually necessary to supply the potentiometer circuits from a phase-shifting transformer, i.e. the potentiometer is preferably of the polar-co-ordinate form.

The switch S is then thrown over to contact 2, and both the "in-phase" and "quadrature" potentiometers are adjusted to give zero galvanometer deflection. The reading of the "in-phase" potentiometer then gives the value of $I_e R_s$, where I_e is the loss component of the current in the primary winding and R_s is the value of the standard resistance. The reading of the "quadrature" potentiometer gives the value $I_M R_s$, where I_M is the magnetizing component of the primary current.

Hysteresis Loss in Small Specimens. In development work on electrical sheet steels, it may be necessary to determine both the rotational and alternating hysteresis loss in the sheet when the available samples are quite small. Owing to the considerable variation in magnetic properties in different directions relative to the direction of rolling, it is also convenient to be able to carry out tests upon the same sample with different directions of magnetization. F. Brailsford (Refs. (39) and (40)) has described methods of measurement of such rotational and alternating hysteresis losses, utilizing for the purpose a modified torque magnetometer. The samples used are in the form of discs of $1\frac{1}{4}$ in. diameter, three such

discs being used as one sample. These are mounted on a brass rod which is suspended, by a phosphor-bronze suspension, in the air gap of an electromagnet with which field strengths up to $H = 4 \times 10^4$ A/m can be obtained. Since the method of testing does not involve continuous rotation, eddy-current losses are absent.

The reader is referred to Brailsford's papers for a full description of the apparatus and for the theory of the method.

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CHAPTER X

ILLUMINATION

In the subject of illumination we are concerned with the light which is obtained from incandescent bodies of various forms. The light emitted from such bodies depends very largely upon their temperature. A body which is hotter than the medium surrounding it radiates energy into that medium. This energy is radiated in the form of heat waves only, if the temperature of the body is below that at which it becomes "red-hot." At the "red-hot" temperature the body is emitting light waves in addition to heat waves. The wavelength of the heat waves emitted at lower temperatures is comparatively great, but, as the temperature rises, the wavelength of the shortest wave which is being emitted diminishes until it enters the band of wavelengths which are classed as light waves and gives the impression of red light. At higher temperatures still, the wavelength of the shortest wave emitted from the body gives the impression of white light, when the body is said to be "white-hot." Throughout, heat waves are emitted as well as light waves. The velocity of all these radiated waves—heat and light alike—is the same, namely, 3×10^{10} cm per sec. The wavelengths of the visible light waves are between 0·00078 mm and 0·00038 mm. The ratio

$$\frac{\text{Energy radiated in the form of light}}{\text{Total energy radiated by the body}}$$

is called the *radiant efficiency* of the body, and obviously depends upon the temperature of the body. The maximum radiant efficiency is obtained at such a temperature that the wavelength of the shortest wave radiated by the body is 0·00038 mm, since further increase in temperature will only increase the total energy radiated without increasing the amount of energy which is radiated in the form of light.

Before proceeding with the consideration of the laws of illumination, some definitions of the quantities to be dealt with must be given (see Ref. (15)).

Definitions. *Light* is defined as that part of the energy, radiated from a body, which produces the sensation of light through the eye. Light is thus *energy*.

Luminous Flux (symbol F or Φ) is the *light energy radiated per second* from a luminous body. Luminous flux is therefore a form of *power*.

The *Luminous Intensity* (symbol I) in any given direction is the

luminous flux radiated per unit solid angle (i.e. the solid angular density of flux) measured in the direction in which the intensity is required. Referring to Fig. 10.1, if O is a point source of light and OX is the direction in which the intensity is required, then the luminous flux contained within the solid angle $\delta\omega$, divided by $\delta\omega$, is the intensity in the direction OX. The angle $\delta\omega$ is a very small solid angle containing the direction OX. If a is the area, at radius r, which subtends the solid angle $\delta\omega$ at O, then $\delta\omega = \frac{a}{r^2}$. If F is the flux within the solid angle, then intensity in direction OX is $\frac{F}{\delta\omega} = \frac{F}{a}r^2$.

The *Lumen* (abbreviation lm). The unit of flux is the *lumen*, which is the flux emitted in unit solid angle from a point source having a uniform intensity of one candela.

The *Candela* (abbreviation cd) is defined as the unit of luminous

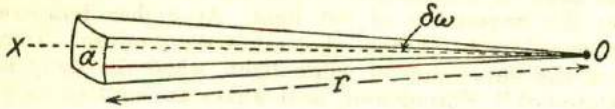


FIG. 10.1

intensity "such that the luminance of a full radiator at the temperature of solidification of platinum is 60 units of luminous intensity per square centimetre."

In the past the luminous intensity of a source was reckoned in candle-power or candles. One candela is equal to 0·982 candles.

Luminance (symbol L) which now replaces the old term "brightness" is defined, for a point of a surface and in a given direction, as "the quotient of the luminous intensity in the given direction of an infinitesimal element of the surface containing the point under consideration, by the orthogonally projected area of the element on a plane perpendicular to the given direction."

The preferred metric unit is the *nit* or candela per square metre. Another unit is the foot-lambert (ft-L) which is "the luminance of a uniform diffuser emitting one lumen per square foot." 1 nit $= \frac{1}{3{\cdot}426}$ foot-lambert.

The *Mean Horizontal Intensity* of a source of light is the average value of the intensities in all directions in a horizontal plane passing through the source of light.

The *Mean Spherical Intensity* of a source is the mean of the intensities in all directions from the source of light, and is the same as the intensity of a source of light which radiates the same total flux uniformly in all directions.

The *Lumen-hour* is also a unit of quantity and is the light emitted when a flux of one lumen continues for one hour.

The *Reduction Factor* of a source of light is the ratio

$$\frac{\text{Mean spherical intensity}}{\text{Mean horizontal intensity}}$$

for the source.

The *Illumination* (symbol E) of a surface is measured by the flux received per unit area of surface, in lumens per square foot (abbreviation lm/ft^2).

The preferred metric unit is the *Lux*, or one lumen per square metre (abbreviated lx or lm/m^2).

Laws of Illumination. (1) The illumination of a surface is inversely proportional to the square of the distance of the surface

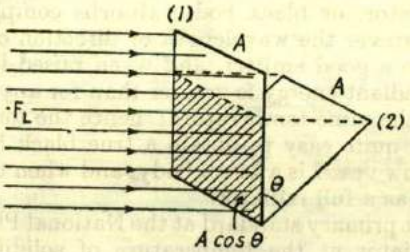


FIG. 10.2. LAMBERT'S COSINE LAW

from the source of light, provided the latter is a *point* source or is sufficiently distant from the surface to be regarded as such.

This is obviously true, since the solid angle subtended at the point source by the surface is $\frac{A}{r^2}$ (where A is the area of the surface and r its distance from the source), and hence, for a given flux per unit solid angle from the source, the flux per unit area $\propto \frac{1}{r^2}$.

(2) LAMBERT'S COSINE LAW. This law states that, if a surface is inclined at an angle $90° - \theta$ to the direction of the luminous flux, then the illumination of the surface is reduced from that given according to law 1 (above) in the ratio $\frac{\cos\theta}{1}$. From Fig. 10.2 it can be seen that, if F is the luminous flux falling upon the surface when in position 1, then the flux falling upon the surface when in position 2 will be $F\cos\theta$, where θ is the angle between the two positions. Hence, if A is the area of surface, the illumination in position 1 will be $\frac{F}{A}$, and in position 2, $\frac{F\cos\theta}{A}$.

Standards. The earliest standard was the simple candle, which was carefully specified, but which was incapable of providing the accuracy required by developments in lighting techniques. It was superseded by flame standards such as the Hefner Standard Lamp, which burnt pure amyl acetate, and the Harcourt Pentane Lamp. The first of these was the German standard and the second the British standard. These flame standards, although an improvement upon the simple candle, had many disadvantages, and subsequently, until 1948, a group of incandescent lamps, operating under prescribed conditions, were used to define the international candle.

The present primary standard was adopted in 1948 and utilizes the radiation from a specified area of a "full radiator" (black body) at a definite temperature. A fairly full description of the establishment of the Photometric Scale with this new standard is given in Ref. (22).

A full radiator, or black body, absorbs completely all incident radiation whatever the wavelength or direction of incidence. Such a body is also a good emitter, and when raised in temperature, its emission of radiant energy is greater than for any other body of the same size at the same temperature; hence the term "full radiator." It is actually quite easy to obtain a true black body since a small hole in a hollow vessel is a black body, and when the vessel is heated the hole acts as a full radiator.

The present primary standard at the National Physical Laboratory is a full radiator at the temperature of solidification of molten platinum and it is illustrated in Fig. 10.3. The radiator is a cylinder of fused thorium oxide, 45 mm long and 2·5 mm internal diameter. It is immersed in the platinum, which is contained in a fused thorium oxide crucible. The crucible is in a refractory pot packed with powdered thorium oxide to provide thermal lagging. The platinum is heated by eddy currents which are produced in it by a coil carrying alternating current at a frequency of 1·6 Mc/s. When the platinum is allowed to cool, it remains at its freezing point for a short time, sufficient to enable measurements to be taken. The candela is defined as $\frac{1}{60}$th of the luminous intensity of 1 sq. cm of this source.

This source is used to calibrate standard lamps, using a photometer head, and to facilitate the comparison the source is provided with a prism and lens as shown. The lens is required to produce an enlarged image of the aperture in the radiator on one of the comparison surfaces in the photometer head. The "transmission factor" of the prism and lens have to be very carefully measured. The transmission factor of a material is defined as the ratio of the luminous flux transmitted by it to that incident upon it.

The platinum solidifies at 2,042° K and its colour is different from most practical sources. The *colour temperature* of a source is the temperature of a black body which matches the source in

colour and emits radiation of substantially the same spectral distribution within the visible region. A tungsten filament lamp is about 2,360° K and gas-filled lamps are about 2,800° K to 3,000° K. The colour difference encountered in comparing sources of different colour temperature is overcome by the use of a special blue filter which increases the colour temperature of the source. The transmission factor of these filters is usually about 50 per cent.

Clearly this primary standard is unsuitable for day-to-day working and it is therefore used to calibrate working substandards

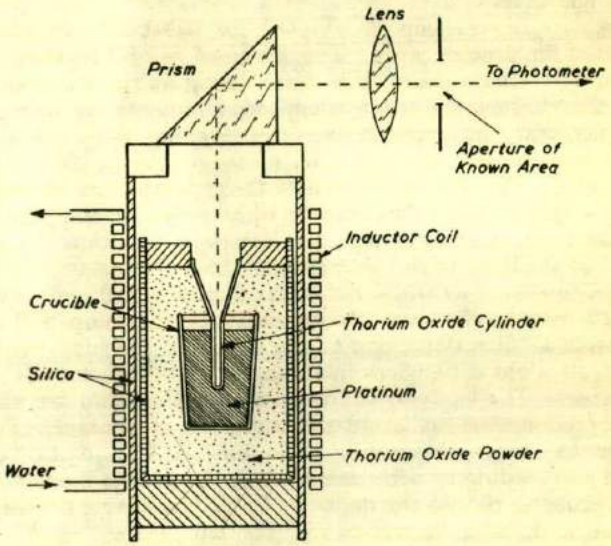


FIG. 10.3. PRIMARY STANDARD OF LUMINOUS INTENSITY

in the following manner: the primary standard is used to determine the luminous intensity of specially-made, tungsten filament lamps which are under-run to match the primary standard in colour. These lamps are then compared with other lamps which operate at a colour temperature of 2,360° K, the colour difference being overcome by means of the special blue glass filter. Lamps of this second group constitute the secondary working substandards of luminous intensity. Another group of tungsten filament lamps form the substandards of luminous flux. Their luminous intensities, in a large number of directions, are measured by comparison with the substandards of luminous intensity and the luminous flux for each lamp is deduced. Substandards of luminous flux at 2,800° K are established by comparison with those operating at 2,360° K using the blue glass filter again.

Substandards. Standards for use in general laboratory measurements must be conveniently portable and robust. Incandescent lamps are now used for this purpose, their luminous intensity being first of all obtained by comparison with an N.P.L. calibrated standard and thereafter checked from time to time against such a standard.

The principal requirement of such substandard lamps is that their intensity shall not vary appreciably with time, and that the light which reaches the photometer from the lamp shall be of a uniform character.

A special construction is adopted for substandards which are to be used in precise work. The filament is of tungsten, and is mounted in a single plane, this plane being at right angles to the axis of the photometer bench when measurements are being made. This is necessary in order to fix, definitely, the direction in which the intensity of the lamp is that measured when the lamp was standardized. For the same reason these lamps are permanently fixed in a special holder which can be attached to the photometer bench, and which ensures that the direction in which the light is radiated by the lamp to the photometer "head," or measuring device, is always the same as when the lamp was standardized. This precaution is necessary, since the intensity of the lamp will not be the same in all directions, due to the effects of the supports for the filament, to slight differences in thickness of the glass bulb, and to other causes. The leads to the terminals of the lamp are also permanently connected to avoid unsteadiness of contact. Further, in order to ensure constancy of intensity, the joints between filament and leading-in wires are welded, the bulb is made specially large in order to reduce the density of any blackening deposit upon the glass, and, after manufacture, the lamps are "aged" before standardization by being run for 100 hours or more. In use, these lamps must be run at a specified voltage, which must not be exceeded by even a small percentage, if the intensity is to remain constant.

Substandards of luminous flux differ in construction from intensity standards. The glass envelope is spherical, interrupted only by the support, and the filament approximates to a full circle. Normally the filament forms an octagonal figure with the connections at two opposing corners so that there are two parallel paths for the current. This eliminates variations in the light distribution which would be caused by a discontinuity in the filament.

In all lamps used as standards or substandards the filament supports are frosted to prevent reflections.

In order that substandard lamps shall not be used any more than is absolutely necessary, comparison lamps are often used for laboratory purposes, these lamps being such that their luminous intensity remains reasonably constant. Such lamps may be checked at frequent intervals against the substandard.

The Measurement of Luminous Intensity. For this purpose, apparatus consisting of a photometer bench and a "photometer head" is used, in conjunction with a standard or substandard lamp. The principle upon which most of the methods of measurement are based is the inverse square law. Fig. 10.4 represents, diagrammatically, two lamps S and T set at a distance apart, with some type of screen in line with, and between, them. Let S be a standard lamp (of known intensity) and T the lamp whose intensity, in the direction of the line joining it to the standard lamp, is required. If the screen is moved in between the lamps until the illumination is the same on both sides of it, then, from the inverse square law,

$$\frac{\text{Intensity of } T}{\text{Intensity of } S} = \frac{l_2^2}{l_1^2}$$

where l_1 and l_2 are the distances of the screen from S and T respectively.

The screen or detecting device, used for determining the point at

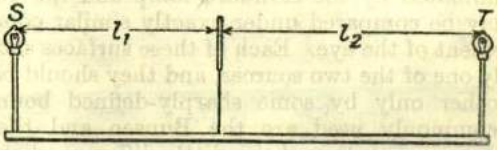


FIG. 10.4. MEASUREMENT OF LUMINOUS INTENSITY

which the lamps give equal illumination, is called the "photometer head," and the graduated bench upon which it slides is called the "photometer bench." Many different types of photometer head can be used in conjunction with one photometer bench. A number of these will be described, but, first of all, the photometer bench merits description.

THE PHOTOMETER BENCH. The most usual type of photometer bench consists of two steel rods, some three or four metres long, which carry the stands or saddles holding the two lamps to be compared, the carriage for the photometer head, and for any other apparatus which may be used in making measurements. One of the bars carries a brass strip which bears a scale graduated in millimetres. The main requirements of such a bench are that it shall be rigid—in order that the lamps being compared may be free from vibration—and that the carriage which holds the photometer head may be moved along smoothly and with very little effort. Since the method of measurement involves the *square* of distances, it is necessary that these distances shall be measured very accurately if the results of the luminous intensity measurement are to be accurate. The carriages which slide upon the bench have, except in the case

of the one which carries the photometer head, a circular table which can be rotated in a horizontal plane and clamped in any position. This table carries a scale, graduated in degrees, round its edge, for measurement of the angle of rotation. The carriages also carry sockets, etc., for the attachment of the lamp under test. Since it is very important that no light, other than that from the lamps under comparison, shall reach the screen of the photometer head, a system of opaque black screens is often used. For the same reason the photometry room is usually darkened and its walls are painted dead black.

In using the bench, the photometer head is moved along until the illumination from the two sources is the same as nearly as can be judged. To obtain the distance more exactly, two points are found at which there is a perceptible difference in the illumination from the two sides, and the point half-way between them is then taken as the position for equal illumination.

PHOTOMETER HEADS. Most photometer heads consist of some device by means of which the illumination of two surfaces, side by side—one illuminated by the standard lamp and the other by the test lamp—may be compared under exactly similar conditions and without movement of the eye. Each of these surfaces should receive light from only one of the two sources, and they should be separated from one another only by some sharply-defined boundary. The heads most commonly used are the Bunsen and the Lummer-Brodhun (contrast type), there being little difference between these two types from the point of view of accuracy of measurement. When the lamps to be compared give lights of the same, or very similar, colour, these two photometer heads are the best for the purpose, but if the colours of the two lights are appreciably different a Flicker photometer will probably give better results.

Bunsen Head. This device consists essentially of a piece of thin opaque paper which has a translucent "spot" at its centre, this "spot" being obtained by treating the centre portion of the paper with oil or wax. The device is often called the "Grease Spot Photometer." If the paper is placed between the two lamps to be compared, with its plane perpendicular to the line joining them, the opaque part of the paper will be illuminated from one source only, while the transparent part will be illuminated from both sources. The positions at which the transparent spot is no longer perceptible are then obtained, viewing the paper first from one side and then from the other. The intensity of the test lamp may then be obtained from the expression

$$I_T = I_S \cdot \frac{l_1 l_1'}{l_2 l_2'},$$

where I_T and I_S are the luminous intensities of the test lamp and standard respectively, l_1 and l_1' being the two distances of the

photometer head from the former, and l_2 and l_2' the two distances of the head from the latter.

A second method of using the Bunsen head is to illuminate one side of the paper from a fixed lamp, the photometer head also being kept in a fixed position, and then to measure the distances, on the opposite side of the paper, at which, first the test lamp, and then a standard lamp, must be placed in order to cause the grease spot to disappear. If l_1 and l_2 are these distances, then

$$\frac{I_T}{I_S} = \frac{l_1^2}{l_2^2}$$

A third, and perhaps the best, method of using the photometer head is to use two mirrors, placed behind the paper as shown in Fig. 10.5, so that the two sides may be viewed at the same time.

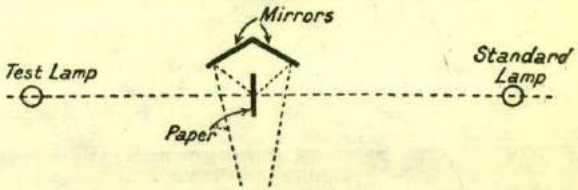


FIG. 10.5

The position of the head for equal *contrast* in illumination between the opaque and transparent portions of the paper on the two sides is then determined, and the intensity of the test lamp calculated from the same expression as before. It may be necessary to reverse the photometer head and repeat the experiment, taking the geometric mean of the two results as the correct value.

Lummer-Brodhun Photometer Head. This head may be considered as an improved "grease spot" photometer in which the spot is completely transparent. There are two types of this head, the equality-of-brightness type and the contrast type, the latter being the better, and being very generally used in photometric measurements.

Fig. 10.6 illustrates the equality-of-brightness type. In the photometer head there are two mirrors, M_1 and M_2, a plaster-of-paris screen S, and a compound prism P. This compound prism is made up of two right-angled glass prisms. The principal surface of one of these prisms is spherical but has a small flat portion at its centre, this flat surface making optical contact with the flat surface of the other prism as shown. Light entering the head from the two lamps illuminates both sides of the screen S. The light from the test lamp (say) is reflected by the screen surface upon which it falls, to mirror M_1, which again reflects it to the compound prism. Only that portion of this reflected light which falls on the flat central portion of

the spherical prism is allowed to pass through to the telescope, the rest of the light being reflected back. Again, light from the standard lamp, after being reflected from S and M_2, falls on the compound prism. That portion of its light which falls on the surface of contact of the two prisms passes through the compound prism, the rest being reflected through the telescope as shown. The effect in the telescope is to show the centre portion of a circular area illuminated by the test lamp and a surrounding area illuminated by the standard (Fig. 10.7). Thus the arrangement gives the same effect as the

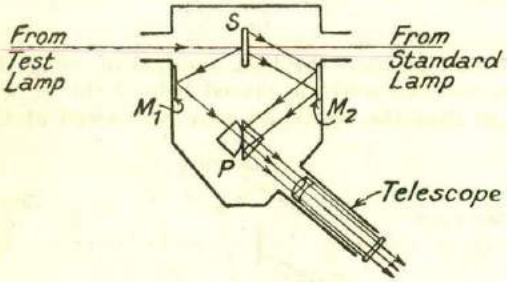


FIG. 10.6. LUMMER-BRODHUN PHOTOMETER HEAD: EQUALITY-OF-BRIGHTNESS TYPE

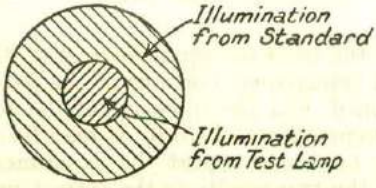


FIG. 10.7

"grease spot" photometer. The photometer head is moved until the dividing line between the centre portion and surrounding ring of light disappears. This will disappear completely only if the two lights being compared are of the same colour.

In the "contrast" type of Lummer-Brodhun head the compound prism is arranged as shown in Fig. 10.8 (*a*). The two joining surfaces of the component right-angled prisms are flat, but the left-hand one has its hypotenuse surface etched away at *a*, *b*, and *c*, so as to form a pattern as shown in Fig. 10.8 (*b*). The light falling on the compound prism from the two sides of the screen passes through the unetched portions of the joining surface and is reflected at the etched surfaces. This means that, with the weaker source of light on the right and the stronger on the left (the sources referred to being the illuminated surfaces of the screen in the photometer head),

the etched surface will be less brightly illuminated than the unetched portion, as shown by the shaded and unshaded portions in Fig. 10.8 (*b*). *G* and *G'* are sheets of glass which give a little reflected light for the purpose of maintaining some *difference* between the illumination of the etched and unetched areas in all positions of the photometer head. This difference is about 8 per cent when the photometer head is in the balance position. In this balance position the *difference* in illumination between the etched and unetched portions should be the same on each half of the circular area. Upon moving away from the balance position the contrast in illumination between area *c* and its surrounding area will become less (say), and the contrast between the illumination of *ab* and the inner trapezium

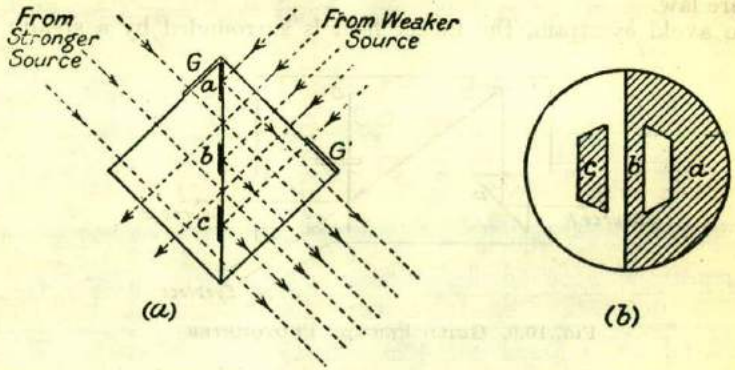


FIG. 10.8. LUMMER-BRODHUN PHOTOMETER HEAD: CONTRAST TYPE

will become greater. The balance position is, therefore, that which gives *equal contrast*, and not equal brightness.

An accuracy of within 1 per cent can be obtained with this type of head when comparing lights of similar colour, and for such a purpose it is the most accurate photometer head available.

There are many other types of photometers, amongst which are several *polarization* photometers, in which the two fields of light which are being compared are made equal by weakening one of them by the use of polarizing prisms. The Martens photometer is an instrument of this type. For descriptions of this and other types of photometers, Refs. (1), (2), (3) should be consulted.

Flicker Photometers. These photometers depend upon the fact that, if two illuminated surfaces are presented to the eye alternately, the alternations being rapid, flicker disappears when the surfaces are of equal brightness. Colour differences between the two lights under comparison do not affect these photometers to the same extent as they do photometers of the steady comparison type, since the colour difference between two alternating fields of light disappears at a lower

speed of alternation than does a difference of brightness. The speed of alternation used should be the lowest speed with which disappearance of flicker can be obtained over the smallest possible range of variation of brightness, this speed depending upon the difference in colour of the two lights.

The Guild flicker photometer is illustrated in Fig. 10.9. One light source illuminates a white diffusing screen S through a prism P. The other source illuminates a rotating sector D which also has a white diffusing surface. When the sector is rotated, the eye sees the surface S and the surface of the sector D alternately. In use, the distances of the sources are varied until minimum flicker is obtained, and the intensity of the test source is deduced from the inverse square law.

To avoid eyestrain, the flicker field is surrounded by a steady

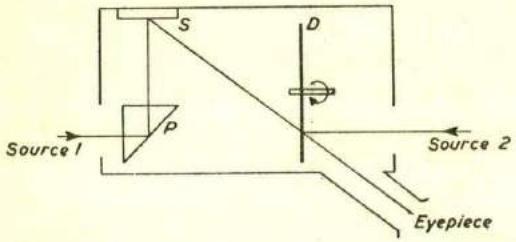


FIG. 10.9. GUILD FLICKER PHOTOMETER

field of slightly lower brightness—this avoids extreme contrast between the field and its surroundings. This is achieved by inserting a spherical diffusing enclosure in the eyepiece, the interior surface of this enclosure being illuminated by a small lamp.

H. E. Ives and E. T. Kingsbury (Ref. (7)) have investigated the subject of flicker photometers very fully, and have stated the conditions necessary for their satisfactory use.

Distribution of Luminous Intensity. In the above, the intensity of a lamp in one given direction only has been considered. It is often important that the intensity distribution in a number of directions round the source shall be known. The distribution measurements usually made are for the determination of the mean horizontal intensity and the mean spherical intensity.

MEAN HORIZONTAL INTENSITY. This may be determined for any lamp by turning the lamp about a vertical axis (by means of the rotating table on the photometer bench carriage) and measuring the intensity in the direction of the line joining the test lamp and standard after every 10° or 15° rotation. A polar curve may be plotted from these measurements, this curve being the *horizontal distribution curve* of intensity (see Fig. 10.10 (*a*)).

To obtain the mean horizontal intensity, the intensity may be plotted against degrees rotation on rectangular axes, as shown in Fig. 10.10 (*b*). The mean height of this curve gives the mean horizontal intensity. The m.h.i. may also be determined from a single measurement by using a piece of apparatus which spins the lamp round at about 200 revolutions per minute. The intensity is measured by a photometer in the ordinary way. If the speed of

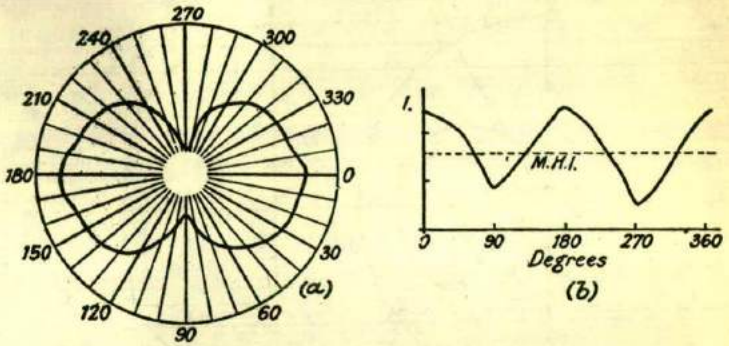


FIG. 10.10. DETERMINATION OF MEAN HORIZONTAL INTENSITY

rotation is high enough there is no appreciable flicker on the photometer head screen.

MEAN SPHERICAL INTENSITY. This may be obtained from the intensity distribution curve in a vertical plane. The vertical distribution curve of a lamp may be obtained by tilting the lamp in a vertical plane, using a specially constructed apparatus which keeps the centre of the test lamp stationary during the rotation through 180°. Measurements of intensity are then made in the ordinary way every 10° or 15°. The assumption is made that the intensity of the lamp is not altered by movement from its normal position. If this assumption is not justifiable in a particular case, measurements of intensity must be made by using an arrangement of mirrors in conjunction with a bodily movement of the lamp in a vertical direction (see Ref. (1)).

It may be necessary to make measurements in two or more vertical planes, and to obtain the curve of mean vertical distribution from these measurements, but in many cases measurements in one vertical plane only are sufficient.

The mean vertical distribution curve of the lamp having been obtained, the mean spherical intensity may be obtained graphically by one of several constructions.

Rousseau's Construction. This construction is illustrated in Fig. 10.11. The polar curve *P* is the mean vertical distribution curve for

the lamp. A semicircle of any convenient radius is drawn with the pole O of the polar diagram as centre, XY being its diameter. The line $X'Y'$ is drawn equal and parallel to this vertical diameter, and from this line are set up ordinates which are equal to corresponding radii on the polar curve (e.g. ordinate CD is made equal to radius OQ, and so on), these ordinates being set up along projections from the ends of radii of the polar curve. The curve joining the ends of

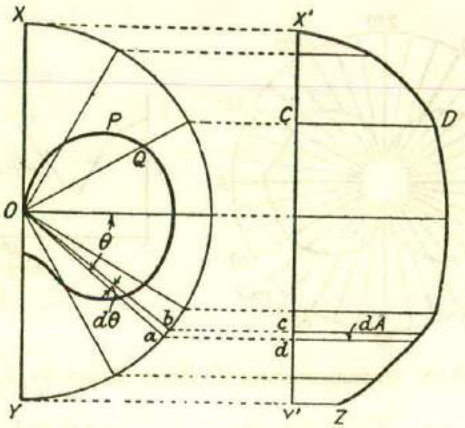


FIG. 10.11. ROUSSEAU'S CONSTRUCTION

these ordinates (i.e. curve $X'DZ$) is the Rousseau curve. The mean height of this curve $\left(\text{i.e. } \frac{\text{area } X'DZY'}{\text{length } X'Y'}\right)$ gives the mean spherical intensity of the lamp to the same scale as that to which the polar curve is drawn.

Theory.

$$\text{Mean spherical intensity} = \frac{\text{Total flux radiated}}{\text{Total solid angle}} = \frac{\text{Total flux}}{4\pi}$$

(Solid angle being defined by $\frac{\text{Area subtending the angle}}{\text{Radius}^2}$, for a sphere the solid angle is $\frac{4\pi r^2}{r^2} = 4\pi$.)

Now, consider the luminous intensity, in a direction making an angle θ with the horizontal, as in Fig. 10.11. Let this intensity be I. This intensity will apply to the surface of a zone of a sphere produced by the rotation of the small length ab about the line XY. Length ab is $rd\theta$, where r is the radius of the semicircle whose diameter is XY and $d\theta$ a very small angular element. Now, the area of this zone is $2\pi r \cos\theta \,.\, rd\theta$ or $2\pi r^2 \cos\theta d\theta$, so that the solid angle subtended by it at O is $\frac{2\pi r^2 \cos\theta d\theta}{r^2}$, i.e. $2\pi \cos\theta \,.\, d\theta$.

The flux contained within this solid angle is, by definition of luminous intensity,

$$I \times \text{the solid angle}$$

or

$$2\pi I \cos\theta \, . \, d\theta$$

Thus, the total flux radiated from the lamp is

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\pi I \cos\theta \, . \, d\theta$$

and the m.s.i. is $\frac{1}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\pi I \cos\theta \, . \, d\theta = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} I \cos\theta \, . \, d\theta$.

Now, the small length cd is equal to $rd\theta \, . \cos\theta$, and hence the area dA (to scale) is

$$Ir \cos\theta \, . \, d\theta$$

Thus the total area under the Rousseau curve is

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} Ir \cos\theta \, . \, d\theta$$

and its mean height is

$$\frac{1}{2r} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} Ir \cos\theta \, . \, d\theta = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} I \cos\theta \, . \, d\theta$$

which is the expression derived above for the mean spherical intensity of the lamp.

The mean hemispherical intensity of the lamp may be obtained by using Rousseau's construction for either the upper or lower half of the polar curve of mean vertical distribution.

The Integrating Sphere. This piece of apparatus is now commonly used for the measurement of mean spherical intensity. (See B.S. 354 : 1945, Photometric integrators.) The method of measurement consists essentially of a measurement of the total flux of light radiated by the lamp, from which the m.s.i. may be obtained by dividing by 4π. The intensities of the lamp in *all* directions from the source are taken into account in this apparatus, which thus gives a better value of m.s.i. than the method previously described, which is based upon the assumption that the intensity distribution is the same in all vertical planes—an assumption which may not always be justifiable.

The integrating sphere consists of a hollow sphere, a metre or more in diameter, the inside surface of which is painted with a white paint which is as perfectly diffusing as possible. The sphere contains a small window of translucent glass which is illuminated by reflection from the inner surface of the sphere when a lamp is placed inside. Light is prevented from reaching the window directly from the lamp by the interposition of a small screen in between the two.

It can be shown that the illumination of every part of the internal spherical surface is the same (Refs. (1), (2), (3)). This illumination is proportional to the total flux of light emitted by the lamp, and hence to the m.s.i. of the lamp.

To measure the mean spherical intensity of a lamp, the lamp is placed inside the sphere and the brightness of the glass window measured. A substandard lamp, whose m.s.i. is known, is then substituted for the test lamp, and the brightness of the window again measured. The mean spherical intensities of the two lamps are proportional to the corresponding brightnesses of the sphere window.

The measurements of window brightness may be made by the use of some form of illuminometer to compare the brightness of the window with that of a surface whose illumination can be varied and is known. A Lummer-Brodhun photometer head can also be used for the purpose. A mirror is placed so as to reflect light from the sphere window into the compound prism of the photometer, and balance is obtained, using a lamp on the other side of the photometer for comparison purposes. Balance is obtained (by moving the comparison lamp) first with the substandard lamp in the sphere and then with the test lamp inside, the squares of the corresponding distances of the comparison lamp from the photometer head being inversely proportional to the mean spherical intensities of the lamps.

Ulbricht, in 1900, first suggested this use of the internally illuminated sphere, and it is therefore named, after him, the Ulbricht sphere.

The arrangement, together with the photometer for brightness measurement, is shown in Fig. 10.12.

Simple Theory of the Integrating Sphere. Let the indirect illumination on the surface of the sphere be E lux, then the total reflected flux on the inside is $4\pi R^2 E$, where R is the radius of the sphere in metres. Suppose the direct flux is F lumens, and the reflection factor, defined as the ratio of the reflected to incident luminous flux, is ρ. Then the flux absorbed is

$$(F + 4\pi R^2 E)(1 - \rho)$$

Now, the flux emitted = the flux absorbed, and therefore

$$F = (F + 4\pi R^2 E)(1 - \rho)$$

from which

$$E = \frac{\rho F}{4\pi R^2 (1 - \rho)}$$

Thus the indirect illumination E on the surface is directly proportional to the luminous flux of the source.

Illumination Photometers. These photometers are used for the measurement of the illumination of surfaces. They may be required to measure the illumination in rooms or in streets, and should therefore be, in general, portable instruments. The general principle of such instruments is that of the comparison of the illumination of a white matt surface placed at the point at which the

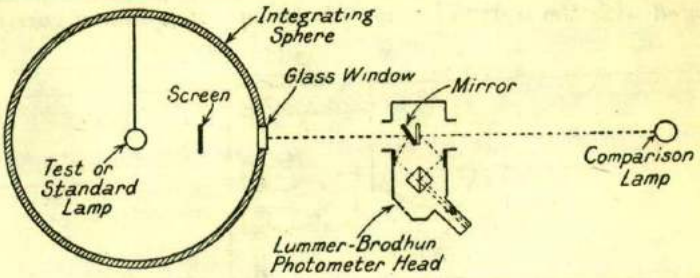


FIG. 10.12. MEASUREMENT OF MEAN SPHERICAL INTENSITY BY MEANS OF THE INTEGRATING SPHERE

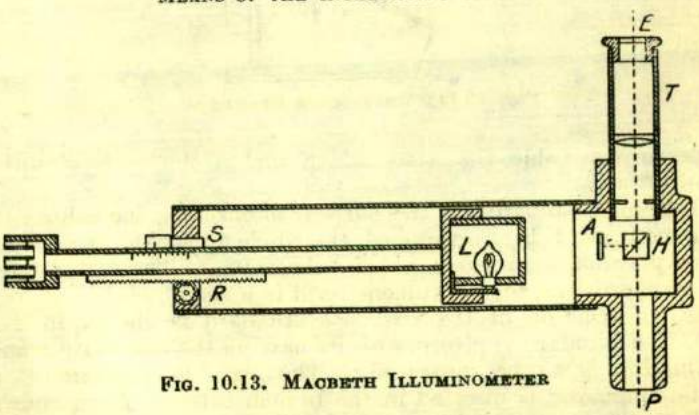


FIG. 10.13. MACBETH ILLUMINOMETER

illumination is to be measured, with the illumination of another surface forming part of the instrument, whose illumination is known and variable.

MACBETH ILLUMINOMETER. The operation of this instrument is based upon the inverse square law, and it is one of the most accurate instruments of this type. The construction of the instrument is shown in Fig. 10.13.

A Lummer-Brodhun compound prism H is mounted in a box and viewed through a telescope T. On the opposite side of the prism from the telescope is a tube P, which is pointed towards a test plate of opal glass, situated at the point at which the illumination is to be

measured. A is a screen of opal glass, which is illuminated by a lamp L in an enclosure having an aperture in the side nearest the screen A. The lamp is moved along a tube which is at right angles to the telescope, by a rack and pinion R, until balance is obtained at the photometer head. A scale S then gives the illumination of the test plate directly. The lamp L is supplied from a dry battery or accumulator, and a milliammeter and rheostat are provided with the instrument for adjustment of the lamp current

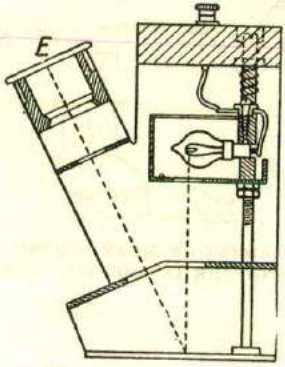


FIG. 10.14. REFERENCE STANDARD

to its correct value (i.e. that which makes the scale S direct reading).

A reference standard, for the purpose of checking the calibration of the instrument, is also supplied, the whole apparatus being made up in a portable form which includes rheostats, milliammeter, reference standard, and the instrument itself in a small case

The construction of the reference standard is shown in Fig. 10.14. The standard is placed with its base on the test plate whose illumination is to be measured. The sighting aperture P of the illuminometer is inserted in the branch tube at E as shown. The lamp in the standard is supplied from the same battery as the illuminometer lamp, and its current is adjusted to give some known value of illumination (corresponding to the lamp current) on the test plate. The illuminometer lamp is then moved until its scale reading is that corresponding to the test-plate illumination, and its current is adjusted until balance is obtained at the photometer head with the illuminometer lamp in this fixed position. The current so obtained is then maintained in the illuminometer lamp during the illumination measurements. The normal range of this illuminometer is 1 to 25 lm/ft^2.

HOLOPHANE LUMETER. Fig. 10.15 shows the schematic arrangement of the "Lumeter," while Fig. 10.16 gives a view of the inside

of the instrument itself. When the photometer is used the brightness of an internal white comparison surface, illuminated by light from a lamp inside the box, is adjusted, by rotating a disc having a sector-shaped opening, until it matches the brightness of the test surface. The light from the lamp, whose output is maintained at a fixed value by a resistor, is diffused by an opal glass window. Colour filters may be interposed between this window and the comparison surface to match illumination by yellow or blue-green light.

One scale of the instrument is marked to read the illumination of the test surface in lumens per square foot and another is graduated

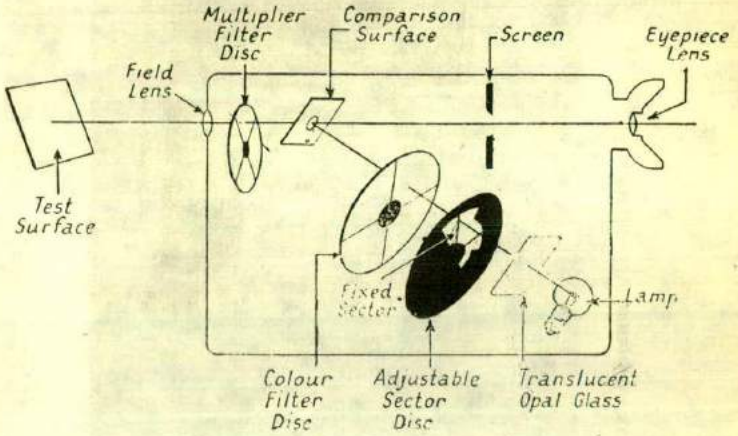


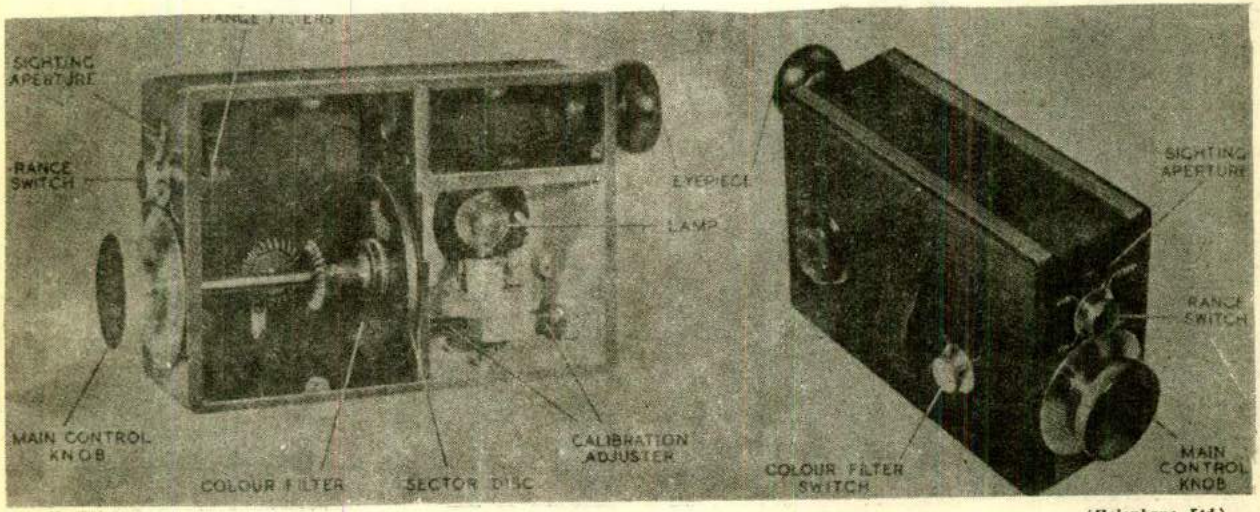
FIG. 10.15

to read the luminance (brightness) of this surface in foot-lamberts. The range of illumination which can be covered is from 0·002 to 8,000 lumens per square foot.

Physical Photometry. Photometric measurements which do not depend upon the human eye for the judgment of equality of brightness utilize a photo-electric cell.

A photo-electric cell is a piece of apparatus which, when connected in an electric circuit, allows a current to pass only when light falls upon the cell. The magnitude of the current varies also with the intensity of the light.

A difficulty experienced with many of the earlier types of photo-electric cells was that the relationship between the photo-electric current and the luminous flux falling on the cell was not linear, nor did it remain constant over a long period. More recently the construction of such cells has been improved to overcome this difficulty, and photo-electric methods of testing lamps for commercial purposes have been developed.



(Holophane Ltd.)

FIG. 10.16. HOLOPHANE LUMETER

A circuit for the measurement of luminous flux by a bridge, or null, method is shown in Fig. 10.17. *P.C.* is a photo-electric cell and V_1 and V_2 are amplifying valves. The bridge being initially balanced with no light falling on *P.C.*, the introduction of light will produce a voltage across the grid leak *G.L.* which unbalances the bridge. Balance is restored by applying a voltage from the potential divider *P.D.*, this voltage being directly proportional to the photo-electric current, and therefore to the luminous flux falling on *P.C.* (assuming *P.C.* to have a linear current/luminous-flux characteristic).

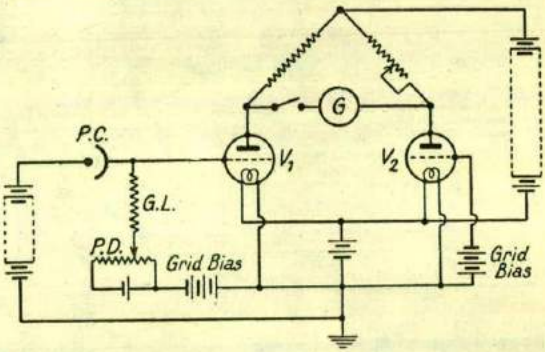


FIG. 10.17

P.D. is thus calibrated in lumens. The lamp under test is placed inside an integrating cube, the aperture of which allows light (directly proportional to the total lumens of the lamp under test) to fall on the photocell which is contained within a small adjoining integrator.

For consistent results the insulation of the grid circuits of V_1 and V_2 must be kept very high and constant and the apparatus must be screened from stray electrostatic and electromagnetic disturbances.

Fig. 10.18 shows a modern light meter (*Megatron* Type C) which is suitable for street lighting or interior lighting measurements. The light-sensitive device is a selenium barrier-layer photo-electric cell coupled directly to a sensitive indicating instrument. These instruments normally have three ranges, typical values being 0–5, 0–50, and 0–150 lux, but ranges up to 0–1,000 lux are available. The light-sensitive cell can be positioned at some distance from the indicating instrument so that the operator does not affect the incident light on the cell when taking readings. It is essential that the photo-electric cells used in light meters should integrate correctly all the light flux incident on them; such cells usually have a cosine error due to light at large incident angles from the vertical being partially reflected from the lacquered surface of the cell. The instrument illustrated is "cosine-corrected," the correction being

effected by using an unlacquered cell covered with a Perspex hemisphere. This arrangement gives a drop in response of 10 per cent for an incident angle of 80° as compared with the 35 per cent drop encountered with an uncorrected cell.

Weston exposure meters and *Eel* photometers (Evans Electro-selenium, Ltd.) also use "Photo-voltaic" or barrier-layer photocells. The construction and principle of the Weston *Photronic* cell of the barrier-layer selenium-rectifier type are shown in Fig. 10.19 (*a*) and (*b*). The cell, which is fitted behind a glass window, consists of a

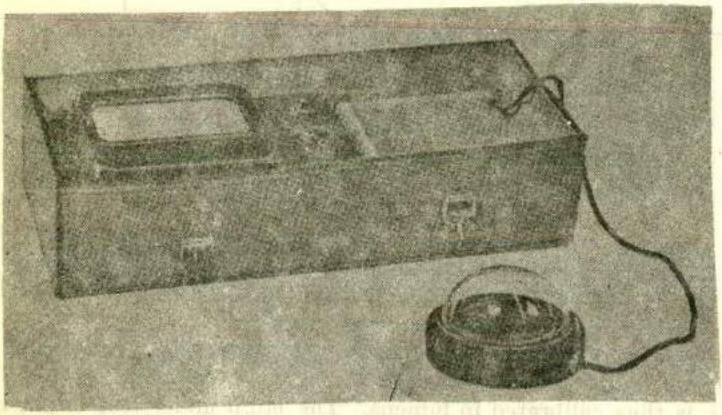


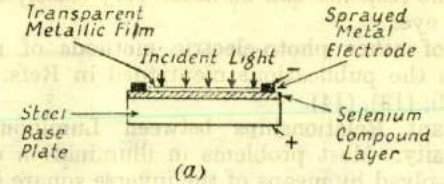
FIG. 10.18. COSINE-CORRECTED LIGHT METER

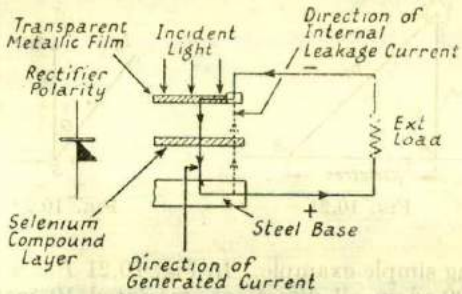
layer of a selenium compound prepared on a metal base plate. A transparent metallic conducting film is deposited on the layer, which is first heat-treated to convert it into a sensitive crystalline state. A metallic ring is sprayed on to the top surface for ease of contact. The conducting film, within this ring, is finally sprayed with a thin film of transparent lacquer.

Light, falling on the cell, causes a movement of electrons from the selenium compound surface which acts as a photo-sensitive cathode. The electrons are collected by the transparent metal film—the anode —and passed to the external circuit. This is equivalent to an electric current flowing into the ring as shown in Fig. 10.19 (*b*)

Such cells have a non-linear voltage/current characteristic and act as rectifiers. The steel base acts as the positive pole of the cell and there is internal leakage current as shown. The e.m.f. generated in the cell is directly proportional to the intensity of illumination, but the output current is proportional to this illumination only when the external resistance is low compared with the internal resistance. A range of cells gives an output, at 40 lm/ft^2 and 1,600 ohms external resistance, of 80–90 μA to 130–140 μA.

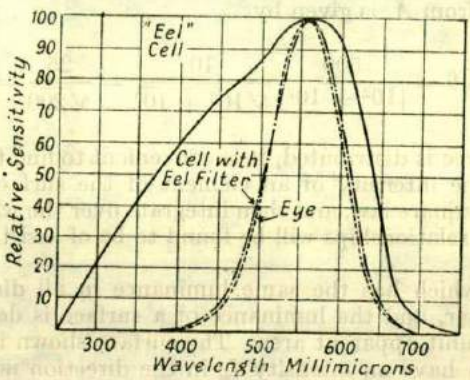
Since the barrier layer has a large capacitance (about 0·1 μF) this type of cell cannot be used in pulsating light over 10,000 cycles





(Sangamo Weston, Ltd.)

FIG. 10.19



(Evans, Electro-selenium, Ltd.)

FIG. 10.20. SPECTRAL RESPONSE OF THE *Eel* PHOTO-CELL

per second; the external resistance must in any case, be low with pulsating light to avoid serious reduction of response due to the shunting effect of this capacitance.

The spectral response of the *Eel* cells is shown in Fig. 10.20. With a correction filter, having a transmission factor of about 50 per cent for white light, the response can be made very closely in agreement with that of the eye.

Descriptions of other photo-electric methods of measurement will be found in the publications mentioned in Refs. (1), (2), (3), (5), (9), (11), (12), (13), (14).

Some Important Relationships between Luminous Flux and Luminous Intensity. Most problems in illumination due to point sources may be solved by means of the inverse square law as shown

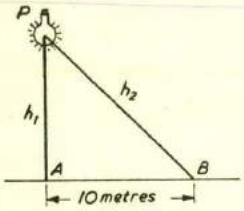


FIG. 10.21

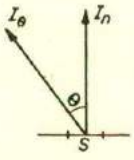


FIG. 10.22

by the following simple example. In Fig. 10.21 P is a point source, of intensity 500 cd in all directions, mounted 10 metres above a road. Then the illumination at a point A, directly under the lamp, is given by $E_A = \frac{I}{h_1^2} = \frac{500}{100} = 5$ lux. The illumination at a point B, 10 metres from A, is given by

$$E = \frac{I}{h_2^2}\cos\theta = \frac{500}{[10^2 + 10^2]}\frac{10}{\sqrt{10^2 + 10^2}} = \frac{25}{\sqrt{200}} = 1{\cdot}77 \text{ lux}$$

When the source is distributed, it is convenient to find the illumination due to the intensity of an element of the surface by means of the inverse square law, and then integrate over the whole surface. The following relationships will be found to be of assistance in such calculations.

A surface which has the same luminance in all directions is a uniform diffuser, and the luminance of a surface is defined as the intensity per unit apparent area. The surface shown in Fig. 10.22 is assumed to have an intensity I_n in the direction normal to the surface, and the intensity at angle θ to the normal is I_θ. If S is the area of the small element of the surface, then the luminance in the direction θ is

$$L = \frac{I_\theta}{S\cos\theta}$$

but, since the surface is a uniform diffuser, the luminance is the same in all directions. Hence

$$L_\theta = L = \frac{I_n}{S}$$

Therefore $I_\theta = I_n \cos\theta$.

If this intensity distribution is plotted on a Rousseau diagram it will be found that the flux, in lumens, of a plane uniform diffuser is given by

$$\Phi = \pi I_n$$

A fluorescent lamp may be considered to be a uniformly diffuse cylinder, and again we may take the intensity perpendicular to the axis of the cylinder to be I_n. Then the intensity at an angle θ to the perpendicular is again $I_\theta = I_n \cos\theta$. Using the Rousseau diagram it will be found that in this case the flux emitted by the cylinder is

$$\Phi = \pi^2 I_n$$

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CHAPTER XI

HIGH-VOLTAGE MEASUREMENTS AND TESTING

General Classification. The rapid advance in the use of high voltages for transmission purposes, during the last few years, has necessarily been accompanied by extensive research upon high-voltage phenomena. The subject of high voltage testing is, therefore, a very important one at the present time, and includes a large variety of testing methods.*

These may be classified as follows—

1. Sustained low-frequency tests.
2. Constant direct-current tests.
3. High-frequency tests.
4. Surge, or impulse tests.

Purpose of Various Test Methods. 1. SUSTAINED LOW-FREQUENCY tests are the commonest of all high-voltage tests. The frequency used is almost always the standard frequency—50 cycles per second in Britain and most European countries, and 60 cycles per second in America.

Voltages of 2 or 3 kilovolts (50 c/s) are used for routine voltage tests upon motors, switchgear, and other apparatus after manufacture, and in some cases after installation, in accordance with specifications of the British Standards Institution.

British Standard Specification 116 : 1952, "Oil Circuit-breakers for Alternating Current Systems," states that when such apparatus is tested at the maker's works, the test voltages for the various components shall be as given in Table XI.

"The test voltage shall be alternating, of any frequency between 25 and 100 cycles per second, and approximately of sine waveform."

Other tests are specified for the apparatus after erection on site.

Low-frequency tests are made upon specimens of insulation material for the determination of their dielectric strength and dielectric losses, for the routine testing of supply mains, and for works tests upon high-voltage transformers, porcelain insulators, and other apparatus. Voltages of 2,000 kilovolts and upwards are used for research work and for the testing of porcelain insulator strings and of high-tension cables.

2. CONSTANT DIRECT-CURRENT TESTS. To transmit large amounts of power efficiently, high transmission voltages are necessary.

* Owing to the impossibility of covering, fully, the whole ground of the subject in a single chapter, an extensive list of recent papers on high-voltage measurements is given at the end of the chapter. These should be referred to for more detailed information.

TABLE XI

Component	Rated Service Voltage	Test Voltage		
		All Circuit-breakers for Systems up to 88 kV	Circuit-breakers for Non-effectively Earthed Systems of 110 kV and above	Circuit-breakers for Effectively Earthed Systems of 110 kV and above
	kV	kV	kV	kV
Circuit-breaker including any series coils	Up to 0·66	2	—	—
	3·3	9·5	—	—
	6·6	17	—	—
	11	27	—	—
	22	52	—	—
	33	76	—	—
	44	100	—	—
	66	150	—	—
	88	200	—	—
	110	—	250	200
	132	—	300	250
	165	—	365	300
	220	—	485	400
	275	—	—*	485
Neutral earthing circuit breaker	1·91	6·5	—	—
	3·81	10·5	—	—
	6·35	16	—	—
	12·7	31	—	—
	19·1	45	—	—
	25·4	60	—	—
	38·1	88	—	—
Operating coils and small wiring	—	2	2	2
Operating motors	In accordance with B.S. 168 and B.S. 170.			

* All 275-kV systems are considered for this specification to be effectively earthed.

Overhead lines have been erected with a working voltage of 500 kV (and up to 750 kV experimentally) while cables for 275 kV have been made and tested.

The electricity supply regulations state that no extra-high-voltage main shall be brought into use "unless, after it has been placed in position, and before it is used for the purposes of supply, the insulation of every part thereof has withstood the continuous application, during half an hour, of pressure exceeding the maximum pressure to which it is intended to be subjected in use, that is to say, in the case of every electric line to be used for a pressure not exceeding 10,000 volts, twice the said maximum pressure, and in the case of a line to be used for a pressure exceeding 10,000 volts, a pressure exceeding the said maximum pressure by 10,000 volts; and the undertakers shall record the results of the tests of each main or section of a main."

If such voltage tests were carried out with an alternating high-voltage supply, the transformer for the purpose would have to be of

inconveniently large capacity, owing to the heavy charging current taken by such lengths of cable. Transport difficulties would be encountered, also, in moving the testing transformer to the site of the cable system, and for this reason high-voltage direct current is used for testing purposes. The plant for such a supply need only be of small capacity, and may be constructed in a portable form.

High-voltage direct current is also used in research work upon insulation and upon X-rays and electrical precipitation.

3. High-frequency Tests. Such tests, at frequencies varying from several thousand cycles per second to a million or more cycles, are important in work upon insulators for radio purposes where

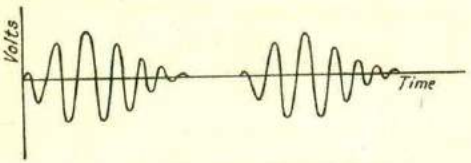


Fig. 11.1. Damped Oscillations

such frequencies are usual. It has been found, also, that in the case of porcelain insulators in service on transmission lines, breakdown, or flashover, occurs in most cases as a result of high-frequency disturbances in the line, these being due either to switching operations or to external causes. Thus, insulators which may be satisfactory under high voltage of low frequency are not necessarily satisfactory under the different conditions existing when the frequency is of a much higher order.

It has been found that undamped high-frequency oscillations—i.e. oscillations of approximately constant amplitude—cause failure of insulators at comparatively low voltages, due to high dielectric loss and heating. Such oscillations do not, however, occur in power systems in practice. Damped high-frequency oscillations having a waveform such as that shown in Fig. 11.1 do occur in practice, due to switching operations, or to arcing grounds.

For this reason, high-voltage tests at high frequencies are made in insulator manufacturing works, in an endeavour to obtain a design of insulator which will satisfactorily withstand all conditions in service.

4. Surge, or Impulse Tests. These tests are carried out in order to investigate the influence of "surges" in a transmission line upon breakdown of the line insulators, and of the end turns of transformers connected to the line. In order to appreciate the importance of such tests and to realize what are the requirements of the testing plant, the nature of these surges must be understood.

Such surges are usually produced by lightning in the neighbourhood of the transmission line, and are the result of the sudden discharge of the electricity on the suspended particles of a thunder cloud, which takes the form of a lightning stroke A direct lightning stroke on the line is comparatively rare, but a charged cloud above, and near to, the line, will induce charges, of opposite sign to that of its own charge, in the line. These charges are "bound" (i.e. held in that portion of the line nearest the cloud) so long as the cloud remains near without discharging its electricity by a lightning stroke. If, however, the cloud is suddenly discharged—as it is when the lightning stroke occurs—the induced charges in the line are no longer bound, but travel, with the velocity of light, along the line to equalize the potential at all points of the line.

This means that a steep-fronted voltage wave travels along the

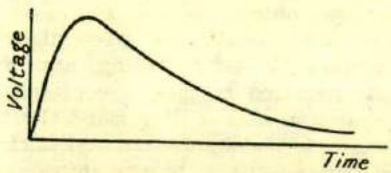


FIG. 11.2. SURGE WAVEFORM

line, its form being as shown in Fig. 11.2.* Due to this travelling wave the potential of any point along the line will rise very suddenly from its normal value by an amount equal to the amplitude of the wave. The time taken for this voltage rise is of the order of one millionth of a second. The effect is to impose very severe stresses upon the line insulators and upon the transformer windings (if the wave is allowed to reach them), the stresses depending upon the steepness of the wave front.

Violent rupture of insulators is often caused by such surges, and the investigation of their effect upon different types of insulators and other apparatus, as well as the testing of surge-absorbers designed for transformer protection, is obviously of great importance.

The shapes of voltage waves for impulse testing have now been standardized in several countries. Thus in Britain, in B.S. 923 : 1940 the standard testing wave is expressed as "1/50 microsec," which indicates that it is to rise to its peak in 1 microsecond and is to fall to one-half of its peak value in 50 microseconds.

The testing plant for this purpose must be capable of producing a high, unidirectional voltage, suddenly applied to the apparatus under test.

Testing Apparatus. Certain apparatus is common to all types of high-voltage tests. Special apparatus is required, in addition, for

* See also Refs. (8), (9), (10), (11), (12).

direct-current, high-frequency, and impulse tests. For all tests, the following apparatus is usually essential.

(*a*) A high-voltage transformer.
(*b*) Apparatus for voltage regulation.
(*c*) Control gear and high-voltage connections, including safety protective devices.
(*d*) Apparatus for voltage measurement.

(*a*) HIGH-VOLTAGE TRANSFORMERS. Such transformers, for testing purposes, require careful design, owing to the fact that they are subjected to transient voltages and surges during their normal operation when the insulation under test breaks down. To withstand the internal stresses set up by such disturbances the transformer insulation must be carefully proportioned. The transformers are usually single-phase core type, and are oil-immersed, Bakelite cylinders being used for insulation between high- and low-voltage windings. Transformers for cable testing, and certain other purposes, may also be required to give a considerable current, and the question of regulation and cooling must therefore be carefully considered. In insulation testing the current taken from the transformer, when the test specimen breaks down, is limited by the insertion of water resistances in the test circuit, so that for such purposes the transformer need not have a large kVA capacity. The following table gives the order of the capacities required for various purposes, and also the approximate values of maximum voltage used.

TABLE XII

Purpose of Transformer	Approximate Capacity (kVA)	Maximum Voltage (kV)
Routine voltage tests upon motors and switchgear	Small	2 or 3
Insulation testing	10 to 20	50
Routine testing of cables	50	10 to 30
High-voltage transformer and porcelain insulator testing and research	20 to 50	100 to 200
Research and testing of strings of insulators	½ to 1 kVA per kilovolt	500 to 3,000
High-voltage cable testing	100 to 500	100 to 500

Equipments giving a maximum voltage up to 500 kV have usually a single high-voltage transformer. For the higher voltages—1,000 kV and upwards—two or more transformers are generally

used, connected *in cascade*—a method of connection due to Prof. Dessauer. This arrangement is found to be more convenient than the single transformer for very high voltages, owing to the very large size and high cost of a single transformer for the purpose. A common method of connecting transformers in cascade is shown in Fig. 11.3. The low-voltage supply is connected to the primary winding of transformer I, the tank of which is earthed. One end of the h.v. secondary winding of this transformer is connected to

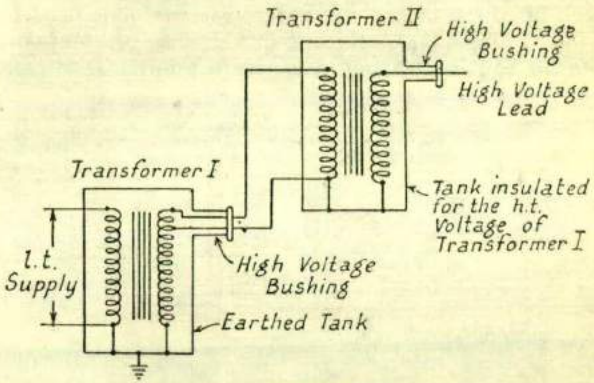


FIG. 11.3. CASCADE CONNECTION OF TRANSFORMERS

the earthed tank From the other end of the secondary winding, a lead passes through a high-voltage bushing, which provides insulation for the full secondary voltage between this lead and the tank. Through this bushing also runs a second lead from a tapping point on the secondary winding, the voltage between this tapping point and the high-voltage end of the secondary winding being that required for the primary winding of transformer II. One end of the secondary winding of transformer II is connected to its tank, which is insulated from earth for the full secondary voltage of transformer I. The other end of the secondary winding provides the high-voltage terminal of the equipment, the total voltage—to earth—obtainable being approximately the sum of the secondary voltages of the two transformers.

(*b*) VOLTAGE REGULATION. In order to avoid surges on the high-voltage side of the transformer, and also for accuracy of voltage measurement, it is essential that the regulation of voltage shall be smooth. Sudden variations of the testing voltage *must* be avoided in most tests. Another requirement of voltage-regulating apparatus is that it shall not cause distortion of the voltage waveform.

The transformer secondary voltage is regulated by variation of

the voltage applied to its primary winding. This may be done either—

(i) By variation of the alternator field current, or
(ii) By insertion of either resistance or inductance in the supply circuit from the alternator, or
(iii) By means of an induction regulator, or
(iv) By means of a tapped transformer.

(i) *Variation of the Alternator Field Current.* This method can only be used, of course, when a separate alternator is used for the supply to the testing plant. Except in the case of comparatively small plants for routine testing, a separate alternator is generally

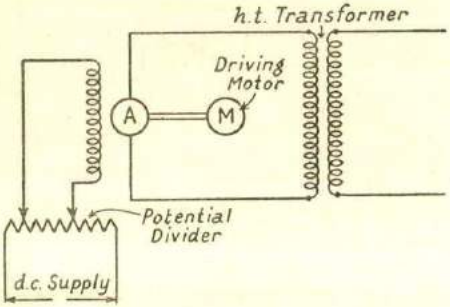


FIG. 11.4. ALTERNATOR FIELD CURRENT METHOD OF VOLTAGE CONTROL

used. The alternator should have a voltage waveform which is as nearly as possible sinusoidal on no load, and the distortion under load conditions should be small. This is achieved by making the air gap of the alternator large; by special design of the armature windings; and by suitable regulation of the number, and shape, of the slots.

This method of voltage regulation has the advantages of smooth control of the voltage from zero to the full voltage, absence of impedance for regulation purposes in the transformer primary circuit (which may produce waveform distortion), good waveform, convenience, simplicity, and freedom from appreciable disturbance by frequent short-circuiting of the transformer during testing.

The field current of the alternator is varied directly in the case of fairly small machines, but when a large alternator is used the field current of the exciter may be varied, thus varying the alternator field current indirectly. Fig. 11.4 shows the connections for direct variation of the alternator field current. A potential divider, connected across a steady d.c. supply, is used, and the connections

are arranged so that a small field current in a reverse direction to the normal one may be obtained. By this arrangement zero voltage may be obtained by neutralization of the residual magnetism in the field. Smooth and gradual voltage regulation is obtained by a special design of the potential divider, which should have a large number of turns and some provision for very gradual and steady movement of the sliding contact.

(ii) *Resistance, or Inductance, Control.* When a separate alternator is not used—in the case of small equipments—resistance or inductance must be inserted in the a.c. supply circuit for voltage regulation.

If resistance is used, it is better to make the connections as in

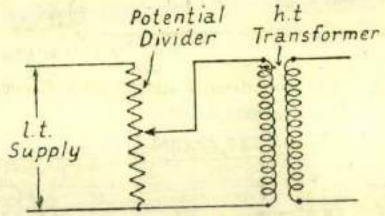


FIG. 11.5. POTENTIAL DIVIDER-CONTROL

Fig. 11.5, using the resistor as a potential divider, than to insert it in series with the transformer primary winding. Zero voltage can be obtained in this way. A slider resistor should be used for smooth voltage regulation.

The disadvantages of the resistance method are the loss of power in the resistor and the impracticability of its use for high powers, owing to the very large size and great cost of resistors for the purpose. For small equipments (2 or 3 kVA) it has the advantages of cheapness, convenience, smooth voltage variation, and small distortion of the voltage waveform.

A choke coil, connected in series with the transformer primary, may be used instead of resistance. Voltage regulation is then obtained by the withdrawal or insertion of the iron core of the coil. This method has the advantage of greater efficiency than the resistance method, on account of its low power factor, but has several serious disadvantages. These are—

(*a*) the large size of coil required if the power to be dealt with is high;

(*b*) the distortion of waveform owing to the iron core;

(*c*) the fact that increase of its inductance will *increase* the primary voltage of the transformer instead of decreasing it if the power factor of the load on the secondary side of the testing transformer is leading, as is often the case.

(iii) *Induction Regulator Method.* This method of voltage regulation has the advantage of smooth regulation from zero to full voltage, and may be used for all loads and power factors. It is, therefore, much to be preferred to the resistance and inductance methods described above.

An induction regulator is, essentially, a transformer, or mutual

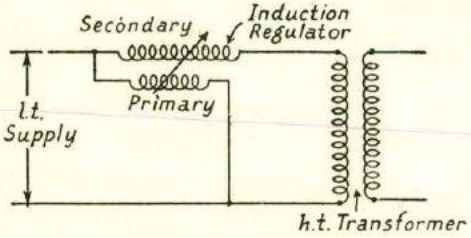


FIG. 11.6. INDUCTION-REGULATOR CONTROL

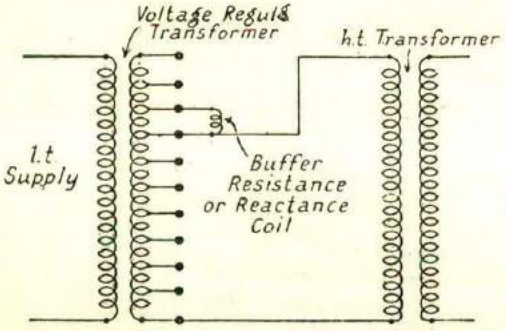


FIG. 11.7. TAPPED-TRANSFORMER REGULATION

inductor, the secondary voltage of which can be varied by the rotation of the primary through any required angle up to 180°. The connections are shown in Fig. 11.6. If the primary and secondary have equal numbers of turns, by rotation of the primary winding the voltage induced in the secondary may be varied from $-E$ to $+E$ (where E is the low-voltage supply). Thus the voltage applied to the h.v. transformer primary may be varied from zero to $2E$.

Careful design, with distributed windings on the rotor, are necessary to prevent waveform distortion. This method of regulation is often used in cable-testing equipments, since its gradual voltage variation at loads of any magnitude is advantageous for such work.

(iv) *Voltage Variation by Means of a Tapped Transformer.* Fig. 11.7 gives the connections of this method of regulation. An intermediate regulating transformer is used, having its primary supplied

from the l.v. supply. The secondary winding of the transformer has a large number of tappings whereby the voltage applied to the primary of the h.v. transformer may be varied. In order to avoid surges, due to the opening of the secondary circuit of the regulating transformer when the tapping switch is moved, two contact brushes are used, making contact with adjacent studs, and with a "buffer" resistor or reactance coil between them to prevent short-circuit of a section of the transformer winding. An auto-transformer may be used instead of the double-wound transformer shown in the diagram. The operation is similar to that in the case of the double-wound transformer.

For gradual regulation a number of coarse tappings are used together with fine tappings. The method has the advantages of high efficiency and small waveform distortion, but the regulation is not smooth unless a very large number of tappings is used, and when the power of the equipment is large, the switchgear must be large and is expensive.

(c) CONTROL GEAR AND CONNECTIONS. On the low-voltage side—i.e. in the primary circuit of the h.v. transformer, there should be—

A *main switch*, to isolate the testing apparatus from the supply.

Fuses.

A *circuit-breaker*, with arrangements for tripping the breaker over a wide current range, and having a no-volt coil in order to protect apparatus from damage in the event of failure of the supply.

An *over-voltage relay*, which short-circuits the no-volt coil of the circuit-breaker if the supply voltage exceeds a predetermined value.

Interlocks. These are arrangements which guard against the supply to the transformer being switched on except at a low value, or which make it impossible for the operator to pass inside the screen-work surrounding the high-voltage testing area while the supply is on. The latter takes the form of a switch on the gate of the screen.

Earth connections. Both for safety and for the protection of apparatus it is necessary that all metal parts forming part of the testing equipment, which should be at earth potential, should be definitely earthed. This applies to the framework of switchgear and to the guard screen, transformer tanks (except where a cascade arrangement is used), and to one end of each of the transformer windings.

On the high-voltage side, care must be taken in deciding the size and shape of the leads and connections, in order to avoid "corona" loss, which may distort the voltage waveform and may influence the breakdown or flashover voltages measured, on account of the surges in the circuits, and because of ionization of the air in the test room, which it may produce. "Corona" is the name given to the luminous glow which surrounds a high-voltage conductor when the potential gradient at its surface exceeds the disruptive strength of air. Breakdown of the air at the conductor surface occurs first of

all, and this spreads outwards, ionizing the air immediately surrounding the conductor, and producing both a glow and a hissing sound. The result of such corona is a loss of power, with the possible effects mentioned above. The voltage at which corona occurs depends upon the diameter of the high-voltage conductor, the voltage increasing with increasing diameter. Thus, to avoid such effects the conductors in the high-voltage circuit should be made much larger than is necessitated by current-carrying considerations.

E. T. Norris and F. W. Taylor (Ref. (13)) recommend for the diameters of high-voltage conductors "1 in. for 100 kV and about

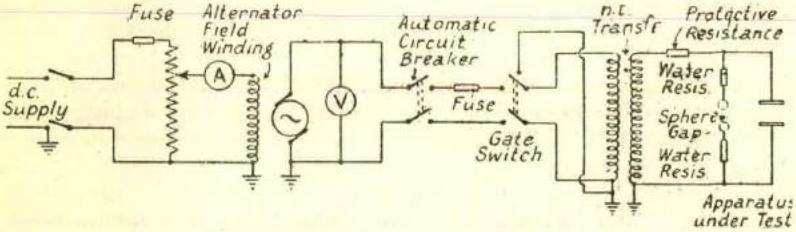


FIG. 11.8. CONNECTIONS OF TESTING EQUIPMENT

12 in. for 1,000 kV, if the atmospheric conditions are normal and the high-tension clearances are ample." High-voltage conductors are almost always bare, the surrounding air forming the principal insulation. The supports for these conductors are usually Bakelite tubes or rods, or porcelain insulators. Since the current to be carried is usually very small, good contact at joints is not often essential, mere touch being sufficient. Sharp edges, which produce concentration of electrostatic field, must be avoided throughout the circuit. Terminals are usually fitted with spherical metal caps, and sharp bends in conductors are fitted with spherical elbow pieces to distribute the electric stress.

As a protection from surges which may be produced when a sphere-gap, or the apparatus under test, breaks down, a high resistance (about $\frac{1}{2}$ ohm per volt), or choke coil, is inserted in the main high-voltage circuit. A sphere-gap is also connected across the high-voltage winding as a protection in the event of excessive voltage being applied to the circuit. This may be the same gap as is used for voltage measurement. A water resistance, consisting of a glass tube or tubes containing water, with some common salt added, is connected in the sphere-gap circuit to prevent pitting of the sphere surfaces by limiting the current flowing at breakdown. These resistances should be about 1 ohm per volt to limit the current to 1 amp.

Fig. 11.8 gives a simplified diagram of connections for a high-voltage testing equipment having a separate supply alternator.

(*d*) APPARATUS FOR VOLTAGE MEASUREMENTS. Owing to the difficulty of designing electrostatic voltmeters, for the measurement of extra high voltages, which will be free from errors due to corona effects within the instrument and to external electrostatic fields, a number of special methods of measurement have been devised for the purpose. Some method must be available, also, for calibrating such voltmeters when constructed. Work by the British Electrical and Allied Industries Research Association (Ref. (14)), and the American Institute of Electrical Engineers, led to the standardization of the sphere-gap method as the most reliable one.

(i) *Sphere-gap.* Two metal spheres are used, separated by an air gap. The potential difference between the spheres is raised until a spark passes between them. The value of the potential difference required for spark-over depends upon the dielectric strength of air, the size of the spheres, their distance apart, and upon a number of other factors. As a result of the research work already mentioned, the British Standards Institution has drawn up a list of "Rules for the Measurement of Voltage with Sphere-gaps" (B.S. 358 : 1939). Some years ago the American Institute of Electrical Engineers drew up a list of rules for the measurement of voltages up to 50 kV, using needle points for a spark-gap, instead of spheres as at present used. The disadvantages of this form of gap were the errors caused by variation in sharpness of the needles; by variation of the humidity of the atmosphere; and by the corona which formed at the points before the gap actually sparked over.

In B.S. 358:1939, twelve sizes of spheres are to be used to cover a range of voltage from 2 to 2,500 kV, namely, of diameters 2, 5, 6·25, 10, 12·5, 15, 25, 50, 75, 100, 150 and 200 cm. The following are extracts from these rules—

1. "A spark-gap may be used for the determination of the peak value of a voltage-wave, and for the checking and calibration of voltmeters and volt-measuring devices for high-voltage tests.

2. "Peak voltages may be measured from about 2 kilovolts to about 2,500 kilovolts by means of sphere-gaps used in conjunction with the calibrations provided herewith."

3. "The appropriate Tables may also be used for the measurement of impulse voltages of either polarity and of the standard wave-forms, having fronts of at least one microsecond and times-to-half-value of at least 5 micro-seconds."

4. "The length of any diameter shall not differ from the correct value by more than 1 per cent for spheres of diameter up to 100 cm, or by more than 2 per cent for larger spheres."

5. "One sphere may be earthed or both may be insulated, the voltages being symmetrical with respect to earth in the latter case."

Subject to certain specified conditions,

"no conductor or body having a conducting surface (except the supporting shanks) shall be nearer the sparking point of the high-voltage sphere than the distance given by the expression $\left(0{\cdot}25 + \frac{U}{300}\right)$ metres, where U is the voltage in kV (peak) which is to be measured."

6. "It is recommended that spheres of brass, bronze, steel, copper, aluminium or light alloys be used, and that the surface of the spheres should be cleaned immediately before use. A high degree of polish should be avoided."

The rules provide that the current at spark-over shall be limited to about 1 amp and that the interval between consecutive discharges shall be great enough to prevent appreciable heating of the spheres.

The density of the air affects the spark-over voltage for a given gap-setting. The "relative air density," is given by

$$d = \frac{p}{760} \cdot \frac{273 + 20}{273 + t} = 0{\cdot}386 \left(\frac{p}{273 + t} \right) . \quad . \quad (11.1)$$

where p = barometric pressure in millimetres of mercury

t = temperature in degrees Centigrade

A correction factor, by which the spark-over voltage for a given gap-setting, under the standard conditions (760 mm pressure and 20° C) must be multiplied, in order to obtain the actual spark-over voltage, may be obtained from a table given in the B.S.I. rules. This factor is approximately equal to d for values of d above 0·9.

With regard to humidity, the rules state that over the sparking distances covered by the calibration tables, the breakdown voltage of the sphere-gap is independent of the humidity of the atmosphere, but deposited dew on the surface of the spheres lowers the breakdown voltage and invalidates the calibrations.

For the calibration of a spark-gap from a standard sphere-gap the two gaps should not be connected in parallel. "Equivalent spacings should be determined by comparing each gap in turn with a suitable indicating instrument."

F. W. Peek (Ref. (15)), Russell (Ref. (16)), and others have obtained formulae by which the spark-over voltage for a given gap-setting and sphere-diameter may be calculated, but the use of the calibrations given in the B.S.I. rules is to be recommended instead of the use of such formulae, the application of which is limited.

The spark-over voltages given in the rules are in terms of r.m.s. (sine-wave) voltage. To obtain the corresponding peak values—which are important since it is the peak value of the voltage, rather than the r.m.s. value, which determines the flash-over, or breakdown of insulation—the r.m.s. values given must be multiplied by $\sqrt{2}$.

A disadvantage of the sphere-gap method of measurement is that it cannot be used to give a continuous record of voltage. It is generally used to calibrate some indicating instrument, or other apparatus, which does give such a continuous record. In the following methods of measurement, wherever calibration is referred

to, it is to be understood that the sphere-gap method is implied unless otherwise stated.*

(ii) *Transformer-ratio Methods of Measurement.* In this method the primary voltage of the high-voltage transformer is measured by a calibrated voltmeter, and this is multiplied by the turns ratio of the transformer to obtain the secondary voltage. The assumption is made that the transformer ratio is unaffected by the transformer impedance and by variations in the load. The method also gives r.m.s. values of voltage, and the secondary voltage waveform must be determined in order that peak values of voltage may be obtained from the "peak factor" of the wave, i.e. the ratio

$$\frac{\text{Peak voltage}}{\text{r.m.s. voltage}}$$

Some high-voltage transformers carry a separate voltmeter-coil having a number of turns which is a definite fraction of the number of turns on the secondary winding. The voltage induced in this coil (measured by a low-voltage voltmeter), when multiplied by the turns ratio, gives the secondary voltage of the transformer. This method cannot be used with the cascade arrangement of transformers and may only be relied upon to within 1 or 2 per cent.

(iii) *Potential-divider Methods.* In such methods, either a high-value resistor or an air capacitor must be used, connected across the high-voltage winding, a definite fraction of the total secondary voltage being measured by means of a low-voltage electrostatic voltmeter.

High-value resistors for this purpose have the disadvantage of residual phase-angle errors due to the effects of distributed capacitance, unless special precautions are taken in their design (Ref. (17)). Such resistors have been constructed, however, and used in several high-voltage equipments.

The advantages of capacitors as compared with resistors are: their comparatively simple construction, their freedom from heating, and the fact that they are much more easily shielded from extraneous capacitance effects than are high-value resistors. B. G. Churcher and C. Dannatt (Refs. (18), (19)) have described the construction and use of such capacitors.

In the capacitor method an electrostatic voltmeter, whose capacitance is large compared with that of the high-voltage standard capacitor, is connected in series with the latter, and is shunted by a capacitance of much greater value than its own. The capacitance of the standard capacitor must be accurately known, and it is important that it shall be free from dielectric loss. For the latter

* The calibration of the sphere-gap itself is described by S. Whitehead and A. P. Castellain (Ref. (14)), who used a number of different methods for the purpose, and obtained consistency between them.

reason air capacitors are always used for this purpose, their capacitance usually being from 50 to 100 $\mu\mu$F.*

If C is the capacitance of the standard capacitor, and C_1 that of the capacitor which shunts the electrostatic voltmeter (see Fig. 11.9), then the voltmeter reading v is given by

$$v = \frac{i}{\omega C_1}$$

where i is the current passing through the capacitors and $\omega = 2\pi \times$ frequency.

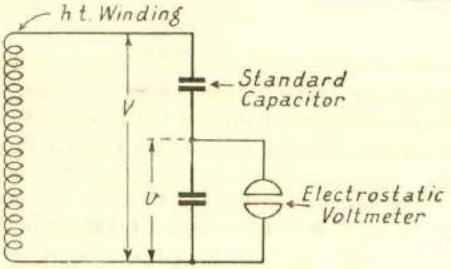


FIG. 11.9. CAPACITOR POTENTIAL DIVIDER

Neglecting the capacitance of the voltmeter, the effective capacitance of C and C_1, in series, is $\frac{CC_1}{C + C_1}$, and the voltage V of the high-voltage circuit is

$$V = \frac{i}{\omega\left(\frac{CC_1}{C + C_1}\right)}$$

Thus

$$V = \frac{C + C_1}{C} \cdot v \qquad . \qquad . \qquad (11.2)$$

This method measures r.m.s. voltages.

Other methods utilizing a standard impedance involve the measurement of the current which flows through the impedance—either a standard air capacitor or high-value resistor—when it is connected across the high-voltage winding of the transformer. The current may be measured by means of a thermo-junction and galvanometer; by measuring the voltage drop across a known resistance, in series with the standard high impedance, using an electrometer; or by a valve-voltmeter method described by S. Whitehead and A. P. Castellain (*loc. cit.*).

A *thermo-junction* consists of a small "heater" of fine wire,

* The effect of humidity of the atmosphere and of dust deposits upon the purity of capacitors is given by B. G. Churcher and C. Dannatt (*loc. cit.*).

through which the current to be measured is passed. To the centre of this heater is attached a thermo-junction of two dissimilar metals, the attachment being by means of a small bead of a material which, while being electrically insulating, is a good thermal conductor. An e.m.f. is set up, in the circuit containing the thermo-junction, when a current is passed through the heater. This e.m.f. depends upon the magnitude of the heating effect of the current, and thus the r.m.s. value of a small alternating current may be measured by comparison with a known direct current. The thermo-junction, which may be of the vacuum type—in which the heater and junction are enclosed in a small evacuated glass bulb with terminals brought out—or air type, is used in conjunction with some form of galvanometer. The arrangement is calibrated with direct current, and is then used for the measurement of alternating currents of the same order of magnitude as the direct, calibrating current. Such thermo-junctions may be obtained for 2·5 mA, 5 mA, 10 mA, and so on, up to 10 A. Below 1 A they are vacuum type, and above 1 A air type.

From the measured value of the current flowing through a standard impedance, the r.m.s. values of the high voltage may be obtained by multiplication by the value of the impedance.

(iv) *Measurement of Peak Voltage.* Owing to the fact that the *maximum* or *peak* value of the applied voltage is that which produces the actual breakdown stress in the material under test, it is important that this value should be known in most cases. If methods of voltage measurement which give r.m.s. voltage values are used, the peak voltage may be obtained from the peak factor of the voltage wave. It is often more satisfactory, however, to use some method of voltage measurement which gives the peak value of the voltage directly. From the fact that the breakdown of air is involved in the sphere-gap method, this is obviously one method which depends upon the peak values, and the calibration curves for sphere-gaps, already referred to, give such peak values.

Other methods of measurement of peak voltage have been devised and are as follows—

1. *Rectified Capacitor-charging-current Method.* This method depends upon the fact that the peak voltage value is proportional to the average charging current of a standard air capacitor. The charging current is therefore rectified, either by a mechanical rectifier, by specially designed thermionic valves or by metal rectifiers, and its average value is then measured by a d.c. moving-coil permanent-magnet milliammeter, or by a d'Arsonval galvanometer.

The valves used depend for their action upon the fact that, if two electrodes are enclosed within a highly evacuated tube, one of these electrodes—the anode—being comparatively cold, whilst the other —the filament—is heated, current is passed only in the direction anode to filament. The amount of current which it is possible to rectify by this means is dependent upon the filament temperature.

One-half of each wave must be entirely suppressed by such a valve. In order to prevent ripples in the output current wave, two valves are used with a suitable proportioning of inductance and capacitance in the circuit.

The capacitor used may be either of the type already mentioned, the capacitance being calculated from the dimensions of the capacitor —when the method is an absolute one—or it may take the form of a capacitor bushing, fitted to the transformer, in which case cali bration is needed.

Theory. If v is the voltage across the capacitor, of capacitance C, at any instant, and Q is the quantity of electricity in the capacitor at that instant, we have

$$\int i dt = Q = Cv$$

where i = charging current and t = time.

Now, during one whole period, the voltage rises to a positive maximum, falls to zero, rises to a negative maximum, and falls again to zero, thus giving a total change in voltage, during the cycle, of $4V_{max}$, where V_{max} is the peak, or maximum, value. Meanwhile, the quantity of electricity supplied to the capacitor during this period is $\int_0^T i dt$, T being the periodic time.

Thus
$$\int_0^T i dt = C \cdot 4V_{max}$$

or
$$\frac{1}{T}\int_0^T i dt = \frac{C \cdot 4V_{max}}{T}$$

Now, $\frac{1}{T}\int_0^T i dt$ is the average value of the charging current, i_{av}. Hence

$$i_{av} = \frac{4CV_{max}}{T} = 4Cf \cdot V_{max}$$

where f is the frequency.

Thus
$$V_{max} = \frac{i_{av}}{4Cf} \quad . \quad . \quad . \quad . \quad (11.3)$$

which gives the peak voltage value if the capacitance of the capacitor and the frequency (which must be kept constant throughout the test) are known.

Fig. 11.10 shows the connections of an arrangement for peak voltage measurement using full-wave rectification (see Ref. (20)).

This method of peak voltage measurement was first described by Chubb and Fortescue, and its accuracy is considered by R. Davis, G. W. Bowdler, and W. G. Standring (Ref. (21)), who state that, for a sine wave of voltage, the method is accurate provided that the impedance of the rectifier is negligibly small compared with that of the capacitor; that the rectifier is efficient in entirely suppressing one-half of the alternating wave; and that the milliammeter, or galvanometer, indicates the mean current correctly.

With regard to wave-form, the same authors state that: "the method . . . is satisfactory for all wave-shapes with the exception of (1) wave-shapes with different positive and negative maxima, and (2) wave-shapes with more than one peak in each half-cycle."

2. *Ryall Crest Voltmeter.* L. E. Ryall (Ref. (22)) has developed a simple form of peak voltmeter which is independent of frequency. It consists of a neon lamp, used in conjunction with a capacitor potential-divider. Two capacitors, one—a variable one—of much greater capacitance than the other, are connected in series across

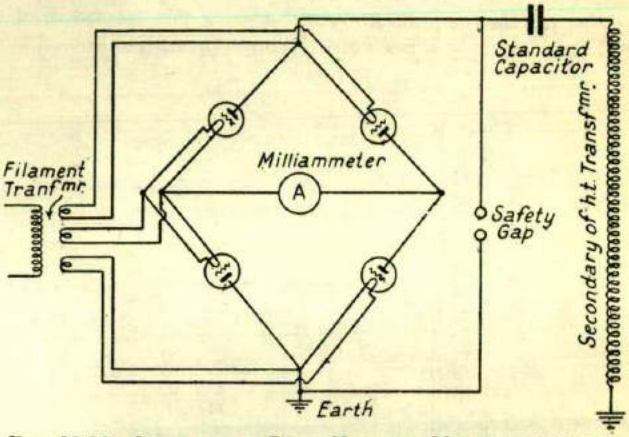


FIG. 11.10. CIRCUIT FOR PEAK VOLTAGE MEASUREMENT BY RECTIFIED CAPACITOR-CHARGING CURRENT

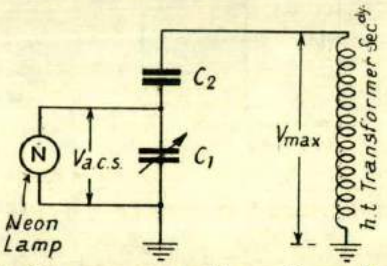


FIG. 11.11. CONNECTIONS OF RYALL CREST VOLTMETER

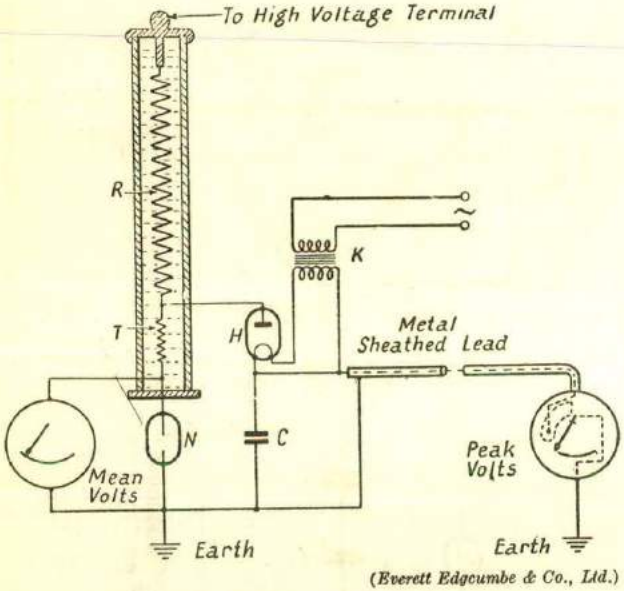
the voltage whose peak value is to be measured. A neon lamp of a specially chosen type is connected across the variable capacitor (see Fig. 11.11). Neon lamps have the characteristic that they only commence to glow when the voltage applied to them reaches a certain definite value, called the "striking" voltage.

In this method of peak voltage measurement the variable air capacitor, across which the lamp is connected, is adjusted until the lamp "strikes." Then the peak value of the high voltage is given by

$$V_{max} = V_{a.c.s.}\left(1 + \frac{C_1}{C_2}\right) \quad . \quad . \quad . \quad (11.4)$$

where $V_{a.c.s.}$ is the a.c. striking voltage of the lamp (which is a constant quantity if the type of lamp is suitably chosen), and C_1 and C_2 are the capacitances of the variable and fixed capacitors respectively.

Ryall found that the a.c. striking voltage for a neon lamp "is constant for all frequencies above 25 cycles per second, and can be measured to within $\pm \frac{1}{2}$ per cent if about 5 min is allowed to elapse



(*Everett Edgcumbe & Co., Ltd.*)

FIG. 11.12. EVERETT EDGCUMBE PEAK KILOVOLTMETER

between successive flashes of the lamp, so that the lamp can return to its original ionized condition.

"The a.c. extinguishing voltage, $V_{a.c.e.}$, is also constant for all frequencies above 25 cycles per second, and can be measured to within $\pm \frac{1}{4}$ per cent."

The accuracy of the method obviously depends upon the accuracy with which $V_{a.c.s.}$ may be determined, and upon the accuracy with which the capacitances C_1 and C_2 are known.

Both the lamp and capacitor C_1 are enclosed within metal boxes for screening purposes, and the lamp, in addition, is screened by a wrapping of lead foil, which is earthed, as the lamp glows when placed in a strong electrostatic field. A design of such a crest voltmeter for 150 kV (max.) is given by Ryall (*loc. cit.*).

Fig. 11.12 shows the principle of the Everett Edgcumbe peak

voltmeter for high-voltage measurements up to 500 kV (peak). This instrument gives a direct and continuous indication of the peak voltage under all conditions of loading and its readings are independent of waveform.

A resistor R is connected between the high-voltage terminal and earth. Across a section T of this resistor is joined a high-voltage rectifying valve H in series with a capacitor C and an electrostatic voltmeter reading direct in peak volts. For safety, the connecting leads are enclosed in an earthed flexible metal sheathing.

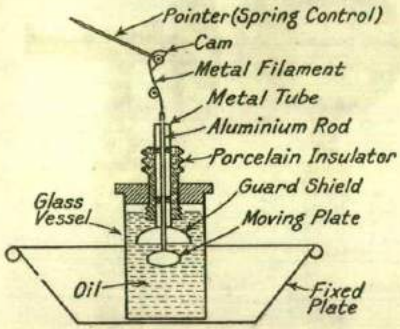


FIG. 11.13. SIEMENS AND HALSKE HIGH-VOLTAGE VOLTMETER

In the diagram of connections a mean voltmeter is also included.

(v) *High-voltage Voltmeters.* For the measurement of voltages up to about 200 kV, several forms of voltmeter have been designed which may be connected across the high-voltage circuit directly, without any potential-dividing device. Most of these are of the "attracted-disc" type, based on Lord Kelvin's volt balance. The latter is an absolute instrument. It consists of two flat discs, one fixed, and the other suspended from one arm of a balance. The fixed disc is insulated from the case of the instrument and is charged to a high potential. The pull between the two discs is given by

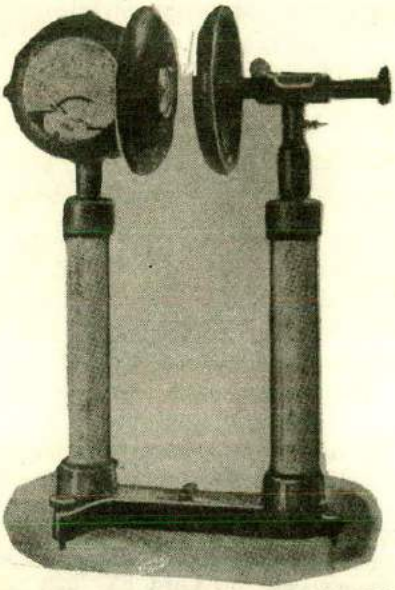
$$F = \frac{\varepsilon . A . V^2}{2d^2} \text{ newtons} \quad . \quad . \quad . \quad (11.5)$$

where A is the area of the moving disc, V the voltage between the discs, d the distance between them, and ε the permittivity of the medium. This force is balanced by weights on the balance arm, the magnitude of the weights giving the force F, and hence V.

Fig. 11.13 shows the construction of the Siemens and Halske instrument, based on the above principle, while Fig. 11.14 shows the Abraham voltmeter, the latter being commonly used in high-voltage testing equipments.

In the former instrument the high-voltage plate is in the form of a

hollow metal spheroid, suspended, by means of an aluminium rod and metal filament, in oil. The oil tends to prevent the formation of an arc between the high-voltage plate and the earthed plate, and by enabling the two plates to be brought nearer together, increases the working forces of the instrument. The earthed plate is in the form of a shallow metal pan, which also screens the instrument from



(Everett Edgcumbe & Co., Ltd.)

FIG. 11.14. ABRAHAM-VILLARD VOLTMETER

external electrostatic effects. Another guard screen is provided in the oil just above the high-voltage plate, as shown.

Control of the instrument is by a spiral spring on the pointer spindle, and damping is by fluid friction of the high-voltage plate in the oil. The cam on the pointer spindle gives a scale which is almost uniform over the upper 70 per cent of its range.

In the Abraham instrument—manufactured by Messrs. Everett Edgcumbe—there are two hollow metal mushroom-shaped discs arranged as shown. The right-hand disc forms the high-voltage plate, while the centre portion of the left-hand disc is cut away and encloses a small disc which is movable and is geared to the pointer of the instrument. The two mushroom-shaped discs are fixed in position except that the right-hand one may be set to different distances from the other, to alter the range of the instrument. A scale is provided, for this setting, on the right-hand support. The

two large discs form adequate protection for the working parts of the instrument against external electrostatic disturbances. This instrument is made in six sizes covering the range of voltage 3 kV to 500 kV.

Most of the high-voltage electrostatic voltmeters which are absolute instruments depend upon a direct measurement of the forces brought into play by the voltage applied. But there is another class of absolute instrument for this purpose which operates through the indirect action of these forces, i.e. through their influence upon the period of oscillation of a conducting body. The best known example of such instruments is the ellipsoid voltmeter (see Ref. (66)); E. Bradshaw, S. A. Husain, N. Kesavamurthy and K. B. Menon (Ref. (88)) have discussed their general theory and have considered their scope and limitations.

Ionic Wind Voltmeter. When a highly-charged point is situated in air or other gas, a movement of the air surrounding the point is observed. This is referred to as the "electric wind," and is brought about by the repulsion of ions from the surface of the point by the intense electrostatic field. These ions, colliding with uncharged molecules of air in the neighbourhood of the point, carry the latter with them and set up the "electric wind." If an earthed electrode is placed near to the highly-charged one, an intense electric field exists, of course, between the two, and a similar wind is observed also at the earth electrode.

This phenomenon has been investigated by W. M. Thornton and by M. Waters and W. G. Thompson, and has been put to a useful purpose in their Ionic Wind Voltmeter (Ref. (23)).

Fig. 11.15 shows the principle of the instrument. A hot wire, of platinum-gold alloy, included in one arm of a Wheatstone bridge network, is used as the earthed electrode of a high-voltage gap. Before the high voltage is applied to the gap the bridge network is balanced—i.e. the galvanometer G is set to zero. When the voltage across the gap exceeds a certain value—called the "threshold voltage"—the electric wind cools the hot wire and hence reduces its resistance. A reduction of 25 per cent in resistance is obtainable under suitable conditions, and this is sufficient to cause an appreciable out-of-balance voltage in the bridge network, as indicated by the galvanometer deflection. The cooling, and hence the galvanometer deflection, depends upon the potential gradient (and hence upon the applied voltage), as well as upon the temperature of the electrode, the nature and pressure of the gas in which the electrostatic field exists, and upon the frequency.

The "threshold voltage" is the voltage which is required before the potential gradient is sufficient for ionization to commence. When the applied voltage is alternating, therefore, only that portion of the voltage wave which lies above the threshold value is effective from the point of view of cooling of the hot wire. The area of the

effective portion of the voltage wave—and hence the cooling—varies for different wave shapes, even though the waves have the same r.m.s. value. Voltage waveform, therefore, influences the instrument readings, but the authors have shown how this influence may be predetermined. Calibration of the instrument is on a sine waveform.

The voltmeter can be used to indicate either peak or r.m.s. alternating voltages, or direct voltages. Several forms of the instrument have been constructed, both for outdoor and laboratory use, and for voltages up to 300 kV.

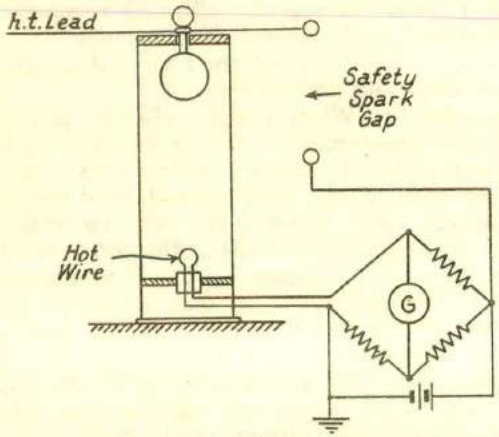


FIG. 11.15. IONIC WIND VOLTMETER

The principle advantages are that the high voltage may be measured by an observer at some distance from the charged conductors, and the robust construction, and freedom from disturbance by weather and temperature conditions which make it suitable for outdoor use.

A description of the vacuum-enclosed electrostatic voltmeters recently developed by Associated Electrical Industries, Ltd. is given in Chapter XVIII, p. 689.

Special Apparatus for Tests other than Low-frequency Alternating Current Tests. The apparatus described in the foregoing pages is, to a great extent, common to all methods of high-voltage testing. Except for a few special pieces of apparatus which may be required in individual cases, according to the purpose for which the equipment is to be used, no further apparatus is required for sustained low-frequency high-voltage tests. A complete diagram of connections for an equipment of this type is shown in Fig. 11.16.

Apparatus, in addition to that already described, is required

however, for the other types of test—namely, constant direct-current tests, high-frequency tests, and surge tests.

CONSTANT DIRECT-CURRENT TESTS. As previously mentioned, high-voltage direct current is chiefly used for the testing of high-voltage cables after installation. For the generation of high-voltage direct current a number of comparative low-voltage generators can

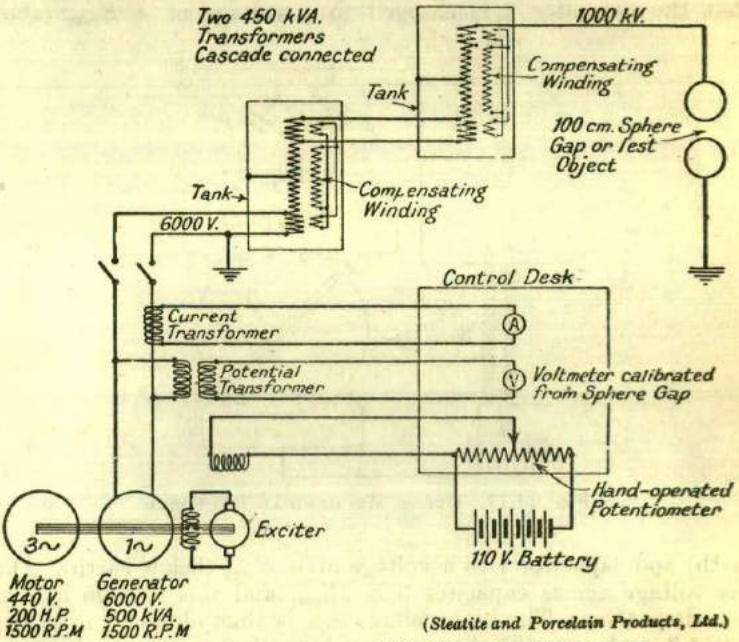


FIG. 11.16. LOW-FREQUENCY H.V. TESTING EQUIPMENT

be connected in series. A specially designed machine, called a "transverter" (Ref. (24)), can also be used to obtain d.c. from a three-phase a.c. supply. Both of these methods are, however, only of use when a large amount of power is to be dealt with, and cannot be adapted to a small power-testing set, which, for the purpose of cable testing, should be portable.

The method of obtaining the direct current now used is by the rectification of high-voltage alternating current, the latter being obtained from a high-voltage transformer.

The rectification may be done either by a mechanical rectifier, by a thermionic valve rectifier or by selenium or copper-oxide rectifiers. Metal rectifiers are now in common use.

Delon Mechanical Rectifier. Fig. 11.17 illustrates the principle of this apparatus. An insulated disc D carries a conducting bar B,

the ends of which come, in turn, opposite to one pair of the collecting brushes aa' and bb'. This disc is driven in synchronism with the alternating current supply, so that the rotating bar B is opposite to collecting brushes bb' (say) at the instant when the high-voltage wave is at its positive maximum point, and opposite aa', when the voltage is negative maximum. Sparks pass between the collecting brushes and the connecting bar at these instants, the result being that the capacitor 2 is charged to a voltage of $+ E_{max}$ (above

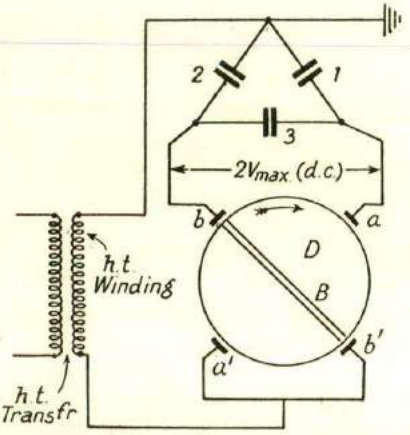


FIG. 11.17. DELON MECHANICAL RECTIFIER

earth) and capacitor 1 to a voltage of $- E_{max}$ (below earth). Thus the voltage across capacitor 3 is $2E_{max}$ and this voltage is *direct* not alternating. The high-voltage d.c. is thus obtained from capacitor 3, and since the charging is taking place continuously, direct current can be supplied to a circuit under test.

In tests upon cables the internal capacitance between the cable cores, and from cores to ground, form the capacitors 1, 2, and 3, but the spark-over which takes place at the collecting brushes in the mechanical rectifier tends to set up surges and oscillations which, by weakening the cable dielectric, may cause breakdown at a lower indicated voltage than would otherwise be obtained.

For this reason the mechanical rectifier has been largely displaced by thermionic valves or metal rectifiers.

Tests with Thermionic or Metal Rectifiers. Fig. 11.18 shows the connections for a high-voltage d.c. test upon a three-core cable—cores to ground.

Current passes through the valve, or metal rectifier which now commonly replaces it, only during one half-wave of the voltage cycle. During this time the capacitor formed by the cores and the sheath is

charged to a voltage equal to that of the maximum value, V_{max}, of the high-voltage transformer secondary voltage. When this voltage reverses—during the next half-wave—the voltage of this cable-capacitor remains the same, while the voltage of the valve filament rises to a maximum of V_{max} in the opposite direction. Thus the valve must be able to withstand a voltage of $2V_{max}$ between its anode and filament. The filament transformer must, also, be insulated for a voltage of V_{max} between windings. The

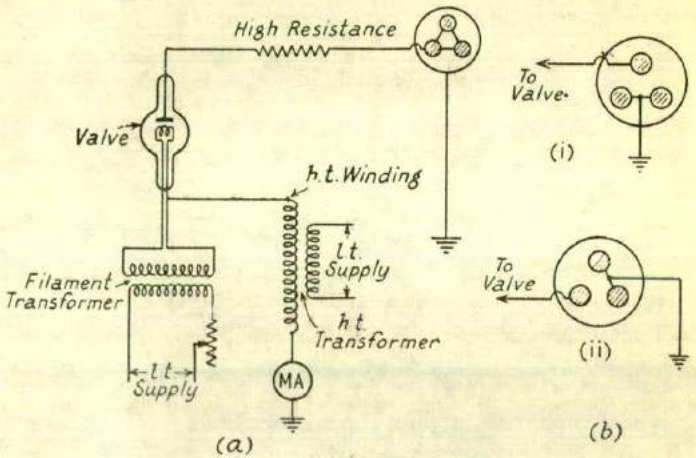


FIG. 11.18. CONNECTIONS FOR D.C. HIGH-VOLTAGE CABLE TESTS

milliammeter *MA* reads the charging and leakage currents through the capacitor formed by the cable.

The voltage for the valve filaments is usually 8 volts, and care must be taken that the correct voltage is used, since departure from it, by a comparatively small percentage, causes a considerable fall in the output current of the valves if the filament voltage is low. Resistances of about $\frac{1}{4}$ megohm are connected in the anode circuits of these valves for protection against surge effects.

Tests are usually made upon three-core cables with connections as shown in Fig. 11.18 (*b*) as well as with the connections of Fig. 11.18 (*a*).

Two valves are used for test voltages up to 200 kV. The connections for such a test are shown in Fig. 11.19, the test being one between cores *A* and *B* of a three-core cable. Valve V_a allows current to flow only in the direction anode to filament. Thus, during one half-cycle current flows through it in this direction, and core *A* of the cable is charged to a potential V_{max} below earth, V_{max} being the maximum value of the voltage wave of the h.v. transformer secondary winding. Meanwhile, valve V_b is allowing no

current to flow through it. During the next half-cycle the process is reversed, current flowing through valve V_b and charging core B to a potential V_{max} above earth. Thus the d.c. voltage between cores is $2V_{max}$, and from each core to earth V_{max}.

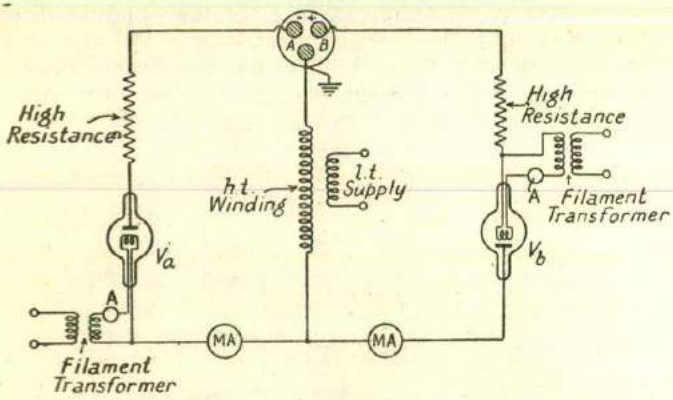


FIG. 11.19. CONNECTIONS FOR HIGH-VOLTAGE D.C. TEST UP TO 200 kV

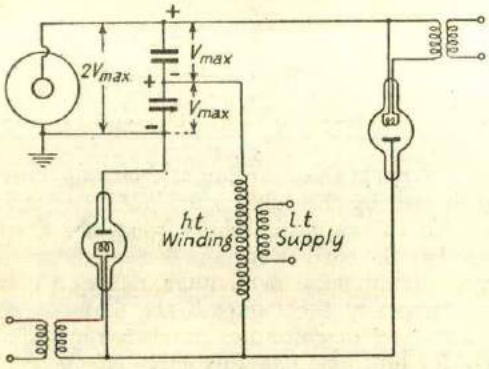


FIG. 11.20 CONNECTIONS IN THE CASE OF A SINGLE-CORE CABLE

In the case of a single-core cable, if the same voltage ($2V_{max}$) is to be applied between core and sheath as between cores in the three-core cable, two auxiliary capacitors must be used, connected as shown in Fig. 11.20.

In some portable high-voltage d.c. testing sets a single-phase low-voltage alternator, driven by a petrol engine, is used for the supply. The primary of the high-voltage transformer is supplied through an auto-transformer with a variable-inductance choke coil

for fine adjustment. Voltage measurement is made by an electrostatic voltmeter of the Abraham Villard type, directly connected across the cable cores under test, although a sphere-gap is often included. In order to avoid trouble from surges, which may cause breakdown of apparatus, it is advisable to increase the testing voltage to its full value gradually, and also to discharge the cable through a high resistance. Owing to dielectric absorption it is necessary, also, to maintain the discharging connection for a considerable length of time, in order to avoid a subsequent dangerous rise of voltage.

Other connections for high-voltage d.c. cable testing are given by J. Urmston (Ref. (25)).

Equivalence of D.C. and A.C. Test Voltages. Owing to the "electric osmosis" effect, any moisture which may exist within the cable dielectric tends to move towards the negatively-charged electrode (the sheath or one of the cores, depending upon the test), when a d.c. voltage is applied. Although the amount of such moisture is usually small, it may yet be sufficient to cause breakdown due to its concentration near the negative electrode. If the applied voltage is alternating, no such movement of moisture occurs, the moisture remaining uniformly distributed. Again, breakdown may occur when testing cable-samples due to surges which are produced by spark-discharge and corona effects at the cable ends. These effects are more severe with a.c. than with d.c.

For these reasons it is obvious that there are other considerations beyond mere equivalence of potential gradient which must determine to what alternating voltage a given direct voltage is equivalent from the point of view of insulation breakdown.

N. A. Allen, to whom the authors are indebted for much of their information upon cable-testing with direct currents, gives (Ref. (24)) a table showing the ratios of d.c. to a.c. test voltages, quoted by various authorities as giving an equivalent breakdown-test upon cables and dielectrics. For paper-insulated cables, although a d.c./a.c. ratio of 2·5 has been used, Allen suggests that a ratio of 1·5 to 2·0 would be more satisfactory. Owing to the fact, also, that this ratio increases with increasing insulation thickness, he suggests the adoption of a ratio 1·5 for cables up to 33 kV, and a ratio of about 2·0 for voltages above this. Table XIII—taken from the same paper—gives the d.c./a.c. ratios usually adopted by English cable makers.

Localization of Faults in High-voltage Cables. Another application of the rectifying valve is in the localization of faults in long lengths of high-voltage cable. Such faults are often of very high resistance and cannot easily be located, with accuracy, by the ordinary methods of fault localization given in the following chapter.

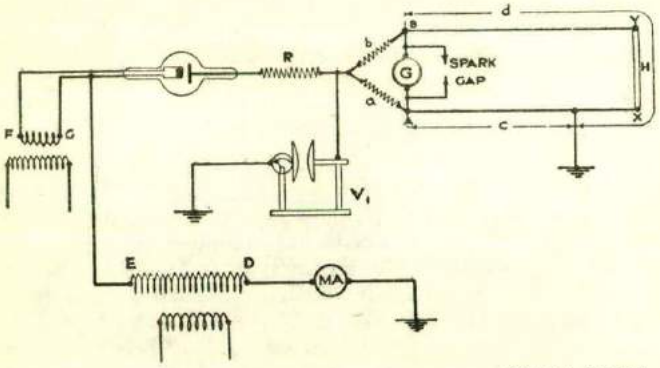
In high-voltage cables the large amount of oil present in the cables and joints causes faults in the cable (which may be obvious—owing

TABLE XIII

Working Voltage	Standard Test Voltage		Ratio D.C./A.C.	Average Thickness of Dielectric
	A.C.	D.C.		
11,000	20,000	30,000	1·50	0·30 in. (7·62 mm)
22,000	44,000	75,000	1·71	0·40 in. (10·2 mm)
33,000	66,000	100,000	1·52	0·50 in. (12·7 mm)

to their low resistances—when the high testing voltage is applied) to seal up rapidly when this voltage is withdrawn, resulting in their having, again, a high resistance. One method of dealing with such a fault is to reduce its resistance by, the continued application of a high testing voltage which "burns out" the fault, and enables location to be made by ordinary methods.

The method which has been largely adopted, as being more



(*Watson, London*)

FIG. 11.21. CONNECTIONS OF AN EQUIPMENT FOR LOCALIZATION OF FAULTS IN HIGH-VOLTAGE CABLES

satisfactory than this, is to use a valve-rectifier high-tension d.c. testing set for the supply of current for a "Murray loop" test. This and similar h.v. tests are described by Allen (Ref. (24)) and by Urmston (Ref. (25)).

Fig. 11.21 gives the connections for such a test, using a single-valve d.c. high-voltage testing set whereby a testing voltage of about 60 kV—which is usually sufficient for the purpose—can be obtained. A two-valve set may be used, if necessary, for higher testing voltages.

A loop is made by connecting together (by means of a short-circuiting strap) the two distant ends of a sound cable and the

faulty one. A highly insulated slide-wire, having a sliding contact which can be operated by a long insulating handle, is connected across the near ends of the loop, and a galvanometer and spark-gap —the latter for protection of the galvanometer—are also connected across these ends as shown. The d.c. testing voltage is applied to an intermediate point on the slide-wire, and an Abraham voltmeter is used to indicate the applied voltage. A high resistance and milliammeter are connected in the supply circuit for the limitation and measurement, respectively, of the supply current—i.e. of the fault current.

The procedure, as given by Urmston (Ref. (25)) is as follows: "Pressure is then applied, and when the breakdown occurs the fault current is limited to 3 or 4 mA, and a preliminary balance made. The current is then increased to 40 or 50 mA, and the balance readjusted."

The cable voltage is usually low when the fault is fully broken down, but the voltage may rise very suddenly if the fault clears, and this necessitates the precautions regarding insulation of the slide-wire in order to safeguard the operator. The spark-gap, which is set to about 0·01 in. by micrometer, is for the protection of the galvanometer and slide-wire when the fault resistance falls, at breakdown, resulting in the discharge of the cable. The inductance of the galvanometer and bridge circuit cause this discharge current to pass across the spark-gap rather than through them.

If a and b are the resistances of the slide-wire ratio arms at balance (see Fig. 11.21), and c and d are the resistances of the cable lengths to the fault, then

$$\frac{b}{a} = \frac{d}{c}$$

or

$$\frac{a+b}{a} = \frac{c+d}{c}$$

$$\therefore c = \frac{a(c+d)}{a+b}$$

Thus, assuming the resistance of the cable-loop, per unit length, to be uniform, and calling the total length of the loop L, we have—

$$\text{Distance of fault from point } A = \frac{aL}{a+b}$$

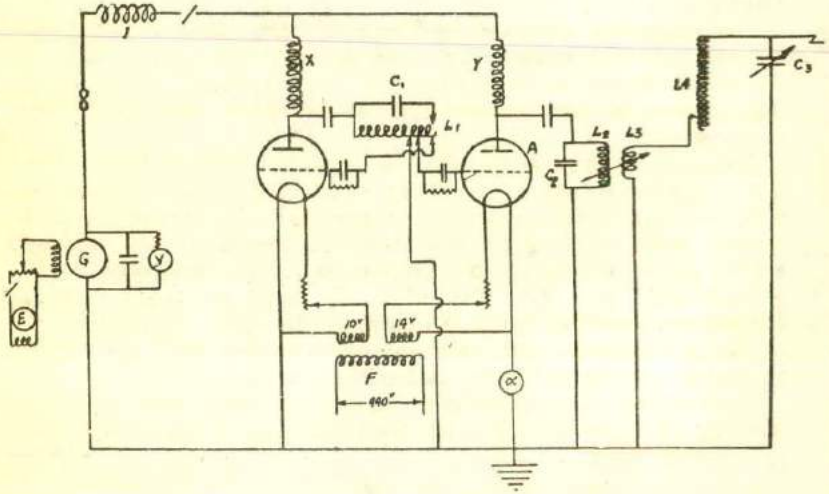
The resistance and length of the short-circuiting strap may be assumed negligible.

HIGH-FREQUENCY TESTS. It has been stated already that the behaviour of insulating materials at high frequencies is very different from that at ordinary commercial frequencies. This is largely due to the very much greater dielectric power loss, within the material,

at the high frequency. The heat produced by this power loss tends to produce breakdown of the insulation at voltages which are smaller than those at which breakdown occurs when the frequency is low. Such tests are useful, also, in the detection of lack of homogeneity in compound-filled porcelain insulators.

Two kinds of high-frequency high-voltage tests are carried out—

(*a*) Tests with apparatus which produces undamped high-frequency oscillations.



(*Associated Electrical Industries, Ltd.*)

FIG. 11.22. HIGH-FREQUENCY TESTING EQUIPMENT FOR 150 KV

(*b*) Tests with apparatus producing damped high-frequency oscillations.

(*a*) *Undamped oscillations* do not occur in power systems, but are useful for insulation testing purposes, especially for insulation to be used in radio work.

High-frequency alternators have been used for frequencies up to 30,000 cycles per second, and high-frequency arc generators have also been used, but these have the disadvantage that smooth voltage variation is difficult.

Fig. 11.22 shows a valve circuit used by Associated Electrical Industries, Ltd., for such tests. Voltage variation is effected by variation of the coupling between the anode circuit of the main valve and the secondary circuit, by means of the variometer, or by variation of the anode voltage of the valves. The voltage obtainable is 150 kV, and the frequency 100,000 cycles per second.

(*b*) *Damped high-frequency oscillations* are obtained by the use of

a Tesla coil, together with a circuit containing a quenched spark-gap, as shown in Fig. 11.23. The Tesla coil constitutes the high-voltage transformer. It consists of two air-cored coils which are placed concentrically. The high voltage secondary coil has a large number of turns, and is wound on a frame of insulating material, the insulation between turns being air, or in some cases, oil. If the Tesla coil is oil-immersed, the spacing between turns can be made smaller than when air is the insulation. The primary winding has only a few turns, wound on an insulating frame. The supply is usually 50 c/s a.c. to the primary of a high-voltage testing transformer,

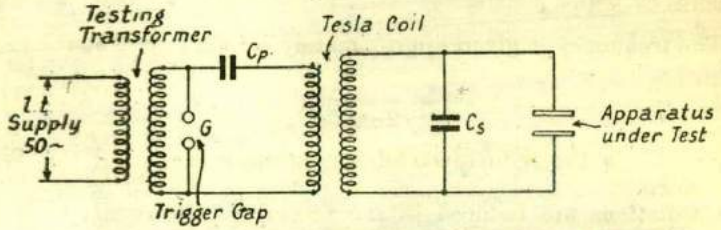


FIG. 11.23. CIRCUIT FOR HIGH-VOLTAGE TESTS WITH DAMPED HIGH-FREQUENCY OSCILLATIONS

although a valve rectifier may be used on the secondary side of this transformer to give a d.c. supply to the primary side of the Tesla transformer. Two capacitors, C_p and C_s, are connected in the primary and secondary circuits respectively, of the Tesla transformer. C_p is an air capacitor. C_s is usually made up of a sphere-gap, for voltage measurement purposes, the internal capacitance of the secondary winding of the Tesla coil, and the capacitance of the apparatus under test, the latter being usually small in comparison with the two others. The primary circuit of this transformer also contains a trigger spark-gap.

Assuming the supply to the primary of the Tesla transformer to be alternating—as it usually is—the capacitor C_p is charged to some maximum voltage, which depends upon the voltage on the secondary side of the supply transformer, and upon the setting of the "trigger-gap." At this voltage value the trigger-gap breaks down, the capacitor C_p discharges, and a train of damped oscillations, of high frequency, is produced in the circuit containing C_p, the spark-gap, and the primary winding of the Tesla transformer. During the time taken for this train of oscillations to die away (this time being a very small fraction of a second), the spark-gap is conducting, due to the formation of an arc across it.

This charge and discharge of capacitor C_p takes place twice in one voltage cycle—i.e. in $\frac{1}{100}$ sec for 50 c/s supply. Thus there will be 100 of these trains of damped oscillations per second.

The frequency of the oscillations themselves is very high—100,000 cycles per second being a usual value—its actual value depending upon the inductance and capacitance of the oscillatory circuit.

B. L. Goodlet (Ref. (26)) states, in connection with the test frequency to be adopted, that "to the author's knowledge the highest frequency oscillation from which any serious trouble has been experienced on any transmission system is 80,000 cycles." He also gives results of tests which show that the spark-over voltage of an insulator is very little higher at about 600,000 c/s than it is at 100,000 c/s, and suggests that 100,000 c/s is therefore the maximum required for testing purposes.

The frequency is given approximately by the expression

$$f = \frac{1}{2\pi\sqrt{L_p C_p}} \quad . \quad . \quad . \quad . \quad (11.6)$$

where L_p is the inductance of the primary circuit of the Tesla transformer.

Oscillations are induced in the secondary circuit of the Tesla transformer by oscillatory current in the primary, and these will be of the same frequency as those in the primary circuit if the secondary inductance and capacitance are adjusted so that the two circuits are in tune—i.e. if $L_p C_p = L_s C_s$. L_s and C_s are the inductance and capacitance of the secondary oscillatory circuit. In this way a series of trains of damped oscillations is applied to the apparatus under test, connected as shown.

To prevent the energy of the oscillatory discharge from surging backwards and forwards between the primary and secondary circuits of the Tesla transformer, the trigger spark-gap must be quenched by air-blast cooling. This is helped by using a rotating spark-gap.

A sphere-gap is used for voltage measurement, and if the wave-form of the Tesla secondary voltage is required, a cathode-ray oscillograph must be used. Accurate measurements of the high-frequency voltage are not always required, as such tests are often carried out by allowing high-frequency discharge to take place across the surface of an insulator for a certain time, and observing its effect upon the insulator. The frequency of the discharge is obtained from the constants of the oscillatory circuits.

Voltage and Frequency Values in the Oscillatory Circuits. The voltage relationships in the primary and secondary circuits of the Tesla transformer can be determined approximately as follows (Ref. (26))—

Let V_p = maximum voltage to which the primary capacitor C_p is charged.
V_s = maximum voltage to which the secondary capacitor C_s is charged.

Then,

Energy in primary capacitor at breakdown of trigger-gap = $\frac{1}{2} C_p V_p^2$

This energy is passed on to the secondary circuit, with some loss due to resistance and dielectric losses, and to the fact that the electromagnetic coupling between the two circuits is not perfect.

Energy given to secondary circuit $= \frac{1}{2} C_s V_s^2$. If ε is the efficiency of the energy transfer, then

$$\frac{1}{2} C_s V_s^2 = \varepsilon \cdot \frac{1}{2} \cdot C_p V_p^2$$

or

$$\frac{V_s^2}{V_p^2} = \varepsilon \frac{C_p}{C_s}$$

Thus,

$$\frac{V_s}{V_p} = \sqrt{\varepsilon \cdot \frac{C_p}{C_s}}$$

To determine the oscillation frequencies in the two circuits, let M be the mutual inductance between the two windings of the Tesla transformer, and let I_p and I_s be the currents in the primary and secondary oscillatory circuits, respectively. Then, neglecting the resistances of the two circuits, since these are usually small compared with the other impedances in the circuits, we have—

For the primary circuit,

$$I_p \left(j\omega L_p - \frac{j}{\omega C_p} \right) - j\omega M I_s = 0 \quad . \quad . \quad . \quad (11.7)$$

ω being $2\pi \times$ frequency).

In the secondary circuit,

$$I_s \left(j\omega L_s - \frac{j}{\omega C_s} \right) - j\omega M I_p = 0 \quad . \quad . \quad . \quad (11.8)$$

Then

$$\frac{I_p}{I_s} = \frac{j\omega M}{j\omega L_p - \frac{j}{\omega C_p}} = \frac{j\omega L_s - \frac{j}{\omega C_s}}{j\omega M}$$

Hence,

$$\omega^2 M^2 - \omega^2 L_p L_s - \frac{1}{\omega^2 C_p C_s} + \frac{L_s}{C_p} + \frac{L_p}{C_s} = 0$$

or

$$\omega^2 (M^2 - L_p L_s) + \frac{L_s}{C_p} + \frac{L_p}{C_s} - \frac{1}{\omega^2 C_p C_s} = 0$$

Let $\frac{M^2}{L_p L_s} = K^2$, or $K = \frac{M}{\sqrt{L_p L_s}}$

K is the "coefficient of coupling" of the circuits.)

Then

$$\omega^2 (K^2 L_p L_s - L_p L_s) + \frac{L_s}{C_p} + \frac{L_p}{C_s} - \frac{1}{\omega^2 C_p C_s} = 0$$

or

$$\omega^2 (1 - K^2) - \frac{1}{L_p C_p} - \frac{1}{L_s C_s} + \frac{1}{\omega^2 L_p L_s C_p C_s} = 0$$

Factorizing, we have,

$$\left[\omega (1 - K) - \frac{1}{\omega L_p C_p} \right] \left[\omega (1 + K) - \frac{1}{\omega L_s C_s} \right] = 0$$

since $L_p C_p = L_s C_s$ when the two circuits are tuned.

Thus, $\omega^2 = \dfrac{1}{L_p C_p (1 - K)}$

or $f_1 = \dfrac{1}{2\pi\sqrt{L_p C_p (1 - K)}}$ (f_1 being one value of the frequency)

or $\omega^2 = \dfrac{1}{L_s C_s (1 + K)}$

or $f_2 = \dfrac{1}{2\pi\sqrt{L_s C_s (1 + K)}}$ (f_2 being another value of the frequency)

Now, K is usually small compared with $L_p C_p$ and $L_s C_s$, so that as previously stated, the frequency is given, approximately, by

$$f = \frac{1}{2\pi\sqrt{L_p C_p}} = \frac{1}{2\pi\sqrt{L_s C_s}} \quad . \quad . \quad (11.9)$$

With regard to the closeness of the coupling between the two circuits of the Tesla transformer, Goodlet (*loc. cit.*) points out that

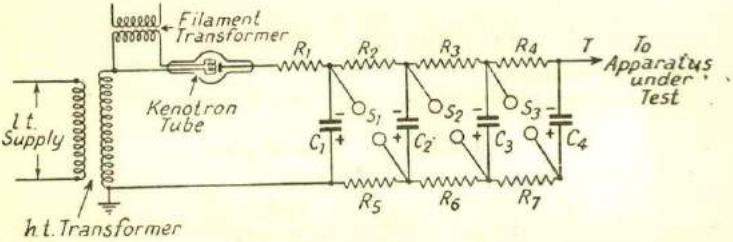


FIG. 11.24. CONNECTIONS OF MULTI-STAGE SURGE GENERATOR

if this is very close an impulse effect will be obtained rather than an oscillatory one, and if the coupling is very loose the conditions existing with undamped oscillations will be obtained.

SURGE OR IMPULSE TESTS. In surge tests it is required to apply to the circuit or apparatus under test a high direct voltage whose value rises from zero to maximum in a very short time and dies away again comparatively slowly—i.e. a voltage having a very steep wave-front and a flat tail is to be applied. This is done by means of a circuit due to Prof. E. Marx (Refs. (89), (90), and (91)) and called the Marx circuit after him. To illustrate the principles of the method the connections of a multi-stage surge generator are given in Fig. 11.24. Many modifications of the circuit have been used.

C_1, C_2, C_3, and C_4 are capacitors which are connected in parallel with high resistances, R_1, R_2, etc., between them. S_1, S_2, and S_3 are trigger spark-gaps.

The capacitors are charged, in parallel, from a high-voltage transformer, through a rectifying valve. At a certain voltage, depending upon their setting, the trigger-gaps break down and connect the capacitors in series instantaneously. Thus the voltage

between the point T and earth, at this instant, is the sum of the voltages to which the capacitors were charged. The discharge takes place in so short a time that the energy loss in the resistors is negligible. Thus, the whole of the energy which is stored in the capacitors during charging is suddenly discharged through the test circuit when the trigger-gaps break down. The gaps are set so that S_1 breaks down at a slightly lower voltage than S_2, S_2 at a slightly lower voltage than S_3, and so on. Then S_1 breaks down first and S_2 and S_3 follow instantly.

Although only three gaps are shown in the figure, the number of gaps and capacitors may be increased to give any desired multiple of the charging voltage, and it has been found feasible in practice to operate a 50-stage impulse generator. The number which can be used successfully is limited to some extent, however, by the fact that the high resistance between the supply and the distant capacitors, when a large number are used, may prevent them from receiving a full charge. A large number of gaps in series also reduces the impulse voltage obtainable.

F. S. Edwards, A. S. Husbands and F. R. Perry (Ref. (87)) have reviewed subsequent developments of the Marx circuit and described the construction of impulse generators as well as their practical application. In this paper the simplified circuit shown in Fig. 11.25 (*a*) is given for a single-stage impulse generator. C_1 = discharge capacitance of the generator, C_2 = capacitance of the load, L_1 = internal inductance of generator, L_2 = external inductance of load and connections, R_1 or R'_1 = resistance for control of wave-tail (there are two alternative positions for this resistance), R_2 = resistance for control of wave-front. These authors state, however, that if the tail of the wave is long compared with its front—as it is for a standard wave—very little error is caused by ignoring the wave-tail resistance during the calculation of the wave-front duration. The circuit can then be further simplified to that of Fig. 11.25 (*b*).

The critical resistance of the circuit (see Chapter XVI) is

$$R = \sqrt{\frac{4L}{C}}, \text{ where } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

The voltage across the load C_2, for a charging voltage E across C_1, is then given by

$$V = \frac{CE}{C_2}\left[1 - \left(1 + \frac{2t}{CR}\right)e^{-\frac{2t}{CR}}\right] \quad . \quad . \quad (11.10)$$

or, if L is reduced to zero, R remaining the same,

$$V = \frac{CE}{C_2}\left(1 - e^{-\frac{t}{CR}}\right) \quad . \quad . \quad . \quad . \quad (11.11)$$

V approaches $\frac{CE}{C_2}$ as t approaches infinity.

The "nominal wave-front" defined in B.S. 923 : 1940 is equal to $2{\cdot}75CR$ when L is zero, and to $2{\cdot}1CR$ when the circuit is critically damped. In calculating the resistances the value of R_2 needed to make the circuit of Fig. 11.25 (*a*) non-oscillatory with R_1 absent is obtained. Then the value of R_2 (if $> \sqrt{\frac{4L}{C}}$) required to give the

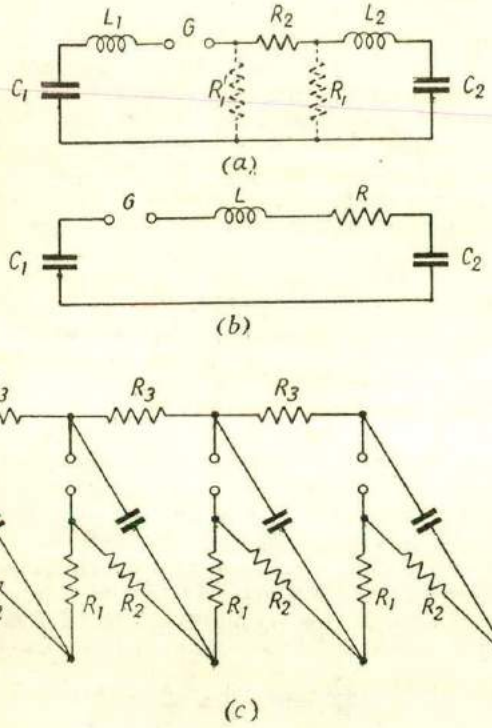


FIG. 11.25

desired wave-front is computed, and finally the value of R_1 to give the required wave-tail when R_1 is on the generator side of R_2.

The efficiency of the generator can then be approximately estimated as $\frac{C_1}{C_1 + C_2}$ multiplied by a factor of about 0·95 for a 1/50-microsec wave.

Wave-front-control resistors can be connected either outside or within the generator, or partly in and partly outside. The best arrangement is probably to have about half of the resistance

TABLE VI

Capacitances of Various Systems of Conductors

($\epsilon = \epsilon_0\epsilon_r$ = permittivity of the medium and $\epsilon_0 = \frac{1}{36\pi \times 10^9}$. Dimensions in metres. Capacitance in farads)

No.	System of Conductors	Capacitance	Explanation of Symbols	Assumptions, etc.	Source
1	Isolated sphere	$4\pi\epsilon R$	R = radius of sphere		
2	Concentric spheres	$\frac{4\pi\epsilon R_1 R_2}{R_2 - R_1}$	R_1 and R_2 = radii of spheres	Spheres isolated from other conductors	
3	Two spheres distant from one another	$\frac{4\pi\epsilon R_1 R_2 D}{D(R_1 + R_2) - 2R_1R_2}$	R_1 and R_2 = radii of spheres D = distance between centres	Spheres isolated from other conductors	
4	Two spheres at same potential	$4\pi\epsilon\left(R_1 + R_2 - \frac{2R_1R_2}{D}\right)\frac{D^2}{D^2 - R_1R_2}$	As in (3)	Approx. formula (see Ref.)	Russell (Ref. (12))
5	Two equal spheres close together	$2\pi\epsilon R\left(1 + \frac{x}{6R}\right)\left(1{\cdot}2704 + \frac{1}{2}\log_e\frac{R}{x} + \frac{x}{18R}\right)$	R = radius of spheres x = shortest distance between their surfaces	Approx. formula (see Ref.)	Russell (Ref. (12))
6	Two parallel conducting plates	$\frac{\epsilon A}{D}$	A = area of one plate D = distance between plates	D small. Edge effect negligible	
7	N parallel plates	$\frac{\epsilon(N-1)A}{D}$	As in (6)	As in (6)	
8	Two parallel plates with several dielectrics between them	$\frac{A}{\left(\frac{D_1}{\epsilon_1} + \frac{D_2}{\epsilon_2} + \frac{D_3}{\epsilon_3} + \dots\right)}$	A = area of one plate D_1, D_2, D_3 = thicknesses of dielectrics ϵ_1, etc. = permittivity of dielectrics	As in (6)	
9	Two similar long parallel conducting cylinders	$\frac{\pi\epsilon l}{\log_e\frac{2D-d}{d}}$	l = axial length of cylinders d = diam. of cylinders D = distance between centres	d small compared with D Cylinders isolated	
10	Two similar long parallel conducting cylinders	$\frac{\pi\epsilon l}{\log_e\left[\frac{D + \sqrt{D^2 - d^2}}{d}\right]}$	As in (9)	d not small compared with D	
11	Two coaxial conducting cylinders	$\frac{2\pi\epsilon l}{\log_e\frac{D}{d}}$	l = axial length d and D = diam. of cylinders		
12	Long straight conductor parallel to earth	$\frac{2\pi\epsilon l}{\log_e\frac{4H-d}{d}}$	l = axial length d = diam. of conductor H = height above earth		
13	Two long similar parallel conductors parallel to earth	$\frac{\pi\epsilon l}{\log_e\frac{D}{r}\left(\frac{2H}{\sqrt{4H^2 + D^2}}\right)}$	l = axial length r = radius of conductors D = distance apart H = height above ground		
14	Thin circular disc	$8\epsilon r$	r = radius of disc	Thickness small compared with r. Disc isolated	
15	Two similar coaxia parallel discs	$4\pi\epsilon\left[\frac{r^2}{4D} + \frac{r}{4\pi}\left(\log_e\frac{16\pi(D+t)r}{D^2} + \frac{t}{D}\log_e\frac{D+t}{t} + 1\right)\right]$	t = thickness of discs r = radius of discs D = distance apart	t and D small compared with r	Kirchhoff (Ref. (32))
16	Single vertical wire	$\frac{2\pi\epsilon}{\log_e\frac{l}{r}}$	l = length of wire r = radius of wire	Distant from earth and other conductors	
17	Distributed capacitance of two-layer solenoid	$\frac{2\pi\epsilon Rl}{3D}$	R = radius of section of solenoid D = distance between layers l = axial length of solenoid	D small compared with R	Morecroft (Ref. (4))
18	Distributed capacitance of solenoid of N layers	$C_0 \times \frac{4}{3}\left(\frac{N-1}{N}\right)^2$	C_0 = capacitance between innermost and outermost layers		Morecroft (Ref. (4))
19	Distributed capacitance of short single-layer solenoid	$0{\cdot}88\epsilon l$	l = length of one turn of wire on solenoid	Approx. formula	Breit (Ref. (33))

outside the generator. Wave-tail-control resistors can be used as the charging resistors within the generator circuit.

Fig. 11.25 (*c*) shows the connections of a widely used multi-stage circuit which combines high efficiency with distributed wave-front resistors. The value R_3 is made large compared with R_1, but R_2 is as small as is necessary to give the required length of wave-tail. Since the current through R_2 does not flow through R_1 it does not reduce the initial generator output voltage, however small the value of R_2 or however large R_1 may be.

A sphere-gap is used for voltage measurement, and measures the peak value of the impulse wave. Owing to the fact that the sphere-gap and apparatus under test are in parallel, care must be taken in using the former for voltage measurement. The conditions of its use for this purpose are specified in B.S. 358 : 1939. This states that the accuracy of measurement is within $\pm$ 3 per cent provided that the duration of the impulse voltage is not less than that of a 1/5-microsec wave.

A single-beam cathode-ray oscillograph is used, with a simple capacitance-type potential divider, to obtain either a visual or photographic record of the impulse wave-shape. A. K. Nuttall (Ref. (92)) has described the construction of an oscillograph for the measurement of moderately high impulse voltages directly.

Impulse Ratio. The "impulse ratio" of an insulator is the ratio

$$\frac{\text{Minimum spark-over voltage when tested with an impulse voltage}}{\text{Spark-over voltage when tested at power frequency}}$$

This is not constant for any given insulator, but is always greater than unity. It depends upon—

(*a*) The polarity of the impulse voltage. The spark-over voltage is usually less, by some 10 per cent, when the insulator pin (in the case of a porcelain insulator) is positive than when it is negative.

(*b*) The steepness of the impulse wave-front and the time of decay of the impulse voltage. The highest spark-over voltage is obtained with an impulse voltage which rises most rapidly to its peak value and also falls away again rapidly. Goodlet (*loc. cit.*) gives the figures 1·3 to 1·5 for the mean impulse ratio in the case of pin-type porcelain insulators, and 1·2 to 1·4 for suspension insulators.

Notes on the Testing of Insulators, Insulating Materials, and Cable Lengths. While impulse and high-frequency tests are carried out as above described, for research purposes, and by manufacturers in order to ensure that their finished products will give satisfactory performance in service, the most general tests upon insulating materials are carried out at power frequencies. Such tests may be carried out in accordance with the purchaser's specifications, and their exact nature then depends upon individual requirements. So many factors, such as barometric pressure, temperature, time of application of the testing voltage, and so on, influence the results of these tests that the British Standards Institution, and

similar authorities in other countries, have drawn up standard specifications which state standard test conditions for various types of manufactured apparatus and materials.

TESTING OF PORCELAIN INSULATORS. Such insulators are designed so that spark-over occurs at a lower voltage than puncture, thus safeguarding the insulator, in service, against destruction in the case of line disturbances. Flash-over tests are thus very important in this case. Flash-over, or surface breakdown, is really due to a breakdown of the air at the insulator surface, and the voltage at which it occurs for a given insulator depends upon—

(*a*) The barometric pressure.

(*b*) The temperature.

(*c*) The shape of the electrostatic field.

(*d*) The humidity.

(*e*) The nature of the contact between the insulator and the electrodes.

Of these factors, the first two can be taken into account by assuming that the flash-over voltage is directly proportional to the "air density correction factor" $\left(\frac{0{\cdot}386p}{273+t}\right.$, p being the pressure in millimetres of mercury and t the temperature in degrees Centigrade), provided this is not very different from unity.

If the electrostatic field has no component in a direction normal to the surface of the insulator, the breakdown voltage is simply that for the air alone, and is practically independent of the material of the insulator. Usually, however, the electrostatic field lies partly in the surrounding air and partly in the insulator, and the shape of this field becomes very important. It is thus necessary to take into account the disposition of any metal parts, used for mounting purposes, when considering results.

The flash-over voltage decreases with increasing humidity and correction factors are given in B.S. 137 : 1941. Dirt of any kind upon the surface has a similar effect to that of deposited moisture in lowering the flash-over voltage, and thus for consistent results with a given insulator it should be both clean and dry.*

Unless the contacts between the electrodes and insulator are good, ionization of the air near the contacts will take place, and the breakdown voltage will be reduced.

The time of application of the voltage may also affect the results. If the applied voltage is high, and is maintained for some time at a value a little below the normal flash-over voltage, breakdown often occurs.

In addition to the dry flash-over test, rain tests, and tests in a misty or smoke-laden atmosphere, are often carried out, the rain test being specified in British Standard Specification No. 137 : 1941,

* See also Ref. (84).

for "Porcelain and Toughened Glass Insulators for Overhead Power Lines" (3·3 kV and upwards).

The specification gives a table of standard insulator rating numbers and test voltages, the latter being separated into "Minimum dry flash-over voltage," "Minimum wet flash-over voltage," "One-minute dry test voltage," "One-minute rain test voltage" and "Puncture test voltages."

The various tests are then specified and the following notes give an indication of their nature (the specification should be referred to for full details).

POWER FREQUENCY TESTING VOLTAGE. This shall be of frequency between 25 and 100 c/s and approximately sine wave-form. The peak value, determined by sphere-gap, oscillograph or other approved method, shall not exceed 1·45 × r.m.s. value.

FLASH-OVER TESTS. These are (i) 50 per cent dry impulse flash-over test, using an impulse generator delivering a positive 1/50-microsecond impulse wave. "The voltage shall then be increased to the 50 per cent impulse flash-over voltage, i.e. the voltage at which approximately half of the impulses applied cause flash-over of the insulator." (ii) Dry flash-over and dry one-minute test in which the test voltage given in the table (mentioned above) is applied. "The voltage shall be raised gradually to this value in approximately ten seconds and shall be maintained for one minute. The voltage shall then be increased gradually until flash-over occurs." (iii) Wet flash-over and one-minute rain test in which the insulator is "sprayed throughout the test with artificial rain drawn from a source of supply at a temperature within 10° C of the ambient temperature in the neighbourhood of the insulator." The resistivity of the water is to be between 9,000 and 11,000 ohm-cm.

SAMPLE TESTS. Specifications are given for temperature-cycle tests, mechanical tests, electro-mechanical tests, puncture tests and porosity tests.

ROUTINE TESTS. These are to be applied to all insulators and "shall be commenced at a low voltage and shall be increased rapidly until a flash-over occurs every few seconds. The voltage shall be maintained at this value for a minimum of five minutes or, if failures occur, for five minutes after the last punctured piece has been removed. At the conclusion of the test the voltage shall be reduced to about one-third of the test voltage before switching off."

POTENTIAL DISTRIBUTION ALONG A SUSPENSION INSULATOR STRING. Owing to the earth capacitances between the metal fittings (caps and pins) of an insulator string and the supporting tower or pole, the potential distribution across the various component units of the string is by no means uniform. The potentials across the units adjacent to the line conductor are much greater than those across units nearer to the supporting cross-arm. If all the units are identical, the consequence is that the "string efficiency," i.e. the ratio $\frac{\text{actual breakdown voltage of the complete string}}{\text{breakdown voltage per unit} \times \text{number of units}}$ is low.*

Measurement of the potential distribution along such a string may be carried out in the laboratory by either (*a*) direct measurement or (*b*) a potential-divider or null method. These methods may be understood from the simplified diagrams of Fig. 11.26, in which (*a*) shows the direct method and (*b*) the potential-divider method.

* For a fuller discussion of the question, see Ref. (78); also F. W. Peek, *Trans. A.I.E.E.*, Vol. XXXI, p. 907; or H. Cotton, *The Transmission and Distribution of Electrical Energy*.

In (*a*) a small test spark-gap is employed, set to spark over at a voltage *e* across it. The voltage *V* across the string is brought up to such a value as to cause the gap to spark. The connection *P* is moved from one unit to another down the string, and for each position the required value of *V* for gap spark-over is determined. The results give the percentage voltage drops *e*/*V* for the various portions of the string.*

Although this method is very simple and involves only a capacitance current, which does not cause pitting of the spheres, it is rather rough, and the presence of the exploring wire, unless this is

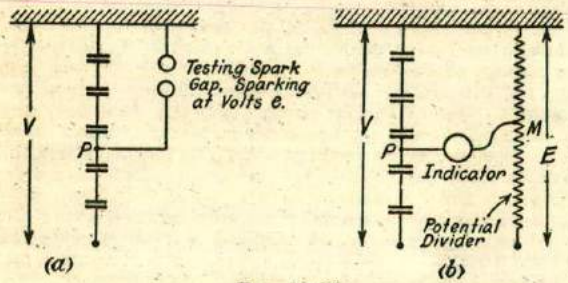


FIG. 11.26

correctly placed, may influence the electrostatic field distribution. Again, the capacitance of the gap itself may be comparable to that of the insulator units. In method (*b*) the contact *M* is moved until the indicator shows zero, when the potentials of *M* and *P* are obviously the same; thus that of *P* is known, for a given applied voltage *V*, from the calibration of the potential divider.

The potential divider may be a resistor or capacitor, or it may be replaced by a separate source of supply. This source, either from an induction regulator† or a second alternator driven in synchronism with that supplying the insulator string, must be in the same phase and of the same wave-form as the supply voltage *V*.

The indicator‡ may be: (i) a spark-gap; (ii) a neon tube of special construction; (iii) an electrometer across a high resistance; (iv) a vibration galvanometer in series with a high resistance; or (v) a triode.

The use of an electrometer for the measurement of high alternating voltages has been described by P. W. Baguley and H. Cotton.§

* See R. H. Marvin, *Trans. A.I.E.E.*, January and June, 1916.

† Drewnowsky, *Archiv für Elektrotechnik*, Vol. XXVII, p. 229.

‡ See also A. Schwaiger, "Voltage Distribution for Insulator Suspension Chains," *E. and M.*, No. 50, 1919; and "Theory of the High-voltage Insulator," *E.T.Z.*, No. 43, 1920; and *E. and M.*, No. 38, 1920; also Schering and Raske, *E.T.Z.*, Vol. LVI, p. 75, 1935.

§ *World Power*, Vol. XIX, No. CXI, March, 1933.

They used a Lindemann electrometer, which is a very useful instrument for the measurement of very small currents. The instrument, described by F. A. Lindemann and T. C. Keeley,* is manufactured by the Cambridge Instrument Co.

TESTING OF INSULATING MATERIALS. In the case of such materials it is not the voltage that produces spark-over breakdown which is important, but rather the voltage for puncture of a given thickness (i.e. the dielectric strength). The measurements made upon insulating materials are usually, therefore, those of dielectric strength and of dielectric loss and power factor, the latter being intimately connected with the dielectric strength of the material.

The breakdown of solid insulating materials is a complex process, and does not occur simply when the applied potential gradient, or field strength, exceeds a certain critical value, regardless of other conditions.

It is found that the dielectric strength of a given material depends, apart from the chemical and physical properties of the material itself, upon many factors including—

(*a*) The thickness of the sample tested.

(*b*) The shape of the sample.

(*c*) The previous electric and thermal treatment of the sample.

(*d*) The shape, size, material, and arrangement of the electrodes.

(*e*) The nature of the contact which the electrodes make with the sample.

(*f*) The waveform and frequency of the applied voltage (if alternating).

(*g*) The rate of application of the testing voltage and the time during which it is maintained at a constant value.

(*h*) The temperature and humidity when the test is carried out.

(*i*) The moisture content of the sample.

It is obviously very necessary, therefore, that tests shall be carried out under standard conditions, and with standard sizes and shapes of both sample and electrodes if the results are to have any real significance.

The Nature of Dielectric Breakdown. The theory of breakdown which was most generally adopted in the past was the *thermal* theory. K. W. Wagner first attempted to give a definite mathematical theory of thermal breakdown, and Prof. Miles Walker, during a discussion on E. H. Rayner's paper on "High-voltage Tests and Energy Losses in Insulating Materials" (*Jour. I.E.E.*, Vol. XLIX, p. 3) stated the essentials of the theory, which are as follows—

Dielectric losses occur in insulating materials when an electrostatic field is applied to them. These losses result in the formation of heat within the material. Most insulating materials are bad thermal conductors, so that, even though the heat so produced is small, it is not rapidly carried away by the material. Now, the conductivity of such materials increases considerably with increase of temperature, and the dielectric losses, therefore, rise and produce more heat, the temperature thus building up from the small initial

* *Phil. Mag.*, 1924, Vol. XLVII, p. 577.

temperature rise. If the rate of increase of heat dissipated, with rise of temperature, is greater than the rate of increase of dielectric loss with temperature rise, a stable condition (thermal balance) will be reached. If, however, the latter rate of increase is greater than the former, the insulation will break down owing to the excessive heat production, which burns the material.

Now, the dielectric losses per cubic centimetre in a given material and at a given temperature are directly proportional to the frequency of the electric field and to the square of the field strength. Hence the decrease in breakdown voltage with increasing time of application and increasing temperature, and also the dependence of this voltage upon the shape, size, and material of the electrodes and upon the form of the electric field.

Much research has been done, particularly by the British Electrical and Allied Industries Research Association, during recent years on this question of the dielectric breakdown of insulating materials. S. Whitehead (Ref. (93)) has reviewed the subject comprehensively, and this work, together with those mentioned in Refs. (2), (94), and (95), should be consulted by readers wishing to study the subject. H. F. Church and C. G. Garton (Ref. (95)) list five well-established causes of failure in solid insulation and state that two others are now under investigation.

Importance of Dielectric Loss Measurements. From the above it will be realized that the measurements of dielectric loss in insulating materials are very important, and give a fair indication as to comparative dielectric strengths of such materials. In the case of cables, dielectric loss measurements are now generally recognized as the most reliable guide to the quality and condition of the cable.

The measurement of dielectric losses was dealt with in Chapter IV.

Directions for the Testing of Solid Specimens. A number of reports issued by the British Electrical and Allied Industries Research Association* give directions for the study of insulating materials of all classes, including solid dielectrics, papers, fabrics, varnishes, and oils.

The following extracts from the B.S.I. Standard Specification B.S. 234 : 1957, "Loaded and Unloaded Ebonite for Electrical Purposes," will serve to indicate what precautions are to be taken and what methods adopted in the testing of ebonite, which may be taken as a typical solid dielectric.

ELECTRIC STRENGTH. When five test pieces are tested in accordance with Appendix E, at least four shall withstand, for one minute without breakdown, the appropriate test voltage—

For Type 1—40 kV
For Type 2—25 kV
For Type 3—30 kV

Appendix E gives the following information—

TEST PIECE. The test piece shall be a flat disc with smooth surfaces not less than 100 mm in diameter and $3{\cdot}2 \pm 0{\cdot}2$ mm in mean thickness. The variation in thickness shall not exceed 0·2 mm.

ELECTRODES. The lower electrode shall consist of a solid cylinder of brass 3 in. in diameter and not less than 1 in. thick, and the upper electrode shall consist of a solid cylinder of brass $1\frac{1}{2}$ in. in diameter and not less than $1\frac{1}{2}$ in. thick, the plane faces of the electrodes being in contact with the test piece and

* See references at end of chapter.

concentric with each other during the test. The sharp edges shall be removed from the electrodes, the radius at the edge being approximately $\frac{1}{32}$ in.

It is laid down that the test pieces shall be conditioned at a temperature of $20 \pm 2°$ C and a relative humidity of 65 ± 5 per cent, for not less than 18 hours immediately before test. The test shall be made under insulating oil complying with B.S. 148 : 1959. The applied voltage is to be raised as rapidly as is consistent with avoiding transient over-voltages.

It is also laid down that the permittivity and power factor determined at a frequency between 800–1,600 c/s shall not exceed the following values—

	Type 1	Type 2	Type 3
Permittivity	3·0	3.3	3·8
Power factor	0·006	0·008	0·008

A Schering bridge method for carrying out these tests is described in Appendix C.

The above extract is reproduced by permission of the British Standards Institution, from whom copies of the complete standard may be obtained.

THE TESTING OF INSULATING OILS. In British Standard Specification (B.S. 148 : 1959) for "Insulating Oil for Transformers and Switchgear," the method of applying the testing voltage (which must be alternating, of approximately sine wave-form, of frequency between 25 and 100 cycles per second and with a peak factor of $\sqrt{2} \pm 5$ per cent) which is recommended, is shown in Fig. 11.27 taken from the specification.

It is specified that—

"The minimum internal dimensions of the test-cell shall be 55 mm × 90 mm × 100 mm high.

"The electrodes shall be polished spheres of 12·7–13 mm diameter, preferably of brass, arranged horizontally with their axis not less than 40 mm above the bottom of the cell. For the test, the distance between the spheres shall be $4 \pm 0{\cdot}02$ mm. A suitable gauge shall be used for checking the gap.

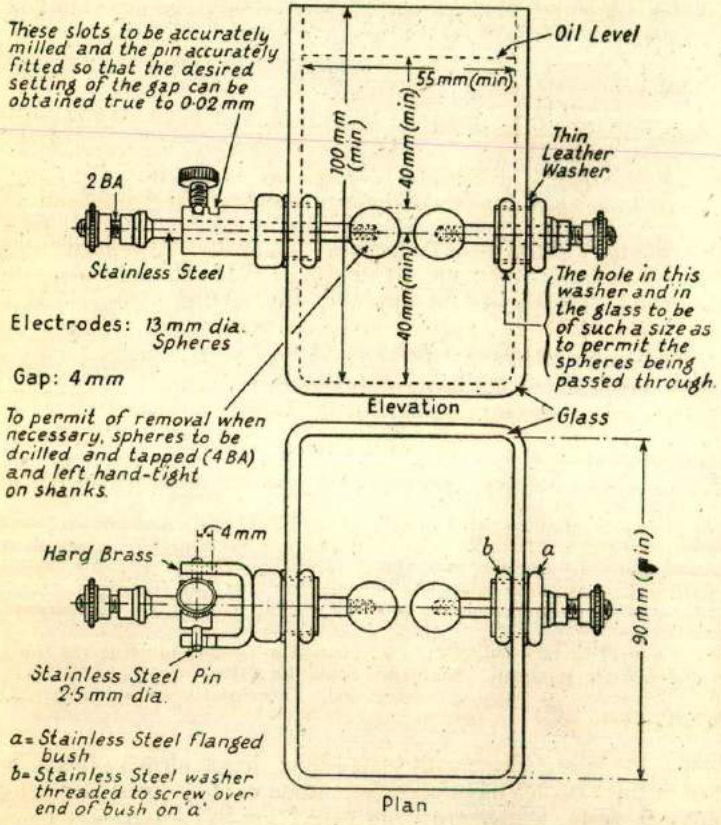
"Spheres shall be replaced as soon as pitting caused by discharges is observed."

On account of the serious effect of suspended solids and moisture in the oil upon the electric strength, great care must be taken in preparing the oil sample for test and in cleaning the test-cell. These matters are dealt with in the specification.

CABLE TESTING. The cable tests which have already been described in this chapter have been tests upon cables already installed. Acceptance tests, which are tests called for by purchasers before accepting cables from the manufacturers, are also of importance. The results of such tests must be a reliable guide as to the probability of the cables being satisfactory in service.

At one time breakdown tests upon short lengths of cable with alternating voltages of commercial frequency, rapidly applied, were called for, but it was later realized that such tests gave little or no reliable information regarding the cable, since the breakdown value when the voltage is applied for a considerable time is usually considerably smaller than that obtained with rapid application of voltage. Thus, tests with a voltage applied for 15 or 30 min were

carried out in addition to the rapid tests. As a development of these tests, time/voltage curves for short lengths of cable are now plotted by some manufacturers (Refs. (29), (30)). These are obtained by first determining the breakdown voltage for one length with rapid application of voltage, and then, with other lengths, finding the



(*By courtesy of the B.S.I.*)

FIG. 11.27. APPARATUS FOR THE MEASUREMENT OF THE DIELECTRIC STRENGTH OF OIL

length of time required before breakdown occurs, with applied voltages of gradually decreasing magnitudes for each test length. For example, if 100 kV produces breakdown of sample 1 when rapidly applied, and it is found that it takes 5 min for 90 kV to produce breakdown of sample 2, 12 min for 85 kV to produce breakdown of sample 3, and so on, a time/voltage curve for the

cable can be plotted. This method is suggested by Dunsheath (Ref. (6)). E. A. Beavis (Ref. (30)) has investigated the question of the preparation of the ends of the cable lengths for such tests, in order to avoid breakdown by flash-over instead of by puncture.

Dielectric-loss and power-factor tests are regarded as giving the most reliable information as to the quality of the cable, and such tests are of greater importance than breakdown tests upon short lengths.

It is found that the dielectric losses in high-voltage cables do not increase with the square of the voltage, as is the case with most simple dielectrics, nor does the power factor remain constant. The losses increase in proportion to some power of the voltage greater than the second, and the power factor rises with voltage, these effects being due to ionization caused by air which is entrapped within the cable insulation. Measurements of power factor with different voltages applied to the cable are therefore made, and the power-factor/voltage curve is plotted. This should be practically flat if the cable under test is such as will be satisfactory in service. The variation of power factor with temperature is also of importance. In a good cable the power factor should increase very little with increase of temperature. To investigate this power factor, measurements are made at different dielectric temperatures, obtained by heating the cable by the passage of current through its cores. The power-factor/voltage curve is also obtained when the cable has cooled after a heat run. This should not be appreciably different from the curve obtained before the heat run.

Live Conductor Detection. While this scarcely comes under the heading of a high-voltage measurement it is perhaps worthy of mention here.

It is very important to be able to determine whether a high-voltage conductor is live or dead prior to earthing it, and a difficulty arises from the fact that, if the detecting device used is defective, it may indicate that the conductor under test is dead when in fact it is not.

The requirements for a satisfactory potential indicator have been discussed by G. F. Shotter and E. E. Hutchings (Ref. (97)). They are—

"(i) To indicate reliably if a conductor is not at earth potential.

"(ii) To discriminate whether the voltage on a conductor is due to conductive connection with a source of supply, or to electrostatic charge, or electrostatic or inductive coupling with a live circuit. This involves a quantitative indication.

"(iii) To be of a form suitable for use by a linesman either indoors or outside under all weather conditions.

"(iv) To be such that it can be tested before use to ensure that it is operating satisfactorily."

This report made recommendations for the construction of a potential indicator of the resistance type and such an instrument is

now made by Everett Edgcumbe and Co., Ltd. It has a series resistor comprising a number of ceramic resistance units in a tube of Bakelized material and a robust moving-coil indicator having high sensitivity. A permanently attached flexible lead is used for earthing. The indicator reads direct in kilovolts to earth and has a scale which is closed up at higher readings. This increases the effective range and enables the detector to be checked, before use, on a medium-voltage circuit.

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CHAPTER XII

LOCALIZATION OF CABLE FAULTS

THE routine testing of cables with high voltages and the localization of faults in high-voltage cables, using high-voltage d.c., have already been dealt with in the previous chapter. Only the localization of faults in cables which are in service, and for use with the lower distribution voltages, will be considered here.

The faults which are most likely to occur are: (*a*) a breakdown of the insulation of the cable which allows current to flow from the core to earth or to the cable sheath—called a "ground" fault; (*b*) a "cross" or short-circuit fault, in which case the insulation between two cables, or between two cores of a multi-core cable, is faulty; and (*c*) an open-circuit fault where the conductor becomes

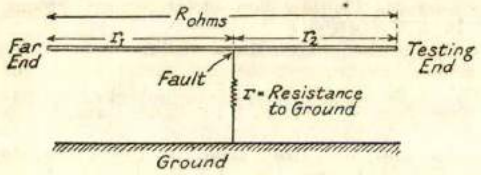


FIG. 12.1. GROUND FAULT ON A SINGLE CABLE

broken or a joint pulls out. The methods used for locating an open-circuit fault differ from those used in the two other cases.

The causes of such faults are numerous and need not concern us. It is important that their exact position shall be determined, however, in order that repairs may be undertaken without loss of time and effort.

In the case of multi-core cables it is advisable, first of all, to measure the insulation resistance of each core to ground and also between cores. If the fault is a "ground" this will enable the faulty core to be discovered; and if a short-circuit, the cores which are involved can be determined.

Blavier and Earth Overlap Tests. These tests enable one to find the position of a ground on a single cable—i.e. when no other cables run along with the faulty one.

The case is illustrated by Fig. 12.1. The total resistance of the cable, before the occurrence of the fault, is assumed to be known. Let this be R ohms. Suppose also that r is the resistance of the fault to ground, and that r_1 and r_2 are the resistances of the lengths of cable, "far end" to fault, and "testing end" to fault, respectively.

In the Blavier test the resistance between line and ground is

measured, first with the "far end" disconnected from earth, and then with the "far end" earthed.

These measurements may be made with the aid of a low-voltage supply, and either an ammeter and voltmeter or a bridge network.

Theory. Let the two measured values of resistance be M_1 and M_2.

Then $$M_1 = r_2 + r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

$$M_2 = r_2 + \frac{rr_1}{r + r_1} \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

and $$R = r_1 + r_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

From (i), $$r = M_1 - r_2$$

Hence, in (ii), $$M_2 = r_2 + \frac{(M_1 - r_2)\, r_1}{(M_1 - r_2) + r_1}$$

from which $$(M_2 - r_2)(M_1 - r_2 + r_1) = (M_1 - r_2)\, r_1$$

$$\therefore M_1M_2 - M_1r_2 - M_2r_2 + r_2^2 + r_1M_2 = M_1r_1$$

Substituting from (iii), we have, after simplifying,

$$M_1M_2 - 2M_2r_2 + r_2^2 + RM_2 = M_1R$$

or $$r_2^2 - 2M_2r_2 + M_2^2 - M_2^2 + M_1M_2 = R(M_1 - M_2)$$

$$(r_2 - M_2)^2 = (M_1 - M_2)(R - M_2)$$

Thus $$r_2 = M_2 \pm \sqrt{(M_1 - M_2)(R - M_2)} \quad . \quad . \quad (12.1)$$

Since r_2 is obviously less than M_2, the negative sign in this expression is the one to be taken, i.e.

$$r_2 = M_2 - \sqrt{(M_1 - M_2)(R - M_2)} \quad . \quad . \quad (12.2)$$

In the "earth overlap" test, the two measurements made are: first, the resistance M_1 between line and ground, measured from the "testing end," with the "far end" earthed; and then the resistance M_2, line to ground, measured at the "far-end," with the "testing end" earthed.

Then $$M_1 = r_2 + \frac{rr_1}{r + r_1}$$

$$M_2 = r_1 + \frac{rr_2}{r + r_2}$$

and, as before, $R = r_1 + r_2$.

By elimination, as in the Blavier test, we have

$$r_2 = M_1\left(\frac{R - M_2}{M_1 - M_2}\right)\left\{1 - \sqrt{\frac{M_2(R - M_1)}{M_1(R - M_2)}}\right\} \quad . \quad (12.3)$$

and also $$r_1 = M_2\left(\frac{R - M_1}{M_2 - M_1}\right)\left\{1 - \sqrt{\frac{M_1(R - M_2)}{M_2(R - M_1)}}\right\} \quad . \quad (12.4)$$

In each of the above tests, the distance of the fault from either end is obtained from the resistance between the fault and the end by using the known value of the resistance, per unit length, of the cable. The accuracy of these two methods is not high, mainly because it is often difficult to make connections to earth, at the ends of the cable, having a low enough resistance for this to be considered negligible—as it is in the theory of the tests (see Ref. (8)).

Voltage-drop Tests. These tests can be used when a second cable, free from faults, runs along with the faulty cable, the sound cable being used, either as part of the current circuit as in Fig. 12.2, or as a potential lead to the voltmeter as in Fig. 12.3.

Referring to Fig. 12.2, a large *steady* current is passed through the loop formed by the sound and faulty cables, joined together at the

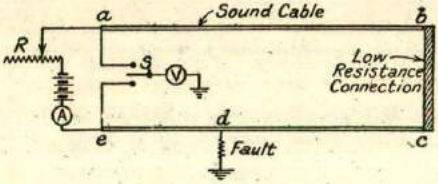


FIG. 12.2. SOUND CABLE USED IN CONJUNCTION WITH THE FAULTY ONE

distant end, as shown, by a low-resistance connection. The current is from a number of accumulators, and is regulated and measured by the resistor R and ammeter A. By means of the throw-over switch S the voltmeter V, one terminal of which is earthed, is connected first across the section *ed* of the loop, and then across the section *abcd*. Let the two readings obtained be V_1 and V_2. Since the same current flows through both of these loop-sections (neglecting the voltmeter current) we have

$$\frac{V_1}{V_2} = \frac{\text{Resistance of section } ed}{\text{Resistance of section } abcd}$$

or

$$\frac{V_1}{V_1 + V_2} = \frac{\text{Resistance of section } ed}{\text{Resistance of the whole loop } abcde}$$

Now, if the cross-section of the cable is the same throughout the length, we have

$$\frac{\text{Distance of fault from } e}{\text{Length of the whole loop}} = \frac{V_1}{V_1 + V_2}$$

from which the position of the fault can be found. If the cable cross-section is not uniform, a correction must be applied to allow for the fact.

The resistance of the voltmeter should be large compared with the resistance of the fault, since the latter forms part of the voltmeter circuit. This fact constitutes an objection to the method. Another reason for the high-resistance voltmeter is that the instrument would otherwise take an appreciable current and so introduce errors in measurement.

In the circuit shown in Fig. 12.3, the fault resistance does not enter into the resistance of the voltmeter circuit. In this case the voltmeter measures the voltage drop across the length *ed* of the cable, and the resistance of this length is obtained by dividing by the current through it (as indicated by the ammeter *A*).

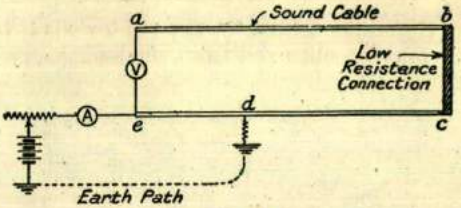


FIG. 12.3

The voltmeter resistance should be high as compared with the resistance of both the length *ed* and of the length *abcd* of the cable, if corrections for voltmeter current are to be avoided. If the voltmeter is electrostatic, no corrections are, of course, necessary.

The length *ed* is calculated, as before, from the known resistance per unit length. Care must be taken that the current passed through the length *ed* of the cable is not sufficient to produce appreciable heating as the resistance per unit length will then be different from that used in the calculation of length.

The same precaution must be taken in the previous method if the cable cross-section is not uniform. If the cross-section is the same throughout, however, the proportionality renders the precaution less necessary, although it is not advisable to pass such a current as will produce appreciable heating, even in this case.

Loop Tests. These tests can be carried out for the location of either a ground or a short-circuit fault, provided that a sound cable runs along with the grounded cable or with the two cables (or cores in a multi-core cable) which are short-circuited. Such tests have the advantage that the resistance of the fault does not affect the results obtained, provided this resistance is not very high. If it is high it may adversely affect the sensitivity.

MURRAY LOOP TEST. The connections for this test are shown in Fig. 12.4. Connections (*a*) are for a test for a ground fault, and (*b*) for a short-circuit fault. Both circuits are essentially Wheatstone bridge networks, *G* being a galvanometer and *P* and *Q* resistors,

or a slide-wire, forming two ratio arms. Referring to Fig. 12.4(*a*), the bridge is balanced by adjustment of P and Q until G indicates zero deflection.

Then
$$\frac{P}{Q} = \frac{R}{X}$$

or
$$\frac{P+Q}{Q} = \frac{R+X}{X}$$

$$\therefore \frac{Q}{P+Q} = \frac{X}{R+X} = \frac{X}{2r} \quad . \quad . \quad . \quad (12.5)$$

where r is the resistance of one of the cables when free from faults. The value of r may be obtained from the lengths, cross-sections and

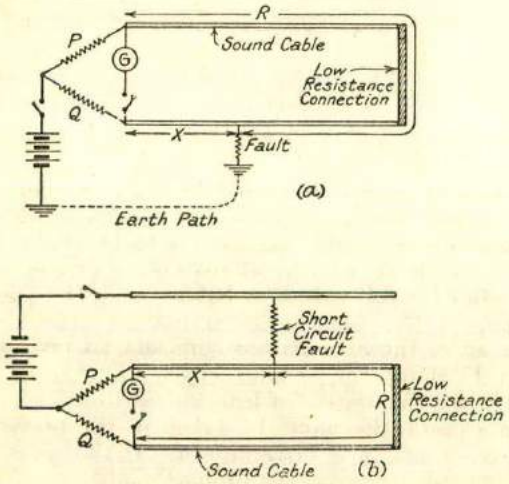


FIG. 12.4. MURRAY LOOP TEST

temperatures of the two cables, all of which are assumed to be the same for each. The distance of the fault from the lower end of resistor Q may then be obtained from the value of X obtained as above.

It should be noted that the resistance of the fault enters only in the battery supply circuit, and provided it is not sufficiently large to reduce the sensitivity, will not affect the results.

In Fig. 12.4 (*b*), the connections are practically the same as in the ground test, except that a portion of one of the short-circuiting cables is substituted for an earth path in the battery circuit. Balance

is obtained as before, by the adjustment of P and Q, and at balance,

$$= \frac{R}{X}$$

or

$$\frac{P}{P+Q} = \frac{X}{R+X} \quad . \quad . \quad . \quad . \quad (12.6)$$

The total resistance $(R + X)$ of the loop is assumed to be known, so that X, and hence the distance of the fault from the upper end

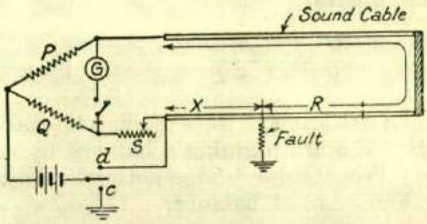


FIG. 12.5. VARLEY LOOP TEST FOR GROUND FAULT

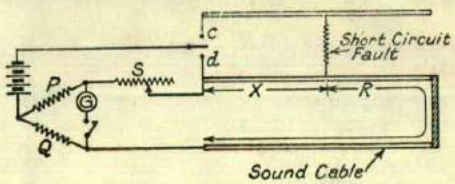


FIG 12.6. VARLEY LOOP TEST FOR SHORT-CIRCUIT FAULT

of P, may be calculated. Again, the fault resistance enters only into the battery circuit.

VARLEY LOOP TEST. This test makes provision for the measurement of the total loop resistance instead of obtaining it from the known lengths of cable and their resistance per unit length. The connections are shown in Figs. 12·5 and 12.6 for the ground and short-circuit tests respectively. In this loop test the ratio arms P and Q are fixed, balance being obtained by adjustment of a variable resistor S, placed in series with the section of the loop having the smaller resistance. When balance is obtained with the throw-over switch, in the battery circuit, on contact c, then, in either test, the magnitude of the resistance X may be obtained from the setting of S for balance, together with the values of P and Q and of the resistance $R + X$ (i.e. the total resistance of the loop).

At balance, in the ground test,

$$\frac{P}{Q} = \frac{R}{X+S}$$

or

$$\frac{P+Q}{Q} = \frac{R+X+S}{X+S}$$

From which,

$$X = \frac{Q(R+X) - SP}{P+Q} \quad . \quad . \quad . \quad (12.7)$$

In the short-circuit test,

$$\frac{Q}{P} = \frac{R}{X+S}$$

Now, P, Q, and S are known. $R + X$ may be measured by throwing over to contact d and obtaining a balance by adjustment of S as in the ordinary Wheatstone bridge network. In the ground test, as connected in Fig. 12.5, at balance,

$$\frac{P}{Q} = \frac{R+X}{S_1}$$

where S_1 is the new setting of S. Thus $R + X$ can be found. In Fig. 12.6, at balance,

$$\frac{P}{Q} = \frac{S_2}{R+X}$$

where S_2 again is the required setting of S for balance. The measured value of $R + X$ is then used in the calculation of X, from whose value the position of the fault is obtained as before. The use of the Evershed and Vignoles Bridge-Megger Testers for the location of cable faults was mentioned on p. 324.

FISHER LOOP TEST. In this test, developed by H. W. Fisher, two sound conductors, running from the testing end to the far end of the faulty cable, must be available. The lengths and resistances of these two conductors need not be known, but the length and resistance of the faulty cable must be known.

Two balances of the bridge network are necessary, the two connections for these being as shown in Fig. 12.7 (*a*) and (*b*). In the first test, one of the sound cables—of resistance R_1—is left disconnected, and in the second test it is merely used as a lead from the battery to the far end of the faulty cable. R_2 is the resistance of the other sound cable, while x and r are the resistances between the fault and the testing end, and between fault and the far end, respectively.

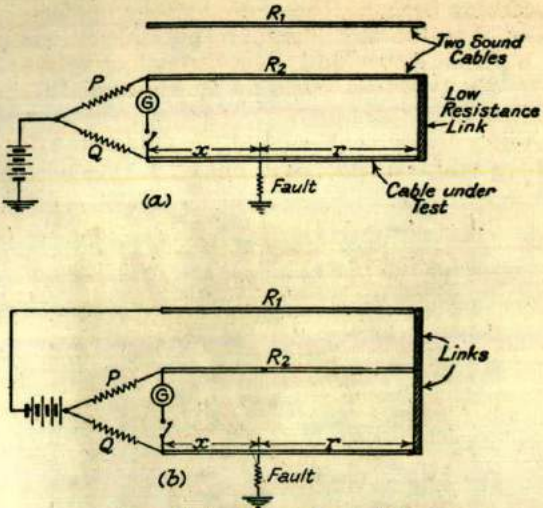


FIG. 12.7. FISHER LOOP TEST

Theory. Let P_1 and Q_1 be the balance values of P and Q in the first test, and P_2 and Q_2 their values in the second test.

Then $$\frac{P_1}{Q_1} = \frac{R_2 + r}{x}$$

and $$\frac{P_2}{Q_2} = \frac{R_2}{x + r}$$

R_2 is eliminated as follows—

$$\frac{P_1}{Q_1} + 1 = \frac{R_2 + r}{x} + \frac{x}{x} = \frac{R_2 + r + x}{x}$$

$$\frac{P_2}{Q_2} + 1 = \frac{R_2}{x + r} + \frac{x + r}{x + r} = \frac{R_2 + r + x}{x + r}$$

$$\therefore \frac{\frac{P_2}{Q_2} + 1}{\frac{P_1}{Q_1} + 1} = \frac{x}{x + r}$$

$$\therefore x = (x + r) \frac{\left(\frac{P_2}{Q_2} + 1\right)}{\left(\frac{P_1}{Q_1} + 1\right)} \quad . \quad . \quad . \quad . \quad . \quad (12.8)$$

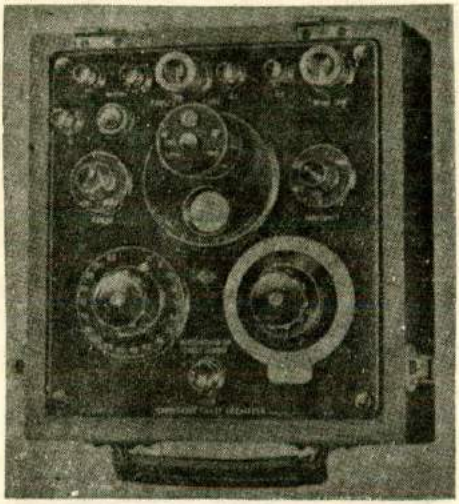
If the resistance per unit length of the faulty cable is uniform, we have

Distance of fault from testing end

$$= \left(\frac{\frac{P_2}{Q_2} + 1}{\frac{P_1}{Q_1} + 1} \right) \times \text{Total length of faulty cable}$$

Fault Localizing Bridges. These usually take the form of portable self-contained Wheatstone bridges incorporating switching arrangements which permit either Murray or Varley loop tests to be carried out. The D_1–30–A test set made by Muirhead and Co., Ltd., is a typical instrument of this kind.

The Cambridge Instrument Co., Ltd., manufacture a direct-reading fault-localizing bridge illustrated in Fig. 12.8. The operation of this



(Cambridge Instrument Co., Ltd.)

FIG. 12.8. DIRECT-READING FAULT-LOCALIZING BRIDGE

instrument is based on the Murray loop test (Fig. 12.4 (*a*)), but in this case the ratio $\frac{Q}{P+Q}$ is read directly on two dials, one calibrated from 0 to 95 and the other from 0 to 5. The dials are calibrated so that the sum of their readings gives the distance to the fault as a percentage of the length of the section under test, and not as a fraction of the total length of loop.

Corrections to be Applied in Loop Tests. If a ratio such as

$$\frac{\text{Resistance to fault}}{\text{Resistance of the whole loop}}$$

is, in one of these tests, determined in terms of resistances in the bridge ratio arms, it is obviously equal to

$$\frac{\text{Distance to fault}}{\text{Length of the whole loop}}$$

only if the cable section (and also the temperature) is uniform throughout the loop. If this is not so, corrections must be applied.

Correction when the Cross-section and Length of both Faulty and Sound Cables are Known.

Let L_f = length of faulty cable.

a_f = cross-section of faulty cable.

L_s = length of sound cable.

a_s = cross-section of sound cable.

Then, equivalent length of the whole loop is $L_f + L_s \cdot \frac{a_f}{a_s}$ and this length must be used in calculating the distance to the fault, instead of the actual length of loop. If resistances of the two cables, per unit length, r_f and r_s, are used instead of cross-sections, the equivalent length is $L_f + L_s \frac{r_s}{r_f}$. Temperature corrections are applied in a similar way if the temperatures of the two cables are known or can be estimated with reasonable accuracy.

Correction when the Cross-section of the Faulty Cable is Not Uniform. Suppose the faulty cable consists of a number of sections in series, these sections having different resistances per unit length. Let L_1, L_2, L_3, etc., be the lengths of those sections, and r_1, r_2, r_3, etc., be their resistances per unit length. Then the first section has resistance L_1r_1, the second L_2r_2, and so on. If x is the resistance—obtained by measurement—of the cable from testing end to the fault, it is first necessary to determine in which section the fault exists. Thus, if x is greater than $L_1r_1 + L_2r_2$, but less than $L_1r_1 + L_2r_2 + L_3r_3$, the fault is in the third section, and its distance L along this section from the point of junction with section 2 is given by

$$L = \frac{x - L_1r_1 - L_2r_2}{r_3} \quad . \quad . \quad . \quad . \quad . \quad (12.9)$$

As mentioned above, temperature corrections must also be applied if any appreciable difference of temperature exists between the various sections. It may be necessary, also, to correct for the resistances of joints if these are numerous.

Although other cases requiring corrections exist, enough has been said to indicate the method of applying such corrections in any particular case.

Tests for an Open-circuit Fault. If a complete disconnection occurs in a cable—either by a fault burning clear without causing a ground fault, or on account of the cable being pulled out at a joint—its position may be found by a capacitance test. The capacitance of a cable, to ground or to another parallel conductor, is proportional to the length of the cable. If the capacitance C of the whole length of the cable, when sound, is known, and the capacitance C_x of the length between one end and the fault is measured, the distance of the fault from the testing end is $\frac{C_x}{C} \times$ length of the whole cable. If the capacitance C is not known, tests must be carried out from each end of the cable, the sum of the two capacitances so measured giving the capacitance C. The distance of the fault from either end can then be obtained as above.

For the measurement of capacitance, either a ballistic galvanometer or an alternating-current bridge method, using a high-frequency generator and telephone detector, may be employed.

Connections for the measurement of the capacitance of the cable

up to the fault are shown in Fig. 12.9, two alternatives being given. The first method of measurement is by direct deflection, using a ballistic galvanometer. C is a standard capacitor. The galvanometer BG is shunted by an Ayrton shunt. The galvanometer throw is first observed when the capacitance represented by the cable-length is charged from the battery, the standard capacitor being then out of circuit. This capacitor is then substituted for the cable by means

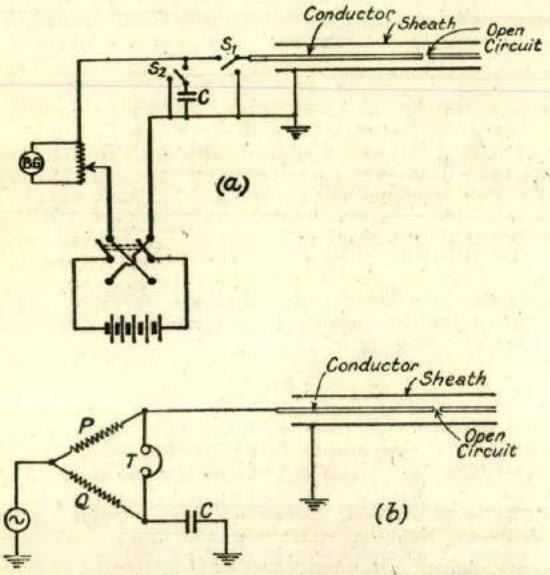


FIG. 12.9. TEST FOR AN OPEN-CIRCUIT FAULT

of the switches shown, and the galvanometer throw produced, when it is charged from the same battery, is observed. If D_c and D_s are these two throws—corrected if necessary for variations of the shunting powers in the two cases—we have

$$\frac{\text{Capacitance of cable-length to the fault}}{\text{Capacitance of standard capacitor}} = \frac{D_c}{D_s}$$

In the second method a capacitor bridge is employed, the supply being a high-frequency generator, and the detector T a telephone. The cable capacitance is then measured in terms of the standard capacitor C, the non-inductive resistors P and Q being adjusted to give a balance of the bridge, when

$$\frac{\text{Capacitance of cable-length to the fault}}{\text{Capacitance of standard capacitor}} = \frac{Q}{P}$$

The distance to the fault is then obtained, as described above, from this measured value of the capacitance, up to the fault, compared with the capacitance of the whole cable.

Induction Method of Testing. This method, which is used for the localization of ground faults in cables, can only be successfully employed when the cable has no metallic sheath or armouring. It is, however, useful for the localization of such faults in vulcanized-bitumen cables, and has the merit of discovering the fault directly without reliance having to be placed upon calculations, or upon assumptions as to cable resistance.

A battery and interrupter are used to send an interrupted direct current through the faulty cable. An exploring, or search coil, to

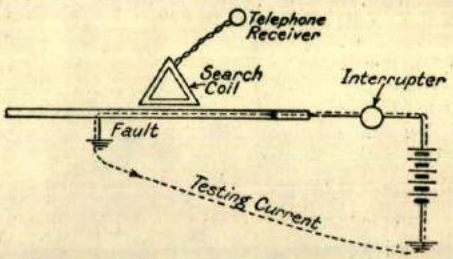


FIG. 12.10. INDUCTION METHOD OF LOCATING A GROUND FAULT

the terminals of which a telephone receiver is connected, is then placed with its plane parallel to the direction of the cable as shown in Fig. 12.10. This coil consists of about two hundred turns of 26 or 30 S.W.G. wire, wound on an equilateral-triangle former of about 3 ft side.* The intermittent current in the cable induces an e.m.f. in the coil, and a note is thus heard in the telephone receiver. If the search coil is moved along the cable in the direction of the fault, this note will cease immediately the fault is passed, since the testing current there passes to earth, and from thence passes to the earthed terminal of the testing battery. The position of the fault is thus directly determined from the cessation of the telephone note.

If the cable is lead-covered or is armoured, the method is rendered uncertain, owing to the currents flowing in the sheath (which is connected to the cable at the fault) and to the magnetic shielding effect of the steel armouring in the latter case.

A test set on this principle, containing an impulse generator as well as two different forms of search coil, is the "P & B" Cable Tracer and Fault Locator, made by Price and Belsham, Ltd. From the point of view of tracing the run of an underground cable the magnetic shielding effect of the armouring is negligible.

* These dimensions are taken from Raphael's *Localization of Faults*.

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CHAPTER XIII

ELECTRICAL METHODS OF MEASURING TEMPERATURE

General. The most important methods of measuring temperature are—

(*a*) By mercury thermometers.
(*b*) By gas thermometers.
(*c*) By pyrometers.

Mercury thermometers are generally used for temperatures up to about 300° C, mercury remaining liquid over the range – 39° C to 357° C. If the mercury is under pressure of an inert gas such as nitrogen or carbon dioxide, the boiling point is increased so that temperatures up to 500° C may be measured. Borosilicate glass is used for high-range thermometers. To avoid trouble from softening of the glass, quartz tubes containing mercury, under gas pressure, can be used up to 700° C. Tin which melts at 232° C, has been used instead of mercury for temperatures up to 1,000° C.

Gas thermometers depend for their operation upon the fact that for certain gases the change in volume at constant pressure, or the change in pressure at constant volume, as their temperature varies, obeys a regular law. They are therefore used as standards for the measurement of temperature, and although such thermometers are not suitable for commercial and ordinary laboratory measurements, they have been used to determine the melting points of a number of metals. These melting points, once established, may be used in the calibration of other—and more convenient—instruments for temperature measurement. Gas thermometers have been used with hydrogen up to about 400° C and with nitrogen up to 1,600° C. Their probable error has been given by A. L. Day* as ± 2° C. He also gives the melting points of a number of metals, including that of pure platinum, at 1,755° C (although the temperature now accepted is 1,773·5° C).

Gas thermometers have been used to measure very low temperatures and one using helium is applicable down to a temperature of about one degree above absolute zero.

Kelvin showed that it was possible to formulate a temperature scale to be independent of the properties of any particular substance. This scale would be the same as that for a "perfect" gas. The departures of the gases, used in gas thermometers, from perfection, in this sense, are small and can be calculated.

The present International Temperature Scale was adopted at the Ninth General Conference of Weights and Measures in 1948. This

* *Transactions of the Faraday Society*, November, 1911, p. 142.

scale is described in a National Physical Laboratory publication, *The International Temperature Scale of* 1948 (H.M. Stationery Office). The Kelvin scale is recognized as the fundamental thermodynamic scale to which all temperature measurements should ultimately be referable. On this scale temperatures are designated as °K and denoted by the symbol T, and the interval between the ice point T_0 and the steam point T_{100} is 100 degrees.

In the Celsius scale, which is in general use, the temperature is $T - T_0$ and is usually denoted by the symbol t. This scale is the same as the centigrade scale, but it was decided at this conference to secure international conformity by abandoning the word centigrade in favour of the name Celsius. The symbol °C is now understood to mean degrees Celsius. It should be mentioned that the word centigrade is still in common use; some considerable time usually elapses before a change in terminology is generally adopted.

The scale is based upon a number of fixed points, all the temperatures being found at a pressure of one standard atmosphere defined as $1{\cdot}013250 \times 10^5$ newtons/m^2.

Boiling point of oxygen .	− 182·97° C
Melting point of ice . .	0° C exactly
Boiling point of water . .	100° C exactly
Boiling point of sulphur . .	444·600° C
Freezing point of silver . .	960·8° C
Freezing point of gold . .	1,063·0° C

Pyrometers are instruments, or pieces of apparatus, which are generally used for the measurement of high temperatures and for the indication and recording of temperatures under circumstances which could not easily be met by mercury thermometers. There are four types of pyrometers, namely, platinum resistance, thermo-electric, radiation, and optical pyrometers. Of these, the first three are electrical instruments, and it is with these that we are most concerned in this chapter, although in the commonest form of optical pyrometer electrical measurements are involved.

The change in electrical resistance, corresponding to any given temperature change, is found to be a perfectly constant quantity for any given piece of metal. The thermo-electric e.m.f., set up in a given thermo-junction of two dissimilar metals, for any given change in temperature, is also quite definite. Since both electrical resistance and e.m.f. can be conveniently measured with great precision, electrical pyrometers, depending upon these effects, form very convenient and sensitive methods of measuring temperature.

Much useful information is given in the British Standard *Code for Temperature Measurement*, B.S. 1041 : 1943. See also Refs. (19), (26).

Electrical Resistance Pyrometers. These pyrometers may be used to cover a range from − 200° C to 1,200° C, readings to 1° C being possible when the resistance material is platinum, as it usually is.

The law of resistance variation with temperature has been fully investigated by Callendar and Griffiths, using an air thermometer as standard. Copper and nickel have also been used for low temperature ranges—up to about 100° C. The chief requirement of the materials used in such thermometers is that their resistance at a certain temperature must be the same after subsequent heating as it was before the heating. Again, the change in resistance, per degree alteration in temperature, should be as large as possible; but this change is not uniform over any large temperature range for any of the resistance metals in use. Provided, however, the law of variation is known for any particular specimen, this can be used as a standard for the calibration of other resistance thermometers. If the material is suitably chosen, and is properly treated before being used, constancy of resistance at any given temperature can be obtained.

Platinum has been found very satisfactory in this respect, and in fact the International Temperature Scale is realized between the oxygen point −182·970° C and the freezing point of antimony 630·5° C by means of a standard platinum resistance thermometer which is calibrated at the fixed points on the scale.

Laws of Resistance Variation. The resistance of a platinum thermometer above 0° C is given by

$$R_t = R_0 (1 + At + Bt^2) \quad . \quad . \quad . \quad . \quad (13.1)$$

where R_t = the resistance at temperature t° C,

R_0 = the resistance at temperature 0° C,

and A and B are constants. R_0 and A and B are determined by calibration at the melting point of ice 0° C, the boiling point of water 100° C, and the boiling point of sulphur 444·60° C. In the particular case of standard thermometers the purity of the platinum should be such that the ratio R_t/R_0 is not less than 1·390 for $t = 100$° C.

The temperature t may be deduced by inserting the measured value of R_t in Equation (13.1).

When the highest accuracy is not required and a platinum thermometer not of standard accuracy is in use, it is sufficient to introduce a single constant k called the "fundamental coefficient" which can be used to express the law of resistance in the form

$$R_t = R_0 (1 + kt) \quad . \quad . \quad . \quad . \quad (13.2)$$

with sufficient accuracy for small temperature changes. This constant may be determined by measuring R_t at the ice point and the steam point, i.e. at 0° C and 100° C.

Then, if R_{100} and R_0 are its resistance at 100° C and 0° C respectively, we have

$$R_{100} = R_0 (1 + 100k)$$

from which

$$k = \frac{R_{100} - R_0}{100 R_0}$$

Then, for the resistance R at any temperature $t°$ C we have

$$R = R_0 (1 + k \,.\, T_p)$$

$$= R_0 \left(1 + \frac{(R_{100} - R_0)}{100 R_0} T_p\right)$$

or

$$R = R_0 + \frac{R_{100} - R_0}{100} \,.\, T_p \quad . \quad . \quad . \quad . \quad (13.3)$$

T_p is called the "platinum temperature," and a correction must be applied in order to determine the temperature in degrees Celsius.

From the above expression, we have

$$T_p = \frac{R - R_0}{R_{100} - R_0} \times 100$$

$R_{100} - R_0$ is the "fundamental interval," and is constant for any particular pyrometer. The correction to be applied to convert platinum temperatures to Celsius temperatures is rendered necessary when higher temperatures than 100° C are to be determined, on account of the importance of the constant B at these temperatures.

If R is the resistance at an actual temperature of $t°$ C, this temperature is obtained from the platinum temperature T_p, from the following expression, due to Callendar,

$$t - T_p = \delta \left[\left(\frac{t}{100}\right)^2 - \left(\frac{t}{100}\right)\right] \quad . \quad . \quad . \quad (13.4)$$

This "difference formula" is correct to within $\frac{1}{10}$° C up to 500° C, and to within $\frac{1}{2}$° C up to 1,000° C. The constant δ depends upon the purity of the platinum, its value being 1·5 if the platinum is very pure. This constant is determined from measurements of the resistance of the pyrometer at three known temperatures, namely, those of melting ice, boiling water, and boiling sulphur, which are 0° C, 100° C, and 444·6° C respectively.

A calibration curve for the particular pyrometer is used, in practice, to obtain the Celsius temperatures, instead of using the formula.

C. F. Marvin* found that the resistance of pure nickel varies with temperature, according to the law

$$\log_e R = a + bt \quad . \quad . \quad . \quad . \quad (13.5)$$

where R = resistance in ohms

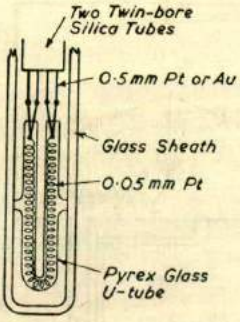
t = temperature in degrees Celsius

and a and b are constants of the order of 1 and 0·0017 respectively, depending upon the purity. For a temperature range of − 25° C to 350° C the assumption that this law is true will not introduce an error greater than 1° C. If the range of temperature is small, the error will be under 0·1° C.

Construction of Platinum Resistance Thermometers. A bare platinum wire (usually about 0·2 mm in diameter) is wound on a mica frame, and is enclosed within a porcelain, or fused silica, tube. This tube may also be enclosed within a steel tube for protection, its use being dependent upon the temperature to be measured and upon the situation of the instrument when in use. The bridge, or measuring circuit, is often situated at a considerable distance from the

* *Physical Review*, April, 1910, p. 522.

body whose temperature is to be measured, and for this reason resistance changes with temperature in the long leads required may introduce errors of measurement, unless compensating leads are used. The latter are in the form of a loop of wire, of the same material as the leads to the thermometer, running along with these leads and ending in a loop of platinum wire, inside the thermometer tube but not connected to the resistance coil. In use, these compensating leads are connected in the opposite side of the bridge to that in which the thermometer itself is connected. Any temperature



(*J. Sci. Instrum.*)

FIG. 13.1. BULB OF STANDARD PLATINUM THERMOMETER

changes which affect the leads to the thermometer also affect the compensating leads equally, and thus the balance of the bridge network is not affected by them. Copper or steel tubes are sometimes used, instead of porcelain or silica ones, for temperatures up to about 700° C. It is essential that the platinum should be protected from fumes which may corrode it and cause change of resistance. Joints within the thermometer itself should be welded, since metallic solderings give off fumes, when hot, which cause deterioration of the platinum. Also, the platinum wire is subjected to a special annealing treatment, to avoid subsequent changes of resistance due to strain when in use.

These pyrometers may be used for continuous work up to about 900° C, and intermittently for temperatures up to 1,200° C. Above 900° C trouble may be experienced due to deterioration of both the platinum and of the mica insulation. The accuracy obtainable is of the order of 0·01° C up to 100° C, 0·5° C up to 500° C, and 3° C up to 1,000° C.

STANDARD PLATINUM THERMOMETERS. Fig. 13.1 shows the construction of a standard platinum resistance thermometer designed by C. R. Barber (Ref. 24). A stability of $\pm$ 0·001° C has been achieved with this design. A strain-free construction has been

adopted, the resistance element consisting of a helix of 0·05 mm platinum wire which is mounted in a fine tube; this tube is then bent into a hairpin form and enclosed in an outer sheath. The resistance element is connected as a four-terminal resistor; the four leads are of gold and are supported in two twin-bore silica tubes. The normal temperature range for this thermometer is from − 180 to + 600°C. The bulb length is 3 cm and the sheath diameter 5·6 mm.

A special bridge known as the Smith Bridge, Type 3, is necessary

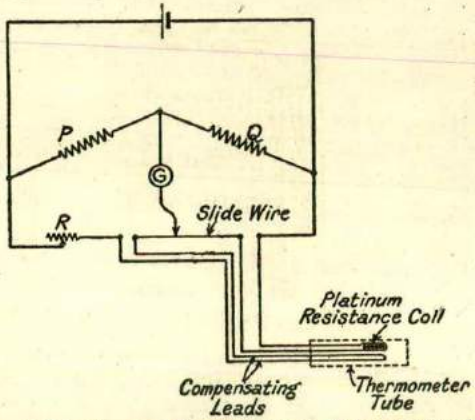


FIG. 13.2. BRIDGE NETWORK FOR TEMPERATURE MEASUREMENTS

to realize the full accuracy of this thermometer, and details of the most recent construction will be found in Ref. (25).

Method of Using Platinum Thermometers. These pyrometers may be used either in a Wheatstone bridge network of the ordinary type, or in conjunction with an indicating instrument, such as the Whipple Indicator, which gives the temperature directly. They may be used, also, with a recording instrument such as the Callendar Recorder, which records temperature variations on a revolving drum. A Kelvin double bridge is used instead of a Wheatstone bridge when the thermometer resistance is low.

Fig. 13.2 shows the connections for the measurement of the change in resistance of the thermometer with temperature by a Wheatstone bridge network. P and Q are equal ratio arms. In a third arm are a variable resistance R and the compensating leads. The thermometer coil is contained in the fourth arm, with a slide-wire in between these last two arms, as shown.

A special bridge, designed for the purpose, is often used, and the slide-wire is graduated in degrees, the position of the slide-wire contact for balance of the bridge giving the temperature directly.

Temperature Indicators and Recorders. The Whipple indicator consists of a bridge of this type, the slide-wire being wound spirally on a drum. The indicator is calibrated so that the position of the drum, for balance, gives the temperature directly in degrees.

Other forms of temperature indicators are in the form of a bridge network, but do not require adjustment of resistance after the initial balance. If the bridge is balanced at the lowest temperature of the range of the thermometer, variation of temperature, producing resistance variation, will cause a galvanometer current to flow. This current will depend upon the resistance variation—i.e. upon the temperature—so that the galvanometer can be calibrated to read temperature directly. Errors due to changes of battery voltage in this form of instrument are avoided by the provision of a standard resistance, which can be substituted for the thermometer resistance, and the bridge adjusted to give a standard indication upon the indicator scale with this standard resistance in circuit.

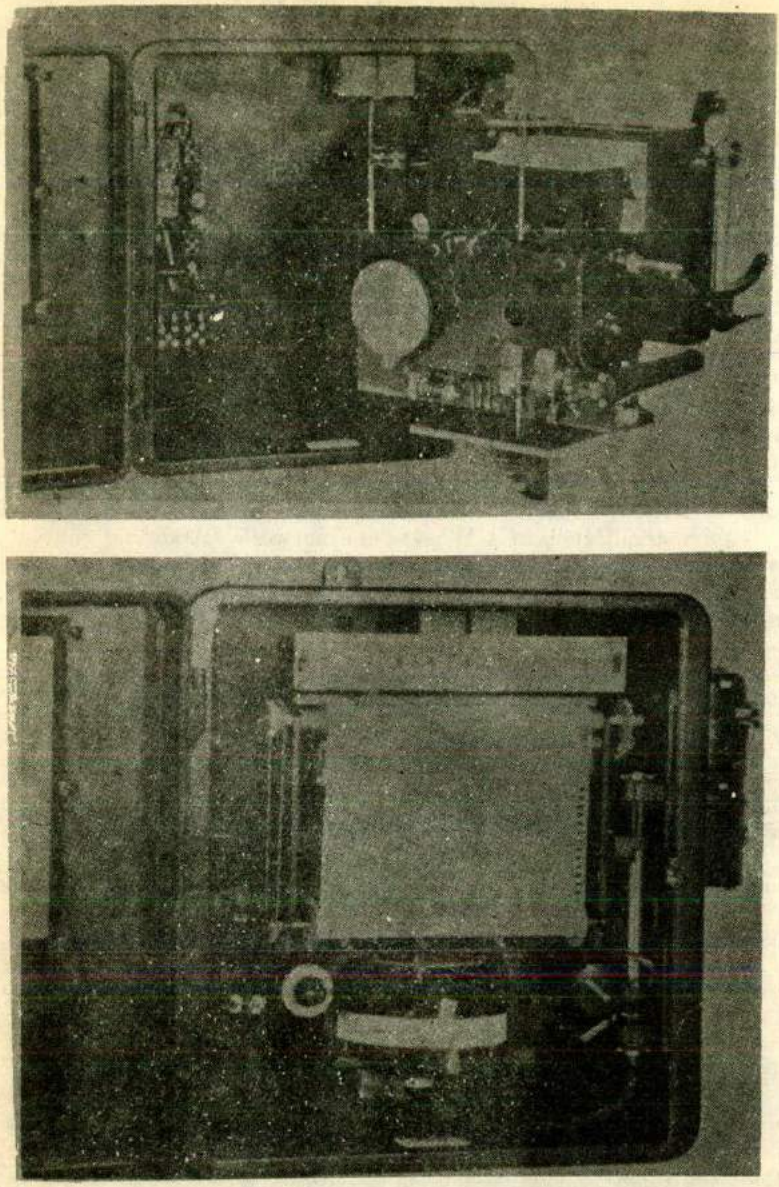
In the Callendar Recorder, the bridge is kept balanced automatically by sensitive relays which are operated by the current in the galvanometer branch of a Wheatstone network. A sliding contact is moved along a slide-wire by these relays. A drum, whose axis is parallel to the slide-wire, revolves slowly and carries a chart upon which a pen, carried by the sliding contact arm, traces the movements of the sliding contact. These movements are proportional to the temperature variations, and thus a record of the temperature it made upon the chart.

Fig. 13.3 shows the construction of a modern form of Callendar Recorder made by the Cambridge Instrument Co., Ltd. It consists essentially of a Wheatstone bridge in which balance is automatically maintained. Up to six simultaneous records can be taken on a monthly chart.

Thermo-electric Pyrometers. These pyrometers, which can measure temperatures up to 1,400° C, are generally used for less precise work than that for which resistance thermometers are used, and have the advantage of being cheaper than the latter. They also have the advantage of following temperature changes with very little time-lag, and are thus suitable for recording comparatively rapid changes. They are very convenient, also, for measuring the temperature at one particular point in a piece of apparatus.

If two wires, of different metals, are joined together at each end so as to form a complete electrical circuit, it is found that a current flows in the circuit if one of the junctions is at a higher temperature than the other.* This current is the result of an e.m.f. which is set up in the circuit and which is a function of the temperature of the "hot" junction if the other—the "cold" junction—is maintained at a constant temperature. If, instead of the two wires being

* The "Seebeck" effect.



(*Cambridge Instrument Co., Ltd.*)

FIG. 13.3. CALLENDAR RECORDER

directly joined together at the "cold" junction, two leads are taken from these ends of the wires to an indicating instrument, the thermo-electric effect is the same, provided the ends are maintained at the same "cold" temperature.

In thermo-electric pyrometers the e.m.f. set up is either measured by a potentiometer or is allowed to send a current through a galvanometer, connected in the circuit as just described, the galvanometer deflection then being proportional to the thermo-electric e.m.f. if the resistance of the circuit is constant. Since the thermo-electric e.m.f. is really dependent upon the *difference* of temperature between the two junctions, it is necessary to keep the temperature of the "cold" junction constant if the temperature of the "hot" junction is to be accurately measured.

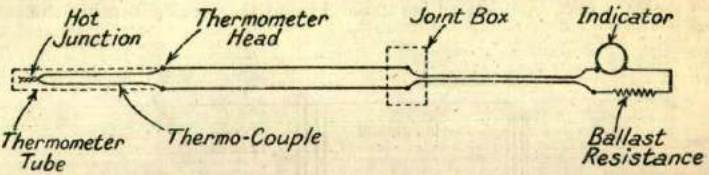


FIG. 13.4. CONNECTIONS OF THERMO-ELECTRIC PYROMETER

Compensating Leads. The tube containing the thermocouple itself is necessarily fairly short. This means that the temperature of the two open ends of the couple may be very far from constant. For this reason, "compensating leads" are used to connect these ends to the indicating or measuring instrument. The materials of these leads are chosen so that the thermo-electric e.m.f.s set up at their junctions with the open ends of the thermocouple are equal and opposite, and so neutralize one another. The effect obtained is therefore that of removing the cold junction from the thermometer head to the terminals of the indicating instrument, or to some point where the temperature is reasonably constant and can be controlled. In the latter case, flexible copper leads are taken from the constant-temperature point to the indicating instrument. A "ballast" resistor is also included in the circuit. This is a resistor of material having a negligibly small temperature coefficient, and its value is made large compared with the resistance of the rest of the circuit so as to make changes of resistance of the latter with temperature of no importance.

The complete circuit is as shown in Fig. 13.4. If a high-resistance galvanometer is used, the ballast resistance may be unnecessary. This method of use is most suitable when the temperature to be measured is very high. The absolute constancy of the temperature of the cold junction is not then of such importance as when the temperature to be measured is lower. For measurements of lower temperatures the cold junction may be contained within a vacuum flask, and maintained at a known low temperature, the connections then being as in Fig. 13.5. It is customary to use copper leads to the indicating instrument, and if neither of the thermocouple

wires is of copper, both of the junctions between the thermocouple wires and the copper leads must be in the vacuum flask. Crushed ice may be used in the cold junction flask. Alternatively, the reference junction may be maintained at a steady, though higher, temperature in a small electrically-heated chamber.

Thermo-electric E.M.F.s. The law of e.m.f. produced with variation of the temperature difference between hot and cold junctions may be written

$$e = \alpha (t - t_0) + \frac{\beta}{2}(t^2 - t_0^2) \quad . \quad . \quad . \quad (13.6)$$

where e = thermo-electric e.m.f. in volts,

t and t_0 = the temperatures of the hot and cold junctions respectively in degrees Celsius,

and α and β are constants which depend upon the metals forming the couple.

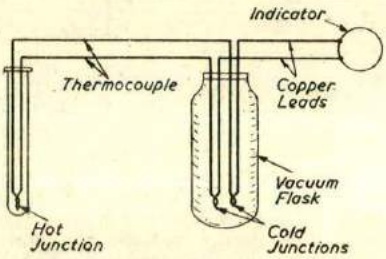


FIG. 13.5

Differentiating Equation (13.6) with respect to t gives

$$\frac{de}{dt} = \alpha + \beta t \quad . \quad . \quad . \quad . \quad . \quad (13.7)$$

$\frac{de}{dt}$ is termed the thermo-electric power, and if β is negative in sign, $\frac{de}{dt}$ is zero for one value of t, which is known as the neutral temperature.

Values of α and β are given in many books of physical tables for metals and alloys when used as a thermo-junction with lead as the other metal of the junction. When a thermocouple is formed with two metals, the values of α and β appropriate to the combination are obtained by taking the differences of the constants for the materials with lead. For example, compared with lead the thermo-electric powers in microvolts per °C for copper and constantan (60 Cu 40 Ni alloy) with the cold junction at 0° C are

Copper . .	+ 2·76	$+ 0{\cdot}8 \times 10^{-2}t$
Constantan . .	− 38·1	$- 7{\cdot}3 \times 10^{-2}t$

The thermo-electric power for the thermocouple is then

$$\frac{de}{dt} = + 40{\cdot}86 + 8{\cdot}1 \times 10^{-2}t$$

and the e.m.f. equation is

$$e = 40{\cdot}86\,(t - t_0) + \frac{8{\cdot}1}{2} \times 10^{-2}(t^2 - t_0^2)$$

$$= [40{\cdot}86 + 8{\cdot}1 \times 10^{-2} t_{av}]\,(t - t_0) \text{ microvolts}$$

where $t_{av} = \frac{t_1 + t_0}{2}$ is the average temperature of the hot and cold junctions.

When the cold junction is at 0° C, t_0 is zero.

The e.m.f. of a thermocouple is not linear in terms of temperature because of the term in β, but with most common thermocouples β is quite small and there is a range of temperature over which the departure from linearity is small.

The tabulated values of the constants α and β are not completely reliable because they vary with impurities in the metal and its treatment. It is usual, therefore, for manufacturers of thermocouples to supply tables of e.m.f. in millivolts against temperature for their thermocouples.

The e.m.f. produced for any given temperature difference between hot and cold junctions is constant, at a given hot-junction temperature, for any pair of metals, regardless of the size of the two wires and of the areas in contact at the junctions.

Metals used for Thermo-junctions. Two classes of metals are used in thermo-junctions—base metals and rare metals. The advantages of the base-metal couples are that such metals are comparatively cheap and the couples, therefore, may be made more robust than when expensive metals are used. They have also a high thermo-electric e.m.f. The rare metals—usually platinum and platinum alloys—are more durable for a given size, and can withstand higher temperatures, than the base metals. Their thermo-electric e.m.f.s are somewhat small (about one-fifth as great as those with base metals) but are more constant than those obtained with the base metals.

Amongst the latter are couples of copper-constantan, silver-constantan and iron-constantan. These can be used for temperature measurements up to 500° C, 700° C, and 900° C, respectively, and their thermo-electric e.m.f.s in millivolts at 500° C (the cold junction being at 0° C) are approximately 27·8, 27·6, and 26·7 respectively. The Hoskins couple—a nickel wire and a wire made of an alloy of 90 per cent nickel and 10 per cent chromium—is also used up to 1,100° C, its thermo-electric e.m.f. at 500° C being approximately 10·0 millivolts when the cold junction is at 0° C.

Platinum with platinum-rhodium (alloy), and platinum with platinum-iridium, are rare-metal couples which are commonly used for temperatures (continuous) up to 1,400° C and 1,000° C respectively, their thermo-electric e.m.f.s at 500° C being 4·4 millivolts and 7·4 millivolts respectively.

The rare-metal couple formed between platinum and the alloy platinum–10 per cent rhodium is of special interest. It is capable of very high accuracy and is used to interpolate the International

Temperature Scale between 630·5° C and the gold point 1,063·0° C. The equation for this thermocouple is taken as

$$e = a + bt + ct^2 \quad . \quad . \quad . \quad (13.8)$$

The constants a, b and c are determined by calibration at the freezing point of antimony (630·5° C) and at the silver (960·8° C) and gold (1,063° C) points.

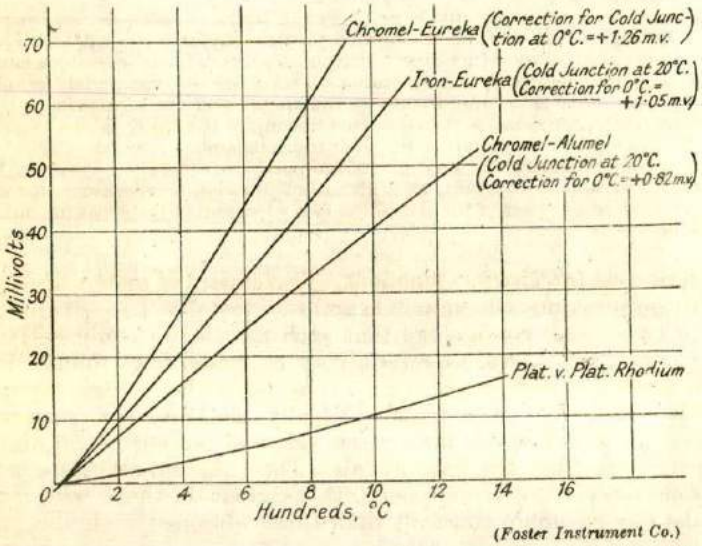


FIG. 13.6. VOLTAGE–TEMPERATURE CURVES FOR THERMOCOUPLES

For temperature measurements up to about 80° C a bismuth-antimony couple is often used.

A graph, showing the results of tests upon various thermocouples by the Foster Instrument Co., is given in Fig. 13.6.

Construction and Use of Thermo-electric Pyrometers. The construction of the thermo-electric pyrometer is illustrated in Fig. 13.7, which shows a pyrometer manufactured by the Cambridge Instrument Co. for high-temperature measurements (up to 900° C) in which the couple is iron-constantan.

To protect the thermocouple, a sheath, surrounding the hot junction, is almost always used. This may be of steel or nickel-chrome for base-metal couples employed at the lower temperatures, but is usually of porcelain or quartz in the case of the rare-metal pyrometers for use at the highest temperatures. Alundum (fused alumina and fireclay) sheaths are sometimes used also at the very high temperatures. In rare-metal thermocouples the wires forming

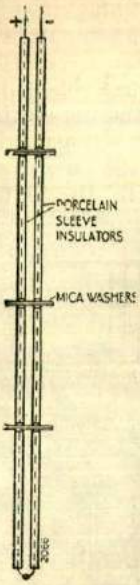
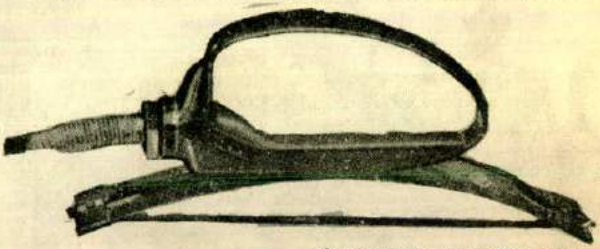


FIG. 13.7
THERMO-ELECTRIC PYROMETER

the couple may be about 0·02 in. in diameter, but in the base-metal couples the wire diameter is usually about 0·1 in. The junction itself may be formed by either twisting or fusing the two wires together.

Fig. 13.8 shows a thermo-electric pyrometer manufactured by the Cambridge Instrument Co. for the measurement of the temperatures of surfaces. The thermocouple takes the form of a thin strip (about 0·01 in. thick and 0·25 in. wide), consisting of pieces of two dissimilar metals welded together end to end. This strip is flexible, and if pressed



(*Cambridge Instrument Co., Ltd.*)

FIG. 13.8. SURFACE PYROMETER

lightly upon a hot surface such as a heated metal roller, it conforms to the shape of the latter and makes intimate contact with it.

The e.m.f. set up in the strip is measured on an indicator whose scale is graduated directly in degrees Celsius.

As previously stated, thermo-electric e.m.f.s may be measured either directly by a galvanometer or by a potentiometer. The former is the more convenient for most practical purposes, since the galvanometer may be calibrated—for a given temperature of the cold junction—to give temperatures directly. If the galvanometer has a high resistance, changes of resistance of the thermocouple circuit with temperature will introduce no appreciable errors. Calibration of the galvanometer may be by comparison with another instrument (already calibrated), or may be carried out by observing the deflection for a number of known temperatures, such as the boiling points or melting points of various substances.*

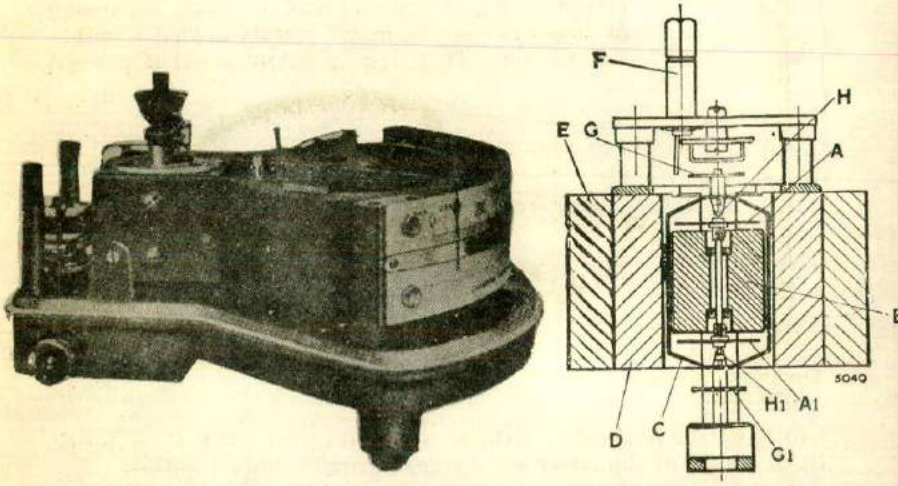
This method of using the thermocouple may be applied either to the indication or to the continuous recording of temperatures.

The potentiometer method, in which the thermo-electric e.m.f., is measured by comparison with the e.m.f. of a standard cell, is, however, more precise than the above method.

* For tables of such temperatures, see Ref. (2).

Special potentiometers for such purposes are manufactured by various scientific instrument manufacturers, and have already been mentioned in Chapter VIII.

Fig. 13.9 shows the construction of an indicator manufactured by the Cambridge Instrument Co. for use with their thermo-electric pyrometers. The moving coil, C, of the indicator is suspended magnetically in the field of the magnet of which D is the pole block. A and A_1 are iron discs, B is an iron core, G and G_1 are control



(Cambridge Instrument Co., Ltd.)

FIG. 13.9. TEMPERATURE INDICATOR

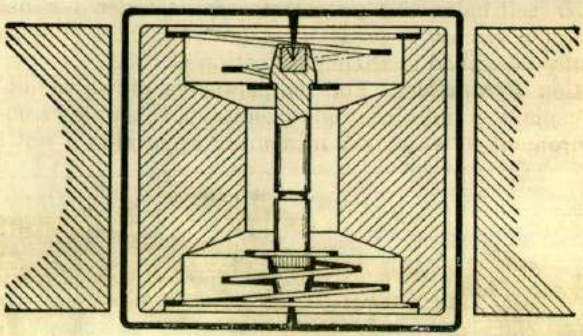
springs, and H and H_1 pivots. The magnetic suspension ensures resilience and protects the pivots against the effects of vibration.

An indicator for the same purpose, manufactured by the Foster Instrument Co., is illustrated in Fig. 13.10. The "Resilia" patent design of the moving system protects the latter against vibration by ensuring that the pivots of the coil and the jewels will not be separated by such vibration. The pivots are turned inwards from the coil and rest in conical jewels carried by a very light staff. This staff is held in position, relative to the core, by light springs, so that when the instrument is subjected to mechanical vibration the coil, pivots, jewels, and staff move together and the pivots are uninjured.

For the recording of temperature variations, a recorder, similar in construction to that already described, in dealing with resistance thermometers (see page 529), is used. Instead of the moving contact, which carries the pen, moving over the slide-wire of a Wheatstone

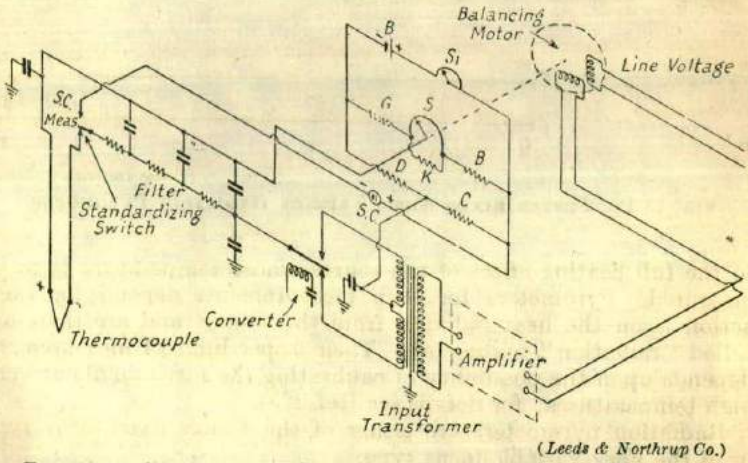
bridge network, it now moves over a slide-wire in a potentiometer circuit, the latter being included in the recorder itself.

Fig. 13.11 shows the principle of operation of a quick-response recorder which employs an electrical balancing system with a



(*Foster Instrument Co.*)

FIG. 13.10. 'RESILIA" INDICATOR MOVEMENT



(*Leeds & Northrup Co.*)

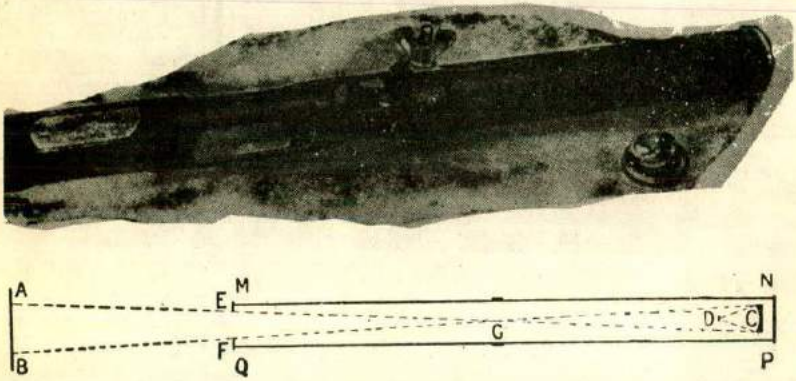
FIG. 13.11. "SPEEDOMAX" POTENTIOMETER CIRCUIT FOR MEASURING TEMPERATURE WITH THERMOCOUPLES

potentiometer circuit. The system consists of two main elements: a converter-amplifier unit and a two-phase balancing motor. When the potentiometer becomes unbalanced due to a change of temperature, which alters the thermocouple e.m.f., the converter contacts alternately connect the unbalance potential across the primary winding of the amplifier input transformer. The current induced in the transformer secondary winding is amplified to energize the

control winding of the balancing motor. This starts the motor, which moves the contact along the measuring slide-wire, and the indicating pointer and recording pen across the scale and chart, until balance is obtained when the motor torque becomes zero.

Another self-balancing potentiometer recorder for use with a thermocouple circuit is made by Elliott Brothers (London), Ltd., and is fully described in their publication G.126A.

Radiation Pyrometers. For temperatures above about 1,200° C it is convenient, if not absolutely necessary, to employ some method of measurement in which the measuring apparatus is not subjected



(*Foster Instrument Co.*)

FIG. 13.12. FOSTER FIXED-FOCUS TYPE OF RADIATION PYROMETER

to the full heating effect of the source whose temperature is to be measured. Pyrometers for such measurements depend for their action upon the heat radiated from the source, and are therefore called "radiation" pyrometers. Their upper limit of measurement depends upon the possibility of calibrating the instruments at very high temperatures; for details see Ref. (23).

Radiation pyrometers are either of the Foster fixed-focus type or of the Féry variable-focus type.

FIXED-FOCUS TYPE. Fig. 13.12 shows a pyrometer of this type. It consists essentially of a long tube $MNPQ$ containing a concave mirror C which is adapted to focus the heat rays which pass into the tube (from the source AB, through the narrow aperture EF) on a sensitive thermocouple D. The image formed at D is a "heat image," and is always in focus, without adjustment, over a wide range of distances between the pyrometer and the source of heat. This is brought about by making the tube long and the aperture small.

Suppose the solid angle subtended at G is ϕ; then, provided the radiating surface of the hot body is large enough to subtend this

angle ϕ at G at any given distance of the pyrometer tube from the hot body, the temperature measured will be the true one. If, however, the distance between the hot body and pyrometer is so great that the total radiating surface subtends an angle less than ϕ at G, the measured temperature will be low.

The e.m.f. produced in the thermocouple is measured as previously described. If a galvanometer or millivoltmeter is used for this purpose, it may be calibrated to read temperatures directly.

The Stefan-Boltzmann Law of Radiation states that the energy radiated by a heated black body (i.e. a body for which the radiated energy for a given temperature is a maximum) is proportional to the fourth power of its absolute temperature.

If T_2 is the absolute temperature of the hot body and T_1 is the absolute temperature of a colder body near to it, then the law of radiation is

$$W = k(T_2^4 - T_1^4) \quad . \quad . \quad . \quad . \quad (13.9)$$

where W is the energy, per sq. cm per sec, received by the cooler body and k is a constant. The experimental value of k is $5{\cdot}70 \times 10^{-12}$ watts per sq. cm per deg.^{4}

Again, the energy W' received by the body at absolute temperature T_1, from a body at temperature T_3, is given by

$$W' = k(T_3^4 - T_1^4)$$

Thus,
$$\frac{W'}{W} = \frac{T_3^4 - T_1^4}{T_2^4 - T_1^4}$$

If T_1 is small compared with T_2 and T_3, we have, very approximately,

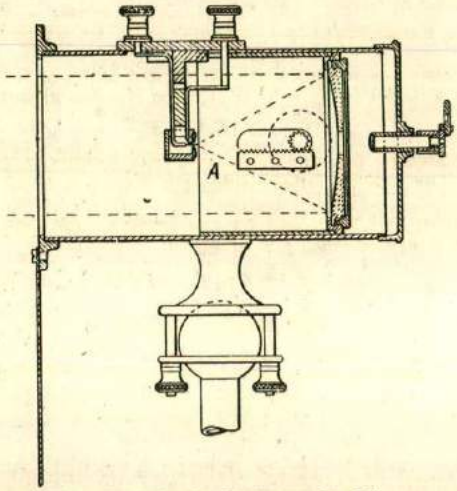
$$\frac{W'}{W} = \frac{T_3^4}{T_2^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.10)$$

The receiver at D is blackened so as to approach as nearly as possible to the theoretical "black body" and absorbs all, or almost all, of the radiation falling upon it. Its temperature rise is thus proportional to the fourth power of the absolute temperature of the hot body whose temperature is to be measured. For this reason the scale of the indicating instrument used with radiation pyrometers is cramped at the lower end, and open at the upper end.

FERY VARIABLE-FOCUS PYROMETER. The construction of the Fery variable-focus pyrometer, manufactured by the Cambridge Instrument Co., is shown in Fig. 13.13. A concave mirror, with an opening at its centre, through which the hot body may be observed through an eyepiece, is used to focus the heat radiation upon a sensitive thermocouple A. The focusing is carried out by means of a rack and pinion, which moves the mirror as required. A small mirror is placed directly behind the thermocouple The position of the concave mirror is adjusted, by the focusing arrangement, until the two images of the hot body, which are reflected by the concave mirror upon the small mirror behind the thermocouple, overlap the couple itself. The distance of the pyrometer from

the hot body is not important, provided it is small enough for this overlap to be obtained. As in the fixed-focus instrument, the e.m.f. set up in the thermocouple is measured by a sensitive moving-coil instrument whose scale may be graduated in temperatures directly.

Use of Radiation Pyrometers. In both of the above pyrometers the terminals which form the cold junction of the couple must be protected from the heat from the hot body whose temperature is to



(Cambridge Instrument Co., Ltd.)

FIG. 13.13. FÉRY VARIABLE-FOCUS PYROMETER

be measured. Base-metal couples, such as copper-constantan, are usually employed in both cases.

Although the laws of radiation stated above only hold, strictly, for perfectly black bodies, only small errors are usually introduced, since the departure of the actual hot bodies from perfect black-body conditions is slight in most cases for which such pyrometers are used.

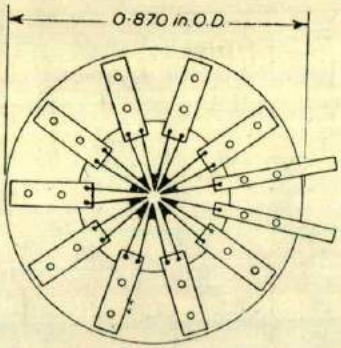
Radiation pyrometers are used for temperatures above 600° C, since at lower temperatures errors may be introduced by the fact that the temperature of the pyrometer itself may not be negligible compared with that of the hot body. They are often used for the measurement of furnace temperatures. If viewed through a small opening, a furnace which is surrounded by walls which are at approximately the same temperature as itself behaves as a perfectly black body.

Such pyrometers may be used for recording purposes by using

the thermocouple in conjunction with a recorder of the type described earlier in the chapter.

The advantages of these pyrometers are their high upper limit of temperature measurement and their comparative independence upon the distance of the instrument from the hot body. They have, however, the disadvantage of having to be calibrated individually, using hot bodies whose temperatures are known.

Thermopiles for Radiation Pyrometers. Fig. 13.14 illustrates a thermopile suitable for use in a radiation pyrometer (Ref. (27)).



(*Reinhold Publishing Corp., N.Y.; See Ref.* (26), *p.* 1219)

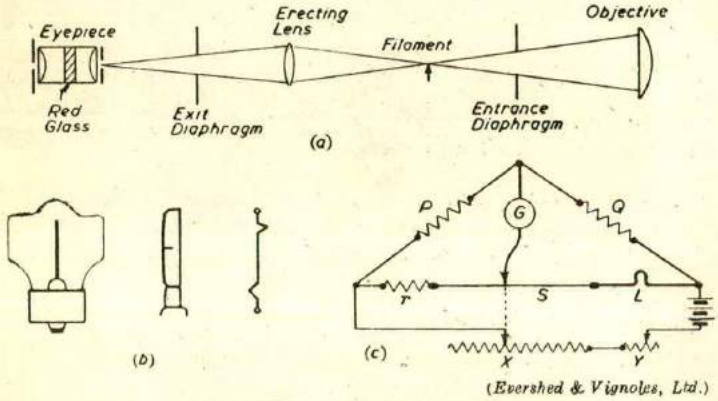
FIG. 13.14. THERMOPILE FOR RADIATION PYROMETER

This gives a greatly increased sensitivity as compared with a single thermocouple on a heat-absorbing surface. The thermopile consists of ten V-shaped thermocouples spot-welded to a terminal assembly consisting of thin flat strips of metal affixed to an annular ring of mica. The hot junctions of the thermocouples are flattened and spaced at the centre of the ring to form the radiation receiver. The hot junctions are blackened in order to absorb the incident radiation. The cold junctions of the thermocouples are at the points of attachment to the metal strips. In use the mica ring is clamped in the pyrometer housing, thin mica covering rings being used for insulating the cold junctions. The thin cold-junction strips have a low heat capacity, and if the pyrometer casing in contact with them is of large thermal mass, they will all be at the temperature of the casing.

Optical Pyrometers. One of the best methods of measuring the temperature of hot bodies radiating energy in the visible spectrum is to compare the intensity of the hot body with that of a known source. The "disappearing filament" pyrometer uses this principle. In this instrument light from the hot body is focused upon the plane of a lamp filament which is viewed through a piece of red glass. The red glass has an effective wavelength of 650 mμ, and the comparison between the intensity of the lamp and hot body is therefore

carried out at one wavelength only. The intensity of the light of any one wavelength depends upon the temperature of the hot body.

The current through the lamp filament is adjusted until the brightness of the filament is equal to that of the source as viewed through the red glass. When these two brightnesses are equal the outline of the filament disappears into the surrounding field of light from the hot body. Otherwise the filament shows either brighter or duller than the surrounding light, according to whether its temperature is greater or less than that required for equality of brightness.



(*Evershed & Vignoles, Ltd.*)

FIG. 13.15. OPTICAL PYROMETER

(*a*) Optical System.
(*b*) Detail of lamp and filament.
(*c*) Measuring circuit.

The optical system of a modern "disappearing filament" pyrometer made by Evershed and Vignoles, Ltd., is shown in Fig. 13.15 (*a*). This device was developed as a result of a report by C. R. Barber of the National Physical Laboratory on the design and performance of commercial optical pyrometers (Ref. 28), and it represents a marked advance in technique. The special lamp used is illustrated in Fig. 13.15 (*b*). It has a flat strip filament, 0·006 mm thick by 0·004 mm wide, constructed in the manner shown to minimize the end effects at the anchoring points. A small index marks the operative point of the filament. The glass envelope of the lamps has a flat window on each side so that undistorted images may be obtained.

This instrument is calibrated in terms of the resistance of the lamp filament, whereas most earlier instruments used the lamp current for this purpose. The operation may be understood by reference to Fig. 13.15 (*c*). A Wheatstone bridge circuit is used in which P and Q are ratio arms, L is the lamp resistance and r a balancing resistor. S is a slide-wire which is calibrated directly in

terms of temperature, X is a control rheostat and Y a trimming rheostat. Rapid balancing is achieved by coupling the rheostat X and slide-wire contact together so that, in matching the brightness, the slide-wire contact is automatically set close to the balance point of the bridge. The rheostat has to be graded to maintain this proximity. The final balance is obtained by opening the mechanical coupling and adjusting S to balance. The trimmer Y is to compensate for changes in the battery voltage. This adjustment is carried out by setting the rheostat X and slide-wire to centre scale and adjusting Y for balance.

The complete instrument is quite portable and is held in the hands when in use. It covers the ranges 850° C to 1,300° C and 1,200° C to 2,050° C, the higher range being obtained by introducing an absorption glass between the objective and the lamp filament.

Optical pyrometers should be calibrated under black-body conditions, but it is customary to calibrate them against tungsten-ribbon-filament lamps of known emissivity.

A particular advantage of the optical pyrometer is the ease with which it can be used to measure and compare the temperature of small hot bodies. In cases where the test object is not under black-body conditions, there will be a discrepancy between the measured and true temperatures. Nevertheless, the pyrometer reading can be used to control the temperature. A table of corrections which can be used when the emissivity of the body is known will be found in Ref. (23).

The measurement of the increase in temperature of coils in electrical machinery and apparatus, by measurement of the increase of resistance and otherwise, has not been discussed in the foregoing pages, but such methods are discussed in several of the publications mentioned below.

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CHAPTER XIV

EDDY CURRENTS

The Nature of Eddy Currents. The term "eddy currents" is applied to those electric currents which circulate within a mass of conducting material when the latter is situated in a varying magnetic field. The conducting material may be considered as consisting of a large number of closed conducting paths, each of which behaves like the short-circuited winding of a transformer of which the varying magnetic field is the working flux.

"Eddy e.m.f.s" are induced in these elemental paths, by the varying magnetic field, giving rise to the eddy currents. Fig. 14.1 illustrates this induction.

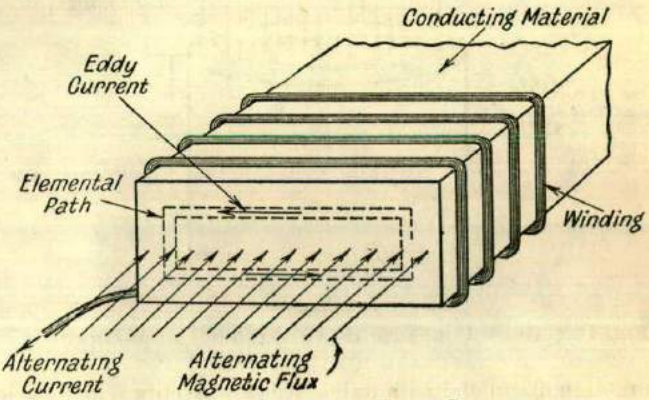


FIG. 14.1. INDUCTION OF EDDY CURRENTS

Effects of Eddy Currents. Eddy currents result in a loss of power, with consequent heating of the material, and the magnitude of this power loss is often a matter of considerable importance in electrical engineering.

The eddy currents, since they flow in closed paths in the material —usually iron—have an axial magnetic field of their own which is in opposition to the inducing magnetic field, and so reduces its strength. The reduction is greatest at the centre of the core since the eddy currents in *all* the elemental paths, from the centre of the material to the outside surface, are effective in producing the opposing magnetic field there. This results in a flux distribution which is not uniform, the flux density in the outer portions of the conductor being greater than that at its centre, which is screened

by the eddy currents. There is thus a reduction of the effective cross-section of the core.

The effect of eddy currents upon the flux distribution is chiefly of importance in transformers and other apparatus where the iron used would otherwise be worked at uniform flux density. Not only is the distribution of flux in such cores affected by eddy currents, but its magnitude, for a given value of magnetizing current, is obviously reduced thereby, and the phase of the resultant flux is not the same as that of the magnetizing current.

Eddy Current Loss in Cylindrical Conductors. Fig. 14.2 represents a portion of a conductor which carries an alternating magnetic flux

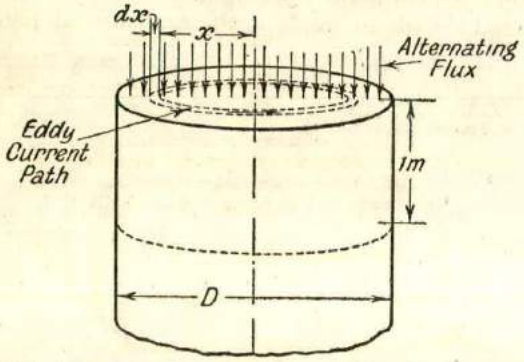


FIG. 14.2 EDDY CURRENTS IN A CYLINDRICAL CONDUCTOR

in a direction parallel to its axis. Eddy currents will flow in elemental circular paths, as shown.

Assume—as an approximation—that the flux distribution is uniform and that it alternates according to the law

$$B = B_{max} \sin \omega t$$

where B is the flux density at any time t and

$$\omega = 2\pi \times \text{frequency} = 2\pi f$$

Consider, first, 1 m axial length of the conductor. The flux enclosed by an elemental circular path, of radius x and radial width dx, at any instant is

$$\pi x^2 \,.\, B$$

Thus, the e.m.f. induced in this path is given by

$$e = \frac{d(\pi x^2 \,.\, B)}{dt} \text{ volts}$$

$$= \pi x^2 \,.\, B_{max}\, \omega \cos \omega t$$

$$= 2\pi^2 x^2 f B_{max} \cos \omega t$$

The r.m.s. value of this voltage is

$$E_x = \sqrt{2}\pi^2 x^2 f B_{max}$$

If ρ is the resistivity of the material of the conductor (in ohm-metres), the resistance of the elemental path is

$$r_x = \frac{2\pi x \times \rho}{dx \times 1} \text{ (for an axial length of 1 m)}$$

Neglecting its inductance, the r.m.s. value of the eddy current in this path is

$$i_x = \frac{\sqrt{2}\pi^2 x^2 f B_{max}}{\frac{2\pi x \rho}{dx}}$$

$$\therefore i_x = \frac{\pi x f B_{max} dx}{\sqrt{2}\rho} \text{ amp}$$

The eddy current loss in watts in this elemental path is thus

$$i_x^2 r_x = \frac{\pi^2 x^2 f^2 B_{max}^2 dx^2}{2\rho^2} \times \frac{2\pi x \rho}{dx}$$

$$= \frac{\pi^3 . x^3 f^2 B_{max}^2}{\rho} dx \text{ watts}$$

The total loss per metre axial length, obtained by integration between the limits $\frac{D}{2}$ and zero, is

$$P = \int_{x=0}^{x=\frac{D}{2}} \frac{\pi^3 x^3 f^2 B_{max}^2}{\rho} . dx$$

or

$$P = \frac{\pi^3 f^2 B_{max}^2 D^4}{64\rho} \text{ watts per metre axial length} \qquad (14.1)$$

If the flux wave-form is not sinusoidal, but has a form factor k_f instead of $\frac{\pi}{2\sqrt{2}}$ (as in the case of the sine wave), the above expression becomes

$$P = \frac{\pi k_f^2 . f^2 B_{max}^2 D^4}{8\rho} \qquad (14.2)$$

Now, the eddy e.m.f. is independent of the length of the cylinder (provided the flux is parallel to the axis throughout), and since the eddy current loss in watts $= i_x^2 r_x = \frac{E_x^2}{r_x}$ this loss is proportional to $\frac{1}{r_x}$.

Thus, eddy current loss $\propto \frac{1}{r_x}, \propto \frac{1}{\frac{2\pi x \rho}{dx \,.\, l}} \propto l$

where l is the total axial length of the cylinder and r_x the resistance of an elemental path of radius x, radial thickness dx, and axial length l.

Summarizing, the total eddy current loss in a cylindrical conductor, of any given diameter and material, is directly proportional to its axial length.

The eddy current loss per cubic metre of the conductor is, therefore, in the general case,

$$P = \frac{\frac{\pi k_f^2 \,.\, f^2 \,.\, B_{max}^2 D^4}{8\rho}}{\frac{\pi D^2}{4}}$$

or
$$P = \frac{k_f^2 \,.\, f^2 \,.\, B_{max}^2}{2 \,.\, \rho} D^2 \text{ watts per cubic metre} \qquad (14.3)$$

In general, the eddy current loss per cubic metre, for a given material and form factor of the flux wave is proportional to

$$f^2 B_{max}^2 D^2$$

Eddy Current Loss in Thin Sheets. Fig. 14.3 (*a*) shows a thin plate, of thickness t and width b, the thickness being small in comparison with the width. Suppose that this plate carries an alternating magnetic flux $B = B_{max} \sin \omega t$, and that the flux density is uniform over the cross-section, the flux running parallel to the axis of the plate as shown. Eddy currents will flow in the plate in elemental paths, as shown in Fig. 14.3 (*b*).

Consider 1 m axial length, and also 1 m of the width of the plate. In the case of an elemental path whose long sides are each distant x metres from the centre of the plate, and whose depth is dx (Fig. 14.3 (*b*)), the flux enclosed within 1 m width of this path is, at any instant,

$$2x \times 1 \times B = 2xB_{max} \sin \omega t$$

Thus, the e.m.f. induced in this portion of the elemental path is

$$e_x = \frac{d}{dt}(2xB_{max} \sin \omega t)$$

or $$e_x = 2xB_{max}\omega \cos \omega t \text{ volts}$$

Its r.m.s. value is

$$E_x = \frac{4\pi f x B_{max}}{\sqrt{2}} \text{ volts}$$

If ρ is the resistivity of the material of the plate (in ohm-metres), the resistance of this portion of the path is $\frac{2 \times \rho}{dx \times 1}$ of $\frac{2\rho}{dx}$ ohms. Then,

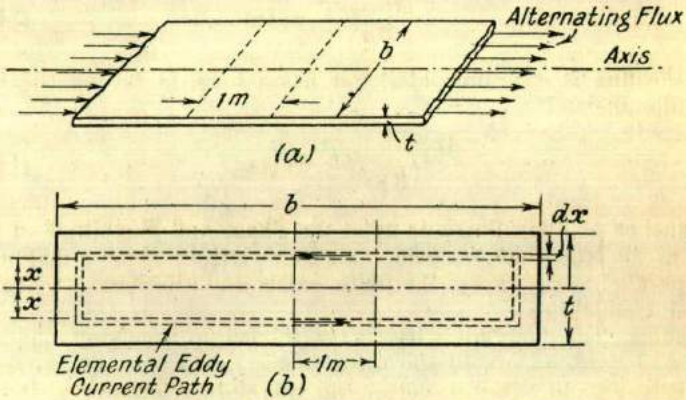


FIG. 14.3. EDDY CURRENTS IN THIN SHEETS

neglecting the end portions of the path (parallel to the edges of the plate), the eddy current in the path is

$$i_x = \frac{E_x}{\frac{2\rho}{dx}} = \frac{4\pi f x B_{max}}{\frac{2\sqrt{2}\rho}{dx}} \text{ amp}$$

or $$i_x = \frac{\sqrt{2}\pi f x B_{max} dx}{\rho} \text{ amp}$$

Thus, the eddy current loss in this path per metre length and breadth of the plate is

$$P_x = \frac{2\pi^2 f^2 x^2 B_{max}^2 dx^2}{\rho^2} \times \frac{2\rho}{dx}$$

$$= \frac{4\pi^2 f^2 x^2 B_{max}^2 \, dx}{\rho} \text{ watts}$$

Integrating between the limits $x = \frac{t}{2}$ and $x = 0$, we have, for the total loss per metre length and breadth of the plate (i.e. for t cu. m),

$$P = \int_{x=0}^{x=\frac{t}{2}} \frac{4\pi^2 f^2 B_{max}^2}{\rho} x^2 \, . \, dx$$

or

$$P = \frac{\pi^2 f^2 B_{max}^2}{\rho} \times \frac{t^3}{6} \quad . \quad . \quad . \quad . \quad (14.4)$$

Thus the loss per cubic metre is

$$\frac{\pi^2 f^2 B_{max}^2 t^2}{6\rho} \text{ watts} \quad . \quad . \quad (14.5)$$

If the flux is not sinusoidal, but has a form factor k_f, the loss per cubic metre is

$$\frac{4 k_f^2 f^2 B_{max}^2 t^2}{3\rho} \text{ watts} \quad . \quad . \quad . \quad (14.6)$$

Influence of Eddy Currents upon the Phase and Magnitude of the Flux in an Iron Core. Consider an iron plate in which a magnetic flux, parallel to the axis of the plate, exists and alternates according to a sinusoidal law.

Assume also, for simplicity in the following discussion, that all the eddy currents are in the same phase relative to the working magnetic flux in the iron core—and therefore relative also to the eddy e.m.f. Although this assumption is not correct, owing to the different phase angles of the various eddy current paths in the core, it will suffice for the discussion of the effect of the eddy currents upon the core flux. Referring to Fig. 14.4 (*a*), let i represent, in magnitude and phase, the resultant of all the eddy currents in a portion of the iron core 1 m wide and of 1 m axial length (see Fig. 14.3). This resultant is actually the vector resultant of the eddy currents in the core and could be obtained by constructing a polygon, as in Fig. 14.4 (*b*), if the magnitudes and phases of the individual eddy currents were known. The closing line of the polygon gives the resultant eddy current.

It will be observed that the vector diagram of Fig. 14.4 (*a*) is similar to that of a transformer. The similarity is explained by the fact that the iron core, with its magnetizing winding, does actually constitute a transformer, the secondary "winding" of which is permanently short-circuited and exists *inside* the core itself. This secondary "winding" is, of course, the eddy current path in the core.

The vector ϕ represents the working flux in the core. It is the flux which the magnetizing current I_m—supposed to be flowing in a coil of N turns wound on the iron core—would produce if no eddy currents were present. The eddy e.m.f.s, represented by the single vector E_e, will lag 90° behind ϕ as shown. The resultant eddy current i lags behind this voltage, owing to the inductance of the eddy current paths, and has, in phase with it, a flux ϕ_e—produced by the eddy currents. Thus the resultant flux in the iron is the vector sum of ϕ and ϕ_e, and is represented by ϕ_r.

The effect of the eddy currents upon the flux in the iron is, therefore, twofold; they both reduce the working flux—ϕ_r being less than

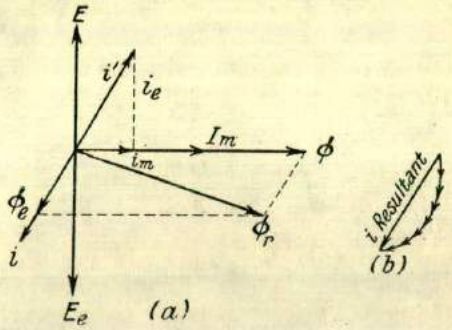


FIG. 14.4

ϕ—and also cause the resultant flux to lag behind the magnetizing current.

The vector E represents the e.m.f. required to overcome the inductive voltage drop in the magnetizing winding—i.e. E is the vector difference between the e.m.f. applied to this winding and the voltage drop in the resistance of the winding. On account of the eddy current i, the magnetizing winding must carry a current i'—in phase opposition to i—in addition to the magnetizing current I_m. (The current required to supply the power lost in hysteresis in the iron is omitted in the diagram to avoid complication.) The current i' has two components, i_m in phase with ϕ and i_e in phase with E. The former compensates for the demagnetizing effect of the eddy currents, and maintains the flux ϕ at the same value as it would have if there were no such currents, while the latter supplies the power which is lost in eddy current heating of the iron.

If there are n turns per metre axial length on the magnetizing winding, the e.m.f. induced in this winding per metre breadth of the core is

$$\sqrt{2}\pi f t n B_{max} \text{ volts (r.m.s.)}$$

As already seen, the power loss per metre axial length and breadth of the core plate (of thickness t metres) is

$$\frac{\pi^2 f^2 B_{max}^2 t^3}{6\rho} \text{ watts (assuming uniform flux distribution)}$$

Thus, the component i_e of the current in the magnetizing winding is given by

$$i_e = \frac{\frac{\pi^2 f^2 B_{max}^2 t^3}{6\rho}}{\sqrt{2}\pi f t n B_{max}} = \frac{\pi f B_{max} t^2}{\sqrt{2} \times 6\rho n}$$

$$= \frac{0{\cdot}118\pi f B_{max} t^2}{\rho n} \text{ amp} \quad . \quad . \quad . \quad (14.7)$$

If all the eddy currents are assumed to be in phase, the resultant eddy current i is given by

$$\int_0^{\frac{t}{2}} i_x dx = \int_0^{\frac{t}{2}} \frac{\sqrt{2}\pi f x B_{max}}{\rho} dx$$

$$= \frac{\sqrt{2}\pi f B t^2_{max}}{8\rho} \text{ amp}$$

Thus $$i' = \frac{\sqrt{2}\pi f B t^2_{max}}{8\rho n}$$

$$= \frac{0{\cdot}177\pi f B_{max} t^2}{\rho n} \text{ amp} \quad . \quad . \quad . \quad (14.8)$$

Since $$(i')^2 = i_e^2 + i_m^2$$

$$i_m = \frac{0{\cdot}132\pi f B_{max} t^2}{\rho n} \text{ amp} \quad . \quad . \quad . \quad (14.9)$$

The above discussion is based upon the following assumptions—

(a) that uniform flux distribution exists,

(b) the eddy currents are all in phase,

(c) the thickness of the iron plate is small compared with its length and breadth.

The flux distribution, when such assumptions are not made, will now be discussed.

Flux Distribution in Thick Iron Plates. Consider 1 m axial length and breadth of an iron plate with an alternating magnetic flux (of sinusoidal wave-form) running parallel to the axis of the plate. Let ϕ_x be the maximum value of the flux enclosed per metre breadth within the two inner sides of an elemental path of width dx, these sides each being parallel to the centre line of the plate and at distance

x from it, as in Fig. 14.3. Then the total e.m.f. induced, per metre breadth, for the two inner sides is

$$E_{x\,max} = 2\pi f \phi_x \text{ volts}$$

Let $B_{x\,max}$ be the maximum value of the flux density—considered uniform—over the width dx. This is justifiable if dx is very small. The maximum e.m.f. induced in the two outer sides of the path is then

$$E_{(x+dx)\,max} = 2\pi f(\phi_x + 2B_{x\,max}\,dx)$$

The difference between the e.m.f.s induced at distances $(x + dx)$ and x from the centre of the plate is thus

$$E_{(x+dx)\,max} - E_{x\,max} = 4\pi f B_{x\,max}\,dx$$

Now, the maximum value of the eddy current at distance x from the centre of the plate is

$$I_{x\,max} = \frac{E_{x\,max}}{\frac{2\rho}{dx}} = \frac{E_{x\,max}}{2\rho}\,dx$$

The maximum m.m.f., per metre axial length of the plate, due to this eddy current, is

$$I_{x\,max} = \frac{E_{x\,max}}{2\rho}\,dx$$

The maximum value of the flux density, $dB_{x\,max}$, produced by this m.m.f., is given by

$$dB_{x\,max} = \frac{E_{x\,max}}{2\rho}\,dx \,.\, \mu$$

where $\mu = \mu_0\mu_r$ is the permeability of the iron.

Since the flux density and eddy e.m.f. are not in phase with one another, symbolic notation is necessary for simplification of the calculation. Thus, denoting the symbolic value by a bar thus, $\bar{B}$, $\bar{E}$, we have

$$\frac{d\bar{B}_{x\,max}}{dx} = \frac{\bar{E}_{x\,max}}{2\rho} \,.\, \mu$$

and

$$\frac{d\bar{E}_{x\,max}}{dx} = -j\,4\pi f\,\bar{B}_{x\,max}$$

whence by differentiation of $\frac{d\bar{B}_{x\,max}}{dx}$ and by substitution for $\frac{d\bar{B}_{x\,max}}{dx}$ we have

$$\frac{d^2\bar{B}_{x\,max}}{dx^2} = -j\,\frac{2\pi f \bar{B}_{x\,max}\,\mu}{\rho}$$

$$= -j \,.\, 2\gamma^2 \bar{B}_{x\,max}$$

where

$$\gamma = \sqrt{\frac{\pi f \mu}{\rho}}$$

The solution of this differential equation for $\bar{B}_{x\ max}$ is

$$\bar{B}_{x\ max} = \bar{M} e^{x\sqrt{-j2\gamma^2}} + \bar{N} e^{-x\sqrt{-j2\gamma^2}} \quad . \quad . \quad (14.10)$$

$\bar{M}$ and $\bar{N}$ are complex constants which are equal, since $\bar{B}_{x\ max}$ has the same value, but is opposite in sign, on the two sides of the centre line of the plate and at distances of x from this centre line.

Let $\bar{B}_{max}$ = the flux density at the surface of the plate $\left(\text{i.e. when } x = \frac{t}{2}\right)$

Then $$\bar{B}_{max} = \bar{M}\left(e^{\frac{t}{2}\sqrt{-j2\gamma^2}} + e^{-\frac{t}{2}\sqrt{-j2\gamma^2}}\right)$$

and $$\frac{\bar{B}_{x\ max}}{\bar{B}_{max}} = \frac{e^{\gamma x\sqrt{-2j}} + e^{-\gamma x\sqrt{-2j}}}{e^{\frac{t}{2}\gamma\sqrt{-2j}} + e^{-\frac{t}{2}\gamma\sqrt{-2j}}}$$

Now, $$\sqrt{-2j} = \sqrt{2}\sqrt{-j} = \sqrt{2}\sqrt{e^{-j\frac{\pi}{2}}} = \sqrt{2}\, e^{-j\frac{\pi}{4}}$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4} - j\sin\frac{\pi}{4}\right)$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) = 1 - j$$

Hence $$\frac{\bar{B}_{x\ max}}{\bar{B}_{max}} = \frac{e^{\gamma x(1-j)} + e^{-\gamma x(1-j)}}{e^{\frac{t}{2}\gamma(1-j)} + e^{-\frac{t}{2}\gamma(1-j)}} \quad . \quad . \quad (14.11)$$

Using the expansions,

$$e^{+j\theta} = \cos\theta + j\sin\theta$$

and $$e^{-j\theta} = \cos\theta - j\sin\theta$$

we have

$$\frac{\bar{B}_{x\ max}}{\bar{B}_{max}} = \frac{(e^{\gamma x} + e^{-\gamma x})\cos\gamma x - j(e^{\gamma x} - e^{-\gamma x})\sin\gamma x}{\left(e^{\frac{t}{2}\gamma} + e^{-\frac{t}{2}\gamma}\right)\cos\frac{t}{2}\gamma - j\left(e^{\frac{t}{2}\gamma} - e^{-\frac{t}{2}\gamma}\right)\sin\frac{t}{2}\gamma}$$

or $$\bar{B}_{x\ max} = \frac{\bar{B}_{max}[(e^{\gamma x} + e^{-\gamma x})\cos\gamma x - j(e^{\gamma x} - e^{-\gamma x})\sin\gamma x]}{\left(e^{\frac{t}{2}\gamma} + e^{-\frac{t}{2}\gamma}\right)\cos\frac{t}{2}\gamma - j\left(e^{\frac{t}{2}\gamma} - e^{-\frac{t}{2}\gamma}\right)\sin\frac{t}{2}\gamma}$$

. . . . (14.12)

This gives the law of variation of flux density across the section of the plate in terms of the flux density at the surface.

Oberbeck and J. J. Thomson first investigated this effect of eddy currents upon the flux density in plates of various thicknesses. The curves of Fig. 14.5 show the order of the variation in maximum flux density across the section of the plate for plates of three different thicknesses, the material of the plates being transformer iron of ordinary grade and the frequency 100 cycles per second. The relative permeability μ_r (assumed constant) is taken as 2,500, and the resistivity $\frac{1}{10^7}$ ohm-metres.

It can be seen from these curves that, if approximately uniform flux distribution is to be obtained, the thickness of plate used, at

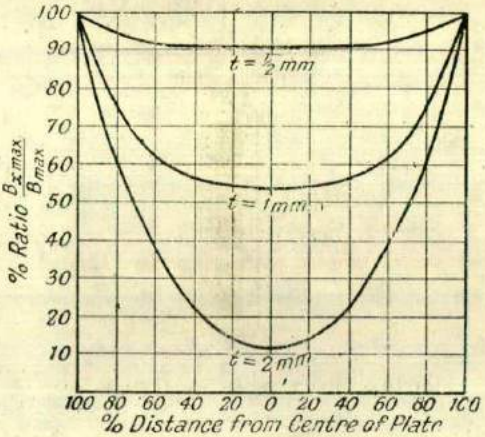


FIG. 14.5 VARIATION IN FLUX DENSITY IN THE CROSS-SECTION OF A THICK PLATE

100 cycles per second frequency, should be less than $\frac{1}{2}$ mm. Actually, plates about 0·35 mm (0·014 in.) are used for this frequency, and for 50 cycles per second, although the variation in flux density is less than that shown when the frequency is less than 100 cycles per second.

By the use of thin plates, or laminations, the adjacent sheets being insulated from one another, the iron is used most economically, since almost uniform flux density is obtained—which means that no portion of the iron core is under-worked—and also the eddy current loss, which is proportional to the square of the thickness of the plate, is thereby reduced.

When plates of this thickness are used for work at commercial frequencies, the assumption that the flux density is uniform throughout the cross-section is justifiable, and the eddy current loss may be calculated with reasonable accuracy by the methods previously given.

For further consideration of the question the reader should refer to the works quoted in Refs. (1), (2), (5). The first of these three works discusses also the eddy current loss in iron sheets when the magnetization is rotating, as is the case in rotating electrical machinery.

Eddy Current Losses in Armature Conductors. Fig. 14.6 shows the distribution of flux in and around a stator slot on an alternator. The slot shown contains three conductors, and the flux is shown as

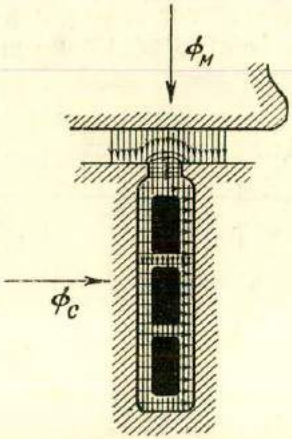


FIG. 14.6. FLUX IN AN ARMATURE SLOT

though it consisted of two quite independent components. Actually, these components combine to form one resultant flux.

Both of these fluxes will induce eddy currents in the conductors. The flux ϕ_M, flowing parallel to the sides of the slot, will be small unless the teeth on each side of the slot are highly saturated, in which case the reluctance of the path in the slot becomes comparable with that of the paths in the teeth. In this case the conductors should be made narrow in the direction across the slot—perpendicular to the slot sides.

The flux ϕ_c is produced by the current in the conductors in the slot. Some of the flux surrounding these conductors, and proportional to the current in them at any instant, will be carried in the iron of the teeth and will pass, also, across the mouth of the slot and across the iron under the bottom of the slot. The remainder passes across the slot itself, and is responsible for the induction of eddy currents in the conductors.

The eddy currents in the individual conductors depend, therefore, upon the position, in the slot, of the conductor considered, and also upon the ratio of the width of the conductor to the width of the

slot. The conductor nearest the mouth of the slot has the greatest eddy-current loss.

Messrs. A. B. and M. B. Field (Refs. (9) and (10)) have both investigated the question of eddy-current losses in such conductors. The former, in his paper (*loc. cit.*) gives sets of curves from which the eddy-current loss in any particular conductor in a slot may be determined. Each curve in the set refers to the conductor in some particular position in the slot, one curve referring to the bottom conductor, one to the bottom but one, and so on. These curves give the ratio $\frac{R_{eff}}{R}$ for the conductor, corresponding to various values of ad, where d is the depth of the conductor itself in centimetres, and a is a number which depends upon the ratio r of the width of the conductor to the width of the slot, and upon the frequency.

S. P. Smith (Ref. (12)) has redrawn these curves so that they give the mean value of the ratio $\frac{R_{eff}}{R}$ for all the conductors in the slot, instead of the ratio for each individual conductor. This set of curves is reproduced, by permission, in Fig. 14.7.

R_{eff} is the effective resistance of the conductor as influenced by eddy currents—which have the effect of increasing the effective resistance, since they increase the copper loss. R is the resistance of the conductor with direct current flowing in it (i.e. when there are no eddy currents). The power loss in the conductor with alternating current of r.m.s. value I flowing in it is thus I^2R_{eff}, whereas the power loss with an equal direct current I flowing is I^2R, the eddy current loss being responsible for the increase of the a.c. power loss over that with direct current.

The number a is given by

$$\frac{2\pi}{10}\sqrt{\frac{\mu_r fr}{1{,}000\rho}}$$

where μ_r is the relative permeability of the material of the conductor, ρ its resistivity in ohm-metres, and f is the frequency of the current in the conductors.

Thus, for copper, $a = 0{\cdot}145\sqrt{fr}$.

In using these curves, dimensions are calculated in centimetres.

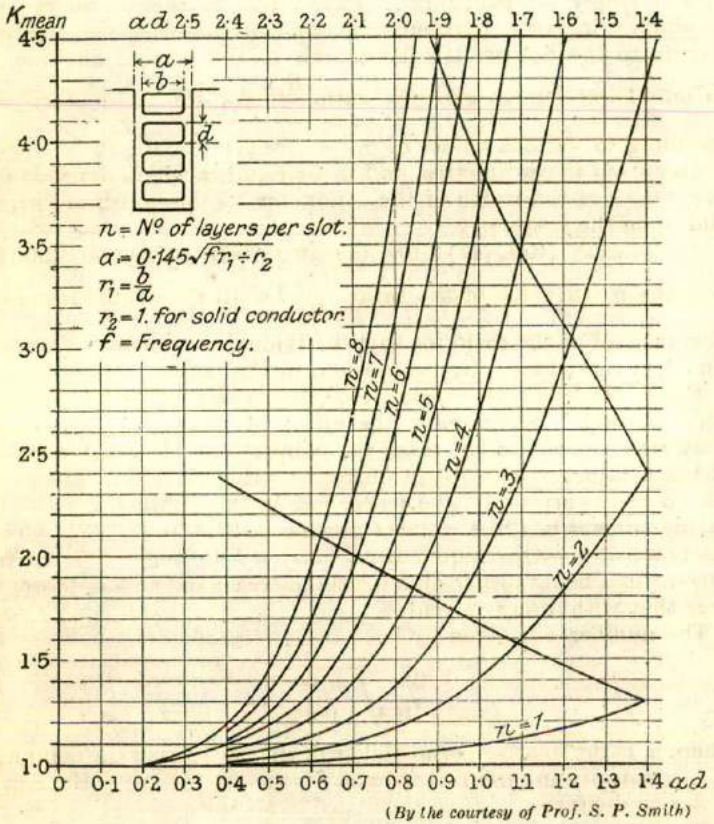
As an example of the use of such curves, suppose we have three conductors, each 0·5 cm wide, and 1·25 cm deep, in a slot of width 0·9 cm, and that the frequency is 50 cycles per second.

Then $$r = \frac{0{\cdot}5}{0{\cdot}9} = 0{\cdot}555$$

$$\therefore\ a = 0{\cdot}145\sqrt{0{\cdot}555 \times 50} = 0{\cdot}764$$

Then the ratio K_{mean} $\left(\text{i.e. the ratio } \frac{R_{eff}}{R}\right)$ is from the curves, 1·8.

Thus the mean value of R_{eff} for the three conductors is $1{\cdot}8R$ where R is the d.c. resistance of each conductor. This means that the copper loss with alternating current of 50 cycles per second



(By the courtesy of Prof. S. P. Smith)

FIG. 14.7

is 1·8 times the copper loss with direct current. Hence, the eddy current loss is 0·8 of the d.c. copper loss.

Eddy Current Damping. Advantage is taken of the fact that eddy currents are induced in metal parts when they cut through magnetic lines of force by using these currents as a means of damping electrical indicating instruments. In the induction type instruments, for alternating current work, the eddy currents induced in a thin metal

disc, together with the magnetic field of an electromagnet, produce also the operating, or deflecting, torque.

The two commonest forms of eddy-current damping device are a metal former, upon which the working coil of the instrument is wound, or a thin aluminium disc which is attached to the moving system and moves in the field of a permanent magnet.

EDDY CURRENT DAMPING TORQUE WITH A METAL FORMER. Fig. 14.8 gives an outline diagram of the working portion of a moving-coil, permanent-magnet, instrument whose coil is wound upon a metal

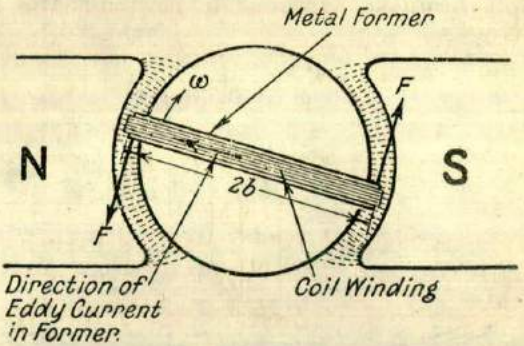


FIG. 14.8. EDDY CURRENT DAMPING IN A MOVING-COIL INSTRUMENT

former. Suppose that the axial length of the former is l and its breadth $2b$ metres, and let it rotate in a permanent-magnet field which is radial and of uniform flux density B webers per square metre. If the former moves with angular velocity ω radians per second, its linear velocity is $b\omega$ metres per second, and the e.m.f. induced in each side of it is $Blb\omega$ volts. Thus the total e.m.f. induced in the former is

$$2Blb\omega \text{ volts}$$

The resistance of the path of the resulting current in the former is $\frac{(4b + 2l)\rho}{td}$, where t is the thickness and d the width of the section of the metal of the former, td being thus the cross-sectional area of the current-path. ρ is the resistivity in ohm-metres of the material of the former.

Then, the eddy current is given by

$$I_e = \frac{2Blb\omega td}{(4b + 2l)\rho} \quad . \quad . \quad . \quad (14.13)$$

The force on each side in the radial field is $F = BI_e l$ newtons

These forces produce a torque which opposes the motion of the former. This torque is $2bF$ newton-metres, or

$$\text{Damping torque } T_D = 2bF = 2b \,.\, BI_e l$$

$$= 2b \,.\, B \,.\, l\left(\frac{2Blb\omega td}{(4b + 2l)\rho}\right)$$

$$= \frac{4B^2b^2l^2\omega td}{(4b + 2l)\rho} \text{ newton-metres} \quad . \quad . \quad (14.14)$$

The *damping constant*, expressed in newton-metres per radian per second is

$$K_D = \frac{T_D}{\omega} = \frac{2B^2b^2l^2td}{(2b + l)\rho} \quad . \quad . \quad . \quad (14.15)$$

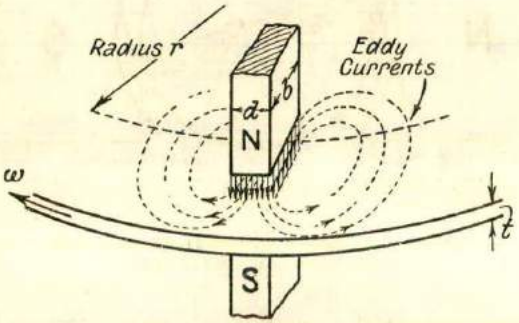


FIG. 14.9. EDDY CURRENT DAMPING WITH A METAL DISC

The damping can obviously be varied by varying the thickness t of the metal of the former.

EDDY CURRENT DAMPING TORQUE WITH A METAL DISC. Although, in this case, the exact calculation of the eddy current is a matter of considerable difficulty, simple methods of calculating the approximate values of such currents, and of their damping effect, have been given by Evershed (Ref. (11)) and by Drysdale and Jolley (Ref. (7)). The method given by the latter is followed below.

Fig. 14.9 shows a metal disc of thickness t metres, which rotates with angular velocity ω radians per second, and cuts through the magnetic field of a permanent magnet whose poles are NS.

Then, if the flux density under the pole is B webers per square metre, an e.m.f. $e = Bb\omega r$ volts will be induced in the portion of the disc which is, at any instant, in the inter-polar gap. The radius r is measured from the centre of the disc to the centre of the pole face. If we consider only the portion of the disc which is immediately under the pole, the resistance of the eddy-current path

is $\frac{b\rho}{dt}$, where b is the radial length of the pole (which is the length of the portion of the disc in which the e.m.f. is induced), and d is the width of the pole. ρ is the resistivity of the material of the disc in ohm-metres.

The actual resistance of the total eddy-current path depends upon the radial position of the pole, and is somewhat greater than $\frac{b\rho}{dt}$. Let this resistance be $k \cdot \frac{b\rho}{dt}$, where k is a constant (greater than unity) for any given radial position of the poles.

The eddy current is thus given by

$$i_e = \frac{Bb\omega r dt}{k \cdot b \cdot \rho} = \frac{B\omega r dt}{k \cdot \rho} \text{ amp} \quad . \quad . \quad . \quad (14.16)$$

and the retarding force by

$$F = Bi_e \cdot b \text{ newtons}$$

The damping torque is thus

$$T_D = Fr = \frac{B^2 \cdot b \cdot \omega r^2 dt}{k \cdot \rho} \text{ newton-metres} \quad . \quad (14.17)$$

Substituting ϕ for the total flux passing between the poles, we have, since $\phi = Bbd$

$$T_D = \frac{\phi^2 r^2 t\omega}{k \cdot \rho \cdot A} \quad . \quad . \quad . \quad . \quad (14.18)$$

where A is the area of the pole face and equals bd.

The damping constant is therefore

$$K_D = \frac{T_D}{\omega} = \frac{\phi^2 r^2 t}{k \ \rho \cdot A} \quad . \quad (14.19)$$

newton-metres per radian per second.

The damping may be altered by adjustment of the radial position of the poles, and is zero when the centres of the poles are at the edge of the disc. Drysdale and Jolley (*loc. cit.*) give the results of measurements of damping constants for discs of different materials and thicknesses, and their book should be consulted for further information on the subject.

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CHAPTER XV

WAVE-FORMS AND THEIR DETERMINATION

Wave-form. Alternating current calculations generally are carried out upon the assumption that the wave-form of the voltage supplied to the circuit is sinusoidal; i.e. the law of variation of the voltage with time may be written

$$e = E_{max} \sin \omega t$$

e being the instantaneous value of the voltage at any time t and E_{max} being its maximum value. ω is, of course, $2\pi \times$ frequency.

Special precautions must, however, be taken, in constructing the supply alternator, if the voltage wave-form is to approximate reasonably closely to a purely sinusoidal form.

Consider an alternator with a revolving field and with an armature —or stator—the surface of which is smooth instead of containing slots and teeth, which render its iron surface discontinuous. In such a machine a sinusoidal voltage wave-form could be obtained by shaping the poles so that the length of the air-gap at any point is proportional to $\frac{1}{\cos \theta}$, where θ is the angle—measured in electrical degrees—between the point in question and the centre of the pole.

If, however, the stator has open slots, these will affect the flux distribution from the poles, the air-gap reluctance at any point being dependent upon the position of the point, at any instant, relative to the adjacent stator teeth. Thus, the movement of the poles past the teeth is accompanied by small variations of flux distribution which are superposed upon the main flux, and these produce corresponding small e.m.f.s in the conductors, the frequency of which is higher than, and some multiple of, that of the main e.m.f. These small e.m.f.s produce "harmonics" in the e.m.f. wave, and cause it to depart from purely sinusoidal wave-form. Again, the magnetizing current of a transformer, the core of which is worked at a flux density sufficiently high to produce saturation, contains a pronounced "third harmonic." This means that it consists of a pure sine wave of current of frequency f (say), together with another sine wave of frequency $3f$. This distortion of the current wave is a result of hysteresis in the iron core of the transformer. One example of a distorted wave-form is shown in Fig 15.1.

This wave can be split up into the two pure sine curves shown dotted.

Thus $$e_1 + e_2 = e$$

The sine curve whose frequency is the same as the "complex" wave (i.e. the original wave) is called the "fundamental," and the other sine curve is called the "third harmonic," since its frequency is three times that of the fundamental.

In the figure, the maximum value of the fundamental is four times

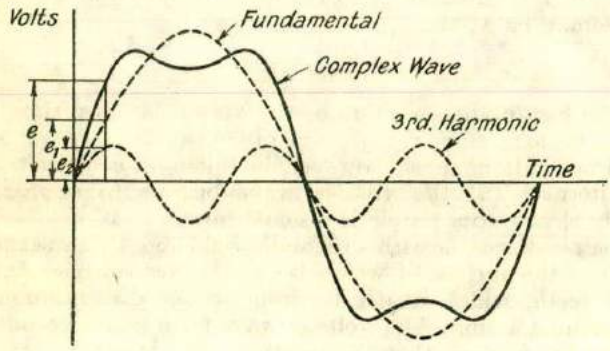


FIG. 15.1. WAVE-FORM WITH A THIRD HARMONIC

that of the third harmonic. Hence, calling the former E_1, the equation for the complex wave may be written

$$e = E_1 \sin 2\pi ft + \frac{E_1}{4} \sin 6\pi ft$$

where f is the frequency of the fundamental,

or $$e = E_1 \sin \omega t + \frac{E_1}{4} \sin 3\omega t \quad . \quad . \quad . \quad (15.1)$$

If the third harmonic had been displaced in phase by 180° relative to the fundamental, the effect would have been to make the wave-form peaked as shown by Fig. 15.2, instead of flat.

The Composition of Complex Wave-forms*. *Any single-valued periodic function* (i.e. a function which can have only one value for any given value of the independent variable) *can be resolved into a fundamental periodic function and a number of harmonics.* This is known as Fourier's theorem, and is true no matter how complicated the complex function may be.

* It is now common practice to use the term "non-sinusoidal" as a preferred alternative to "complex."

Expressed mathematically, this means that the equation of any complex wave e may be written

$$e = E_0 + E_1 \sin(\omega t + \phi_1) + E_2 \sin(2\omega t + \phi_2) + E_3 \sin(3\omega t + \phi_3) + \ldots E_n \sin(n\omega t + \phi_n) \quad (15.2)^*$$

where E_1, E_2, etc., are the maximum values of the various component sine waves, and the angles ϕ_1, ϕ_2, etc., are the phases of these waves

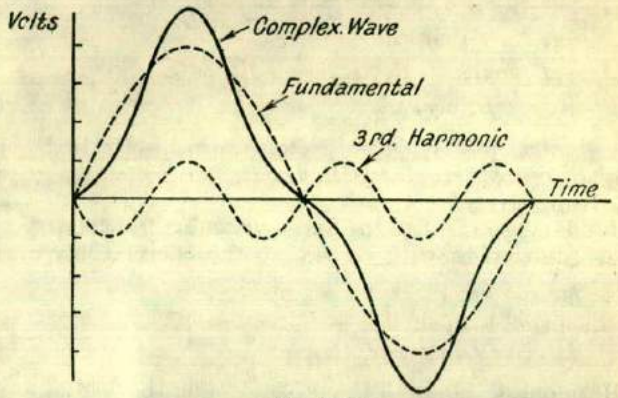


FIG. 15.2.

on the angular scales ωt, $2\omega t$, etc., respectively, when time is zero.

$\omega = 2\pi \times$ frequency, and therefore the frequencies of the terms from the second onwards are in the ratio $1 : 2 : 3 : \ldots : n$. $E_1 \sin(\omega t + \phi_1)$ represents the fundamental wave, $E_2 \sin(2\omega t + \phi_2)$ the second harmonic, and so on.

E_0 is independent of time and represents a constant component of the complex wave. If the time axis is drawn so that the mean ordinate of the complex wave is zero, E_0 is zero. This is the case in alternating current and e.m.f. waves.

The above general expression may be rewritten in another form if the trigonometrical expansion

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

is applied to it.

* The expansion (15.2) can also be conveniently written in the exponential form

$$e = \sum_{k=-n}^{n} E'_k e^{jk\omega t}$$

where the coefficients E'_k are generally complex numbers and E'_{-k} is the conjugate of E'_k.

Then, omitting the E_0 term, as being zero,

$$\begin{aligned} e = {} & E_1(\sin \omega t \cos \phi_1 + \cos \omega t \sin \phi_1) \\ & + E_2(\sin 2\omega t \cos \phi_2 + \cos 2\omega t \sin \phi_2) \\ & + E_3(\sin 3\omega t \cos \phi_3 + \cos 3\omega t \sin \phi_3) + \\ & \ldots E_n(\sin n\omega t \cos \phi_n + \cos n\omega t \sin \phi_n) \end{aligned}$$

or,

$$\begin{aligned} e = {} & A_1 \sin \omega t + A_2 \sin 2\omega t + A_3 \sin 3\omega t \\ & + \ldots A_n \sin n\omega t \\ & + B_1 \cos \omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t \\ & + \ldots B_n \cos n\omega t \quad . \quad . \quad . \quad . \quad . \quad (15.3) \end{aligned}$$

where the coefficients A_1, A_2, A_3, etc., are respectively $E_1 \cos \phi_1$, $E_2 \cos \phi_2$, $E_3 \cos \phi_3$, etc. and B_1, B_2, B_3, are $E_1 \sin \phi_1$, $E_2 \sin \phi_2$, $E_3 \sin \phi_3$, respectively.

If the fundamental and all the harmonics start from zero together, their phase differences with respect to the complex wave are all zero and

$$\begin{aligned} e = {} & E_1 \sin \omega t + E_2 \sin 2\omega t + E_3 \sin 3\omega t + \ldots \\ & + \ldots E_n \sin n\omega t \quad . \quad . \quad . \quad . \quad . \quad (15.4) \end{aligned}$$

Even Harmonics. Since all the poles forming the field of an alternator are similarly constructed and shaped, it follows that the half-wave of e.m.f. generated in a conductor during the passage of (say) a south pole past it, is exactly similar in shape to the half-wave generated during the passage of the preceding north pole. In general, therefore, it follows that the negative halves of the wave-forms of alternating currents and voltages are identical in shape with the positive halves.

It will be shown below that the effect of even harmonics in a complex wave is to make the shape of the negative half-wave different from that of the positive half. From the above reasoning, it is thus obvious that *alternating current and voltage wave-forms do not, in general, contain even harmonics.*

Consider two sine waves, of frequency f and $2f$, such as those shown dotted in Fig. 15.3. When added together, these give a complex wave whose law of variation with time is given by

$$e = E_1 \sin 2\pi ft + E_2 \sin (4\pi ft - \phi_2)$$

where E_1 and E_2 are the amplitudes of the fundamental and second harmonic respectively. (The time is, in this case, reckoned from the instant when the fundamental is zero and the phase angle ϕ_2 is measured on the scale of degrees applying to the second harmonic.)

It is seen that the resultant complex wave has a negative half which is of different shape from the positive half. A similar effect is produced whatever the relative phases of the fundamental and

harmonic, and with all even harmonics, whatever their frequency. Odd harmonics do not produce the effect.

Since there can be no even harmonics present in alternating current and voltage waves whose two halves are identical in shape, the equations of such waves may be written in the form

$$e = E_1 \sin(\omega t + \phi_1) + E_3 \sin(3\omega t + \phi_3) + E_5 \sin(5\omega t + \phi_5) + \ldots E_n \sin(n\omega t + \phi_n) \quad . \quad . \quad . \quad . \quad (15.5)$$

where n is odd.

Harmonic Analysis. The process of splitting-up a complex wave into its fundamental and harmonics is called "harmonic analysis."

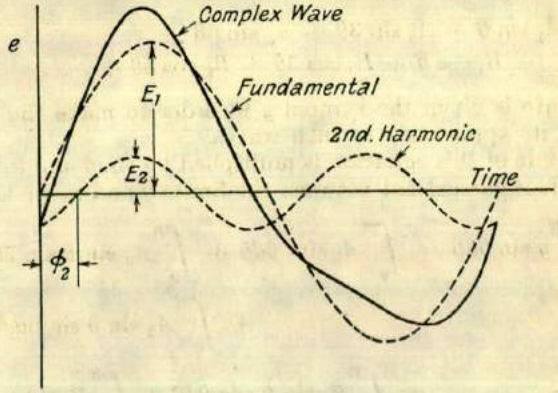


FIG. 15.3. WAVE-FORM CONTAINING A SECOND HARMONIC

The number of terms to be obtained in such an analysis depends upon the accuracy with which it is desired to express, mathematically, the complex wave. The third and fifth harmonics are the most important in alternating current and voltage wave-forms, and the analysis is, therefore, often carried no further than the third term of the expression for the complex wave, although in special cases it may be necessary to determine the amplitudes of harmonics up to (say) the seventeenth.

METHOD 1. This method was given originally by Perry (Ref. (9)). It depends upon the following theorems in the integral calculus.

(*a*) $\int_0^{\pi} \sin\theta d\theta = 2$

(*b*) $\int_0^{\pi} \sin^2\theta d\theta = \frac{\pi}{2}$

(*c*) $\int_0^{\pi} \cos^2\theta d\theta = \frac{\pi}{2}$

$$(d) \int_0^{\pi} \sin m\theta \cos n\theta d\theta = 0$$

$$(e) \int_0^{\pi} \sin m\theta \sin n\theta d\theta = 0$$

$$(f) \int_0^{\pi} \cos m\theta \cos n\theta d\theta = 0$$

where m and n are positive integers which are unequal.

Consider a complex wave whose ordinate y varies with the abscissa θ according to the law

$$y = A_1 \sin \theta + A_3 \sin 3\theta + A_5 \sin 5\theta + \ldots + B_1 \cos \theta + B_3 \cos 3\theta + B_5 \cos 5\theta + \ldots$$

The ordinate is given the symbol y in order to make the equation general in its application to such waves.

If each side of this equation is multiplied by $\sin \theta$ and integration of each side is carried out between the limits 0 and π, we have

$$\int_0^{\pi} y \sin \theta d\theta = \int_0^{\pi} A_1 \sin^2 \theta d\theta + \int_0^{\pi} A_3 \sin \theta \sin 3\theta d\theta + \int_0^{\pi} A_5 \sin \theta \sin 5\theta d\theta + \ldots + \int_0^{\pi} B_1 \sin \theta \cos \theta d\theta + \int_0^{\pi} B_3 \sin \theta \cos 3\theta d\theta + \int_0^{\pi} B_5 \sin \theta \cos 5\theta d\theta + \ldots \quad (15.6)$$

From the theorems already quoted, all terms on the right-hand side of the equation, except the first, are equal to zero. Therefore

$$\int_0^{\pi} y \sin \theta \, . \, d\theta = \int_0^{\pi} A_1 \sin^2 \theta \, . \, d\theta = A_1 \frac{\pi}{2}$$

Hence,

$$A_1 = \frac{2}{\pi} \int_0^{\pi} y \sin \theta \, . \, d\theta$$

Now, $\frac{1}{\pi} \int_0^{\pi} y \sin \theta \, . \, d\theta$ is obviously the mean value of the function $y \sin \theta$ for one half-period of the complex wave, and therefore the amplitude A_1 of the fundamental of the complex wave is given by twice the mean value of $y \sin \theta$ for one half-wave.

In the same way the values of A_3, A_5, etc., are obtained by multiplying each side of the equation of the complex wave by $\sin 3\theta$ $\sin 5\theta$, and so on, and integrating each side between the limits 0

and π. To obtain B_1, B_3, etc., multiply each side of the equation by $\cos\theta$, $\cos 3\theta$, and so on, and integrate.

The values of the coefficients so determined are—

$A_1 = 2 \times$ mean value of $y \sin\theta$ for $\frac{1}{2}$-period
$A_3 = 2 \times$ „ „ $y \sin 3\theta$ „ „
$A_5 = 2 \times$ „ „ $y \sin 5\theta$ „ „
. . = ...
$B_1 = 2 \times$ mean value of $y \cos\theta$ for $\frac{1}{2}$-period
$B_3 = 2 \times$ „ „ $y \cos 3\theta$ „ „

and so on.

If the equation is required in the form

$$y = Y_1 \sin(\theta + \phi_1) + Y_3 \sin(3\theta + \phi_3) + \ldots Y_n \sin(n\theta + \phi_n)$$

the coefficients Y_1, Y_3, etc., and the phase angles ϕ_1, ϕ_3, etc., may be obtained from the values of the predetermined coefficients A_1, A_3, . . . and B_1, B_3, . . . , as follows—

From Equation (15.3),

$$A_1 = Y_1 \cos\phi_1$$
$$A_3 = Y_3 \cos\phi_3$$
. . = ...

also
$$B_1 = Y_1 \sin\phi_1$$
$$B_3 = Y_3 \sin\phi_3$$
. . = ...

Hence $A_1^2 + B_1^2 = (Y_1 \cos\phi_1)^2 + (Y_1 \sin\phi_1)^2 = Y_1^2$

or
$$Y_1 = \sqrt{A_1^2 + B_1^2}$$

Similarly,
$$Y_3 = \sqrt{A_3^2 + B_3^2}$$
$$Y_5 = \sqrt{A_5^2 + B_5^2} \text{ and so on.}$$

Also,
$$\frac{B_1}{A_1} = \frac{Y_1 \sin\phi_1}{Y_1 \cos\phi_1} = \tan\phi_1$$

so that
$$\phi_1 = \tan^{-1}\frac{B_1}{A_1}$$

Similarly,
$$\phi_3 = \tan^{-1}\frac{B_3}{A_3}$$
$$\phi_5 = \tan^{-1}\frac{B_5}{A_5} \text{ and so on.}$$

In this way, the equation may be written in the desired form by substituting these values of the amplitudes and phase angles in the equation.

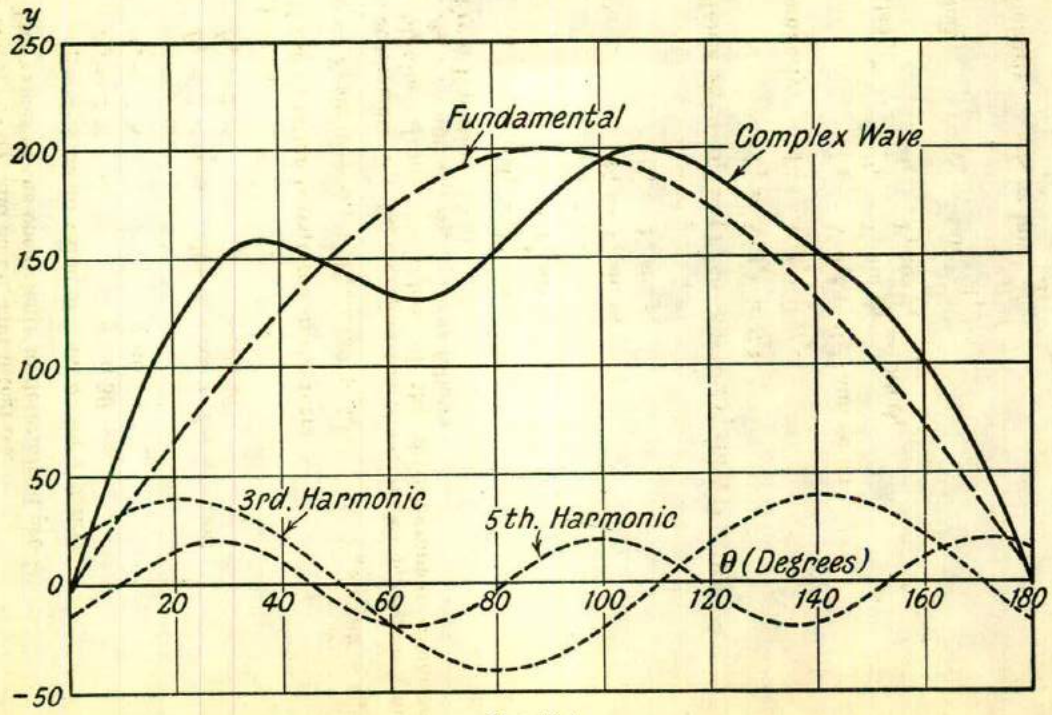

y
250
200
150
100
50
0
-50
Fundamental
Complex Wave
3rd. Harmonic
5th. Harmonic
θ (Degrees)
20
40
60
80
100
120
140
160
180

FIG. 15.4

Procedure for the Determination of the Coefficients by a Summation Process. From the fact that

$$A_1 = 2 \times \text{mean value of } y \sin\theta \text{ for one half-period}$$

it is obvious that A_1 could be obtained by a graphical method as follows.

Take any ordinate y_1, corresponding to an angle θ_1 and multiply it by $\sin\theta_1$, setting up an ordinate representing $y_1 \sin\theta_1$, at abscissa θ_1. By repeating the process for a large number of values of y, a new curve $y \sin\theta$, plotted against θ, is obtained. The area under one half-wave of this curve could be measured, to scale, using a planimeter, and its mean value obtained by dividing this area by the length of the base π. By this means A_1 could be found. The other coefficients $A_3, A_5, \ldots$, and also $B_1, B_3, \ldots$, could also be found by similar methods. Such a procedure would, however, be very laborious and a tabulation method is therefore used, as below, in order to simplify it somewhat. In the tabular method the base of the half-wave is divided into a number of equal parts—the greater the number of parts the greater the accuracy of the analysis—and ordinates are set up at the dividing points.

Let the number of these parts be n. Then each part corresponds to an angle of $\frac{\pi}{n}$. If $y_{\frac{\pi}{n}}, y_{\frac{2\pi}{n}}, \ldots$, are the ordinates at the dividing points, then the corresponding products—when A_1 is being determined—are $y_{\frac{\pi}{n}} \sin\frac{\pi}{n}, y_{\frac{2\pi}{n}} \sin\frac{2\pi}{n}, \ldots$.

The mean value of $y \sin\theta$ for the half-wave is then obtained by dividing the sum of these products by the number of parts into which the base is divided.

The other A coefficients are obtained in a similar way, but the y ordinates are then multiplied by $\sin 3\theta$, $\sin 5\theta \ldots$. The B coefficients are determined in the same way, when the y ordinates are multiplied by $\cos\theta$, $\cos 3\theta \ldots$.

The accuracy of the results of such an analysis may be checked from the facts that—

(i) The ordinate of the complex wave corresponding to $\theta = 0$ should equal $B_1 + B_3 + B_5 + \ldots$.

(ii) The ordinate corresponding to $\theta = \frac{\pi}{2}$ should equal $A_1 - A_3 + A_5 - A_7 + \ldots$.

These relations are obtained as below—

When $\theta = 0$

$$y = A_1 \sin 0 + A_3 \sin 0 + A_5 \sin 0 + \ldots + B_1 \cos 0 + B_3 \cos 0 + B_5 \cos 0 + \ldots$$

or $$y = B_1 + B_3 + B_5 + \ldots$$

TABLE XIV

ANALYSIS OF WAVE SHOWN IN FIG. 15.4

Complex Wave		Analysis of Sine Terms						Analysis of Cosine Terms					
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(j)	(k)	(l)	(m)	(n)	(p)
θ	y	$\sin\theta$	$y\sin\theta$	$\sin 3\theta$	$y\sin 3\theta$	$\sin 5\theta$	$y\sin 5\theta$	$\cos\theta$	$y\cos\theta$	$\cos 3\theta$	$y\cos 3\theta$	$\cos 5\theta$	$y\cos 5\theta$
15°	97	0·259	25	0·707	68·5	0·966	94	0·966	94	0·707	68·5	0·259	25
30°	152	0·5	76	1·0	152	0·5	76	0·866	132	0	0	− 0·866	− 132
45°	152	0·707	107·5	0·707	107·5	− 0·707	− 107·5	0·707	107·5	− 0·707	− 107·5	− 0·707	− 107·5
60°	134	0·866	116	0	0	− 0·866	− 116	0·5	67	− 1·0	− 134	0·5	67
75°	142	0·966	137	− 0·707	− 100·5	0·259	36·8	0·259	36·8	− 0·707	− 100·5	0·966	137
90°	176	1·0	176	− 1·0	− 176	1·0	176	0	0	0	0	0	0
105°	200	0·966	193	− 0·707	− 141·4	0·259	51·8	− 0·259	− 51·8	0·707	141·4	− 0·966	− 193
120°	189	0·866	164	0	0	− 0·866	− 164	− 0·5	− 94·5	1·0	189	− 0·5	− 94·5
135°	161	0·707	114	0·707	114	− 0·707	− 114	− 0·707	− 114	0·707	114	0·707	114
150°	133	0·5	66·5	1·0	133	0·5	66·5	− 0·866	− 115	0	0	0·866	115
165°	82	0·259	21·2	0·707	58	0·966	79·5	− 0·966	− 79·5	− 0·707	− 58	− 0·259	− 21·2
180°	0	0	0	0	0	0	0	− 1·0	0	− 1·0	0	− 1·0	0
		Sum	1196·2	Sum	215·1	Sum	79·1	Sum	− 17·5	Sum	112·9	Sum	− 90·2
		Mean Value	99·7	Mean Value	17·9	Mean Value	6·6	Mean Value	− 1·46	Mean Value	9·4	Mean Value	− 7·5
		$A_1 = 199{\cdot}4$		$A_3 = 35{\cdot}8$		$A_5 = 13{\cdot}2$		$B_1 = -2{\cdot}92$		$B_3 = 18{\cdot}8$		$B_5 = -15$	
Check		$A_1 - A_3 + A_5 = 176{\cdot}8 \doteqdot$ the value of y at $\theta = 90°$ (i.e. 176)						$B_1 + B_3 + B_5 = 0{\cdot}88 \doteqdot$ the value of y at $\theta = 0°$ (i.e. 0)					

Equation of wave: $y = 200\sin(\theta - 0°\,50') + 40{\cdot}3\sin(3\theta + 27°\,42') + 20\sin(5\theta - 48°\,39')$

When $\theta = \frac{\pi}{2}$

$$y = A_1 \sin\frac{\pi}{2} + A_3 \sin\frac{3\pi}{2} + A_5 \sin\frac{5\pi}{2} + \ldots$$
$$+ B_1 \cos\frac{\pi}{2} + B_3 \cos\frac{3\pi}{2} + B_5 \cos\frac{5\pi}{2} + \ldots$$
$$= A_1 - A_3 + A_5 - \ldots$$

To illustrate the method of tabulation for the determination of the coefficients, consider, as an example, the complex wave shown in Fig. 15.4, the analysis of which is to be carried out for the determination of harmonics up to, and including, the fifth.

In Table XIV, the base of the half-wave is divided into twelve equal intervals, each of 15°, as shown in column (*a*). The corresponding values of y, taken from the curve, are shown in column (*b*). The coefficients A_1, A_3, and A_5 are determined from the mean values of the products given in columns (*d*), (*f*), and (*h*), and the coefficients B_1, B_3, and B_5 from the products given in columns (*k*), (*m*), and (*p*). As previously explained, checks are applied in order to test the accuracy of the determination. The curve to be analysed contains, in this case, only third and fifth harmonics, in addition to the fundamental. This is shown by the checks, since $A_1 - A_3 + A_5$ is equal to the ordinate of the complex wave at $\theta = 90°$, and $B_1 + B_3 + B_5$ equal to the ordinate at 0° (to within the limits of error obtainable with a graph of this size).

If other harmonics had been present, $A_1 - A_3 + A_5$ could not have equalled the ordinate at 90°, nor would $B_1 + B_3 + B_5$ have equalled the ordinate at 0°. These checks give a guide as to whether further analysis, beyond the fifth harmonic, is necessary, in order to represent the complex wave mathematically with sufficient accuracy.

The equation of the complex wave is, therefore, from the results obtained in the table,

$$y = 199{\cdot}4 \sin\theta + 35{\cdot}8 \sin 3\theta + 13{\cdot}2 \sin 5\theta - 2{\cdot}92 \cos\theta$$
$$+ 18{\cdot}8 \cos 3\theta - 15 \cos 5\theta \quad . \quad . \quad . \quad . \quad (15.7)$$

In order to write the expression for y in terms of sine functions only, the coefficients Y_1, Y_3, and Y_5 must be determined, and also the phase angles ϕ_1, ϕ_3, and ϕ_5, relative to the phase of the complex wave.

Then,
$$Y_1 = \sqrt{A_1^2 + B_1^2} = 200$$
$$Y_3 = \sqrt{A_3^2 + B_3^2} = 40{\cdot}3$$
$$Y_5 = \sqrt{A_5^2 + B_5^2} = 20$$

Also,
$$\phi_1 = \tan^{-1}\frac{B_1}{A_1} = -0°\ 50'$$
$$\phi_3 = \tan^{-1}\frac{B_3}{A_3} = 27°\ 42'$$
$$\phi_5 = \tan^{-1}\frac{B_5}{A_5} = -48°\ 39'$$

The equation of the complex wave is thus

$$y = 200 \sin(\theta - 0°\ 50') + 40{\cdot}3 \sin(3\theta + 27°\ 42') + 20 \sin(5\theta - 48°\ 39') \quad . \quad . \quad . \quad . \quad (15.8)$$

The three component waves are shown dotted in Fig. 15.4.

METHOD 2. *Summation Method with Grouped Terms.* This method is similar to the one given above, but the labour involved in the summation for the determination of the coefficients is reduced by the grouping of terms. The method is due to S. P. Thompson (Ref. (10)) and to Runge (Ref. (11)).

If the nth harmonic is to be determined, the base of the half-wave is divided into $n + 1$ equal parts, giving n equidistant ordinates. These ordinates are treated in supplemental pairs, the ordinate y_1 at θ_1 being used in conjunction with the ordinate y_n at $(180 - \theta_1)$ as below.

First consider the sum of two such ordinates,

$$\begin{aligned} y_1 + y_n = {} & A_1 \sin\theta_1 + A_3 \sin 3\theta_1 + A_5 \sin 5\theta_1 + \ldots \\ & + B_1 \cos\theta_1 + B_3 \cos 3\theta_1 + B_5 \cos 5\theta_1 + \ldots \\ & + A_1 \sin(180 - \theta_1) + A_3 \sin 3(180 - \theta_1) \\ & + A_5 \sin 5(180 - \theta_1) + \ldots \\ & + B_1 \cos(180 - \theta_1) + B_3 \cos 3(180 - \theta_1) \\ & + B_5 \cos 5(180 - \theta_1) + \ldots \end{aligned}$$

All the terms containing coefficients B disappear, since

$$\cos\theta_1 = -\cos(180 - \theta_1)$$
$$\cos 3\theta_1 = -\cos 3(180 - \theta_1)$$
$$\ldots = \ldots.$$

Also,
$$\sin\theta_1 = \sin(180 - \theta_1)$$
$$\sin 3\theta_1 = \sin 3(180 - \theta_1)$$
$$\ldots = \ldots.$$

Hence,

$$y_1 + y_n = 2A_1 \sin\theta_1 + 2A_3 \sin 3\theta_1 + 2A_5 \sin 5\theta_1 + \ldots \quad . \quad . \quad . \quad . \quad . \quad . \quad (15.9)$$

Similarly, if the difference of these ordinates is considered, we have

$$y_1 - y_n = 2B_1 \cos\theta_1 + 2B_3 \cos 3\theta_1 + 2B_5 \cos 5\theta_1 + \ldots \quad (15.10)$$

Ordinates are grouped in this way before multiplying by the sine functions for the determination of the individual coefficients. This obviously saves labour.

Taking as an example the curve (Fig. 15.4) previously analysed, the procedure, in an analysis up to the fifth harmonic, is to divide the base into six equal parts, the five ordinates set up at the dividing points being measured and arranged in rows as under—

	y_{30}	y_{60}	y_{90}
	y_{150}	y_{120}	
Sum . . .	a_1	a_2	a_3
Difference . .	d_1	d_2	

The notation "y_{30}" means the ordinate at $\theta = 30°$
From the previous method, we have

$A_1 = 2 \times$ the mean value of $y \sin\theta$ for half-period

In this case, therefore,

$$A_1 = \frac{2\,[y_{30}\sin 30° + y_{60}\sin 60° + y_{90}\sin 90° + y_{120}\sin 120° + y_{150}\sin 150° + y_{180}\sin 180°]}{6}$$

or, since sin 180° and y_{180} are both zero,

$$A_1 = \frac{1}{3}\,[y_{30}\sin 30° + y_{60}\sin 60° + y_{90}\sin 90° + y_{120}\sin 120° + y_{150}\sin 150°]$$

$$= \frac{1}{3}\,[a_1 \sin 30° + a_2 \sin 60° + a_3 \sin 90°]$$

since $\sin 30° = \sin 150°$

and $\sin 60° = \sin 120°$

In the same way it can be shown that

$$A_3 = \frac{1}{3}\,(a_1 - a_3)\sin 90°$$

and $$A_5 = \frac{1}{3}\,(a_1 \sin 30° - a_2 \sin 60° + a_3 \sin 90°)$$

Again,

$$B_1 = \frac{2\,[y_{30}\cos 30° + y_{60}\cos 60° + y_{90}\cos 90° + y_{120}\cos 120°}{6}$$

$$\frac{+\,y_{150}\cos 150° + y_{180}\cos 180°]}{6}$$

$$= \frac{1}{3}\,[y_{30}\cos 30° + y_{60}\cos 60° + y_{120}\cos 120° + y_{150}\cos 150°]$$

since cos 90° and y_{180} are both zero.

Hence $$B_1 = \frac{1}{3}\,(d_1\cos 30° + d_2\cos 60°)$$

It can be shown also that

$$B_3 = \frac{1}{3}\,d_2\cos 180° = -\frac{1}{3}\,d_2$$

and $$B_5 = \frac{1}{3}\,(-d_1\cos 30° + d_2\cos 60°)$$

Proceeding with the analysis we have, therefore,

	$y_{30} = 152$	$y_{60} = 134$	$y_{90} = 176$
	$y_{150} = 133$	$y_{120} = 189$	
Sum	$a_1 = 285$	$a_2 = 323$	$a_3 = 176$
Difference	$d_1 = 19$	$d_2 = -55$	

The values a_1, a_2, a_3, d_1 and d_2 are then used in the table as shown.

Using the coefficients A_1, A_3, etc., as obtained in the table, we have, for the equation of the complex wave,

$$y = 199{\cdot}5\sin\theta + 36{\cdot}3\sin 3\theta + 12{\cdot}8\sin 5\theta - 3{\cdot}68\cos\theta + 18{\cdot}33\cos 3\theta - 14{\cdot}65\cos 5\theta$$

From these coefficients

$$Y_1 = \sqrt{A_1^2 + B_1^2} = 200$$

$$Y_3 = \sqrt{A_3^2 + B_3^2} = 40{\cdot}6$$

$$Y_5 = \sqrt{A_5^2 + B_5^2} = 19{\cdot}5$$

Also, $$\phi_1 = \tan^{-1}\frac{B_1}{A_1} = -1°\,3'$$

$$\phi_3 = \tan^{-1}\frac{B_3}{A_3} = 26°\,42'$$

$$\phi_5 = \tan^{-1}\frac{B_5}{A_5} = -48°\,45'$$

Hence, the equation of the wave, written in sine terms only, becomes

$$y = 200 \sin (\theta - 1° 3') + 40{\cdot}6 \sin (3\theta + 26° 42') + 19{\cdot}5 \sin (5\theta - 48° 45') \quad . \quad . \quad . \quad (15.11)$$

The checks in this analysis (see Table XV) show that there are no other components of the wave beside the fundamental and 3rd and 5th harmonics.

It will be noted that the above equation is not quite the same as that obtained in the previous method of analysis, but the differences are within the limits of error to be expected when such methods are applied to a wave of the size shown.*

In the above, it has been assumed that the wave-form of a current or voltage is known and that the wave is to be split up, by mathe-

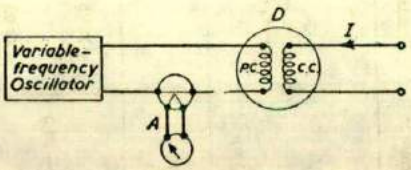


FIG. 15.5. HARMONIC ANALYSIS OF A CURRENT WAVE

matical or graphical methods, into its component fundamental and harmonic waves.

Harmonic Analyser. A complex wave can be split up into its components by experimental methods, the magnitudes of the various harmonics being measured directly. Harmonic analyses at power frequencies can be made very simply with the circuit shown in Fig. 15.5. *D* is a dynamometer wattmeter movement having a centre zero, and its voltage coil carries an analysing current supplied from a variable-frequency oscillator. The current being analysed is passed through the current coil. The oscillator is first adjusted to the frequency of the fundamental of the wave-form under analysis, and, as the oscillator frequency approaches that of the fundamental, the wattmeter pointer will commence to oscillate about its zero at the difference frequency. The oscillator frequency is adjusted until the slowest possible beat is obtained, and the maximum swing of the wattmeter pointer observed. It is convenient to adjust the oscillator output until the wattmeter pointer swings full scale. The voltage-coil current is read by means of a thermal milliammeter *A*.

The action depends upon the fact that the maximum deflection

* For graphical methods of analysis and for other methods, including analyses up to the eleventh or seventeenth harmonic, the reader is referred to the works quoted in Refs. (2), (3), (5), (6).

TABLE XV

Analysis of Wave Shown in Fig. 15.4 (p. 570)

θ	sin θ	cos θ	Sine Terms			Cosine Terms		
			Fundamental	5th Harmonic	3rd Harmonic	Fundamental	5th Harmonic	3rd Harmonic
0°	0	1·0						
30°	0·50	0·866	142·5 [$= a_1 \sin 30°$]				16·45 [$= d_1 \cos 30°$]	55 [$= -d_3 \cos 0°$]
60°	0·866	0·50		280 [$= a_5 \sin 60°$]		− 27·5 [$= d_3 \cos 60°$]		
90°	1·0	0	176 [$= a_3 \sin 90°$]		109 [$= (a_1 - a_3) \sin 90°$]			
Total, 1st Column . .			318·5		109	− 27·5		55
Total, 2nd Column . .			280			16·45		
Sum of 1st and 2nd Column totals			598·5 $\therefore A_1 = \frac{598·5}{3} = 199·5$		$A_3 = \frac{109}{3} = 36·3$	− 11·05 $\therefore B_1 = \frac{-11·05}{3} = -3·68$		55
Difference of 1st and 2nd Column totals . .			38·5 $\therefore A_5 = \frac{38·5}{3} = 12·8$			− 43·95 $\therefore B_5 = \frac{-43·95}{3} = -14·65$		$\therefore B_3 = \frac{55}{3} = 18·33$

Check $A_1 - A_3 + A_5 = 176$ = ordinate of complex wave at $\theta = 90°$. $B_1 + B_3 + B_5 = 0$ = ordinate of complex wave at $\theta = 0°$

Equation of complex wave: $y = 200 \sin (\theta - 1° 3') + 40·6 \sin (3\theta + 26° 42') + 19·5 \sin (5\theta - 48° 45'$

of the dynamometer wattmeter occurs only when the current waves in the fixed and moving coils are at the same frequency and in phase with one another. It is practically impossible to adjust the oscillator to attain a steady maximum deflection, but measurement of the maximum value of a very slow swing suffices. Since the calibration of the wattmeter will be known, the r.m.s. value of the fundamental of the wave being analysed may be determined from its maximum indication.

Higher harmonics may be measured simply by tuning the oscillator to the appropriate harmonic frequency, and measuring the maximum swing again. It is convenient to keep the analysing current in the voltage-coil circuit constant when carrying out an analysis, because then the ratios of the maximum indications for the harmonics to that for the fundamental give the harmonic analysis directly.

Let the current in the moving coil be

$$i = I_0 + I_1 \sin(\theta + \phi_1) + I_2 \sin(2\theta + \phi_2) + I_3 \sin(3\theta + \phi_3) + \ldots I_n \sin(n\theta + \phi_n)$$

If the analysing current supplied to the voltage coil by the oscillator is

$$i_a = I_a \sin(m\theta + \phi_a)$$

(i.e. the analysing current has the frequency of the mth harmonic) then the instantaneous torque is

$$T \propto i i_a$$

and the mean torque is

$$T_m \propto \frac{1}{2\pi} \int_0^{2\pi} i i_a \, d\theta$$

$$\propto \frac{1}{2\pi} \int_0^{2\pi} I_a \sin(m\theta + \phi_a)[I_0 + I_1 \sin(\theta + \phi_1) + I_2 \sin(2\theta + \phi_2) + \ldots] \, d\theta$$

$$\propto \frac{1}{2\pi} \int_0^{2\pi} I_a I_m \sin(m\theta + \phi_a) \sin(m\theta + \phi_m) \, d\theta$$

since all such terms as $\int_0^{2\pi} I_a I_r \sin(m\theta + \phi_a) \sin(r\theta + \phi_r) \, d\theta$ are zero, r being different from m (see page 568).

Evaluating the integral we have

$$T_m \propto \frac{I_a I_m}{2} \cos(\phi_m - \phi_a)$$

and the maximum value is

$$T_m \propto \frac{I_a I_m}{2} \text{ when } \phi_a = \phi_m$$

This may be written as

$$D = K I_a' I_m'$$

where D is the deflection of the instrument, K its calibration constant, and I_a' and I_m' are the r.m.s. values of the analysing current and the mth harmonic of the test current, respectively.

When analysing voltage wave-forms, it is usual to connect the wattmeter movement supply under test through a capacitor, as shown in Fig. 15.6. This has the effect of magnifying the harmonics. Then if R is the total resistance of the circuit, L its inductance, and

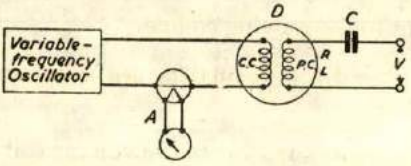


FIG. 15.6. HARMONIC ANALYSIS OF A VOLTAGE WAVE

C the series capacitance (all of which must be known) the mth harmonic of the applied voltage wave is given by

$$V_m = I_m' \sqrt{R^2 + \left(m\omega L - \frac{1}{m\omega C}\right)^2}$$

A similar principle has been used in instruments described by J. D. Cockcroft, R. T. Coe, J. A. Tyacke, and Miles Walker (Ref. (12)) and by Coe (Ref. (18)).

While the method just described is useful in laboratory work, instruments using this principle have been largely superseded by heterodyne analysers and analysers using resistance-capacitance tuned feedback amplifiers.

A wave analyser made by Marconi Instruments, Ltd., measures the component frequencies and amplitudes of wave-forms over the frequency range 20 c/s to 16 kc/s for a range of inputs from 100 μV to 300 V. The instrument operates on the heterodyne principle. The input signal is fed to a balanced ring-bridge modulator which is also fed from a variable-frequency oscillator. The oscillator is adjusted so that the sum of its frequency and that of the input component being measured is 50 kc/s—the frequency to which the subsequent system is tuned. The output from the modulator is passed to a double-crystal 50 kc/s band-pass filter having a pass bandwidth of 4 c/s. This is followed by a 3-stage amplifier tuned to 50 kc/s, and the output is measured by a rectifier voltmeter. The

internal oscillator is calibrated so that it indicates the frequency of the component under examination directly.

A wave analyser made by Dawe Instruments, Ltd., measures the component frequencies and amplitudes of wave-forms over the frequency range 50 to 16,000 c/s with four input ranges of 0·5, 5, 50 and 500 V maximum.

The circuit uses a local oscillator and balanced modulator which work on the heterodyne principle giving a constant intermediate frequency from the selected component of the input. The subsequent amplifier is sharply tuned to this intermediate frequency and the output is measured by a valve voltmeter. The selectivity can be varied over a wide range with negligible change in the gain of the amplifier, and the use of toroidal inductors and silvered-mica capacitors, with negative feedback, ensures permanent stability of the tuned amplifier.

Muirhead and Co., Ltd., have recently developed the Muirhead-Pametrada wave analyser, designed primarily for analysing wave-forms of complex vibrations, but being applicable also to audio and power frequency wave-forms up to 21 kc/s. The design is quite different from that of the heterodyne type of analyser, which produces an output frequency that is almost constant whatever the frequency under test. The Muirhead analyser acts as a tunable filter and feeds to the output meter a signal of the same frequency as that of the selected component. The output can be applied to an oscillograph.

The instrument is described by the makers as of the negative-feedback type with resistance-capacitance tuning. It uses two selective amplifier units connected in cascade, and tuning is by varying simultaneously two resistors in the arms of the Wien bridges which provide the negative feedback path for each amplifier.

The Shape of the Current Wave-form when the E.M.F. Wave-form contains Harmonics. The shape of the current wave-form when an e.m.f. whose wave contains harmonics is applied to a circuit is not, in general, the same as that of the impressed e.m.f. wave, but depends upon the resistance, inductance, and capacitance in the circuit. The consideration of the question is simplified if the complex e.m.f. wave is thought of as consisting of a fundamental wave and its harmonic waves, each of which exists separately and produces its own component of the current wave.

Different forms of circuits will now be considered.

1. RESISTANCE ONLY. Suppose an e.m.f. whose equation is $e = E_1 \sin \omega t + E_3 \sin 3\omega t + E_5 \sin 5\omega t + \ldots$ is impressed upon a circuit of resistance R and whose inductance and capacitance are both negligible. Then the equation of the current is

$$i = \frac{E_1}{R} \sin \omega t + \frac{E_3}{R} \sin 3\omega t + \frac{E_5}{R} \sin 5\omega t + \ldots \quad . \quad (15.12)$$

Thus, the current wave-form is of the same shape as that of the impressed e.m.f.

2. Inductance Only. Suppose the same e.m.f. is impressed upon a circuit of inductance L and having negligible resistance and capacitance. The phase of each component of the current will lag 90° behind the e.m.f. component producing it. Also, the reactance of the circuit will not be the same to all the e.m.f. components.

Hence, the components of the current wave are

$$i_1 = \frac{E_1}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i_3 = \frac{E_3}{3\omega L} \sin\left(3\omega t - \frac{\pi}{2}\right)$$

$$i_5 = \frac{E_5}{5\omega L} \sin\left(5\omega t - \frac{\pi}{2}\right)$$

and so on.

Thus, the equation of the current wave is

$$i = \frac{E_1}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) + \frac{E_3}{3\omega L} \sin\left(3\omega t - \frac{\pi}{2}\right) + \frac{E_5}{5\omega L} \sin\left(5\omega t - \frac{\pi}{2}\right) + \ldots \quad (15.13)$$

The wave-shape of the current is thus different from that of the e.m.f. due to the facts that, first, the amplitudes of the harmonics in the current wave are not in a constant ratio to the e.m.f. harmonics to which they are due, but decrease with increasing order of the harmonic; and second, that the phases of the current harmonics are different from those of the e.m.f. harmonics.

The general effect is, however, that inductance in a circuit *reduces* the amplitudes of the harmonics in the current wave and causes the latter to approach more nearly to the sinusoidal wave-shape than the e.m.f. wave.

3. Capacitance Only. If an e.m.f. wave

$$e = E_1 \sin \omega t + E_3 \sin 3\omega t + E_5 \sin 5\omega t + \ldots$$

is impressed upon a circuit having capacitance C and whose resistance and inductance are both negligible, the current produced is

$$\frac{e}{\text{The reactance of the capacitor}}$$

The reactance due to the capacitance C is not the same to all the components of the e.m.f. wave, since it is given by the expression

$$\frac{1}{2\pi \times \text{frequency} \times C}$$

and the frequency differs for different harmonics. The current is thus given by

$$i = E_1\omega C \sin\left(\omega t + \frac{\pi}{2}\right) + 3E_3\omega C \sin\left(3\omega t + \frac{\pi}{2}\right) + 5E_5\omega C \sin\left(5\omega t + \frac{\pi}{2}\right) + \ldots . \quad (15.14)$$

each component of the current leading the e.m.f. component to which it is due by the angle $\frac{\pi}{2}$.

It can be seen that the effect of capacitance upon the current wave is to increase the amplitudes of the harmonics in it, relative to the fundamental, as compared with the relative amplitudes in the e.m.f. wave.

Thus, the ratio

$$\frac{\text{Amplitude of 3rd harmonic of e.m.f. wave}}{\text{Amplitude of the fundamental of e.m.f. wave}} = \frac{E_3}{E_1}$$

whereas, in the current wave,

$$\frac{\text{Amplitude of 3rd harmonic}}{\text{Amplitude of the fundamental}} = \frac{3E_3\omega C}{E_1\omega C} = \frac{3E_3}{E_1}$$

i.e. the 3rd harmonic is magnified three times in the current wave. Obviously the higher the order of the harmonic the greater the magnification.

Fig. 15.7 shows a complex voltage wave whose equation is

$$e = 100 \sin \omega t - 10 \sin 5\omega t$$

its fundamental and fifth harmonic components being shown dotted.

Three current wave-forms are also shown. The first, i_r, is the wave-form of the current when the e.m.f. e is impressed upon a circuit of resistance 5 ohms, the inductance and capacitance of the circuit both being negligible;

$$i_r = \frac{100}{5} \sin \omega t - \frac{10}{5} \sin 5\omega t \qquad i_r = 20 \sin \omega t - 2 \sin 5\omega t$$

This current is in phase with the e.m.f. The second wave-form, i_L, is that of the current obtained when the e.m.f. is applied to a circuit containing inductance only, the reactance, at the frequency of the fundamental, being 5 ohms.

Then

$$i_L = \frac{100}{5} \sin\left(\omega t - \frac{\pi}{2}\right) - \frac{10}{5 \times 5} \sin\left(5\omega t - \frac{\pi}{2}\right)$$

$$= 20 \sin\left(\omega t - \frac{\pi}{2}\right) - 0{\cdot}4 \sin\left(5\omega t - \frac{\pi}{2}\right)$$

This current lags by $\frac{\pi}{2}$ behind the e.m.f. wave, and its wave-form is seen to be much more nearly sinusoidal than that of the e.m.f.

The third wave-form, i_c, is that of the current in a circuit having capacitance only, when the e.m.f. e is impressed upon the circuit, the capacitance being such that the reactance of the circuit, at the frequency of the fundamental, is again 5 ohms.

Then
$$i_c = \frac{100}{5}\sin\left(\omega t + \frac{\pi}{2}\right) - \frac{10}{5/5}\left(\sin 5\omega t + \frac{\pi}{2}\right)$$
$$= 20\sin\left(\omega t + \frac{\pi}{2}\right) - 10\sin\left(5\omega t + \frac{\pi}{2}\right)$$

The distortion in the current wave i_c, when capacitance, only, exists in the circuit, is very noticeable.

Resonance with Harmomcs. If an e.m.f. of complex wave-form such as

$$e = E_1\sin(\omega t + \phi_1) + E_3\sin(3\omega t + \phi_3) + \ldots . \qquad (15.15)$$

is impressed upon a circuit containing resistance R, inductance L, and capacitance C, in series, the equation of the current in the circuit is

$$i = \frac{E_1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\sin(\omega t + \phi_1 - \theta_1)$$
$$+ \frac{E_3}{\sqrt{R^2 + \left(3\omega L - \frac{1}{3\omega C}\right)^2}}\sin(3\omega t + \phi_3 - \theta_3) + \ldots \qquad (15.16)$$

where θ_1, θ_3, . . . , are the phase angles by which the components of the current wave lag behind the e.m.f. components which produce them.

Thus
$$\tan\theta_1 = \frac{\omega L - \frac{1}{\omega C}}{R}, \quad \tan\theta_3 = \frac{3\omega L - \frac{1}{3\omega C}}{R}$$

and so on.

When the inductive reactance ωL, in any circuit, is equal to the capacitive reactance $\frac{1}{\omega C}$, the condition of "resonance" exists, and the current in the circuit is then given by $\frac{\text{e.m.f.}}{\text{resistance}}$. This current may be very large, and as a result, the voltage across either the

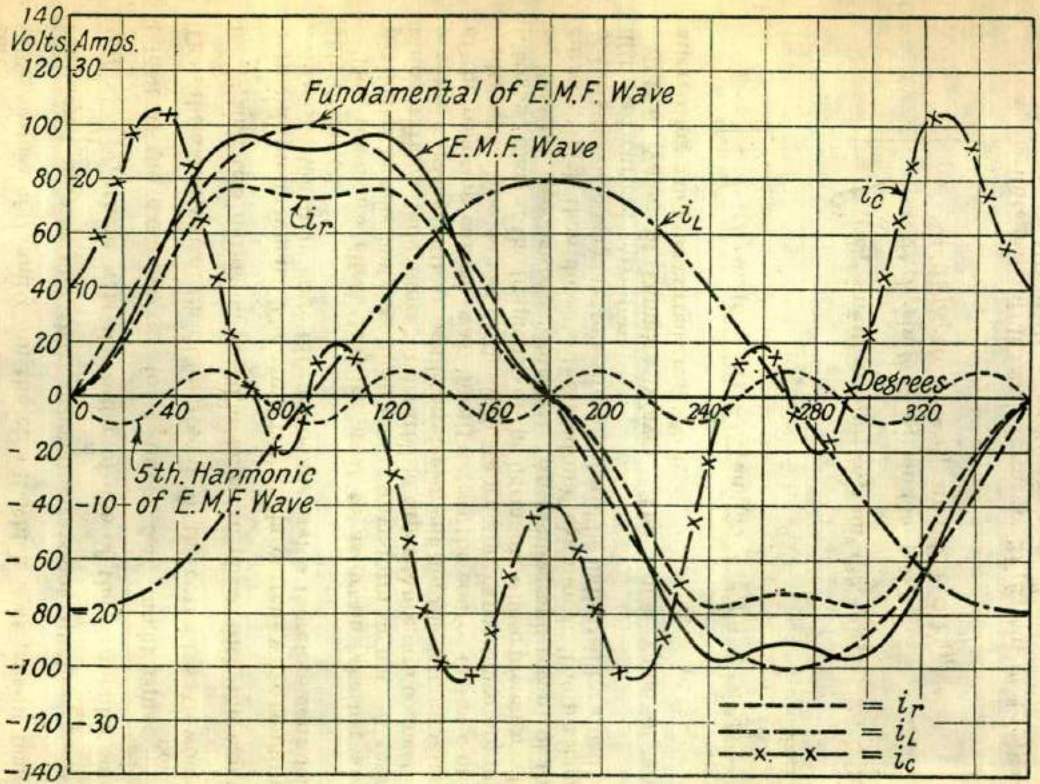

Volts
Amps.
Fundamental of E.M.F. Wave
E.M.F. Wave
i_r
i_L
i_C
Degrees
5th. Harmonic of E.M.F. Wave
$= i_r$
$= i_L$
$= i_C$

FIG. 15.7

inductance or the capacitance may become seriously high, although the voltage across the inductance and capacitance together is zero.

The values of L and C, in the circuit which is at present being considered, may be such that, although ωL is not equal to $\frac{1}{\omega C}$, yet $n\omega L = \frac{1}{n\omega C}$, where n is the order of one of the harmonics contained in the e.m.f. wave; for example, $5\omega L$ might equal $\frac{1}{5\omega C}$. Then the amplitude of the 5th harmonic in the current wave would be $\frac{E_5}{R}$, and this harmonic would, also, be in phase with the e.m.f. harmonic producing it. The result is that the current wave contains a disproportionately large fifth harmonic and is therefore considerably distorted.

This resonance effect may be used to ascertain whether any particular harmonic exists in an e.m.f. wave-form or not. A resistance, variable inductance, and variable capacitance, are connected in series, together with an oscillograph element (for the determination of the resulting current wave-form) across the source of the e.m.f. under test. The values of the inductance and capacitance are adjusted so that, at the frequency of the harmonic whose presence is to be detected, resonance is obtained. An oscillograph record of the current which then flows is obtained and examined. The presence of the harmonic which is to be detected will be indicated by a badly-distorted current wave-form.

R.M.S. Value of Complex Waves. The r.m.s. value of any alternating e.m.f. or current is given by

$$E = \left[\frac{1}{\pi}\int_0^{\pi}(E_1 \sin(\theta + \phi_1) + E_3 \sin(3\theta + \phi_3) + \ldots)^2 d\theta\right]^{\frac{1}{2}} \quad \ldots \quad (15.17)$$

$$= \left[\frac{1}{\pi}\int_0^{\pi}\Big(E_1^2 \sin^2(\theta + \phi_1) + E_3^2 \sin^2(3\theta + \phi_3) + \ldots + 2E_1E_3 \sin(\theta + \phi_1)\sin(3\theta + \phi_3) + 2E_1E_5 \sin(\theta + \phi_1)\sin(5\theta + \phi_5) + \ldots\Big)d\theta\right]$$

Now, the integrals of all such terms as $2E_1E_3 \sin(\theta + \phi_1) \sin(3\theta + \phi_3)$ between the limits π and 0 are zero. Thus

$$E = \left[\frac{1}{\pi}\int_0^{\pi}(E_1^2 \sin^2(\theta + \phi_1) + E_3^2 \sin^2(3\theta + \phi_3) + \ldots)d\theta\right]^{\frac{1}{2}} \quad \ldots \quad (15.18)$$

Now, $\int_0^{\pi} \sin^2(\theta + \phi)d\theta = \frac{\pi}{2}$

$$\therefore E = \left[\frac{1}{\pi}\left(E_1^2\frac{\pi}{2} + E_3^2\frac{\pi}{2} + E_5^2 \cdot \frac{\pi}{2} + \ldots\right)\right]^{\frac{1}{2}}$$

$$= \sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{2}} \quad . \quad (15.19)$$

Now, E_1, E_3, E_5, . . . , are maximum values of the various components of the complex wave. All these components are sine waves of various frequencies, and therefore the r.m.s values of the components are, in all cases, given by

$$\frac{\text{Maximum value of the component}}{\sqrt{2}}$$

Thus, if E_1', E_3', etc., are the r.m.s. values of the harmonics composing the complex wave, the r.m.s. value of this wave is

$$E_s = \sqrt{(E_1')^2 + (E_3')^2 + (E_5')^2 + \ldots} \quad . \quad (15.20)$$

Example. Calculate the r.m.s. value, of the current whose equation is $i = 50 \sin 157t + 20 \sin 471t + 5 \sin 785t$

$$I = \sqrt{\frac{50^2 + 20^2 + 5^2}{2}}$$

$$= \sqrt{\frac{2500 + 400 + 25}{2}} = 38{\cdot}2$$

Power in Circuits in which the E.M.F. and Current Waves are not purely Sinusoidal. It has already been seen that when the e.m.f. impressed upon a circuit is

$$e = E_1 \sin(\omega t + \phi_1) + E_3 \sin(3\omega t + \phi_3) + \ldots$$

the most general expression for the current is

$$i = I_1 \sin(\omega t + \phi_1 - \theta_1) + I_3 \sin(3\omega t + \phi_3 - \theta_3) + \ldots$$

The instantaneous power p in such a circuit is given by

$$\begin{aligned} p = ei = {} & E_1 I_1 \sin(\omega t + \phi_1)\sin(\omega t + \phi_1 - \theta_1) \\ & + E_3 I_3 \sin(3\omega t + \phi_3)\sin(3\omega t + \phi_3 - \theta_3) \\ & + \ldots \\ & + E_1 I_3 \sin(\omega t + \phi_1)\sin(3\omega t + \phi_3 - \theta_3) \\ & + E_3 I_1 \sin(3\omega t + \phi_3)\sin(\omega t + \phi_1 - \theta_1) \\ & + \ldots \quad . \quad (15.21) \end{aligned}$$

The mean power in the circuit is

$$P = \frac{1}{\pi}\int_0^{\pi} ei \,.\, d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi} [E_1 I_1 \sin(\omega t + \phi_1)\sin(\omega t + \phi_1 - \theta_1)$$
$$+ E_3 I_3 \sin(3\omega t + \phi_3)\sin(3\omega t + \phi_3 - \theta_3)$$
$$+ \ldots .$$
$$+ E_1 I_3 \sin(\omega t + \phi_1)\sin(3\omega t + \phi_3 - \theta_3)$$
$$+ E_3 I_1 \sin(3\omega t + \phi_3)\sin(\omega t + \phi_1 - \theta_1)$$
$$+ \ldots .] \, d(\omega t) \quad . \quad . \quad . \quad (15.22)$$

The integrals between 0 and π of all such terms as $E_1 I_3 \sin(\omega t + \phi_1)\sin(3\omega t + \phi_3 - \theta_3)$, where the frequencies of the two sine waves which are multiplied together are different, are zero.

Thus

$$P = \frac{1}{\pi}\int_0^{\pi} [E_1 I_1 \sin(\omega t + \phi_1)\sin(\omega t + \phi_1 - \theta_1)$$
$$+ E_3 I_3 \sin(3\omega t + \phi_3)\sin(3\omega t + \phi_3 - \theta_3)$$
$$+ \ldots .] \, d(\omega t)$$

$$= \frac{1}{\pi}\left[\frac{\pi}{2}(E_1 I_1 \cos\theta_1 + E_3 I_3 \cos\theta_3 + \ldots .)\right]$$

$$= \frac{1}{2}(E_1 I_1 \cos\theta_1 + E_3 I_3 \cos\theta_3 + \ldots .)$$

or $$P = E_1' I_1' \cos\theta_1 + E_3' I_3' \cos\theta_3 + \ldots . \quad . \quad . \quad (15.23)$$

where E_1', E_3', . . . , and I_1', I_3', . . . , are the r.m.s. values of the component harmonics of the voltage and current.

Thus, the mean power in the circuit is the sum of the mean powers due to the various components of the current and voltage, since $E_1' I_1 \cos\theta_1$ is the mean power due to the fundamentals of the two waves, $E_3' I_3' \cos\theta_3$ that due to the 3rd harmonic, and so on.

Power Factor. The power factor of a circuit in which the voltage and current waves are non-sinusoidal requires special definition.

Consider the usual definition of power factor as the cosine of the angle of phase-difference between the voltage and current waves. If these waves are complex—and possibly differently-shaped—this angle of phase difference between them is somewhat indefinite. This difficulty is overcome by defining the power factor of the circuit as the cosine of the angle of phase-difference between two "equivalent sine waves" having respectively r.m.s. values equal to those of the voltage and current in the circuit. Let ϕ_e be the

phase difference between the two equivalent sine waves; then the power factor is

$$\cos \phi_e = \frac{P}{EI}$$

where P is the mean power in the circuit, and E and I are the r.m.s. values of the equivalent sine waves of voltage and current. P is the value of the power which would be measured by a wattmeter connected in the circuit, while E and I are the values of voltage and current which would be measured by alternating-current instruments which give the equivalent sine wave values.

Determination of Wave-form. Early methods of determining the wave-form of an alternating current or voltage were contact or point-by-point methods. Those due to Joubert and to Hospitalier are described in Refs. (14) and (15) respectively. Oscillographs are now almost universally used.

Oscillographs. Oscillographs are used for the observation and measurement of transient and other phenomena, as well as for the determination of alternating current and voltage wave-forms. There are three types of oscillograph in use—

(*a*) Electromagnetic.
(*b*) Electrostatic.
(*c*) Cathode-ray.

They all must consist of a moving system, which is caused to deflect when the current or voltage under test is applied to the instrument, and an optical system for the indication or recording of the wave.

The deflection of the moving system must, at any instant, be strictly proportional to the applied voltage or to the current at that instant. Thus the moving system must possess very little inertia in order that it may respond instantly to changes of current or voltage. The natural period of the system must be much smaller than that of the alternating wave under test, and the damping must be critical. Eddy current and hysteresis effects in the instrument should be negligible, and its inductance and capacitance must be very small.

(*a*) Electromagnetic Oscillograph. Two forms of electromagnetic oscillograph have been developed—the moving-magnet type, by Blondel, and the moving-coil type, by Duddell. The Duddell instrument, which has the advantages of negligible inductance and freedom from hysteresis errors, was commonly used for years but has now been displaced, for many purposes, by the cathode-ray oscillograph. Electromagnetic oscillographs are best suited to work at low voltages and comparatively low frequencies. They are, of course, vibration galvanometers having an especially low natural period of vibration.

The Duddell oscillograph is particularly useful for the simultaneous

recording of a number of wave-forms. It is possible to mount up to 12 vibrator elements in a compact unit and to record on photographic film or paper. Such units have been found to be invaluable in stress analysis in aircraft.

Duddell Oscillograph. The principle of this instrument is illustrated by Fig. 15.8. A single loop of thin phosphor-bronze strip forms the vibrator, or moving system This is situated in the field of a powerful magnet—either a permanent magnet or an electromagnet—having specially-shaped pole-pieces to obtain an intense field between them. If high sensitivity is required an electromagnet is used, the magnet being worked with a high degree of saturation

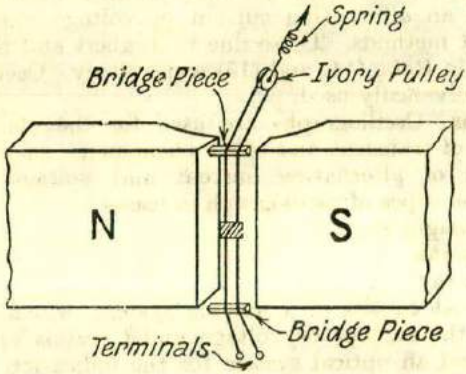


FIG. 15.8. DUDDELL OSCILLOGRAPH

in order that small changes in magnetizing current shall have a negligible effect upon the strength of the magnetic field.

The vibrator loop is formed by passing the phosphor-bronze strip round a small ivory pulley, to which a spring and tension-adjusting device are attached. The loop passes over two ivory bridge-pieces which confine the vibrating portion of the loop to the section which is situated in the magnetic field. A small, light mirror is cemented to the loop midway between the bridge pieces. A piece of soft iron, cut away to clear the mirror, is usually fitted between the two sides of the loop, so that each side has its own air gap. The clearances between the sides of the loop and the pole pieces are very small—of the order of 0·01 cm.

If a current of I amperes passes through the loop which is situated in a magnetic field of flux density B, forces of BIl newtons act on each side, causing one side to move inwards and the other outwards, thus causing the loop as a whole to deflect, as shown in Fig. 15.9. l is the length of loop between the bridge pieces. If the current I is alternating, the loop is caused to vibrate. In this type of oscillograph

the undamped natural period of the loop is about $\frac{1}{10,000}$ second. (It may be as low as $\frac{1}{17,000}$ sec.) Damping is obtained by immersing the moving system in oil, each side of the loop vibrating in a small oil-filled chamber formed by the pole-pieces and the central soft iron piece. Wave-forms of frequencies up to about 300 cycles per second can be faithfully recorded by such an instrument.

These oscillographs are often fitted with several vibrators, side by

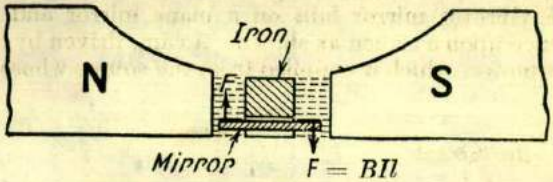


FIG. 15.9.

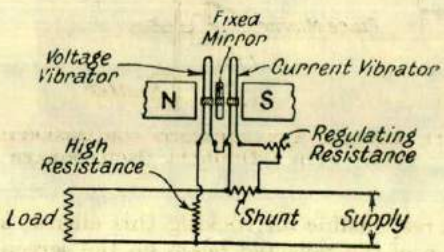


FIG. 15.10. CONNECTIONS OF DUDDELL OSCILLOGRAPH

side. (The Cambridge Instrument Co. make both six- and twelve-element oscillographs of this type.) A fixed mirror, between them, is used to give a zero line on the film upon which the wave-forms are recorded. The connections of two vibrators for obtaining current and voltage wave-forms are shown in Fig. 15.10.

The resistances of the loops are usually about 4 or 5 ohms, and currents of 1 milliamp can be detected, the safe working current of the vibrators being about 100 milliamp. It is thus necessary to shunt the current vibrator and to connect a high resistance in series with the voltage vibrator, as shown in the figure.

A beam of light cast upon the mirror of one of these loops will be reflected, and the reflected beam will move backwards and forwards in a horizontal plane (assuming the loops to be in a vertical position). In recording the wave-shape, the photographic film is passed at constant speed—the actual speed depending upon the frequency of

the waves being recorded—in a direction perpendicular to that of the movement of the reflected beam of light, the result of these two perpendicular motions being the wave-shape required. If time measurements are to be made upon the "oscillogram" so obtained, a current of known frequency is passed through one of the loops, giving a wave on the oscillogram which can then be used as a time scale.

The oscillograph can be calibrated by observing the deflection produced by a known direct current in the vibrator loop.

If a wave-form is to be observed, but not recorded, the optical arrangements are as shown in Fig. 15.11. The reflected beam of light from the vibrator mirror falls on a plane mirror and is reflected from thence upon a screen as shown. A cam, driven by a small synchronous motor, which is supplied from the source whose wave-form

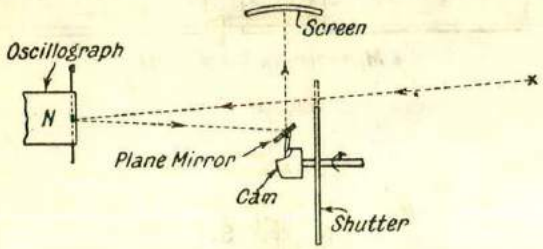


FIG. 15.11. OPTICAL ARRANGEMENTS FOR OBSERVING WAVE-FORMS WITH A DUDDELL OSCILLOGRAPH

is required, is responsible for rocking this mirror, and so giving a forward motion of the reflected beam on the screen. This forward motion is continued for about $1\frac{1}{2}$ cycles of the wave, and the plane mirror is returned, by the cam, to its initial position during the next half-cycle, during which period the light is cut off by a rotating shutter attached to the motor shaft. Persistence of vision gives the effect of a continuously-existing wave ($1\frac{1}{2}$ cycles long) on the screen.

Theory of the Duddell Oscillograph. Since the instrument is, essentially, a moving-coil vibration galvanometer, its equation of motion is

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = Gi$$

as shown when the theory of the vibration galvanometer was discussed (see page 274).

In considering the vibration galvanometer the current i was assumed to be purely sinusoidal, its equation being $i = I_{max} \cos \omega t$. In the case of the oscillograph, however, this assumption cannot be

made. The current i must be assumed to contain harmonics and to be given by the expression

$$i = \sum_{n=1}^{n=a} I_n \sin (n\omega t \pm \phi_n) \quad . \quad . \quad (15.24)$$

according to Fourier's theorem. n is the order of the various harmonics and varies from 1 to the order of the highest harmonic present in the wave.

The general expression for the deflection θ was found, when discussing the theory of the vibration galvanometer, to contain a transient term and a term corresponding to a steady vibratory motion. In the oscillograph the damping is critical ($b^2 = 4ac$) and the transient term rapidly falls to zero. The expression representing the motion of the vibrator is

$$\theta = G \sum_{n=1}^{n=a} \frac{I_n}{c + n^2\omega^2 a} \sin (n\omega t \pm \phi_n - \beta) \quad . \quad (15.25)$$

where $\beta = \tan^{-1} \dfrac{2n\omega\sqrt{ac}}{c - an^2\omega^2}$.

This expression follows from the vibration galvanometer theory. Owing to the term $an^2\omega^2$ in the denominator, the higher harmonics have a disproportionately small effect upon the deflection, as compared with the lower harmonics. The harmonics also, in the expression for the deflection θ, have not the same relative phases as those in the current wave in the loop, and thus the oscillograph does not give a true reproduction of the current wave down to the harmonics of a high order. It can, however, be made to give a reproduction which is sufficiently accurate for practical purposes.

Obviously, if the damping and inertia are both zero—i.e. if the constants a and b are both zero—the deflectional equation becomes

$$c\theta = G \sum_{n=1}^{n=a} I_n \sin (n\omega t \pm \phi_n) \quad . \quad . \quad . \quad (15.26)$$

and the deflection is directly proportional to the current at any instant. Under these conditions the amplitude of the deflection produced by the nth harmonic is

$$\theta_n = \frac{GI_n}{c}$$

When inertia and damping are present the amplitude of the deflection produced by the nth harmonic is

$$\theta_n' = \frac{GI_n}{c + n^2\omega^2 a}$$

The ratio of the actual amplitude of the deflection produced by the nth harmonic, to the amplitude produced under ideal conditions, is thus

$$\frac{\theta_n'}{\theta_n} = \frac{\dfrac{GI_n}{c + n^2\omega^2 a}}{\dfrac{GI_n}{c}} = \frac{c}{c + n^2\omega^2 a} = \frac{1}{1 + \dfrac{n^2\omega^2 a}{c}}$$

Now, from Equation (6.50), page 280, the periodic time of the damped natural oscillations of the moving system is that value of t which makes $\frac{\sqrt{4ac - b^2}}{2a} \cdot t$ equal to 2π. Let this value of t be T', then $T' = \frac{4\pi a}{\sqrt{4ac - b^2}}$. If the damping is zero (i.e. $b = 0$) the free period T_0 of the moving system, is given by

$$T_0 = 2\pi\sqrt{\frac{a}{c}}$$

Let T be the period of the fundamental of the wave whose form is to be determined by the oscillograph, then

$$\omega = 2\pi f = \frac{2\pi}{T}$$

or

$$T = \frac{2\pi}{\omega}$$

Substituting T_0 and T in the expression for the ratio $\frac{\theta_n'}{\theta_n}$ we have

$$\frac{\theta_n'}{\theta_n} = \frac{1}{1 + n^2 \dfrac{4\pi^2}{T^2} \cdot \dfrac{T_0^2}{4\pi^2}} = \frac{1}{1 + n^2\left(\dfrac{T_0}{T}\right)^2}$$

Calling this ratio R_n, we have

$$R_n = \frac{1}{1 + n^2\left(\dfrac{T_0}{T}\right)^2} \quad \text{or} \quad \frac{T_0}{T} = \frac{1}{n}\sqrt{\frac{1}{R_n} - 1} \quad . \quad . \quad (15.27)$$

Example. A Duddell oscillograph has a moving system whose natural period of oscillation is $\frac{1}{10{,}000}$ sec. The wave-form of a complex wave, the frequency of whose fundamental is 50 cycles per second, is to be determined. What are the percentage errors in the reproduction of the 3rd, 5th, 7th, and 11th harmonics?

$$T_0 = \frac{1}{10{,}000} \text{ sec} \qquad T = \frac{1}{50} \text{ sec}$$

Then $$R_3 = \frac{1}{1 + 3^2\left(\frac{\frac{1}{10{,}000}}{\frac{1}{50}}\right)^2} = \frac{1}{1 + 9\left(\frac{50}{10{,}000}\right)^2} = 0{\cdot}9998$$

$$R_5 = \frac{1}{1 + 25\left(\frac{50}{10{,}000}\right)^2} = 0{\cdot}9994$$

$$R_7 = \frac{1}{1 + 49\left(\frac{50}{10{,}000}\right)^2} = 0{\cdot}9988$$

$$R_{11} = \frac{1}{1 + 121\left(\frac{50}{10{,}000}\right)^2} = 0{\cdot}9970$$

Thus the percentage errors in reproduction are—

In the 3rd harmonic	.	.	0·02 per cent
,, 5th ,,	.	.	0·06 ,,
,, 7th ,,	.	.	0·12 ,,
,, 11th ,,	.	.	0·30 ,,

Phase Displacement of Harmonics during Reproduction. It has been seen (Equation (15.25)) that a phase displacement β occurs, in the expression for the deflection of the oscillograph, in the case of any particular harmonic.

This angle β is given by

$$\beta = \tan^{-1}\frac{2n\omega\sqrt{ac}}{c - an^2\omega^2}$$

Substituting $\frac{2\pi}{T}$ for ω and $\frac{T_o}{2\pi}$ for $\sqrt{\frac{a}{c}}$ we have, after simplifying,

$$\beta = \tan^{-1}\frac{2n}{\frac{T}{T_o} - n^2 \cdot \frac{T_o}{T}} \quad . \quad . \quad . \quad . \quad (15.28)$$

Thus, in the above example, the phase displacement in the reproduction of the seventh harmonic, relative to the actual phase of this harmonic, is

$$\tan^{-1} \cdot \frac{14}{\left(\frac{1}{50}\Big/\frac{1}{10{,}000}\right) - 49 \cdot \left(\frac{1}{10{,}000}\Big/\frac{1}{50}\right)}$$

$$= \tan^{-1} 0{\cdot}0699$$

$$= 4°$$

This angle must be measured on the scale of degrees corresponding to the seventh harmonic. Thus 360° of the fundamental wave corresponds to 7 × 360 degrees of the seventh harmonic.

(*b*) ELECTROSTATIC OSCILLOGRAPH. This instrument, due to Ho and Koto (Ref. (21)), is really an electrometer of special design. It is best suited to low-frequency high-voltage work (above 2,000 volts) and has the advantages of consuming no energy, and of requiring only a very small current. With the development of the cathode-ray oscillograph the electrostatic type has largely fallen out of use and will not, therefore, be described, although it was fully discussed in earlier editions of this book.

(*c*) THE CATHODE-RAY OSCILLOGRAPH. The cathode-ray oscillograph differs fundamentally from the previous instruments, as the moving metallic conductors are absent. They are replaced by a beam of electrons, which itself represents a current, and can be deflected by an electrostatic or an electromagnetic field. The inertia of this beam is very considerably less than that of a metallic conductor. Consequently the c.r.o. can be used at frequencies much above the highest which can be observed on the Duddell oscillograph.

The c.r. tube has two main parts: the "electron gun," and the deflecting arrangements.

The *electron gun* is a device for producing and focusing a concentrated beam of fast electrons. The source of the electrons in the Crookes tube, from which the cathode-ray tube was eventually derived, is a piece of metal at room temperature. The field required to tear the electrons out of the metal is large, and high voltages must therefore be used. Modern c.r. tubes use a heated cathode coated with oxides of the rare-earth elements, which emit electrons very readily. The emitted electrons are accelerated by an electric field set up between the cathode and an anode placed near to it. The gun is so proportioned that the majority of the electrons form a concentrated beam. Electrons which are outside this beam are usually intercepted by a diaphragm.

The operation of the c.r. tube may be understood by reference to Fig. 15.12. The cathode K is surrounded by a cylindrical control electrode G containing a diaphragm, and this electrode has a negative potential with respect to the cathode. Its action is the same as that of the grid of a thermionic valve: varying its potential varies the cathode emission current and hence the brilliance of the spot. The anode A is an open cylinder maintained at a high positive potential with respect to the cathode: the electrons are accelerated by the electrostatic field existing between the anode and cathode, and, because of their high speed, they pass through the anode cylinder and impinge upon the fluorescent screen S.

The electron beam must be focused to form a spot on the screen, and the focusing arrangements may be either electrostatic or electromagnetic. The c.r. tube shown in Fig. 15.12 has electrostatic focusing, the focusing being carried out by means of an "electron lens." The electron lens is formed between the electrode F, which consists of a cylinder with an aperture, and the anode A. Fig. 15.13

shows how the electron lens is formed by maintaining F at a slightly lower potential than A. The equipotential surfaces formed between the electrodes are of the form indicated, and it is seen that they bear some resemblance to a simple convex optical lens. Since the force on a charged particle is always at right angles to the equipotential surface, the beam of electrons crossing the surfaces will experience forces as shown by the arrows. The field on the lower potential side converges the beam, whilst that on the higher potential side diverges the beam. The electrons are passing up a potential gradient and are therefore accelerating; they will be travelling faster in the higher potential region and the divergence

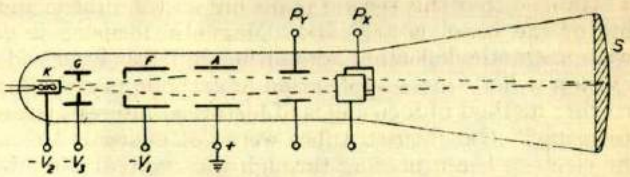


FIG. 15.12. CATHODE-RAY TUBE

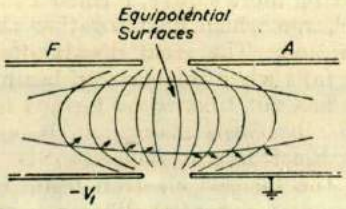


FIG. 15.13. ELECTRON LENS

will be less than the convergence, since the deflection due to a given radial force will be less in the shorter transit time. When the beam is correctly focused it converges into a small spot on the screen.

The design of electron lenses has advanced considerably, but is more difficult than that of optical lenses, and the performance of the electron lenses which are available is much inferior to that of optical lenses. In consequence, the electron beam must not deviate far from the direction of the axis or aberrations of the image will be produced. The last electrode of the lens system is normally at earth potential, and preceding electrodes and the cathode are made successively more negative. If this were not done the screen of the tube would have to be maintained at a positive potential equal to that of the last electrode of the lens. This leads to difficulties when an earthed body approaches the screen, since the potential of the latter is momentarily reduced by the capacitive effect and the electrons are repelled. This shows itself in practice as a defocusing of the beam when the tube face is touched

Electromagnetic focusing depends upon the fact that an electron which enters a constant magnetic field perpendicular to its path is deflected and moves in a circular path. The narrow axial beam of electrons produced by the electron gun is caused to enter a magnetic field parallel to the axis of the tube. Electrons which are travelling parallel to this axisare unaffected by the magnetic field, while electrons which have a component of velocity away from the axis move in a spiral path which eventually returns to the axis. Provided that the axial velocity of all the electrons starting from a single point (the diaphragm) is the same, they will all, no matter what their radial velocity, return to the axis together at another single point. If it is arranged that this second point lies on the tube of the screen, focusing of the beam is achieved. Magnetic focusing is generally used with magnetic deflection, and although it has been widely used in television it finds little application in cathode-ray oscillographs.

One other method of focusing is of historical interest, the so-called "gas-focusing." Gas-focused tubes were not evacuated completely, and the electron beam passing through the residual gas produced a large concentration of ions. These, being much heavier than the electrons, and moving more slowly, formed a more or less persistent core of positive charge which held together the electron beam by electrostatic attraction. The great disadvantage of this means of focusing is that it fails when the electron beam is rapidly deflected, since the ion-core has not time to be formed in each new position.

The *deflecting arrangements* may again be either electrostatic or electromagnetic. Electrostatic deflection is accomplished quite easily by causing the focused electron beam to pass between two plates between which a potential difference exists. The electrons are attracted to the positive plate, and repelled from the negative. The deflecting plates usually open out towards the screen, so that they do not intercept the deflected beam. Two sets of plates are fitted, producing deflections at right angles. The plates which produce a deflection to left or right of the screen, as normally mounted, are termed the X-plates, while those which produce a vertical deflection are termed the Y-plates. The deflecting plates are supplied with voltages which are balanced about earth, so that no further acceleration or deceleration of the electron beam leaving the electron lens is produced. Without this precaution the beam would be defocused by the deflecting voltage applied to the plates.

Electromagnetic deflection is achieved by producing a magnetic field perpendicular to the axis of the tube, by means of coils outside the tube. The electrons entering this field travel in circular paths, and emerge from the field with their paths deflected in a plane perpendicular to the deflecting field. Two fields at right angles are used to produce deflections in the X-direction (horizontally) and in the Y-direction (vertically). A greater deflection of the beam can be produced magnetically than electrostatically, and this enables

a shorter tube to be used. The power required is considerably greater, however, and the inductance of the deflecting coils leads to difficulty when deflections are very rapid. In consequence the majority of cathode-ray oscillographs use electrostatic deflection.

The beam of electrons, after deflection, travels along the c.r. tube until it impinges on the "screen." This is a coating of fluorescent materials on the end face of the tube, which shows a luminous spot at the point where the electron beam strikes it.

Theory of Electrostatic Deflection. If the mass of an electron is m, its charge q, the potential difference between the cathode and last anode V, the potential gradient between the deflecting plates E, and L_1 and L_2 are respectively the axial length of the deflecting

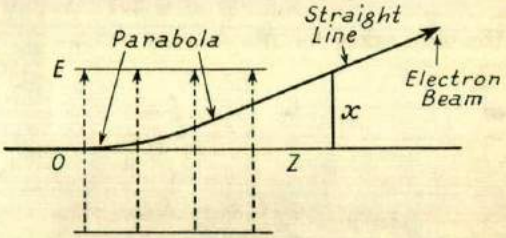


FIG. 15.14

plates and the distance from the centre of the deflecting plates to the screen, then the electron beam will be travelling at a velocity given by

$$qV = \frac{1}{2}mv^2 \quad . \quad . \quad . \quad . \quad (15.29)$$

Here v is the velocity of the beam, and the two sides of the equation give respectively the energy acquired by an electron in moving through the p.d. V, and the kinetic energy of the electron.

Referring to the Fig. 15.14, an electron entering the field between the plates will be accelerated with an acceleration f given by

$$mf = qE \quad . \quad . \quad . \quad . \quad . \quad (15.30)$$

The radial velocity of the electron will then be

$$v' = \int_0^t f \, . \, dt = \frac{qE}{m} \, . \, t \quad . \quad . \quad . \quad (15.31)$$

and its displacement from the axis,

$$x = \int_0^t v' \, . \, dt = \frac{qE}{2m} \, . \, t^2 \, . \quad . \quad . \quad (15.32)$$

Since the distance which the electron has moved along the axis is

$$z = vt = \sqrt{\frac{2qV}{m}} \cdot t$$

it follows that the path of the electron is a parabola. It is a property of the parabola that the tangent at any point (x, z) intersects the z-axis at the point $\frac{1}{2}z$. Thus if the tangent to the emergent beam is produced back it meets the axis at the mid-point of the plates.

The angle α which the tangent to the emergent beam makes with the axis is given by

$$\tan \alpha = \frac{v'}{v} = \frac{qE}{m} \cdot t \sqrt{\frac{m}{2qV}}$$

where t is the time taken for the electron to traverse the length L_1 of the plates, i.e.

$$t = \frac{L_1}{v} = L_1 \sqrt{\frac{m}{2qV}}$$

Thus

$$\tan \alpha = \frac{qEL_1 m}{m \ 2qV} = \frac{EL_1}{2V}$$

The radial distance of the beam from the axis when it arrives at the screen will be

$$y = L_2 \tan \alpha = \frac{EL_1 L_2}{2V} \quad . \quad . \quad (15.33)$$

all lengths being in metres, and E in volts per metre.

This simplified analysis has assumed that E is constant, i.e. it has neglected fringing of the field at the edge of the plates, and the non-uniformity of E which is produced when the plates are not parallel. In practice, the tube is calibrated with a steady potential applied between the plates, and the sensitivity in volts per cm is thus obtained under working conditions.

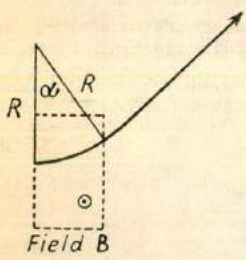


FIG. 15.15

Theory of Magnetic Deflection. An electron with charge q travelling with velocity v in the field B will experience a force qvB newtons, perpendicular to v and to B, in the sense given by the left-hand rule. The speed of the electron will evidently be unaffected, since the force acting upon it has no component in the direction of v. The electron thus has a constant velocity, and a constant acceleration perpendicular to its path. It follows that this path must be a

circle. To find the radius of the circular path, we equate the force on the electron to the centrifugal force corresponding to its motion—

$$qvB = \frac{mv^2}{R}$$

$$R = \frac{mv}{q \,.\, B}$$

Thus, if the length of the circular path is S, the angle α will be

$$\alpha = \frac{S}{R} = \frac{SqB}{mv}$$

$$= \frac{SqB}{m}\sqrt{\frac{m}{2qV}},$$

using the value of v already found on page 599. Note that $S \fallingdotseq L_1$.

The tangent to the emergent beam will not intersect the axis exactly at the mid-point of the field, so that strictly L_2 should not be measured from the mid-point. For small values of α, however, the error is negligible, and we may write for the deflection $y' = L_2 \tan \alpha = L_2\alpha$ approximately, when α is small, so that

$$y' = \sqrt{\frac{q}{2m}}\frac{BSL_2}{\sqrt{V}} \quad . \quad . \quad . \quad (15.34)$$

The value of $\frac{q}{m}$, the ratio of the charge to the mass of an electron, is $1{\cdot}77 \times 10^{11}$ coulomb/kg, so that $y' \fallingdotseq 3 \times 10^5 \frac{BSL_2}{\sqrt{V}}$ if V is in volts and all lengths are in metres. As before, the treatment is only approximate, since it neglects the variations of B at the edges of the field; and, as before, the tube is calibrated with a known steady voltage.

Electromagnetic deflection can sometimes be used with advantage when a deflection proportional to a current is required, but the deflecting coils require an appreciable power, and their inductance may interfere with the circuit in which the current is to be measured. It is usual practice to obtain a voltage proportional to the current, and to apply this, with amplification if necessary, to a tube with electrostatic deflection.

Post-deflectional Acceleration. When a c.r. tube is used to display high-frequency phenomena the transit time of the electron beam across the screen is very small, and it is essential, therefore, that a high anode voltage be used to give adequate spot intensity. It can be seen from the derived expressions on p. 600 that the voltage applied to the deflecting plates must be in proportion to the anode

voltage to attain a given sensitivity. Normally, in cathode-ray oscilloscopes, the deflection plates are driven through electronic amplifiers, and it is essential that these amplifiers should have a uniform frequency response. The higher the gain required in an electronic amplifier the more difficult it becomes to achieve uniform frequency response: this has, in the past, been a limiting factor in the overall frequency range which could be covered. The post-deflectional-acceleration tube has led to a marked improvement in the response of cathode-ray oscilloscopes. The electrons are accelerated after passing through the deflecting plates and therefore the beam deflection due to a given voltage on the plates is comparatively large. The essential requirement for satisfactory post-deflectional acceleration is that the beam should cross equipotential surfaces normally whatever its position inside the tube. One method of obtaining the correct field configuration is to surround the tapering glass envelope of the tube with a close-wound spiral resistance film. A large potential difference is applied between the ends of the spiral, giving a uniform gradient in field inside the tube. It is usual, with this type of tube, to restrict the operating area on the screen so that optimum linearity is achieved. Tube sensitivities as high as 1 cm per volt on the deflecting plates are attainable.

Recording with a C.R. Tube. Three different techniques are used for recording the information from a c.r. tube, according to the particular circumstances.

One method is to use a moving film on to which the image of the tube face is projected by a lens. The electron beam is deflected only in one plane, perpendicular to the direction of motion of the film, and the latter is either a continuous roll, or, for higher speeds, is wound upon a drum. If a drum is used, arrangements must generally be made to extinguish the beam until the record is required, and again when the length of film on the drum has been exposed. Alternatively, the shutter may be opened for one revolution of the drum at the appropriate time. If the beam remains at rest before and after the event to be recorded, however, the requirements are less critical, and it is only necessary to ensure that the shutter is opened in time to record the event, and not long enough for the record to be blurred by excess of light.

A second method is to project the face of the tube upon a stationary film, and to traverse the electron beam at a constant speed from left to right, at the same time as it is deflected vertically by the quantity to be measured. The horizontal movement, or "scan" is started at the beginning of the period which is to be recorded, and the beam moves only once across the screen. It is usual to extinguish the beam except during the period of the scan, since otherwise the intense spot stationary at the side of the screen "burns" the latter, i.e. renders it insensitive at that spot. Arrangements

must be made to start the scan at the correct moment, but the shutter of the camera is left open until the trace has been recorded. This method of "single-stroke" working can be used for visual observation if the material of the screen has some delay, i.e. if the screen continues to give out light at the points where it was excited for some time afterwards.

The third method of recording can be used only with cyclic phenomena, and it consists in scanning the electron beam horizontally for an integral number of cycles of the phenomenon to be observed, and then returning it very quickly to the left-hand side of the screen, where it starts on a second scan. Thus a stationary picture of one or more waves is presented. This picture can be observed, and the arrangement forms the commonest application of the c.r. tube, namely the c.r. oscilloscope. Photographic records can be obtained by exposing a film for a suitable length of time to the stationary picture.

The third method has the advantage that a very faint and consequently well-focused trace can be used, with a relatively long exposure, but the method is naturally limited to repetitive phenomena. Recording by the first method on continuous chart avoids the difficulties of synchronization, but at the expense of producing great lengths of unwanted record. The maximum chart speed for continuous feed is about 100 in./sec. With drum recording, chart speeds up to 1,000 in./sec can be obtained, and frequencies up to about 10 kc/s can be recorded. With single-stroke working the highest writing speed which (to the authors' knowledge) has been achieved is 20,000 km per second (or 20,000 mm per microsecond) (Ref. (58)), though this is not necessarily the limit.

Synchronization of the record is quite easily carried out, either for drum recording or single-stroke recording, if the event can be controlled. In this case the electron beam is started by a pulse which also starts the event. If the event cannot be controlled, it is possible to use the signal which it produces to start the electron beam. The difficulty is then to keep down the unavoidable delay in starting the beam, which causes the beginning of the event to go unrecorded. The techniques involved are complicated, and resemble those used in radar.

Time-base Circuits. It was seen above that it is often necessary to traverse the electron beam at a constant speed from left to right, either once or continually at definite intervals. This is accomplished by the time-base circuit. Only circuits suitable for electrostatic deflection will be considered here.

All time-base circuits depend fundamentally upon the charging of a capacitor. The voltage between the plates of a capacitor is

$$V = \frac{Q}{C}$$

so that the rate of change of voltage is

$$\frac{dv}{dt} = \frac{1}{C} \cdot \frac{dQ}{dt} = \frac{I}{C} \quad . \quad . \quad . \quad (15.35)$$

Thus if the current flowing into a capacitor is constant, the voltage across it increases uniformly with time. The design of time-base circuits is directed towards obtaining a current which remains constant whatever the charge on the capacitor.

If the time-base is to be repetitive, as is usual, arrangements must also be made to return the beam rapidly to the left-hand side of the screen as soon as it reaches the right-hand side. The frequency of the time-base will depend on the rate of charging of the capacitor, and this must be variable over a wide range, in order to

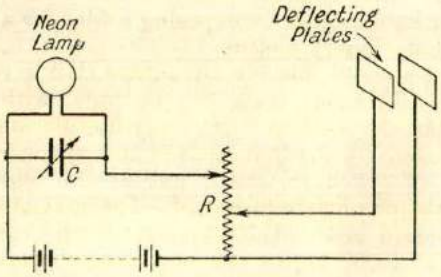


FIG. 15.16 CIRCUIT FOR OBTAINING THE TIME-BASE

present input wave-forms of various frequencies. In addition it is necessary to synchronize the time-base with the input, to ensure that each sweep of the time-base begins at exactly the same part of the cycle of the input wave. Without this precaution the presentation of the wave-form on the screen of the c.r.o. would move erratically from side to side, or drift gradually to left or right as the time-base frequency altered.

The simplest possible time-base is obtained by charging a capacitor through a high resistance from a high-voltage supply. The current through the resistor depends on the voltage across it, and if the maximum voltage across the capacitor is small compared with the supply voltage, this current will remain nearly constant during the charging. The rate of rise of the deflecting voltage will thus be constant and a linear time-base will result.

A method of producing a linear time-base, due to N. V. Kipping, employs a neon lamp with a variable capacitor across its terminals, the full connections being as shown in Fig. 15.16.

A neon lamp has the property of "striking" (i.e. commencing to pass a current) only when the voltage applied to it reaches a certain

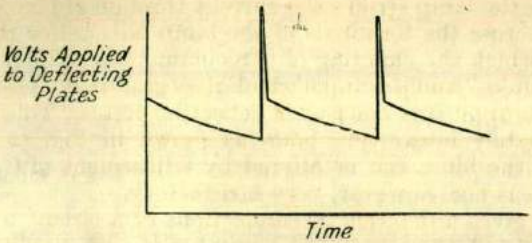
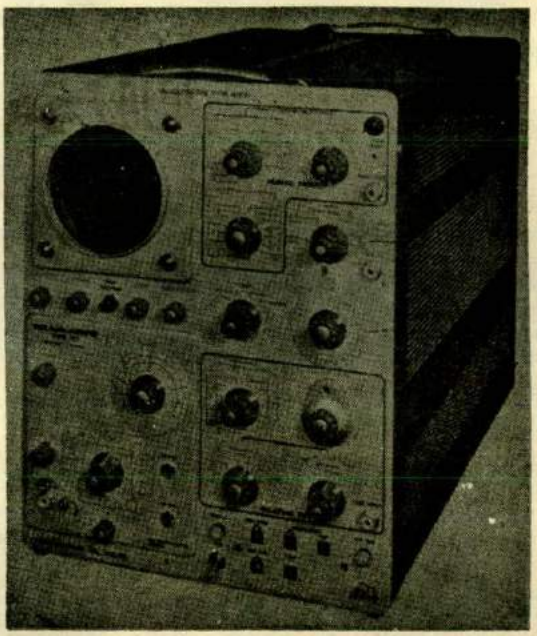


FIG. 15.17.



(E.M.I. Electronics Ltd.)

FIG. 15.18. E.M.I. CATHODE-RAY OSCILLOSCOPE

value and of "failing" (i.e. ceasing to pass a current) at a considerably lower voltage. Thus, referring to the figure, the capacitor C builds up to a voltage given by $e = E\,(1 - e^{-\frac{t}{RC}})$, where E is the voltage of the battery supplying the lamp (200 volts) and R is the resistance in series with the capacitor and lamp, before the lamp strikes. The time taken for it to build up to the striking voltage of the lamp obviously depends upon the value of C (for a given value

of R). When the lamp strikes the current through R increases, and the voltage across the terminals of the lamp falls below the failing value, after which the charging of C recommences. The neon lamp therefore "blinks" and a voltage which rises gradually and then falls rapidly is thus applied to one pair of deflecting plates. This produces an approximately linear time-base as shown in Fig. 15.17. The frequency of the blink can be altered by adjustment of C. Such a simple circuit is not, however, very satisfactory.

Fig. 15.19 gives a diagram of connections of a circuit utilizing a gas-filled triode which is typical of a number of such circuits devised

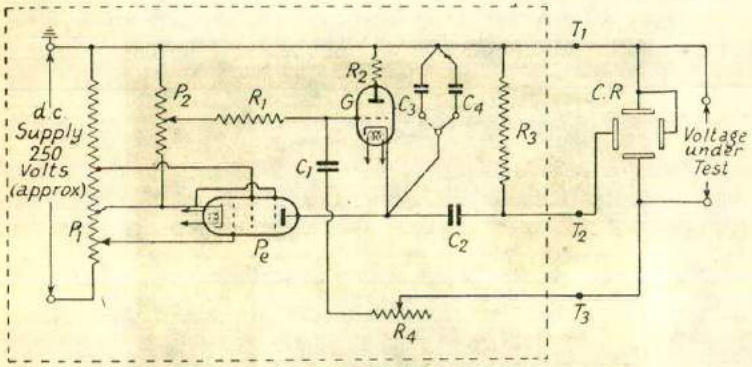


FIG. 15.19

for the production of a linear time-base. The time-base, or *sweep*, circuit itself is enclosed within the frame (shown dotted), the terminals for connection to the cathode-ray oscillograph being T_1, T_2, and T_3.

P_e is a pentode and G a gas-filled triode (the gas may be mercury vapour, neon, etc.). P_1 and P_2 are potential dividers. R_1 (about 50,000 ohms) is a resistor in the grid circuit of the triode and R_2 (about 500 ohms) is a limiting resistor in its anode circuit. R_3 (about 2 megohms) and C_2 (about 2 microfarads) are for the purpose of isolating d.c. potentials so that these do not produce a steady deflection of the cathode ray.

The action of the circuit is as follows. When the d.c. supply is first switched on, the capacitor C_4 (or C_3) acts instantaneously as a short-circuit and the full d.c. voltage is applied to the pentode. C_4 charges (at almost constant current due to the shape of the anode-current/anode-voltage characteristic of the pentode) and the voltage across it rises until it is sufficient for the triode G to discharge. The voltage required for discharge depends upon the grid bias of G and is therefore controlled by the potential divider P_2.

Again, the charging current of C_4 depends upon the grid bias of the pentode and is controlled by potential divider P_1. Now, as already shown, the rate of sweep on the oscillograph depends upon the rate of increase of voltage across the time-base pair of plates, i.e. upon the rate of increase of the voltage across C_4 and, since i is constant, $\frac{dv}{dt}$ (and therefore the rate of sweep) is constant, giving a linear time-base.

The rate of sweep depends upon the setting of P_1, and the amplitude of sweep upon the setting of P_2.

The capacitances C_3 and C_4 are usually of the order of 1 microfarad and 0·1 microfarad respectively.

An important feature of the circuit is the synchronizing branch circuit consisting of capacitor C_1 (about 0·1 microfarad) and R_4 (about 2 megohms). This enables the sweep frequency to be "locked" to some sub-multiple of the frequency of the voltage under test by applying to the grid circuit of the triode a small alternating voltage of this frequency. The result is to maintain the trace of the wave stationary upon the oscillograph screen.

Such a circuit gives almost constant velocity of sweep (i.e. linear time-base) with almost instantaneous return, for frequencies from (say) 0·5 per second to about 10,000 per second.

A great variety of c.r. oscilloscopes are made commercially by a large number of manufacturers. These instruments vary from relatively inexpensive ones having a limited frequency range to elaborate and costly special-purpose equipment. The oscilloscope Type W.M.16, made by E.M.I. Electronics, Ltd., and illustrated in Fig. 15.18, is an example of the most advanced practice in the design of general-purpose oscilloscopes. This instrument has a bandwidth from d.c. to 40 Mc/s, a sensitivity from 50 mV/cm to 50 V/cm covered in nine continuously variable ranges, and a rise-time to a step input of 10 millimicroseconds. The time-base has sweep times from 0·02 microsec/cm to 0·5 sec/cm. Facilities are provided to enable the initiation of the main sweep to be delayed so that the initial part of a transient may be studied. In addition, single-sweep operation is possible to permit photographic recording of non-recurrent phenomena. It is claimed that time and voltage measurement may be made with an accuracy of $\pm$ 3 per cent.

High-voltage C.R. Oscillographs. E. L. White (Ref. (59)) has described the use of h.v. oscillographic tubes for the study of fast transient phenomena, particularly in the work of the Electrical Research Association on high-voltage surges and on spark breakdown in small gaps. The paper shows examples of single-sweep traces recorded on a sealed-off tube at 10 kV (Fig. 15.20). One of the traces is of a 1,000 megacycles/sec oscillation.

The connections of the E.R.A. high-speed sweep circuit are given in Fig. 15.21. This circuit, described fully in Ref. (57), obtains the drive

from a pulse transformer. The generator valve is an EF55 having its base modified to give greater clearance around the anode pin.

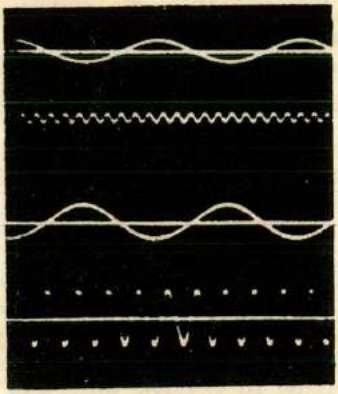


FIG. 15.20

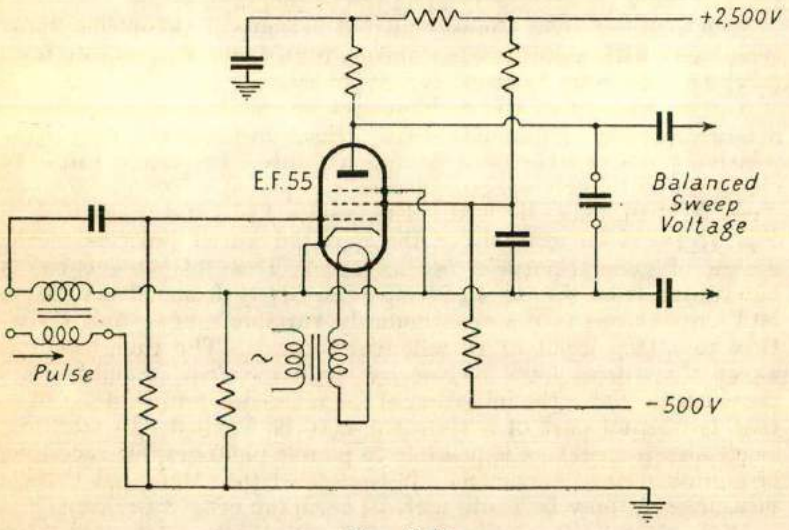


FIG. 15.21

The generator produces a maximum rate of change of sweep voltage of 4×10^{10} volts/sec.

The works mentioned in Refs. (7), (23), (24), (25) and (53) give, also, very full information regarding cathode-ray oscillographs and their applications, while Ref. (52) is concerned especially with the various forms of time-base.

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CHAPTER XVI

TRANSIENT PHENOMENA

PHENOMENA which are not simple periodic functions of time, and which usually exist for only a short time, are referred to as "transients" or "transient phenomena." We have already had several examples of such phenomena in previous chapters.

In this chapter, transients produced by the sudden opening or closing of various circuits will be considered.

Initial Conditions when a Steady Unidirectional Voltage is Impressed upon a Circuit. When such a voltage is applied to a closed circuit the current which will *eventually* flow in the circuit will, of course, be given by $\frac{\text{Applied voltage}}{\text{Resistance of the circuit}}$. At the instant of switching the voltage on to the circuit, however, the current is zero. The initial, or transient, conditions with which we are now concerned are those which exist while the current is changing from its initial value to its final steady value. These conditions depend upon the resistance, inductance, and capacitance of the circuit under consideration, and various possible cases will now be discussed.

Resistance Only. If a steady voltage E, from a battery (say), is applied to a circuit, of resistance R, whose inductance and capacitance are negligibly small, the current in the circuit rises to the value $\frac{E}{R}$ instantaneously, without any transient conditions.

Resistance and Inductance in Series. Let E be the impressed steady voltage, and R and L the resistance and inductance of the circuit; then, when the voltage is switched on to the circuit, we have

$$E = Ri + L\frac{di}{dt} \quad . \quad . \quad . \quad . \quad (16.1)$$

where i is the instantaneous value of the current in the circuit at any time t. The sum of the voltage drop due to the resistance of the circuit and the back e.m.f. of self-induction is equal and opposite to the impressed voltage at all instants.

The instantaneous current i can be obtained as follows—

$$E - Ri = L\frac{di}{dt}$$

or

$$\frac{dt}{L} = \frac{di}{E - Ri}$$

Integrating each side we have

$$\int \frac{dt}{L} = \int \frac{di}{E - Ri}$$

$$\frac{t}{L} = \frac{-1}{R} \log_e (E - Ri) + A$$

where A is a constant.

$$-\frac{Rt}{L} = \log_e (E - Ri) - AR$$

$$= \log_e (E - Ri) + \log_e B$$

where B is another constant such that $\log_e B = -AR$.

Hence $$e^{-\frac{Rt}{L}} = B(E - Ri)$$

B can be found from the conditions at the instant of switching. Thus, when $t = 0$, $i = 0$, so that

$$e^0 = B(E) \text{ or } B = \frac{1}{E}$$

Substituting this value for B we have

$$e^{-\frac{Rt}{L}} = \frac{1}{E}(E - Ri)$$

so that $$i = \frac{E}{R}(1 - e^{-\frac{Rt}{L}}) \quad . \quad . \quad . \quad . \quad (16.2)$$

This means that the current rises according to an exponential law, the current/time graph being as shown in Fig. 16.1, curve (1). The maximum, or final, value is attained when $t = \infty$

Then

$$i = \frac{E}{R}(1 - e^{-\infty}) = \frac{E}{R}$$

Thus, if a circuit possesses inductance the current only reaches its steady value—as given by Ohm's law—after infinite time, although it may rise to within a very small percentage of this value in quite a short period of time.

Example. Calculate the value of the current after 1 second, when a voltage of 100 volts is applied to a circuit having a resistance of 20 ohms and an inductance of 100 millihenries.

$$i = \frac{100}{20}(1 - e^{-\frac{20}{0{\cdot}1}})$$

$$= 5\left(1 - \frac{1}{e^{200}}\right) = 5\left(1 - \frac{1{\cdot}38}{10^{87}}\right) = 5$$

In this example, the ratio $\frac{R}{L}$ is large, and the ratio $\frac{L}{R}$—which is called the "time-constant" of the circuit—is thus small. The current therefore very rapidly attains a value which, for all practical purposes, is the same as its final steady value of 5 amp.

If the resistance had been 0·2 ohm instead of 20 ohms, the time-constant would have been 0·5 instead of 0·005, and the current, after 1 second, would have been

$$i = \frac{100}{0{\cdot}2}\left(1 - e^{-\frac{0{\cdot}2}{0{\cdot}1}}\right) = 500\left(1 - \frac{1}{e^2}\right)$$
$$= 432{\cdot}3 \text{ amp}$$

which is appreciably less than its final value of 500 amp.

Obviously, in all such circuits, after a time $\frac{L}{R}$ seconds the current is given by

$$i = \frac{E}{R}(1 - e^{-1}) = 0{\cdot}6322\frac{E}{R}$$

Transient Effect when the Impressed Voltage is Removed. If the impressed voltage is suddenly removed from the circuit (without opening the circuit) when a current $\frac{E}{R}$ amp is flowing in it (a considerable time having passed since the instant of switching on) the current does not disappear instantaneously, just as it did not rise to its final steady value instantaneously. The energy which was absorbed by the magnetic field of the inductive part of the circuit during the initial rise of the current to its maximum value, and which has been stored in the field while the steady current has been flowing, is now discharged again and maintains a current for some time after the removal of the impressed voltage.

The current variation with time, after removing the voltage, is obtained as below. The voltage equation of the circuit is now

$$L\frac{di}{dt} + Ri = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (16.3)$$

$$\therefore -\frac{dt}{L} = \frac{di}{Ri}$$

Integrating, we have

$$-\int\frac{dt}{L} = \int\frac{di}{Ri}$$

$$-\frac{t}{L} = \frac{1}{R}\log_e i + A'$$

where A' is a constant.

$$\therefore e^{-\frac{Rt}{L}} = B'i$$

where B' is a constant such that $\log B' = A'R$.

If $$i = \frac{E}{R} \text{ when } t = 0,$$

$$e^0 = \frac{B'E}{R} \text{ or } B' = \frac{R}{E}$$

Hence, the law of die-away of the current is

$$i = \frac{E}{R} e^{-\frac{Rt}{L}} \quad . \quad . \quad . \quad . \quad (16.4)$$

This curve is shown in Fig. 16.1, curve (2).

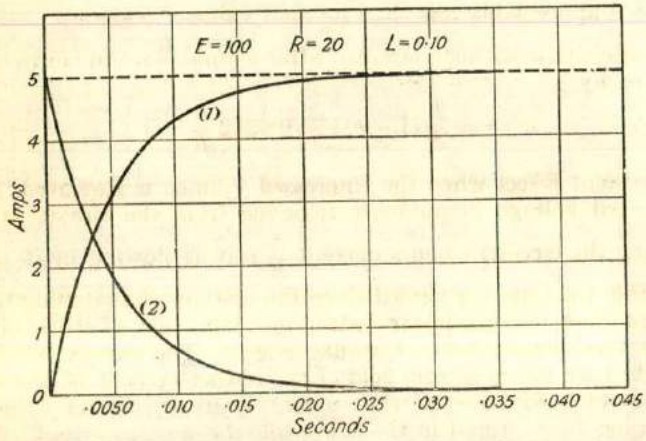


FIG. 16.1. CURVES OF RISE AND DECAY OF CURRENT IN AN INDUCTIVE CIRCUIT

Circuit containing Resistance, Inductance, and Capacitance. Suppose a steady voltage E is suddenly switched on to a circuit containing resistance R, inductance L, and capacitance C. The voltage equation of the circuit when current of instantaneous value i begins to flow is

$$Ri + L\frac{di}{dt} + \frac{\int idt}{C} = E \quad . \quad . \quad . \quad (16.5)$$

The expression $\int idt$ represents the quantity of electricity which has been given to the capacitor during the first t seconds after switching on the voltage, t being variable. Hence $\frac{\int idt}{C}$ represents the voltage across the capacitor terminals after t seconds.

Differentiating both sides of the above equation with respect to t we have

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

or, rearranging,

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0 \quad . \quad . \quad . \quad (16.6)$$

It has been shown already that the solution of an equation of this form is

$$i = Ae^{m_1 t} + Be^{m_2 t}$$

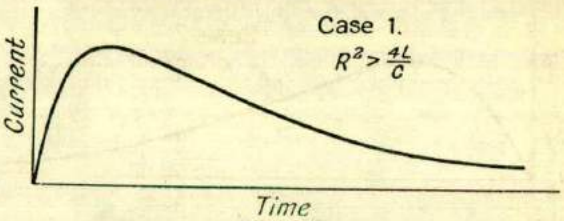


FIG. 16.2

where A and B are constants, and m_1 and m_2 are the roots of the auxiliary equation $Lm^2 + Rm + \frac{1}{C} = 0$.

Thus,

$$m_1 = \frac{-R + \sqrt{R^2 - \frac{4L}{C}}}{2L}$$

and

$$m_2 = \frac{-R - \sqrt{R^2 - \frac{4L}{C}}}{2L}$$

The final form of the expression for the current i depends upon whether the term under the square-root sign is positive, zero, or negative—i.e. upon whether R^2 is greater than, equal to, or less than $\frac{4L}{C}$. There are, therefore, three cases to be considered.

Case 1. $R^2 > \frac{4L}{C}$.

In this case, m_1 and m_2 are both real quantities. To find A and B, consider the initial conditions of the circuit. Suppose $i = 0$ when $t = 0$ and that the capacitor is uncharged (i.e. $\int idt = 0$). Then, from the original voltage equation, when $t = 0$,

$$L\frac{di}{dt} = E \text{ or } \frac{di}{dt} = \frac{E}{L}$$

From the current equation we have

$$0 = Ae^0 + Be^0$$
$$= A + B$$
$$\therefore A = -B$$

also

$$\frac{di}{dt} = Am_1e^0 + Bm_2e^0 = \frac{E}{L}$$

$$\therefore Am_1 - Am_2 = \frac{E}{L}$$

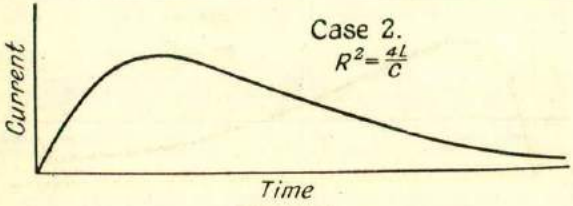


FIG. 16.3

or

$$A = \frac{E}{L(m_1 - m_2)}$$

and

$$B = \frac{-E}{L(m_1 - m_2)}$$

Hence

$$i = \frac{E}{L(m_1 - m_2)}(e^{m_1 t} - e^{m_2 t}) \text{ amp}$$

Substituting for m_1 and m_2 in terms of R, L, and C, we have

$$i = \frac{E}{\sqrt{R^2 - \frac{4L}{C}}}\left[e^{\left(\frac{-R + \sqrt{R^2 - \frac{4L}{C}}}{2L}\right)t} - e^{\left(\frac{-R - \sqrt{R^2 - \frac{4L}{C}}}{2L}\right)t}\right] \qquad (16.7)$$

Case 2. $R^2 = \frac{4L}{C}$.

The most general solution of the equation for the current i is now

$$i = (A + Bt)e^{-\frac{Rt}{2L}}* \qquad (16.8)$$

If, initially, $i = 0$ and $\int idt = 0$ when $t = 0$, then, when $t = 0$,

$$\frac{di}{dt} = \frac{E}{L}$$

* See Piaggio, *Differential Equations*.

Thus $$A = 0$$

and $$\frac{di}{dt} = B = \frac{E}{L}$$

$$\therefore i = \frac{E}{L} te^{-\frac{Rt}{2L}} \quad . \quad . \quad . \quad . \quad (16.9)$$

Case 3. $R^2 < \frac{4L}{C}$

The roots m_1 and m_2 are then imaginary and can be written

$$m_1 = -k_1 + jk_2$$
$$m_2 = -k_1 - jk_2$$

where $j = \sqrt{-1}$, $k_1 = \frac{R}{2L}$ and $k_2 = \frac{\sqrt{\frac{4L}{C} - R^2}}{2L}$.

The method of deriving the solution in this case has been previously described (page 277), and applying this technique, the expression for the current is—

$$i = Fe^{-\frac{R}{2L}t} \sin\left(\frac{\sqrt{\frac{4L}{C} - R^2}}{2L} t + \beta\right) \quad . \quad (16.10)$$

where F and β are constants which may be determined from the initial conditions.

At $t = 0$, $i = 0$ and $\int i dt = 0$; hence $\frac{di}{dt} = \frac{E}{L}$ when $t = 0$.

From the final equation for i, when $t = 0$

$$0 = F \sin \beta$$

Therefore either $F = 0$ or $\sin \beta = 0$.

Now F cannot be zero since this would cause the current to be zero for all values of t, which it is not. Thus $\beta = 0$. Now, when $t = 0$, by differentiation,

$$\frac{di}{dt} = F \frac{\sqrt{\frac{4L}{C} - R^2}}{2L} = \frac{E}{L}$$

Hence $$F = \frac{2E}{\sqrt{\frac{4L}{C} - R^2}}$$

Substituting for F and β in Equation (16.10) and rearranging gives

$$i = \frac{E}{\sqrt{\frac{L}{C} - \frac{R^2}{4}}} e^{-\frac{R}{2L}t} \sin\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\right)t \quad . \quad (16.11)$$

The general shapes of the current/time graphs for these three cases are shown in Figs. 16.2, 16.3, and 16.4. The current, in the first two cases, is non-oscillatory, and in the third case oscillatory. In all three cases the final value of the current will, of course, be

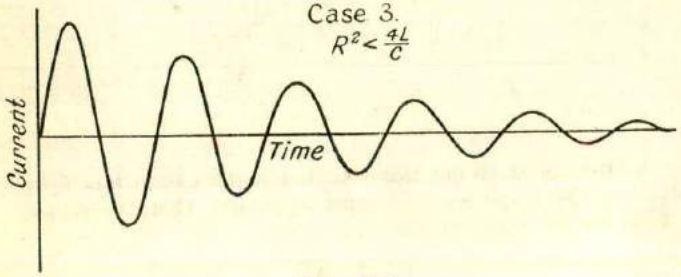


FIG. 16.4

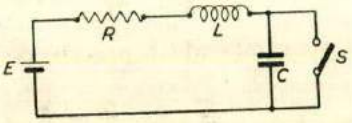


FIG. 16.5

zero, since there is a capacitor in the circuit and the applied voltage is unidirectional.

Other Initial Conditions. It has been assumed in the case just considered that the capacitor is initially uncharged. If the capacitor is charged, before switching, to a voltage V, Equation (16.5) becomes, at the instant $t = 0$,

$$L\frac{di}{dt} \pm V = E$$

and the initial conditions are, at $t = 0$, $i = 0$ and $\frac{di}{dt} = \frac{E \mp V}{L}$.

The sign of V will depend upon the polarity of the initial charge, being positive if V is opposed to E.

The Case where Part of a Circuit is Switched. Referring to Fig. 16.5,

the switch S across the capacitor C is normally closed. A d.c. voltage E is applied to the circuit, and when the current in R and L has reached its steady value, the switch S is opened. Given that $R^2 < \frac{4L}{C}$, it is required to find the current in the circuit after the switch S is opened.

The voltage equation for the circuit when S is open is

$$Ri + L\frac{di}{dt} + \int\frac{i\,dt}{C} = E \quad . \quad . \quad . \quad (16.12)$$

Since $R^2 < \frac{4L}{C}$ the oscillatory solution is required—

$$i = Fe^{-\frac{R}{2L}t} \sin\left(\frac{\sqrt{\frac{4L}{C} - R^2}}{2L}t + \beta\right) \quad . \quad (16.13)$$

At the instant of switching, when $t = 0$, i is the steady current and is equal to $\frac{E}{R}$. The capacitor is uncharged so $\int i\,dt = 0$, and by insertion of these two conditions in Equation (16.12) it is apparent that $\frac{di}{dt} = 0$ when $t = 0$.

We have, therefore, when $t = 0$,

$$\frac{E}{R} = F \sin \beta \quad . \quad . \quad . \quad . \quad (16.14)$$

and

$$\frac{di}{dt} = -\frac{R}{2L}F \sin \beta + F\frac{\sqrt{\frac{4L}{C} - R^2}}{2L} \cos \beta = 0 \quad . \quad (16.15)$$

From Equation (16.15),

$$\tan \beta = \frac{\sqrt{\frac{4L}{C} - R^2}}{R} \quad \text{or} \quad \beta = \tan^{-1}\frac{\sqrt{\frac{4L}{C} - R^2}}{R}$$

and, from the properties of a right-angled triangle, $\sin \beta = \sqrt{1 - \frac{CR^2}{4L}}$

Substituting in Equation (16.14),

$$F = \frac{E}{R\sqrt{1 - \frac{CR^2}{4L}}}$$

Thus the two constants have been evaluated, and if they are substituted in the general expression for the current, the solution will be found to be

$$i = \frac{E}{R} e^{-\frac{R}{2L}t} \left[\frac{R}{\sqrt{\frac{4L}{C} - R^2}} \sin at + \cos at \right]$$

where

$$a = \frac{\sqrt{\frac{4L}{C} - R^2}}{2L}$$

Circuits Containing a Number of Meshes. The transient response of networks containing a number of meshes can be obtained by applying Kirchhoff's laws to the network in terms of instantaneous

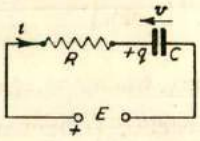


FIG. 16.6

values. In this manner a differential equation may be derived whose solution gives the required response. Insertion of the initial conditions in the solution enables the constants to be determined.

It is essential when dealing with such networks that the normal conventions for current and voltage directions be strictly adhered to. These conventions are as follows.

CIRCUITS CONTAINING INDUCTANCE AND RESISTANCE. The self-induced e.m.f. is taken as acting in the direction of the current but is written with a negative sign, i.e. $e = -L\frac{di}{dt}$. The equation for the circuit is

$$E + e = iR \quad \text{or} \quad E - L\frac{di}{dt} = iR$$

Clearly, when the current is rising, $\frac{di}{dt}$ is positive and the induced e.m.f. opposes the current rise.

CIRCUITS CONTAINING CAPACITANCE AND RESISTANCE. Consider Fig. 16.6 in which the capacitor is charging; then

$$q = Cv = \int i\,dt$$

and

$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

The equation for the circuit is

$$E = iR + v$$

or

$$E = CR\frac{dv}{dt} + v$$

Fig. 16.7 shows the case where the capacitor is discharging, when

$$v = \frac{q}{C} \text{ and } i = -\frac{dq}{dt} = -C\frac{dv}{dt}$$

The equation for the circuit is

$$v = iR$$

or

$$v = -CR\frac{dv}{dt}$$

Application to a Network Containing Two Meshes. Referring to Fig. 16.8, C_1 is initially charged and a potential difference V volts

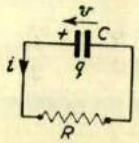


Fig. 16.7

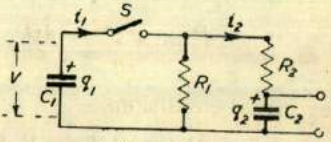


Fig. 16.8

exists across its terminals. C_2 is initially uncharged and an expression is required for the voltage appearing across C_2 after the switch S is closed. The instantaneous current directions after closure of S are inserted as shown and, applying Kirchhoff's laws, the following equations may be derived—

$$\frac{q_1}{C_1} = (i_1 - i_2)R_1 = \left(-\frac{dq_1}{dt} - \frac{dq_2}{dt}\right)R_1 \quad . \quad (16.16)$$

and

$$\frac{q_1}{C_1} = i_2R_2 + \frac{q_2}{C_2} = \frac{dq_2}{dt}R_2 + \frac{q_2}{C_2} \quad . \quad . \quad (16.17)$$

It is convenient when dealing with such equations to use the operator D; thus $\frac{dq}{dt}$ may be written as $D \,.\, q$.

After effecting this alteration and rearranging, Equation (16.16) becomes

$$0 = (1 + C_1R_1D)q_1 + C_1R_1Dq_2 \quad . \quad . \quad (16.18)$$

and Equation (16.17) becomes

$$0 = -q_1 + \left(\frac{C_1}{C_2} + C_1R_2D\right)q_2 \quad . \quad . \quad (16.19)$$

These two equations may be treated as linear simultaneous differential equations. Multiplying Equation (16.19) by $(1 + C_1R_1D)$ and adding the result to Equation (16.18) gives

$$0 = C_1R_1Dq_2 + \left[\frac{C_1}{C_2} + C_1R_2D\right][1 + C_1R_1D]q_2$$

Cancelling out the common C_1 and rearranging gives

$$0 = C_1R_1R_2D^2q_2 + \left(R_1D + R_2D + \frac{C_1R_1D}{C_2}\right)q_2 + \frac{q_2}{C_2}$$

Divide by $C_1R_1R_2$:

$$0 = D^2q_2 + \left(\frac{1}{C_1R_2} + \frac{1}{C_1R_1} + \frac{1}{C_2R_2}\right)Dq_2 + \frac{q_2}{C_1C_2R_1R_2} \quad (16.20)$$

Write as

$$0 = D^2q_2 + P \,.\, Dq_2 + Qq_2 \quad . \quad . \quad (16.21)$$

The solution is $q_2 = Ae^{m_1t} + Be^{m_2t}$

where $m_1 = -\dfrac{P + \sqrt{P^2 - 4Q}}{2}$ and $m_2 = -\dfrac{P - \sqrt{P^2 - 4Q}}{2}$

Inserting initial conditions—

$$\text{At } t = 0,\ q_2 = 0, \text{ i.e. } A + B = 0$$

$$\therefore A = -B$$

Also at $t = 0$, since C_2 is uncharged, it acts instantaneously as a short circuit, hence

$$i_2 = \frac{V}{R_2} = \frac{dq_2}{dt}$$

Differentiating the general solution,

$$\left(\frac{dq_2}{dt}\right)_{t=0} = m_1A + m_2B = \frac{V}{R_2}$$

from which, by substituting $A = -B$,

$$B = \frac{-V}{R_2(m_1 - m_2)} \text{ and } A = \frac{+V}{R_2(m_1 - m_2)}$$

The solution for q_2 is then

$$q_2 = \frac{V}{R_2(m_1 - m_2)}\left[e^{m_1t} - e^{m_2t}\right] \quad . \quad . \quad (16.22)$$

and the voltage across C_2 is given by

$$v_2 = \frac{q_2}{C_2} = \frac{V}{C_2R_2(m_1 - m_2)}\left[e^{m_1t} - e^{m_2t}\right] \quad . \quad (16.23)$$

It is interesting to note that the solution is the difference of two exponential decay terms, both m_1 and m_2 being negative. The ·age across C_2 is a typical impulse wave-form as discussed in .apter XI. The circuit shown is the equivalent of a simple impulse generator.

Circuit containing Resistance and Inductance, when the Impressed Voltage is Alternating. Let R and L be the resistance and inductance of the circuit and let the impressed voltage be given by $e = E_{max} \sin \omega t$

The voltage equation for the circuit is then

$$e = Ri + L\frac{di}{dt} = E_{max} \sin \omega t \quad . \quad . \quad (16.24)$$

or
$$\frac{di}{dt} + \frac{Ri}{L} = \frac{E_{max}}{L} \sin \omega t^*$$

Hence,
$$i\, e^{\int \frac{R}{L} dt} = \int \left(\frac{E_{max}}{L} \sin \omega t \right) e^{\int \frac{R}{L} dt} + A$$

or
$$i\, e^{\frac{Rt}{L}} = \frac{E_{max}}{L} \int e^{\frac{Rt}{L}} \sin \omega t + A$$

$$\therefore i = \frac{E_{max}}{L} \cdot e^{-\frac{Rt}{L}} \int e^{\frac{Rt}{L}} \sin \omega t + A e^{-\frac{Rt}{L}} \quad . \quad (16.25)$$

The term
$$\int e^{\frac{Rt}{L}} \sin \omega t = e^{\frac{Rt}{L}} \frac{\left[\frac{R}{L} \sin \omega t - \omega \cos \omega t \right]}{\frac{R^2}{L^2} + \omega^2}$$

$$\therefore i = \frac{E_{max}}{L\left[\frac{R^2}{L^2} + \omega^2 \right]} \left(\frac{R}{L} \sin \omega t - \omega \cos \omega t \right) + A e^{-\frac{Rt}{L}}$$

* This differential equation is of the form

$$\frac{dy}{dx} + Py = M$$

which is solved by first multiplying each side by $e^{\int P dx}$ and then integrating each side with respect to x, when we obtain the equation

$$y e^{\int P dx} = \int M e^{\int P dx} + A$$

where A is a constant.

$$= \frac{E_{max}}{L\left[\frac{R^2}{L^2} + \omega^2\right]} \frac{(R \sin \omega t - \omega L \cos \omega t)}{L} + Ae^{-\frac{Rt}{L}}$$

$$= \frac{E_{max}}{R^2 + \omega^2 L^2} (R \sin \omega t - \omega L \cos \omega t) + Ae^{-\frac{Rt}{L}}$$

$$= \frac{E_{max}}{\sqrt{R^2 + \omega^2 L^2}} \left[\frac{R}{\sqrt{R^2 + \omega^2 L^2}} \sin \omega t - \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} \cos \omega t \right] + Ae^{-\frac{Rt}{L}}$$

or
$$i = I_{max} \sin (\omega t - \alpha) + Ae^{-\frac{Rt}{L}} \quad . \quad . \quad . \quad . \quad (16.26)$$

where $\alpha = \tan^{-1} \frac{\omega L}{R}$ and I_{max} is the maximum value of the current under steady conditions and equals $\frac{E_{max}}{\sqrt{R^2 + \omega^2 L^2}}$.

The first term of the above expression for i obviously represents the current which flows under steady conditions (i.e. after the voltage has been switched on for some time). The second term represents a transient current which disappears comparatively rapidly, but which exists for a short time after closing the switch. This transient current is the link between the conditions existing at the instant of switching and the steady-state current. It is possible to deduce the value of A simply from an understanding of the behaviour of the circuit. The current through the inductor in this case cannot change instantaneously, and if we consider switching to occur at $t = 0$, then A is equal to the difference between the current at the instant of switching and the instantaneous value of the steady-state current for the particular point on the voltage wave at which switching occurs. In the case where the circuit is switched on to the supply, the current at the instant of switching is zero, but in the case where part of a circuit is shorted by a switch, then the initial current is finite.

It is possible to arrange the switching instant always to occur at $t = 0$ by writing $E_{max} \sin (\omega t + \theta)$ for $E_{max} \sin \omega t$; the current in the circuit is then

$$i = I_{max} \sin(\omega t + \theta - \alpha) + Ae^{-\frac{R}{L}t} \quad . \quad (16.27)$$

and the value of θ may be chosen so that switching occurs at any required point on the voltage wave.

When the simple R and L circuit is switched on to the supply the initial current is zero, i.e. $i = 0$ when $t = 0$ and we have

$$0 = I_{max} \sin (\theta - \alpha) + A$$

The general equation of the current is therefore

$$i = I_{max} \sin (\omega t + \theta - \alpha) - I_{max} e^{-\frac{R}{L}t} \sin (\theta - \alpha) \quad (16.28)$$

The general shape of the current/time graph is as shown in Fig. 6.9, from which it can be seen that the effect of the transient term is to produce dissymmetry in the first few cycles of the current wave; this dissymmetry rapidly dies away so that the wave approaches very closely to its normal sinusoidal form after a few cycles.

If switching occurs when $\theta = \alpha$ the transient term is zero.

When $\theta - \alpha = \frac{\pi}{2}$ the transient term in Equation (16.28) is initially equal to I_{max}, the current wave is displaced an amount

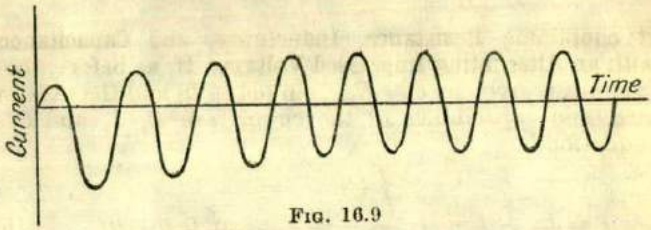


FIG. 16.9

equal to its peak value, and the peak current in the circuit, half a cycle later, is nearly equal to $-2I_{max}$, provided that the transient term has not decayed appreciably in the half-cycle. This is known as the "current-doubling" effect.

Circuit containing Resistance and Capacitance Only. If the same alternating voltage $e = E_{max} \sin(\omega t + \theta)$ is impressed upon a circuit having resistance R and capacitance C, but no inductance, the voltage equation is

$$e = Ri + \frac{\int i dt}{C} = E_{max} \sin(\omega t + \theta) \quad . \quad (16.29)$$

By differentiation we have

$$\omega E_{max} \cos(\omega t + \theta) = R\frac{di}{dt} + \frac{i}{C}$$

This is an equation of the same form as that obtained when resistance and inductance were contained in the circuit, as in the previous section.

It can be solved in the same way, giving

$$i = \frac{E_{max}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \sin(\omega t + \theta + \alpha) + Ae^{-\frac{t}{RC}} \quad . \quad (16.30)$$

or

$$i = I_{max} \sin(\omega t + \theta + \alpha) + Ae^{-\frac{t}{RC}} \quad . \quad . \quad (16.31)$$

If the capacitor is uncharged at the instant of switching, $\int i dt = 0$ when $t = 0$, and inserting this initial condition into Equation (16.29) gives the condition

$$i = \frac{E_{max}}{R} \sin \theta, \text{ when } t = 0$$

Substituting this condition in Equation (16.31) gives the value of the constant

$$A = \frac{E_{max}}{R} \sin \theta - I_{max} \sin (\theta + \alpha)$$

Circuit containing Resistance, Inductance, and Capacitance in Series, with an Alternating Impressed Voltage. If, as before, the impressed voltage is given by $e = E_{max} \sin (\omega t + \theta)$ and the resistance, inductance, and capacitance of the circuit are R, L, and C, the voltage equation is

$$e = Ri + L\frac{di}{dt} + \frac{\int i dt}{C} = E_{max} \sin (\omega t + \theta) \quad . \quad (16.32)$$

Differentiating, we have

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = \omega E_{max} \cos (\omega t + \theta)$$

The solution of this equation will give an expression for i which is the sum of a particular integral and the complementary function, the latter being obtained by solving the equation

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

The solution of this equation has already been shown to be

$$i = Ae^{m_1 t} + Be^{m_2 t}$$

where A and B are constants and m_1 and m_2 are the roots of the equation

$$Lm^2 + Rm + \frac{1}{C} = 0 \quad \text{(see page 615)}$$

As regards the particular integral, by analogy with the differential equation for the motion of the moving system of a vibration galvanometer, given on page 277, we have the solution

$$i = \frac{\omega E_{max}}{\sqrt{\left(\frac{1}{C} - L\omega^2\right)^2 + R^2\omega^2}} \cos (\omega t + \theta - \beta) \quad . \quad (16.33)$$

where $\beta = \tan^{-1} \dfrac{R\omega}{\frac{1}{C} - L\omega^2} = \tan^{-1} \dfrac{R}{\frac{1}{\omega C} - \omega L}$

This may be written as

$$i = \frac{E_{max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t + \theta + \alpha)$$

where $\tan \alpha = \dfrac{\frac{1}{\omega C} - \omega L}{R}$, i.e. $\alpha = 90 - \beta$,

or, otherwise,

$$i = \frac{E_{max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t + \theta - \gamma)$$

where $\gamma = \tan^{-1} \dfrac{\omega L - \frac{1}{\omega C}}{R} = -\alpha$.

The complete equation for the current i is therefore

$$i = Ae^{m_1 t} + Be^{m_2 t} + \frac{E_{max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t + \theta - \gamma)$$

or
$$i = Ae^{m_1 t} + Be^{m_2 t} + I_{max} \sin(\omega t + \theta - \gamma) \quad . \quad (16.34)$$

The last term obviously represents the current in the circuit when the transient current—represented by the first two terms—has died away.

Now,
$$m_1 = \frac{-R + \sqrt{R^2 - \frac{4L}{C}}}{2L}$$

and
$$m_2 = \frac{-R - \sqrt{R^2 - \frac{4L}{C}}}{2L}$$

so that the final form of the equation for the current depends, as before, upon whether R^2 is greater than, equal to, or less than, $\frac{4L}{C}$. These three cases will now be considered in turn.

Case 1. $R^2 > \dfrac{4L}{C}$.

Both m_1 and m_2 are now real quantities, and

$$i = Ae^{m_1 t} + Be^{m_2 t} + I_{max} \sin(\omega t + \theta - \gamma)$$

To evaluate the constants A and B, the initial conditions in the circuit must be known.

The switch is closed when $t = 0$, and the angle θ may be chosen so that switching occurs at any desired point on the voltage wave. Since the circuit contains inductance the current must be zero at the instant of closing the switch, so that $i = 0$ when $t = 0$. Suppose there is no initial charge in the capacitor so that $\int i dt = 0$ when $t = 0$. The use of the second initial condition as it stands involves integration of the general solution; this may be avoided by inserting these conditions into Equation (16.32), giving

$$L\frac{di}{dt} = E_{max} \sin \theta \text{ when } t = 0 \quad . \quad . \quad (16.35)$$

which is a simpler condition.

We have then from Equation (16.34), when $t = 0$,

$$0 = A + B + I_{max} \sin (\theta - \gamma) \quad . \quad . \quad (16.36)$$

and, differentiating the general solution,

$$\frac{di}{dt} = m_1 A e^{m_1 t} + m_2 B e^{m_2 t} + \omega I_{max} \cos (\omega t + \theta - \gamma)$$

but (from Equation (16.35))

$$\frac{di}{dt} = \frac{E_{max}}{L} \sin \theta \text{ when } t = 0$$

hence

$$\frac{E_{max}}{L} \sin \theta = m_1 A + m_2 B + \omega I_{max} \cos (\theta - \gamma) \quad . \quad (16.37)$$

The values of A and B can now be derived from Equations (16.36) and (16.37).

Case 2. $R^2 = \dfrac{4L}{C}$

In this case $m_1 = m_2 = -\dfrac{R}{2L}$ and the most general expression for the transient, when these roots are equal, is

$$e^{-\frac{R}{2L}t} (A + Bt)$$

Thus the equation for i is now

$$i = e^{-\frac{R}{2L}t} (A + Bt) + I_{max} \sin (\omega t + \theta - \gamma) \quad . \quad (1638)$$

If the switch is closed at $t = 0$ when $e = E_{max} \sin \theta$, and if the capacitor is initially uncharged, the initial conditions are as before.

At $t = 0$, $i = 0$, and $L\frac{di}{dt} = E_{max} \sin \theta$. Therefore

$$0 = A + I_{max} \sin (\theta - \gamma) \tag{16.39}$$

and, by differentiating the expression for i,

$$\frac{di}{dt} = -\frac{R}{2L} e^{-\frac{R}{2L}t} (A + Bt) + Be^{\frac{R}{2L}t} + \omega I_{max} \cos (\omega t + \theta - \gamma)$$

Substituting the condition $\frac{di}{dt} = \frac{E_{max}}{L} \sin \theta$ at $t = 0$ gives

$$\frac{E_{max}}{L} \sin \theta = -\frac{R}{2L} A + B + \omega I_{max} \cos (\theta - \gamma) . \tag{16.40}$$

The constants A and B may be evaluated from Equations (16.39) and (16.40).

Case 3. $R^2 < \frac{4L}{C}$.

In this case, the roots m_1 and m_2 are imaginary. If $m_1 = -k_1 + jk_2$ and $m_2 = -k_1 - jk_2$ $\left(\text{where } k_1 = \frac{R}{2L} \text{ and } k_2 = \frac{\sqrt{\frac{4L}{C} - R^2}}{2L}\right)$ the transient terms may be combined to give the expression

$$e^{-\frac{R}{2L}t} \left[F \sin \left(\frac{\sqrt{\frac{4L}{C} - R^2}}{2L} t + \beta \right) \right]$$ (see page 278)

where $F = \sqrt{P^2 + Q^2}$

$P = A + B$, $Q = j(A - B)$

$\beta = \tan^{-1} \frac{P}{Q}$

Hence, the whole expression for the current takes the form

$$i = Fe^{-\frac{Rt}{2L}} \sin \left(\frac{\sqrt{\frac{4L}{C} - R^2}}{2L} t + \beta \right) + I_{max} \sin (\omega t + \theta - \gamma) \tag{16.41}$$

If, as in the previous cases, the switch is closed when $t = 0$, the constants F and β may be evaluated from the initial conditions—

$$\text{At } t = 0,\ i = 0 \text{ and } L\frac{di}{dt} = E_{max} \sin \theta$$

Applying these conditions to Equation (16.41) gives two equations in F and β—

$$0 = F \sin \beta + I \sin (\theta - \gamma) \quad . \quad . \quad (16.42)$$

$$\frac{E_{max}}{L} \sin \theta = -\frac{R}{2L} F \sin \beta + F \frac{\sqrt{\frac{4L}{C} - R^2}}{2L} \cos \beta + \omega I_{max} \cos (\theta - \gamma) \quad . \quad (16.43)$$

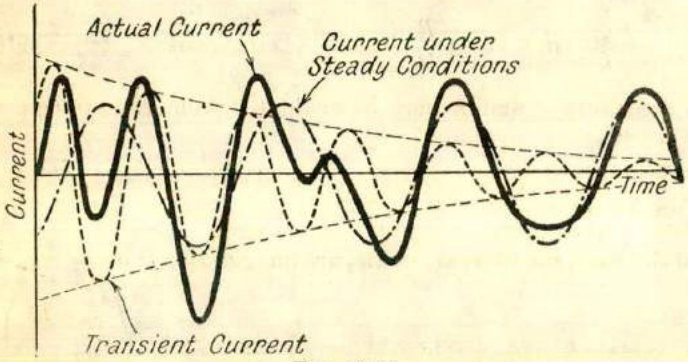


FIG. 16.10.

Whilst these equations can be solved to give general expressions for F and β, the mathematics is rather unwieldy and it is much simpler to substitute numerical values for the constants before solution.

The general shape of the current wave, immediately after closing the switch which applies the voltage to the circuit, is shown in Fig. 16.10. As the transient current dies away, the wave approaches the form obtained under steady conditions, as shown.

The Calculation of Transients by the use of Laplace Transformations. As already seen in this chapter, the relationship between the voltage and current in an electric circuit is, in general, a differential equation the solution of which has two parts—the particular integral, representing the steady state to which the quantities tend after a long time, and the complementary function, representing the transient phenomena which die away.

The steady-state solution can be found by Ohm's law, using resistance only in d.c. circuits and impedance in a.c. circuits.

Laplace transformations, which are fully discussed by J. C. Jaeger (Ref. (4)), may be used to provide a convenient method of calculating both parts of the solution—steady state and transient —when the voltage or the current in a circuit is suddenly changed. The notes on the method, given here, are intended merely as an introduction. The works mentioned in Refs. (4) and (5) should be referred to by readers wishing to make a fuller study of the subject.

Note 1. The Laplace transform is defined by the equation

$$\overline{V}(p) = \int_0^\infty e^{-pt} \, . \, V(t)dt$$

where $\overline{V}(p)$ is the transform of $V(t)$.

Note 2. Table XVI gives a list of the transforms which are most useful for the calculation of electrical transients.

TABLE XVI

Voltage	Transform
Zero up to time $t = 0$, then a constant V	V/p
" " " " $Ve^{-\alpha t}$	$V/(p + \alpha)$
" " " " $Vt \, . \, e^{-\alpha t}$	$V/(p + \alpha)^2$
" " " " $V \sin \omega t$	$V\omega/(p^2 + \omega^2)$
" " " " $V \cos \omega t$	$Vp/(p^2 + \omega^2)$
" " " " $Ve^{-\alpha t} \, . \, \sin \omega t$	$V\omega/[(p + \alpha)^2 + \omega^2]$
" " " " $Ve^{-\alpha t} \, . \, \cos \omega t$	$V(p + \alpha)/[(p + \alpha)^2 + \omega^2]$

The quantities in the first column (which are written for voltage but could be written similarly for current) are zero up to time $t = 0$, so that their value for a negative value of t is *not* that obtained by inserting this negative value in the expression given.

Note 3. The transform of any voltage V (not necessarily constant) is written $\overline{V}$, and the transform of current, I. The "generalized impedance" is written $\overline{Z}$ and this is obtained by writing p in place of $j\omega$ in the complex expression for the impedance of the circuit; e.g. for a circuit containing resistance, inductance and capacitance,

$$\overline{Z} = R + pL + \frac{1}{pC}$$

Note 4. To obtain the solution to any given problem, when the voltage or current is suddenly changed from zero to some other (constant or alternating) value all that is needed is to solve the subsidiary equation $\overline{I} = \dfrac{\overline{V}}{\overline{Z}}$, and then to consult Table XVI to find the corresponding value of I (or V).

Example 1. A constant voltage of 50 V is suddenly applied to a circuit containing a resistance of 10 Ω and inductance of 5 H in series. What will be the current?

The applied voltage is 50 V and, from Table XVI, $\bar{V} = \frac{50}{p}$. The generalized impedance is

$$\bar{Z} = R + pL$$

Thus

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{50}{p(R + pL)}$$

Splitting this expression into partial fractions we have

$$\bar{I} = \frac{50}{Rp} - \frac{50L}{R(R + pL)}$$

and, from lines 1 and 2 of Table XVI it follows that

$$I = \frac{50}{R} - \frac{50}{R} e^{-\frac{R}{L}t}$$

$$= 5(1 - e^{-2t})$$

Thus I is zero up to time $t = 0$, and for positive values of t,

$$I = 5(1 - e^{-2t})$$

If the given voltage, or current, is not zero up to time $t = 0$, we adopt a simple artifice. For example, if the voltage of 50 V had been applied up to $t = 0$ and then removed, we should consider this applied voltage to be made up of a voltage of 50 V applied continuously and a voltage of − 50 V applied suddenly at $t = 0$. The current produced by the first voltage is, by Ohm's law,

$$I = \frac{V}{R} = \frac{50}{10} = 5$$

while the current produced by the voltage − 50 is, as above,

$$I = -5(1 - e^{-2t})$$

The total current flowing is therefore

$$I = 5 - 5(1 - e^{-2t}) = 5e^{-2t}$$

Example 2. A constant voltage V is suddenly applied to a circuit containing resistance R, inductance L and capacitance C. What is the current which will flow?

The applied voltage V is transformed to $\bar{V} = \frac{V}{p}$.

The generalized impedance is

$$\bar{Z} = R + pL + \frac{1}{pC}$$

Thus

$$\bar{I} = \frac{\frac{V}{p}}{R + pL + \frac{1}{pC}}$$

which can be written

$$\bar{I} = \frac{V}{L\left[p^2 + \left(\frac{R}{L}\right)p + \frac{1}{LC}\right]}$$

By completing the square we can write

$$I = \frac{V}{L[(p+\mu)^2 + n^2]}$$

where

$$\mu = \frac{R}{2L} \text{ and } n^2 = \frac{1}{LC} - \mu^2$$

$$= \frac{1}{LC} - \frac{R^2}{4L^2}$$

From line 6 of Table XVI, the expression for the current is thus

$$I = \frac{V}{nL} e^{-\mu t} . \sin nt$$

This is correct if $n^2 > 0$. If $n^2 = 0$,

$$I = \frac{V}{L} t e^{-\mu t} \quad \text{(see Ref. (4)}$$

and if $n^2 = -k^2 < 0$,

$$I = \frac{V}{kL} e^{-\mu t} . \sinh kt$$

These expressions—although of slightly different form—will be found to agree with those obtained for these circuit conditions earlier in the chapter.

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CHAPTER XVII

MEASURING INSTRUMENTS

In this and the succeeding chapters, we shall be concerned with the principles and construction of the various measuring instruments which are used for the measurement of current, voltage, power, and energy. Certain features are common to all such instruments and these will now be discussed.

Classification. In the broadest sense, instruments may be divided into two classes—

(*a*) Absolute instruments.

(*b*) Secondary instruments.

(*a*) Absolute Instruments give the value of the electrical quantity to be measured in terms of the constants of the instrument and of its deflection, no comparison with another instrument being necessary. For example, the tangent galvanometer gives the value of the current to be measured in terms of the tangent of the angle of deflection produced by the current, the radius and number of turns of the galvanometer coil, and the horizontal component of the earth's magnetic field. No calibration of the instrument is thus necessary. Another example of an absolute instrument is the Rayleigh current balance, which has also been discussed in an earlier chapter.

(*b*) Secondary Instruments are so constructed that the value of current, voltage, or other quantity to be measured can be determined from the deflection of the instrument, only if the latter has been calibrated by comparison with either an absolute instrument or one which has already been calibrated. The deflection obtained is meaningless until such a calibration has been made.

This class of instrument is in most general use, absolute instruments being seldom used except in standards laboratories and similar institutions, and it is principally, therefore, with secondary instruments that we shall deal in this and the following chapters.

Effects Utilized in Measuring Instruments. Secondary instruments may be classified according to the various effects of electric current or voltage upon which their operation depends. The effects utilized are—

(*a*) Magnetic effect.

(*b*) Heating effect.

(*c*) Chemical effect.

(*d*) Electrostatic effect.

(*e*) Electromagnetic induction effect.

Table XVII shows in what kinds of instruments these various effects are most generally used in practice.

TABLE XVII

Effect	Instruments Utilizing the Effect
Magnetic effect	Ammeters, voltmeters, wattmeters, integrating meters, and most other electrical instruments.
Heating effect	Ammeters and voltmeters.
Chemical effect	Integrating meters (D.C. ampere-hour meters).
Electrostatic effect	Voltmeters (indirectly, ammeters and wattmeters).
Electromagnetic induction effect	Alternating-current ammeters, voltmeters, wattmeters, and integrating meters.

Of these effects the first and last have been most commonly used.

The use of the term "integrating meter" in the above table may perhaps require explanation. An integrating instrument is one which measures the total amount, either of quantity of electricity, or of electrical energy, supplied to a circuit over a period of time. Its readings give, therefore, either the number of ampere-hours, or of watt-hours, supplied.

The introduction of the term "integrating meter" leads to another method of classifying secondary instruments, namely—

(*a*) Indicating.
(*b*) Recording.
(*c*) Integrating.

Ammeters, voltmeters, and wattmeters, belong to the first of these three classes. As stated aboye, ampere-hour and watt-hour meters belong to the third class. Recording instruments give a continuous record of the variations of some electrical quantity, such as current or power, by means of a path traced out by a pen (attached to the moving system of the instrument) on a sheet of paper carried by a revolving drum.

Indicating Instruments. In most indicating instruments it is essential that three distinct forces shall act upon their moving system in order that they shall indicate satisfactorily. These forces are—

(i) A deflecting force.
(ii) A controlling force.
(iii) A damping force.

Thus, the deflecting, or operating, force causes the moving system of the instrument to move from its "zero" position (i.e. its position when the circuit in which it is placed is disconnected from the supply). The magnitude of this movement would be somewhat indefinite (in most cases) unless some controlling force existed which limits the movement and ensures that the magnitude of the deflection is always the same for a given value of the quantity to be measured. A "damping" force is also necessary in order to bring the moving

system to rest in its deflected position quickly. Without such damping, owing to the inertia of the moving parts, the pointer of the instrument would oscillate about its final deflected position for some time before coming to rest, and this would cause waste of time in taking readings, as well as preventing any but very slow variations of the quantity to be measured from being observed.

Curves showing the effect of damping upon the variation of position, with time, of the moving system of an instrument are given in Fig. 17.1. Oscillation about the final position is shown in the under-damped curve. If the instrument is over-damped, the moving system rises slowly from zero to its final deflection. When the

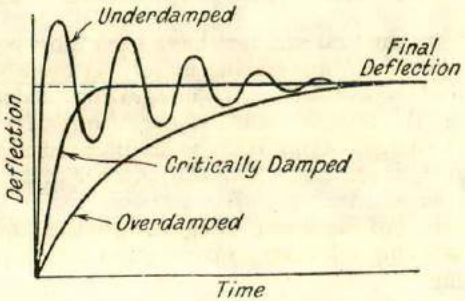


FIG. 17.1. DAMPING CURVES

instrument rises quickly to its deflected position without oscillation the damping is said to be "critical," and the instrument "dead-beat." In practice it is found that the best results are obtained when the damping is slightly less than the critical value.

The damping force must operate only while the moving system of the instrument is actually moving. The final deflection of the instrument must not be affected by the damping.

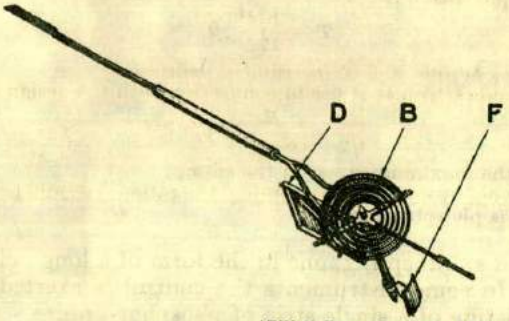
The deflection of the moving system of an indicating instrument will rise from zero to such a value that the controlling torque at this deflected position is equal (and opposite) to the deflecting torque. The deflecting torque produced by any given value of the quantity to be measured is constant, and thus the controlling torque must increase in magnitude with the deflection until it balances the constant deflecting torque.

The operating or deflecting torque is produced by utilizing one or other of the effects already mentioned. The actual method of production of this torque depends upon the type of instrument and will be discussed later.

The controlling torque in indicating instruments is almost always obtained by a spring; much less commonly, by gravity.

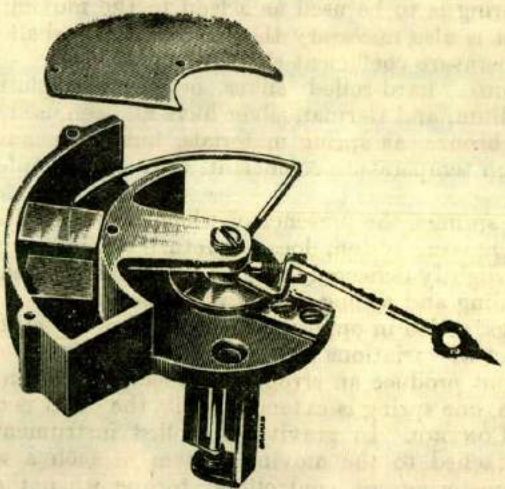
SPRING CONTROL. A hair-spring—usually of phosphor-bronze—

attached to the moving system, is used in indicating instruments for control purposes, the arrangement being as shown in Fig. 17.2 —in which B is the control spring, F the moving iron, and D a damping vane—and in Fig. 17.3.



(Elliott Bros. (London), Ltd.)

FIG. 17.2. MOVING SYSTEM OF A SPRING-CONTROLLED INSTRUMENT



(Nalder Bros. and Thompson, Ltd.)

FIG. 17.3. MOVING SYSTEM OF A MOVING-IRON INSTRUMENT SHOWING THE AIR-DAMPING CHAMBER

To give a controlling torque which is directly proportional to the angle of deflection of the moving system the number of turns on the spring should be fairly large, so that the deformation per unit length is small. The stress in the spring must be limited to such a value that there is no permanent set.

Suppose that a spiral spring is made up of a total length of L in. of strip whose cross-section is rectangular, the radial thickness being t and the depth b, both in inches. Let E be Young's modulus (in pounds per square inch) for the material of the spring. Then, if θ radians be the deflection of the moving system to which one end of the spring is attached, the law connecting the controlling torque with θ is

$$T = \frac{Ebt^3}{12L} \cdot \theta \quad . \quad . \quad . \quad . \quad . \quad (17.1)$$

Thus, controlling torque $\propto \theta \propto$ instrument deflection.

Another formula which is of use in connection with the design of springs is

$$\frac{L}{t/2} = \frac{E}{S_{max}} \times \theta \quad . \quad . \quad . \quad . \quad . \quad (17.2)$$

where S_{max} is the maximum stress in the spring.

In order to avoid overstressing the spring, the ratio $\frac{L}{t/2}$ should be about 2,000 if the material is phosphor-bronze.

Instead of a spiral spring, one in the form of a long helix is sometimes used. In some instruments the control is exerted by a suspension consisting of a single strip of phosphor-bronze.

The requirements of materials used for springs are that they shall be non-magnetic, not subject to appreciable fatigue, and, in cases where the spring is to be used as a lead to the moving system (as it often is), it is also necessary that its resistance shall be low and that its temperature coefficient shall be small.

Silicon-bronze, hard-rolled silver or copper, platinum-silver, platinum-iridium, and German silver have all been used, in addition to phosphor-bronze, as spring materials, but on account of either fatigue or high temperature coefficient, they are all inferior to the latter.

Fatigue in springs, the presence of which may be detected by the fact that the moving system does not return to zero after deflection, but remains slightly deflected, may be avoided to a great extent by proper annealing and ageing during manufacture.

Two springs, coiled in opposite directions, are sometimes fitted so that the effect of variations of temperature upon the length of the spring may not produce an erroneous deflection. When the instrument deflects, one spring is extended while the other is compressed.

Gravity Control. In gravity-controlled instruments, a small weight is attached to the moving system in such a way that it produces a restoring, or controlling, torque when the system is deflected. This is illustrated in Fig. 17.4. The controlling torque, when the deflection is θ, is $Wl \sin \theta$, where W is the control weight and l its distance from the axis of rotation of the moving system, and it is therefore proportional only to the sine of the angle of deflection, instead of, as with spring control, being directly proportional to the angle of deflection. The controlling torque can be varied quite simply by adjustment of the position of the control weight upon the arm which carries it. Gravity-controlled instruments must obviously be used in a vertical position in order that

the control may operate. They must also be level, or their zero position will be affected. For these reasons, control by gravity is not so well suited to indicating instruments generally. It is now practically obsolete as a form of control and is of historical interest only.

COMPARISON OF SPRING AND GRAVITY CONTROL. As against the disadvantages mentioned above, gravity control has the advantages, when compared with spring control, of cheapness, and of independence upon temperature, and freedom from deterioration with time.

Consider an instrument in which the deflecting torque T_D is directly proportional to the current (say) to be measured.

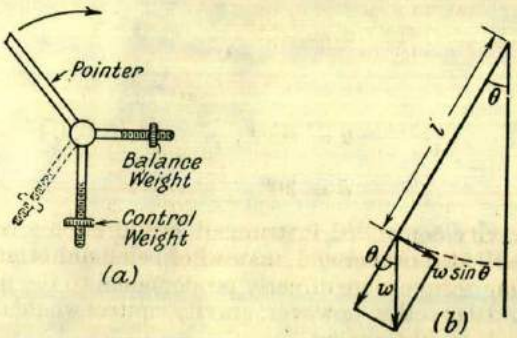


FIG. 17.4. GRAVITY CONTROL

Thus, if I is the current,

$$T_D = kI \quad . \quad . \quad . \quad . \quad . \quad (17.3)$$

If the instrument is spring-controlled, the controlling torque being T_C, then when the deflection is θ,

$$T_C = k_s\theta \quad . \quad . \quad . \quad . \quad . \quad (17.4)$$

Also $T_C = T_D$

or $k_s\theta = kI$

$$\therefore \theta = \frac{k}{k_s} . I \quad . \quad . \quad . \quad . \quad (17.5)$$

Thus the deflection is proportional to the current throughout the scale.

If the instrument is gravity controlled,

$$T_C = k_g \sin\theta \quad . \quad . \quad . \quad . \quad (17.6)$$

and $T_C = T_D = kI$

$$\therefore k_g \sin\theta = kI$$

$$\sin\theta = \frac{k}{k_g} . I$$

$$\theta = \sin^{-1}\left(\frac{k}{k_g} I\right) \quad . \quad . \quad . \quad (17.7)$$

For example, if the maximum deflection, produced by a current I_{max}, is 90°, then when the current is $\frac{I_{max}}{2}$, the deflection if the instrument is spring controlled is 45°.

If gravity control is used, we have

$$\sin 90° = \frac{k}{k_g} \cdot I_{max} = 1$$

or

$$I_{max} = \frac{k_g}{k}$$

Thus when

$$I = \frac{I_{max}}{2}$$

$$\theta = \sin^{-1} \cdot \frac{\frac{I_{max}}{2}}{I_{max}} = \sin^{-1} \cdot 0{\cdot}5$$

or

$$\theta = 30°$$

Thus, a gravity-controlled instrument would have a scale which was "cramped" at its lower end, instead of being uniformly divided, if the deflecting torque were directly proportional to the quantity to be measured. In practice, however, gravity control would not usually be used in such an instrument.

Balancing of Moving Parts. In order that the deflection of a spring-controlled instrument shall be independent of its position, and also that the wear on the bearings shall be uniform, it is essential that the centre of gravity of the moving system shall lie on the axis of rotation. This condition is attained by the use of balance weights carried on fairly short arms attached to the moving system. In the gravity-controlled system shown in Fig. 17.4, both the "control" weight and the "balance" weight take a part in balancing the weight of the pointer and the rest of the moving system.

In determining suitable weights and distances from the axis of rotation for control and balancing purposes, attention must be paid to their effects upon the weight and inertia of the moving system. If large weights at short radii are used, the weight of the moving system is made large, whereas small weights at a large radii add considerably to the inertia of the system and necessitate a large damping torque. A compromise is usually effected.

Torque and Weight Ratio. In order to reduce the load on the bearings and to reduce the friction torque, which is proportional to the pressure on the bearing surface, the weight of the moving parts should be made as small as possible.

Expressing the deflecting torque in terms of the force which, acting at a radius of 1 cm, would produce full-scale deflection, the ratio of this torque to the weight of the moving system should be,

if possible, not less than 0·1. The torque/weight ratio is influenced by whether the axis of the moving system is vertical or horizontal.

DAMPING. There are three systems of damping in general use. These are as follows—

(*a*) Air friction damping.
(*b*) Fluid friction damping.
(*c*) Eddy current damping.

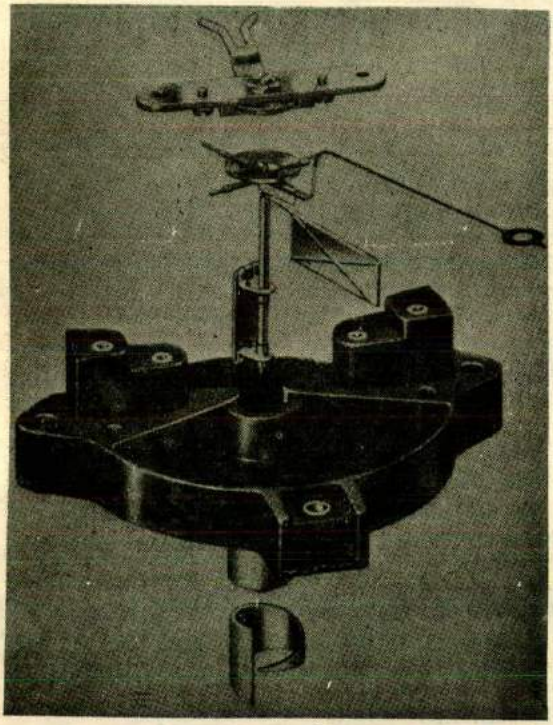
The system used varies with the type of instrument. Eddy current damping is perhaps the most efficient form, and is used in cases where the introduction of a permanent magnet—necessary for the induction of the eddy currents—will not produce errors due to distortion of an existing magnetic field which is being used for operating purposes. It is a very convenient form of damping when either a permanent magnet or some metal part, such as a disc, already forms part of the operating system of the instrument. For these reasons this form of damping is used in "hot-wire," moving-coil, and induction instruments.

The disadvantages of fluid friction damping are that it can only be used in instruments which are used in a vertical position, and also that, owing to "creeping" of the oil used for the purpose, it is difficult to keep the instrument clean. An advantage of this method is, however, that the damping oil can be used for insulation purposes, as well as for damping, in some forms of instrument in which the whole moving system is immersed in the oil. When the system is so immersed the up-thrust of the oil on the system reduces the load on the bearings or suspension.

Air Friction Damping. Two methods of damping by air friction are illustrated in Figs. 17.3 and 17.5. In one case a light aluminium piston is attached to the moving system and moves in an air chamber closed at one end, as shown in Fig. 17.3. The cross-section of this chamber may be either circular or rectangular. The clearance between the piston and the sides of the chamber should be small (a few thousandths of an inch) and uniform. If the piston is moving rapidly into the chamber, the air in the closed space is compressed and the pressure opposes the motion of the piston (and therefore of the whole moving system). If the piston is moving out of the chamber rapidly, the pressure in the closed space falls, and the pressure on the open side of the piston is greater than that on the opposite side. Motion is thus again opposed. With this damping system care must be taken to ensure that the arm carrying the piston is not bent, or the piston will touch the sides of the chamber during its movement. The solid friction which thus occurs may result in a serious error in the deflection. When once bent, it is often difficult to straighten the piston arm so that it does not touch the sides of the chamber at any point during the deflection. The second method utilizes a vane, mounted on the spindle of the moving system. This vane is of thin aluminium sheet and moves in a closed, sector-shaped box as shown in

Fig. 17.5, illustrating the air damping system (and the other working parts) of an Evershed and Vignoles moving-iron instrument.

Fluid Friction Damping. In this method of damping, no very careful fitting, as in the previous method, is necessary. A light vane, attached to the spindle of the moving system, dips into a pot



(*Evershed & Vignoles, Ltd.*)

FIG. 17.5. AIR DAMPER

of damping oil and should be completely submerged by the oil. Fig. 17.6 illustrates the method.

The frictional drag on the disc in the first system is always in the direction opposing motion, and increases with the speed of rotation of the disc. There is no friction force when the disc is stationary. The suspending stem of the disc should be cylindrical and of small diameter where it penetrates the oil surface, so that surface tension effects may be negligible. In the second system (Fig. 17.6 (*b*)), increased damping (as compared with the previous system) is obtained by the use of vanes, in vertical planes, carried on a spindle and immersed in oil, as shown.

The principal requirements of oil for damping purposes are that it shall not evaporate quickly, shall not have any corrosive action upon metals, and that its viscosity shall not change appreciably with temperature. It should also be a good insulator.

Eddy Current Damping. When a sheet of conducting material moves in a magnetic field so as to cut through lines of force, eddy

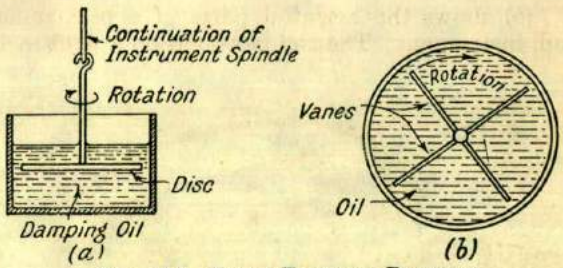


FIG. 17.6. FLUID FRICTION DAMPING

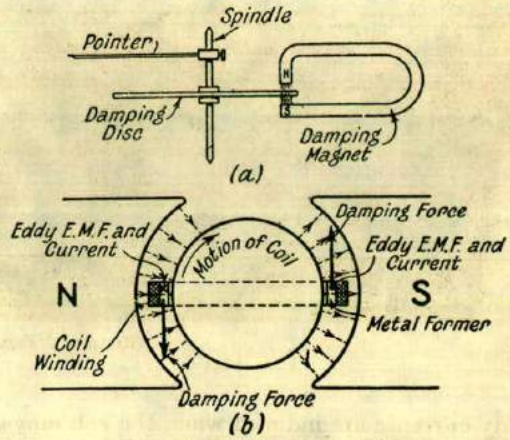


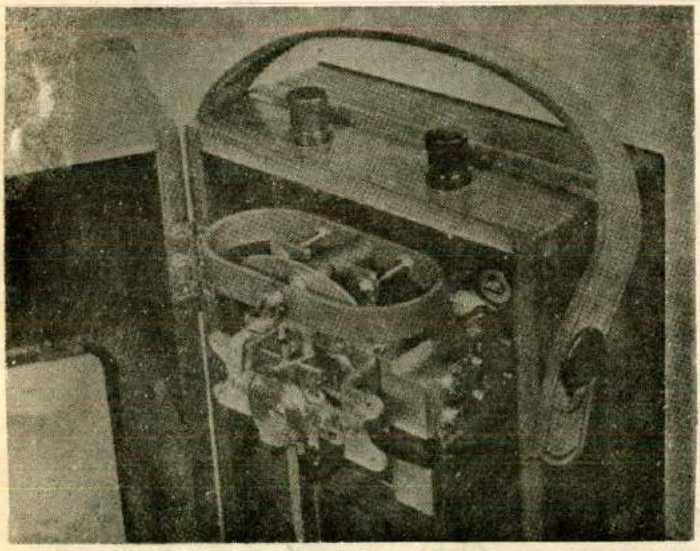
FIG. 17.7. EDDY CURRENT DAMPING

currents are set up in it and a force exists between these currents and the magnetic field, which is always in the direction opposing the motion. This force is proportional to the magnitude of the current and to the strength of field. The former is proportional to the velocity of movement of the conductor, and thus, if the magnetic field is constant, the damping force is proportional to the velocity of the moving system and is zero when there is no movement of the system. The theory is discussed in Chapter XIV.

Fig. 17.7 shows two methods of applying this method of damping. In diagram (*a*) a thin disc of conducting, but non-magnetic, material

—usually copper or aluminium—is mounted on the spindle which carries the pointer of the instrument. When the spindle rotates, the edge of the disc cuts through the lines of force in the gap of a permanent magnet, and eddy currents, with consequent damping, are produced. An arrangement similar to this is often used in hot-wire instruments.

Fig. 17.7 (*b*) shows the essential parts of a permanent-magnet, moving-coil, instrument. The coil is wound on a light metal former



(*Evershed & Vignoles, Ltd.*)

FIG. 17.8

in which eddy currents are induced when the coil moves in the permanent-magnet field. The directions of the eddy currents, and of the damping forces produced as a result of them, are shown in the figure.

Fig. 17.8 shows an adjustable magnetic damping device used in the latest type of Evershed and Vignoles recorder.

CONSTRUCTIONAL DETAILS OF INDICATING INSTRUMENTS.

(i) *Methods of Supporting Moving Systems.* The two commonest methods of supporting the moving system of an instrument are—

(*a*) By pivoting. (*b*) By thread suspension.

When the system is pivoted, the ends of the spindle are conical, and should be of hardened steel. These ends fit into conical holes in jewels (preferably sapphire), which form the bearings, on the fixed part of the instrument.

To reduce friction at the pivots the contact area should be small,

but the pressure per unit area must be carefully considered, since, if the pivots are sharply pointed, this may exceed the crushing strength of the material of the pivot. Although the weight of the moving system may be only a few grammes, if the area of the point of the pivot is very small, the pressure per square inch may be many tons. For example, this pressure may be 80 tons per sq. in. or even more.

In mounting the moving part, a small amount of play should be allowed so that the pivots are not forced hard on to the jewels.

The question of the wear on instrument pivots and bearings is so important—particularly in electricity meters, in which the rotating system is almost continuously in movement—that fundamental studies extending over many years have been made, especially by G. F. Shotter and by the Electrical Research Association. Refs. (29), (30), (32) deal very fully with the matter. It has been shown that the relation between the direction of pressure and the optic axis is very important in the steel-sapphire combination. Tests have confirmed that, with a set frictional limit as basis, jewels having an active surface at an angle of 90° to the optic axis have a much longer life than those cut at other angles.

Glass has been proved inferior to sapphire as a bearing material (except, perhaps, when the direction of the load may vary), but German work (Ref. (30)) during World War II has indicated that hardened spinel bearings may be almost as good.

Thread suspension is advantageous when the operating forces are small compared with the weight of the moving part, since bearing friction is avoided. Such suspensions are, however, delicate, and protection from vibration and shock is necessary. Phosphor-bronze strip is most commonly used for these suspensions.

(ii) *Permanent Magnets.* In most cases when permanent magnets are used in instruments it is essential that their strength shall not vary with time.

Such magnets may be of very hard steel, containing a small percentage of tungsten or of cobalt and chromium together. The coercive force of cobalt-chromium steel is high, and thus magnets made of this steel are not subject to self-demagnetization to the same extent as tungsten-steel magnets.

Within the last two decades great advances have been made in the development of magnetic materials having properties suiting them to the construction of permanent magnets. In particular, Alnico (in the United States) or Alcomax (in England), alloys of iron, nickel and aluminium, have especially good properties with coercive forces which may be as high as 60,000 A/m.

R. M. Bozorth (Ref. (20)) gives data for modern permanent-magnet materials. The energy product $(BH)_{max}$ (see Chap. I, p. 46) ranges from 2,400 for tungsten steel to as much as 36,000 for Alnico (or Alcomax) having a composition of 24 Co, 14 Ni, 8 Al, 3 Cu.

The coercive force H_c for this material is given as 46,000 A/m and the residual induction B_{rem} as 1·25 Wb/m².

During manufacture, permanent magnets are artificially aged by being placed in a weak alternating magnetic field, or by heating. This reduces their strength somewhat, but ensures permanence of the magnetism remaining.

(iii) *Pointers and Scales.* The shape and size of pointer used depend upon the type of instrument. In all cases, however, the weight and inertia of the pointer must be reduced as far as possible, both to reduce the load on the bearings of the moving system and to avoid the high degree of damping which would be necessary if the moving system had considerable inertia.

For the sake of lightness, aluminium strip or tube is used for the pointer, a truss construction being used in some cases for rigidity.

In some instruments where precision in reading, at close range, is aimed at, a strip of mirror is fitted on the plate which bears the scale. The end of the pointer is flattened so that, when viewed from above, it appears as a narrow strip or edge. The eye of the observer must be moved until the end of the pointer and its image in the mirror are coincident, before a reading is taken. This avoids error due to parallax.

The moving system of most indicating instruments rotates through an angle of about 90° (for full-scale deflection), although some makers have designed instruments having angles of deflection of 120° or even greater. The length of scale in many instruments is about 6 in. Thus, for an accuracy, in the reading, of (say) ½ per cent, it is necessary to observe the position of the pointer to within 0·03 in. at full-scale deflection and to within smaller fractions of an inch when the deflection is less than the full-scale value. Hence the necessity for a clearly-marked and carefully divided scale and for a sharply-pointed pointer. In the case of good-class instruments the card upon which the scale is to be marked is blank when first fitted. The main divisions of the scale are then marked in by comparison of the instrument with a sub-standard one, after which the scale is completed by means of some dividing instrument. Stiff card, mounted on a metal sheet, is generally used for the scale. An excellent review of the various types of pointers and scales in use in electrical measuring instruments, with comments on their relative effectiveness and on possible future developments, with many illustrations, has been given by L. B. S. Golds (Ref. (14)).

(iv) *Cases.* These may be of hard wood or brass, but are most often of cast iron or pressed steel. The steel cover is an advantage for magnetic screening in the case of instruments which are affected by external magnetic fields. The base which carries the operating portion of the instrument is, also, often of steel for the same reason. The cover should be fitted so as to exclude dust and moisture from the instrument. When steel covers are used, the moving system of

the instrument should be mounted in a position as far away from the case as possible, in order to avoid errors due to hysteresis and eddy current effects in the case.

Recording Electrical Quantities. The value of an electrical quantity which varies with time may be recorded,

(*a*) to preserve information which could be obtained at any instant from indicating instruments—as when the voltage or power of a supply circuit is recorded, or

(*b*) to give information which could not be obtained readily from indicating instruments. Examples are wave-forms and transient phenomena or the phase relationships in different parts of a circuit.

It is often necessary to record an electrical quantity which is being used to study some other physical quantity such as strain or pressure. The speed of response of the recording instrument is then important, whereas a recorder which merely replaces an indicating instrument is not usually required to have a quick response. The recording instruments considered below are given in (roughly) ascending order of response speed.

Recorders. These instruments generally resemble the corresponding indicating instruments—voltmeters, wattmeters, etc., but have a light arm, carrying a pen, in place of the pointer. The pen rests lightly on a chart which is moved, at a low and uniform speed, in a direction perpendicular to that of the deflection of the pen. This chart is unwound from a drum on to another similar drum by clockwork. The path traced out by the inked pen gives a continuous record of the variations of deflection of the instrument.

Owing to the friction of the pen on the chart and to the necessarily greater weight of the moving system, the design of an indicating instrument must be somewhat modified if it is to be used for recording purposes.

In order that friction shall not introduce serious errors, the operating torque must be increased in proportion to the increased friction torque The controlling torque must, therefore, also be increased, and increased damping is necessary on account of the greater inertia of the moving system. It may be necessary also to increase the size of the bearings on account of the greater load on them.

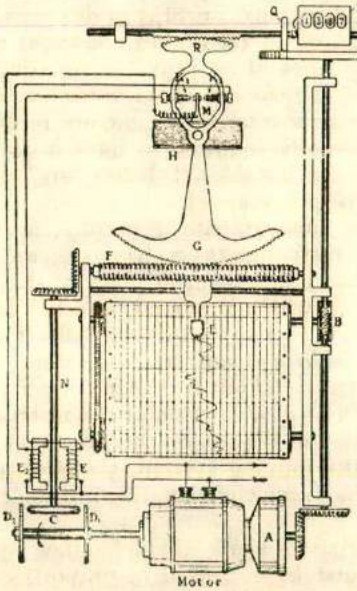
The line diagram in Fig. 17.9 shows the construction of a relay-operated recording instrument by Messrs. Everett Edgcumbe. The tongue M is moved by any variation in the quantity to be measured. This closes the circuit of one of the electromagnets E_1, E_2, causing the shaft N to deflect. The friction wheel C engages with one of the discs D_1, D_2, and causes the worm F to rotate. This moves both the toothed quadrant G and the pen L.

The chart is driven by the small motor, whose speed is maintained constant by the centrifugal governor A.

Instruments of this kind can be divided into two classes: those in which the pen-arm is vertical, and those in which it is horizontal.

The former has the advantage that the time-lines on the chart are straight, whereas in the second they are segments of circles. The pen, however, is generally heavier in the instruments with a vertical pen, since the ink must be carried in the pen at the end of the arm, or in a trough into which the pen dips. Instruments with a horizontal pen generally carry the ink in a well at the axis about which the pen arm rotates, and ink flows along the arm to the writing point by capillary action.

Instruments which are intended to record information normally



(Everett Edgcumbe & Co., Ltd.)

FIG. 17.9. RECORDING INSTRUMENT

shown on indicating instruments have a response time, from zero to full scale, of several seconds. For the purposes for which the recorders are used this time is unimportant, and it is not generally quoted.

Milliammeter Recorders with Quick Response. A pen recorder of normal pattern having a moving-coil movement driving a horizontal pen can be made to have a response time, from zero to full-scale, down to about 0·1 sec by careful design. The moving parts must be as light as possible and the driving power must be increased as the response time is reduced. Such an instrument is made by Messrs. Everett Edgcumbe. The four-inch-wide chart runs normally at 1 inch per second but can run at up to 4 inches per second. The

response time is 0·1 second and the instrument consumes 1 watt for full-scale deflection.

By still further reducing the weight of the pen and by considerably reducing its movement, the response of such an instrument can be improved further. The recorder made by Kelvin and Hughes (Industrial), Ltd., has a pen movement of 7·5 mm on each side of the centre zero. Several models of different coil resistance are made and one example has a uniform frequency response up to 90 c/s while it consumes 1 watt for full deflection to either side.

Milliammeter Recorder driven by Amplifier. The power required to drive a recorder having a milliammeter movement with a fast

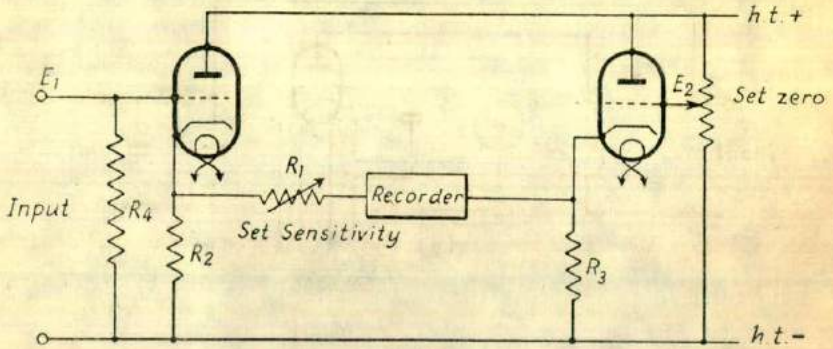


FIG. 17.10

response is often greater than is available from the circuit providing the quantity (whether voltage or current) which is to be recorded. In such cases the recorder may be driven by an electronic amplifier which requires a very small input power.

An amplifier which can be used for this purpose is shown in Fig. 17.10.

The circuit shows a differential cathode follower. The resistance of the recorder and the series resistance R_1 are small compared with R_2 or R_3. If the input voltage E_1 is the same as the voltage E_2 applied to the grid of the second valve, and if the characteristics of the valves are the same, no current will flow through the recorder. If the applied voltage is then increased the current through the first valve increases and the voltage of its cathode rises. This causes a current to flow through the recorder, and raises the voltage of the cathode of the second valve, which in turn decreases the current through that valve. Thus the final effect of raising the input voltage E_1 is to increase the current through the first valve, decrease the current through the second, and cause an equalizing current to flow through the recorder.

The current flowing through the recorder for a given change of

the input voltage is dependent mainly on the circuit resistances, and is very little affected by changes of the valve characteristics or of the h.t. voltage. It can be adjusted by varying the value of R_1.

Since the resistance R_4 may be high—of the order 0·5 MΩ—the power required to drive the amplifier is very small.

The principle of the Tinsley (H. Tinsley and Co., *Brit. Pat. No.* 537,784 and others) d.c. amplifier, which is free from drift or instability and is operated entirely from a.c. mains, is illustrated in Fig. 17.11 (see also Ref. (23)). A thyratron T is used and its anode is supplied with an alternating voltage. Its grid is supplied in anti-phase through capacitor C so that the thyratron is maintained below

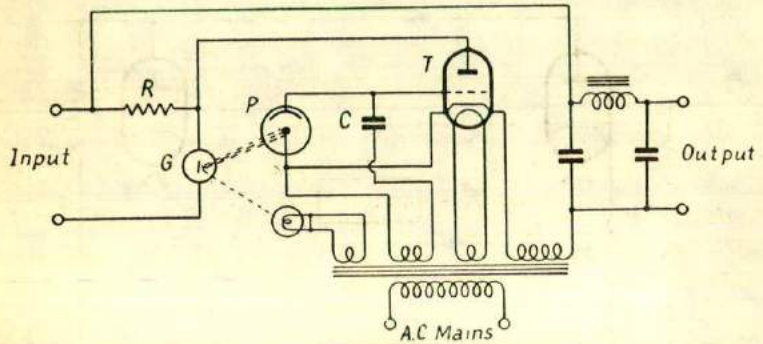


FIG. 17.11

the critical firing potential. A vacuum photocell P shunts the grid to the cathode.

The voltage to be measured is applied to a reflecting galvanometer G and the spot of light is deflected over the photocell, this causing the latter to act as a variable resistance across C. The phase of the potential applied to the grid is thus changed and the thyratron fires for part of each positive cycle. The resulting current pulses are fed through the feedback resistor R so as to oppose the applied voltage. Equilibrium is reached when the product of the output current and the resistance R equals the input voltage (less a small quantity required to move the galvanometer the necessary 3 or 4 mm). If R is very small the output current is very large and must be exactly proportional to the applied voltage, provided the galvanometer movement is small.

The unidirectional pulses are fed to the recorder through a smoothing circuit.

The maximum output current of the amplifier is 20 mA into a load of 2,500 Ω (max.) for all inputs.

Servo-operated Pen Recorders. If a recorder must be driven by an amplifier, it is inherently more accurate to use the amplifier in a

servo-system. An example of such an instrument is that manufactured by Messrs. Evershed and Vignoles. The operation of the recorder can be understood from Fig. 17.12.

The moving coil is wound in two equal halves, which are supplied with currents in opposition by a balanced amplifier. There is no control spring, and when the currents in the two halves of the moving coil are equal, the pen will remain at rest. The pen-arm carries a contact which bears lightly on a small potentiometer. The input to the amplifier is taken from the centre-tap of a resistor connected between the input terminal and the pen-arm contact.

If the voltage picked up by the pen-arm contact is equal and opposite to the input voltage, there is no input to the amplifier, the

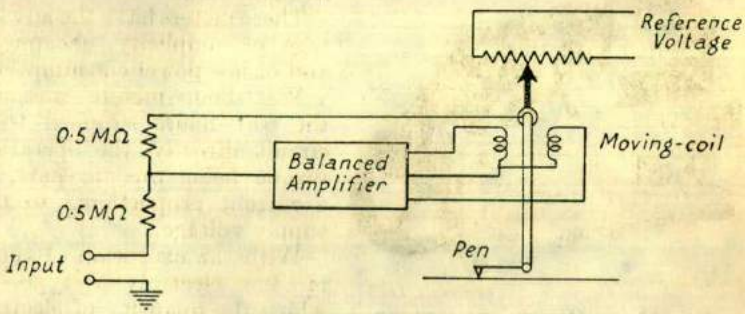


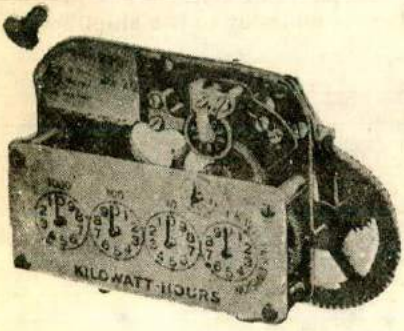
FIG. 17.12

currents in each half of the moving-coil are equal, and the pen remains at rest. If the input voltage is increased, the currents in the two parts of the moving coil became different, and a torque is set up which drives the pen in a direction to increase the negative voltage picked up by the pen-arm contact. When this becomes equal and opposite to the applied voltage there is again no input to the amplifier, and the pen comes to rest.

The recorder is thus essentially a self-resetting potentiometer, and the accuracy is unaffected by the amplifier characteristics. The instrument has a chart width of 2⅞ in., and has a robust vertical pen arm. The response is uniform up to about 10 c/s for an amplitude of 1½ in. peak-to-peak.

Integrating Instruments. These instruments measure, and register, either the total quantity of electricity, in ampere-hours, or the total amount of energy, in kilowatt-hours, supplied to a circuit in a given time. They give no direct indication as to the rate at which the energy is being supplied. Their registrations are independent of the rate at which a given quantity of energy is supplied, provided that the current flowing is sufficient to cause the instrument to operate.

Ampere-hour meters are used on direct-current circuits where the supply voltage is constant. The number of watt-hours supplied to such a circuit is obtained by multiplying the measured number of ampere-hours by the supply voltage. The registering dials (or scales) are usually marked in kilowatt-hours, which means that the instrument only gives correct readings when connected in a circuit whose voltage is that for which the instrument was calibrated. Obviously, when such instruments are used, the constancy of the voltage is depended upon. This is justifiable in most cases, since, by law, the voltage in public supply systems must be maintained within 6 per cent of the nominal value.



(*Ferranti, Ltd.*)

FIG. 17.13. METER REGISTERING MECHANISM

These meters have the advantage of simplicity, cheapness, and of low power consumption.

Watt-hour meters measure the watt-hours supplied to a circuit, directly, the operating torque being due in part, to a current proportional to the supply voltage.

With the exception of those of the electrolytic type—in which the quantity of electricity supplied is indicated by the level of liquid in a graduated tube—integrating meters register by means of a train of gear wheels and dials similar to that shown in Fig. 17.13.

Meters of the "motor" type are most generally used, and in these instruments the train of wheels is driven from the spindle of the rotating system of the instrument. This spindle has a worm cut on it, and this engages with a pinion and thus drives the wheel-train. The spindles of the wheels in the train carry hands which move over the dials (five or six in number) which register units, tens, hundreds, and so on.

The essential parts of such meters are –

(*a*) An operating system which produces a torque proportional to the current or power, and causes the rotating system to rotate.

(*b*) A braking device—usually a permanent magnet—which produces a braking torque proportional to the speed of rotation, and thus causes the rotating system to run at a steady speed such that the braking torque is equal to the operating torque.

(*c*) A device for registering the number of revolutions of the rotating system, this usually taking the form of a train of wheels operating the hands of dials.

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CHAPTER XVIII

AMMETERS AND VOLTMETERS

AMMETERS and voltmeters are classed together because there is no essential difference in principle between them. Except in the case of electrostatic instruments, a voltmeter carries a current which is proportional to the voltage to be measured, and this current produces the operating torque. In an ammeter this torque is produced by the current to be measured, or by a definite fraction of it. Thus, the only real difference between the two instruments is in the magnitude of the current producing the operating torque.

An ammeter is usually of *low resistance*, so that its connection in series with the circuit in which the current is to be measured does not appreciably alter this current. A voltmeter, on the other hand, is connected across the voltage to be measured, and must therefore have a *high resistance* so that the current taken by it may be small. A low-range ammeter—i.e. one which gives full-scale deflection for a very small current—may thus be used as a voltmeter if a high resistance is connected in series with it. The current which flows through it when it, together with its series resistance, is connected across the voltage to be measured, must be within its range when used as an ammeter.

Example. A milliammeter, whose resistance is 5 ohms, gives full-scale deflection for a current of 15 milliamp. Calculate the resistance which must be connected in series with it in order that it may be used as a voltmeter for voltages up to 100 volts.

Let R be the required series resistance. The current flowing through the instrument when 100 volts are applied to the instrument and resistance in series must be 0·015 amp.

$$\frac{100}{R+5} = 0{\cdot}015$$

or

$$R + 5 = \frac{100}{0{\cdot}015} = 6{,}666 \text{ ohms}$$

$$\therefore R = 6{,}661 \text{ ohms}$$

Power Loss. If R_A is the resistance of an ammeter in which a current I flows, the power loss in the instrument is I^2R_A watts.

Again, if R_V is the resistance of a voltmeter to which a voltage E is applied, the power loss in the instrument is $\frac{E^2}{R_V}$.

Obviously, in order that the power loss in the instruments shall be small, R_A must be small and R_V large.

Operation of Ammeters and Voltmeters on Alternating-current Circuits. If an ammeter or voltmeter, whose operating torque is

proportional to the current passing through it, is used on an alternating current circuit, the torque acting on the moving system is alternating. This means that the moving system tends to oscillate about its zero position. If the current through it alternates rapidly, the moving system of the instrument cannot, in general, follow these alternations, and no deflection will be observed.

It is thus essential that the torque, in an instrument to be used in an alternating-current circuit, shall be proportional to the *square* of the current passing through the instrument. The deflection will then be proportional to the *mean value of the square of the current* (assuming that spring control is used), and the instrument can thus be used for the measurement of r.m.s. values of current or voltage.

Types of Instruments. The following types of ammeters and voltmeters are in common use—

(*a*) Moving iron.
(*b*) Moving coil—
  (i) permanent-magnet form,
  (ii) dynamometer form.
(*c*) Thermal.
(*d*) Electrostatic (voltmeters only).
(*e*) Induction.

Of these the permanent-magnet moving-coil type can be used for direct-current measurements only, and the induction type for alternating-current measurements only. The other types can be used with either direct or alternating current.

The moving-iron and moving-coil types both depend for their action upon the magnetic effect of current. The former is the most generally used form of indicating instrument, as well as the cheapest. It can be used for either direct- or alternating-current measurements and, if properly designed, is very accurate. The permanent-magnet moving-coil instrument is the most accurate type for direct-current measurements, and instruments of this type are frequently constructed to have substandard accuracy.

Thermal instruments have the advantage that their calibration is the same for both d.c. and a.c. They are particularly suited to alternating-current measurements, since their deflection depends directly upon the heating effect of the alternating current, i.e. upon the r.m.s. value of the current. Their readings are thus independent of the frequency or wave-form of the current, and of any stray magnetic fields which may exist in their vicinity.

As voltmeters, electrostatic instruments have the advantage that their power consumption is exceedingly small. They can be made to cover a large range of voltage, and can be constructed to have substandard accuracy. Their main disadvantage is that the electrostatic principle is only directly applicable to voltage measurements.

The induction principle is more generally used for watt-hour meters than for ammeters and voltmeters owing to the comparatively high cost, and inaccuracy, of induction instruments of the latter types.

Errors in Ammeters and Voltmeters. There are certain errors which occur in most types of instruments, while other errors occur only in those of a particular type. These latter errors will be dealt with later, together with the instruments in which they occur.

Of the errors common to most types of instrument, friction and temperature errors are perhaps the most important. To reduce the effect of friction torque, and consequently the error produced by it, the weight of the moving system must be made as small as possible compared with the operating forces, i.e. the ratio of torque to weight must be large (about $\frac{1}{10}$ for full deflection).

A vertical spindle is generally to be preferred to a horizontal one from the point of view of a small friction torque.

The most serious error produced by the heat generated in the instrument, or by changes in room temperature, is that due to a change in the resistance of the working coil. Such a change of resistance is of little importance in ammeters, but in voltmeters, in which the working current should be directly proportional to the applied voltage, it is essential that the resistance of the instrument shall remain as nearly constant as possible.

Thus, the power loss in the instrument should be small, and resistance coils which are likely to produce appreciable heating should be mounted, if possible in such a position that they are well ventilated.

To eliminate temperature errors, the working coil is wound with copper wire and is of comparatively low resistance. A high "swamping" resistance, of material whose temperature coefficient is small, is connected in series with the coil, so that, although the resistance of the coil may change considerably, the change in total resistance is small.

Other errors resulting from heating may be caused by expansion of the control spring, or of other parts of the instrument, although such errors are usually small. Lack of balance in the moving system and changes in the strength of permanent magnets (if used) are other possible sources of error which are common to several types of instrument.

Permissible Errors in Ammeters and Voltmeters. British Standard Specification B.S. 89 : 1954 grades indicating instruments on the basis of their limits of error. These grades are—

Precision (Pr.). Industrial (In.).

The limits of error over the effective range, expressed as a percentage of the maximum scale value, may be summarized* from the specification as follows—

PRECISION. *Voltmeters and ammeters:* From 0·3 per cent for a single-range voltmeter of the permanent-magnet moving-coil type up to 0·5 per cent for a multi-range, self-contained ammeter of the same type.

Wattmeters: From 0·5 per cent for a single-element single-range electrodynamic wattmeter to 0·6 per cent for a multi-range instrument. For a double-element multi range instrument the limit is 1·0 per cent.

INDUSTRIAL: The limits of error for industrial *ammeters and voltmeters* are from 1 per cent to 3 per cent in the case of switchboard instruments, and from 0·75 per cent to 3 per cent for portable instruments, the higher errors being for thermocouple and rectifier instruments. *Wattmeter* errors are between 1·0 and 2·5 per cent.

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TABLE XVIII

Type of Instrument	Pattern	Accuracy of which the Instrument is Capable	Ammeter Ranges for which this Accuracy may be Expected	Voltmeter Ranges for which this Accuracy may be Expected
Permanent-magnet moving-coil (d.c. only)	Laboratory	Precision	Up to 600 A	Up to 750 V
	Combined ammeter and voltmeter testing set	Precision or Industrial		
Moving-iron	Laboratory	Precision, d.c. and a.c. up to 100 c/s	0·1 to 10 A	50 V to 750 V
	Testing sets	Industrial, d.c. and a.c. up to 100 c/s		
	Switchboard	Industrial, d.c. and fixed-frequency a.c.		
Electrodynamic ammeters and voltmeters	Laboratory and testing sets	Precision, d.c. and a.c. up to 60 c/s	0·5 to 5 A shunts and c.t.s for higher ranges	Above 75 V
Electrodynamic wattmeters	Laboratory	Precision, d.c. and a.c. 25 to 150 c/s	0·25 to 20 A	75 V to 750 V
		Industrial	Up to 100 A	Up to 750 V
Thermocouple	With self-contained thermocouples	Industrial	On d.c. and a.c. up to 5×10^6 c/s 10 mA to 1 A	
Induction ammeters, voltmeters and wattmeters		Industrial	(At standard or marked frequency and temperature)	
Electrostatic voltmeters		Industrial		Up to 7,000 V d.c. and a.c. up to 10,000 c/s
Rectifier voltmeters and milliammeters		Industrial	1 mA to 50 mA for full-scale deflection	Not less than 10 V for full-scale deflection
			Frequency 25 to 2,000 c/s. Indications affected by wave-form	

The limits of error, and capabilities of different types, of recording instruments are given in B.S. 90 : 1940.

Moving-iron Instruments. There are two general types of such instruments, namely the *attraction* type and the *repulsion* type. In all moving-iron instruments the current to be measured (or a current proportional to the voltage to be measured) is passed through a coil of wire, the number of turns on which depends upon the current to be passed through it. A certain number of ampere turns is required for the operation of the instrument, and this number can be made up by having a few turns and a large current, or vice versa.

In the attraction form of instrument, a small piece of iron is drawn

into the core of the coil when the current flows. In the repulsion form there are two rods, or pieces, of iron inside the coil—one fixed and one movable. These are similarly magnetized when the current flows through the coil, and repulsion of the moving iron from the fixed one ensues. The force of repulsion is obviously roughly proportional to the square of the current in the coil, since each iron is magnetized thereby.

Whatever the direction of the current in the coil of the instrument, the magnetization of the moving iron is always such that attraction takes place in the attraction form and repulsion in the repulsion form. They are thus "unpolarized" instruments (i.e. instruments

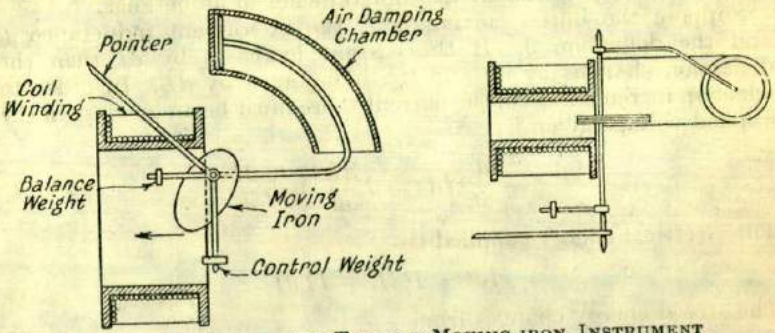


FIG. 18.1. ATTRACTION TYPE OF MOVING-IRON INSTRUMENT

which are independent of the direction in which current passes through them).

The principle of a moving-iron instrument of the attraction type is illustrated in Fig. 18.1, and of the repulsion type in Fig. 18.2.

In the instruments shown, gravity control is used. In the past, this method of control was very generally used for moving-iron instruments, but now spring control is used almost universally. The method of damping moving-iron instruments is by air friction, two different forms of damping chambers being shown in the figures.

In the attraction type of instrument the moving iron is eccentrically pivoted, and consists of thin discs of soft iron. This iron tends to move, when the current flows, from the weaker magnetic field outside the coil into the stronger field inside it. The shape of the disc is such that a suitably divided scale is obtained.

In the repulsion type, the shapes of the irons vary in different makes of instrument. They may be rods, or the fixed iron may consist of a tongue-shaped piece of sheet iron bent into a cylindrical form, the moving iron being another piece of sheet iron, bent and mounted so as to move parallel to the fixed iron and towards its narrower end. Such an arrangement is shown in Fig. 18.3. These shapes of the irons give a more uniform scale than is

obtained with plain rods. Figs. 18.4, 18.5 and 18.6 show the construction of moving-iron instruments by different makers. The Record Electrical Co., Ltd., make a "Cirscale" moving-iron instrument (see also p. 667) of the repulsion type, with two separately pivoted repulsion movements, to give a 270° scale.

THEORY OF THE MOVING-IRON INSTRUMENT. An expression for the torque of a moving-iron instrument may be derived by considering the energy relations when there is a small increment in current supplied to the instrument. When this happens there will be a small deflection $d\theta$ of the pointer and work will be done. This is given by $Td\theta$, where T is the instrument torque. There will also be a change in the energy stored owing to inductance.

Suppose the initial current is I, the instrument inductance L and the deflection θ. If the current increases by dI then the deflection changes by $d\theta$ and the inductance by dL. In order to effect an increment dI in the current there must be an increase in the applied voltage given by

$$e = \frac{d}{dt}(LI) = I\frac{dL}{dt} + L\frac{dI}{dt}$$

The electrical energy supplied is

$$eIdt = I^2dL + ILdI$$

The stored energy changes from

$$\tfrac{1}{2}I^2L \quad \text{to} \quad \tfrac{1}{2}(I + dI)^2(L + dL)$$

Hence the change in stored energy is

$$\tfrac{1}{2}(I^2 + 2IdI + dI^2)(L + dL) - \tfrac{1}{2}I^2L$$

Neglecting second- and higher-order terms in small quantities this becomes

$$ILdI + \tfrac{1}{2}I^2dL.$$

From the principle of the conservation of energy,

Electrical energy supplied = increase in stored energy + mechanical work done

$$I^2dL + ILdI = ILdI + \tfrac{1}{2}I^2dL + Td\theta$$

Thus

$$Td\theta = \tfrac{1}{2}I^2dL$$

or

$$T = \tfrac{1}{2}I^2\frac{dL}{d\theta} \quad . \quad . \quad . \quad . \quad (18.1)$$

T is in newton-metres, I in amperes, L in henrys, and θ in radians.

Errors in Moving-iron Instruments. The causes of errors in these instruments may be divided into those which occur with either direct or alternating current and those which occur only with alternating current.

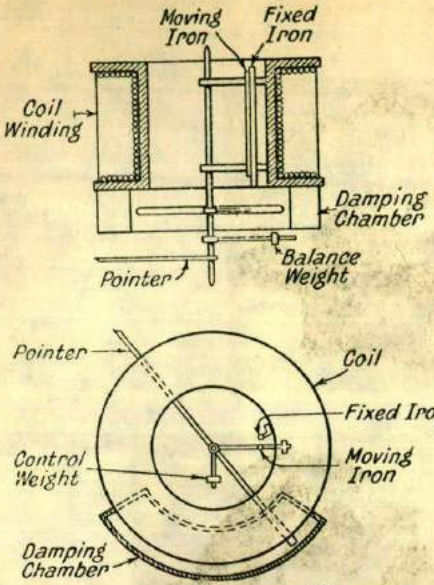


FIG. 18.2. REPULSION TYPE OF MOVING-IRON INSTRUMENT

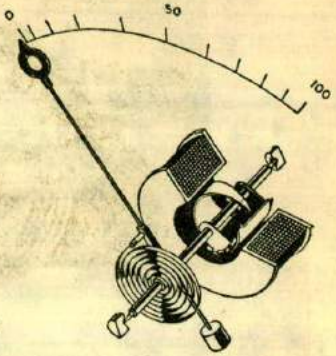
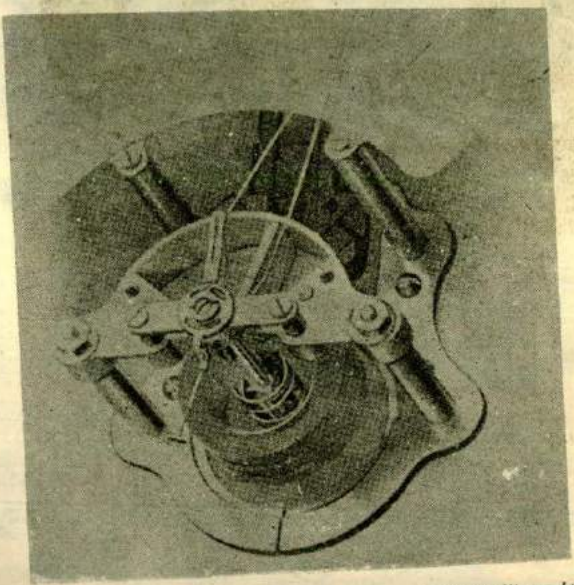
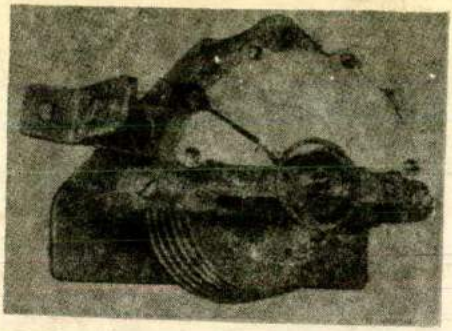


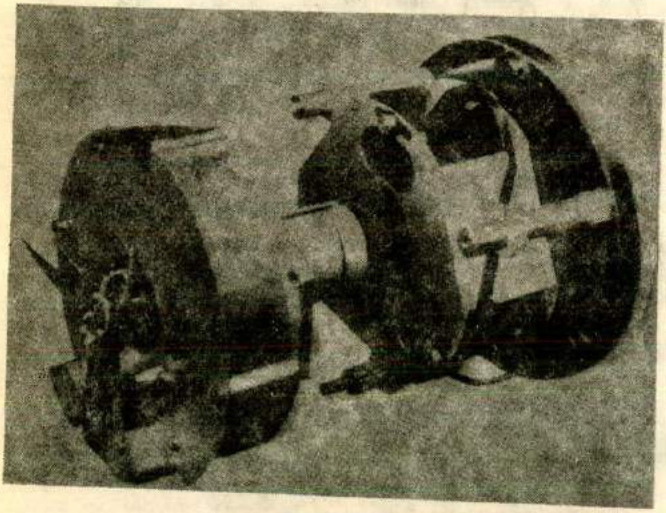
FIG. 18.3

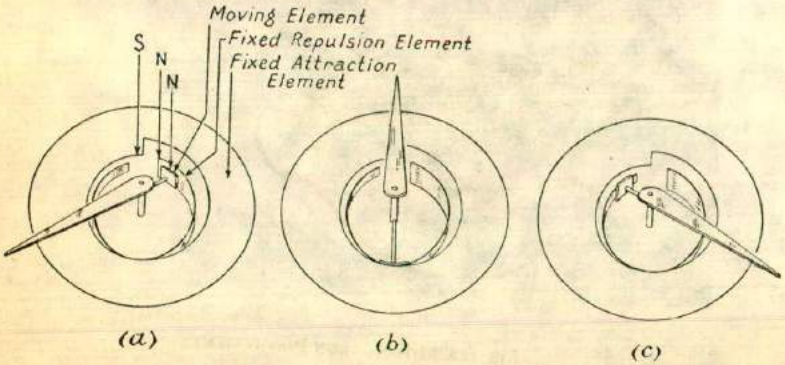


(Sangamo Weston Ltd.)

FIG. 18.4. MOVING-IRON INSTRUMENT

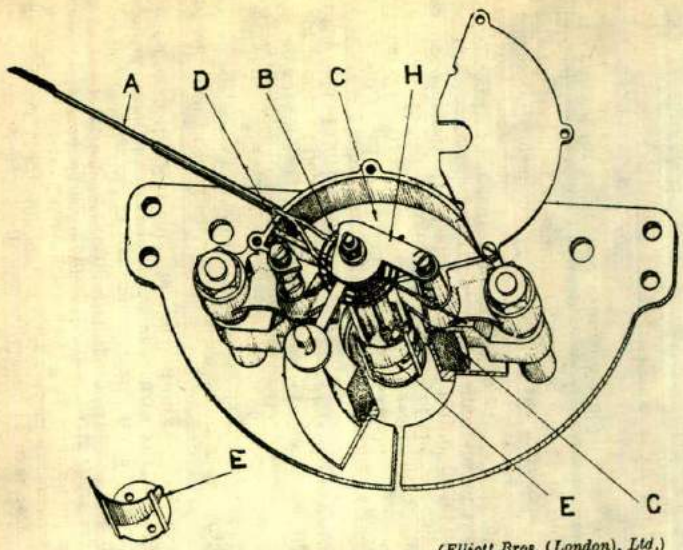


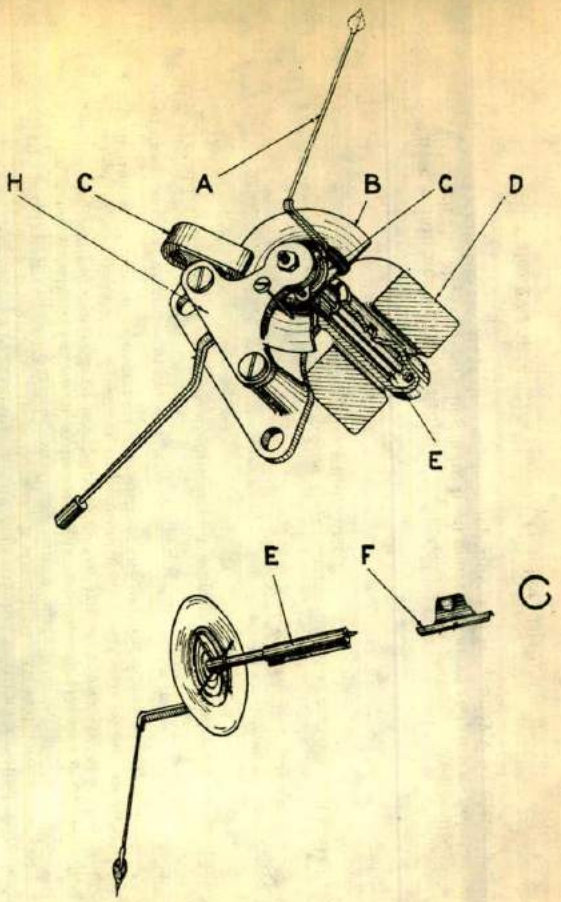




(*Crompton Parkinson, Ltd.*)

FIG. 18.5. CIRCULAR SCALE MOVING-IRON INSTRUMENT SHOWING (CENTRE) ARRANGEMENT OF SHIELDING IRONS





(Elliott Bros. (London), Ltd.)

FIG. 18.6. DETAILS OF MOVING-IRON INSTRUMENT (REPULSION TYPE)

(*a*) WITH BOTH D.C. AND A.C. (i) *Hysteresis Error.* This is a serious source of error in moving-iron instruments. Owing to hysteresis in the iron of the operating system the readings are higher when descending values of current or voltage are measured than when ascending values are observed.

The error is reduced by making the iron parts short so that they demagnetize themselves, or by choosing such (low) values of flux density in the iron that the hysteresis effect in the iron is small. A low flux density is, however, detrimental when the instrument is used with alternating current owing to the flattening of the *B-H* curve at the bottom end. A compromise is usually made.

Another effect of hysteresis is to cause an error due to the change in the position of the poles in the moving iron as its position changes. This "position" error is usually small.

(ii) *Stray Magnetic Fields.* Errors due to this cause may be serious if not guarded against, owing to the weakness of the operating magnetic field. The error produced depends upon the direction of the stray field relative to that of the field of the instrument.

Such errors are minimized by magnetic screening of the working part of the instrument by an iron case or a thin iron shield.

(*b*) WITH A.C. ONLY. (i) *Frequency Errors.* Changes of frequency may produce errors due to changes of reactance of the working coil and also to changes in the magnitudes of eddy currents set up in metal parts of the instrument near to the working portion. The magnitudes of these eddy currents, and consequently their effect upon the magnetic field of the instrument, vary with frequency.

The change of impedance of the coil of the instrument, of resistance R (together with its series resistance r), due to change of frequency, is only of importance in the case of voltmeters. If L is the inductance of this coil circuit, the current for an applied voltage E will be given by

$$I = \frac{E}{\sqrt{(R+r)^2 + \omega^2 L^2}}, \text{ where } \omega = 2\pi \times \text{frequency}$$

Thus, if the frequency changes, the current for a given applied voltage changes, and hence an error in deflection is produced. In order that the error shall be small, L must be made small compared with $R + r$.

The time-constant for such voltmeters is usually of the order of 0·0005. There are several methods of compensating for frequency error, one of which consists in connecting a capacitor in parallel with r. It can be shown that the impedance of the whole circuit (including the working coil) is independent of frequency if $C = \frac{L}{r^2}$, where C is the the capacitance of the capacitor.

Total impedance of the instrument circuit when r is shunted by a capacitor C is

$$R + j\omega L + \frac{r}{1 + j\omega Cr} = R + j\omega L + \frac{r}{1 + \omega^2 C^2 r^2} - \frac{j\omega Cr^2}{1 + \omega^2 C^2 r^2}$$

For independence of frequency this should equal $R + r$, and equating real and imaginary terms, we have the conditions

$$r = \frac{r}{1 + \omega^2 C^2 r^2} \text{ and } C = \frac{L}{r^2}$$

These conditions are only fulfilled if $\omega^2 C^2 r^2$ is small compared with unity. Although the above theory had long been generally accepted, it was pointed out* that while the circuit is thus made independent of frequency the resistance

* See *The Wireless Engineer*, October, 1940, p. 429.

of the voltmeter is reduced from $R + r$ to $R + \frac{r}{1 + \omega^2 C^2 r^2}$ and that $\omega^2 C^2 r^2$ is not usually sufficiently small for this reduction to be negligible.

Both graphical and calculation methods of obtaining the correct value of C to avoid this are given in the article.

DESIGN DATA FOR MOVING-IRON INSTRUMENTS. Table XIX, which is compiled from data given by various writers on measuring instruments and from figures supplied by instrument manufacturers, gives average values of the various quantities involved in the design of moving-iron instruments (see also Ref. (1)).

TABLE XIX

Quantity	Voltmeters	Ammeters
Weight of moving system	4 g	3 g
Torque at full deflection	0·2 g-cm	0·2 g-cm
$\frac{\text{Torque}}{\text{Weight}}$ ratio	0·05	0·067
Resistance	20 ohms per volt of range	0·01 ohm
Inductance	1 henry	—
Watts lost	8	4
Coil surface per watt	16 sq. cm	
Ampere-turns at full deflection	300	300

Moving-coil Instruments. There are two types of moving-coil instruments, namely the permanent-magnet type, which can only be used for direct-current measurements, and the dynamometer type, which can be used on either direct or alternating current circuits.

1. PERMANENT-MAGNET TYPE. The principle of this type of instrument is the same as that of the moving-coil, or D'Arsonval, galvanometer. A light rectangular coil is pivoted so that its sides lie in the air gaps between the two poles of a permanent magnet and a soft-iron cylinder. When current passes through the coil a deflecting torque is produced owing to the reaction between the permanent-magnet field and the magnetic field of the coil. This is illustrated in Fig. 18.7. The air gap between the magnet poles and iron core is small (about 0·05 in.), and the flux density is uniform and is in a radial direction. If a current I flows in the moving coil in the direction shown, forces F, F, will act on the two sides of the coil which are in the field. The torque causing the coil to rotate is thus $2Fr$, where r is the mean distance of the wires forming the sides of the coil from the axis of rotation.

If there are N turns on the coil and the flux density in the air gap is B, the force F (newtons), for a given current I (amperes) in the coil is given by

$$F = NBIl$$

where l is the active length in metres of the sides of the coil in the air gap.

Since B is uniform, the deflecting torque is constant for all positions of the moving coil, provided its sides are within the pole arcs of the magnet.

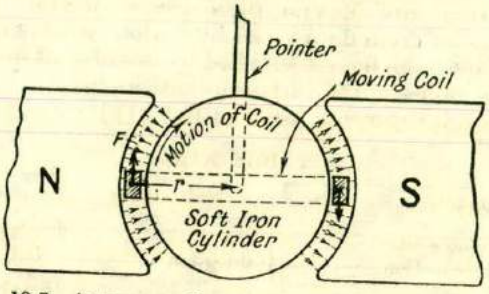
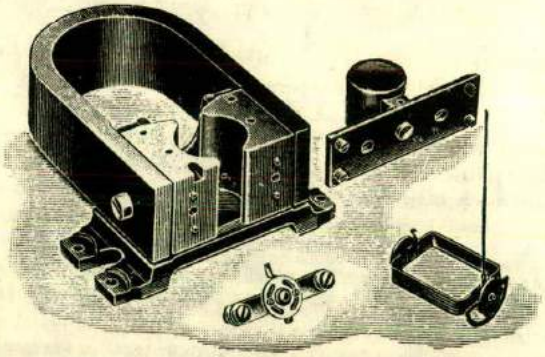


FIG. 18.7. ACTION OF PERMANENT-MAGNET MOVING-COIL INSTRUMENT



(*Nalder Bros. and Thompson, Ltd.*)

FIG. 18.8. PARTS OF MOVING-COIL INSTRUMENT

The full expression for the torque in newton-metres is

$$T = 2r \times NBIl \quad . \quad . \quad . \quad . \quad . \quad . \quad (18.2)$$

$$= \text{Ampere-turns on coil} \times \text{Area of coil} \times B$$

Constructional Details. Moving-coil instruments are spring-controlled. Two phosphor-bronze hair springs are used, these also serving as leads to the moving coil. Fig. 18.8 shows the construction of a moving-coil instrument. A bracket of non-magnetic material carries the iron cylinder. which fits inside the coil. Since the deflecting torque is directly proportional to the current and

the controlling torque of the springs is proportional to the deflection θ, we have

$$T_D \propto I$$

and

$$T_C \propto \theta$$

But

$$T_D = T_C$$

when the moving system is at rest in its deflected position;

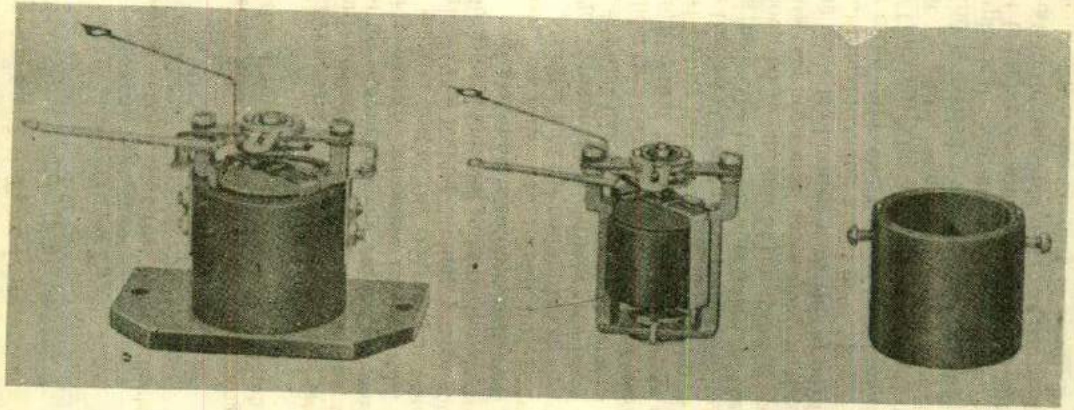
$$\therefore \theta \propto I$$

Thus the scale is uniformly divided. Damping is by eddy currents induced in the aluminium former upon which the moving coil is wound.

Types of Permanent-magnet Moving-coil Instruments. The form of instrument already described is perhaps the commonest. Its development for commercial use was largely the work of Dr. Weston.

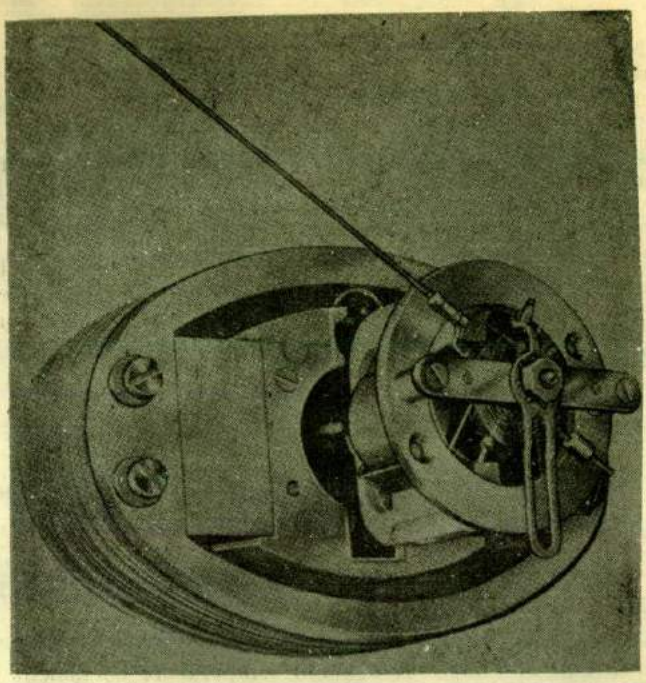
The other types are the same in principle, but different in the form of the magnet and in the relative positions of the magnet and moving system. The Nalder-Lipman patent moving-coil movement shown in Fig. 18.9 is unusual in that there is, inside the coil, a cylindrical magnet of nickel-aluminium-cobalt-steel alloy with very high coercive force. An outer ring of soft iron provides the return path for the magnetic flux and the coil moves in the uniform field between the two. The instrument is entirely free from interference by stray magnetic fields and has a very low power consumption. Figs. 18.10 and 18.11 show the internal construction of two Crompton-Parkinson instruments, the first having a circular scale and the second being of the self-shielding pattern. Fig. 18.12 shows the original arrangement of the Record "Cirscale" instrument. One pole of the permanent magnet lies inside the two parallel halves of the other pole. The coil, which is mounted to one side of the spindle, fits round the inner pole as shown, its two horizontal sides lying in the two inter-polar gaps. A very long scale is obtained with this type of instrument, the total deflection obtainable being about 300°. Fig. 18.13 shows the working system of an improved form of instrument of this type.

In the "Unipivot" instrument manufactured by the Cambridge Instrument Co., Ltd., the moving coil is circular instead of rectangular, and is carried by one pivot in the top of the iron core, which is spherical. The construction is shown in Fig. 18.14. The instrument is highly accurate, the friction torque being reduced by the elimination of one pivot. It is frequently used for current measurements when the current is small, but is not very robust, and needs rather careful handling. The current is led into the moving coil by the single control spring and passes out by a ligament at the bottom of the coil. Such instruments must, of course, be used only in a horizontal position.



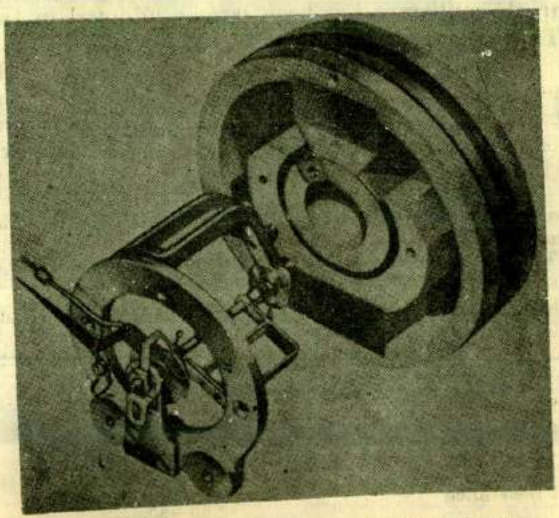
(*Nalder Bros. & Thompson, Ltd.*)

FIG. 18.9. CONSTRUCTION OF NALDER-LIPMAN PATENT MOVING-COIL INSTRUMENT



(*Crompton Parkinson, Ltd.*)

FIG. 18.10. MAGNET SYSTEM OF SELF-SHIELDING-PATTERN MOVING-COIL INSTRUMENT WITH MOVEMENT EXTRACTED



(*Crompton Parkinson, Ltd.*)

FIG. 18.11. MOVEMENT OF CIRCULAR-SCALE MOVING-COIL INSTRUMENT

Extension of Range of Moving-coil Instrument. Owing to the low current-carrying capacity of the moving coil, these instruments are very largely used in conjunction with shunts—when used as ammeters—and with a high series resistance when used as voltmeters. The extension of range of instruments will be discussed more fully in the next chapter. Ammeter shunts are low-value resistors which are connected in parallel with the instrument so that only a small fraction of the current to be measured actually passes through the latter. The shunt itself is usually of manganin strip or tube, and has, therefore, a very small temperature coefficient. The

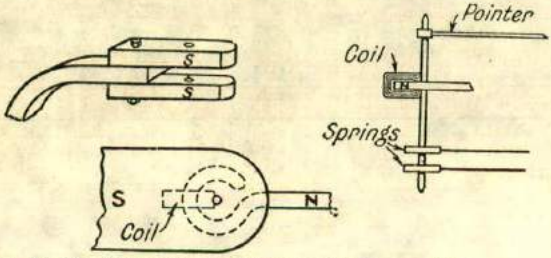


FIG. 18.12. DETAILS OF RECORD "CIRSCALE" INSTRUMENT

instrument must also have a small temperature coefficient if the ratio $\frac{\text{Instrument current}}{\text{Total current}}$ is to remain constant.

B.S. 89 : 1954 specifies a voltage drop, across the shunt and instrument in parallel, of 0·075 volt. An operating current of 0·015 amp (15 milliamp) has been largely adopted in these instruments. Thus, the total resistance of the instrument (including the leads which connect it to the potential terminals of the shunt) must be 5 ohms. Of this, only about 1 ohm should be in the coil and copper leads, the remaining 4 ohms being provided by a "swamping" resistor of manganin. The advantage of this arrangement is shown in the following example.

Example. A moving-coil instrument whose resistance is 5 ohms and whose working current (for full-scale deflection) is 0·015 amp is to be used with a manganin shunt, to measure up to 100 amp. Calculate the error caused by a 10° C rise in temperature—

(*a*) when the whole of the 5 ohms is in the copper of the instrument coil and leads,

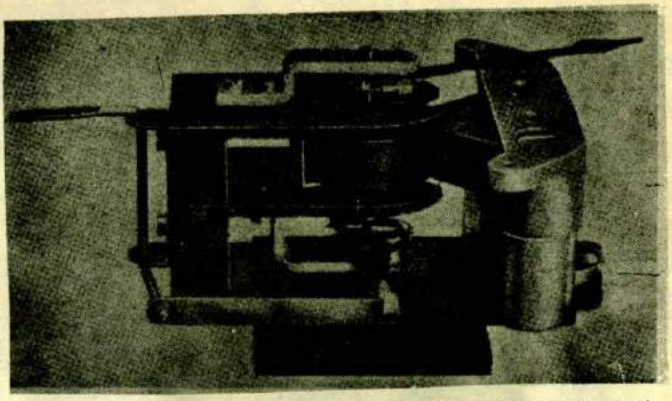
(*b*) when a 4 ohm manganin swamping resistor is used with a coil and leads of 1 ohm resistance.

Instrument current = 0·015 amp

∴ Shunt current = 99·985 amp

Voltage drop across the shunt = 0·075 volt

∴ Shunt resistance $= \frac{0{\cdot}075}{99{\cdot}985} = 0{\cdot}00075$



(Record Electrical Co., Ltd.)

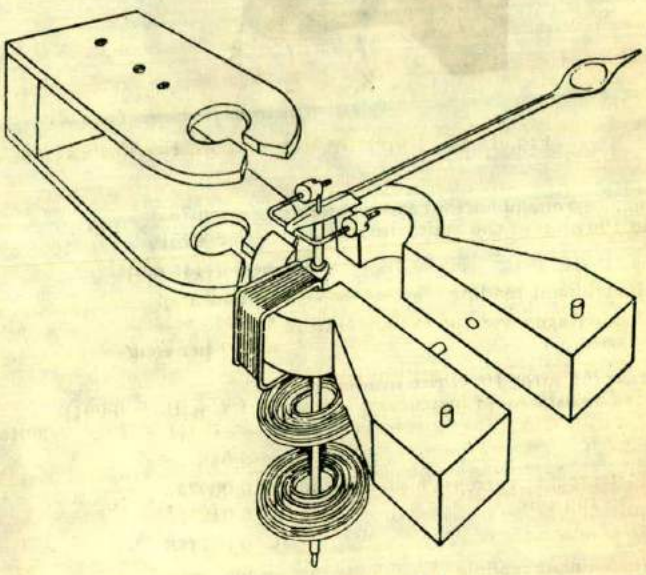


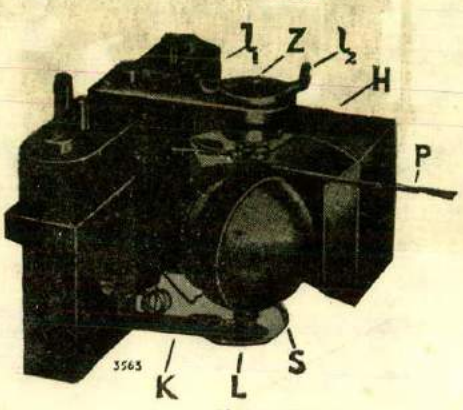
FIG. 18.13. RECORD "CIRSCALE" INSTRUMENT

(*above*) Movement complete
(*below*) Details of pole irons

Take the temperature coefficient of copper as 0·0040 ohm/ohm/°C, and of manganin 0·00015 ohm/ohm/°C.

Then shunt resistance at 10° C. rise in temperature $= 0{\cdot}00075\,(1 + 10 \times 0{\cdot}00015$
$= 0{\cdot}000751$

In case (*a*), instrument resistance after 10° C rise in temperature $= 5\,(1 + 10 \times 0{\cdot}004)$
$= 5{\cdot}2$



(*Cambridge Instrument Co., Ltd.*)

FIG. 18.14. UNIPIVOT MOVING-COIL INSTRUMENT

Then, instrument current corresponding to 100 amp in the main circuit $= \dfrac{0{\cdot}000751}{5{\cdot}200751} \times 100$
$= 0{\cdot}01444$ amp

$\therefore$ Instrument reading $= 96{\cdot}3$ amp

$\therefore$ Percentage error due to temperature rise $= 3{\cdot}7$ per cent

In case (*b*), after 10° C rise in temperature, resistance of instrument circuit $= 1(1 + 10 \times 0{\cdot}004) + 4(1 + 10 \times 0{\cdot}00015)$
$= 5{\cdot}046$

$\therefore$ Instrument current corresponding to 100 amp $= \dfrac{0{\cdot}000751}{5{\cdot}046751} \times 100$
$= 0{\cdot}001488$

$\therefore$ Instrument reading $= 99{\cdot}2$ amp

$\therefore$ Percentage error due to temperature rise $= 0{\cdot}8$ per cent

Thus, the effect of using the manganin swamping resistor is to reduce the percentage error for 10° C rise in temperature from 3·7 per cent to 0·8 per cent.

Fig. 18.15 shows the connections of a moving-coil instrument when used, (*a*) as an ammeter for large currents, (*b*) as a voltmeter. If the voltage drop across the shunt is 0·075 volt and the working current of the instrument 0·015 amp, the total power loss is $0{\cdot}075I$ when used as an ammeter (I being the current to be measured), and $0{\cdot}015V$ when used as a voltmeter, V being the voltage to be measured. In general, the power loss in permanent-magnet moving-coil instruments is lower than that in other types of instruments.

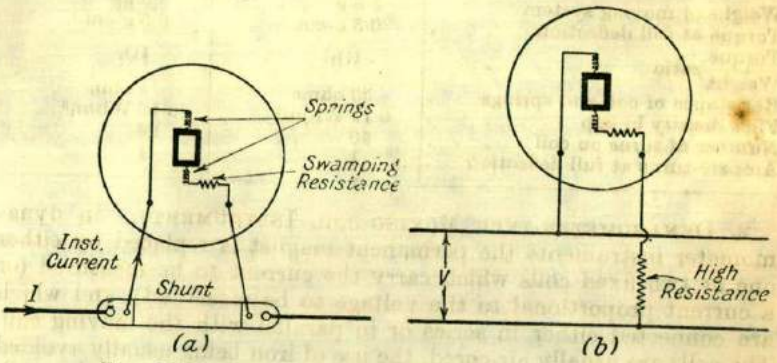


FIG. 18.15. CONNECTIONS OF MOVING-COIL INSTRUMENT

Principal Advantages of Permanent-magnet Moving-coil Instruments. These are—

(*a*) Low power consumption.

(*b*) High $\frac{\text{Torque}}{\text{Weight}}$ ratio.

(*c*) Uniformity of the scale and the possibility of a very long scale.

(*d*) The possibility of a single instrument being used, with shunts or series resistors, to cover a large range of current or voltage.

(*e*) Freedom from hysteresis errors and, very largely, from errors due to stray magnetic fields.

(*f*) Perfect damping, simply afforded by eddy currents induced in the metal former of the moving coil.

Errors. Friction and heating errors are, of course, present as in other types of instrument. The weakening of the permanent magnet with the passage of time may introduce a considerable error unless the magnet is carefully aged during manufacture. Thermo-electric e.m.f.s may introduce errors when these instruments are used shunted, for current measurements, but with a well-designed shunt such error should be small.

Design Data. The following table gives approximate values of various quantities in connection with the design of permanent-magnet moving-coil instruments.

TABLE XX

Quantity	Voltmeters	Ammeters
Weight of moving system	3·5 g	3·5 g
Torque at full deflection	0·5 g-cm	0·5 g-cm
$\frac{\text{Torque}}{\text{Weight}}$ ratio	$\frac{1}{4}$th	$\frac{1}{4}$th
Resistance of coil and springs	50 ohms	1 ohm
Flux density in gap	0·12 Wb/m^2	0·12 Wb/m^2
Number of turns on coil	50	20
Ampere-turns at full deflection	1	1

2. Dynamometer-type Moving-coil Instruments. In dynamometer instruments the permanent magnet is replaced by either one or two fixed coils which carry the current to be measured (or a current proportional to the voltage to be measured), and which are connected either in series or in parallel with the moving coil. The coils are usually air-cored, the use of iron being usually avoided in such instruments owing to its introduction of hysteresis, eddy current, and other errors when the instrument is used for a.c. measurements. The general arrangement is shown in Fig. 18.16, in which the connections of a dynamometer ammeter (*b*) and of a dynamometer voltmeter (*c*) are also shown.

The torque of the instrument is dependent upon the strengths of the magnetic fields of both fixed and moving coils—i.e. in an ammeter the torque is roughly proportional to the current squared, and in a voltmeter to the voltage squared. Dynamometer instruments can thus be used in alternating-current circuits, for which a square law is essential.

Two hair-springs are used for the control and as leads to the moving coil. Damping is often by air piston or enclosed vane, although in some cases eddy-current damping, by an aluminium disc rotating in a permanent-magnet field, is used. In ammeters of this type the current which can be measured without the use of a shunt is small owing to the difficulty of leading-in the heavy currents to the moving system, and also on account of the heavy moving coil which would be necessitated if this were to have a high current-carrying capacity. Thus the moving coil is usually connected, in series with its swamping resistors, across a shunt together with the fixed coils, as shown in Fig. 18.16 (*b*). By this means the time-constants of the two parallel branches can be more easily made equal—an essential requirement if the division of current between

the two paths is to be independent of frequency when the instrument is used for alternating-current measurements.

Disadvantages for Direct-current Use. Although useful for precise measurements on alternating-current circuits, these instruments compare unfavourably with the permanent-magnet type owing to the following disadvantages—

(*a*) The magnetic field strength obtainable is small, owing to the absence of iron. This means that a comparatively large number of ampere-turns must

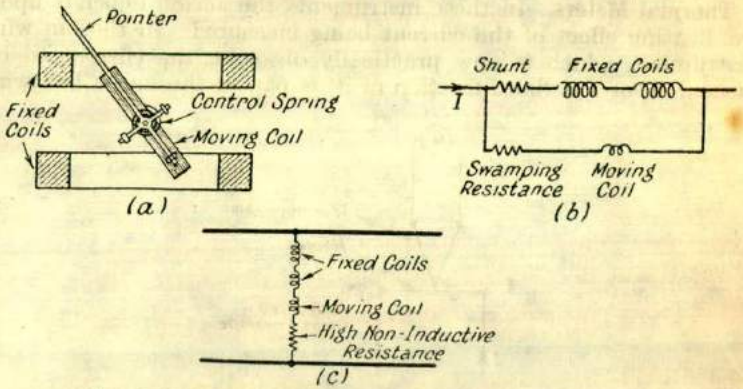


FIG. 18.16. DYNAMOMETER MOVING-COIL INSTRUMENT

be used on the moving coil in order to obtain the necessary deflecting torque. A heavy moving system and high power loss are the results. The $\frac{\text{torque}}{\text{weight}}$ ratio is small, and hence friction errors tend to be serious as well as those due to internal heating.

(*b*) Owing to the deflecting torque varying with current according to an approximate square law, the scale is not uniform.

(*c*) Such instruments are more expensive than the types of ammeters and voltmeters already described.

For these reasons, dynamometer ammeters and voltmeters are not in common use, especially in direct-current circuits. The most important application of the dynamometer principle is the dynamometer wattmeter.

For full descriptions of the various types of dynamometer instruments and for the theory of the instruments, the reader is referred to the works mentioned in Refs. (1) and (2).

A. H. M. Arnold (Ref. (31)) has discussed the performance limits of dynamometer instruments at audio frequencies. He gives curves showing the frequency errors of dynamometer ammeters both with air cores and with nickel-iron cores, and shows the improvement which can be obtained through compensation by connecting a capacitance and resistance in series across the instrument terminals.

Arnold shows that dynamometer voltmeters with nickel-iron

cores are superior to those with air cores because of their greater inductance, and that the effective frequency range can be extended by connecting a capacitor in parallel with the series resistor. K. A. Macfadyen and N. D. Hill (Ref. (32)) have developed a negative-feedback amplifier circuit for use with a dynamometer instrument in measurements at 50 c/s with distorted wave-forms. This dispenses with the need for a series resistor for the instrument.

Thermal Meters. In these instruments the action depends upon the heating effect of the current being measured. In the hot-wire instrument, which is now practically obsolete, the current to be measured, or a definite fraction of it, is passed through a fine wire

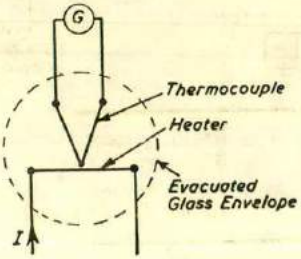


FIG. 18.17. THERMAL CONVERTER

which expands due to the heating effect of the current. If the resistance and coefficient of expansion of the wire are constant, the heating and expansion are both directly proportional to the square of the current. The expansion of the hot wire is used to produce a deflection of a pointer, this deflection being magnified by an ingenious arrangement of coupled wires held taut by a spring. Such an instrument has many disadvantages, however, and it has been superseded by instruments using thermal converters.

THERMAL CONVERTERS. The thermal converter consists of a heater through which the current to be measured passes, a thermocouple sensing the temperature rise at the mid-point of the heater. A sensitive low-resistance permanent-magnet moving-coil instrument is connected to the thermocouple. The heater is a straight wire or strip mounted between relatively massive conducting supports. The device is shown schematically in Fig. 18.17. Provided that the heat loss from the heater wire, due to convection and radiation, is small—i.e. the heat flow is mainly by conduction along the wire into the heat sinks constituted by the massive supports—the temperature distribution along the wire is parabolic, and the temperature rise at its mid-point is proportional to the square of the current to within close limits.

Thermal converters for use with heater currents below about 1 ampere are normally enclosed in evacuated glass bulbs, shown

by the broken circle in Fig. 18.17. In this form they are commonly known as vacuo-thermo-junctions or vacuum thermocouples. Enclosure of the device *in vacuo* eliminates thermal loss due to air convection, gives mechanical protection, ensures freedom from corrosion due to atmospheric effects, and gives protection from draughts. The thermocouple is in thermal contact with the heater, but should be electrically insulated from it. Otherwise, undesirable reversal errors may occur, when the instrument is calibrated on direct current, due to passage of part of the heater current across the thermo-junction.

Vacuo-thermo-junctions are manufactured for full rated heater currents down to 1·25 mA. The voltage drop in the heater ranges typically from 0·15 V for a 0·5 A heater to 2 V for a 1·25 mA heater. The mid-point temperature rise is about 150° C, and the thermocouple e.m.f. at full heater current is in the region of 6 mV. It is apparent, therefore, that a sensitive indicator must be used with them, but there are many modern pointer-type permanent-magnet moving-coil instruments which are perfectly suitable for this purpose; the Cambridge Unipivot instrument is a typical example.

Frequency Range. Instruments using vacuo-thermo-junctions have an extensive frequency range and are completely free from waveform error. The heater consists of a thin straight wire; its inductance and capacitance are very low, and the upper frequency limit is usually determined by the connections to the heater. Special forms, in which the heater and supports are in a straight line, will give accurate results at frequencies in the region of 20 Mc/s and above. The upper frequency limit in this case is usually ultimately determined by the so-called "skin effect" (see Chapter V), i.e. the current tends to crowd towards the outside of the conductor at high frequencies, giving an increase in heating effect, due to the increase in effective resistance of the conductor owing to the diminished area in which the current flows. The skin effect is related to the diameter of the conductor: the smaller the diameter of the conductor used at a given frequency the less the skin effect. Hence the larger the heater rating the lower is the upper frequency limit.

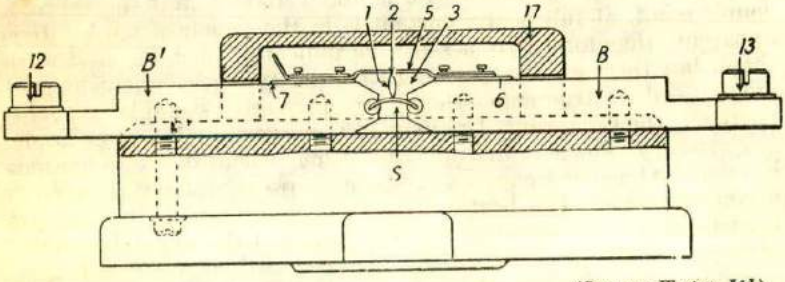
Ranges. Vacuo-thermo-junctions are made in heater ratings between 1·25 mA and 1 A, the current range being doubled at each step. Above 1 A the thermal converters are no longer *in vacuo* and are made for currents up to 50 A.

The Cambridge Instrument Co., Ltd., make a multi-range instrument, suitable for use up to 10 Mc/s, in which the range is changed by switching-in any one of six vacuo-thermo-junctions.

Multi-range instruments for use in the audio-frequency range can have a single vacuo-thermo-junction, and the current and voltage ranges are changed by means of shunts and multipliers in a similar manner to that used in direct-current instruments. The heater

resistance changes slightly from zero to full current, and some additional swamping resistance in series with the heater may be necessary when shunts are used.

Temperature Effects. Vacuo-thermo-junctions usually have temperature coefficients in the region of 0·2 per cent per °C. This temperature coefficient is believed to be mainly due to uncertainty in the position and temperature of the cold junction of the thermo-couple. Goodwin (Ref. (23)) has shown that if the cold junction is at the mean temperature of the heater supports, the temperature coefficient largely vanishes. It is impracticable to use this principle in the vacuo-thermo-junction, but it has been successfully employed in the heavy-current converters which are not *in vacuo*. A 50 A thermal converter using this principle is shown in Fig. 18.18.



(*Sangamo Weston, Ltd.*)

FIG. 18.18. HEATING ELEMENT FOR THERMO-AMMETER (ELEVATION)

Between the terminals BB' a platinum-alloy resistor strip S is hard-soldered. The hot junction of a thermo-couple is hard-soldered, or welded, to the centre of this strip. The cold ends of this thermo-couple are soldered to the centre points of two copper compensating strips, which are in thermal contact with the terminals BB', but are insulated from them by thin mica strips. The compensating strips are so proportioned that they are thermally equivalent to the strip S, and the cold junctions are therefore at the mean temperature of the supports. A surrounding case (17) shields the heater from external air currents, and this case, together with the compensating strips, ensures that the indications of the instrument are independent of external temperature variations and of air currents.

Disadvantages. These devices are fragile and have very limited overload capacity, 50 per cent being the safe limit.

Since they are true square-law devices the scale tends to be very cramped below half full-scale current. Range multipliers therefore must use a scale of two.

They are somewhat sensitive to ambient temperature changes unless some compensating arrangement is provided.

Although the power consumption is substantially greater than that of a permanent-magnet moving-coil instrument, it is not so large as to be classed as a disadvantage.

PRECISION MEASUREMENTS WITH THE VACUO-THERMO-JUNCTION. The vacuo-thermo-junction can be used for the most precise measurement of alternating currents and voltages (Ref. (33)), and accuracies of 1 part in 10^4 in current measurement in the audio-frequency range are possible. The procedure may be understood by reference to Fig. 18.19.

The alternating current being measured is supplied to the heater

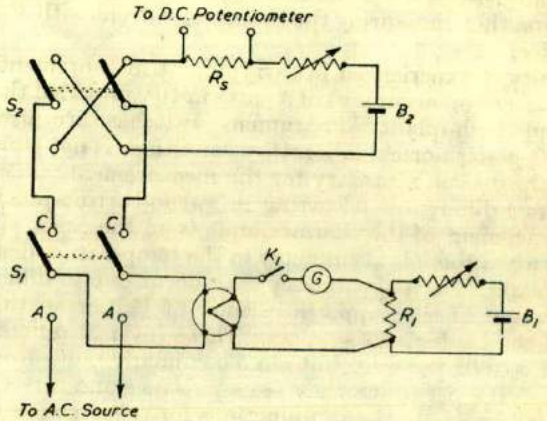


FIG. 18.19. PRECISE ALTERNATING-CURRENT MEASUREMENT

of the vacuo-thermo-junction when the switch S_1 is in position AA. The output e.m.f. from the thermocouple is balanced against the potential difference across the resistor R_1, by adjusting the current supplied from the battery B_1 to R_1. In practice, only a rough balance is necessary, the deflection of the galvanometer G being noted. The sensitivity of the galvanometer should be such that a change of heater current of 2 parts in 10^5 can be detected, i.e. the galvanometer must detect a change of about 0·25 μV in a couple resistance of 9 ohms. The heater is next connected to a direct-current source by changing S_1 to CC. The galvanometer circuit must be opened during the switching operation to protect the galvanometer. The direct current from the battery B_2 is then adjusted until the same deflection is obtained on the galvanometer. This current is then measured with the aid of the standard resistor R_S and a precision d.c. potentiometer, and the r.m.s. value of the alternating current will then be equal to the direct current.

Certain difficulties which arise in practice complicate the procedure

slightly. If the polarity of the direct current through the heater is reversed, a different indication will be obtained on the galvanometer. This is because the vacuo-thermo-junction has so-called *reversal difference*. The true r.m.s. value of the alternating current is given by the mean of the forward and reverse direct currents for the same indication. In the measuring procedure outlined, the direct-current measurement must be corrected by an amount equal to half the reversal difference to obtain the true alternating current. The reversal difference can be readily measured at any current setting by operating the reversing switch, noting the change of galvanometer deflection, adjusting the heater current to restore the original deflection, and then measuring the change of current with R_S and the potentiometer.

No difficulty is experienced in carrying out measurements with a precision (but not an accuracy) of 5 parts in 10^5, provided that stable supplies and high-quality instrument switches are used. The thermal drift under normal laboratory conditions is not troublesome during the short time necessary for the measurement.

The reversal differences occurring in vacuo-thermo-junctions are due to displacement of the thermocouple from the exact mid-point, and appear when there is asymmetry in the temperature distribution along the heater. Such asymmetry may occur due to differences in the temperature of the supports because of Peltier heating at the junction of the heater and support. These reversal differences can be as low as $\pm$ 0·01 per cent, but more commonly range from $\pm$ 0·05 per cent to $\pm$ 0·2 per cent; they are a nuisance and it is a common practice to select vacuo-thermo-junctions for precise work to obtain the lowest possible reversal difference.

It has been stated earlier that the mean of the direct-current calibrations gives the alternating-current calibration. There is, however, a residual error known as the *transfer error*. This transfer error is known to be due to the presence of Thomson heating in the heater wire during the direct-current test. The Thomson effect is named after Lord Kelvin, who showed that, when a current flows through a wire along which there is a temperature gradient, heat is absorbed or liberated in addition to Joule heating, depending upon the direction of the current flow and the heater material. This effect causes asymmetry in the direct-current test which is not present, due to thermal inertia, in the alternating-current test. It is known that a heater of 80 Ni, 20 Cr (an alloy with a small Thomson coefficient) gives a residual transfer error of less than one part in 10^4.

With the direct measurement described, an accuracy of $\pm$ 1 in 10^4 is possible, but if shunts and multipliers are used to extend the range, the accuracy falls off to about $\pm$ 5 in 10^4. Further information of these techniques will be found in Refs. (33), (34), (37).

An instrument using a similar principle for the precise measurement of alternating currents and voltages is made by the Croydon

Precision Instrument Co. This device has voltage ranges from 3 to 200 volts and current ranges from 10 mA to 5 A. An accuracy of $\pm$ 0·05 per cent in voltage and current measurement is claimed over the frequency range 25 c/s to 10 kc/s.

Electrostatic Instruments. Such instruments are essentially voltmeters, and although they may be applied to the measurement of current and power, such applications involve the measurement of the voltage drop across a known impedance.

Their advantages are that they give equally correct measurements in a.c. and d.c. circuits, and that since no iron is present in their working system, they are free from all errors in connection with magnetic fields in iron, such as those due to hysteresis and eddy

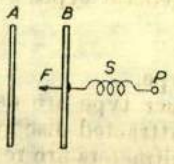


FIG. 18.20. PRINCIPLE OF ELECTROSTATIC VOLTMETER

currents. Wave-form and frequency variations, also, are unimportant, and the power loss in such instruments is also extremely small.

They have the disadvantage, however, that the operating forces are very small, especially for low voltages (of the order of several hundred volts), their most useful range being from about 500 volts upwards to several hundred kilovolts.

General Theory of the Electrostatic Voltmeter. All electrostatic instruments rely for their operation upon the fact that a force exists between two plates having opposite charges. This force may be deduced by using the principle that, if there is relative motion of the plates, the mechanical work done is equal to the change in stored energy. This principle can be simply justified as follows—

Referring to Fig. 18.20, plate A is fixed and plate B is movable and restrained by a spring S connected to the fixed point P. Let a potential difference V volts be applied to the plates; then a force of attraction F newtons exists between them. The plate B moves towards A until this force is balanced by that of the spring S. The capacitance between the plates is then C farads and the stored energy is $\frac{1}{2}CV^2$ joules.

Now let there be a small increment dV in the applied voltage; then the plate B will move a small distance dx towards A. When the voltage is being increased, a capacitance current will flow, given by

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} + V\frac{dC}{dt}$$

The input energy is

$$Vidt = V^2dC + CVdV$$

The change in stored energy is

$$\begin{aligned} &\tfrac{1}{2}(C + dC)(V + dV)^2 - \tfrac{1}{2}CV^2 \\ &= \tfrac{1}{2}(C + dC)(V^2 + 2V\,dV + dV^2) - \tfrac{1}{2}CV^2 \\ &= \tfrac{1}{2}V^2\,dC + CV\,dV \end{aligned}$$

neglecting the higher order terms in small quantities.
From the principle of the conservation of energy,

$$\text{Input electrical energy} = \text{increase in stored energy} + \text{mechanical work done}$$

$$V^2\,dC + CV\,dV = \tfrac{1}{2}V^2\,dC + CV\,dV + F\,dx$$

from which

$$F = \tfrac{1}{2}V^2\frac{dC}{dx}. \qquad . \quad . \quad . \quad . \quad (18.3)$$

TYPES. There are two general types of electrostatic voltmeters, namely—

(*a*) The quadrant type.
(*b*) The attracted-disc type.

Instruments of the former type are used for voltages up to 10 or 20 kilovolts, while the attracted-disc type is general for voltages above this. Since these voltmeters are really modifications of Lord Kelvin's quadrant and attracted-disc electrometers, the theory of these electrometers will now be discussed, the same theory holding for the voltmeters under consideration.

Quadrant Electrometer. The principle of this instrument is illustrated by the line diagrams in Fig. 18.21. There are four fixed metal double quadrants arranged so as to form a shallow circular box with short air gaps between the quadrants. Inside this (incompletely closed) box a thin metal "needle" is suspended by means of a thread of phosphor-bronze or silvered quartz. The needle is of a double-sector shape, and is suspended so as to be equidistant from the quadrant plates, which are above and below it. A plan view is shown in diagram (*a*) of Fig. 18.21. In diagrams (*b*) and (*c*), two methods of connection to the quadrants and needle are shown. In diagram (*b*) a high-voltage battery is used to charge the needle to a potential considerably above that of the quadrants to which the negative of the voltage to be measured is connected. When so connected, the electrometer is said to be used "heterostatically." When the needle is connected directly to one pair of quadrants, as in diagram (*c*), the electrometer is used "idiostatically."

The idiostatic connection is generally used in commercial instruments. With the polarities shown in diagram (*b*), end *A* of the needle is repelled by the fixed quadrant adjacent to it, while end *B* is attracted by its adjacent fixed quadrant, so that rotation of the needle is produced. In diagram (*c*), end *B* of the needle is repelled, and end *A* attracted, by the fixed quadrants near them.

The torque producing rotation will be shown (below) to be proportional to the square of the voltage to be measured in the case of

the idiostatic connection. Hence, the instrument can be used to measure alternating voltages.

Actually, of course, the forces of attraction and repulsion are not

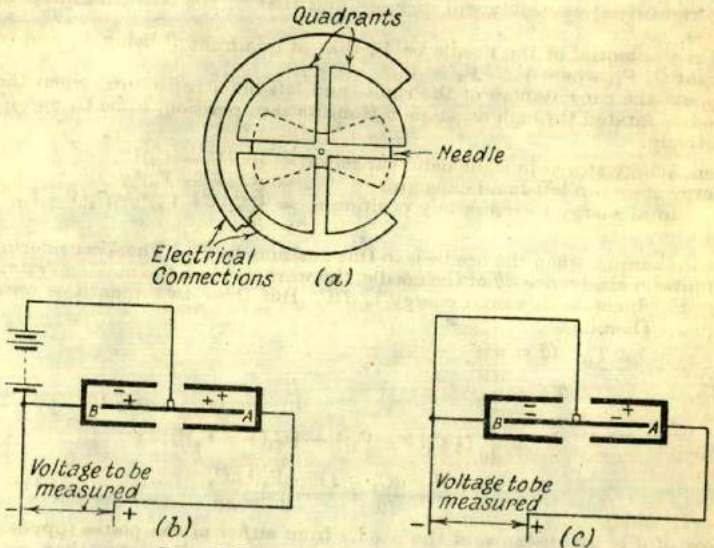


FIG. 18.21. QUADRANT ELECTROMETER

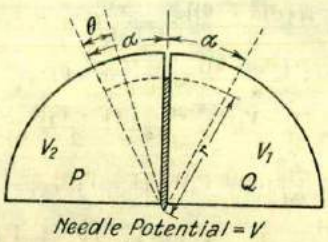


FIG. 18.22

purely rotational, but have components in the direction perpendicular to the needle—one upwards and one downwards at each end of the needle—but these components neutralize one another, leaving only the rotational components.

Theory. In considering the theory of the instrument it is simpler to consider one-half of the needle only, with the two (double) quadrants adjacent to it. Suppose the connection to be heterostatic.

Then, referring to Fig. 18.22, in which the half of the needle is taken simply as a sector of a circle whose radius is r, we have an arrangement which is essentially two capacitors side by side. Each of these capacitors is composed

of portions of the upper and lower plates of one of the double quadrants, and a portion (both sides) of the needle. When the needle rotates, the capacitances will change, one becoming less and the other greater. Thus, in Fig. 18.22, suppose the needle rotates in a clockwise direction; then the capacitance of the right-hand capacitor will increase and that of the left-hand one will decrease.

Let the potential of the needle be V, that of quadrant P being V_2 and of quadrant Q, V_1, where $V > V_2 > V_1$.

Suppose the capacitances of the right- and left-hand capacitors, when the needle has rotated through an angle θ from its zero position, to be C_1 and C_2 respectively.

Then, energy stored in right-hand capacitor $= \frac{1}{2}C_1(V - V_1)^2$

Energy stored in left-hand capacitor $= \frac{1}{2}C_2(V - V_2)^2$

Thus, total energy stored in any position θ $= \frac{1}{2}[C_1(V - V_1)^2 + C_2(V - V_2)^2]$

$= W$

Let the torque, when the needle is in this position, be T_θ. Then, considering an infinitesimal advance $d\theta$ of the needle, the work done on the moving system is $T_\theta \cdot d\theta$. Increase in stored energy is dW. But these two quantities must be equal. Therefore

$$T_\theta \, d\theta = dW$$

or

$$T_\theta = \frac{dW}{d\theta}$$

$$= \frac{d}{d\theta}\left[\tfrac{1}{2}C_1(V - V_1)^2 + \tfrac{1}{2}C_2(V - V_2)^2\right]$$

$$= \frac{(V - V_1)^2}{2}\frac{dC_1}{d\theta} + \frac{(V - V_2)^2}{2}\frac{dC_2}{d\theta} \quad . \quad . \quad . \quad (18.4)$$

Now, if d is the distance of the needle from either of the plates (upper or lower) of the quadrants, and if 2α is the angle of the needle sector, then, since the plates are in air,

$$C_1 = \frac{2[\frac{1}{2}r^2(\alpha + \theta)]}{d}\varepsilon_0 = \frac{r^2}{d}(\alpha + \theta)\,\varepsilon_0$$

and

$$C_2 = \frac{2[\frac{1}{2}r^2(\alpha - \theta)]}{d}\varepsilon_0 = \frac{r^2}{d}(\alpha - \theta)\,\varepsilon_0$$

$$\therefore \quad T_\theta = \frac{(V - V_1)^2}{2} \times \frac{r^2}{d}\varepsilon_0 - \frac{(V - V_2)^2}{2} \times \frac{r^2}{d}\varepsilon_0$$

$$= \frac{r^2\varepsilon_0}{2d}[(V - V_1)^2 - (V - V_2)^2]$$

or

$$T_\theta = \frac{r^2}{2d}\varepsilon_0(V_2 - V_1)[2V - (V_1 + V_2)] \quad . \quad . \quad (18.5)$$

This expression for the torque is positive only when $2V > V_1 + V_2$, and its magnitude, for given values of V_1 and V_2, obviously depends upon the value of V.

In the idiostatic connection the needle and quadrant Q are connected together, so that $V = V_1$ and the expression for the torque then becomes

$$T_\theta = -\frac{r^2}{2d}\varepsilon_0(V_2 - V_1)^2$$

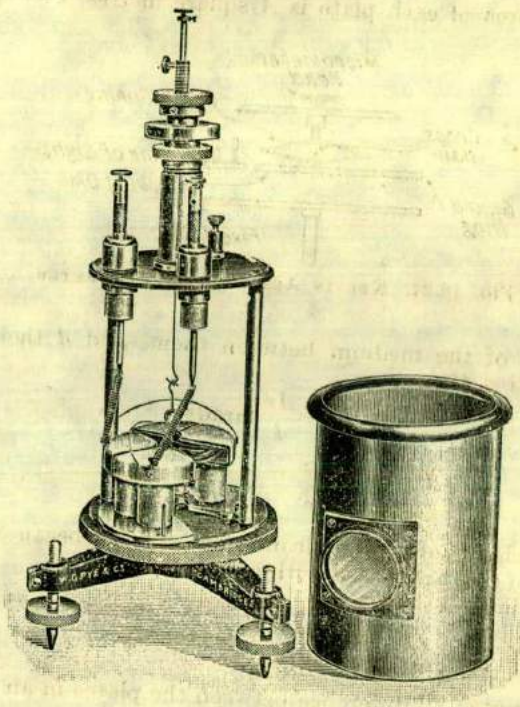
$$= -\frac{r^2\varepsilon_0}{2d} \times v^2 \quad . \quad . \quad . \quad . \quad (18.6)$$

where v is the potential difference to be measured, and equals $V_2 - V_1$.

The torque is negative in this case, which means that the needle will rotate in an *anti-clockwise* direction as shown previously.

Considering now four quadrants and a needle of double-sector shape, the torque is given by

$$T_\theta = -\frac{r^2\varepsilon_0}{d} v^2 . \quad . \quad . \quad . \quad (18.7)$$



(*W. G. Pye & Co., Ltd.*)

FIG. 18.23. DOLEZALEK ELECTROMETER

being double the above value. The torque is in newton-metres if the potentials are expressed in volts, r and d being in metres. Converting to gramme-centimetres, we have

$$T_\theta = -\frac{r^2v^2}{1{\cdot}11 \times 10^7 d} \text{ g-cm}$$

where v is in volts. Fig. 18.23 shows the construction of a quadrant electrometer of the Dolezalek pattern made by W. G. Pye & Co., Ltd.

In the figure the fixed quadrants are shown opened out to disclose the needle.

Attracted-disc Electrometer. It was shown earlier that the force of attraction between two parallel plates, when there is a potential difference V between them, is

$$F = \tfrac{1}{2}V^2 \frac{dC}{dx} \text{ newtons}$$

Now, if the area of each plate is A square metres, $\varepsilon = \varepsilon_0\varepsilon_r$ is the

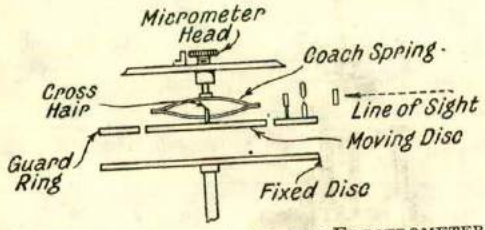


FIG. 18.24. KELVIN ABSOLUTE ELECTROMETER

permittivity of the medium between them, and d their distance apart in metres, then

$$C = \frac{A\varepsilon}{d} \text{ farads}$$

and

$$\frac{dC}{dx} = -\frac{A\varepsilon}{d^2}$$

The minus sign is of no significance and appears because a positive increment of C is associated with a decrease in d.

Substituting,

$$F = \frac{A\varepsilon V^2}{2d^2} \text{ newtons}$$

Hence the potential difference between the plates in air is

$$V = d \times 10^5 \sqrt{\frac{7{\cdot}2\pi F}{A}} \text{ volts} . \qquad . \quad (18.8)$$

ε_0 being $\dfrac{1}{36\pi \times 10^9}$.

This attraction between parallel plates is used as a measure of potential difference in the attracted-disc electrometer.

Kelvin Absolute Electrometer. This was one of the earliest instruments employing the attracted-disc principle. The essential parts of the instrument are shown in Fig. 18.24

The moving disc is carried by a spring, and is thus suspended

from a micrometer head so that it is above the centre of a fixed disc. Surrounding the moving disc is a guard ring separated from the disc by a short air gap. The purpose of this guard ring is to render the field between the moving and fixed discs uniform by transforming the non-uniform fringing field from the edge of the moving disc to its own outer edge. It is electrically connected to the moving disc. The effective area of the moving disc is its actual area plus half the area of the air gap.

A fine cross-hair is carried by the moving disc, so that, by means of a sighting device consisting of lenses and two finely pointed rods, the zero setting of the disc may be accurately determined.

In use, the potential difference to be measured is applied between the two discs. The moving one is attracted downwards and is brought back to its zero position by turning the micrometer head, the movement required to return the disc to zero being observed.

The spring and micrometer head are calibrated by first short-circuiting the instrument, then setting the moving disc to its zero position, and adding known weights to the disc. The movements of the micrometer required to bring the disc back to zero are observed for different weights, a calibration being thus obtained. Thus, actually, the attractive force produced by a certain potential difference between the discs is measured by the instrument, and the potential difference itself is then determined in terms of the dimensions of the instrument and of this measured force.

The disadvantage of the electrometer, when used as above, is that when the potential difference to be measured is only a few hundred volts, the two discs must be very near together for any appreciable attractive force to be obtained. In such cases the measurement of their distance apart is difficult to carry out accurately. To overcome this difficulty, Lord Kelvin used an auxiliary high potential in conjunction with the instrument, the heterostatic connection, previously mentioned, being employed.

In secondary instruments of the attracted-disc pattern, the guard ring is omitted and the voltage corresponding to a given deflection is obtained by calibration instead of by calculation from the dimensions of the instrument.

Commercial Forms of Electrostatic Voltmeters. Fig. 18.25 shows, diagrammatically, the construction of the Kelvin Multicellular Voltmeter. This instrument is essentially a quadrant electrometer with a large number of needles and of fixed quadrants, instead of the single needle and four fixed quadrants. It is suitable for a voltage range of about 100 to 1,000 volts, although instruments of this type having as low a range as 40 volts have been constructed.

The large number of quadrantal cells is necessary in order to obtain a sufficiently high working force with such low voltages.

By suspending the moving system, bearing friction is avoided. The coach-spring is fitted as a protection against fracture of the

suspension due to vibration. A clamp is fitted to be used when transporting the instrument. The torsion head, which can be moved very slowly by a worm-wheel attachment, is for zero adjustment. The pointer and scale are of the "edgewise" pattern, and damping is by a vane dipping into an oil dashpot.

There are several other features which have been omitted from the diagram for the sake of clearness. The suspended system has a safety collar just above the pointer to prevent such movements of the system as would cause the moving needles to touch the fixed

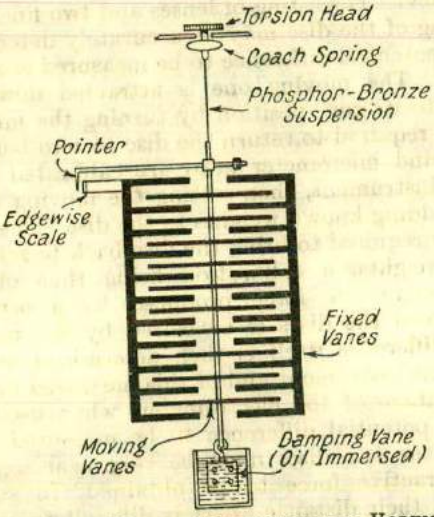


FIG. 18.25. KELVIN MULTICELLULAR VOLTMETER

quadrants and produce a short-circuit. Two vertical guard plates of tin-foil are fitted inside the case of the instrument. These plates are electrically connected to the moving system and to the case, which is of metal.

Precision Forms of Kelvin Electrostatic Voltmeters. Work at the National Physical Laboratory during the past two or three decades has resulted in the development of this form of instrument as a transfer instrument, from direct to alternating current, of standard accuracy.

There is no standard for alternating voltages comparable with the standard cell for direct voltages, so that the approach to standardization in the former case must be through the use of a transfer instrument which reads the same on both a.c. and d.c. Although both thermal and dynamometer transfer instruments can be used, at N.P.L. the greatest reliance is placed on this electrostatic standard instrument developed by E. H. Rayner (Ref. (28)).

The instrument, which is a reflecting one with a scale about four metres long, is calibrated on d.c. by potentiometer and standard cell. Optimum sensitivity is around the 100-V point and the readability on the scale is there better than 1 part in 20,000. Applying the appropriate corrections, alternating voltages of about 100 V can be measured in terms of the absolute units to an accuracy of 1 part in 10,000 (Ref. (27)).

The use of electrostatic instruments for a.c. power measurements is discussed in Refs. (29) and (30).

Kelvin Electrostatic Voltmeter for Voltages up to 10,000 *volts.* A simple form of voltmeter, which has the same principle as the one described above, but which has only one moving needle and is suitable for a voltage range of about 1,000 to 10,000 volts, is shown in Fig. 18.26.

The needle is of thin aluminium sheet, and is carried on a spindle which is supported by knife edges. This needle is connected to one terminal of the instrument, and is mounted in the space between two parallel, fixed, vanes, which are connected to the other terminal of the instrument. It carries a pointer as shown, and is gravity-controlled. Mechanical damping, by a wire passing through the case of the instrument, is employed. The controlling torque may be increased by adding small weights to the bottom of the needle. This, of course, increases the range of the instrument.

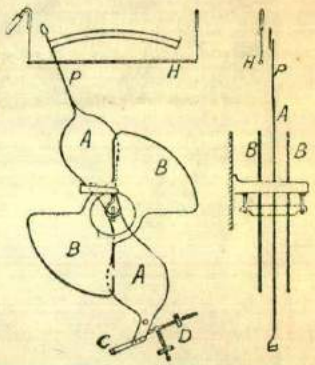
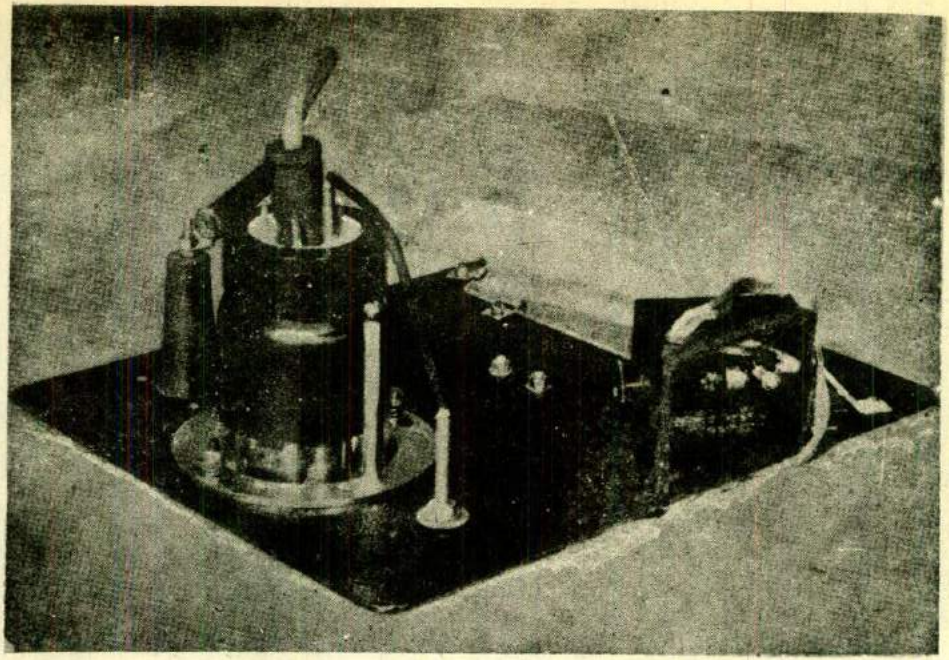


FIG. 18.26. PARTS OF ELECTROSTATIC VOLTMETER
(From "Theory and Practice of Alternating Currents," Dover.)

The instrument case has a glass front to which a strip of tin-foil is pasted, in order to lead away to earth any electrostatic charges which may accumulate on the glass and so affect the reading of the instrument.

Vacuum-enclosed Electrostatic Voltmeters. Associated Electrical Industries, Ltd., have developed instruments of this type for the measurement of high voltages. The construction of one for 12 kV is shown in Figs. 18.27 and 18.28. The active vanes are segments of cylinders, concentrically mounted on a framework. The fixed vane is at high voltage, and the moving vane, mounted between jewel bearings, is earthed. The control springs are made from temper-hardening spring alloy, and a concave mirror is carried by the moving system, a light pointer being used. For damping there are two permanent magnets operating on a radial portion of the moving vane.

The weight of the moving system is 1·25 grammes and the torque is 0·0125 gramme-centimetres per degree deflection. The full-scale deflection of the movement is 30° (60° on the scale) and the



(*Associated Electrical Industries, Ltd.*)

FIG. 18.27. ELECTROSTATIC VOLTMETER

torque/weight ratio is about 1/4. The movement is aperiodic with a response time from 1/5 to 1 second and the instrument is robust. Such instruments are calibrated to give an accuracy of 0·1 per cent or better.

Other Forms. Fig. 18.29 shows the moving system of an Everett Edgcumbe single-vane voltmeter, three such movements being used in one case as a three-phase "leakage indicator" for 2,000 volts.



(Associated Electrical Industries, Ltd.)

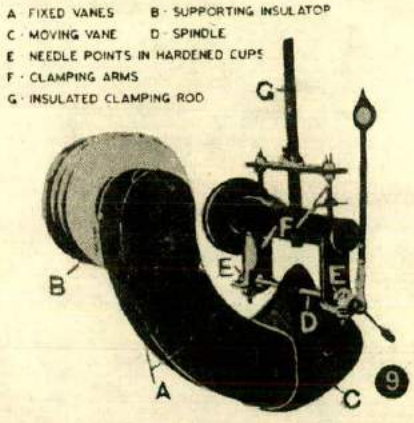
FIG. 18.28. INTERNAL CONSTRUCTION OF THE ELECTROSTATIC VOLTMETER

A leakage indicator is for the purpose of detecting a ground fault on a system. In the three-phase indicator one terminal of each movement is connected to earth and the remaining three terminals are connected to the three insulated lines of the system under observation. A difference between the readings of the three instruments contained in the indicator shows that a fault exists on one or more of the lines. The advantage of electrostatic instruments for this purpose is that they themselves do not take any appreciable current, and therefore do not constitute a "leak" when installed.

The instrument shown consists of two fixed curved vanes about 1 in. wide and mounted parallel to one another about $\frac{1}{2}$ in. apart.

A pivoted vane is pulled inside them when the potential difference is applied. The pivots are attached to the spindle at right angles to its axis, and rest in polished steel cups. Gravity control is employed.

Voltmeters for Extra-high Voltage. Voltmeters for the measurement of voltages in excess of about 10,000 volts are almost always of the attracted-disc type. Several forms of such instruments have already been described in Chapter XI. The Abraham voltmeter, which is manufactured by Messrs. Everett Edgcumbe & Co., for voltages up to 500,000 volts, is perhaps the most generally used form.



(Everett Edgcumbe & Co., Ltd.)

FIG. 18.29. MOVING SYSTEM OF SINGLE-VANE VOLTMETER

Induction Instruments. These instruments can only be used on alternating-currents circuits. Their chief advantage is that a full-scale deflection of some 300° can be obtained, giving a long and open scale. The effect of stray magnetic fields upon their readings is small and the damping is good.

They have, however, several serious disadvantages which, for most purposes, outweigh their advantages. The large deflection means a greatly increased stress in the control spring, since this stress is proportional to the deflection. A serious error may be introduced by a variation of the supply frequency, unless a compensating device is employed. Temperature variations, also, may produce considerable errors unless, again, compensation is employed. Other drawbacks are the fairly high power consumption and high cost of such instruments.

PRINCIPLE. All induction instruments depend for their action upon the torque produced by the reaction between a flux, whose magnitude depends upon the value of current or voltage to be

measured, and eddy currents which are induced in a metal disc or drum by another flux, whose value again is dependent upon the current or voltage to be measured. Since the magnitude of the eddy current is proportional to that of the flux inducing it, the torque at any instant is proportional to the square of the current or voltage to be measured, and the mean torque is proportional to the mean-square value of this current or voltage.

Consider a flux $\phi = \phi_{max} \sin \theta$ producing a torque, by the force which it exerts upon an eddy current lagging in phase by an angle α behind this flux, i.e. whose law of variation is $i = I_{max} \sin (\theta - \alpha)$.

Then, since the instantaneous torque is proportional to the product of the instantaneous current and instantaneous flux, we have

$$\text{Instantaneous torque, } T_{inst} \propto \phi i$$

The mean torque T_M is therefore proportional to $\frac{1}{\pi}\int_0^{\pi} \phi i d\theta$,

or
$$T_M \propto \frac{1}{\pi}\int_0^{\pi} \phi_{max} I_{max} \sin\theta \sin(\theta - \alpha)\, d\theta$$

$$\propto \frac{1}{\pi}\int_0^{\pi} \phi_{max} I_{max} \left[\frac{\cos\alpha - \cos(2\theta - \alpha)}{2}\right] d\theta$$

$$\propto \frac{\phi_{max} I_{max}}{2\pi}\left[\theta\cos\alpha - \frac{\sin(2\theta - \alpha)}{2}\right]_0^{\pi}$$

$$\propto \frac{\phi_{max} I_{max}}{2\pi}(\pi \cos\alpha)$$

Thus
$$T_M \propto \phi I \cos\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (18.9)$$

where ϕ and I are r.m.s. values of flux and current.

Hence, in all induction instruments, some means must be provided for producing an eddy current which is either appreciably less than, or appreciably greater than, 90° out of phase with the flux with which it reacts, since, if α is 90°, cos α—and hence T_M—is zero, the torque being small, also, if α is not far from 90°.

There are two general methods of fulfilling this condition. One method is by splitting the winding of the electromagnet in which the flux exists into two portions, one of which is highly inductive and the other non-inductive. The other method is by splitting the phase of the working flux by a copper band placed round a portion of the poles of the electromagnet. This leads us to the two general types of induction instruments, which are—

(*a*) The Ferraris type.
(*b*) The "shaded-pole" type.

(*a*) FERRARIS TYPE. This instrument operates on the same principle as the induction motor. A rotating field is produced by two

pairs of coils wound upon a laminated magnet system, as shown in Fig. 18.30. These pairs of coils are both supplied from the same source, but a phase displacement of approximately 90° is produced in the currents flowing in them by connecting an inductance in series with one pair and a high resistance in series with the other pair. This rotating field induces currents in an aluminium drum, and causes this drum to follow its rotation. If the drum were free to rotate, it would do so at a speed slightly less than that of the rotating field, and in the same direction as the latter. If a control spring prevents

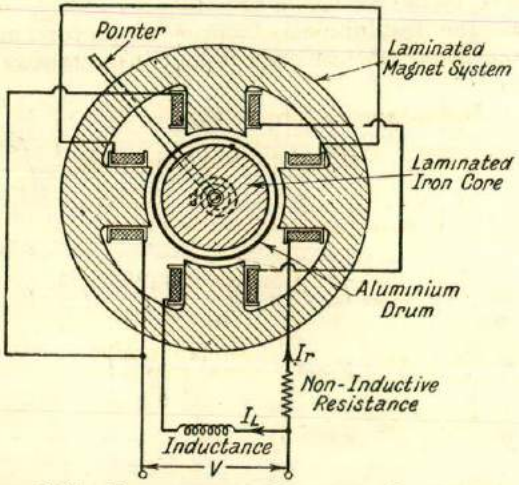


FIG. 18.30. FERRARIS-TYPE INDUCTION INSTRUMENT

such continuous rotation the drum will rotate only through some angle less than 360°—i.e. until the operating torque is balanced by the controlling torque of the spring.

The drum and moving system are carried by a spindle whose ends fit in jewelled cups or bearings. Inside the drum is a cylindrical laminated-iron core, to strengthen the magnetic field cutting the drum. The spindle also carries an aluminium damping disc, the edge of which moves in the air gaps of two permanent magnets.

Theory. The theory of the instrument can be most easily worked out by considering the torque produced by the reactions between the two fluxes and the two eddy currents produced by them.

Saturation of the iron in the instrument, and iron losses, will be neglected, so that the fluxes may be assumed to be in phase with, and proportional to, the currents producing them.

Referring to the vector diagram of Fig. 18.31, the vectors I_R and I_L represent the currents in the non-inductive and inductive windings of the instrument respectively, and the vectors ϕ_R and ϕ_L the fluxes produced by, and in phase, with them. Let the phase angle between these currents (and fluxes) be β.

This angle is large, but cannot be quite 90° owing to the inductance of the so-called "non-inductive" winding and to the resistance of the inductive winding. e_R and e_L—lagging 90° in phase behind ϕ_R and ϕ_L respectively—are the e.m.f.s induced in the rotor drum, and i_R and i_L are the eddy currents set up by them. These currents lag behind the e.m.f.s by a small angle α owing to the inductance of the eddy-current paths.

As regards the torque of the instrument, it can be seen from Fig. 18.32 that there will be two components in opposite directions, one proportional to

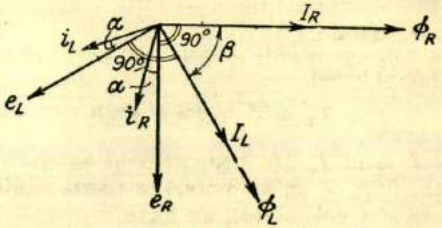


FIG. 18.31

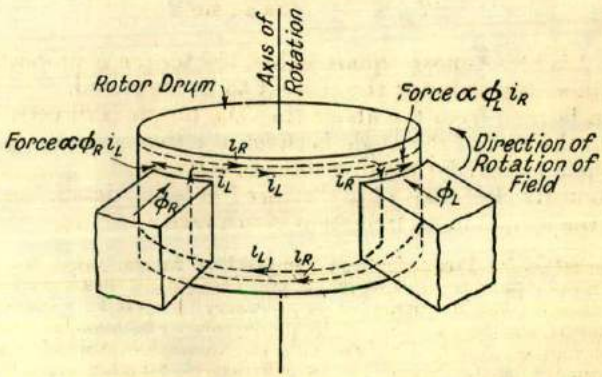


FIG. 18.32. ILLUSTRATING THE ACTION OF THE FERRARIS INDUCTION INSTRUMENT

$\phi_L i_R$ in the direction of rotation of the rotating field, and one proportional to $\phi_R i_L$ in the opposite direction. In this figure both of the fluxes ϕ_R and ϕ_L are assumed to be decreasing, so that, from Lenz's law, the currents i_R and i_L induced by them are in such directions that they tend to maintain the flux (as shown). The directions of the forces acting on the drum are obtained from the left-hand rule.

The resultant torque is thus the *difference* between that due to $\phi_L i_R$ and that due to $\phi_R i_L$. Thus, if ϕ_R, ϕ_L, i_R, and i_L are r.m.s. values, the mean torque is given by

$$T_M = k[\phi_L i_R \cos(90 + \alpha - \beta) - \phi_R i_L \cos(90 + \alpha + \beta)] \quad . \quad (18.10)$$

where k is a constant.

$(90 + \alpha - \beta)$ is the phase angle between ϕ_L and i_R

and $(90 + \alpha + \beta)$ " " " " ϕ_R " i_L

Then, $T_M = k\{-\phi_L i_R \cos[90 - (\alpha - \beta)] + \phi_R i_L \cos[90 - (\alpha + \beta)]\}$

$= k[\phi_R i_L \sin(\alpha + \beta) - \phi_L i_R \sin(\alpha - \beta)]$

Now, $e_R \propto f \cdot \phi_R$ and $e_L \propto f \cdot \phi_L$ where f is the frequency.

Also, $i_R = \frac{e_R}{z}$ and $i_L = \frac{e_L}{z}$, where z is the impedance of the eddy-current paths in the drum.

Hence, $$T_M = \frac{k'\phi_R\phi_L f}{z}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

$$= \frac{2k'\phi_R\phi_L f}{z} \cos\alpha \cdot \sin\beta$$

where k' is another constant.

Again, if ϕ_R and ϕ_L are directly proportional to the currents I_R and I_L (as they are here assumed to be),

$$T_M \propto \frac{I_R I_L f}{z} \cos\alpha \cdot \sin\beta \quad . \quad . \quad (18.11)$$

or, since both I_R and I_L are proportional to the current I to be measured (in the case of an ammeter) or to the voltage to be measured (in the case of a voltmeter), we have

$$T_M \propto \frac{I^2 f}{z} \cdot \cos\alpha \cdot \sin\beta \quad . \quad . \quad . \quad (18.12)$$

Since I is a root-mean-square value, the torque is proportional to the mean-square value of the current to be measured.

It can be seen from the above that the torque is directly proportional to the sine of the angle between the two currents I_R and I_L; hence the necessity for making this angle large.

D. Connelly (Ref. (26)) has discussed, in some detail, the production of the torque in an induction instrument.

Compensation for Frequency and Temperature Errors. Since the torque is directly proportional to the frequency, and also since z, $\cos\alpha$ and I_L are all dependent upon the frequency, it is necessary to consider compensation if serious errors due to variation of frequency are to be avoided. Some compensation can be provided by shunting the ammeter—in which instruments such errors are usually larger than in voltmeters—by a non-inductive shunt. If, then, the frequency increases, the increase in torque which would, as seen from the above expression, take place, is to some extent prevented by the fact that the impedance of the instrument windings increases and hence a greater proportion of the total current is taken by the non-inductive shunt whose impedance remains constant for all frequencies.

In the case of a voltmeter the impedance of the inductive winding of the instrument increases with increasing frequency, and hence this winding takes a smaller current, which tends to compensate for the increase in torque due to the frequency increase.

Variations of temperature may produce serious errors, since the resistances of the eddy-current paths in the rotor are dependent upon temperature. Compensation is obtained by shunting the instrument (in the case of an ammeter) with a shunt of material having a higher temperature coefficient than that of aluminium—of which the rotor is made. This shunt may be the same one as is used for frequency compensation. If, then, the temperature increases, the proportion of the total current which passes through the instrument is increased, and this compensates for the loss of torque due to the reduction of the eddy currents owing to the increased resistance of their path.

In voltmeters a combination of shunt and swamping resistance in series with the instrument is often used.

The frequency errors in induction instruments of all types are so serious, and are so difficult to compensate satisfactorily, that such instruments are most often used for switchboard purposes when frequency changes are small.

SHADED-POLE TYPE. Fig. 18.33 illustrates the principle of the shaded-pole type of induction instrument. A thin aluminium disc is mounted on a spindle which is supported by jewelled bearings. The spindle carries a pointer and a control spring is attached to it. The edge of the disc moves in the air gap of a laminated electromagnet which is energized either by the current to be measured or by a current proportional to the voltage to be measured. A damping

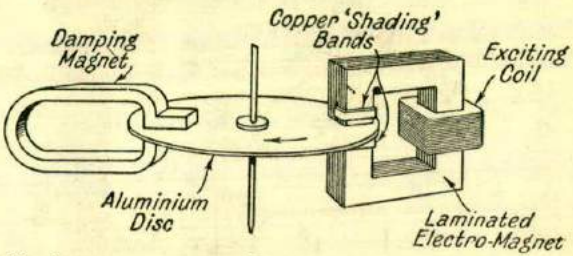


FIG. 18.33. ILLUSTRATING THE PRINCIPLE OF THE SHADED-POLE TYPE OF INDUCTION INSTRUMENT

magnet is placed at the opposite side of the disc from the electromagnet, so that the disc serves for damping as well as for operating purposes. The poles of the electromagnet are split into two halves, with a thin copper band round one-half of each pole (on the same side), as shown. These two bands have currents induced in them by the alternating flux of the electromagnet, and the magnetic fields set up by these induced currents cause the flux, in the portions of the iron surrounded by the bands, to lag in phase by some 40 to 50 degrees behind the flux in the unshaded portions of the poles. This phase displacement is necessary for the production of torque, the shading bands serving the same purpose as the interposition of resistance and inductance between the windings and the terminals in the Ferraris type of instrument.

The disc, instead of being circular, is sometimes made cam-shaped in order to improve the scale of the instrument. The scale is normally cramped at the lower end, since the deflecting torque T_D is proportional to the square of the current or voltage to be measured, and spring control is used. Then

$$T_D \propto I^2 \text{ and } T_C \propto \theta$$

where θ is the deflection.

For steady deflection $T_C = T_D$

so that $\theta \propto I^2$

Theory. Fig. 18.34 gives a vector diagram showing the relative phases of the fluxes, and the induced currents in the disc. The current induced in the shading band produces a number of "back" ampere-turns, and causes the flux ϕ_s, in the shaded portion of the pole to lag in phase behind the flux ϕ_u in the unshaded portion. The resultant flux ϕ, in the main portion of the iron, is the vector sum of ϕ_u and ϕ_s, as shown.

These two fluxes induce voltages e_u and e_s in the disc. each of which is 90° in phase behind the flux inducing it. Eddy currents i_u and i_s flow in the disc as a result of these e.m.f.s, each current lagging by a small angle α behind its voltage owing to the inductance of the path in the disc.

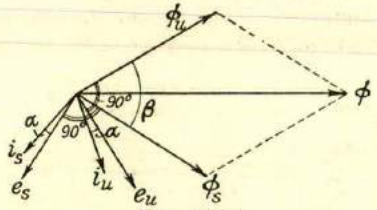


FIG. 18.34

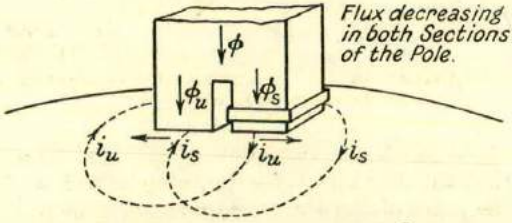


FIG. 18.35. ILLUSTRATING THE ACTION OF THE SHADED-POLE TYPE OF INDUCTION INSTRUMENT

From Fig. 18.35, which illustrates the operation of the instrument, it can be seen that the deflecting torque has two components, one due to ϕ_s reacting with i_u, and one due to ϕ_u and i_s. These torques are in opposite directions, so that the resultant torque is the *difference* between these two component torques.

If ϕ_s, ϕ_u, i_s and i_u are r.m.s. values, the mean torque T deflecting the disc is given by

$$T = k\{\phi_s i_u \cos[90 - (\beta - \alpha)] - \phi_u i_s \cos[90 + (\beta + \alpha)]\} \quad . \quad . \quad (18.13)$$

where k is a constant.

$90 - (\beta - \alpha)$ is the phase angle between ϕ_s and i_u

$90 + (\beta + \alpha)$ " " " " ϕ_u " i_s

$$\therefore T = k[\phi_s i_u \sin(\beta - \alpha) + \phi_u i_s \sin(\beta + \alpha)]$$

As in the theory of the Ferraris instrument it follows that, if ϕ_s and ϕ_u are each directly proportional to the current I in the magnetizing coil, and z is the impedance of the eddy current paths, the torque is

$$T \propto \frac{I^2 f}{z} \cos\alpha \sin\beta \quad . \quad . \quad . \quad (18.14)$$

For any given frequency,

$$T \propto I^2$$

Variation of frequency affects the torque for a given current directly, since the expression for T contains f. The values of sin β and of cos α and z also vary with frequency, so that compensation is applied both for frequency and temperature variations by shunting and by the use of non-inductive series resistance as described in dealing with the Ferraris instrument.

Thermionic Voltmeters. It is not possible to use moving-iron and electrodynamic voltmeters in high-frequency measurements. Such devices have a limited frequency range, and, in addition, the power

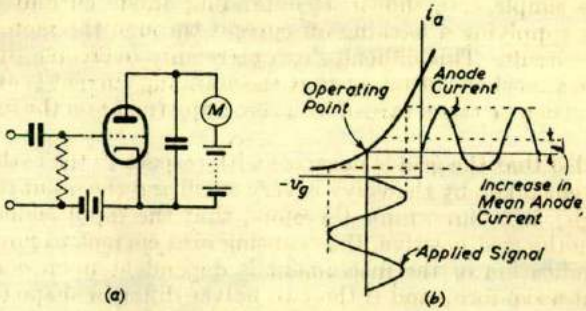


FIG. 18.36. THE ANODE-BEND VOLTMETER
(*a*) Circuit, (*b*) Response to an applied signal

required to operate the instrument usually exceeds that available in electronic circuits. Voltmeters using thermionic valves have a very extended frequency range and a very high input impedance. They are therefore suitable for use at high frequencies and can be used to measure voltage in circuits where the measuring device must absorb negligible power from the circuit. These devices can be calibrated at low frequencies and the calibration holds over the whole frequency range. They are subject to wave-form error, and the nature of this error will be considered with each instrument, but it should be borne in mind that, when these instruments are used in high-frequency circuits, the wave-forms encountered are usually sinusoidal and such errors are then unimportant.

THE ANODE-BEND VOLTMETER. This is possibly the simplest voltmeter and the circuit is given in Fig. 18.36 (*a*). The valve is biased so that it is operating on the curved part of its mutual characteristic as shown in Fig. 18.36 (*b*). This part of the characteristic may be approximately represented by the first three terms of a series expansion as follows—

$$i_a = i_0 + av_g + bv_g^2 \quad . \quad . \quad . \quad (18.15)$$

If the voltage applied to the grid is expressed by $v_g = V_{max} \sin \omega t$, then the increase in mean value of the anode current is $\Delta i_a = \frac{1}{2} b V^2_{max}$, and the change in indication of the anode-circuit milliammeter M is proportional to the mean square of the applied voltage. It seems, therefore, that the milliammeter M can be calibrated in r.m.s. values. In practice, the higher terms which have been neglected in the series expansion for i_a are not negligibly small, and approximate square-law response is obtained only for small signals on the curved part of the mutual characteristic. When the signal is large, a considerable part of the positive signal excursion will be on an approximately linear part of the characteristic, and the device tends to become mean-reading.

In the simple case shown, the standing anode current must be offset by supplying a backing-off current through the meter from a separate circuit. This difficulty can be readily overcome by biasing the valve almost to cut-off, so that the standing current is extremely small, and can be taken care of by a zero adjustment on the indicating meter.

Provided that the grid is negative with respect to the cathode, the grid current taken by the valve is very small and the input resistance very high; it is important, therefore, that the input signal should not drive the grid positive, thus causing grid current to flow.

The indication of the instrument is dependent upon one-half of the input wave-form, and if the two halves differ in shape there will be a different indication according to the way in which the leads are connected. This phenomenon is known as "turnover" and is not peculiar to this instrument: it occurs in many valve voltmeters.

The upper frequency limit is determined by the characteristics of the input circuit. The grid resistor is shunted by the grid–cathode and lead capacitances of the valve, and the leads also have self-inductance. If these could be eliminated, the upper frequency limit would be determined by the transit time of the electrons. Satisfactory operation can be obtained at frequencies up to 10 Mc/s without special circuit arrangements.

The simple circuit shown has some important disadvantages: it is sensitive to changes in cathode temperature, bias and anode voltage. An excellent practical instrument by E. B. Moullin (Refs. (10), (11), (12)) overcomes these disadvantages. The basic circuit is shown in Fig. 18.37. The filament current, bias voltage, and anode supply are all obtained from a single 6 V battery. The mean potential difference between the anode and filament is small, the anode current is therefore small, and the indicating instrument is a sensitive Unipivot pointer galvanometer. The filament current is adjusted to a standard value by means of rheostat B_1; this operation also standardizes the bias and anode current so that variations in the supply can be readily compensated.

THE DIODE VOLTMETER. This device forms the basis of most

modern thermionic voltmeters. The basic circuit of the shunt-diode circuit is shown in Fig. 18.38 (*a*). When an alternating voltage is applied to this circuit, the diode conducts during the positive half-cycle and thus acts as a switch. As a result the capacitor C is charged up to the peak applied voltage every positive half-cycle. During the ensuing part of the cycle the capacitor discharges through the resistor R in series with the input circuit. The anode of the diode is negative with respect to its cathode during discharge and the diode is non-conducting. Discharge continues until the instantaneous

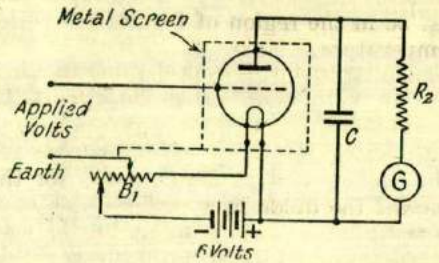


FIG. 18.37. MOULLIN THERMIONIC VOLTMETER

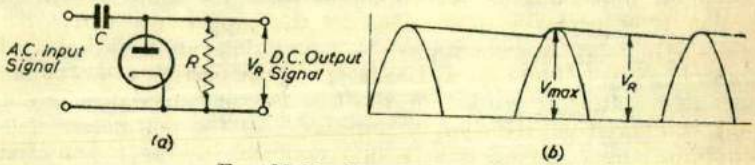


FIG. 18.38. DIODE VOLTMETER
(*a*) Circuit, (*b*) Response

value of the following positive-going wave is greater than the charge voltage on the capacitor. The anode of the diode is then positive with respect to its cathode; it conducts, and the capacitor is recharged to the peak voltage. Provided that the time-constant CR is long compared with the period of the applied wave, the mean voltage across R is equal to the peak value of the applied wave.

The power dissipated in R is $\frac{V^2_{max}}{R} = \frac{2V^2}{R}$, where V_{max} and V are maximum and r.m.s. values of the applied wave respectively. It can be seen therefore that the input resistance of the device is $\frac{1}{2}R$. In order to minimize the loading on the test circuit, R should be high and in practice will be between 50 and 100 MΩ.

The diode circuit converts the input a.c. signal into a d.c. signal in a high resistance, and this d.c. signal must be measured by a high-impedance d.c. valve voltmeter. Before dealing with this aspect of the circuit it is necessary to consider the problems which

arise in the diode circuit itself. It is essential that the forward diode resistance plus the source resistance should be small compared with R, otherwise substantial reading errors may occur. A full analysis of errors due to this cause will be found in Ref. (35). This condition is not difficult to realize in practice since the forward resistance of a typical diode is usually in the region of some hundreds of ohms.

The most troublesome effect is due to the fact that electrons are not all emitted from the cathode of the diode at zero velocity; some have quite a high initial velocity and will reach the anode. These electrons will cause a current in R and develop a voltage across it. This voltage will be in the region of 1 volt, and is dependent upon the cathode temperature. Although it may be offset by a zero adjustment in the instrument, it is preferable to eliminate its effect by balancing it out with a second diode circuit, the two diodes having a common heater.

The leads to the diode introduce self-inductance and capacitance and cause reading errors at high frequencies; for this reason it is customary to mount the diode in a probe which can be connected directly to the test circuit with a minimum of lead. The upper frequency limit is determined by the probe design and at the present moment is in the region of 600 Mc/s for wide-range instruments. The input impedance is very different from the input resistance because it is primarily determined by the capacitance across the input terminals; a capacitance of 1 $\mu\mu F$ has an impedance of approximately 1,500 ohms at 100 Mc/s. Serious errors will occur in the valve voltmeter reading if the test circuit impedance is not small compared with the input impedance of the voltmeter. The two impedances are effectively in series and act as a potential divider across the test voltage.

The basic circuit diagram of a typical diode voltmeter is given in Fig. 18.39. A double diode, D_1 and D_2, is used in the probe, the second diode, D_2, being used to compensate for the effect of the fast electrons previously discussed. The load resistor of the diode circuit is in two parts, R_1 and R_2, one inside the probe and the remainder in the main body of the instrument; the capacitors C_2, C_3, C_4, C_5 and C_6 by-pass any high-frequency ripple. The range is changed by a series of tappings on the diode circuit resistor outside the probe. The remainder of the instrument consists of a Wheatstone bridge circuit in which two of the arms consist of a double-triode valve, V_1 and V_2, and the other arms two equal resistors, R_4 and R_5. The output indicator is a permanent-magnet moving-coil instrument actuated by the bridge out-of-balance current. The bridge is initially balanced by adjusting R_3, and when a signal is applied to the diode probe the grid of V_1 becomes more negative, the valve resistance increases and the bridge becomes unbalanced. The operating point of the two valves is set by the tapping on R_6.

The use of the double-triode bridge tends to eliminate drifts due to changes in anode and heater voltages since both halves of the circuit vary in the same manner.

Rectifier Instruments. Copper-oxide rectifiers are used fairly extensively in conjunction with moving-coil instruments for the measurement of small alternating currents and voltages. They are particularly suited to measurements on communication circuits and for light-current work when the voltage is low and the resistance

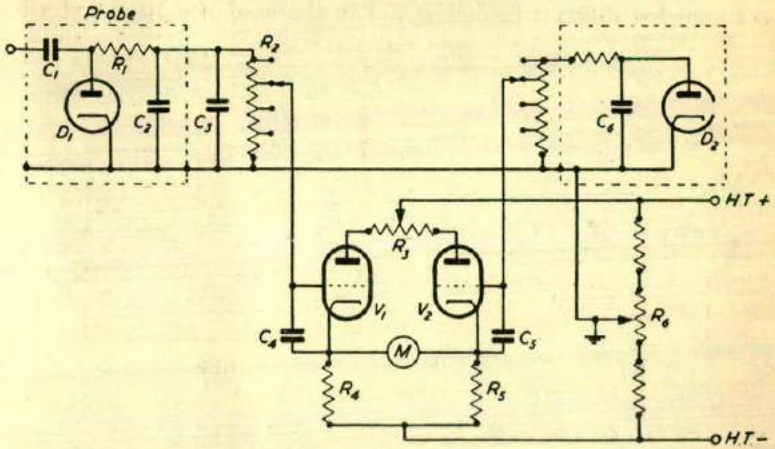


FIG. 18.39. CIRCUIT OF BALANCED VALVE VOLTMETER

high. It is essential in such cases that the current taken by voltmeters should not exceed (say) 1 mA at full scale, and rectifier voltmeters are available having resistances of 1,000 or 2,000 ohms per volt.

These instruments consist of a permanent-magnet moving-coil instrument together with a full-wave copper-oxide rectifier incorporated within the case. The connections are as shown in Fig. 18.40 (*a*). During both half-cycles of the a.c. wave current flows in the moving-coil instrument M in the direction shown, so that its deflection is proportional to the mean value of the current flowing through it.

On the assumption that the current is sinusoidal the scale of the instrument must be marked in terms of 1·11 times the current actually measured, to give r.m.s. values. It is clear at the outset that these instruments must be subject to considerable wave-form errors. A number of current wave-forms having the same mean value I_{av} (say), may have r.m.s. values which vary considerably, but in all cases the rectifier instrument will indicate the same r.m.s. value, namely, 1.11 I_{av}.

The copper-oxide rectifiers used in such instruments are of three types, 1 mA, 5 mA, and 10 mA. As regards their characteristics it is important to realize that their resistance, even to current in the "forward" direction, is not constant, but depends upon the value of the current passing. The resistance in the "reverse" direction varies between 20,000 ohms and 80,000 ohms for a 10-mA rectifier, and between 0·3 and 0 6 megohm for a 1-mA unit. These figures are given by E. Hughes in a valuable paper on the subject of these rectifiers (Ref. (16)). The resistance in the "forward" direction is somewhat difficult to define, but in the case of a 10-mA rectifier

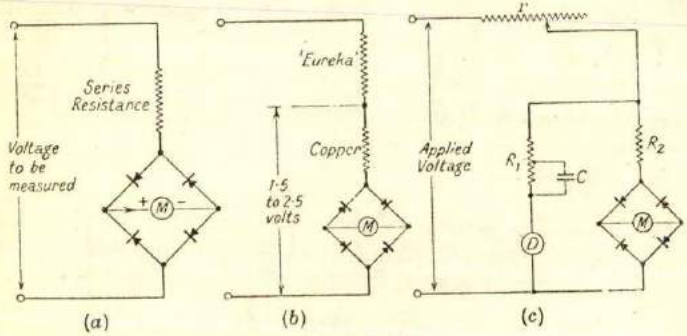


FIG. 18.40

this is of the order of 100 ohms when the rectified current is 6 mA, and between 300 and 500 ohms when the rectified current is 1 mA. The "reverse" current is thus negligible compared with the "forward" current. Dr. Hughes found that the resistance varied approximately as (current)$^{-0\cdot8}$ over a wide range, so that when a sinusoidal voltage $v = V_m \sin \theta$ is applied to a rectifier having a resistance R in series with it, the alternating current flowing is given by

$$i = \frac{V_m \sin \theta}{R + ki^{-0\cdot8}} \qquad . \qquad . \qquad . \quad (18.16)$$

where k is a constant. Alternatively, the d.c. voltage v across the rectifier for a direct current i through it is given approximately by the equation

$$v = c + bi$$

where c and b are constants. On this basis the alternating current through the rectifier may be expressed as

$$i = \frac{V_m \sin \theta - c}{R + b}$$

The effect of the non-linear volt/ampere characteristic of the rectifier is to distort the wave-form of the alternating current flowing through it. Thus, if a sinusoidal voltage is applied the current flowing will be "peaked," its form factor being greater than 1·11.* This means that the actual r.m.s. value of the current flowing will be greater than that indicated by the rectifier instrument, i.e. the instrument reads low. The effect of series resistance is to "swamp" the variable resistance of the rectifier itself, so that the greater the series resistance the less the error in reading on this account. When an alternating voltage is applied to a rectifier instrument having a large series resistance, the reading of the instrument will thus be almost exactly proportional to the mean value of the voltage applied. As already pointed out, if the voltage wave-form is not sinusoidal an error is necessarily introduced. To correct for such non-sinusoidal wave-form, assuming that the instrument reads the true mean value of the wave, the instrument indication must be multiplied by $K_f/1{\cdot}11$, where K_f is the form factor of the voltage wave-form. Dr. Hughes states that rectifier instruments actually read the r.m.s. value within 10 per cent for any shape of wave "except for a certain range and phase of the third harmonic."

The relationship between the applied voltage V and the rectified current I may be expressed in the form

$$I = KV^x, \text{ where } K \text{ is a constant}$$

With comparatively low values of series resistance the exponent x is approximately 2, so that low-range voltmeters will have practically a square-law scale and will read roughly the r.m.s. value of the voltage whatever the wave-form.

Compensation for temperature errors in rectifier voltmeters may be carried out by using a combination of copper and Eureka for the series resistance as shown in Fig. 18.40 (*b*). The copper resistance is of such a value that the total voltage drop across the rectifier and copper together is between 1·5 and 2·5 volts, the remaining series (Eureka) resistance being made up to the total value required, depending upon the applied voltage. The rectifier has a negative temperature coefficient and this is counterbalanced by the positive coefficient of the copper. The lowest voltage range for complete compensation is obviously of the order of 2 volts.

These instruments are subject, also, to frequency errors, the accuracy obtainable being some 2 per cent at full scale for frequencies between 20 and 4,000 cycles per second with ordinary wave-forms. The instrument indications decrease by about 0·5 per cent for every 1,000 cycles up to 35,000 cycles, and applying the necessary correction for this, the accuracy may be within 5 per

* See the contribution by one of the authors to the discussion of Dr. Hughes's paper, *Journal I.E.E.*, Vol. 75, p. 477.

cent when the wave-form approximates closely to a sine wave. By very careful construction to reduce the self-capacitance of the instrument it has been found possible to obtain an accuracy of within 1 per cent over a frequency range of 50 to 10,000 cycles per second and of 5 to 8 per cent between 50,000 and 100,000 cycles. While these errors may appear large, it must be appreciated that the suitability of rectifier instruments for the measurement of small alternating currents having bad wave-forms, such as are met with in telephonic work, lies in their much greater sensitivity as compared with any other type of instrument available for the purpose.

As commonly manufactured, the ranges covered by these instruments are—

Milliammeters 0–0·5 mA to 0–10 mA.
Voltmeters 0–1·0 to 0–300 volts.

The instrument scale is practically linear in the higher voltmeter ranges, but is practically a square-law scale in the lower-range instruments.

A very wide range of germanium and silicon semiconductor rectifiers has become available in recent years; these are either of the point-contact or alloyed-junction form. They have very small junction capacitances and can therefore be used at very high frequencies. The point-contact rectifier can be used in a half-wave configuration with a sensitive moving-coil instrument at frequencies in the region of 3,000 Mc/s. The introduction of these devices has led to the development of the modern amplifier voltmeter, in which a rectifier meter is preceded by a stable wide-band electronic amplifier, the frequency limit then being determined by the amplifier.

A germanium or silicon point-contact rectifier can be used in place of the thermionic diode in the thermionic voltmeter previously described. An upper frequency limit of 1,200 Mc/s is attainable, but the comparatively low reverse breakdown voltage of small semiconductor devices restricts the input voltage range.

The use of rectifier instruments in conjunction with current transformers is also dealt with in the paper by E. Hughes already mentioned.

R. S. J. Spilsbury has described a direct-reading form-factor meter, devised by him (Ref. 17) which utilizes a rectifier instrument. The essential features of the device are shown in Fig. 18.40 (*c*). *D* is a low-range dynamometer voltmeter which is placed in parallel with the rectifier unit, *M* being, of course, a permanent-magnet moving-coil voltmeter. *C* is a capacitor which is connected across a portion of the series resistance of *D* to reduce the frequency error due to the inductance of the dynamometer instrument. R_2 is the series resistor of the rectifier (about 40,000 ohms) and *r* is a variable resistor for the purpose of adjusting the voltage across the combination of *D* and *M* to the same value whatever the voltage applied.

The dynamometer indicates the r.m.s. value of the voltage, and the rectifier instrument its mean value. The resistor r is adjusted until D reads a standard value (actually 20 volts) so that M indicates the mean value of the standard voltage and its scale can thus be marked in terms of form factor.

The range of form factor measurable is from 1·0 to 1·32, and the indications are independent of wave-form, temperature and frequency (between 25 and 800 cycles per second) to an accuracy of 0·2 per cent. An important application of such an instrument is the measurement of form factor in the magnetic testing of sheet steel with alternating currents (see page 422).

H. J. Lindenhovius, G. Arbelet and J. C. van der Breggen (Ref. (24)) describe a millivoltmeter, employing an amplifier and rectifying circuit with a moving-coil instrument, with a full deflection for 1 mV and covering the frequency range 1,000 to 30×10^6 c/s. Using an attenuator, voltages up to 1 kV can be measured.

Testing Sets. This chapter should not be concluded without mention of various "test-sets" which have been developed by a number of manufacturers as multi-range instruments for the measurement of current, voltage, power, resistance and other electrical quantities. Some of these consist of two moving-coil instruments in a common case together with appropriate shunts and multipliers. Others have moving-iron instruments for d.c. or a.c. measurements, and there are also several sets, of unconventional design, intended for special measurements such as those of current in circuits which cannot conveniently be broken.

There is not space here to illustrate these sets or, indeed, to give a full list of them, but the following are some of the sets in common use—

(*a*) Avometers (The Automatic Coil Winder and Electrical Equipment Co., Ltd.). The "Universal" Avometer is an a.c.-d.c. instrument with fifty ranges of current, voltage, resistance, capacitance, power, etc.

A wide variety of instruments by the same makers include an "Electronic" test meter, valve-characteristic meter, high-sensitivity Avometers and a light meter.

(*b*) Weameter Test-set (Elliott Brothers (London), Ltd.). This is an a.c. test set for current, voltage and power with extensions, beyond its normal ranges, through current and voltage transformers.

The Elliott "Multiversal" test-set is particularly for insulation and fault location tests.

(*c*) Ferranti Test-sets (Ferranti, Ltd.) for current and voltage measurements (a.c. or d.c.) by two multi-range instruments contained in a single case.

(*d*) "Vampire" (Everett Edgcumbe & Co., Ltd.) A.C. Test-set for current, voltage, and power (up to 5 kW).

(*e*) The Cambridge A.C. Test-sets (Cambridge Instrument Co.,

Ltd.) are multi-range instruments for current, voltage, and power measurement. Different models are available for use between 20 and 150 c/s and for 20–2,000 c/s.

(*f*) Precision Multi-range Meter (Electronic Instruments, Ltd.). An instrument of sub-standard accuracy for current, voltage and resistance measurements.

(*g*) C.P. "Tong-test" (Crompton-Parkinson, Ltd.). An instrument, for both a.c. and d.c. measurements of current, which is used by placing the "tongs" round the current-carrying conductor.

(*h*) "Selectest" (General Electric Co., Ltd.). A multi-range instrument for d.c. and a.c. measurements of current and voltage at power and audio frequencies, and for resistance measurements up to 1 megohm.

(*i*) Component Tester (Pye, Ltd.). This is an instrument for resistance, capacitance and insulation tests mainly on radio components in the workshop.

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CHAPTER XIX

EXTENSION OF INSTRUMENT RANGE

Ammeter Shunts. The use of low resistance "shunts" to be placed in parallel with an ammeter in order to measure a greater current than that which the instrument itself can carry, has been mentioned in dealing with moving-coil instruments, with which type of ammeter such shunts are most commonly used.

Let the ammeter resistance be R, and its current for maximum deflection be i. Suppose that R_s is the resistance of the shunt which must be placed in parallel with the instrument in order to

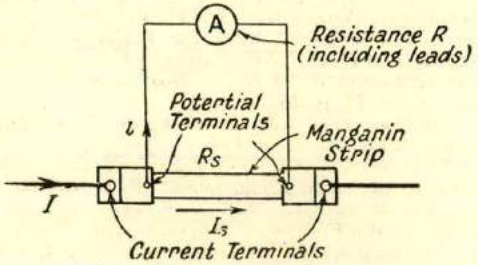


FIG. 19.1. SHUNTED AMMETER

enable a maximum current of I to be measured, and let I_s be the current through the shunt during the measurement (see Fig. 19.1).

Then $$i = I - I_s$$

Also, Voltage drop across shunt = Voltage drop across instrument

Hence, $$I_s R_s = iR$$

The resistance of shunt required is thus given by

$$R_s = \frac{i}{I_s} \cdot R = \frac{i}{I - i} \times R = \frac{1}{\left(\frac{I}{i} - 1\right)} \cdot R \,. \quad (19.1)$$

The ratio of the total current to the instrument current $\left(\text{i.e. } \frac{I}{i}\right)$ is called the "multiplying power" of the shunt. Expressing this ratio as N, we have

$$R_s = \frac{1}{N - 1} \cdot R$$

or $$N = 1 + \frac{R}{R_s} \quad . \quad . \quad . \quad . \quad (19.2)$$

Example. Calculate the resistance of shunt required to make a milliammeter, which gives maximum deflection for a current of 15 milliamp, and which has a resistance of 5 ohms, read up to 10 amp.

$$\text{Multiplying power} = \frac{10}{0{\cdot}015} = 666{\cdot}7 = N$$

$$\therefore R_s = \frac{\cdot 1}{666{\cdot}7 - 1} \times 5$$

$$= \frac{5}{665{\cdot}7} = 0{\cdot}007511 \text{ ohm}$$

or, otherwise.

Voltage drop across instrument for maximum deflection

$$= 5 \times 0{\cdot}015 = 0{\cdot}075 \text{ volt}$$

$$= \text{Voltage drop across shunt}$$

$$= (10 - 0{\cdot}015)\,R_s$$

$$= 9{\cdot}985R_s$$

$$\therefore R_s = \frac{0{\cdot}075}{9{\cdot}985} = 0{\cdot}007511 \text{ ohm}$$

ALTERNATING-CURRENT OPERATION. If a shunt is to be used with an ammeter for alternating-current work, the inductances of both the instrument and the shunt must be considered as well as their resistances.

In order that the division of current between the two parallel branches—instrument and shunt—shall remain the same for all frequencies, the ratio of the *impedances* of the two branches must remain constant.

Thus, let L be the inductance of the instrument and L_s that of the shunt; then the ratio $\dfrac{\sqrt{R^2 + \omega^2L^2}}{\sqrt{R_s^2 + \omega^2L_s^2}}$ must be independent of frequency. To fulfil this condition, the time-constants of the shunt and instrument must be the same, i.e. $\dfrac{L_s}{R_s}$ must equal $\dfrac{L}{R}$.

Then, let $\dfrac{L_s}{R_s} = \dfrac{L}{R} = k$.

We now have $\dfrac{i}{I_s} = \dfrac{\sqrt{R_s^2 + \omega^2L_s^2}}{\sqrt{R^2 + \omega^2L^2}} = \dfrac{\sqrt{R_s^2 + \omega^2k^2R_s^2}}{\sqrt{R^2 + \omega^2k^2R^2}}$

$$\therefore \frac{i}{I_s} = \frac{R_s\sqrt{1 + \omega^2k^2}}{R\sqrt{1 + \omega^2k^2}} = \frac{R_s}{R} \quad . \quad . \quad . \quad (19.3)$$

The general expression for the multiplying power N, in alternating-current circuits, is most easily obtained by the employment of the symbolic notation, owing to the phase difference between i and I_s when the time-constants of the two branch circuits are not equal.

Then, if v is the voltage drop across the instrument and shunt (in parallel),

$$i = \frac{v}{R + j\omega L} \text{ and } I_s = \frac{v}{R_s + j\omega L_s}$$

The total current I in the main circuit is thus given symbolically by

$$\frac{v}{R+j\omega L}+\frac{v}{R_s+j\omega L_s}$$

and the multiplying power $\frac{I}{i}$ by

$$N=\frac{v\left(\frac{1}{R+j\omega L}+\frac{1}{R_s+j\omega L_s}\right)}{v\left(\frac{1}{R+j\omega L}\right)}=\frac{R+R_s+j\omega(L+L_s)}{R_s+j\omega L_s}$$

Expressing this in real values, we have

$$N=\frac{\sqrt{(R+R_s)^2+\omega^2(L+L_s)^2}}{\sqrt{R_s^2+\omega^2L_s^2}} \quad . \quad . \quad . \quad . \quad (19.4)$$

Now, if $\frac{L}{R}=\frac{L_s}{R_s}=k$,

$$N=\frac{\sqrt{(R+R_s)^2+\omega^2k^2(R+R_s)^2}}{\sqrt{R_s^2+\omega^2k^2R_s^2}}=\frac{R+R_s}{R_s}=1+\frac{R}{R_s}$$

General Requirements of Ammeter Shunts. It is essential, whether on a.c. or d.c., that the temperature coefficients of the shunt and instrument shall be low, and as nearly as possible the same, in order that the multiplying power shall be independent of temperature. This question has already been discussed in the previous chapter.

Another requirement is that such shunts shall not vary in resistance with time, and this is ensured by careful annealing during manufacture. Again, the thermo-electric effect in the shunt must be small, and it must be able to carry the necessary current without appreciable heating.

These shunts are usually composed of one or more thin strips of manganin sheet, the ends of which are soldered to two heavy copper blocks which carry two terminals each—one current terminal and one potential terminal—the two current terminals being outermost. When such shunts are used with ammeters, the resistance of the copper leads from the potential terminals of the shunt to the ammeter terminals, forms part of the resistance of the ammeter circuit and must therefore be taken into account in calculating the required shunt resistance. A special pair of leads is usually supplied with ammeters intended to be used shunted, and these leads should always be used for the purpose.

Voltmeter Multipliers. To increase the range of a voltmeter, a non-inductive resistance is connected in series with the instrument. Thus, if the voltmeter gives full-scale deflection when a current i passes through it, and if its resistance is r, V being the voltage to be

measured, and R the resistance connected in series with the voltmeter, we have

$$\frac{V}{R+r} = i$$

or

$$R = \frac{V - ir}{i} = \frac{V}{i} - r \quad . \quad . \quad (19.5)$$

The principal requirement of such multiplying resistances, or "multipliers," when used in d.c. circuits, is that their resistance shall be constant. Thus they must have a very small temperature coefficient, and, on account of the appreciable amount of power absorbed by them, ample provision for cooling must be made.

OPERATION IN A.C. CIRCUITS. When used with alternating currents, the total impedance of the voltmeter and its series resistance must remain as nearly constant as possible for different frequencies. The series resistance should, therefore, have as small an inductance as possible, in order to keep the total inductance of the circuit small. For this reason, the resistance coils are often wound upon flat mica strips to reduce the area enclosed by the turns of wire, and hence to reduce the enclosed flux for a given current. Sometimes non-inductively woven gauze resistors are used.

If r is the voltmeter resistance, l its inductance, and i the current for full-scale deflection, then the voltage drop across the instrument, for full deflection, is

$$v = i\sqrt{r^2 + \omega^2 l^2}$$

where $\omega = 2\pi \times$ frequency.

Let V be the voltage to be measured and R the resistance of the non-inductive series resistor.

Then, total resistance of the circuit $= r + R$

Total inductance of the circuit $= l$

$\therefore$ Total impedance of the circuit $= \sqrt{(r+R)^2 + \omega^2 l^2}$

and the voltmeter current is given by

$$\frac{V}{\sqrt{(r+R)^2 + \omega^2 l^2}} = i$$

Hence,

$$v = \frac{V}{\sqrt{(r+R)^2 + \omega^2 l^2}} \sqrt{r^2 + \omega^2 l}$$

and the multiplying ratio is

$$\frac{V}{v} = \frac{V}{\frac{V\sqrt{r^2 + \omega^2 l^2}}{\sqrt{(r+R)^2 + \omega^2 l^2}}} = \frac{\sqrt{(r+R)^2 + \omega^2 l^2}}{\sqrt{r^2 + \omega^2 l^2}} \quad (19.6)$$

An appreciable error may thus be introduced by neglecting the inductance of the instrument.

Multipliers for Electrostatic Voltmeters. Such multipliers are either in the form of a resistance potential-divider or a capacitor-multiplier.

Resistance potential-dividers are used for voltages of a few thousand volts only. For higher voltages they become expensive to manufacture and are wasteful of power, and capacitor multipliers are therefore used.

Resistance Dividers for such purposes are high non-inductive resistors (often oil-immersed), across a comparatively small portion of which the electrostatic voltmeter is connected (see Fig. 19.2).

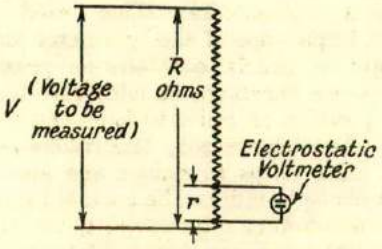


Fig. 19.2. Resistance Potential Divider

If the capacitance of the voltmeter is neglected, the multiplying power is

$$\frac{V}{v} = \frac{R}{r}$$

where V is the voltage to be measured and v the voltage at the voltmeter terminals. R and r are the total resistance (connected across V) and the resistance across which the voltmeter is connected, respectively.

Taking the capacitance of the voltmeter into account, let this be C, and suppose the voltmeter to be used with alternating current of ordinary commercial frequency. Then—

$$\text{Impedance of voltmeter and } r \text{ in parallel} = \frac{r}{1 + j\omega Cr} = z$$

using the symbolic method.

The total impedance connected across the voltage V is thus

$$z_T = R - r + \frac{r}{1 + j\omega Cr}$$

$$z_T = \frac{R + j\omega Cr\,(R - r)}{1 + j\omega Cr}$$

The multiplying power is thus

$$\frac{z_T}{z} = \frac{\dfrac{R + j\omega Cr(R-r)}{1 + j\omega Cr}}{\dfrac{r}{1 + j\omega Cr}} = \frac{R + j\omega Cr(R-r)}{r}$$

$$= \frac{R}{r} + j[\omega C\,(R-r)]$$

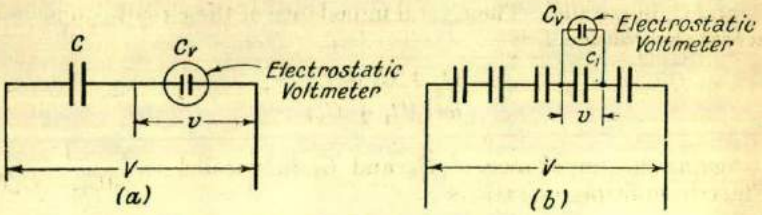


FIG. 19.3. CAPACITOR MULTIPLIERS

Numerically,

$$\frac{z_T}{z} = \sqrt{\left(\frac{R}{r}\right)^2 + \omega^2 C^2\,(R-r)^2}$$

or

$$\frac{z_T}{z} = \frac{R}{r}\sqrt{1 + \frac{\omega^2 C^2 r^2\,(R-r)^2}{R^2}} \quad . \quad . \quad . \quad (19.7)$$

This will have the value $\frac{R}{r}$ if ωCr is small.

CAPACITOR MULTIPLIERS. When used with a capacitor multiplier, the voltmeter may be simply connected in series with a suitable capacitor (as in Fig. 19.3 (*a*)), and the voltage to be measured applied across the whole circuit. Again, a number of capacitors may be connected, in series, across the voltage to be measured, and the voltmeter connected across one of them (as in Fig. 19.3 (*b*)).

Referring to the first method of use, let C_v be the capacitance of the voltmeter and C that of the capacitor in series with it. The total capacitance of the circuit is $\frac{C_v C}{C + C_v}$ and its impedance to a.c. is $\frac{C + C_v}{\omega C_v C}$. The impedance of the voltmeter itself is $\frac{1}{\omega C_v}$. The multiplying ratio is

$$\frac{V}{v} = \frac{\dfrac{C + C_v}{\omega C_v C}}{\dfrac{1}{\omega C_v}} = \frac{C + C_v}{C} = 1 + \frac{C_v}{C} \quad . \quad . \quad (19.8)$$

The capacitance C_v varies with the deflection (i.e. the position of the moving vane) of the voltmeter. Thus the voltmeter must be calibrated together with the series capacitor. It should be noted that, in order that the multiplying ratio shall be large, the capacitance of the series capacitor must be small compared with that of the voltmeter.

Referring to the second method of connection, let C_1 be the capacitance of the capacitor across which the voltmeter—of capacitance C_v—is connected, and let C_s be the capacitance in series with C_1 and C_v in parallel. Then, total impedance of the circuit connected across the voltage V is

$$\frac{C_s + C_1 + C_v}{\omega C_s(C_1 + C_v)}$$

Again, the impedance of C_v and C_1 in parallel is $\dfrac{1}{\omega(C_1 + C_v)}$. Thus the multiplying ratio is

$$\frac{V}{v} = \frac{\dfrac{C_s + C_1 + C_v}{\omega C_s(C_1 + C_v)}}{\dfrac{1}{\omega(C_1 + C_v)}} = \frac{C_s + C_1 + C_v}{C_s}$$

$$= 1 + \frac{C_1 + C_v}{C_s} \quad . \quad . \quad (19.9)$$

In this case, again, C_s is variable, so that the voltmeter should be calibrated together with its multiplier, although if the capacitance C_1 is large compared with that of the voltmeter, the multiplying ratio may be approximately constant.

Instrument Transformers. A very general method of increasing the range of a.c. instruments is to use instrument transformers in conjunction with them. Such transformers are either (*a*) current (or series) transformers, or (*b*) voltage (or potential) transformers. The former are used for increasing the current range of instruments and the latter for increasing the voltage range.

ADVANTAGES OF INSTRUMENT TRANSFORMERS AS COMPARED WITH SHUNTS AND MULTIPLIERS. The most important of these advantages are—

(i) Single-range instruments can be used to cover a large current or voltage range, when used with a suitable multi-range current or voltage transformer, or with several single-range transformers. The usual ranges of ammeters and voltmeters for use with such transformers are 5 amp and 110 volts.

(ii) The indicating instruments can be located at some distance from the circuit, the current or voltage of which is to be measured. This is a great advantage, especially when the circuit under test is a high-voltage one, as far greater safety can be obtained for the

observer than when the instruments are connected directly to the high-voltage circuit. Also, the instruments need not be insulated for high voltages.

(iii) By the use of a current transformer with a suitably split and hinged core, upon which the secondary winding is wound, it is possible to measure the current in a heavy-current busbar without breaking the current circuit. The split core of the transformer is simply fitted round the busbar, which acts as the primary winding of the transformer.

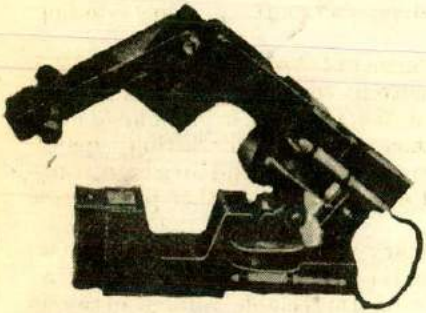
Fig. 19.4 shows a current transformer of this type, while Fig. 19.5 shows a combined ammeter and current transformer which can be used in the same way. In this case the ammeter is, of course, calibrated together with the transformer. Another interesting type of split-core current transformer is made by Price and Belsham, Ltd. This is designed to be fixed round a fuse when no other part of the circuit is available.

Current Transformers. The primary winding of a current transformer is connected in series with the load circuit, the current in which is to be measured. There is no appreciable voltage between the two terminals of this winding, and also—and this is very important—the primary current is obviously not determined by the secondary load on the transformer. The ammeter, or wattmeter current-coil (in the case of power measurements), is connected directly across the secondary winding of the current transformer, as shown diagrammatically in Fig. 19.6.

THEORY OF CURRENT TRANSFORMERS. Fig. 19.7 represents the general vector diagram for a current transformer. The magnitudes of the magnetizing component I_m and the iron-loss component I_e, of the exciting current I_o, are exaggerated for convenience in drawing.

Then n = turns ratio = $\dfrac{\text{number of secondary turns}}{\text{number of primary turns}}$

r_s = resistance of the secondary winding
x_s = reactance of the secondary winding
E_2 = induced secondary voltage
T_p = number of primary turns
T_s = number of secondary turns
E_s = voltage at secondary terminals
I_s = secondary current
I_p = primary current
θ = the "phase angle" of the transformer
ϕ = working flux of the transformer
δ = angle between secondary current and secondary induced voltage
= phase angle of the total burden (including impedance of the secondary winding)
Δ = phase angle of secondary *load* circuit
α = angle between I_e and the working flux



(*Elliott Bros. (London), Ltd.*)

FIG. 19.4. SPLIT-CORE TYPE CURRENT TRANSFORMER



(*Ferranti, Ltd.*)

FIG. 19.5. COMBINED AMMETER AND SPLIT-CORE CURRENT TRANSFORMER

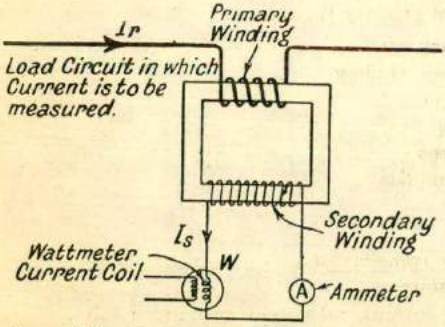


FIG. 19.6. CONNECTIONS OF A CURRENT TRANSFORMER

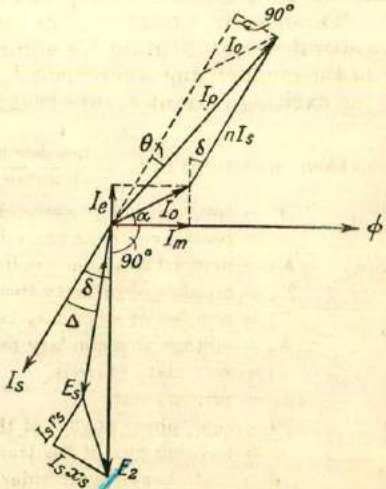


FIG. 19.7. VECTOR DIAGRAM OF A CURRENT TRANSFORMER

Current Transformation Ratio. From the vector diagram,

$$I_p^2 = (I_e + nI_s\cos\delta)^2 + (I_m + nI_s\sin\delta)^2$$
$$= (I_0\sin\alpha + nI_s\cos\delta)^2 + (I_0\cos\alpha + nI_s\sin\delta)^2$$
$$\therefore I_p = [(I_0\sin\alpha + nI_s\cos\delta)^2 + (I_0\cos\alpha + nI_s\sin\delta)^2]^{\frac{1}{2}}$$

$\therefore$ Transformation ratio

$$R = \frac{I_p}{I_s} = \frac{[(I_0\sin\alpha + nI_s\cos\delta)^2 + (I_0\cos\alpha + nI_s\sin\delta)^2]^{\frac{1}{2}}}{I_s}$$
$$= \frac{[I_0^2\sin^2\alpha + 2nI_s\cos\delta\,.\,I_0\sin\alpha + n^2I_s^2\cos^2\delta + I_0^2\cos^2\alpha}{I_s}$$
$$+ \frac{2nI_s\sin\delta\,.\,I_0\cos\alpha + n^2I_s^2\sin^2\delta]^{\frac{1}{2}}}{I_s}$$

Neglecting terms containing I_0^2, this becomes

$$\frac{[n^2I_s^2(\cos^2\delta + \sin^2\delta) + 2nI_sI_0(\cos\delta\sin\alpha + \sin\delta\cos\alpha)]^{\frac{1}{2}}}{I_s}$$
$$= \frac{[n^2I_s^2 + 2nI_sI_0\sin(\alpha+\delta)]^{\frac{1}{2}}}{I_s}$$

which, to a very close approximation,

$$= \frac{nI_s + I_0\sin(\alpha+\delta)}{I_s}$$

since I_e is small compared with nI_s.

$$\therefore R = n + \frac{I_0}{I_s}\sin(\alpha+\delta) \text{ (very nearly)} \quad . \quad (19.10)$$

Although only approximate, this expression is sufficiently accurate for almost all purposes. The above theory refers, of course, to the case when the power factor of the secondary load (or burden) is lagging. This is the most usual condition in practice.

The expression can be expanded still further, as

$$R = n + \frac{I_0}{I_s}(\sin\alpha\cos\delta + \cos\alpha\sin\delta)$$

or

$$R = n + \frac{I_e\cos\delta + I_m\sin\delta}{I_s} \quad . \quad . \quad . \quad (19.11)$$

since $I_0\sin\alpha = I_e$ and $I_0\cos\alpha = I_m$.

PHASE ANGLE. It can be seen from the vector diagram that the secondary current of a current transformer is displaced in phase by almost 180° from the primary current. If this angle were exactly 180°, no phase error would be introduced when a current transformer is used with a wattmeter for power measurements. The existence of the magnetizing and iron-loss components of the primary current causes the angle to be (usually) less than 180°, so that the transformer usually does, in practice, introduce a phase error.

The angle by which the secondary current vector, when reversed, differs in phase from the primary current, is the phase angle of the

transformer; this angle is reckoned as positive if the reversed secondary current *leads* the primary current. On very low power factors the phase angle may be negative.

From the vector diagram (Fig. 19.7),

$$\tan\theta = \frac{I_0 \sin[90-(\alpha+\delta)]}{nI_s + I_0\cos[90-(\alpha+\delta)]}$$

$$= \frac{I_0\cos(\alpha+\delta)}{nI_s + I_0\sin(\alpha+\delta)}$$

from which $$\theta = \frac{I_0\cos(\alpha+\delta)}{nI_s + I_0\sin(\alpha+\delta)} \text{ radians (approx.)} \quad . \quad . \quad (19.12)$$

since θ is a small angle.

The error in the phase angle made by using this approximation is less than $\frac{1}{10}$ per cent up to 3° phase angle, and less than 1 per cent up to 9° phase angle.

Converting this expression to a more convenient form, we have

$$\theta = \frac{I_0(\cos\alpha\cos\delta - \sin\alpha\sin\delta)}{nI_s + I_0\sin(\alpha+\delta)} \text{ radians}$$

$$\doteq \frac{I_m\cos\delta - I_e\sin\delta}{nI_s}$$

since $I_0 \sin(\alpha+\delta)$ is small compared with nI_s.

$$\therefore\ \theta = \frac{180}{\pi}\left(\frac{I_m\cos\delta - I_e\sin\delta)}{nI_s}\right) \text{ degrees} \quad . \quad (19.13)$$

Errors Introduced by Current Transformers. When used for current measurement only, the only essential requirement of a current transformer, in order that it shall not introduce an error into the measurement, is that the secondary current shall be a definite and known fraction of the primary current. It can be seen from the above theory that the current ratio R of the transformer differs from the turns ratio n by an amount which depends upon the magnitude of the exciting current of the transformer, and upon the current and power factor of the secondary circuit. The current ratio is, therefore, not constant under all conditions of load and of frequency, and the error so introduced may be of considerable importance.

Again, in power measurements, it is necessary that the phase of the secondary current shall be displaced by exactly 180° from that of the primary current. It can be seen that, in general, this condition is not fulfilled, but that the transformer has a "phase angle" error θ, which may introduce an appreciable error in power measurements.

In general, it may be said that the ratio error—expressed by $\frac{\text{Nominal ratio} - \text{Actual ratio}}{\text{Actual ratio}}$—is largely dependent upon the value of the iron-loss component I_e of the exciting current, and phase angle error upon the value of the magnetizing component I_m.

Thus, if the angle δ, which is usually fairly small, is assumed zero, then the ratio is

$$R = n + \frac{I_e}{I} \quad . \quad . \quad . \quad . \quad (19.14)$$

and the phase angle $$\theta = \frac{I_m}{nI_s} \quad . \quad . \quad . \quad . \quad . \quad (19.15)$$

The ratio error is considered to be positive when the actual ratio of the transformer is less than the nominal ratio—i.e. when the secondary current, for a given primary current, is high.

DESIGN CONSIDERATIONS. (*a*) *Number of Primary Ampere-turns.* In order that the number of exciting ampere-turns $I_e N_p$ shall be a small proportion of the total primary ampere-turns (this being necessary to keep the ratio and phase-angle errors small), the number of primary ampere-turns should be of the order of 500–1,000. In the case of current transformers having a single bar as their primary winding, the number of primary ampere-turns is, of course, limited by the primary current. When Stalloy is used as the material for the transformer core, satisfactory performance for measurement purposes cannot be expected with this type of transformer when the primary current is less than several hundred amperes, depending upon the magnitude of the burden (see British Standard 81, 1936, p. 30), although if a high-permeability low-loss alloy such as Mumetal is used for the core, the primary current of a bar-primary transformer may be as low as 100 amp and still give a reasonably good performance.

(*b*) *Core.* To minimize the exciting ampere-turns required, the core must have a low reluctance and small iron loss. The flux density used in the core should also be small (not greater than 0·1 weber per square metre). Mumetal cores are now commonly used when it is essential that the transformer errors shall be very small. Mumetal, a nickel-iron alloy containing copper, has the properties of very high permeability, low loss, and small retentivity—all of which are advantageous for current transformer work—but has the disadvantage that its maximum permeability (about 90,000) occurs with a flux density of only about 0·35 weber per square metre, as compared with a maximum permeability of about 4,500 in the case of silicon steel, occurring with a flux density of about 0·5 weber per square metre (Ref. (40)). It is used in the construction of the movements of indicating instruments, for magnetic shields and in other apparatus used in electrical measurements. G. A. V. Sowter (Ref. (43)) has given very full information on Mumetal and other high-permeability alloys, including a series of curves giving magnetization and loss characteristics for these materials together with the variation of their magnetic properties with temperature.

The figures in Table XXI, for Mumetal, are abstracted from Sowter's paper.

TABLE XXI

MAGNETIC PROPERTIES OF MUMETAL

Peak flux density Wb/m²	0·0025	0·005	0·01	0·025	0·05	0·1	0·25	0·5
Magnetizing force at 15° C (A/m)	0·08	0·15	0·28	0·60	1·02	1·76	3·52	10·0
Loss (mW/lb) at 15° C	0·00062	0·00280	0·0122	0·0870	0·385	1·62	9·20	37·5

Incidentally the alloy Permendur (49 per cent iron, 49 per cent cobalt and 2 per cent vanadium) has a very high saturation point of some 2·4 Wb/m² as compared with 0·7–0·8 Wb/m² for other high-permeability alloys.

Its price is a disadvantage of Mumetal, and in this connection Sowter's figures of comparative costs are interesting. These are given in Table XXII.

TABLE XXII

Alloy	Approximate Relative Costs per lb
Hot-rolled silicon iron	0·4
Cold-rolled silicon iron	1·0
36/64 alloy	3·1
Radiometal	3·4
Special radiometal	4·2
Mumetal	5·3

Fig. 19.8* gives curves of magnetizing ampere-turns per metre, at different flux densities, for silicon sheet steel of ordinary grade (Curve I); for special silicon steel (Curve II); and for a nickel-iron alloy.

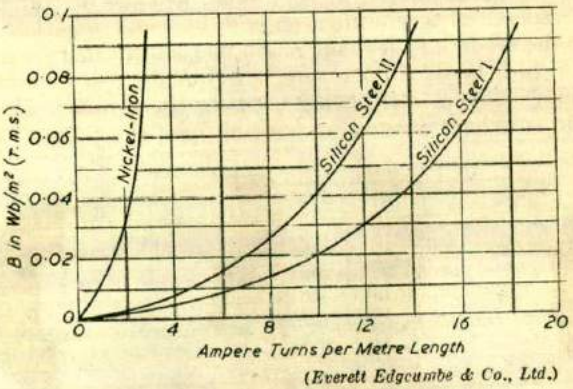
When Stalloy punchings are used, their width is usually about 1 in. to 1½ in., and their individual thickness 0·014 in. The punchings are often annealed, to remove "punching strains." The total width of the punching being small, the strained area may be an appreciable part of the total cross-section (Ref. (10)).

The length of magnetic path in the core should be as small as is consistent with good mechanical construction and with insulation requirements, in order to reduce the core reluctance. For the same reason, joints in the core should be avoided as far as possible, and

* See K. Edgcumbe and F. E. Ockenden, *I.E.E. Journal*, Vol. LXV, p. 553.

such joints as do exist should be rendered as efficient as possible by careful assembly.

Fig. 19.9 shows the three forms of core most commonly used. (*a*) is the rectangular form, built up of L-shaped punchings. The



(*Everett Edgcumbe & Co., Ltd.*)

FIG. 19.8. CURVES OF MAGNETIZING FORCE FOR VARIOUS FLUX DENSITIES IN THE CASE OF THREE DIFFERENT MATERIALS

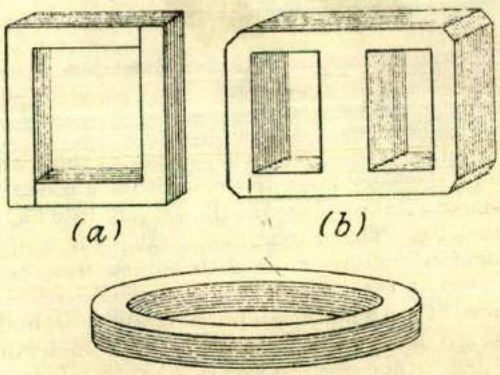


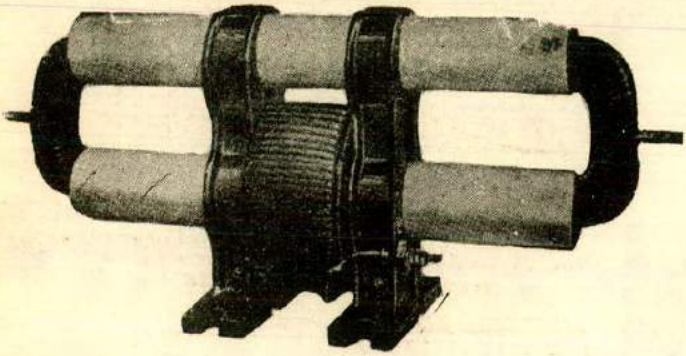
FIG. 19.9 FORMS OF CORES

windings are placed on one of the shorter limbs, with the primary usually wound over the secondary, an advantage being that there is ample space for insulation, so that this form is suitable for high-voltage work.

The "shell" form in Fig. 19.9 (*b*) gives considerable protection to the windings, but is more difficult to build up than the other forms. The windings are placed on the central limb.

The ring form (*c*) is very commonly used when the primary current is large. The secondary winding is uniformly distributed round the ring, and the primary winding is a single bar. This is a very robust construction, and has the further advantages of a jointless core (giving low reluctance) and of very small leakage reactance.

(*c*) *Windings.* The windings should be close together in order to reduce the secondary leakage reactance, as this increases the ratio error. No. 14 S.W.G. copper wire is frequently used for the secondary winding, copper strip being used for the primary winding, the dimensions of which depend, of course, upon the primary current.



(*Chamberlain & Hookham, Ltd.*)

FIG. 19.10. HIGH-VOLTAGE TYPE AIR-COOLED CURRENT TRANSFORMER

The windings must be designed with a view to their withstanding, without damage, the very large forces which are brought into play when a short-circuit takes place on the system in which the transformer is connected. The bar-primary, ring-core construction is generally recognized as the most satisfactory from this point of view.

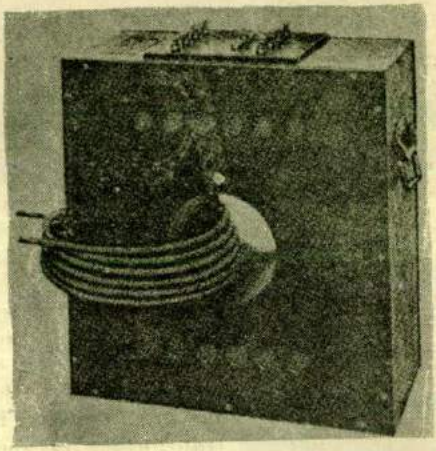
(*d*) *Insulation.* The windings are separately wound, and are insulated by tape and varnish for the lower line voltages. For voltages of 7,000 volts and upwards, the transformers are oil-immersed or compound-filled. In the former case the windings are wound over one another on cylinders of Bakelite or similar material, and the transformer is mounted in a sheet steel tank filled with transformer oil. In the latter case the transformer is enclosed in a tank which is filled up solid with an insulating compound. The compound introduces a difficulty in that the cooling is poor. Some high-voltage transformers have porcelain tubes as the insulation for their primary winding and are not oil-immersed. Such a transformer is shown in Fig. 19.10.

A portable current transformer in which the high current ranges

can be obtained by actually winding turns through the core aperture is illustrated in Fig. 19.11.

Fig. 19.12 shows a special current transformer designed to operate at 275 kV.

DESIGN DATA. The following figures in connection with the design of current transformers do not refer to any particular type, but are



(Everett Edgcumbe & Co., Ltd.)

FIG. 19.11. PORTABLE OMNI-RANGE CURRENT TRANSFORMER

intended to give a general idea of the usual magnitudes of the quantities involved.

Total primary ampere-turns	1,000
Exciting ampere-turns	Not more than 1 per cent of the total primary ampere-turns at full rated current
Full rated secondary current	5 amp
Core flux density	Not more than 0·1 Wb/m²
Length of mean magnetic path in the core	About 40 cm (less than this if the line voltage is low)
Current density in primary winding	1 to 2 amp per sq. mm
Secondary winding	14 S.W.G. (d.c.c.) copper wire
Resistance of secondary winding	About 0·5 ohm
Inductance of secondary winding	Varies according to type from a negligible quantity (in bar-primary, ring-core type) to about 15 millihenrys

(Note that the resistance and inductance of the primary winding are not considered, since they do not affect the performance of the transformer.)

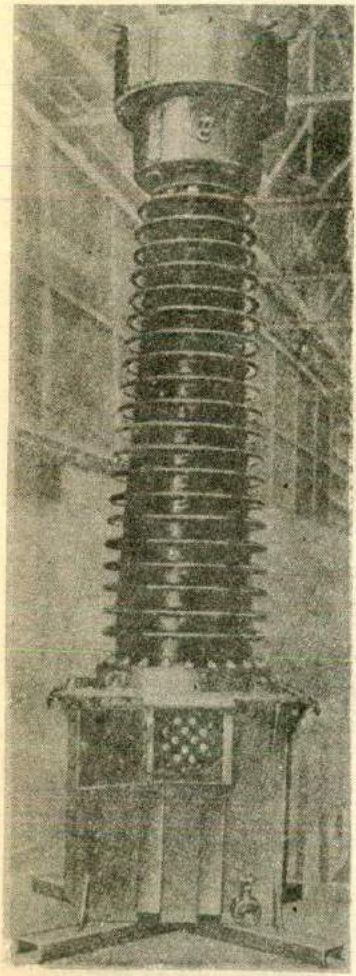
Turns Compensation is used in most current transformers in order to obtain a transformation ratio more nearly equal to the nominal

one than would obtain if the turns ratio were made equal to the nominal ratio of the transformer. It has been shown that, if the phase angle of the secondary circuit is zero, the actual ratio of transformation is given by

$$R = n + \frac{I_e}{I_s}$$

where n is the turns ratio. The effect of reducing the number of secondary turns by (say) 1 per cent will obviously be to reduce the transformation ratio by an approximately equal percentage. By this means, partial compensation for the effect of I_e, in increasing the ratio, can be obtained. Usually, the best number of secondary turns is one or two less than that number which would make n equal to the nominal ratio of the transformer. For example, in a 1,000/5 current transformer of the bar-primary type the number of secondary turns would be either 199 or 198 instead of 200.

The phase angle error is very little affected by a change of one or two in the number of secondary turns.



(*Associated Electrical Industries, Ltd.*)

FIG. 19.12

Outdoor Multiple-secondary Current Transformer for 275 kV.

Standard Ratios, Rating and Permissible Errors. British Standards Institution, Specification B.S. 81 : 1936 for "Instrument Transformers," gives a list of twenty-five rated primary currents, ranging from 5 to 6,000 amp, the rated secondary current normally being 5 amp, although it may be 1 amp or even 0·5 amp under certain conditions.

The burden of the current transformer is expressed as the output in volt-amperes at the rated secondary current. Thus, for a burden of 15 VA, the rated secondary current being 5 amp, the secondary terminal voltage will be 3 volts and the impedance of the connected load 0·6 ohm. Rated burdens for various classes of current transformer are 1½, 2½, 5, 7½ 15, and 30 VA. The standard frequency is 50 cycles per second. There are nine classes of

current transformer mentioned in the specification: Classes A, B, C, and D for use with substandard and commercial instruments of various grades; classes AL and BL for precision testing and laboratory work; and classes AM, BM, and CM for industrial metering.

Various standard rated burdens and permissible errors are stated for these classes, the latter varying from 0·15 per cent ratio error and 3 min phase angle (in the AL class) down to 1 per cent and 120 min. (in class C). No phase-angle error is specified in class D, for which the permissible ratio error is 5 per cent. It should be pointed out that the rating of these transformers is upon an accuracy basis, and is not a thermal rating.

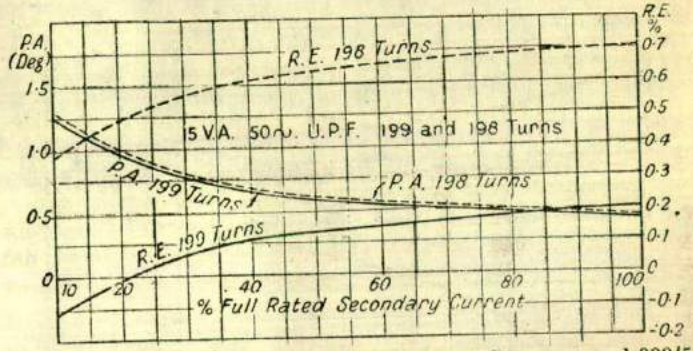


FIG. 19.13. RATIO ERROR AND PHASE ANGLE CURVES FOR 1,000/5 BAR-PRIMARY RING-CORE CURRENT TRANSFORMER

CHARACTERISTICS OF CURRENT TRANSFORMERS. Fig. 19.13 shows the ratio-error and phase-angle characteristics of a 1,000/5 bar-primary ring-core type of current transformer, with a burden of 15 VA, unity power factor, 50 c/s frequency. These errors are plotted against various values of secondary current. Two curves of ratio error and of phase angle are shown, one pair relating to 199 secondary turns and the other to 198 turns. The errors were calculated for a transformer designed to comply with the B.S.I. specification for a class B transformer. Obviously the 199-turn secondary winding gives the better performance.

The shapes of the error curves are characteristic of all current transformers. The "droop" of the ratio-error curve and rise of the phase-angle curve for low values of secondary current are due to the fact that, when this current is reduced—with consequent proportional reduction of the flux density in the core—the iron-loss and magnetizing ampere-turns are not reduced proportionately, and so form a greater proportion of the total primary ampere-turns than they do for higher values of I_s (and of I_p, since the reduction of the secondary current I_s is produced by a reduction of the primary current I_p).

The small effect of the alteration of the number of secondary turns upon the phase-angle error can be seen from these curves. The order

of magnitude of the "droop" of the ratio-error curve depends upon the value of $\frac{I_e}{I_s}$ compared with the value of the turns ratio n, and the rise of the phase-angle curve upon the value of I_m compared with nI_s.

Effect of Variation of Power Factor of the Secondary Burden upon the Performance.

Referring to Fig. 19.7, reduction of the power factor of the load

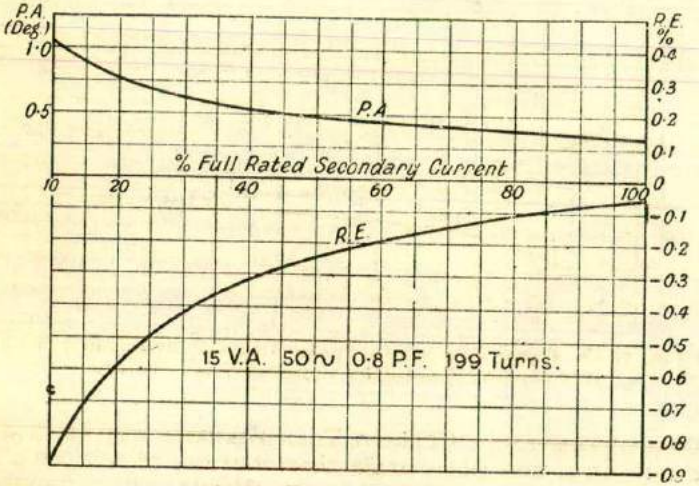


FIG. 19.14

circuit increases the angle Δ—and hence δ—and thus brings the vectors nI_s and I_o more nearly into phase with one another. This increases the value of I_p for any given value of I_s and thus the transformation ratio $\frac{I_p}{I_s}$ is increased. The ratio error is made less positive as the power factor is reduced, up to the point when I_s is in phase with I_o reversed, when the ratio $\frac{I_p}{I_s}$ will be maximum and the ratio error least positive (or most negative).

The phase-angle error is obviously reduced with reduction of power factor, since nI_s moves more into phase with I_o (Fig. 19.7) as δ is increased. This reduces the phase angle θ, which becomes zero when $\delta = 90 - \alpha$, nI_s and I_o then being in phase. Fig. 19.14 shows the ratio and phase-angle curves for the 1,000/5 transformer, to which the curves of Fig. 19.13 refer, when the secondary load power factor is 0·8 lagging, the burden being 15 VA and the frequency 50 c/s.

Fig. 19.15 shows the effect of variation of secondary load power

factor upon the ratio error and phase angle of the same transformer for 10 per cent and 100 per cent full rated secondary current, the burden being 15 VA and frequency 50 c/s.

The ranges of values (between 10 per cent and 100 per cent full rated current) covered by the ratio-error and phase-angle curves for any given power factor can be obtained by taking the intercepts between the two ratio-error or phase-angle curves at the given power factor. For example, at 0·6 power factor the ratio-error curve has a range of om – 0·15 per cent at 100 per cent full rated current to – 1·20 per cent at 10 per cent full current (from intercept *CD*, Fig. 19.15). The phase angle ranges from 0·27° (at 100 per cent full

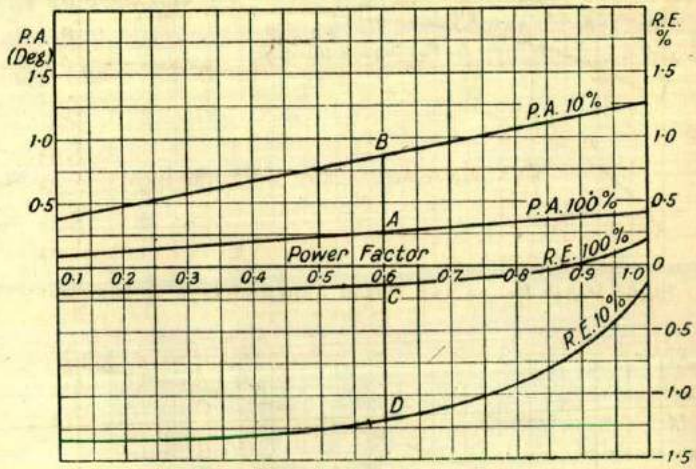


FIG. 19.15. EFFECT OF VARIATION OF THE SECONDARY LOAD POWER FACTOR

rated current) to 0·85° (at 10 per cent full current) as obtained from intercept *AB*.

Effect of Variation of the Secondary Burden upon the Performance of a Current Transformer. Increase of the secondary burden (in volt-amperes) obviously necessitates an increase of secondary terminal voltage for a given value of secondary current. This again means an increase in the induced secondary voltage and consequently in the core flux and flux density. The exciting current I_o (Fig. 19.7) is thus increased, and this increases the ratio of the transformer, causing the ratio error to become less positive for any given values of power factor and frequency. The phase angle, also, is considerably increased by increase of burden.

The ratio-error and phase-angle curves for the 1,000/5 current transformer already mentioned, with burdens of 5, 15, and 40 VA,

are shown together in Fig. 19.16 for comparison. Fig 19:17 shows the general effect of variation of secondary burden upon the ratio and phase-angle errors, the curves referring to the same transformer.

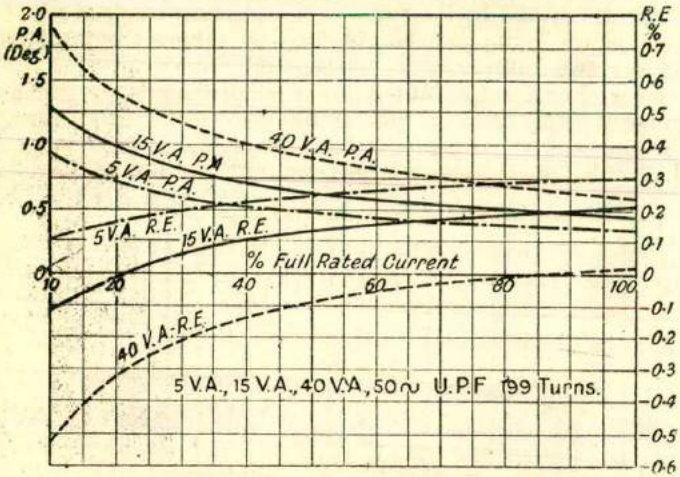


Fig. 19.16. Ratio Error and Phase Angle Curves at Different Burdens

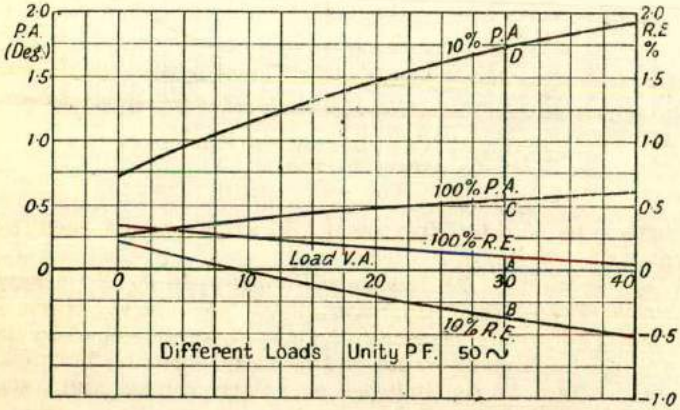


Fig. 19.17. Effect of Variation of Secondary Burden

Effect of Frequency upon Performance. A current transformer is seldom called upon to operate at a frequency very different from that for which it was designed (usually 50 c/s), so that the consideration of the effect of frequency variation is less important than that

of variation of load and power factor. The variation of the magnetizing and iron-loss magnetizing force, with variation of frequency, is comparatively small if the frequency variation does not exceed 10 or 20 cycles per second, so that, *as an approximation*, they may be considered independent of frequency.

For a given volt-ampere load and power factor, the secondary induced voltage is constant. Since this is proportional to the product of frequency and flux density, it follows that an increase of frequency will result in a proportionate decrease in flux density. Thus, in general, the effect of increase of frequency is similar to that

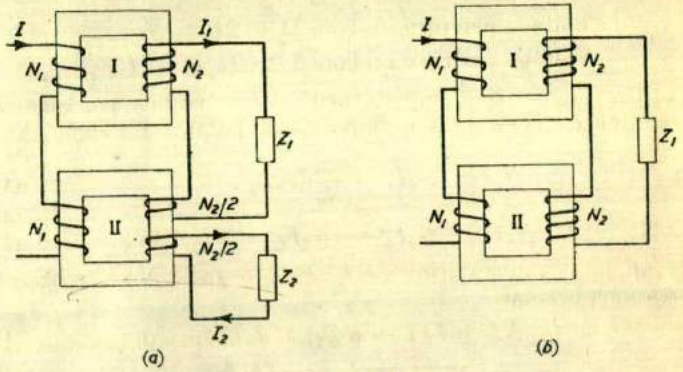


FIG: 19.18

produced by a decrease in the impedance of the secondary burden. It is roughly true that doubling the frequency has the same effect upon the performance of the transformer as halving the secondary burden for a fixed frequency.

In addition to the use of a low-loss, high-permeability, nickel-iron core for the reduction of current transformer errors, several special devices have been suggested by various investigators. H. B. Brooks and F. C. Holtz (Ref. (11)), H. W. Price and C. Kent-Duff (Ref. (12)), and J. G. Wellings and C. G. Mayo (Ref. (13)) have all described methods of minimizing these errors.

A particularly useful device has been described by A. Hobson (Ref. (45)). The error of a current transformer is due to the magnetizing current which is necessary to maintain the core flux. If there is no voltage developed in the secondary winding there is no core flux, and as a result there is no magnetizing current and the transformer is free from error. This condition can be achieved by supplying the burden voltage from a separate source. Fig. 19.18 (*a*) shows Hobson's method of compensation. Two similar cores are used with their primaries in series. The primary windings have the same number of turns, and if the secondary winding on core I has

N_2 turns, the two secondaries on core II each have $\frac{1}{2}N_2$ turns. The actual burden is Z_1, and a similar compensating burden Z_2, equal to Z_1, is connected to one of the secondaries on core II. There must be ampere-turn balance in each core, and the current in Z_2 must therefore be very nearly equal to the current in Z_1. The voltage across Z_2 is then nearly equal to that across Z_1, but this is provided by the half-windings on core II. There is therefore an extremely small voltage across the secondary of core I, and the flux in this core, and the magnetizing current, are negligibly small. The current ratio is then equal to the turns ratio on core I. An expression for the complex ratio may be obtained as follows—

$$\text{Voltage per turn on core II} = 2I_2Z_2/N_2$$

$$\text{Voltage per turn on core I} = (I_1Z_1 - I_2Z_2)/N_2$$

Let the magnetizing ampere-turns $= k \times$ voltage per turn, where k is complex. Then, taking ampere-turn balance for each core,

Core I. $$N_1I + N_2I_1 = \frac{k}{N_2}(I_1Z_1 - I_2Z_2) \quad . \quad . \quad (19.16)$$

Core II. $$N_1I + \tfrac{1}{2}N_2I_1 + \tfrac{1}{2}N_2I_2 = 2kI_2Z_2/N_2 \quad . \quad (19.17)$$

Rearranging and writing $K = k/N_2^2$ and $N = N_1/N_2$ these expressions become

$$NI + I_1[1 - KZ_1] + I_2KZ_2 = 0 \quad . \quad . \quad (19.18)$$

$$NI + \tfrac{1}{2}I_1 + I_2[\tfrac{1}{2} - 2KZ_2] = 0 \quad . \quad . \quad (19.19)$$

Eliminating I_2 gives

$$NI[1 - 6KZ_2] + I_1[1 - KZ_1 - 5KZ_2 + 4K^2Z_1Z_2] = 0$$

from which

$$N\frac{I}{I_1} = \frac{[1 - 6KZ_2 + KZ_2 - KZ_1 + 4K^2Z_1Z_2]}{1 - 6KZ_2}$$

$$= -\left[1 + \frac{K(Z_2 - Z_1) + 4K^2Z_1Z_2}{1 - 6KZ_2}\right]$$

Now, $Z_1 = Z_2$;

hence $$N\frac{I}{I_1} = -\left[1 + \frac{4K^2Z_1^2}{1 - 6KZ_1}\right]$$

KZ_1 is small compared with unity and the complex ratio is then

$$\frac{I}{I_1} = -\frac{1}{N}[1 + 4K^2Z_1^2]$$

and, substituting for K,

$$\frac{I}{I_1} = -\frac{1}{N}\left[1 + \frac{4k^2Z_1^2}{N_2^4}\right] \quad (19.20)$$

This result may be compared with that of a single transformer having the same number of turns with the same core area, by supposing that each core has N_1 turns as in Fig. 19.18 (*b*). For each core the voltage per turn is now $\frac{I_1 Z_1}{2N_2}$, and the magnetizing ampere-turns per core is

$$k\frac{I_1 Z_1}{2N_2} = \frac{KN_2 I_1 Z_1}{2}$$

Taking ampere-turn balance in each core,

$$N_1 I + N_2 I_1 = \frac{KN_2 I_1 Z_1}{2}$$

from which

$$\frac{I}{I_1} = -\frac{1}{N}\left[1 - \frac{1}{2}KZ_1\right]$$

$$= -\frac{1}{N}\left[1 - \frac{kZ_1}{2N_2^2}\right] \quad . \quad . \quad (19.21)$$

In a practical case the error term $\frac{4k^2 Z_1^2}{N_2^4}$ in Equation (19.20) is very much smaller than the corresponding term $\frac{kZ_1}{2N_2^2}$ in Equation (19.21).

Use of Current Transformers. It can be seen from the preceding paragraphs that the errors introduced by a current transformer vary with the magnitude and power factor of the secondary burden and with the frequency. It cannot, therefore, be assumed that the errors so introduced are constant, when the transformer is used with ammeters, wattmeters, etc., whose impedances and power factors differ from the impedance and power factor of the load with which the transformer was calibrated. This must be taken into consideration when selecting instruments for use with the transformer. If, however, the transformer and instrument with which it is to be used are calibrated together, these variations in the characteristics of the former are unimportant. Otherwise, if r is the ratio error—expressed as a percentage—the actual ratio of the transformer R is given by

$$R = \frac{N}{\frac{r}{100} + 1} \quad . \quad . \quad . \quad . \quad (19.22)$$

where N = nominal ratio.

If I_s is the measured value of the secondary current, the current in the circuits to be measured (i.e. the primary current) is

$$\left(\frac{N}{\frac{r}{100}+1}\right) I_s$$

Correction for the phase angle error is unnecessary when current only is to be measured.

Open-circuiting of the Secondary Circuit of a Current Transformer. In a current transformer the number of primary ampere-turns is a fixed quantity (assuming a constant primary current) and *is thus not reduced when the secondary circuit is opened*, as it is in the case of a power transformer. If, therefore, a current transformer has its secondary circuit opened when current is flowing in its primary circuit, a very high flux density is produced in the core owing to the absence of "back" ampere-turns due to current in the secondary winding. This high flux density results in a greatly increased induced voltage in the secondary winding, with possible strain on the insulation and danger to the operator (see Ref. (8)).

Moreover, the high magnetizing forces acting upon the core may, if suddenly removed, leave behind considerable residual magnetism in the core, so that the ratio and phase-angle errors, obtained after such an open circuit, may differ appreciably from those before it occurred. For these reasons, care must be taken to ensure that, even when not in use for measuring purposes, the secondary circuit is *closed* if the primary current is flowing. The secondary winding may, of course, be short-circuited quite safely, since when used for measurement it is practically short-circuited, the impedance of the burden being merely that of the ammeter or wattmeter current coil used.

It should be noted also that a direct current passed through either of the current transformer windings will produce magnetization of the core which may alter the transformer errors. This is especially the case when a direct current is passed through the secondary winding—as in the measurement of its resistance, for example—since the secondary usually has many more turns than the primary winding. An appreciable number of magnetizing ampere-turns may thus result from the passage of even a small current through the secondary winding.

Demagnetization after an Open Circuit. If the secondary circuit of a current transformer is accidentally opened while the primary current is flowing, the transformer should be demagnetized before being used again. There are several simple ways of carrying out such demagnetization. The following two methods appear to be equally satisfactory in reducing the transformer to its original state of magnetization.

The first method is to pass through the primary winding a current at least equal to that which was passing through it when the open circuit took place, the secondary circuit being left open. (This primary current should be supplied from a motor-alternator set which can be shut down as required.) The motor driving the supply alternator is then shut down with the alternator field still on. As the set slows down, the alternator voltage falls gradually to zero, and the iron of the transformer core is passed through a large number of cycles of magnetization, of gradually decreasing amplitude, finishing at zero magnetization.

Another method is to connect, across the secondary winding of the current transformer, a resistance which is sufficiently high to amount almost to an open circuit as regards the secondary "back" ampere-turns obtained. The full current is passed through the primary winding, and the secondary resistance (several hundred ohms) is then gradually reduced to zero as uniformly as possible. By this means the magnetization of the transformer core is reduced from a very high value to its normal value very gradually.

Upon testing a current transformer, after demagnetization, by either of these methods, the authors have found that the alterations of ratio error and phase angle, due to an open circuit, have been entirely eliminated.

Voltage (or Potential) Transformers. The errors introduced by the use of voltage transformers are, in general, less serious than those introduced by current transformers, and the theory of such transformers is essentially the same as that of the ordinary power transformer.

The main difference between the voltage transformer and the power transformer is that, in the former, the secondary current is of the same order as the magnetizing current.

THEORY. Fig. 19.19 shows the vector diagram of a voltage transformer.

ϕ = working flux in the transformer core
I_m = magnetizing component of the no-load current
I_e = iron-loss component of the no-load current
I_0 = no-load current
E_s = voltage induced in the secondary winding
V_s = secondary terminal voltage
I_s = secondary current
$I_s r_s$ = voltage drop in secondary winding resistance r_s (in phase with I_s)
$I_s x_s$ = voltage drop in secondary winding reactance x_s (90° in advance of I_s in phase)
I_p = primary current
= vector sum of I_0 and I_s reversed
E_p = that portion of the primary applied voltage which is available for transformation
= primary terminal (or applied) voltage V_p less the voltage drop in impedance of the primary winding
$I_p r_p$ = voltage drop in the resistance of the primary winding
$I_p x_p$ = voltage drop in the reactance of the primary winding
Δ = the phase angle of the secondary load circuit (usually a voltmeter)

Since the resistance of most voltmeters is very much higher than their reactance, Δ will usually be small.

In the vector diagram the magnitudes of the voltage-drop vectors $I_s r_s$, $I_s x_s$, $I_p r_p$, and $I_p x_p$ have been exaggerated for clearness in the drawing.

The secondary terminal voltage V_s is obtained by subtracting (vectorially) the voltage drops in the secondary winding from the voltage E_s induced in the secondary. Again, the voltage available

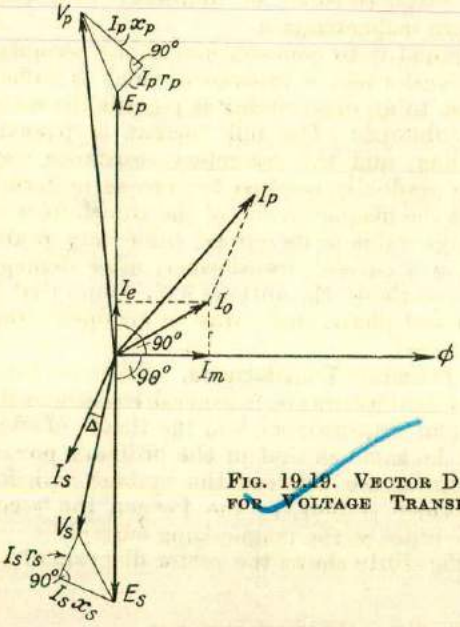


FIG. 19.19. VECTOR DIAGRAM FOR VOLTAGE TRANSFORMER

for transformation in the primary—i.e. the component of the primary applied voltage which is in phase opposition to E_s—is obtained by subtracting (vectorially) the voltage drops in the primary winding from the applied primary voltage V_p.

The rest of the diagram is self-explanatory.

Ratio Expression. The expression for the voltage ratio $\frac{V_p}{V_s}$ of the transformer is best derived by considering the diagram shown in Fig. 19.20, in which all the vectors concerned are referred to the primary side. Secondary voltages, when referred to the primary side, become n times their actual (secondary side) value, n being the turns ratio of the transformer.

$$n = \frac{\text{Number of primary turns}}{\text{Number of secondary turns}} = \frac{E_p}{E_s}$$

When secondary currents are referred to the primary side their actual value must be *divided* by n.

Referring to Fig. 19.20, θ is the phase angle of the transformer—i.e. the phase angle between V_p and V_s reversed. Δ is the phase angle of the secondary load circuit, and β the phase angle between V_s reversed and I_p.

Then, projecting V_p on to V_s (reversed) produced, we have

$$V_p \cos\theta = V_s n + I_s r_s n \cos\Delta + I_s x_s n \sin\Delta + I_p r_p \cos\beta + I_p x_p \sin\beta$$
$$= V_s n + nI_s(r_s \cos\Delta + x_s \sin\Delta) + I_p(r_p \cos\beta + x_p \sin\beta)$$

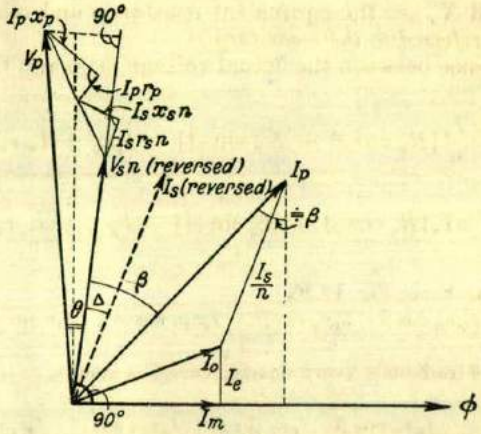


FIG. 19.20

Since θ is usually very small, both V_s (reversed) and V_p may be considered as at right angles to ϕ, so that

$$I_p \cos\beta = I_e + \frac{I_s}{n}\cos\Delta$$

and

$$I_p \sin\beta = I_m + \frac{I_s}{n}\sin\Delta$$

$\cos\theta = 1$ (very nearly) since θ is very small (less than 1°)

$$\therefore V_p = V_s n + nI_s(r_s \cos\Delta + x_s \sin\Delta) + r_p\left(I_e + \frac{I_s}{n}\cos\Delta\right) + x_p\left(I_m + \frac{I_s}{n}\sin\Delta\right)$$

$$V_p = nV_s + I_s \cos\Delta\left(nr_s + \frac{r_p}{n}\right) + I_s \sin\Delta\left(nx_s + \frac{x_p}{n}\right) + I_e r_p + I_m x_p$$

$$V_p = nV_s + \frac{I_s}{n}\cos\Delta(n^2 r_s + r_p) + \frac{I_s \sin\Delta}{n}(n^2 x_s + x_p) + I_e r_p + I_m x_p$$

$\therefore$ the voltage ratio is

$$\frac{V_p}{V_s} = n + \frac{\frac{I_s}{n}[R_p \cos\Delta + X_p \sin\Delta] + I_e r_p + I_m x_p}{V_s} \quad . \quad . \quad (19.23)$$

where R_p = equivalent resistance of the transformer *referred to the primary*

and X_p = equivalent reactance of the transformer *referred to the primary*

Otherwise,

$$V_p = nV_s + nI_s\cos\Delta\left(r_s + \frac{r_p}{n^2}\right) + nI_s\sin\Delta\left(x_s + \frac{x_p}{n^2}\right) + I_e r_p + I_m x_p$$

or
$$\frac{V_p}{V_s} = n + \frac{nI_s[R_s\cos\Delta + X_s\sin\Delta] + I_e r_p + I_m x_p}{V_s} \quad (19.24)$$

where R_s and X_s are the equivalent resistance and reactance of the transformer *referred to the secondary*.

The difference between the actual voltage ratio and the turns ratio n is thus

$$\frac{\frac{I_s}{n}[R_p\cos\Delta + X_p\sin\Delta] + I_e r_p + I_m x_p}{V_s}$$

or
$$\frac{nI_s[R_s\cos\Delta + X_s\sin\Delta] + I_e r_p + I_m x_p}{V_s} \quad (19.25)$$

Phase Angle. From Fig. 19.20,

$$\sin\theta = \frac{I_p x_p\cos\beta - I_p r_p\sin\beta + I_s x_s n\cos\Delta - I_s r_s n\sin\Delta}{V_p}$$

$= \theta$ (radians) (very nearly, since θ is small)

Otherwise,

$$\tan\theta = \frac{I_p x_p\cos\beta - I_p r_p\sin\beta + I_s x_s n\cos\Delta - I_s r_s n\sin\Delta}{V_s n + nI_s r_s\cos\Delta + I_s x_s n\sin\Delta + I_p r_p\cos\beta + I_p x_p\sin\beta}$$

$$= \frac{I_p x_p\cos\beta - I_p r_p\sin\beta + I_s x_s n\cos\Delta - I_s r_s n\sin\Delta}{V_s n}$$

since the terms in the denominator involving I_s and I_p are small compared with $V_s n$, or

$$\tan\theta = \frac{x_p\left(I_e + \frac{I_s}{n}\cos\Delta\right) - r_p\left(I_m + \frac{I_s}{n}\sin\Delta\right) + I_s x_s n\cos\Delta - I_s r_s n\sin\Delta}{V_s n}$$

$$= \frac{I_s\cos\Delta\left(\frac{x_p}{n} + x_s n\right) - I_s\sin\Delta\left(\frac{r_p}{n} + r_s n\right) + I_e x_p - I_m r_p}{V_s n}$$

$$= \frac{\frac{I_s\cos\Delta}{n}(x_p + x_s n^2) - \frac{I_s\sin\Delta}{n}(r_p + r_s n^2) + I_e x_p - I_m r_p}{V_s n} \quad (19.26)$$

Thus, since $\tan\theta = \theta$ (radians) very nearly, θ being small,

$$\theta_{(radians)} = \frac{\frac{I_s\cos\Delta}{n}X_p - \frac{I_s\sin\Delta}{n}R_p + I_e x_p - I_m r_p}{V_s n}$$

$$= \frac{nI_s\cos\Delta\, X_s - nI_s\sin\Delta\, R_s + I_e x_p - I_m r_p}{V_s n}$$

$$= \frac{I_s}{V_s}(X_s\cos\Delta - R_s\sin\Delta) + \frac{I_e x_p - I_m r_p}{V_s n} \quad (19.27)$$

where R_p and X_p are the equivalent resistance and reactance referred to the primary, and R_s and X_s the same referred to the secondary.

ERRORS INTRODUCED BY VOLTAGE TRANSFORMERS. It can be seen from the above that, like current transformers, voltage transformers introduce an error, both of magnitude and of phase, in the measured value of the voltage. The ratio error only is important when measurements of voltage are to be made, the phase angle error being of importance only when power is being measured.

Obviously, the voltage applied to the primary circuit of the transformer cannot be obtained correctly simply by multiplying the voltage measured by the voltmeter in the secondary circuit by the turns ratio n of the transformer.

The divergence of the actual ratio $\frac{V_p}{V_s}$ from n depends upon the reactance and resistance of the transformer windings as well as upon the value of the exciting current of the transformer.

The phase-angle error depends largely upon the same factors.

DESIGN CONSIDERATIONS. In order to reduce the errors introduced by these transformers, the reluctance of the transformer core should be as small as possible, and the flux density in the core should be fairly low (under 1·0 Wb/m^2). The exciting current can then be made small.

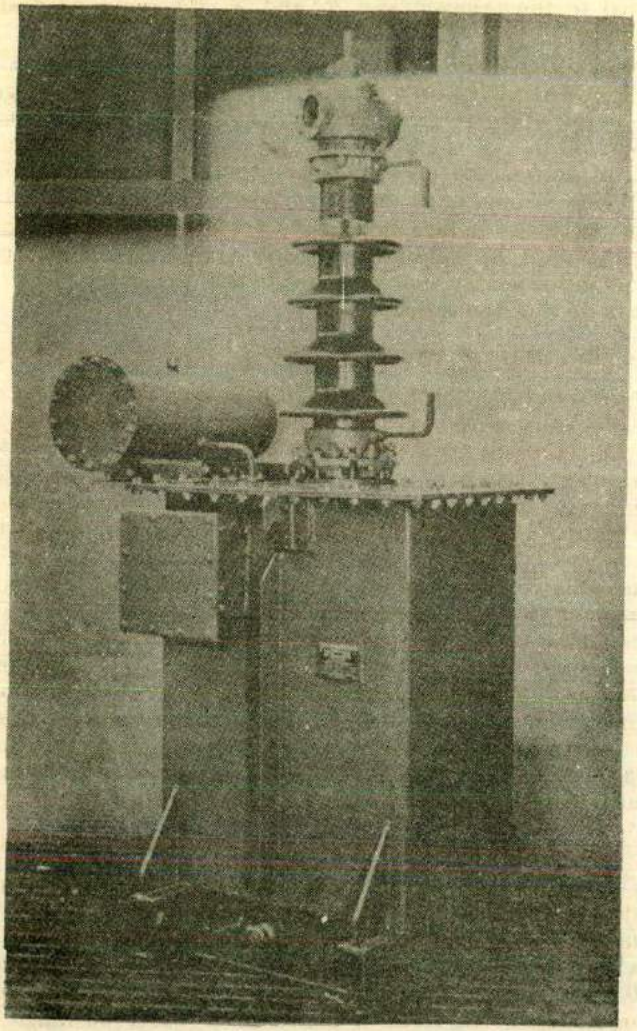
Since the reactance of the transformer windings depends upon the magnitudes of the primary and secondary leakage magnetic fluxes, these should be kept small by placing the two windings as close together as is consistent with insulation requirements. The resistances of the windings should also be as small as possible.

Turns compensation can be applied in the voltage transformer even more successfully than in the current transformer owing to the large number of primary turns, the difference in ratio caused by the addition or removal of one primary turn being thus comparatively small. Since the actual ratio $\frac{V_p}{V_s}$ of the transformer is greater than the turns ratio, n, this actual ratio can be made more nearly equal to the nominal ratio by adjusting the numbers of primary and secondary turns so that n is *less* than the nominal ratio.

As regards insulation, voltage transformers for voltages of 7,000 volts and over are oil-immersed. Adequate insulation must be provided, in the form of porcelain or other bushings, for the primary terminals, since, unlike the case of the current transformer, the full line voltage is applied to the primary winding.

The load on such transformers is usually very small, consisting as it does merely of a voltmeter and/or the voltage coil of a wattmeter, so that there is little heat produced in the transformer.

Fig. 19.21 shows the construction of some types of voltage transformers.



(*Associated Electrical Industries Ltd.*)

FIG. 19.21. OUTDOOR-TYPE VOLTAGE TRANSFORMER, RATIO $\frac{66{,}000}{\sqrt{3}} / \frac{110}{\sqrt{3}}$

Standard Ratio and Permissible Errors. B.S.I. Specification B.S. 81:1936, already referred to, specifies a secondary voltage of 110 volts when the rated primary voltage is applied to the transformer.

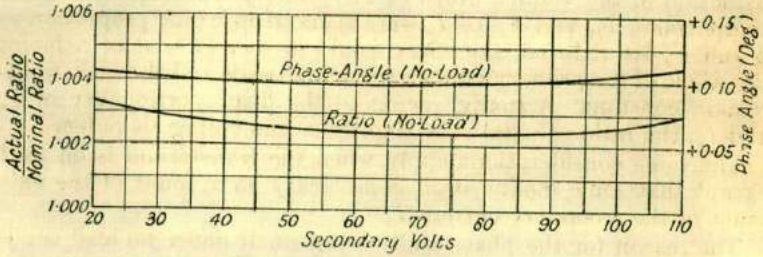


FIG. 19.22. VOLTAGE TRANSFORMER RATIO AND PHASE ANGLE CURVES

The standard ratios of transformation are as follows—

TABLE XXIII

Primary Volts	Secondary Volts	Primary Volts	Secondary Volts
110	110	33,000	110
440	,,	44,000	,,
550	,,	55,000	,,
660	,,	66,000	,,
2,200	,,	88,000	,,
3,300	,,	110,000	,,
5,500	,,	132,000	,,
6,600	,,	165,000	,,
11,000	,,	220,000	,,
22,000	,,		

These voltages apply when the transformer is connected between lines. If connected between line and neutral, the voltages should be divided by $\sqrt{3}$.

Six classes of voltage transformers are mentioned in the specification: A, B, C, D, and AL and BL. The rated burdens range from 15 to 200 VA for classes A, B, and C (no standard burden for class D); while for classes AL and BL, 10 VA burden is specified. Limits of error range from 0·5 per cent ratio error and 20 min phase angle to 2 per cent (5 per cent, class D), and 30 min for classes A, B, and C. For class AL, 0·25 per cent ratio error and 10 min phase angle; and for class BL, 0·5 per cent and 20 min are permissible.

CHARACTERISTICS OF VOLTAGE TRANSFORMERS. Fig. 19.22 shows typical ratio and phase-angle curves for a voltage transformer, the errors being plotted against values of secondary voltage. It can be seen that there is comparatively little change in these errors with change of voltage. The secondary terminal voltage on no load will, of course, be the same as the secondary induced voltage. As the applied primary voltage is reduced, the flux density in the core

will be correspondingly reduced, involving a reduction (although not quite proportional) in I_o (see Fig. 19.19). This, again, means a reduction in the voltage drops in the primary winding. If the two components, I_m and I_e, of I_o were reduced in exact proportion as ϕ and E_s are reduced, the effect would be merely that of reducing the whole of the primary voltage triangle to scale and the ratio would remain constant. Actually, owing to the disproportionality of I_m and I_e, the ratio changes very slightly as the voltage is reduced.

The same considerations apply when the transformer is on load, except that some modification is necessary on account of the existence of the secondary current I_s.

The reason for the phase angle being small under no-load conditions is that the vector $I_p x_p$ representing the primary-reactance voltage drop—which vector is chiefly responsible for the phase displacement—is then perpendicular to the direction of I_o, and is thus more nearly in phase with E_p than when a secondary current exists (see Fig. 19.19).

Effect of Variation of Secondary Burden. Considering the case of full-rated voltage applied to the primary winding, then the secondary voltage will be approximately the same for all values of secondary volt-amperes burden. Increase of this burden will thus mean a proportional increase in the secondary current. Thus, if the burden is 15 VA with full-rated voltage applied to the transformer, the secondary current will be $\frac{15}{110} = 0{\cdot}136$ amp (assuming that the rated secondary voltage is 110 volts). If the burden is increased to 50 VA with the same applied voltage, the secondary current will be $\frac{50}{110} = 0{\cdot}45$ amp.

Thus, with increased burden I_s is increased, and hence the primary current I_p is increased. I_o is reduced slightly, but its variation is unimportant. Both the primary and secondary voltage drops are increased, and thus, for a given value of V_p (Fig. 19.19), E_p, E_s, and V_s are reduced by increase of burden. The effect is therefore to increase the actual ratio $\frac{V_p}{V_s}$ of the transformer as the burden increases.

With regard to the phase angle, the voltage V_p is advanced in phase relative to the flux ϕ (or, more strictly, ϕ is retarded in phase relative to V_p) as the secondary burden increases, owing to the increased voltage drops and to the phase advance of I_p as I_s increases. Again, V_s is retarded in phase, relative to ϕ, on account of the increased secondary voltage drops. Hence, the phase angle is increased (lagging) as the burden increases.

Effect of Power Factor of Secondary Burden. As the power factor of the secondary burden is reduced—i.e. angle Δ (Fig. 19.19) is increased—I_p becomes more nearly in phase with I_o. The voltages V_p and V_s move more nearly into phase with E_p and E_s respectively, and since the voltage drops in the windings, for a given volt-ampere

load, are very little influenced by the power factor of the load, the result is an increase in V_p relative to E_p (or, strictly, a reduction of E_p relative to V_p), and a reduction of V_s relative to E_s. Hence, the ratio of the transformer increases as the power factor of the burden decreases.

Since V_s is advanced in phase and V_p retarded in phase, the phase angle of the transformer is reduced (lagging) or increased (leading).

Effect of Frequency. For a given applied voltage, reduction of frequency results in an increase in the core flux, with a corresponding increase in the exciting current I_e. Since the exciting current does not influence the transformer errors very seriously, the effects of variation of frequency are not so great as in the case of current transformers. The reactance voltage drops in the windings are also proportional to frequency.

Thus, the results of a reduction of frequency are, in the case of ratio error, dependent upon the relative values of I_e and the reactances of the windings, since increase in I_e tends to increase the ratio, whilst the reduction of the reactance voltage drops tends to reduce it.

As regards phase-angle error, both effects retard V_p in phase relative to ϕ, and the reduction of secondary reactance advances V_s in phase. Thus the phase angle is reduced (lagging) as the frequency is reduced.

W. Gray and A. Wright (Ref. (42)), in a very informative paper, discuss the use of voltage and current transformers with switchgear on h.v. circuits. They conclude that capacitor voltage transformers —consisting of a tuned capacitor-reactor divider network together with an electromagnetic voltage transformer—are uneconomical for systems below 110 kV. These authors deal also with the transient performance of current transformers and with details of construction required for high-voltage operation.

A. H. M. Arnold (Ref. (44)) stresses the very low errors introduced by current transformers having nickel-iron cores. He states that voltage transformers for 14 kV have been built with a ratio error of only 0·01 per cent and a phase-angle error of less than 1 minute.

Testing of Instrument Transformers. (*a*) CURRENT TRANSFORMER TESTING. Since it is often necessary, in the precision testing of current transformers, to determine the ratio to within $\frac{1}{10}$ of 1 per cent and the phase angle to within $\frac{1}{20}$ of 1 degree, special apparatus is required.

These testing methods may be divided into two classes: (*a*) absolute methods, and (*b*) secondary, or comparison, methods. In the absolute methods the transformer errors are determined in terms of the constants—resistance, inductance, and capacitance—of the testing circuits; whereas, in the secondary methods, the errors

of the transformer under test are compared with those of a standard current transformer. Examples of both methods are given below.

References to other methods are given at the end of the chapter.

Mutual Inductance Method. This method has been used, with modifications, by Sharp and Crawford, Agnew and Silsbee, and others. It has been developed largely by the American Bureau of Standards. The essential connections are given in Fig. 19.23.

R_1 and R_2 are low-resistance non-inductive shunts, the latter having a slide wire in series with it for fine adjustment. R_1 is of fixed value. Neglecting, for the moment, the question of phase, the

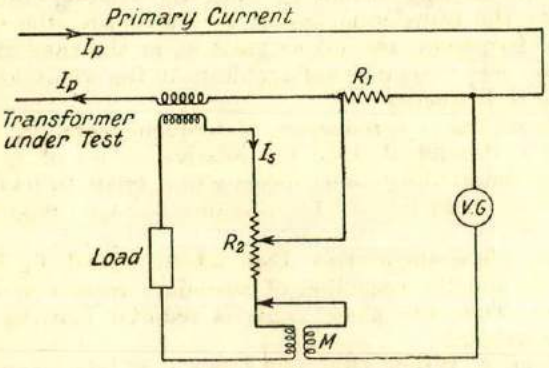


FIG. 19.23. MUTUAL INDUCTANCE METHOD OF CURRENT TRANSFORMER TESTING

vibration galvanometer will indicate zero deflection when the voltage drop I_pR_1 is equal (and opposite) to the voltage drop I_sR_2. Thus, R_1 and R_2 must be chosen so that $\frac{R_2}{R_1}$ is approximately equal to the nominal ratio of the current transformer. Actually, R_1 is suitably chosen to carry the full primary current, and R_2 is adjusted as required to render the two voltage drops equal. In addition, it is necessary, in order to obtain zero galvanometer deflection, to adjust the mutual inductance M, on account of the phase displacement between the currents I_p and I_s.

Theory. Referring to the vector diagram of Fig. 19.24, I_p and I_s are vectors representing the primary and secondary currents of the transformer respectively. I_pR_1 and I_sR_2 are the voltage drops across R_1 and R_2, and are in phase with these currents. The resultant voltage e (vector sum of I_pR_1 and I_sR_2) would exist in the galvanometer circuit if the value of the mutual inductance M were zero. The vector $I_s\omega M$ represents the voltage induced in the secondary of M (which is air-cored) by the current I_s. Obviously this vector lags

in phase 90° behind I_s. When the values of M and R_2 are such that the vectors e and $I_s\omega M$ are equal, and opposite in phase, no current flows through the galvanometer.

Then, from the vector diagram, the phase angle θ of the transformer is given by

$$\tan\theta = \frac{e}{I_sR_2} = \frac{I_s\omega M}{I_sR_2} = \frac{\omega M}{R_2} \quad . \quad . \quad . \quad (19.28)$$

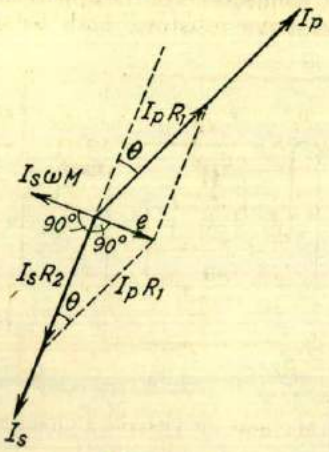


FIG. 19.24

Also,
$$\frac{I_sR_2}{I_pR_1} = \cos\theta$$

or Transformer ratio $= \frac{I_p}{I_s} = \frac{R_2}{R_1\cos\theta} = \frac{R_2}{R_1}$ (very nearly) (19.29)

since θ is usually very small.

Obviously a measurement of the supply frequency is needed for the determination of the phase angle. It should be noted, also, that the impedance of the secondary load circuit includes the resistance R_2 and the impedance of the primary of M, as well as the impedance marked "load." This must be taken into account in stating the burden to which the measured transformer errors apply. R_1 and R_2 are assumed, in the above theory, to be entirely non-inductive.

The Leeds & Northrup Co. manufacture a set of apparatus for this method of testing. The mutual inductor is of the Brooks and Weaver type, and a range of low-resistance non-inductive shunts is supplied for use, as required, in the primary circuit. The current range covered is from 30 to 1,000 amp.

The makers claim for this apparatus that it will give ratio values

which are accurate to within $\frac{1}{10}$ of 1 per cent, and phase angle values to within 5 min.

Biffi Method. This method, described initially by E. Biffi (Ref. (22)), is a very convenient one, since the apparatus required is of a simple character. W. I. Place (Ref. (23)) has investigated the method, and has given a full account of its operation and of the requirements of the apparatus for general current-transformer testing.

Fig. 19.25 shows the connections of the apparatus for this method. r_1 and r_2 are non-inductive resistors, both being of the order of

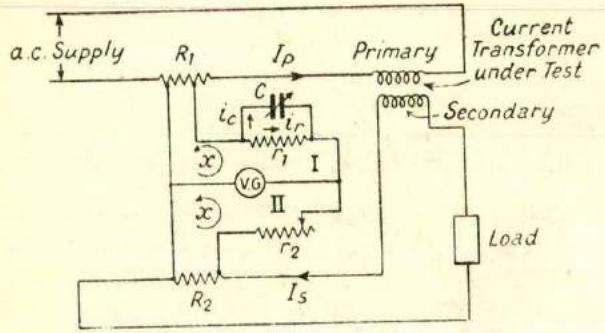


FIG. 19.25. BIFFI METHOD OF TESTING CURRENT TRANSFORMERS

100 to 400 ohms, and the latter being variable. C is a variable capacitor of about 1 microfarad, shunting r_1. R_1 and R_2 are non-inductive shunts, in the primary and secondary circuits of the current transformer respectively, of such magnitudes that the voltage drops across them are approximately equal and are about 0·5 or 0·75 volt. *V.G.* is a vibration galvanometer. The arrangement forms what is essentially a bridge network.

The method of procedure in testing a current transformer, after having tuned the vibration galvanometer, is to adjust r_2 and C until no deflection of the galvanometer is observed. The values of the transformer ratio and phase angle, under the conditions of load and frequency existing during the test, are found by substituting, in the expressions given below, the values of r_2 and C for balance. The supply frequency also should be measured for use in the expression for phase angle.

Ratio Expression

$$\frac{I_p}{I_s} = \frac{R_2(R_1 + r_1)}{R_1(R_2 + r_2)\sqrt{1 + \omega^2 r_1^2 C^2}} \quad . \quad . \quad (19.30)$$

where $\omega = 2\pi \times$ frequency.

Phase Angle

$$\tan\theta = \omega C r_1 \quad . \quad . \quad . \quad . \quad . \quad (19.31)$$
$$= \theta \text{ radians (very nearly)}$$

where θ is the phase angle of the transformer.

Theory. Let x and x be the two mesh currents in the two circuits of the network under balanced conditions (galvanometer current then equals $x - x = 0$).

Then, in mesh I,

$$R_1(x - I_p) + x\left(\frac{r_1}{A} - \frac{j\omega C r_1^2}{A}\right) = 0 \quad . \quad . \quad (19.32)$$

where $A = 1 + \omega^2 r_1^2 C^2$.

NOTE. For r_1 and C in parallel we have,

$$\text{Total impedance} = \frac{r_1}{1 + j\omega C r_1}$$

Rationalizing, we have,

$$\text{Impedance} = \frac{r_1(1 - j\omega C r_1)}{1 + \omega^2 C^2 r_1^2} = \frac{r_1}{A} - \frac{j\omega C r_1^2}{A}$$

In mesh II,

$$R_2(x - I_s) + x r_2 = 0 \quad . \quad . \quad . \quad . \quad (19.33)$$

I_p and I_s are the primary and secondary currents of the transformer under test.

From the above equations we have

$$\frac{I_p}{I_s} = \frac{R_2}{R_1}\left[\frac{R_1 + \frac{r_1}{A} - \frac{j\omega C r_1^2}{A}}{R_2 + r_2}\right]$$

or, converting to scalar quantities from the symbolic,

$$\frac{I_p}{I_s} = \frac{R_2[(R_1 A + r_1)^2 + \omega^2 C^2 r_1^4]^{\frac{1}{2}}}{R_1(R_2 + r_2)A}$$

$$= \frac{R_2[R_1^2(1 + \omega^2 C^2 r_1^2)^2 + 2R_1 r_1(1 + \omega^2 C^2 r_1^2) + \omega^2 C^2 r_1^4 + r_1^2}{R_1(R_2 + r_2)(1 + \omega^2 C^2 r_1^2)}$$

$$= \frac{R_2[(1 + \omega^2 C^2 r_1^2)\{R_1^2(1 + \omega^2 C^2 r_1^2) + 2R_1 r_1 + r_1^2\}]^{\frac{1}{2}}}{R_1(R_2 + r_2)(1 + \omega^2 C^2 r_1^2)}$$

or

$$\frac{I_p}{I_s} = \frac{R_2(R_1 + r_1)}{R_1(R_2 + r_2)\sqrt{1 + \omega^2 C^2 r_1^2}} \text{ (very nearly)} \quad . \quad (19.34)$$

since $\omega^2 C^2 r_1^2$ is very small compared with unity.

The *phase angle expression* is most easily derived from the vector diagram of Fig. 19.26, which refers to balance conditions. In this diagram, the magnitudes of x and of the phase angle θ of the transformer have been exaggerated for clearness. Since, for balance, $(I_s - x)R_2$ must be equal, both in magnitude and phase, to xr_2, it follows that the current x in mesh II is in phase with I_s. At balance also we have that $i_r r_1 = (I_p - x)R_1$, the subtraction of x from I_p being now vectorial, since I_p and x are not quite in phase (actually they are displaced from one another in phase by the phase angle θ). i_r is the component of the current x which passes through the resistor r_1, i_c being the component of x passing through the capacitor C. Since x is usually quite

negligible compared with I_p, its vectorial subtraction from I_p has been omitted in the vector diagram, the value I_p being used instead of the vector difference $(I_p - x)$.

We have, then,

$$\tan\theta = \frac{i_c}{i_r}$$

Now,
$$i_r = \frac{I_p R_1}{r_1}$$

and
$$i_c = \frac{I_p R_1}{\frac{1}{\omega C}} = I_p R_1 \omega C$$

$$\therefore \tan\theta = \frac{I_p R_1 \omega C}{\frac{I_p R_1}{r_1}} = \omega C r_1 \quad . \quad . \quad . \quad . \quad (19.35)$$

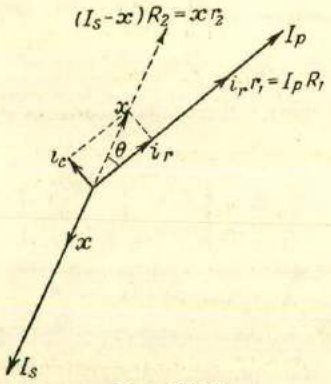


FIG. 19.26

Example. In order to obtain an idea of the quantities involved, suppose a current transformer whose nominal ratio is 300/5 is under test, with a supply frequency of 50 cycles per second. Let r_2 be 400 ohms, and R_2 be 0·15 ohm.

Then, at full rated current, from Equation (19.33),

$$0{\cdot}15(x - 5) + 400x = 0$$

or
$$x = 0{\cdot}002 \text{ amp}$$

Obviously this is quite negligible compared with 300 amp $(= I_p)$, so that no appreciable error is introduced by taking the current through R_1 as equal to I_p as above. The value of R_1 to give a voltage drop equal to that across R_2 (namely, 0·75 volt) at full rated current would be 0·0025 ohm.

Again, assuming a phase angle of 1° in the current transformer,

and supposing the value of C for balance to be exactly $\frac{1}{2}$ microfarad, we have

$$\tan 1° = 2\pi \times 50 \times \frac{0{\cdot}5}{10^6} \times r_1$$

from which

$$r_1 = 111 \text{ ohms}$$

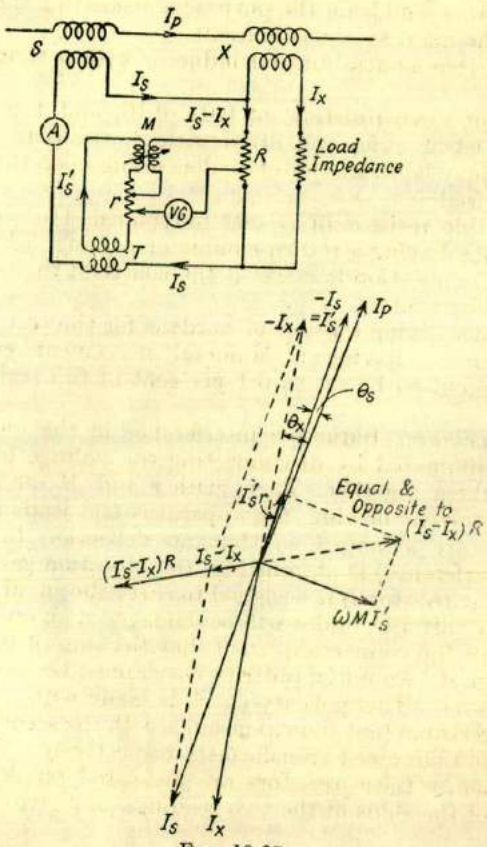


FIG. 19.27

Arnold Method. Unlike the methods previously described, this is a secondary, or comparison, method but it is nevertheless one of the most accurate in use. It has been developed by A. H. M. Arnold (Ref. (38)) at the National Physical Laboratory and is essentially a modification of the Silsbee method (Ref. (24)). The circuit is shown in Fig. 19.27, which also gives the vector diagram for the network under balance conditions. Details of the circuit are as follows.

S and X are the standard and test transformers respectively, these being of the same nominal ratio. In the N.P.L. apparatus, three standard transformers are available, these covering the range of ratios from 5/5 to 12,000/5. Full details of the construction and calibration of the standard transformers are given in Arnold's paper.

T is a 5/5 current transformer having negligibly small ratio and phase-angle errors, and is for the purpose of insulating the measuring circuit from the main secondary circuit.

M is a variable astatic mutual inductor with a range of $\pm$ 2·4 microhenries.

R consists of three resistors, of 0·01, 0·10, and 1 ohm respectively, connected in series and fitted with short-circuiting plugs so that any two may be short-circuited, depending upon the sensitivity and range desired.

r is a variable resistor of $\pm$ 500 microhms. This consists of a constantan tube having a return conductor passing down its centre. One potential connection is made at the centre of the tube and the other is a sliding contact.

An impedance giving a range of burdens for the test transformer and an ammeter, having a Mumetal movement enabling the secondary current to be set to 0·1 per cent of full-scale value, are included.

Testing Procedure. Inductive interference in the measuring circuit is first eliminated by disconnecting the voltage leads from R and joining them together. Then, with r and M set at zero, and full primary current flowing, the apparatus and leads in the measuring circuit are arranged so that no deflection (indicating no inductive interference) is obtained on the vibration galvanometer.

The common resistor R is designed to carry the small difference of the secondary currents, and it will be damaged if an error is made in the polarity of the connections such that the sum of these currents passes through it. An initial polarity check must be carried out with R short-circuited. This polarity check is made with the aid of two 0·0004-ohm resistors (not shown) in series with the secondary circuits of the test and standard transformers respectively. The potential differences across these resistors are measured on M, and r, and comparison of the signs of the two readings on r gives the required check.

Balance is finally obtained, using R as shown, by adjustment of M and r.

At balance,

$$K_x - K_s \doteqdot r/R$$

and

$$\theta_x - \theta_s \doteqdot \omega M/R$$

K_x and K_s are the ratios $\dfrac{\text{actual ratio}}{\text{nominal ratio}}$ for the test and standard transformers respectively, and θ_x and θ_s are their phase angles.

Theory

$$I_s = -\frac{1}{R_s}(\cos\theta_s + j\sin\theta_s) \cdot I_p$$

$$\fallingdotseq -\frac{1}{R_s}[1 + j\theta_s]I_p$$

where R_s is the actual ratio of the standard.

The approximation is justifiable, since θ_s is very small.

Similarly, $$I_x = -\frac{1}{R_x}[1 + j\theta_x]I_p$$

where R_x is the actual ratio of the test transformer.

Thus, $$I_x = \frac{R_s}{R_x}[1 + j(\theta_x - \theta_s)]I_s$$

Now, at balance,

$$R(I_s - I_x) - rI_s + j\omega MI_s = 0 \text{ (since } I'_s = -I_s)$$

Substituting for I_x in terms of I_s,

$$R\left[1 - \frac{R_s}{R_x}\left\{1 + j(\theta_x - \theta_s)\right\}\right] - r + j\omega M = 0$$

Equating real and imaginary terms,

$$R[1 - R_s/R_x] = r$$

or $$1 - R_s/R_x = r/R$$

Hence $$1 - K_sN/K_xN = r/R$$

where N is the nominal ratio.

Thus, to a close approximation, since K_s and K_x are nearly unity.

$$K_x - K_s = r/R$$

Alternatively, if K_s is unity,

$$1 - \frac{1}{K_x} = \frac{r}{R} \quad . \quad . \quad . \quad . \quad (19.36)$$

and $1 - \frac{1}{K_x}$ is the ratio error of the test transformer.

Again, $$\frac{R_s}{R_x}(\theta_x - \theta_s) = \omega M/R$$

or $$\theta_x - \theta_s \fallingdotseq \omega M/R \quad . \quad . \quad . \quad (19.37)$$

Messrs. H. Tinsley & Co. manufacture a precision current-transformer testing equipment based on the above principles. This equipment is designed for a range of ratios of 5/5 to 10,000/5, the accuracy obtainable being 2 in 100,000 in ratio measurements and 0·05 min phase angle. In routine tests the accuracies are 1 in 20,000 and 0·2 min respectively.

Petch-Elliott Testing Set. Messrs. Elliott Bros. have introduced the Petch-Elliott current-transformer testing set for the testing of precision current transformers at a given frequency. As in the Arnold method, the transformer under test is compared with a standard transformer of the same nominal ratio.

The "difference" or "spill" current (called $I_s - I_x$ in the description of the Arnold method above) is passed through a separate

winding on a toroidal transformer which carries three other windings. One of these supplies the vibration galvanometer, and the two others are supplied, through slide-wire potential dividers, from the standard transformer, through an auto-transformer.

This set has the advantages that it is capable of testing transformers designed for either 5 amp, 1 amp, or 0·5 amp secondary currents. It is unaffected by stray magnetic fields, is portable, and has a range of + 0·5 per cent to − 0·5 per cent ratio and + 20 min to − 20 min phase angle on its normal range.

(*b*) Voltage Transformer Testing. Tests for the determination of the ratio and phase-angle errors of voltage transformers may be divided into two classes, just as current transformer tests were divided; namely, absolute tests and comparison tests.

Absolute Methods. In these methods, the voltage of the transformer secondary winding is compared with a suitable fraction of the voltage applied to the primary winding, this fraction being obtained by the use of a non-inductive and non-capacitive resistance potential-divider with a variable tapping. The magnitude and phase of the difference between these two voltages are measured and the transformer errors are obtained therefrom. If the primary voltage of the transformer under test is very high it is very difficult to eliminate the effect of distributed capacitance in the potential divider. A complicated system of screening becomes necessary, and the divider is likely to be a very expensive item in the testing apparatus. B. G. Churcher has described a capacitor potential divider for voltages up to 132 kV (Ref. (25)), and his original paper should be referred to for the description of an absolute method of voltage transformer testing, using such a divider. The question of distributed capacitance in resistance potential dividers for such tests is discussed in a paper by C. Dannatt (Ref. (26)), which also gives details of a testing apparatus designed for routine tests upon voltage transformers.

There are several absolute methods of testing in which the transformer errors are determined in terms of the impedances of the various parts of the testing circuit. These methods are all essentially the same in principle, the transformer secondary voltage being, in all such methods, balanced by a fraction of the primary applied voltage, a vibration galvanometer being used as a detector to indicate exact balance, and various adjustable impedances being used to vary the magnitude and phase of the fraction of the primary voltage until such a balance is obtained.

One such method is described below, references to similar methods being given in Refs. (3), (25), (26), at the end of the chapter.

Fig. 19.28 gives the connections of this method of testing. The burden with which the transformer is to be tested is connected across the secondary winding, and the normal primary voltage, at normal frequency, is applied to the primary winding. One end of

the secondary is connected to one end of the primary winding. A non-inductive potential divider is connected across the primary winding. The lower part of the divider consists of a fixed resistor in series with a slide-wire S, and the upper part consists of a shielded, high-voltage resistor in series with the primary of a variable mutual inductor M. A capacitor C shunts a small part r of the upper part of the divider. VG is a vibration galvanometer. The resistances are chosen so that the nominal ratio of the test transformer is equal to R/R_1.

L is the primary inductance of the mutual inductor and will cause

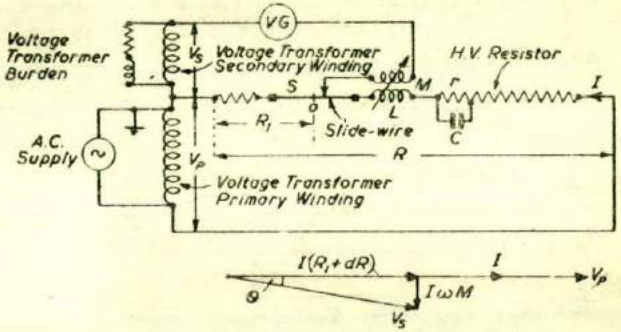


FIG. 19.28. ABSOLUTE METHOD OF TESTING VOLTAGE TRANSFORMER

a phase error in the divider. The capacitor C compensates for this error in the following manner: the impedance of the whole divider is

$$Z = R - r + j\omega L + \frac{r}{1 + j\omega Cr}$$

$$= R - r + j\omega L + \frac{r}{1 + \omega^2 C^2 r^2}(1 - j\omega Cr)$$

$$= R - r + \frac{r}{1 + \omega^2 C^2 r^2} + j\omega\left(L - \frac{Cr^2}{1 + \omega^2 C^2 r^2}\right)$$

By making $\frac{Cr^2}{1 + \omega^2 C^2 r^2} = L$, the divider is non-reactive at the operating frequency, which is usually 50 c/s, and the total impedance is then

$$Z = R - r + \frac{r}{1 + \omega^2 C^2 r^2}$$

The term $\omega^2 C^2 r^2$ is normally small compared with unity and may be neglected, so that the impedance of the divider is equal to R. If this term is not negligible, the resistance of the upper part of the divider

can be increased by a small amount to correct for it at the operating frequency.

In carrying out the test, the contact on the slide-wire and the mutual inductor are adjusted until the vibration galvanometer gives no deflection.

The vector diagram corresponding to the balanced condition is shown in Fig. 19.28, in which $I = \frac{V_p}{R}$ is the current in the divider. The phase angle θ of the test transformer is small so that

$$V_s = I(R_1 + dR) = \frac{V_p}{R}(R_1 + dR)$$

and the transformer ratio is

$$\frac{V_p}{V_s} = \frac{R}{R_1 + dR}$$

$$\text{The ratio error} = \frac{\text{Nominal ratio} - \text{Actual ratio}}{\text{Actual ratio}}$$

$$= \frac{\frac{R}{R_1} - \frac{R}{R_1 + dR}}{\frac{R}{R_1 + dR}}$$

$$= \frac{R_1 + dR}{R_1} - 1 = \frac{dR}{R_1} \quad . \quad . \quad (19.38)$$

The slide-wire can therefore be calibrated directly in values of ratio error.

Phase Angle. From the vector diagram,

$$\tan\theta \fallingdotseq \theta = \frac{\omega M}{R_1 + dR} \fallingdotseq \frac{\omega M}{R_1} \quad . \quad . \quad (19.39)$$

and the mutual inductor can be calibrated directly in phase angle.

Special shielding arrangements have to be adopted for the high-voltage resistor. If it is surrounded by a simple earthed shield, the capacitance currents flowing between the resistor and shield can cause serious errors. Such errors can be reduced by keeping the resistance of the divider as low as possible, and a value of 20 ohms per volt is commonly adopted. The method of shielding is shown in Fig. 19.29. The divider is broken into a number of sections, each enclosed in its own screen, and each screen is held at the mid-potential of the section it encloses by means of an auxiliary divider. There is therefore a considerable reduction in the capacitance current from the divider to the shield. The full theory of shielded high-voltage resistors will be found in Refs. (26), (37).

It is not practical to construct resistance dividers suitable for voltage transformer testing at voltages much above 40 kV; the use of a capacitance divider is preferable at higher voltages.

Methods Employing Capacitance Dividers. A wide variety of methods of testing voltage transformers with the aid of capacitance dividers have been used, and many of these are described in Ref. (37).

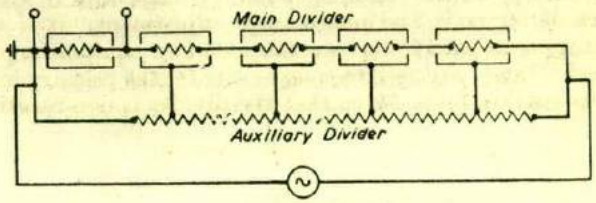


FIG. 19.29. SHIELDED HIGH-VOLTAGE POTENTIAL DIVIDERS

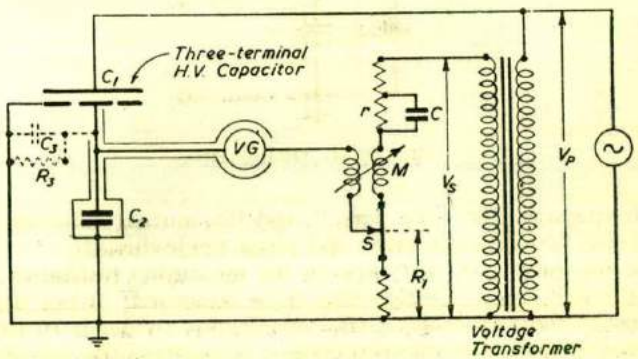


FIG. 19.30. CAPACITANCE DIVIDER METHOD OF TESTING VOLTAGE TRANSFORMER

Fig. 19.30 gives the connections of one method, which usefully illustrates the principles.

C_1 is a high-voltage, compressed-gas, three-terminal capacitor of value 100 $\mu\mu F$, and C_2 is a high-quality mica capacitor having a very small power factor. One of the difficulties with capacitance dividers is the existence of the capacitance C_3 of the low-voltage electrode and guard ring of C_1. This capacitance is often associated with a loss resistance R_3, and it is therefore important that the value of the mica capacitor C_2 should be large so that $C_2 + C_3$ and R_3 have a negligible power factor. The capacitance between the h.v. and l.v. electrodes of C_1 is loss-free; hence the phase error of the divider, which should be negligible, is determined by C_2 shunted by C_3 and R_3. There is therefore a restriction on the minimum value of C_2 and on the minimum ratio of the divider. This difficulty can be overcome

by the modification shown in Fig. 19.31, in which the guard ring is connected to earth through a capacitor C_4 which is adjusted so that $C_1/C_2 = C_{1-E}/C_4$. The guard ring is then at the same potential as the low-voltage electrode.

A resistance divider having a ratio of 10/1 is connected across the secondary winding of the voltage transformer under test. This divider incorporates a variable potential slide-wire S and mutual inductor M for ratio and phase-angle adjustment. It is similar to the arrangement used in the resistance divider equipment previously discussed. C and r are used to compensate for the primary inductance of the mutual inductor M, so that the divider is non-reactive at the

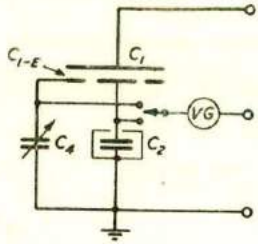


FIG. 19.31. GUARD RING

test frequency. The slide-wire S, and the mutual inductor, can be calibrated to read ratio error and phase angle directly.

The adoption of a 10/1 ratio in the measuring resistance divider permits ratio errors of either sign to be measured. It has the added advantage that the ratio of the capacitance divider is 10 times the nominal ratio of the test transformer, permitting the use of a large value of C_2 and thus minimizing the effect of C_3 and R_3 in the simple arrangement.

The compensation of the measuring resistance divider, and its zero setting, can be very easily checked by comparing it with a simple 10/1 ratio resistance divider. H. Tinsley & Co., Ltd., manufacture a variety of voltage-transformer testing equipment using either resistance or capacitance dividers according to the voltage range.

The Clothier and Medina Method. This method has been developed quite recently and uses a variable-tapping transformer for the measuring device. The method is fully described in Ref. (46), which should be referred to for full details. Fig. 19.32 is a simplified circuit diagram. C_1 and C_2 are three-terminal capacitors whose ratio is equal to the nominal ratio of the test transformer T_1. The windings of T_1 are in series aiding. The errors in T_1 are balanced by means of the transformer T_2 whose ratio can be adjusted a little above or below unity by using decade switches to select the primary turns.

Phase-angle errors are balanced by supplying a current I_R through a high resistance R to the junction of C_1 and C_2. I_R is in phase with the secondary voltage and therefore in quadrature with the current in C_2. The value of I_R is adjusted by a switch on T_2. D is a tuned electronic detector.

Theory. Since the detector voltage is zero at balance, the current in R is aV/R, where aV is the voltage across R with respect to earth and V is the voltage across C_2. aV can have either sign.

At balance $I_1 = I_2 + I_R$.

$$I_1 = j\omega C_1 V_p,\ I_2 = j\omega C_2 V, \text{ and } I_R = aV/R$$

$$\therefore\ j\omega C_1 V_p = j\omega C_2 V[1 + a/j\omega C_2 R]$$

or

$$\frac{V_p}{V_s} = \frac{C_2}{C_1}\frac{V}{V_s}\left[1 - \frac{ja}{\omega C_2 R}\right]$$

If the phase angle of the auxiliary transformer may be neglected, and its voltage is very nearly equal to its turns ratio, then, if N_1 and N_2 are primary and secondary turns on T_2, and if $N = N_2 - N_1$,

$$\frac{V}{V_s} = \frac{N_2}{N_1} = \frac{N_2}{N_2 - N} = \frac{1}{1 - N/N_2} = \frac{1}{1 - D}$$

where

$$D = N/N_2$$

The phase angle $\theta \fallingdotseq \tan\theta = \dfrac{a}{\omega C_2 R}$

The ratio K_a of the test transformer is

$$K_a = \left|\frac{V_p}{V_s}\right| = \frac{C_2}{C_1}\frac{1}{1 - D}\sqrt{1 + \frac{a^2}{\omega^2 C_2^2 R^2}}$$

$$= \frac{C_2}{C_1}\frac{1}{1 - D}\sqrt{1 + \theta^2}$$

If the ratio of the capacitance divider is adjusted so that C_2/C_1 is equal to the nominal ratio K_n of the transformer, then

$$\frac{K_a}{K_n} = \frac{1}{1 - D}\sqrt{1 + \theta^2}$$

$$\fallingdotseq 1 + D + D^2 + \tfrac{1}{2}\theta^2$$

In general,

$$\frac{K_a}{K_n} = 1 + D$$

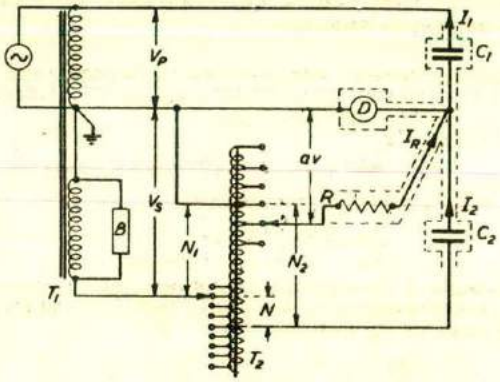
gives sufficient accuracy. . . . . (19.40)

The accuracy of the method depends upon the auxiliary transformer T_2. Its ratio is nearly unity, however, and in these conditions it is not difficult to construct a transformer having very small ratio error and phase angle.

The guard rings which constitute the third terminals of capacitors C_1 and C_2 are not shown in Fig. 19.32; both are connected to earth.

Comparison Methods. The connections for one such method are given in Fig. 19.33 (Ref. (4)). Two wattmeters W_1 and W_2 are used in conjunction with a phase-shifting transformer. The current coils of the wattmeters are connected in series and are supplied with a

current whose phase is variable from the rotor winding of the phase-shifting transformer. The voltage coil of wattmeter W_1 is supplied from the secondary windings of the two voltage transformers—



(*Inst. Elect. Engrs.*)

FIG. 19.32. CLOTHIER AND MEDINA METHOD OF TESTING VOLTAGE TRANSFORMER

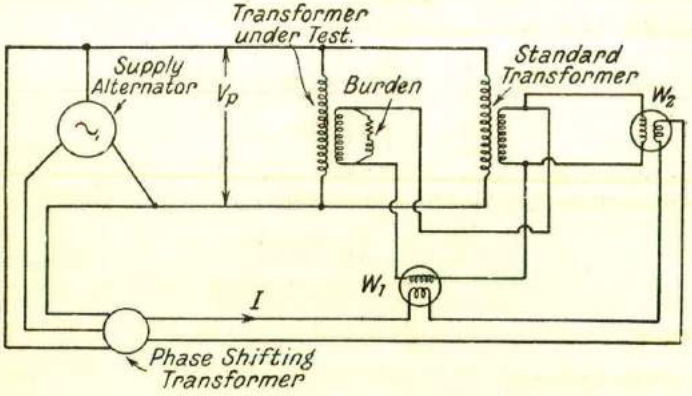


FIG. 19.33. COMPARISON METHOD OF TESTING VOLTAGE TRANSFORMER

one transformer being under test and the other a standard transformer of the same nominal ratio. The secondary windings are connected in opposition so that the voltage applied to the voltage coil of W_1 is the vector difference of the secondary voltages of the two transformers. The burden with which the test transformer is to be tested is connected across its secondary winding. The primary windings of the transformers are connected in parallel, and hence

have the same primary voltage V_p. Wattmeter W_2 is for the purpose of checking the phase of the current I, in the current coils of the wattmeters, relative to the secondary voltage V_s of the standard transformer. The voltage coil of this wattmeter is, for this reason, connected across the secondary winding of this transformer.

As used in this test, wattmeter W_1 is essentially a voltmeter, and its deflection, per volt applied to the voltage coil, corresponding to some given current in the current coil, must be known. Let K be the volts per division for a current I in the current coil, this current being in phase with the applied voltage.

The operation of the method of testing consists in observing the reading D_1 of wattmeter W_1 when the current I in the current coil is in phase with the secondary voltage of the standard transformer. The phase of this current is altered, by adjusting the phase-shifting transformer, until wattmeter W_2 gives a maximum reading, when the current I in its current coil is in phase with the voltage V_s applied to its voltage coil.

The phase-shifter is then adjusted until wattmeter W_2 gives zero reading, when the current I must be 90° out of phase with the voltage V_s. The reading D_0 of wattmeter W_1 is then again observed.

Then, if R_s and R_x are the ratios of the standard and test transformers respectively.

$$R_x = \frac{R_s \cdot V_s}{V_s - KD_1} \quad . \quad . \quad . \quad (19.41)$$

and

$$\theta_x = \theta_s + \tan^{-1} \frac{KD_0}{V_s} \quad . \quad . \quad . \quad (19.42)$$

where θ_x and θ_s are the phase angles of the test and standard transformer respectively.

Theory. Referring to he vector diagram of Fig. 19.34 (*a*), vectors OV_p, OV_s, and OV_x represent the common primary voltage of the two transformers and their secondary voltages respectively, while angles θ_s and θ_x are their phase angles. Ov is the vector difference of V_s and V_x, and β is the phase angle between v and V_s.

Then, when the current I is in phase with V_s, we have, by projecting V_s on to V_x, as in Fig. 19.34 (*b*),

$$V_s \cos(\theta_x - \theta_s) = V_x + v \cos[\beta + (\theta_x - \theta_s)]$$

Now, the ratio R_s of the standard transformer is given by $\frac{V_p}{V_s}$, while the ratio of the transformer under test is $R_x = \frac{V_p}{V_x}$.

Thus,

$$\frac{V_p}{R_s} \cos(\theta_x - \theta_s) = \frac{V_p}{R_x} + v \cos[\beta + (\theta_x - \theta_s)]$$

Again, when I is in phase with V_s, wattmeter W_1 reads D_1, and the power in this wattmeter is then $vI \cos \beta$.

$$\therefore vI \cos \beta = kD_1$$

where k is a constant.

Or $$v \cos \beta = KD_1$$

where K is the constant of the wattmeter in volts per division corresponding to a current I.

Substituting for v in the previous equation, we have

$$\frac{V_p}{R_s} \cos (\theta_x - \theta_s) = \frac{V_p}{R_x} + \frac{KD_1}{\cos \beta} \cos [\beta + (\theta_x - \theta_s)]$$

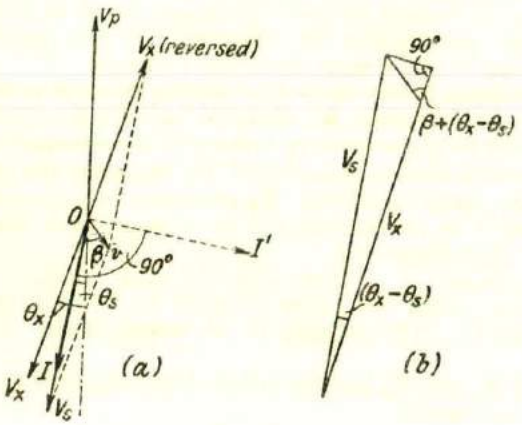


FIG. 19.34

Now, $\theta_x - \theta_s$ is a very small angle, and its cosine may be taken as unity. Hence,

$$\frac{V_p}{R_s} = \frac{V_p}{R_x} + KD_1 \text{ (very nearly)}$$

or $$\frac{1}{R_s} - \frac{KD_1}{V_p} = \frac{1}{R_x}$$

Substituting $R_s V_s$ for V_p, we have,

$$\frac{1}{R_x} = \frac{1}{R_s} - \frac{KD_1}{R_s V_s} = \frac{V_s - KD_1}{R_s V_s}$$

Thus, $$R_x = \frac{R_s V_s}{V_s - KD_1}$$

Again, from Fig. 19.34 (*b*), if $\theta_x - \theta_s$ is neglected in comparison with β,

$$\tan (\theta_x - \theta_s) = \frac{v \sin \beta}{V_s \cos (\theta_x - \theta_s)}$$

$$= \frac{v \sin \beta}{V_s} \text{ (very nearly)}$$

Now, when the current I is in quadrature with V_s (as represented by vector OI'), the power measured by wattmeter W_1 is $vI \cos(90 - \beta)$. If the deflection is now D_θ, then

$$KD_\theta = v \cos(90 - \beta)$$
$$= v \sin \beta$$
$$\therefore \tan(\theta_x - \theta_s) = \frac{KD_\theta}{V_s}$$

i.e.
$$\theta_x - \theta_s = \tan^{-1} \frac{KD_\theta}{V_s}$$

or
$$\theta_x = \theta_s + \tan^{-1} \frac{KD_\theta}{V_s}$$

Other comparison methods are described in the works mentioned in Refs. (2), (3), (27).

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CHAPTER XX

THE MEASUREMENT OF POWER

In direct-current circuits power may be measured either by a wattmeter or by an ammeter and voltmeter, the product of whose readings gives the power in the circuit. In alternating-current circuits the power at any instant is given by

$$p = ei \quad . \quad . \quad . \quad . \quad (20.1)$$

where p = instantaneous power

e = instantaneous voltage

i = instantaneous current

Thus, if both the current and voltage waves are sinusoidal, the current lagging in phase by an angle ϕ, then

$$e = E_{max} \sin \omega t$$

and

$$i = I_{max} \sin (\omega t - \phi)$$

The instantaneous power p is therefore given by

$$p = ei$$
$$= E_{max} I_{max} \sin \omega t \sin (\omega t - \phi)$$

or, writing θ for ωt,

$$p = E_{max} I_{max} \sin \theta \sin (\theta - \phi)$$

The mean power is

$$P = \frac{1}{2\pi} \int_0^{2\pi} E_{max} I_{max} \sin \theta \sin (\theta - \phi) \, d\theta \quad . \quad (20.2)$$

$$= \frac{E_{max} I_{max}}{2\pi} \int_0^{2\pi} \frac{\cos \phi - \cos (2\theta - \phi)}{2} \, d\theta$$

$$= \frac{E_{max} I_{max}}{4\pi} \left[\theta \cos \phi - \frac{\sin (2\theta - \phi)}{2} \right]_0^{2\pi}$$

$$= \frac{E_{max} I_{max}}{2} \cos \phi$$

$$= EI \cos \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (20.3)$$

where E and I are r.m.s. values of voltage and current.

The fact that the power factor ($\cos \phi$) is involved in the expression for the power means that a wattmeter must be used instead of merely an ammeter and voltmeter, since the latter method takes no account of power factor.

Wattmeter Measurements in Single-phase A.C. Circuits. Fig. 20.1 shows a wattmeter connected in such a circuit. The "current coil" of the instrument carries the load current, while the "voltage coil" carries a current proportional to, and in phase with, the voltage. The deflection of the wattmeter depends upon the currents in these two coils and upon the power factor. Inductance in the voltage-coil circuit should be avoided as far as possible, since it causes the voltage-coil current to lag behind the applied voltage. A high non-inductive resistance is connected in series with the voltage coil in order that the reactance of the coil itself shall be small in comparison

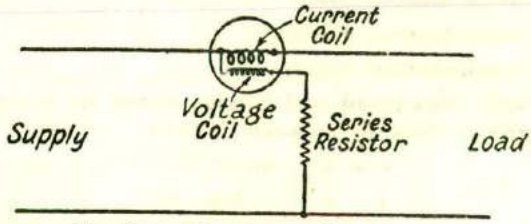


FIG. 20.1. WATTMETER CONNECTIONS

with the resistance of the whole voltage-coil circuit and also, of course, to ensure that the current taken by the voltage coil shall be small.

Wattmeter Errors. (*a*) VOLTAGE-COIL INDUCTANCE

If r_p = resistance of voltage coil

l_p = inductance of voltage coil

R = resistance in series with voltage coil

V = voltage applied to the voltage-coil circuit

then the current through the voltage coil is

$$i_p = \frac{V}{\sqrt{(r_p + R)^2 + \omega^2 l_p^2}}$$

The phase of this current relative to the voltage is lagging by a small angle β such that

$$\tan \beta = \frac{\omega l_p}{r_p + R}$$

By increasing the non-inductive resistance R, this angle is reduced, although the current i_p is reduced by such increase. The effect of an increase of frequency is to increase β (since it increases ω), and to reduce the current i_p, slightly, by its effect in increasing the impedance of the voltage-coil circuit.

Thus, if α is the phase angle (lagging) of the load circuit, the wattmeter deflection is proportional to

$$Ii_p \cos(\alpha - \beta)$$

i.e. to

$$I \frac{V}{Z_p} \cos(\alpha \quad \beta)$$

where Z_p is the impedance of the voltage-coil circuit.

Now,

$$Z_p = \frac{r_p + R}{\cos \beta}$$

Thus the deflection is proportional to

$$I \frac{V}{(r_p + R)} \cos \beta \cos(\alpha - \beta)$$

If the inductance of the voltage-coil circuit were zero, the deflection would be proportional to $\frac{IV}{(r_p + R)} \cos \alpha$, and the wattmeter would read correctly at all frequencies and power factors. The ratio of the true reading of the instrument to the actual reading is, therefore,

$$\frac{\frac{IV}{(r_p + R)} \cos \alpha}{\frac{IV}{(r_p + R)} \cos \beta \cos(\alpha - \beta)} = \frac{\cos \alpha}{\cos \beta \cos(\alpha - \beta)}$$

$$\therefore \text{True reading} = \frac{\cos \alpha}{\cos \beta \cos(\alpha - \beta)} \times \text{Actual reading} \qquad (20.4)$$

Correction Factor. The "correction factor" by which the actual reading must be multiplied, in order to obtain the true reading, is $\frac{\cos \alpha}{\cos \beta \cos(\alpha - \beta)}$. The wattmeter will read high on lagging power factors of the load, since the effect of the inductance of the voltage-coil circuit is to bring the current in it more nearly into phase with the load current than would be the case if this inductance were zero. If the power factor of the load is very low, a serious error may be introduced by voltage-coil inductance unless special precautions are taken to reduce its effect.

Fig. 20.2 gives curves showing the variation in the value of the correction factor as the power factor of the load varies, the phase angle β of the voltage-coil circuit being 1° in one case and 0° 10′ in the other.

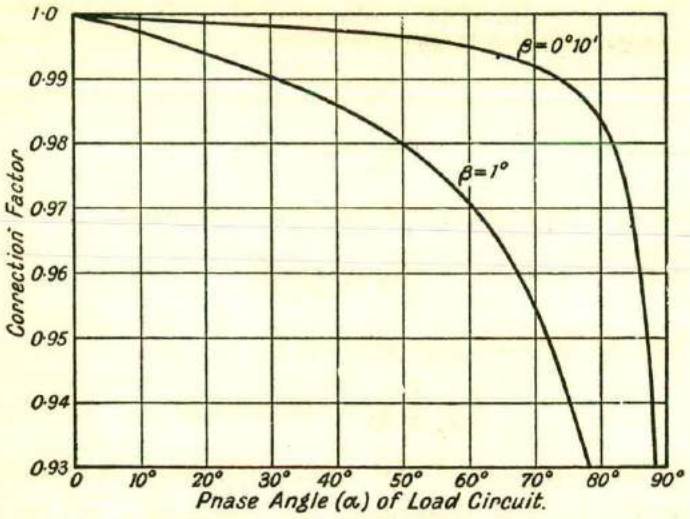


FIG. 20.2. WATTMETER CORRECTION FACTORS

The error, in terms of the actual instrument deflection, is

Actual reading – True reading

$$= \left[1 - \frac{\cos \alpha}{\cos \beta \cos (\alpha - \beta)}\right] \times \text{Actual reading}$$

$$= \left[1 - \frac{\cos \alpha}{\cos (\alpha - \beta)}\right] \times \text{Actual reading}$$

if $\cos \beta$ is assumed equal to unity.

$$\therefore \text{Error} = \left[1 - \frac{\cos \alpha}{\cos \alpha + \sin \alpha \sin \beta}\right] \times \text{Actual reading}$$

$$= \left[\frac{\sin \alpha \sin \beta}{\cos \alpha + \sin \alpha \sin \beta}\right] \times \text{Actual reading}$$

$$= \frac{\sin \beta}{\cot \alpha + \sin \beta} \times \text{Actual reading} \quad . \quad . \quad (20.5)$$

(*b*) VOLTAGE-COIL CAPACITANCE. The voltage-coil circuit may have capacitance as well as inductance, this being largely due to inter-turn capacitance in the high-value series resistor. The effect produced is similar to that of inductance in this circuit, except that the voltage-coil current tends to *lead* the applied voltage instead of to lag behind it. This causes the wattmeter to read low, on lagging

power factors of the load, by increasing the angle between the load and voltage-coil currents.

The effect of frequency will be, of course, to vary the phase angle between V and i_p, the angle increasing with increase of frequency.

If the capacitive reactance of the voltage coil circuit is equal to its inductive reactance, there will be no error due to these effects since the two individual errors will neutralize one another.

(c) EDDY CURRENTS. Eddy currents induced in the solid metal parts of the instrument, by the alternating magnetic field of the current coil, alter the magnitude and phase of this field, and so produce an error. The phase of the induced eddy e.m.f.s will be

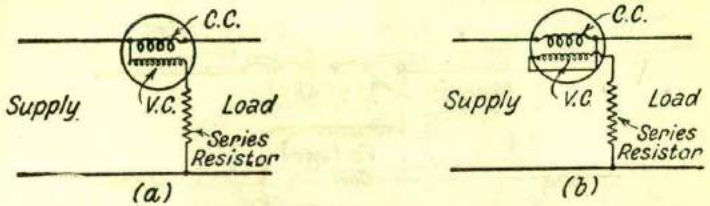


FIG. 20.3. ALTERNATIVE WATTMETER CONNECTIONS

90° behind the inducing flux—i.e. rather more than 90° behind the main current in the current coil. The eddy current is practically in phase with its e.m.f., and this current sets up a magnetic field which, combined with that of the current coil, produces a resultant magnetic field which is less than that of the current coil alone and which also lags behind the current-coil field by a small angle.

This eddy current error is not easily calculable, and may be serious if care is not taken to ensure that any solid metal parts (which should be avoided as far as possible) are well removed from the current coil. If the current coil is designed for heavy currents, it should consist of stranded conductors in order to minimize the eddy currents flowing in the current coil itself.

Methods of Connection in the Circuit. There are two obvious methods of connecting a wattmeter in circuit, as shown in Fig. 20.3, in which *C.C.* and *V.C.* indicate current coil and voltage coil respectively. Neither measures the power in the load directly, without correction, even neglecting the errors discussed above.

In the method of diagram (*a*), in which the voltage coil is connected on the "supply" side of the current coil, the voltage applied to the voltage coil is higher than that of the load on account of the voltage drop in the current coil. In diagram (*b*) the current coil carries the small current taken by the voltage coil, in addition to the load current.

In the first case, the instrument measures the power, I^2R_c, lost in

the current coil, and in the second case the power lost in the voltage coil, as well as the power in the load.

If the load current is small, the voltage drop in the current coil is small, so that the first method of connection introduces a very small error. On the other hand, if the load current is large, the power lost in the voltage coil will be small compared with the power in the load, and the second method of connection is better.

Compensation for Power Loss in Voltage Coil. In some wattmeters a compensating coil is used to eliminate error due to the current coil carrying the voltage-coil current in addition to the load current, when the connections are as in Fig. 20.3 (*b*). This compensating coil is as nearly as possible identical and coincident with

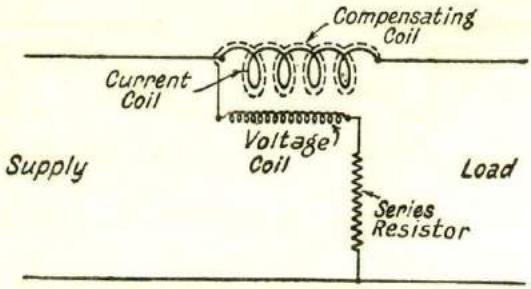


FIG. 20.4. CONNECTIONS OF COMPENSATING COIL

the current coil, so that if it were connected in series with the latter, and a current passed through the two coils—connected so that their magnetic effects were in opposition—the resultant magnetic field would be zero. Actually the compensating coil is connected in series with the voltage coil, but in such a way that its magnetic effect opposes that of the current coil and neutralizes the voltage-coil component of the current in the current coil. Thus, if no-load current flows in the instrument, the deflection should be zero, since the resultant current-coil field should be zero. The connections of this method of compensation are shown in Fig. 20.4.

The Use of Instrument Transformers with Wattmeters. Current and voltage transformers may be used with wattmeters just as they are used with ammeters and voltmeters. By using a number of current transformers, of different ratios, for the supply of the wattmeter current coil, and a number of voltage transformers for its voltage coil, the same wattmeter may be used to cover a very large range of power measurements. The connections of a wattmeter when so used are shown in Fig. 20.5, in which an ammeter and a voltmeter are connected in circuit, supplied from the same transformers as the wattmeter.

When such transformers are used corrections must be applied

to allow for their ratio and phase angle errors. Fig. 20.6 gives the vector diagrams for the currents and voltages of the load, and in the wattmeter coils. Diagram (*a*) refers to a load with a lagging

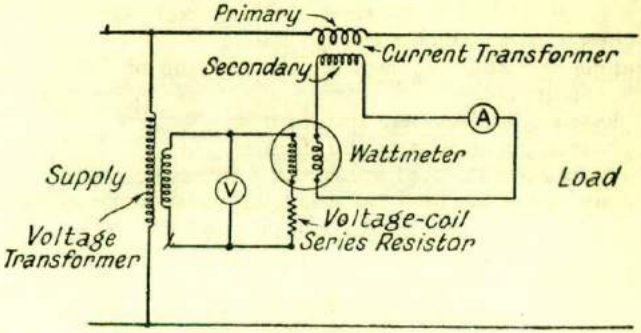


FIG. 20.5. USE OF INSTRUMENT TRANSFORMERS WITH A WATTMETER

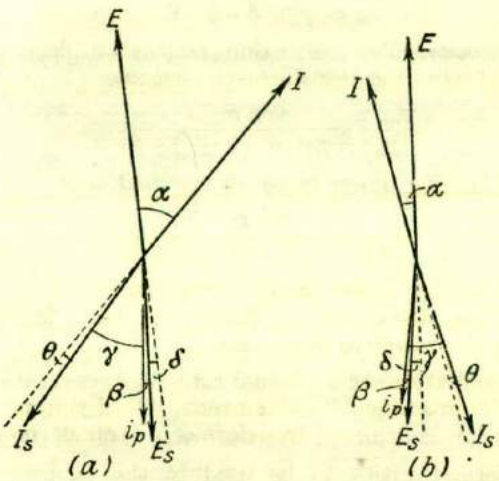


FIG. 20.6

power factor, and diagram (*b*) to a load with a leading power factor It is assumed that both current and voltage transformers are used.

In the diagrams—

E = voltage of the load
I = load current
α = phase angle between load current and voltage

γ = phase angle between the currents in the current and voltage coils of the wattmeter
I_s = current in wattmeter current coil
= secondary current of current transformer
E_s = voltage applied to wattmeter voltage coil
i_p = current in wattmeter voltage coil
β = angle by which i_p lags E_s on account of inductance of voltage coil
δ = phase angle of voltage transformer
θ = phase angle of current transformer
The vectors shown dotted are E and I reversed.
Obviously, when the load has a lagging phase angle α, we have

$$\alpha = \gamma + \theta + \delta + \beta$$

or, more strictly,

$$\alpha = \gamma + \theta \pm \delta + \beta$$

since the phase angle of the voltage transformer may be either lagging or leading.
In the case of a leading power factor of the load,

$$\alpha = \gamma \pm \delta - \theta - \beta$$

Correction Factor. The correction factors—neglecting, for the present, the ratios of the transformers—become

$$\frac{\cos \alpha}{\cos \beta \cos (\alpha - \theta \pm \delta - \beta)}$$

in the case of lagging power factor of the load, and

$$\frac{\cos \alpha}{\cos \beta \cos (\alpha \pm \delta + \theta + \beta)}$$

in the case of leading power factor of the load.
Writing K for the correction factor, we have for the general expression for the power to be measured,

$$\text{Power} = K \times \underset{\text{reading}}{\text{Wattmeter}} \times \underset{\text{transformer}}{\text{Actual ratio of current}} \times \underset{\text{transformer}}{\text{Actual ratio of voltage}} . \quad (20.6)$$

The transformer ratios to be used in the expression are the actual ratios as distinct from the nominal ratios. As was seen in the foregoing chapter, these ratios are not constant but depend upon the load conditions, so that calibration curves of the transformers are necessary if accurate power measurements are to be made. It should be noted also, that in the above the power losses in the instruments are not considered.

Measurement of Power without using a Wattmeter. It is possible to measure the power in a circuit without a wattmeter by using either three ammeters or three voltmeters, in conjunction with

non-inductive resistors. These methods are not, however, of much practical importance.

THREE-VOLTMETER METHOD. The connections are as shown in Fig. 20.7, in which V_1, V_2, and V_3 are the three voltmeters and R is a non-inductive resistor which is connected in series with the load. From the vector diagram of Fig. 20.7 (*b*) we have

$$V_1^2 = V_2^2 + V_3^2 + 2V_2V_3 \cos \phi$$

Neglecting the currents taken by voltmeters V_2 and V_3, the current in R is the same as the load current I. Thus, $V_2 = IR$, and

$$V_1^2 = V_2^2 + V_3^2 + 2IRV_3 \cos \phi$$

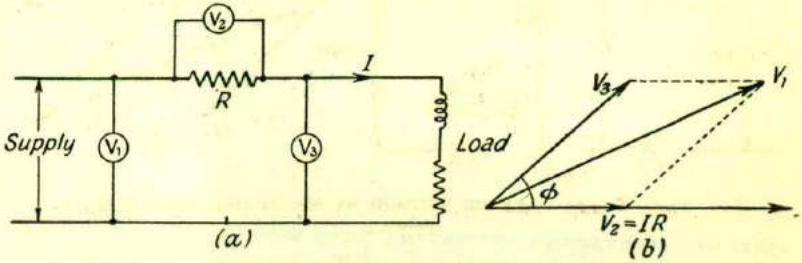


FIG. 20.7. THREE-VOLTMETER METHOD OF MEASURING SINGLE-PHASE POWER

Now, $IV_3 \cos \phi$ is the power in the load, so that

$$\text{Power in load} = IV_3 \cos \phi = \frac{V_1^2 - V_2^2 - V_3^2}{2R} \quad . \quad (20.7)$$

The power factor is given by

$$\cos \phi = \frac{V_1^2 - V_2^2 - V_3^2}{2V_2V_3} \quad . \quad . \quad . \quad (20.8)$$

The assumptions are made that the current in the resistor R is the same as the load current, and that this resistor is entirely non-inductive.

THREE-AMMETER METHOD. This method is somewhat similar to the above. The necessary connections are shown in Fig. 20.8. The current measured by ammeter A_1 is the vector sum of the load current and that taken by the non-inductive resistor R (this latter being in phase with the voltage E). From the vector diagram,

$$I_1^2 = I_2^2 + I_3^2 + 2I_2I_3 \cos \phi$$

But $I_2 = \dfrac{E}{R}$

$$\therefore I_1^2 = I_2^2 + I_3^2 + 2\frac{E}{R} \cdot I_3 \cos \phi$$

Hence, the power $EI_3 \cos \phi$ is given by

$$EI_3 \cos \phi = \frac{(I_1^2 - I_2^2 - I_3^2)\,R}{2} \quad . \quad . \quad (20.9)$$

Also,
$$\cos \phi = \frac{(I_1^2 - I_2^2 - I_3^2)}{2I_2I_3} \quad . \quad . \quad . \quad (20.10)$$

Measurement of Three-phase Power. THREE-WATTMETER METHOD. The connections for this method are shown in Fig. 20.9, in which the

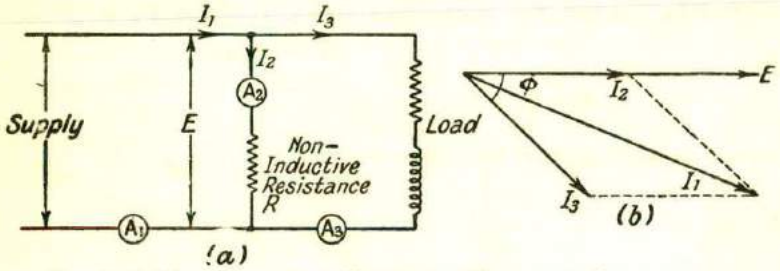


FIG. 20.8. THREE-AMMETER METHOD OF MEASURING SINGLE-PHASE POWER

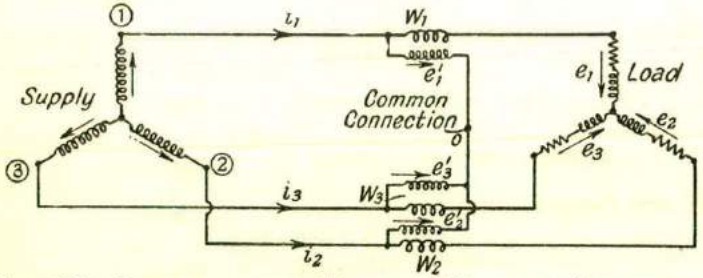


FIG. 20.9. THREE-WATTMETER METHOD OF MEASURING THREE-PHASE POWER

load is star-connected. W_1, W_2, and W_3 are the three wattmeters, connected as shown. The arrows denote the directions of current and voltage which are conventionally considered positive. If the symbols representing currents and voltages denote instantaneous values, then

Instantaneous power in the load $= e_1i_1 + e_2i_2 + e_3i_3$

Let v be the potential difference between the star point of the load and the star point O of the wattmeter voltage coils. Then we have

$$e_1' + v = e_1$$
$$e_2' + v = e_2$$
$$e_3' + v = e_3$$

$\therefore$ Total instantaneous power, by substitution, is

$$(e_1' + v)\, i_1 + (e_2' + v)\, i_2 + (e_3' + v)\, i_3$$
$$= e_1' i_1 + e_2' i_2 + e_3' i_3 + v\,(i_1 + i_2 + i_3)$$
$$= e_1' i_1 + e_2' i_2 + e_3' i_3$$

since $i_1 + i_2 + i_3 = 0$ in any three-phase 3-wire system, whether balanced or not.

Now, $e_1' i_1 + e_2' i_2 + e_3' i_3$ is the total instantaneous power measured by the three wattmeters, and thus the sum of the readings of the wattmeters will give the mean value of the total power.

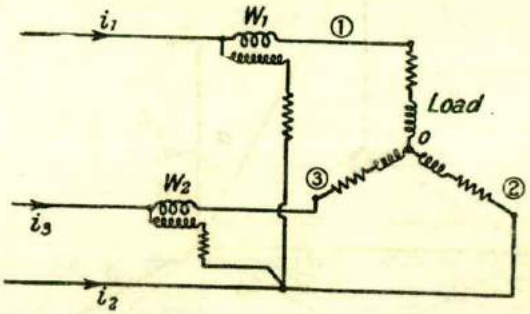


FIG. 20.10. TWO-WATTMETER METHOD OF MEASURING THREE-PHASE POWER

TWO-WATTMETER METHOD. This is the commonest method of measuring three-phase power. It is particularly useful when the load is unbalanced. The connections for the measurement of power in the case of a star-connected three-phase load are shown in Fig. 20.10. The current coils of the wattmeters are connected in lines (1) and (3), and their voltage coils between lines (1) and (2) and (3) and (2) respectively.

Fig. 20.11 gives the vector diagram for the load circuit, assuming a balanced load—i.e. the load currents and power factors are the same for all three phases. E_{10}, E_{20}, and E_{30} are the vectors representing the phase voltages, and are supposed to be equal, while I_1, I_2, and I_3 are vectors representing the line currents. The voltages applied to the voltage-coil circuits of the wattmeters are E_{12} and E_{32}, which are the vector sums of the phase voltages as shown.

Then, total instantaneous power in the load

$$= e_1 i_1 + e_2 i_2 + {}_3 i_3$$

where e_1, e_2, e_3 are the instantaneous phase voltages and i_1, i_2, and i_3 are the instantaneous line currents.

Since $i_1 + i_2 + i_3 = 0$, $i_2 = -i_1 - i_3$

$\therefore$ Total instantaneous power $= e_1 i_1 + e_2(-i_1 - i_3) + e_3 i_2$

$= i_1(e_1 - e_2) + i_3(e_3 - e_2)$

Now, $i_1(e_1 - e_2)$ is the instantaneous power deflecting wattmeter W_1, and $i_3(e_3 - e_2)$ is that deflecting wattmeter W_2. These wattmeters measure $I_1 E_{12} \cos \alpha$ and $I_3 E_{32} \cos \beta$ respectively, where α and β are the phase angles between I_1 and E_{12} and between I_3 and E_{32}. The

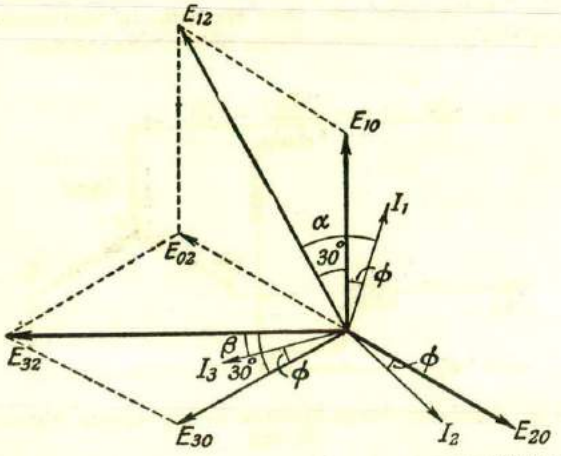


FIG. 20.11. VECTOR DIAGRAM, TWO-WATTMETER METHOD

sum of the wattmeter readings thus gives the mean value of the total power in the load.

Now $\alpha = 30° + \phi$

and $\beta = 30° - \phi$

Also, $E_{12} = E_{32} = \sqrt{3}E$

where E is the phase voltage.

Therefore the sum of the wattmeter readings is

$$W = \sqrt{3}IE \cos (30° + \phi) + \sqrt{3}IE \cos (30° - \phi)$$

If $I_1 = I_2 = I_3 = I$

$$W = \sqrt{3}IE \left[\cos (30° + \phi) + \cos (30° - \phi)\right]$$

$$= \sqrt{3}IE \left[2 \cos 30° \cos \phi\right]$$

$$= 3IE \cos \phi$$

which is, of course, the total power in the load.

It should be noted that, if one of the voltages (such as E_{12}) is more than 90° out of phase with the current associated with this voltage in the wattmeter, the voltage-coil connections must be reversed in order that the instrument may give a forward reading. Under these circumstances the wattmeter reading must be reckoned as negative, and the algebraic sum of the readings of the two instruments gives the mean value of the total power.

Another important point is that, if the power factor of the load

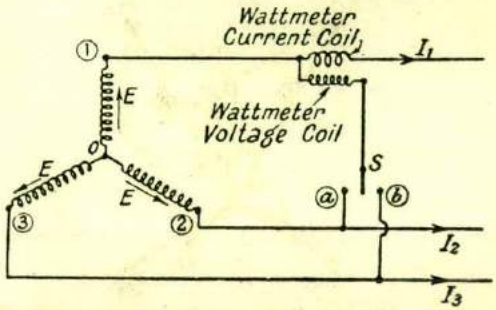


FIG. 20.12. ONE-WATTMETER METHOD FOR A BALANCED THREE-PHASE LOAD

is 0·5—so that I_1 lags 60° behind E_{10} (cos 60° being 0·5)—then the phase angle between E_{12} and I_1 is 90° and wattmeter W_1 should read zero.

Power Factor. If W_1 and W_2 are the two wattmeter readings $W_1 + W_2$ gives the total power (as seen above).

$$\begin{aligned} W_1 - W_2 &= \sqrt{3}\,IE\,[\cos(30° + \phi) - \cos(30° - \phi)] \\ &= \sqrt{3}\,IE\,[-2\sin 30° \sin\phi] \\ &= -\sqrt{3}\,IE\sin\phi \end{aligned}$$

$$\therefore \quad \frac{W_1 - W_2}{W_1 + W_2} = \frac{-\sqrt{3}\,IE\sin\phi}{3\,IE\cos\phi} = -\frac{\tan\phi}{\sqrt{3}}$$

or

$$\tan\phi = \frac{\sqrt{3}\,(W_2 - W_1)}{W_1 + W_2} \qquad . \quad . \quad . \quad . \quad (20.11)$$

From which ϕ and the power factor $\cos\phi$ of the load may be found.

ONE-WATTMETER METHOD. This method can be used only when the load is balanced. The connections for a star-connected system are shown in Fig. 20.12. The current coil of the wattmeter is connected in one of the lines, and one end of the voltage coil is connected to the same line, the other end being connected alternately

to first one and then the other of the remaining two lines by means of the switch S.

The vector diagram for the method of measurement is given in Fig. 20.13, in which E_{01}, E_{02}, and E_{03} represent the three phase voltages, and I_1, I_2, and I_3 the three line currents. In a balanced system these three voltages are each equal to E, and the three currents are each equal to I. The phase angles are also all equal to ϕ. The vector E_{12} is the vector difference between E_{01} and E_{02}, and is the voltage

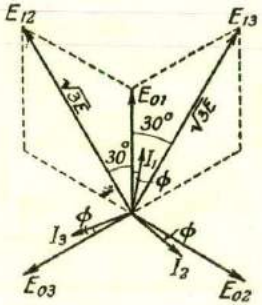


FIG. 20.13. VECTOR DIAGRAM, ONE-WATTMETER METHOD

applied to the voltage coil when the switch S is on contact "a." Similarly, E_{13} is the vector difference of E_{01} and E_{03}, and is applied to the wattmeter voltage coil when switch S is on contact "b."

Then $$E_{12} = E_{13} = \sqrt{3}\,E$$

Wattmeter reading when switch S is on contact "a"

$$= \sqrt{3}\,EI \cos (30° + \phi)$$

$(30° + \phi)$ being the phase angle between E_{12} and I_1. When S is on contact "b" the wattmeter reading is

$$\sqrt{3}\,EI \cos (30° - \phi)$$

Thus, the sum of these readings is $3EI \cos \phi$, as shown in the analysis of the two-wattmeter method, and this is the total power in the circuit.

In the same way the angle ϕ is given by

$$\tan \phi = \frac{\sqrt{3}\,(W_2 - W_1)}{W_1 + W_2}$$

and the power factor is $\cos \phi$ or $\cos \left(\tan^{-1} \dfrac{\sqrt{3}\,(W_2 - W_1)}{W_1 + W_2} \right)$

Method for a Balanced Three-phase Circuit using Transformers with One Wattmeter. Barlow (Ref. (11)) has described a method of

measuring the power in a balanced three-phase circuit, with the load delta-connected, which does not involve the provision of an artificial neutral point and in which the wattmeter gives the total power directly without any multiplying factor.

There are two alternative methods of connection, one using a voltage transformer of 1 : 1 ratio, and the other using two current

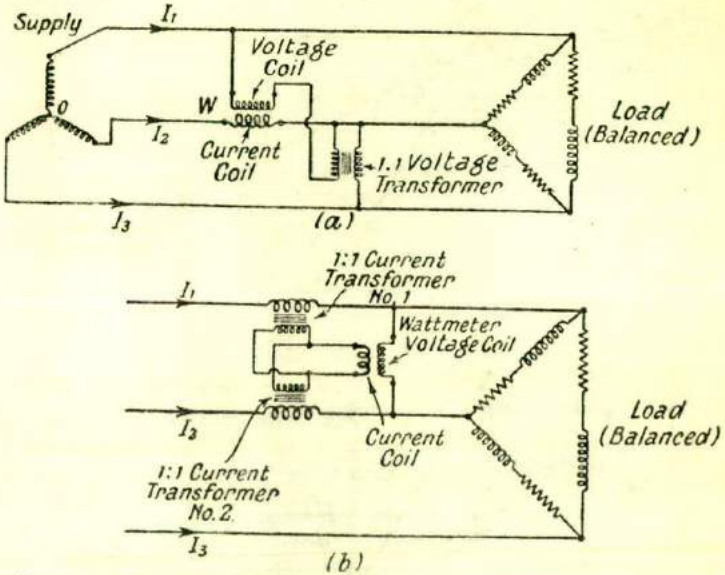


FIG. 20.14. BARLOW METHOD OF MEASURING THREE-PHASE POWER

transformers each of ratio 1 : 1. These connections are shown in Fig. 20.14, (*a*) and (*b*).

The vector diagram of Fig. 20.15 refers to the connections (*a*). E_{10}, E_{20}, and E_{30} are the phase voltages of the supply, and I_1, I_2, and I_3 the line currents. The three voltages and three currents are equal in a balanced system, as are also the three phase angles ϕ. The current coil of the wattmeter carries the current I_2, while the voltage applied to the voltage coil is the vector difference of line voltages E_{21} and E_{32} (reversed). These two voltages are equal, and the phase angle between either of them and their vector difference E' is 30°, and is therefore in phase with E_{20}. The phase angle between E' and I_2 is thus ϕ.

The wattmeter measures $E'I_2 \cos \phi$. Now, if $E_{10} = E_{20} = E_{30} = E$, then $E_{21} = E_{32} = \sqrt{3}E$, and thus $E' = \sqrt{3}E_{21} = 3E$.

The power measured by the wattmeter is $3EI \cos \phi$ (since $I_1 = I_2 = I_3 = I$), which is the mean value of the total power in the

balanced circuit. The ratio error and phase angle of the voltage transformer are, of course, neglected in the above reasoning.

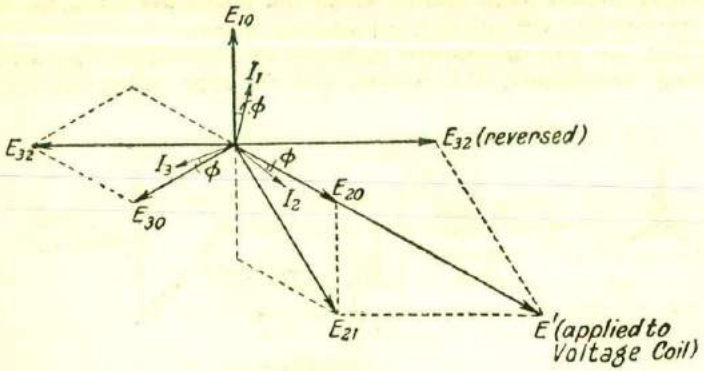


FIG. 20.15

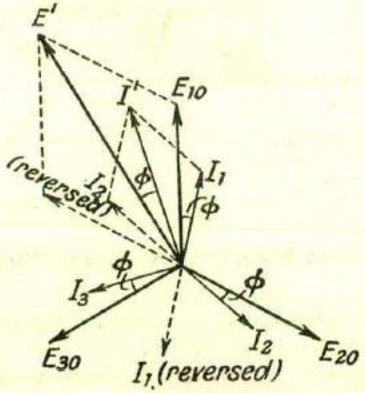


FIG. 20.16

Fig. 20.16 gives the vector diagram for the connections of Fig. 20.14 (*b*). Since the load is balanced,

$$I_1 = I_2 = I_3 = I$$

and

$$E_{10} = E_{20} = E_{30} = E$$

and the phase angles ϕ are all equal.

The vector E', which is the vector difference of the voltages E_{10} and E_{20}, is the voltage applied to the voltage coil and equals $\sqrt{3}\,E$. The currents in the secondary circuits of current transformers 1 and 2 are I_1 reversed and I_2 reversed. The connections to the

current coil of the wattmeter are such that the current in it is the vector sum of I_1 and I_2 reversed, and is represented by the vector I'. The value of this current is $\sqrt{3}\,I$, and the phase angle between I' and E' is ϕ. Then,

$$\begin{aligned}\text{Power measured by wattmeter} &= E'I' \cos \phi \\ &= \sqrt{3}E\sqrt{3}I \cos \phi \\ &= 3EI \cos \phi \\ &= \text{total power in load}\end{aligned}$$

Again, the current-transformer ratio and phase-angle errors are neglected.

Wattmeters. Three types of wattmeter will be considered—

(*a*) Dynamometer.

(*b*) Induction.

(*c*) Electrostatic.

Of these, the first two are most commonly used.

(*a*) DYNAMOMETER TYPE. These instruments are similar in design and principle to the dynamometer ammeter already described (Chapter XVIII). The fixed coils carry the current in the circuit, while the moving coil acts as the wattmeter voltage coil and carries a current proportional to the voltage of the circuit across which it is connected. A high non-inductive resistance is connected in series with the voltage coil.

Dynamometer wattmeters may be divided into two classes—

(i) Suspended-coil, torsion instruments.

(ii) Pivoted-coil, direct-indicating instruments.

(i) *Torsion Wattmeters.* These instruments are used largely as *standard wattmeters.*

The moving, or voltage, coil is suspended from a torsion head by a metallic suspension which serves as a lead to the coil. This coil is situated entirely inside the current, or fixed, coils and the winding is such that the system is astatic. Errors due to external magnetic fields are thus avoided. The torsion head carries a scale, and when in use, the moving coil is brought back to the zero position by turning this head, the number of divisions turned through, when multiplied by a constant for the instrument, giving the power.

Eddy current errors are eliminated as far as possible by winding the current coils of stranded wire and by using no metal parts within the region of the magnetic field of the instrument.

In the Drysdale single-phase astatic wattmeter, manufactured by Messrs. H. Tinsley, the moving coil, which is stitched to a flat strip of mica, is divided into two equal portions wound so that the current (proportional to the applied voltage) circulates in a clockwise direction in one-half and in an anti-clockwise direction in the other This coil is suspended by a silk fibre together with a spiral spring which gives the required torsion.

The fixed coil also is in two halves, which are wound so as to have opposite directions of current circulation in them. The cable used for the fixed coil consists of ten strands, insulated from one another. Thus, in effect, there are ten current coils which run together and are thus as nearly as possible coincident in space. These ten coils, or cores, are brought out to a commutator so that a number of current ranges of the instrument may be obtained by grouping them all in parallel, all in series, or in a series-parallel combination.

The current is led into the moving coil by two fine phosphor-bronze ligaments, the spring, which is of "Iso-elastic" wire and is annealed, merely serving as a torsion control. The spring has a number of turns, and by carefully adjusting its length the constant of the instrument can be made an exact figure. The moving system carries a knife-edge pointer moving over a short scale at the front of the instrument, so that the zero position of the moving coil can be easily determined.

Damping is provided by the mica vane which carries the voltage coil. Drysdale states that, in order to reduce error due to the voltage-coil inductance to a negligible amount, the resistance of the voltage-coil circuit should be at least 3,000 ohms per millihenry of inductance.

Heavy-current Standard Wattmeters. The Drysdale wattmeter described above can be constructed to cover a current range up to 500 amp. Above this value of current, difficulties are encountered owing to the eddy currents in the heavy-section conductors required, and to non-uniform distribution of current over the cross-sections of these conductors.

Both A. E. Moore (Ref. (12)) and P. G. Agnew (Ref. (13)) have developed heavy-current standard wattmeters of the dynamometer type which are capable of dealing with currents up to about 5,000 amp.

The Agnew instrument has a double concentric tube as the "current coil," the tubes being joined at one end. The moving system consists of two astatic coils suspended, within the outer cylindrical tube, one above and one below the inner tube.

In the Moore instrument there is a tubular central conductor which can be water-cooled. This is surrounded by a cylindrical tube, or box, in two halves, bolted together (see Fig. 20.17), the two tubes being connected together at one end. The moving system, which is suspended within the outer tube as in the Agnew instrument, consists of two D-shaped coils with their straight sides horizontal, one above and one below the central tube.

In both instruments the magnetic field of the primary, or "current," conductor—in which field the moving system is situated—is independent of the distribution of current over the cross-sections of the tubular conductors.

(ii) *Direct-indicating Dynamometer Wattmeters.* Like the standard,

or torsion, wattmeters described above, these instruments have a moving voltage coil which is almost entirely embraced by the fixed current coils. The moving coil is carried on a pivoted spindle and the movement is spring-controlled. The moving system carries a pointer and a damping vane, the latter moving in a sector-shaped box. The current coils are usually stranded or laminated, especially

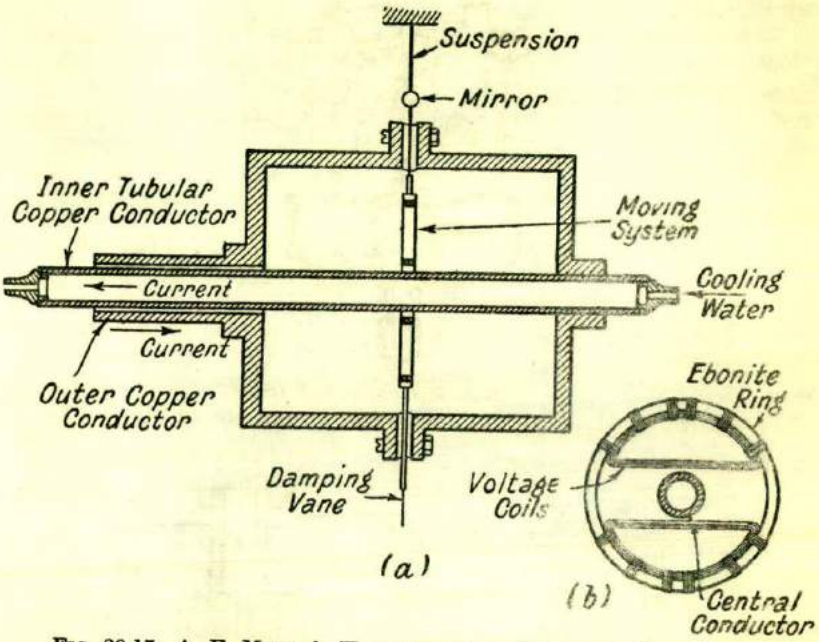
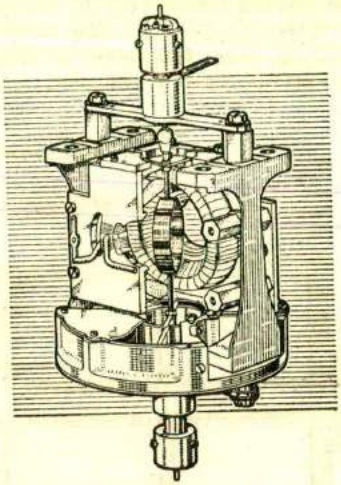


FIG. 20.17. A. E. MOORE'S HEAVY-CURRENT STANDARD WATTMETER

when heavy currents are to be carried. Currents up to about 200 amp can be dealt with in direct-indicating wattmeters of suitable design. For currents above this, a low-range wattmeter is usually employed, in conjunction with current transformers. Similarly, voltages up to about 600 volts are applied to the wattmeter voltage coil directly, but for higher voltages the voltage-coil circuit is designed for 110 volts, and a voltage transformer used to step down the voltage. Metal parts should be removed from the magnetic field of the instrument as far as possible, although care must be taken to ensure that this elimination of metal does not result in any relative movement of the working coils due to warping of the materials substituted for the metal.

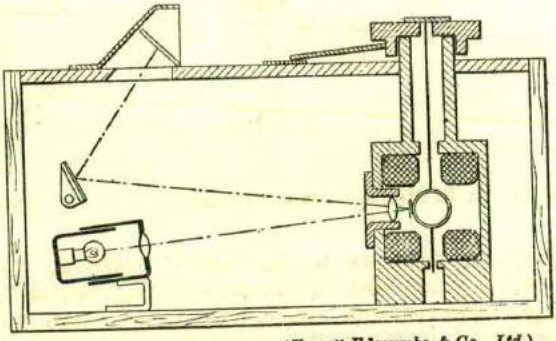
Figs. 20.18, 20.19, 20.20 show the construction of some dynamometer wattmeters. In Fig. 20.18 the details of the suspended

movement of a reflecting dynamometer wattmeter (Elliott Brothers (London), Ltd.) are shown. This instrument is intended for the



(*Elliott Brothers (London), Ltd.*)

FIG. 20.18. MOVEMENT OF REFLECTING DYNAMOMETER WATTMETER

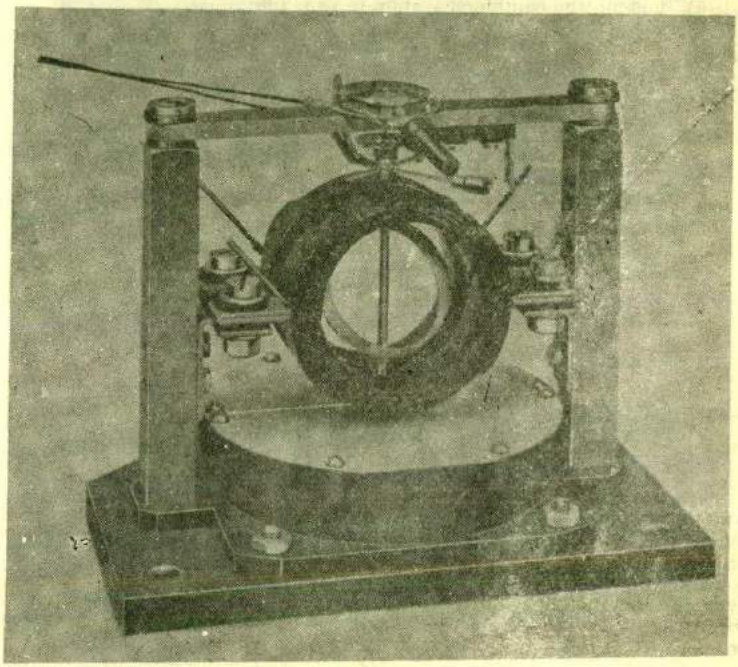


(*Everett Edgcumbe & Co., Ltd.*)

FIG. 20.19. INTERNAL ARRANGEMENT OF PORTABLE DYNAMOMETER WATTMETER

measurement of power when the power consumption of the instrument itself must be small, e.g. when measuring the power taken by a low-power fluorescent lamp. In this particular measurement it is an advantage that the wattmeter has no iron in its magnetic circuit since the lamp voltage has a high harmonic content with an appreciable a.c. component. The wattmeter current coil takes 0·25 VA and the voltage winding is about 750 Ω per volt on 50 c/s.

Fig. 20.19 shows the internal arrangements of the Everett Edgcumbe wattmeter, with a moving system suspended, by a robust ligament, from a torsion head which carries a pointer moving over an evenly divided scale. The instrument is designed for measurements at very low power factors, e.g. iron loss and dielectric loss



(*Crompton Parkinson, Ltd.*)

FIG. 20.20. SINGLE-ELEMENT INDICATING WATTMETER

measurements. The power loss in the current element is very small, e.g. 5 microwatts when carrying a current corresponding to full-scale deflection at unity power factor, the instrument being scaled to 2 watts at 12,000 volts.

In the deflectional type of dynamometer instrument the relative positions of current and voltage coils change with the deflection, whereas in the torsion type the relative positions are the same for all loads, since the moving coil is returned to the zero position in all cases.

Relation between Torque and Mutual Inductance between Fixed and Moving Coils. Referring to Fig. 20.21 the current-coil circuit of the wattmeter is represented by resistance R_1 and inductance L_1, and the voltage-coil circuit by R_2 and L_2. A mutual inductance M exists between the current and

voltage coils. We may write the current and voltage relations in the circuits, in instantaneous values, as follows—

$$e_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$e_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

The work done on the combined system in time t is

$$W = \int_0^t e_1 i_1 \, dt + \int_0^t e_2 i_2 \, dt$$

$$= \int_0^t (R_1 i_1^2 + R_2 i_2^2) \, dt + \int_0^t \left(L_1 i_1 \frac{di_1}{dt} + L_2 i_2 \frac{di_2}{dt} \right) dt$$

$$+ \int_0^t M \left(i_2 \frac{di_1}{dt} + i_1 \frac{di_2}{dt} \right) dt$$

The last term represents energy mutual to both systems, and evaluates as

$$\text{Mutual stored energy} = M i_1 i_2$$

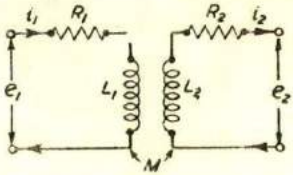


FIG. 20.21

We are only concerned with events when one coil moves relative to the other, so the first two terms can be ignored since neither is affected by relative movement under constant-current conditions.

A force exists between the two coils, and now one of the coils is permitted to move a small distance under the action of this force, with the currents held constant. If the mutual inductance increases by an amount δM due to this movement, then the increase in mutual stored energy is

$$i_1 i_2 \delta M$$

If the mutual inductance increases, there must be an increase in the voltages applied to the two coils in order to maintain the currents constant during the change. Thus the increase in voltage to coil 1 is

$$\delta e_1 = i_2 \frac{dM}{dt}$$

The additional input energy to system 1 is

$$\int_0^t \delta e_1 i_1 \, dt = \int_0^t i_1 i_2 \frac{dM}{dt} \, dt = i_1 i_2 \delta M$$

since i_1 and i_2 are held constant.

Similarly, the additional input energy to system 2 is $i_1 i_2 \delta M$ and the total is $2 i_1 i_2 \delta M$. From the principle of the conservation of energy,

$$\text{Total additional input energy} = \text{change in mutual stored energy} + \text{mechanical work done}$$

If the movement of the coil is rotary, through a small angle $\delta\theta$, under the action of a torque T, then the mechanical work done is $T\delta\theta$.

Hence $$2i_1i_2\delta M = i_1i_2\delta M + T\delta\theta$$

or $$T = i_1i_2\frac{dM}{d\theta} \quad . \quad . \quad . \quad (20.12)$$

The voltage-coil current is given by

$$i_1 = \frac{e_1}{R} = \frac{E_{1max}}{R}\sin\omega t$$

and the current in the current coil is

$$i_2 = I_{2max}\sin(\omega t - \phi)$$

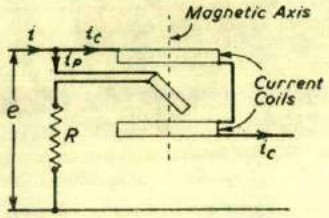


FIG. 20.22. CONNECTIONS OF DYNAMOMETER WATTMETER

Substituting for i_1 and i_2 we have, for the instantaneous torque,

$$T = \frac{E_{1max}\cdot I_{2\,max}}{R}\frac{dM}{d\theta}\sin\omega t\sin(\omega t - \phi)$$

The mean torque is

$$T_M = \frac{E_{1\,max}\,I_{2\,max}}{R}\frac{dM}{d\theta}\frac{1}{2\pi}\int_0^{2\pi}\sin\omega t\sin(\omega t - \phi)\,d(\omega t)$$

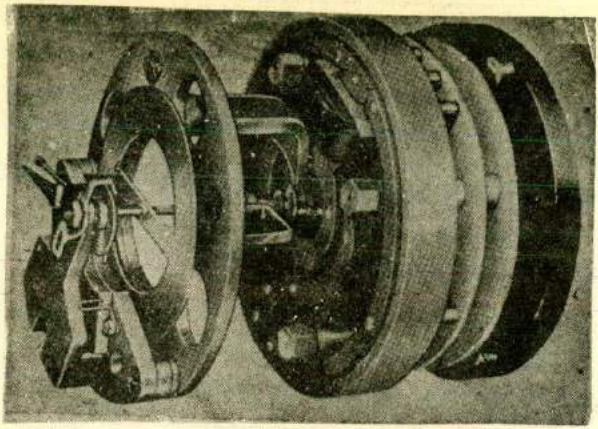
$$= \frac{E_1I_2}{R}\cos\phi\,.\,\frac{dM}{d\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20.13)$$

where E_1 and I_2 are r.m.s. values.

Fig. 20.22 is a diagrammatic representation of a dynamometer wattmeter. The fixed current coils are connected series-aiding, and the voltage coil is shown in its zero position. The voltage coil will normally rotate in a clockwise direction and the current directions necessary to ensure this can readily be determined from the left-hand rule. Alternatively it may be noted that the moving coil turns so that its magnetic effect increases that of the fixed coils.

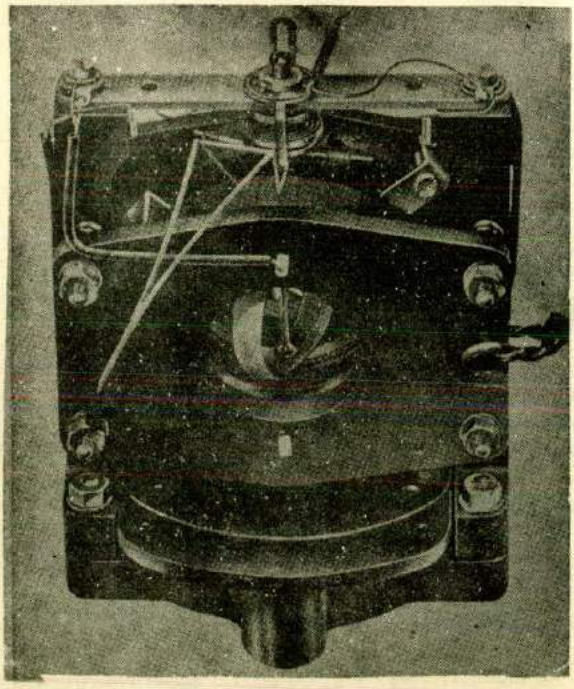
It can be seen that when the plane of the moving coil is perpendicular to the planes of the fixed coils, the mutual inductance between fixed and moving coils is zero, and at this point $\frac{dM}{d\theta}$ is a maximum.

The existence between fixed and moving coils of a mutual inductance which changes both in magnitude and sign as the instrument deflects, means that, when the coils are energized with alternating current, there is a mutually induced e.m.f. in the two coils.



(Crompton Parkinson, Ltd.)

FIG. 20.23. MOVEMENT OF SINGLE-ELEMENT WATTMETER



(Sangamo Weston, Ltd.)

FIG. 20.24. INTERNAL CONSTRUCTION OF DYNAMOMETER WATTMETER

Expressions have been given by A. H. M. Arnold (Ref. (20)) for the indication of a wattmeter taking into consideration the self-capacitances of the windings and the mutual inductance. If the self-capacitances are neglected, it can be shown from these expressions that, when the current coil is between the voltage coil and the load, the instrument indication is given by

$$P_W = P_L + I^2R_C + \left[\frac{L_p + M}{R_p}\right] \omega VI \sin \phi$$

where P_L is the power in the load, R_C is the resistance of the current coil, L_p and R_p are the inductance and resistance of the voltage-coil circuit, M is the mutual inductance between voltage and current coils, V is the voltage, I is the load current, and ϕ the load phase angle.

The mutual inductance therefore introduces an error which varies with the relative positions of the coils. This error is usually negligible at power frequencies but becomes significant at higher frequencies.

Instruments whose coil systems are arranged so that they are always in a position of zero mutual inductance are free from this error, and also develop a maximum torque for every value of current. The Drysdale Torsion-head Wattmeter (see p. 779) is an instrument which makes use of this principle.

The errors due to inductance in the voltage-coil circuit and to other causes have already been considered in the previous pages.

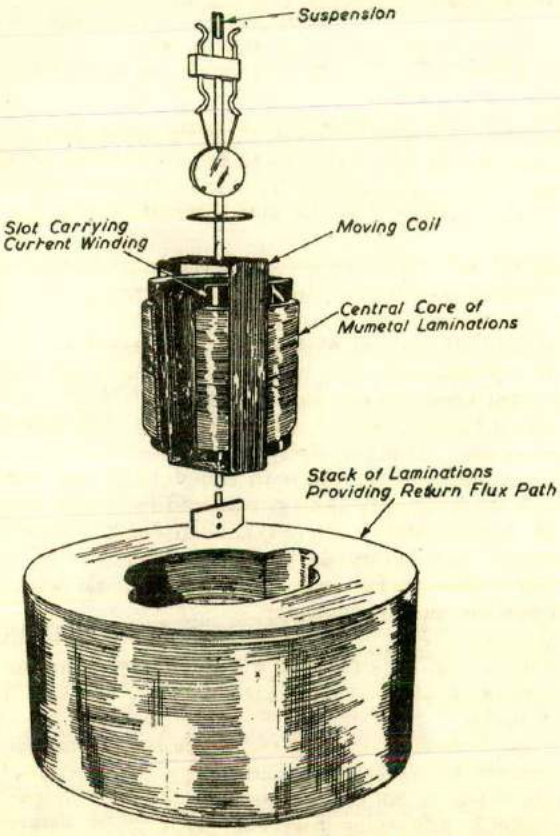
A. H. M. Arnold (Refs. (16), (20)) has discussed the performance of dynamometer wattmeters at audio frequencies (see also p. 743) and, in particular, nickel–iron-cored instruments with a feedback amplifier and current transformer.

J. de Gruchy (Ref. (19)) has pointed out the advantages of the phase-sensitive rectifier bridge as a wattmeter employing a dynamometer instrument, as a power-factor meter and as a rectifier-operated frequency meter.

Cambridge Reflecting Wattmeter. Precision dynamometer wattmeters manufactured by the Cambridge Instrument Co. employ nickel–iron cores arranged as shown in Fig. 20.25 to obtain high electromagnetic efficiency together with good screening and a uniform radial field in which the moving coil can rotate. Because of the high permeability of the material used, the reluctance of the magnetic circuit is almost entirely in the air-gap and hysteresis effects are negligible. By using thin laminations, eddy-current losses, which would cause phase-angle errors, are so reduced that they are easily compensated, over a wide range of frequencies, by connecting a small capacitor across the moving coil and part of the voltage-circuit resistor.

In a reflecting laboratory-type instrument suitable for the calibration of precision-grade meters on d.c. or a.c., a carefully-aged

quartz fibre is used as a control spring to ensure consistency of calibration and low temperature coefficient. The current winding is arranged to provide four current ranges of 0·5, 1, 2·5 and 5 amp, and these, together with nine voltage ranges between 50 and 500



(Cambridge Instrument Co., Ltd.)

FIG. 20.25. CONSTRUCTION OF NICKEL-IRON-CORE DYNAMOMETER

volts, are selected by switches mounted in a separate control box. A further switch increases the sensitivity of the system by a factor of 5, when full-scale deflection will be obtained at 0·2 power factor or with voltages of 1/5 of the marked value.

It is important that wattmeters of this type shall give the same indication on d.c. or a.c., and the best determinations indicate that the difference does not exceed 0·01 per cent.

The scale is 2·5 m long and is in three parallel sections with three separate lamps and optical systems which are brought into use in turn. The makers claim that the accuracy of calibration is 0·05 per cent over the upper three-fourths of the scale length.

(*b*) Induction Wattmeters. Induction wattmeters, the principle of which is the same as that of induction ammeters and voltmeters, can only be used in alternating-current circuits: dynamometer wattmeters can be used in either a.c. or d.c. circuits.

Induction instruments are useful only when the frequency and supply voltage are approximately constant.

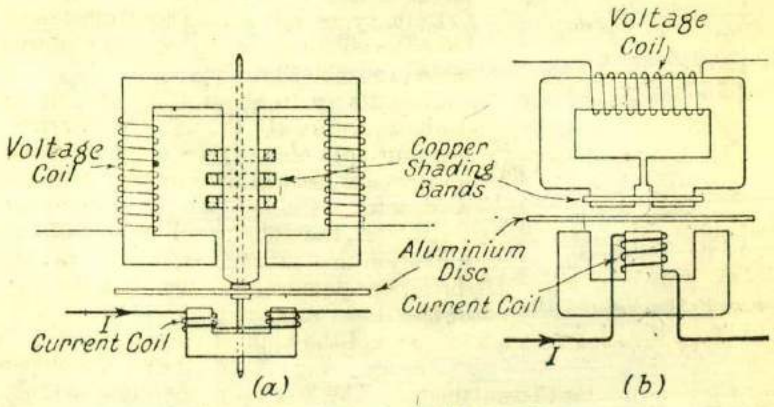


Fig. 20.26. Induction Wattmeters

Construction. These instruments have two laminated electromagnets; one is excited by the load current (or a definite fraction of it), and the other by a current proportional to the voltage of the circuit in which the power is to be measured. A thin aluminium disc is mounted so that it is cut by the flux from both of these magnets, and the deflecting torque is produced by the interactions between these fluxes and the eddy currents which they induce in the disc. One or more copper rings are fitted on one limb of the "shunt" magnet—i.e. the magnet excited by the voltage coil and its current—in order to cause the resultant flux in the magnet to lag in phase by exactly 90° behind the applied voltage.

Fig. 20.26 shows two common forms of magnets with their windings, the magnets being placed, in each case, one above and one below the moving disc of the instrument. The positions and shapes of the magnets are, such that the flux from both "shunt" and "series" magnets cuts the moving disc.

In the form of instrument shown in Fig. 20.26 (*a*), the two voltage coils, connected in series, are wound so that they both send flux through the centre limb. The series magnet in the instrument

carries two small current coils in series, these being wound so that they both magnetize the core, upon which they are wound, in the same direction. The positions of the copper shading bands are adjustable in order that the correct phase displacement between the shunt and series magnet fluxes may be obtained.

In the instrument shown in Fig. 20.26 (*b*) there is only one voltage coil and one current coil. A copper shading band, whose position is adjustable, surrounds the two projecting pole pieces of the shunt magnet.

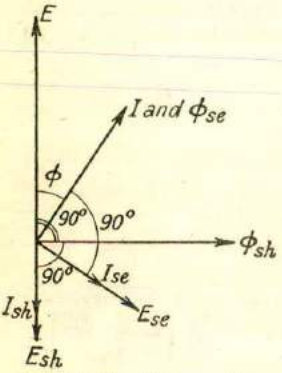


FIG. 20.27. VECTOR DIAGRAM FOR INDUCTION WATTMETER

Both types are spring-controlled and have the advantage of a long and uniform scale (up to 300°).

Currents up to about 100 amp can be dealt with directly in such instruments. For currents above this, current transformers are used in conjunction with the wattmeter. Unlike the dynamometer wattmeter, the voltage-coil circuit of the induction instrument is made as inductive as possible, in order that the flux of the shunt magnet may lag by nearly 90° behind the applied voltage.

Theory. Fig. 20.27 gives a simplified vector diagram for the wattmeter. The flux ϕ_{sh} of the shunt magnet is assumed to lag by exactly 90° behind the applied voltage. As previously stated, this is actually brought about by adjustment of the shading bands, since the angle of lag would be somewhat less than 90° unless such bands were used. The theory of the action of these shading bands in altering the phase of the resultant flux has already been dealt with in considering induction ammeters. It is assumed also that the flux ϕ_{se} of the series magnet is proportional to and in phase with the line current, and that hysteresis and saturation effects in the iron are negligible. Owing to the large air gap in the core these assumptions are justifiable.

Let vector E represent the applied voltage, and vector I, lagging ϕ behind E, the load current. ϕ_{se} is in phase with I.

e_{sh} = eddy e.m.f. induced in the disc by ϕ_{sh}

i_{sh} = eddy current due to and in phase with e_{sh} (the inductance of the eddy current path in the disc being neglected)

e_{se} = e.m.f. induced by the flux ϕ_{se}

i_{se} = eddy current due to and in phase with e_{se}

The instantaneous torque acting upon the disc is, from the theory of the induction ammeter (see page 698), proportional to $(\phi_{sh}i_{se} - \phi_{se}i_{sh})$, these values of current and flux being instantaneous.

Let applied voltage be

$$e = E_{max} \sin \omega t$$

Then the current is

$$i = I_{max} \sin (\omega t - \varphi)$$

The fluxes are $\phi_{se} = kI_{max} \sin (\omega t - \varphi)$

and $\phi_{sh} = k' \int e \, dt = - k' \dfrac{E_{max}}{\omega} \cos \omega t$

(since $e = \dfrac{1}{k'} \dfrac{d\phi_{sh}}{dt}$, being in antiphase to the induced voltage), where k and k' are constants.

The eddy e.m.f. induced by the flux ϕ_{se} is

$$e_{se} = - \frac{d\phi_{se}}{dt} = - k \, I_{max} \, \omega \cos (\omega t - \phi)$$

and $$i_{se} = - \frac{kI_{max}}{Z} \omega \cos (\omega t - \phi - \beta)$$

where Z is the eddy path impedance and β its phase angle. Also

$$e_{sh} = - \frac{d\phi_{sh}}{dt} = - k' E_{max} \sin \omega t$$

and $$i_{sh} = - \frac{k' E_{max}}{Z} \sin (\omega t - \beta)$$

The instantaneous torque on the disc is then

$$\frac{kk'}{Z} E_{max} I_{max} [\cos \omega t \cos (\omega t - \phi - \beta) + \sin (\omega t - \phi) \sin (\omega t - \beta)] \quad (20.14)$$

The mean torque upon the disc is

$$T_M = \frac{kk'}{Z} E_{max} I_{max} \tfrac{1}{2} [\cos (\phi + \beta) + \cos (\phi - \beta)]$$

$$= \frac{kk'}{Z} E_{max} I_{max} \cos \beta \cos \phi$$

$$= \frac{2kk'}{Z} \cos \beta \, . \, EI \cos \phi \quad . \quad . \quad . \quad . \quad (20.15)$$

where E and I are r.m.s. values of voltage and current. Hence

$$T_M \propto \text{the power in the circuit}$$

If the sine and cosine terms in Equation (20.14) are expanded it will be found that the term in the brackets reduces to $\cos \beta \cos \phi$, the terms in ωt cancelling out. The torque on the disc does not therefore fluctuate cyclically.

Comparison with Dynamometer Wattmeters. It appears from the above that the torque is independent of frequency. Actually, however, the torque is not quite independent of frequency, the above theory being simplified by assumptions, regarding the inductances of the various parts of the instrument, which are not justifiable under all conditions. It will, however, be sufficient to indicate the working principles of the instrument.

Since the deflecting torque is directly proportional to the power

in the circuit, and the instrument is spring-controlled, the scale is uniform.

Compared with dynamometer wattmeters, these instruments have the advantages of a greater working torque and length of scale; but they suffer from the inherent disadvantages of less accuracy, greater weight of moving system, greater power consumption, and also that they can only be applied to power measurements in a.c. circuits.

Induction wattmeters are capable of industrial accuracy only at a stated frequency and temperature. The requirements of wattmeters of the various grades of accuracy and the permissible limits of error are stated in British Standard Specification No. 89, "Indicating Ammeters, Voltmeters, Wattmeters, etc." (1954).

Fig. 20.28 (*b*) shows the arrangement of the circuits in the Lipman type of induction wattmeter (Nalder Bros. and Thompson, Ltd.) which has a low VA consumption in its windings.

The voltage winding VV is wound on the main magnet core, which also carries coils FF for power-factor compensation, the resistor R being adjustable for calibration purposes to bring the operating fluxes, F_v and F_c, 90° out of phase with one another. Within the moving element, or rotor, D—an aluminium drum—is a fixed core, of cruciform shape, completing the magnetic circuit. The current coil ML is wound on one limb of this cruciform core. The voltage-coil flux passes horizontally through the rotor, while the flux due to the current coil acts on the rotor in a vertical direction.

(*c*) Electrostatic Wattmeter. This form of wattmeter cannot be regarded as a commercial instrument like the dynamometer and induction forms already discussed. It is, however, a very useful instrument for the measurement of small amounts of power, especially when the voltage is high and the power factor low. Its use for the measurement of dielectric power loss was considered in Chapter IV. It is also useful for the calibration, in the laboratory, of commercial forms of wattmeter and watt-hour meter.

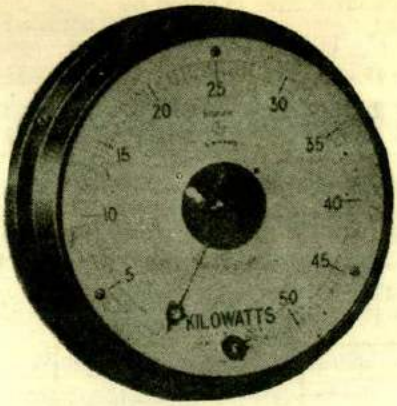
The electrostatic wattmeter consists of a quadrant electrometer used in conjunction with a non-inductive resistor, the essential connections to the load circuit being as shown in Fig. 20.29.

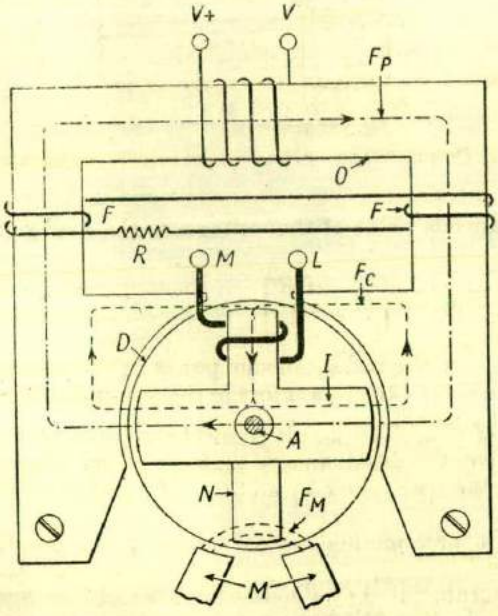
Let the instantaneous load current be i, and let v, v_1, and v_2 be the instantaneous voltages of the needle and of the two pairs of quadrants, as shown in the figure.

It was shown when dealing with the theory of the quadrant electrometer (page 684) that the instantaneous torque is given by

$$T \propto (v - v_1)^2 - (v - v_2)^2 \quad . \quad . \quad . \quad (20.16)$$

Now $(v - v_1)$ is the instantaneous value e of the supply voltage. Also $v - v_2 = (v - v_1) - (v_2 - v_1) = e - iR$, since the potential difference $v_2 - v_1 = iR$.





(*Salder Bros. & Thompson, Ltd.*)

FIG. 20.28. INDUCTION WATTMETER, LIPMAN TYPE

Hence the instantaneous torque is given by

$$T \propto e^2 - (e - iR)^2$$
$$\propto e^2 - e^2 + 2eiR - i^2R^2$$
$$\propto 2ciR - i^2R^2$$

or
$$T \propto 2R\left[ei - \frac{i^2R}{2}\right]. \quad . \quad . \quad . \quad . \quad (20.17)$$

i.e. $T \propto$ the instantaneous power supplied *minus* half the power lost in the non-inductive resistor

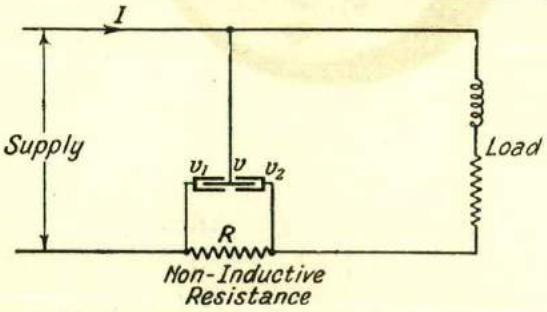


FIG. 20.29. CONNECTIONS OF ELECTROSTATIC WATTMETER

If e = instantaneous value of the voltage on the load side of the instrument,

$$T \propto 2R\left[ei + \frac{i^2R}{2}\right]. \quad . \quad . \quad . \quad . \quad (20.18)$$

$\propto$ the instantaneous power in the load *plus* half the power lost in the non-inductive resistor

Other Methods of Use. The simple connections given in Fig. 20.29 have, as seen above, the disadvantage that the power measured will be different from the true power by an amount equal to half the loss in the resistor R.

Fig. 20.30 gives three methods of use whereby this disadvantage may be overcome.

Method 1. Diagram (*a*) shows connections which are essentially the same as those in Fig. 20.29, except that a two-way switch S is used so that the needle of the electrometer may be connected either to one side of the load or the other, as required.

Now, from expression (20.18) for the instantaneous torque,

$$T \propto 2eiR + i^2R^2$$

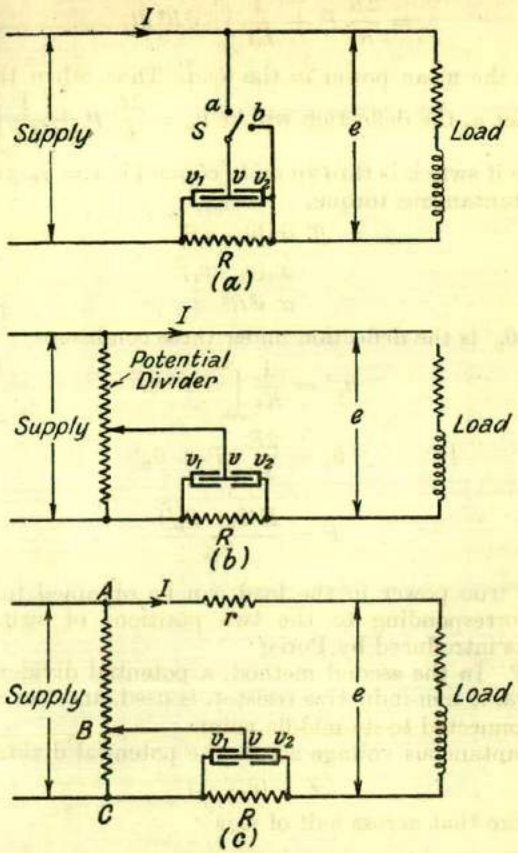


FIG. 20.30. ALTERNATIVE CONNECTIONS OF ELECTROSTATIC WATTMETER

If the controlling torque of the instrument is proportional to the deflection θ, we have, for the steady deflection θ_D of the instrument,

$$K\theta_D = \frac{1}{t'}\int_0^{t'} (2eiR + i^2R^2)\,dt \quad . \quad . \quad . \quad (20.19)$$

where the expression on the right-hand side* is that for the mean value of the torque, K is a constant, and t' is the periodic time of the voltage and current waves.

$$\therefore \theta_D = \frac{2R}{Kt'}\int_0^{t'} eidt + \frac{1}{Kt'}\int_0^{t} i^2R^2\,dt$$

* See p. 785.

$$= \frac{2R}{K} P + \frac{1}{Kt'} \int_0^{t'} i^2 R^2 \, dt$$

where P is the mean power in the load. Thus, when the switch S is on contact a, the deflection will be $\theta_D = \frac{2R}{K} P + \frac{1}{Kt'} \int_0^{t'} i^2 R^2 dt$.

Now, the if switch is thrown on to contact b, $v = v_2$, and we have, for the instantaneous torque,

$$\begin{aligned} T &\propto (v - v_1)^2 \\ &\propto (v_2 - v_1)^2 \\ &\propto i^2 R^2 \end{aligned}$$

Thus, if θ_D' is the deflection under these conditions,

$$\theta_D' = \frac{1}{Kt'} \int_0^{t'} i^2 R^2 \, dt$$

$$\therefore \theta_D = \frac{2R}{K} \cdot P + \theta_D'$$

or

$$P = \frac{K(\theta_D - \theta_D')}{2R} \quad . \quad . \quad . \quad (20.20)$$

Hence the true power in the load can be obtained from the two readings corresponding to the two positions of switch S. This method was introduced by Potier.

Method 2. In the second method, a potential divider, consisting of a high-value non-inductive resistor, is used, and the electrometer needle is connected to its middle point.

The instantaneous voltage across the potential divider is

$$e + (v_2 - v_1)$$

and therefore that across half of it is

$$\frac{e + (v_2 - v_1)}{2}$$

Thus the instantaneous torque is given by

$$\begin{aligned} T &\propto (v - v_1)^2 - (v - v_2)^2 \\ &\propto \left[\frac{e + (v_2 - v_1)}{2}\right]^2 - \left[\frac{e - (v_2 - v_1)}{2}\right]^2 \\ &\propto \tfrac{1}{4} [\{e + (v_2 - v_1) + e - (v_2 - v_1)\} \\ &\qquad \{e + (v_2 - v_1) - e + (v_2 - v_1)\}] \\ &\propto \tfrac{1}{4} [2e] [2(v_2 - v_1)] \\ &\propto e(v_2 - v_1) \\ &\propto eiR \\ &\propto ei \end{aligned}$$

Thus with these connections the instrument measures the true power in the load.

Method 3. If the use of the middle point of the potential divider as the point of connection of the needle is not feasible (possibly owing to the high voltage which would thereby be applied to the instrument), the method of connection shown in diagram (*c*) and suggested by Prof. Miles Walker, may be used.

In this method a non-inductive resistor r is connected in series with the load, and the needle of the electrometer is connnected to a suitable point on the potential divider.

Let $\dfrac{\text{Voltage } A \text{ to } C}{\text{Voltage } B \text{ to } C} = n$

Then, instantaneous torque is given by

$$T \propto (v - v_1)^2 - (v - v_2)^2$$

Now $(v - v_1) = \dfrac{e + ir + iR}{n}$

and $(v - v_2) = v - v_1 - (v_2 - v_1) = v - v_1 - iR = \dfrac{e + ir + iR}{n} - iR$

$$\therefore T \propto \left(\frac{e + ir + iR}{n}\right)^2 - \left(\frac{e + ir + iR}{n} - iR\right)^2 \quad . \quad (20.21)$$

$$\propto 2iR\left(\frac{e + ir + iR}{n}\right) - i^2R^2$$

or $$T \propto \frac{2iRe}{n} + \frac{2i^2rR}{n} + \frac{2i^2R^2 - ni^2R^2}{n} \quad . \quad . \quad . \quad (20.22)$$

Now, if the value of r is made equal to $\dfrac{R}{2}(n - 2)$, we have, by substitution,

$$T \propto \frac{2iRe}{n} + 2i^2\frac{R^2(n-2)}{2n} + \frac{i^2R^2(2-n)}{n} \quad (20.23)$$

$$\propto \frac{2iRe}{n}$$

$$\propto ie$$

$\propto$ true power in the load

It should be noted that in the above discussions the current taken by the electrometer (as a capacitor) is neglected. The potential divider also, although spoken of as a non-inductive resistor, may be the high-voltage winding of the supply transformer, from which tappings are brought out.

Use for Calibration of Commercial Wattmeters or Watt-hour Meters. Fig. 20.31 shows the connections for the calibration of a commercial

instrument by means of the electrostatic wattmeter. The load is, in this case, fictitious. The voltage for the voltage coil of the wattmeter under test is supplied from one phase of the secondary of a phase-shifting transformer, while the current coil of the instrument is fed from one phase of the supply through a step-down transformer. The phase-shifting transformer, which was described on p. 360, requires a three-phase supply. Any required phase displacement between the current and voltage in the wattmeter circuit can be obtained by turning the rotor of the phase-shifting

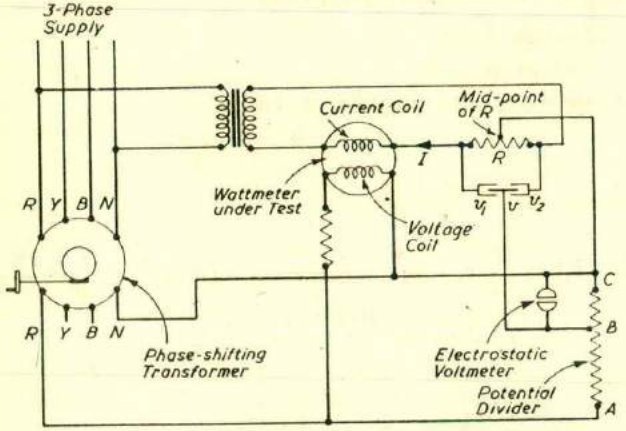


FIG. 20.31. USE OF ELECTROSTATIC WATTMETER FOR WATTMETER TESTING

transformer, by means of a worm and hand-wheel. In this way the apparent power factor of the load is altered.

The needle of the electrostatic instrument is supplied from a tapping point on a potential divider across the voltage coil of the wattmeter. The two pairs of quadrants are connected to the two ends of a non-inductive resistor R, through which the load current flows. The mid-point of this resistor is connected to one end of the potential divider, as shown.

Let $$\frac{\text{Voltage } A \text{ to } C}{\text{Voltage } B \text{ to } C} = n$$

Then, if e is the instantaneous value of the load voltage, the instantaneous value of the voltage B to C is $\frac{e}{n}$, and hence

$$v - v_2 = \frac{e}{n} - \tfrac{1}{2} iR$$

where i is the instantaneous value of the load current, and

$$v - v_1 = \frac{e}{n} + \tfrac{1}{2} iR$$

Thus, the instantaneous torque is given by

$$T \propto (v - v_1)^2 - (v - v_2)^2$$

or
$$T \propto \left(\frac{e}{n} + \frac{iR}{2}\right)^2 - \left(\frac{e}{n} - \frac{iR}{2}\right)^2 \quad . \quad . \quad . \quad (20.24)$$

$$\propto \left(\frac{e}{n} + \frac{iR}{2} + \frac{e}{n} - \frac{iR}{2}\right)\left(\frac{e}{n} + \frac{iR}{2} - \frac{e}{n} + \frac{iR}{2}\right)$$

$$\propto \left(\frac{2e}{n}\right)(iR)$$

$$\propto ei$$

$\propto$ true instantaneous power in the load.

The mean value of the torque is

$$T_m \propto \frac{2R}{nt'} \int_0^{t'} eidt$$

$$\propto \frac{2R}{n} \text{ (mean power in the load)}$$

t' being the periodic time of the current and voltage waves.

If θ_D is the steady deflection of the electrostatic instrument, which is spring-controlled, we have

$$k\theta_D = T_m \text{ where } k \text{ is a constant}$$

or
$$\text{Mean power} = \frac{nk'\theta_D}{2R}$$

where k' is the constant of the electrostatic instrument, and must be determined experimentally by the use of standard resistors and an electrostatic voltmeter.

F. R. Axworthy and J. K. Choudhury (Ref. (17)) have discussed the use of the electrostatic wattmeter, particularly for iron-loss measurements with mixed frequency inductions, and to measure the power absorbed by fluorescent lamps. They describe the construction of a portable electrostatic wattmeter of the quadrant type with a self-contained lamp and scale.

Polyphase Wattmeters. In order that the power in a polyphase circuit may be measured without the use of more than one wattmeter, polyphase wattmeters have been developed. These consist of two separate wattmeter movements mounted together in one case with the two moving coils (assuming the instrument to be of the dynamometer type) mounted on the same spindle, so that the total deflecting torque acting on the moving system will be the sum of the torques produced by the two component wattmeter

working systems. The readings of the instrument thus give the total power in the circuit directly, the addition being carried out by the instrument itself.

Thus, for example, in measuring three-phase power, the connections to the polyphase wattmeter are the same as those for the measurement of power by the two-wattmeter method, using two single-phase instruments. The only difference is that the two single-phase instruments are combined in the polyphase wattmeter and

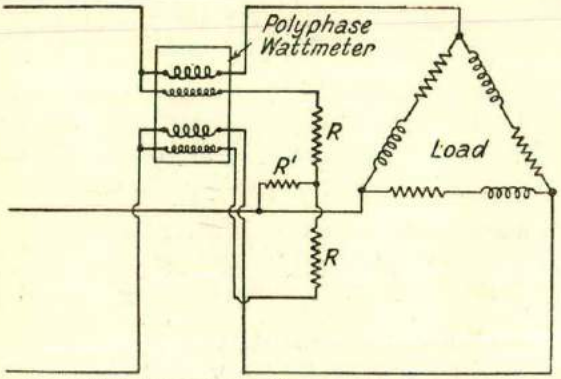


FIG. 20.32. POLYPHASE WATTMETER CONNECTIONS

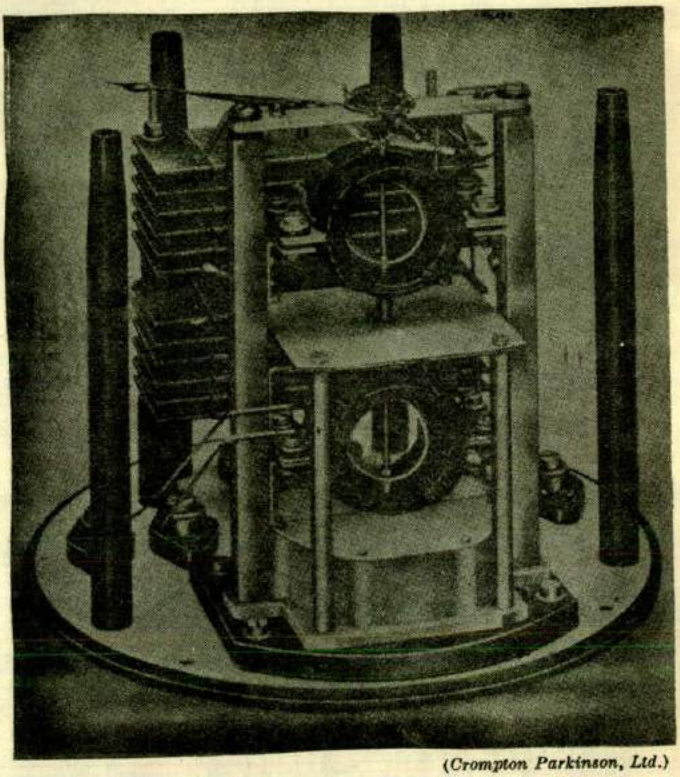
operate a single moving system whose deflection gives the total power directly.

An important point in connection with such double-element wattmeters is that there shall be no mutual interference between the two elements. Thus, the fixed coils of one of the elements must not produce a torque by interacting with the field of the moving coil of the other element. In order to eliminate any such action and ensure that the total torque shall be merely the sum of the torques produced within the two component elements themselves, a laminated iron shield is placed between the two elements to provide magnetic screening.

Compensation for mutual interference can be made by a method due to Weston, the connections for which are shown in Fig. 20.32. Instead of connecting the two voltage coils—each of resistance R (including their series resistances)—directly to the line which does not contain a current coil, they are each connected to one end of a resistor R' whose other end is connected to the line, as shown. By suitably adjusting R' the currents in the voltage-coil circuits can be altered to compensate for any mutual interference between the wattmeter elements (Ref. (2)).

Fig. 20.33 illustrates the internal construction of a commercial type of double-element wattmeter. A nickel-iron plate shields the two

elements from one another. To reduce the weight of the moving system, aluminium is used for many of the parts. The total weight



(Crompton Parkinson, Ltd.)

FIG. 20.33. DOUBLE-ELEMENT WATTMETER

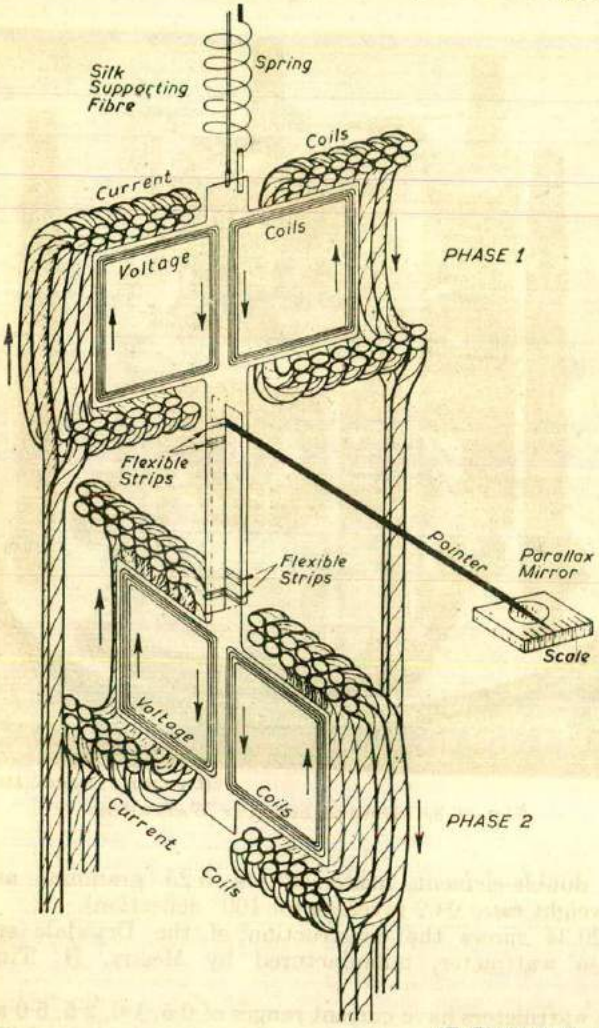
of the double-element movement is 5·25 grammes and the torque/weight ratio 0·09 (average for 100° deflection).

Fig. 20.34 shows the construction of the Drysdale standard polyphase wattmeter, manufactured by Messrs. H. Tinsley & Co.

These wattmeters have current ranges of 0·5, 1·0, 2·5, 5·0 amperes (the range can be extended to 2,400 A by the use of Class AL current transformers). Voltages up to 1,000 V can be used by employing voltage resistance boxes.

Summation Methods. It is often necessary to measure the total power in a number of separate circuits, and it is convenient to have this total indicated upon a single instrument. This is a simple

matter if the circuits are interconnected so that one portion of the system carries the whole load. If, however, there is no part of the



(*H. Tinsley & Co., Ltd.*)

FIG. 20.34. DRYSDALE STANDARD POLYPHASE WATTMETER

system carrying the aggregate load, more complicated methods of measurement must be employed.

A four-element wattmeter, which consists essentially of two polyphase wattmeters like those described in the preceding section,

with all four moving systems (two in each polyphase wattmeter) mounted on a common spindle, can be used for the measurement of the total power in two three-phase systems, but for more than two systems current transformers are used in conjunction with a double-element "summation" wattmeter.

The principle of such summation measurements is illustrated in Fig. 20.35, in which the total power supplied to three circuits A, B

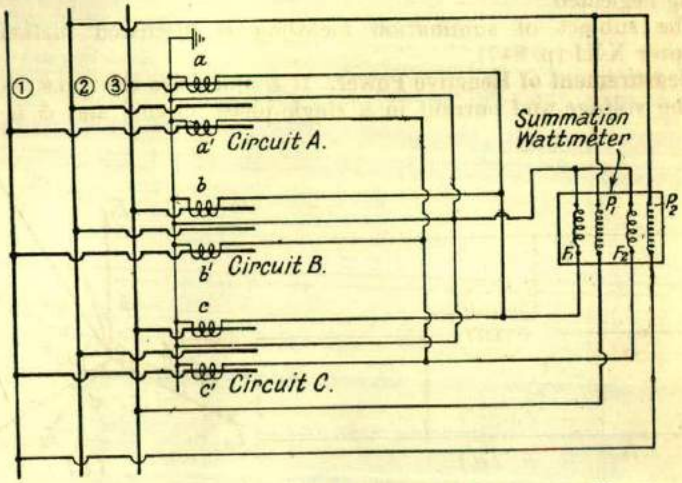


FIG. 20.35. CONNECTIONS FOR SUMMATION METHOD OF POWER MEASUREMENT

and C is measured by a two-element summation wattmeter, in conjunction with six current transformers, all of whose ratios are the same. Actually voltage transformers may also be necessary if the line voltage is too great to be applied directly to the voltage coils of the wattmeter, but these have been omitted for simplicity.

It will be observed that the principle of measurement is essentially that of the two-wattmeter method of measurement of three-phase power.

Current transformers a, b, and c, together supply current coil F_1, while transformers a', b', and c' supply current coil F_2. Neglecting the small phase angles of the current transformers, the currents in their secondary circuits are each 180° out of phase with their primary currents, and hence the vector sum of the former will be equal to the vector sum of the primary currents divided by the ratio of the transformer (the same in all cases), the ratio errors of the transformers also being neglected. It can be seen, therefore, that the current supplied to current coil F_1 is proportional to the vector sum of the currents supplied to the three circuits A, B, and

C by line (3), and the current supplied to current coil F_2 is proportional to the vector sum of the currents supplied by line (1).

The voltage coils P_1 and P_2 are connected between lines (3) and (2) and between (1) and (2) respectively.

The reading of the summation wattmeter is thus equal to the total power supplied to the three circuits A, B, and C, divided by the ratio of the current transformers, the current transformer errors being neglected.

The subject of summation metering is discussed further in Chapter XXI (p. 847).

Measurement of Reactive Power. If E and I are the r.m.s. values of the voltage and current in a single-phase circuit and ϕ is the

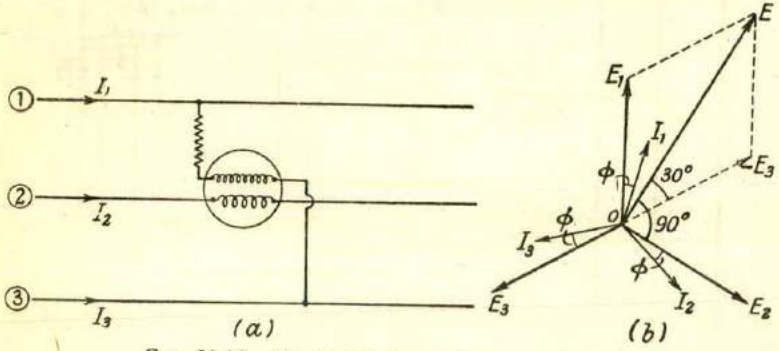


FIG. 20.36. MEASUREMENT OF REACTIVE POWER

phase angle between them, then the actual, or active, power in the circuit is, of course, $EI \cos \phi$.

The active power is equal to the voltage multiplied by the component of the current which is in the phase with it (i.e. $I \cos \phi$). The component of the current which is 90° out of phase with the voltage is $I \sin \phi$, and the product $EI \sin \phi$ is the "wattless" or "reactive" component of the volt-amperes, or the reactive power. The measurement of this reactive component is, however, useful, since the phase angle ϕ of the circuit can be obtained from the ratio $\frac{\text{Reactive power}}{\text{Active power}}$, which equals $\frac{EI \sin \phi}{EI \cos \phi}$, or $\tan \phi$.

It should first be observed that $\sin \phi = \cos (90° - \phi)$, and therefore a wattmeter may be used for the measurement if the current coil carries the load current I and the voltage applied to the voltage coil is such that its phase displacement from the actual voltage of the circuit is 90°. Under these circumstances the wattmeter will read $IE \cos (90° - \phi)$, or $IE \sin \phi$.

It may be more convenient in single-phase measurements to compensate the wattmeter so that the field of its voltage coil lags

90° behind the phase it would have if the wattmeter voltage were used for power measurement.

In the case of a balanced three-phase load when the power is being measured by one wattmeter, it is a simple matter to use this wattmeter for the measurement of the reactive power by connecting its current and voltage coils as shown in Fig. 20.36. The current coil is connected in one line and the voltage coil is connected across the two other lines.

Referring to the vector diagram of Fig. 20.36 (*b*), the vector OI_2 represents the current through the wattmeter current coil. The voltage applied to the voltage coil is the vector difference of OE_1 and OE_3, i.e. the vector OE. Now, the angle between OE_2 and OE_3 reversed is 60°, and since the three phase voltages E_1, E_2, and E_3 are equal, the angle between OE_3 reversed and vector OE is 30°. Hence, the angle between OE_2 and OE is 90°, and the angle between OI_2 and OE is $(90° + \phi)$. This means that the wattmeter reads

$$\begin{aligned} OE \times OI_2 \times \cos(90° + \phi) &= \sqrt{3}\,EI \cos(90° + \phi) \\ &= -\sqrt{3}EI \sin \phi \\ &= -W_r \end{aligned}$$

where E is the phase voltage and I the line current of the system.

The total reactive power of the circuit is

$$\sqrt{3}EI \sin \phi = -\sqrt{3}\,W_r$$

and hence, if W is the measured value of the total active power, the phase angle ϕ of the load is

$$\tan^{-1}\left(\frac{\sqrt{3}\,W_r}{W}\right)$$

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CHAPTER XXI

THE MEASUREMENT OF ENERGY

It may be stated generally that the measurement of energy is essentially the same process as the measurement of power, except that the instrument used must not merely indicate the power, or rate of supply of energy, but must take into account also the length of time for which this rate of supply is continued. Actually, energy, or "supply", meters do not indicate power directly. For a given amount of energy supplied to a circuit, their registrations should always be the same, no matter what the instantaneous values of the power during the time in which the energy is supplied.

Types of Energy, or Supply, Meters. There are three general types of energy, or supply, meters—

(*a*) Electrolytic meters.
(*b*) Motor meters.
(*c*) Clock meters.

The first type is essentially a d.c. instrument although it can be made to function in an a.c. circuit for the measurement of kilovolt-ampere-hours by using a rectifier unit and a current transformer (see Fig. 21.1); the two other types may be used on either d.c. or a.c., according to their construction. The induction form of motor meter can, of course, only be used on a.c. circuits, since its principle is the same as that of the induction wattmeter. As already mentioned in Chapter XVII, supply meters used on d.c. circuits may be either ampere-hour, or watt-hour, meters. In the former case, the registrations of the meter are converted to watt-hours by multiplying by the voltage (assumed constant) of the circuit in which it is used. Usually such meters are calibrated to read direct in kilowatt-hours at the declared voltage, thus rendering the readings incorrect when used on any other voltage.

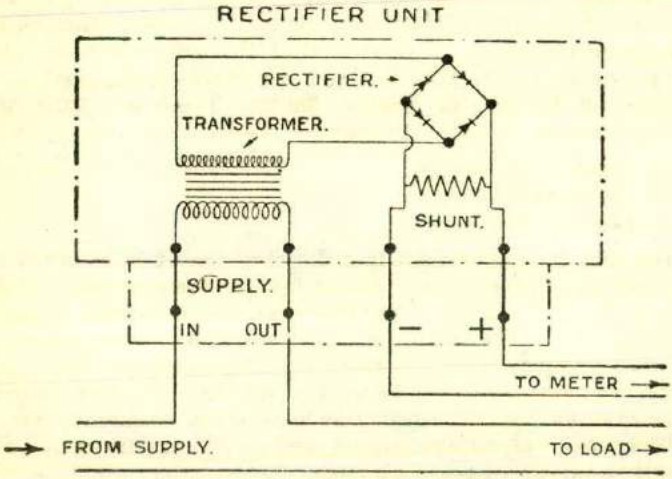
In the same way the question of power factor obviously prevents the use of such ampere-hour meters on a.c.

The advantages of simplicity and cheapness in the case of ampere-hour meters is largely discounted by the fact that variations of circuit voltage are not taken into account by them. For example, suppose that the voltage of a supply, whose nominal voltage is 220, has an average value of only 215 volts for a period of 1 hour, during which a current of 100 amp is being taken by a consumer. Then, if an ampere-hour meter, calibrated for 220 volts, is used to measure the energy supplied to the consumer, the measured quantity of energy will be 220 × 100 watt-hours, or 22 kilowatt-hours; and it is for this amount that the consumer will be charged.

Actually, however, the energy supplied is only 215 × 100 watt-hours, or 21½ kilowatt-hours, so that the consumer loses the cost of ½ kWh in one hour under these conditions.

A watt-hour meter would have taken into account the fall in the supply voltage and would therefore have meant a saving in cost to the consumer. The converse of the above occurs, of course, if the supply voltage is higher than the nominal, when the supply undertaking loses by the use of an ampere-hour meter.

Electrolytic Meters. These are inherently ampere-hour meters, since their readings are proportional to the weight of metal deposited,



(*Reason Manufacturing Co., Ltd.*)

FIG. 21.1. REASON RECTIFIER UNIT FOR USE WITH ELECTROLYTIC METERS

or of gas liberated, from an electrolytic solution. This means that the readings are merely proportional to the number of coulombs, or of ampere-hours, passed through the meter.

In addition to cheapness and simplicity these meters have the advantages that they are accurate even at very small loads; they are unaffected by stray magnetic fields, since they do not depend in any way upon the magnetic effect of the current; and, since there are no moving parts, friction errors are absent. Their general disadvantages are that the potential drop across their terminals is from 1 to 2 volts; the considerable amount of glass used in their construction renders them somewhat fragile; the necessity for frequent inspection to ensure that they do not stand in need of refilling or resetting; and the destruction of the old record of energy supplied when the meter is reset.

For an electrolytic meter to be shunted successfully, the back e.m.f. of polarization must be small.

This can be seen from the following—

Let the back e.m.f. of the meter = e, the resistance of the meter circuit = r, and the resistance of the shunt = R_s. Then, referring to Fig. 21.2,

I = the load current

I_m = the meter current

I_s = the current through the shunt

$$I_s R_s = I_m r + e$$

But $$I = I_s + I_m$$

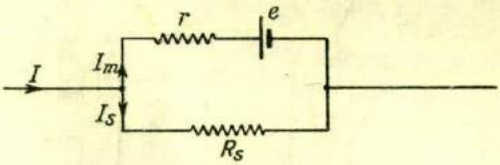


FIG. 21.2

$$\therefore (I - I_m) R_s = I_m r + e$$

$$IR_s = I_m(r + R_s) + e$$

$$I_m = \frac{IR_s}{(r + R_s)} - \frac{e}{(r + R_s)}$$

or

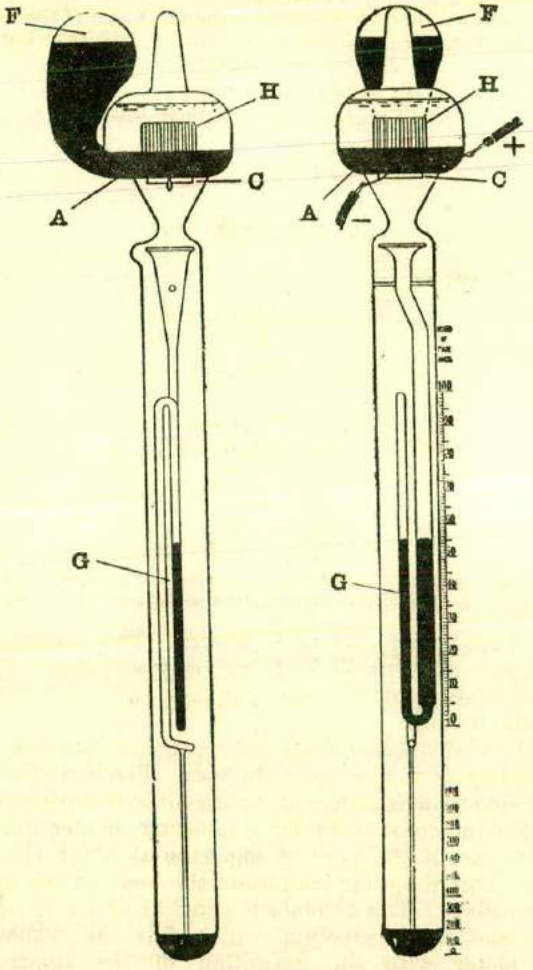
$$\frac{I_m}{I} = \frac{R_s}{r + R_s} - \frac{e}{I(r + R_s)} \quad . \quad . \quad . \quad (21.1)$$

Thus it can be seen that, if e is constant, the meter current is not a constant proportion of the load current, but is reduced as I is reduced.

THE WRIGHT, OR REASON, METER. The construction of this meter is shown in Fig. 21.3. It has been very generally used on d.c. supply systems, and has been perhaps the most successful of all electrolytic meters.

The anode A consists of an annular ring of mercury, contained in a shallow trough at the top of the tube. The loss of mercury by the anode, owing to its transference to the cathode during the electrolytic action, is compensated for by a reservoir of mercury F projecting from the side of the part of the tube at which the anode ring is situated. This reservoir maintains the level of the mercury in the anode constant. The cathode C consists of a ring of sand-blasted iridium, and the electrolyte, which fills the whole of the tube of the meter with the exception of the space occupied by mercury, is a saturated solution of mercury and potassium iodides. The tube is hermetically sealed and external conditions have therefore no effect upon the action of the meter. Standing up from the inner circumference of the annular anode trough is a grid or fence of glass rods H to prevent mercury from the anode spilling over into the measuring tube G, owing to mechanical vibration. This fence does not, however, interfere with the full circulation of

the electrolyte. The measuring tube has a funnel-shaped mouth, and is situated immediately below the cathode ring in order to catch the drops of mercury which fall from the latter after being



(Reason Manufacturing Co., Ltd.)

FIG. 21.3. REASON ELECTROLYTIC METER

deposited from the electrolyte by the action of the current. Alongside this measuring tube is a scale calibrated in kilowatt-hours, and corresponding, of course, to the voltage of the circuit in which the meter is to be used. The measuring tube is bent back on itself twice

so that, upon the mercury in it reaching a certain level, it siphons over into a larger measuring tube below. This siphoning occurs after the passage of 100 units through the meter, and the lower (and larger) measuring tube is thus graduated in hundreds of units. The electrolytic action is as follows. The current is led into and out of the meter by platinum wire leads, one dipping into the mercury anode and the other being welded to the iridium cathode. The passage of the current results in a chemical action which removes mercury from the anode and deposits an equal quantity of mercury upon the cathode, the electrolyte itself being left unchanged. The weight of mercury so deposited is obviously directly proportional to the quantity of electricity passed through the meter; and this results in a strictly uniform scale (assuming uniformity of the sectional area of the measuring tube). The meter is reset by inverting it and allowing the mercury to run back into the upper receptacle.

This type of meter is almost always shunted in order to increase its capacity. Its polarization back e.m.f. is only about 0·0001 volt, and thus does not introduce any appreciable error at low loads due to departure from the correct division of current between meter and shunt. Since the meter has a negative temperature coefficient, a resistor having a fairly large positive temperature coefficient is connected in series with the meter before the connections are made to the shunt, which is usually of manganin and has therefore a very small temperature coefficient. The series resistor takes the form of a coil of wire, part of which is tinned iron and the remainder constantan, and its resistance is adjusted so as to maintain the total resistance of itself and the meter together as nearly constant as possible under all temperature conditions.

Motor Meters. These meters may be for use in direct-current or alternating-current circuits; and in the former case they may be either ampere-hour or watt-hour meters. In this class of meter the moving system is allowed to revolve continuously instead of being allowed merely to rotate through a fraction of one revolution as in an indicating instrument. The speed of revolution is proportional to the current in the circuit in the case of an ampere-hour meter; and to the power in the circuit in the case of a watt-hour meter. It follows, therefore, that the number of revolutions made by the revolving portion in any given time is proportional, in the ampere-hour meter, to the quantity of electricity supplied in that time; and in the case of the watt-hour meter, to the energy supplied. The number of revolutions made by the meter is recorded by a counting mechanism consisting of a train of wheels, to which the spindle of the rotating system is geared.

The control of speed is brought about by a permanent magnet (brake magnet), so placed that it induces currents in some part of the rotating system, these currents producing a retarding torque proportional to their magnitude, which latter is proportional to

the speed of the rotating system. This system attains a steady speed when the retarding torque exactly balances the driving torque produced by the current or power in the circuit.

ERRORS IN MOTOR METERS. The two principal errors common to all motor meters are friction errors and braking errors. The friction error is considerably more important than the corresponding error in most indicating instruments, since it is continuously operative and affects the speed of the rotor for any given value of current or power. The braking action in these meters corresponds to the damping in an indicating instrument. The braking torque in an integrating meter directly affects the speed, for a given driving torque, and hence affects the number of revolutions made in a given time.

Friction forces which exist when the rotor is just starting to revolve (static friction) may prevent it from starting at all if the load is small, and will cause its registration to be low at small loads. This part of the friction torque may be assumed to remain constant when the moving part of the meter is rotating, and may be compensated for by arranging for a small constant driving torque to be applied to the moving system independent of the load. When the meter is running normally, a friction torque is exerted which is proportional to the speed, but this is not of great importance, since it merely adds to the braking action.

In some meters, however, such as those of the mercury motor type, a friction torque proportional to the square of the speed exists, and has to be compensated for. Since the friction torque is proportional to the load on the bearings the weight of the rotating system should be as small as possible.

As regards errors due to variation in the braking action, it can be seen that the steady speed of the meter is such that the braking torque —proportional to the speed—is equal to the driving torque. The braking torque is also proportional to the strength of the brake magnet. Employing symbols, we have

$$T_B \propto \phi i$$

where T_B = braking torque
ϕ = flux of brake magnet
i = current induced by the rotation of the moving system in the field of the brake magnet

Let e = induced voltage
r = resistance of the path of the current i in the rotating disc

The induced voltage is $e \propto \phi n$

where n is the speed of the revolving part of the meter.

$$\therefore T_B \propto \frac{\phi^2 n}{r} \quad . \quad . \quad . \quad . \quad (21.2)$$

This braking torque equals the constant driving torque T_D when

a steady speed of revolution is attained. Thus, if N is the steady speed of the meter,

$$T_B' \propto \frac{\phi^2 N}{r}$$

and

$$T_B' = T_D$$

where T_B' is the braking torque at speed N.

$$\therefore T_D \propto \frac{\phi N}{r}$$

or

$$N \propto \frac{r}{\phi^2} . T_D \quad . \quad . \quad . \quad . \quad (21.3)$$

Hence the steady speed attained by the meter for a constant driving torque T_D is directly proportional to the resistance of the path of the induced (or eddy) current, and inversely proportional to the square of the flux of the brake magnets.

It will be realized from this how very important it is that the strength of the brake magnet shall remain constant throughout the time that the meter is in service. Careful design and treatment during manufacture are necessary to ensure this constancy. In general, the braking torque will be reduced by increase of temperature, since this will increase the resistance r. It is somewhat difficult to compensate completely for this reduction, but in some meters the driving torque is also reduced by increase of temperature, and this brings about automatic partial compensation.

In one ampere-hour meter (see Ref. (23)) two very small, circular, cobalt-steel magnets are used, mounted in opposite polarity. A magnetic shunt is provided, the permeability of which varies with temperature so that an increase of speed of the meter-disc of only 1 per cent is caused by a change of temperature of 30° F.

Motor Meters for D.C. Circuits. There are two classes of such meters—

(*a*) Mercury motor meters.

(*b*) Commutator motor meters.

The most important difference between the two types is in the method of leading current into the armature, or rotating part, of the meter. In type (*a*), the armature usually consists of a thin metal disc rotating in a bath of mercury, this mercury being used to lead the current into and out of the disc instead of the commutator as used in type (*b*).

(*a*) *Mercury Motor Meters.* (i) *Ampere-hour Type.* The operation of the mercury Ah-meter which has been commonly used for d.c. circuits may be understood by referring to Fig. 21.4. The copper disc is immersed in a bath of mercury, and the current is led into the disc, through the mercury, at its circumference on the right-hand side. The current flows radially to the centre, whence it passes out

through the spindle via the mercury to the conducting surround of the jewel bearing, to which external contact is made.

A torque is produced owing to the presence of this current in the field of the right-hand magnet, and the disc rotates as shown in Fig. 21.4. In its rotation the disc cuts through the field of the

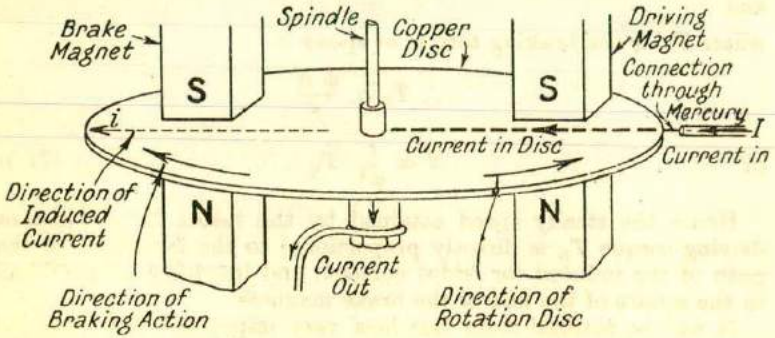


FIG. 21.4. ILLUSTRATING THE PRINCIPLE OF THE D.C. AMPERE-HOUR METER

left-hand magnet and an eddy current is induced in it which results in a braking torque as shown, this torque controlling the speed of rotation of the disc.

As previously shown, the speed is

$$N \propto \frac{r}{\phi^2} . T_D$$

where T_D is the driving torque.

Now, obviously, $T_D \propto$ the current, I

Hence, if r and ϕ are constant,

$$N \propto I$$

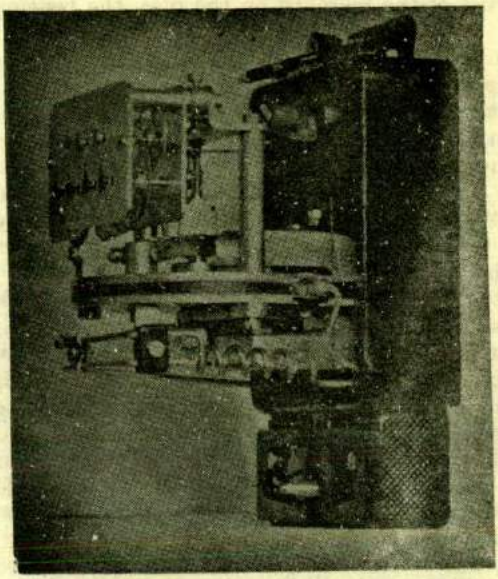
i.e. the speed of the disc is proportional to the current. Thus, the number of revolutions in a given time will be proportional to the quantity of electricity passed in that time, i.e. proportional to $\int I dt$.

In modern instruments it is customary to use a single magnet which provides both the braking and driving torques.

A typical mercury ampere-hour meter, made by Ferranti, Ltd., is shown in Fig. 21.5.

There is a single anisotropic, high-energy, permanent magnet which provides both driving and braking torques. The magnet has iron pole pieces fitting into circular brass plates between which is an insulating fibre ring of the same external diameter as the plates, which are faced with insulating material on their inner sides. The

plates and insulating ring together form a shallow circular box which contains a thin copper disc, the latter being the armature of the meter. The remainder of the space inside this chamber is filled with mercury, which exerts a considerable upthrust on the disc. The disc is mounted at the base of a spindle, pivoted and working in jewelled cup bearings, and the upthrust on the disc due to the mercury is



(*Ferranti, Ltd.*)

FIG. 21.5. FERRANTI D.C. AMPERE-HOUR METER, TYPE FKT

balanced by means of a cylindrical weight on the spindle. The upper part of the spindle has a worm cut on it to engage in the gear wheels of the recording train.

The current enters the meter at the centre contact of the mercury bath, flows through the mercury and disc and leaves at the side contact.

The speed of the disc is adjusted by means of a contact bridge sliding over two wires, thereby changing the resistance and the meter bath current. The range of adjustment is between 20 and 25 per cent.

No fluid friction compensation is necessary in this meter because of the low full-load speed of 18·33 r.p.m.

The rate of registration of the meter element increases with increase in temperature, and meters for small current ratings are compensated by surrounding the lower pole piece of the magnet

with a nickel-iron alloy. The permeability of this alloy decreases when the temperature increases, and there is therefore a change in effective pole area with temperature. Meters for large current ratings are provided with shunts, and when the greater part of the current is shunted the overall temperature coefficient is small in any case.

No compensation is necessary for bearing friction, since, owing to the upthrust of the mercury on the disc, the bearing pressure is very small.

The Chamberlain and Hookham ampere-hour mercury meter, has the same principle as that of the Ferranti meter.

Compensation for fluid friction is provided by an iron bar, wound with a coil in series with the load, the bar being in parallel with the air gap of the permanent magnet. The effect is to reduce the flux of the latter on heavy loads, and since the braking torque is proportional to the square of the flux, while the driving torque is directly proportional to the flux, the result is an increase in the driving torque relative to the braking torque.

(ii) *Watt-hour Mercury Meters.* The construction of the Chamberlain and Hookham type of mercury watt-hour meter is shown in **Fig. 21.6.** In this meter there is an electromagnet K which carries two magnetizing coils connected—in series with a resistor—across the supply mains, so that the current through them is proportional to the voltage. This magnet is situated under the mercury chamber containing the aluminium disc-armature A. Above this disc is an iron ring Q which completes the magnetic circuit of the electromagnet and causes its magnetic field to be perpendicular to the disc. This disc has radial slots cut in it in order to ensure radial flow of the current in it, this current being led into and out of the disc through mercury contacts at diametrically opposite points. These slots prevent the same disc being used for braking purposes, so that another aluminium disc O is used—mounted on the same spindle—in conjunction with a permanent magnet for this purpose.

Within the small limits of variation of voltage to be expected on an ordinary supply system, the flux of the electromagnet may be assumed to be strictly proportional to the current through the magnetizing coils, i.e. to the voltage. Hence, since the torque driving the armature is proportional to the product of the current through it and the flux of the electromagnet, this torque is directly proportional to the power in the circuit. The braking torque is obviously proportional merely to the speed of the armature, and so the steady speed of rotation is proportional to the watts, and the number of revolutions to the watt-hours supplied—i.e. to $\int EI dt$.

Compensation for fluid friction at high speeds of revolution is provided by taking one or two turns of the current lead round the poles of the electromagnet, so that its field is strengthened thereby when the load is heavy.

In calibrating meters of the above type, there are several adjustments which can be made. Adjustable magnetic shunts are often fitted in the case of ampere-hour meters, so that the field strengths of the permanent magnets may be varied. The calibration of watt-hour meters may be varied by adjustment of the position of the

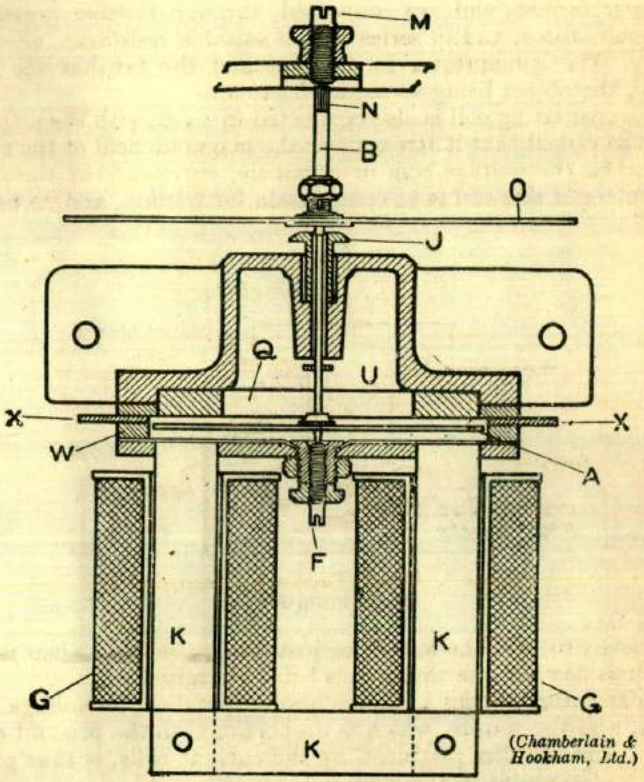


FIG. 21.6. CHAMBERLAIN AND HOOKHAM MERCURY WATT-HOUR METER

brake magnet to give a greater or less braking torque and also by variation of the position of the electromagnet.

Large alterations of the calibration can be made by varying the gear ratios in the recording train, a large number of spare gear wheels being usually stocked in meter-testing laboratories for this purpose.

(*b*) *Commutator Meters.* Fig. 21.7 illustrates the principle of a commutator watt-hour meter, namely the Elihu-Thomson meter, although this type is now mainly of historical interest.

There are two fixed current coils, each consisting of a few turns of heavy copper strip. These produce a magnetic field whose strength is proportional to the load current; in this field rotates the armature, which carries a number of coils connected to the segments of a small commutator. The armature coils are wound upon a non-magnetic former, and are connected, through brushes pressing on the commutator, and in series with a suitable resistance, across the supply. The commutator is of silver and the brushes are silver-tipped, the object being to reduce friction.

A compensating coil is also connected in series with the armature, and is so placed that it strengthens the magnetic field of the current coils when the voltage-coil, or armature, current flows through it. The object of this coil is to compensate for friction, and its position

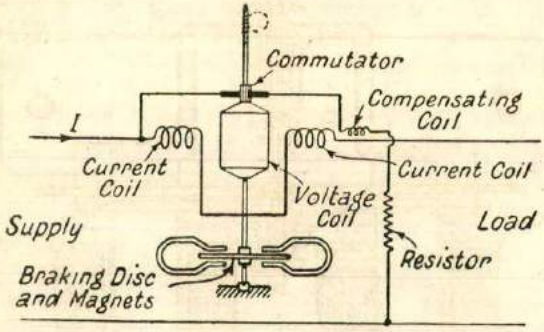


FIG. 21.7. ELIHU THOMSON COMMUTATOR WATT-HOUR METER

is adjusted so that the armature just fails to revolve when no load current is flowing, the shunt coils being energized.

The armature carries a current proportional to the voltage of the circuit, and the torque, which is proportional to the product of this current and the flux produced by the current coils, is thus proportional to the power in the load.

The braking torque is provided by an aluminium disc and two permanent magnets, as shown. As in the mercury meters, the braking torque is proportional to the speed of the disc. Hence, the steady speed attained by the revolving system of the meter is proportional to the watts in the load.

Commutator meters are far less commonly used than the mercury type. The advantages of the latter are greater simplicity in construction; smaller voltage drop across the meter; the capacity for carrying a considerably greater current without shunting; and also small starting friction, owing to the very small pressure on the bearings as a result of the upthrust of the mercury on the rotating system.

The Grassot Fluxmeter as a Quantity Meter. One of the authors has described (Ref. 25) the use of the Grassot fluxmeter for the measurement of a small fraction of an ampere-hour passing through a circuit in (say) a few seconds. In such a case neither an ampere-hour meter nor a ballistic galvanometer can be used successfully. The fluxmeter is used in conjunction with a suitable shunt in the current circuit as shown in Fig. 21.8. When the key is closed on contact b the fluxmeter is short-circuited and the movement is brought quickly to rest owing to the electromagnetic damping action.

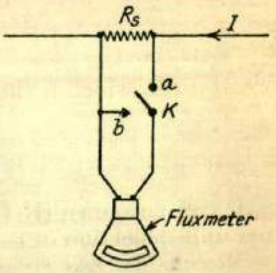


FIG. 21.8

At the beginning of the time interval T during which the quantity of electricity to be measured is passed through R_s, the key K is moved from b to a and is returned to b at the end of the interval. The quantity of electricity is determined by multiplying the fluxmeter deflection which ensues between the two movements of the key K by a constant whose value depends upon that of R_s and upon the constants of the fluxmeter itself.

THEORY. Let I = current in the main circuit; i = current in the fluxmeter circuit at any time during the measuring period; L_s = inductance of the shunt R_s; R and L = resistance and inductance of the fluxmeter circuit. e_s and e_f are as defined on page 385.

Then
$$e_s = R_s(I - i) + L_s\frac{d(I-i)}{dt}$$

$$e_f = K\frac{d\theta}{dt}$$

where K is the fluxmeter constant and $\frac{d\theta}{dt}$ the angular velocity of its coil.

Now,
$$e_s = e_f + L\frac{di}{dt} + Ri$$

or
$$R_s(I-i) + L_s\frac{d(I-i)}{dt} = K\frac{d\theta}{dt} + L\frac{di}{dt} + Ri$$

Since i is very small Ri is negligible and $I - i = I$ (very nearly).

$$\therefore R_sI + L_s\frac{dI}{dt} = K\frac{d\theta}{dt} + L\frac{di}{dt}$$

Integrating with respect to t we have

$$\int_0^T\left(R_sI + L_s\frac{dI}{dt}\right)dt = \int_0^T K\frac{d\theta}{dt}\,.\,dt + \int_0^T L\frac{di}{dt}\,.\,dt$$

$$\therefore R_s\int_0^T I\,dt + L_s\int_{I_1}^{I_2} dI = K\int_{\theta_1}^{\theta_2} d\theta + L\int_{i_1}^{i_2} di$$

I_2 and I_1 are final and initial values of the current in the main circuit, i_2 and i_1 (both zero) are corresponding values for the fluxmeter circuit, while θ_2 and

θ_1 are the final and initial values of the fluxmeter deflection. If L_s is small we may write

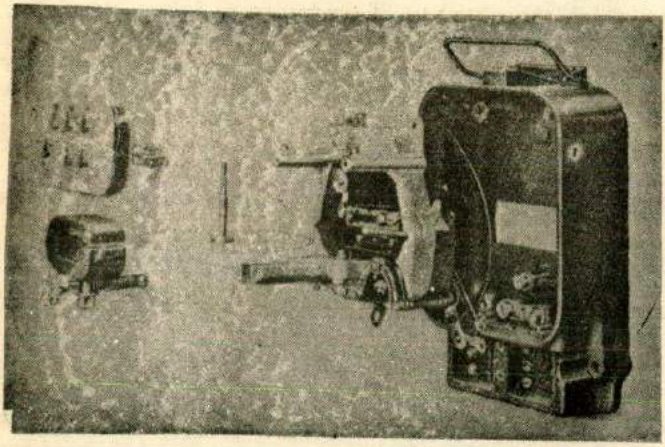
$$R_s \int_0^T i\,dt = K(\theta_2 - \theta_1)$$

or, $\int_0^T i\,dt$, which is the quantity to be measured, is given by

$$\frac{K}{R_s}(\theta_2 - \theta_1)$$

It can be shown (Ref. (25)) that K is the number of flux linkages per unit deflection of the fluxmeter.

Motor Meters for A.C. Circuits. For a.c. circuits the commutator type of meter could be used, but not the mercury type.



(*Associated Electrical Industries, Ltd.*)

Fig. 21.9. A.E.I. Single-phase Watt-hour Meter Type NF

The errors to which the commutator meter would be subject, if so used, would be the same as those of the dynamometer wattmeter, since their principles of operation are essentially the same.

Induction-type meters are, however, almost universally used for a.c. energy measurements, since they possess the definite advantages, as compared with the commutator type, of a higher torque/weight ratio, and the absence of a commutator with its accompanying friction. The induction type is therefore more accurate than the commutator type on light loads. The principle of these meters is almost exactly the same as that of the induction wattmeter. The construction also is very similar, except that the spring control and pointer of the wattmeter are replaced, in the energy meter, by a brake magnet which induces eddy currents in the disc (which now revolves, instead of rotating through only a fraction of a revolution

as in the wattmeter), and by a recording train of wheels driven by a worm on the spindle of the meter.

Single-phase Induction-type Watt-hour Meter. The construction of a typical meter is shown in Fig. 21.9, an exploded view being given for clarity. There is little difference in construction between this instrument and the induction wattmeter illustrated in Fig. 20.26 (previous chapter). The chief alterations are the provision of only one voltage coil, upon the centre limb of the shunt magnet, and only one copper shading band upon this limb.

In this meter the copper shading band is fixed in position and quadrature adjustment is effected by altering the resistance of the shading band. The ends of the shading band are brought out to a loop of resistance wire; the value of the resistance, and hence the quadrature adjustment, is altered by moving a shorting bar on the resistance loop. This adjustment can be seen on the right-hand side of the frame in Fig. 21.9. In addition, there is an adjustable copper loop beneath the pole of the shunt magnet embracing both gaps in the shunt magnet system, the purpose being to compensate for friction in the meter.

Fig. 21.10 shows the construction of the moving element of a single-phase watt-hour meter manufactured by Aron Electricity Meter, Ltd.

It should be observed that, since the disc is revolving continuously when on load, e.m.f.s will be induced in it *dynamically*, as it cuts through the flux of the two electromagnets, in addition to the *statically* induced e.m.f.s due to the alternating flux in these magnets. The full-load speed of rotation in most meters is, however, only about 40 r.p.m., and as this is small compared with the speed of alternation of the static flux (corresponding—say—to a frequency of 50 cycles per second), the torque corresponding to the eddy currents dynamically induced in the disc will be very small compared with the operating torque corresponding to the statically induced eddy currents.

Neglecting, for the present, friction in the meter, and assuming that the flux of the shunt magnet lags by exactly 90° in phase behind the applied voltage, we have, following the theory of the induction wattmeter,

$$\text{Operating (or driving) torque} \propto EI\cos\phi$$

where E and I are the r.m.s. values of voltage and current in the circuit respectively, and ϕ is the phase angle between them. This torque is thus proportional to the power in the circuit.

Now, the retarding torque of the eddy-current brake has been shown to be proportional to the speed of revolution of the disc, i.e.

$$T_B \propto N$$

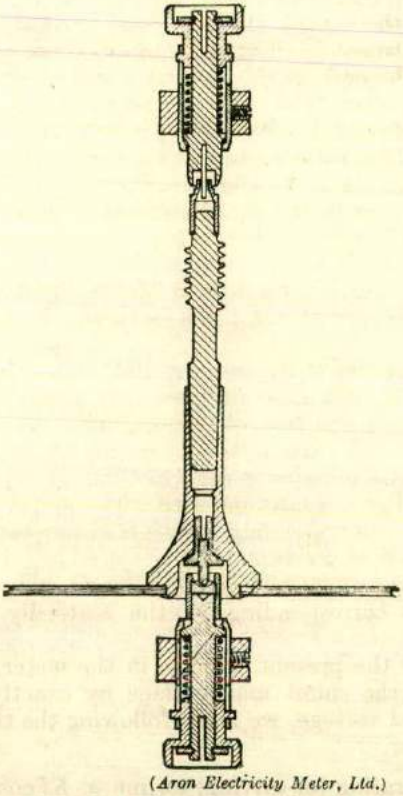
where N = speed of revolution.

Hence, since for a steady speed the driving torque T_D is equal to T_B, we have

$$N \propto EI \cos \phi$$

or Power $\propto$ speed of revolution

Thus the total number of revolutions, which equals $\int N dt$, is proportional to $\int EI \cos \phi \, dt$, i.e. proportional to the energy supplied.



(*Aron Electricity Meter, Ltd.*)

FIG. 21.10. MOVING ELEMENT OF ARON SINGLE-PHASE METER

The voltage and frequency of the supply circuits upon which these meters are principally used are sufficiently constant for errors due to variations of these quantities to be negligible.

METER PHASE ERROR. As pointed out when considering the induction wattmeter, the object of the copper shading band on the shunt magnet is to produce a phase displacement of exactly 90°

(assuming the flux of the series magnet to be in phase with the load current) between the flux of this magnet and the applied voltage.

An error due to incorrect adjustment of the position of this shading band (resulting in an incorrect phase displacement between the voltage and shunt magnet flux) will be evident when the meter is tested on a load whose power factor is less than unity. An error on the "fast" side under these conditions can be eliminated by bringing the shading band farther down the limb of the shunt magnet (i.e. nearer to the disc), or decreasing its resistance.

An error in the speed of the meter when tested on non-inductive load may be eliminated by adjustment of the position of the brake magnet, a movement of the poles of this magnet towards the centre of the disc reducing the braking torque, and vice versa.

The full-load current, the supply voltage, and correct number of revolutions of the disc per kilowatt-hour, are marked on the name-plate of the meter.

Example. For a 20 amp 230 volt meter, the number of revolutions per kWh is 480. If, upon test at the full load (4,600 watts), the disc makes 40 revolutions in 66 sec, calculate the error.

In 1 hr at full load, the meter should make $4{\cdot}6 \times 480$ revolutions.

This corresponds to a speed of $\frac{4{\cdot}6 \times 480}{60} = 36{\cdot}8$ r.p.m.

Thus the correct time for 40 revolutions $= \frac{40}{36{\cdot}8} \times 60 = 65{\cdot}2$ sec.

$\therefore$ Time taken is 0·8 sec too long, and the meter is $\frac{0{\cdot}8}{65{\cdot}2} \times 100 = 1{\cdot}2$ per cent slow.

FRICTION COMPENSATION. The friction compensating loop embraces part of the flux crossing the disc from the shunt magnet, and an eddy current is induced in this loop which causes a phase displacement between the flux embraced and the remainder of the flux. As a result, a small driving torque is exerted on the disc, this torque being adjusted by moving the loop across the face of the shunt magnet, to compensate for friction torque in the instrument.

To check the correctness of this friction compensation, the meter is tested on a light load, under which condition it should run at the correct speed if the compensation is correct. Care must be taken to ensure that the compensating torque is not so great as to cause the meter to run when no load current is flowing, i.e. when only the shunt-magnet flux exists.

CREEP. In some meters a slow, but continuous, rotation is obtained when the voltage coils are excited but with no-load current flowing. This may be due to incorrect friction compensation, to vibration, to stray magnetic fields, or to the fact that the voltage of the supply circuit is in excess of the normal.

To prevent such "creeping," two holes, or slots, are cut in the disc on opposite sides of the spindle. The disc tends to remain

stationary when one of the holes comes to the edge of the field of the shunt magnet.

In some cases, a small piece of iron wire is attached to the edge of the disc. The force of attraction of the brake magnet upon this iron wire is sufficient to prevent continuous rotation of the disc under no-load conditions.

Effect of Temperature Variation. The errors introduced by variations of temperature of an induction meter are usually small, since the various effects produced tend to neutralize one another.

Thus, increase of temperature increases the resistance of the eddy-current paths in the disc, and this reduces both the driving and braking torques. Again, the resistance of the voltage coil is increased with increase of temperature, so that it takes less current, resulting in a reduced flux in the shunt magnet. This reduces the driving torque, but the flux from the brake magnet is also reduced by increase of temperature, and so these effects tend to neutralize one another.

On non-inductive loads the temperature error is generally negligible; on inductive loads, however, especially in the case of large whole-current meters, the error may be of considerable importance. Temperature compensation can be carried out by means of a nickel-iron surround on the brake magnet as described on p. 815. A similar material used in conjunction with a soft-iron piece around a limb of the quadrature loop can be used to compensate for temperature variations on inductive loads.

Before leaving the subject of single-phase watt-hour meters, mention should be made of an interesting article by A. Salzmann (Ref. (41)) describing their use (when special instruments for the purposes (see Chapter XXII) are not available) for power, power factor, polarity and phase-sequence tests.

Polyphase Watt-hour Meters of the Induction Type. As in the case of power measurement with a two-element wattmeter, the energy supplied to a three-phase circuit may be measured by a double-element meter of the induction type, each element being similar in construction to the single-phase meter.

There are two discs mounted on the same spindle, but with a single brake magnet acting on the upper disc only; the spindle drives a single counting train. In addition to the phase-adjustment and friction-compensating device in each element, one or both of the elements has an adjustable magnetic shunt across its shunt magnet, in order that the driving torque for the same power may be made the same in the two instruments. In carrying out this adjustment, the two current coils of the meter are connected in series, and the two voltage coils in parallel, the polarities being such that the two driving torques are in opposition. Adjustment of the magnetic shunt is continued until the meter ceases to rotate with full rated power supplied to both elements.

The adjustments for phase displacement, friction compensation, and for the correct position of the brake magnet are made upon the two elements separately, a single-phase circuit being used. If the torque-balancing and other adjustments have been correctly carried out, both elements will have the same errors for a given position of the brake magnet. Both shunt elements should be energized during the single-phase tests, to allow for interaction.

If the phase compensation is not carried out exactly in each element it may happen that, taken together, the two errors neutralize one another when the load is balanced, giving a net error of zero. Since, however, this condition does not hold for all loads, whether balanced or not, it is essential to observe certain conventional rules with regard to the connections of the meter so that it may, when in service, be connected in the same way as when under test.

In the British Standard Specification No. 37, for electricity meters, a standard system of marking the terminals is laid down, in order that the correct connections may be made. The three lines of the three-phase system are designated the "white," "blue," and "red" lines respectively, the white phase leading the blue by 120°, and the blue leading the red by the same angle. The specification states that in the upper element the current coil shall be connected in the blue phase and the current coil of the lower element in the red phase. The current in the upper current coil therefore leads that in the current coil of the lower element by 120°.

The connections to these polyphase meters are the same as those employed in the two-wattmeter method of measuring three-phase power. Fig. 21.11 shows the construction of a polyphase meter.

Clock Meters. The most important meter of this type is the Aron clock meter. It has the advantages of being unaffected by external magnetic fields and of being comparatively free from frequency and wave-form errors, especially if its voltage-coil circuit is made as far as possible non-inductive. It is especially useful when the power factor of the circuit in which the energy is to be measured is high. The meter will register accurately on very low loads, but its cost and somewhat complicated construction have prevented its use as a house service meter. It can be used on either a.c. or d.c. circuits.

Although many of these clock meters are still in service on d.c. circuits, their cost of manufacture would now be so high that they are no longer being produced. Their novel principle of operation, and their good record of service deserve, however, to be recognized by a brief description.

PRINCIPLE OF OPERATION. The connection diagram of Fig. 21.12 illustrates the principle of the meter. There are two similar flat circular coils P_1 and P_2 carried at the bottom ends of two pendulums which are continuously driven by clockwork, the latter being wound electrically. These coils are connected in series with one another, and with a high resistance having a very small temperature

coefficient, across the line. They thus carry a current proportional to the line voltage. Obviously this circuit must be non-inductive, and must not vary in resistance appreciably with varying temperature, if this proportionality is to remain constant under all conditions of frequency and temperature.

Beneath each of these pendulum coils is a fixed current coil (C_1 and C_2), these being connected in series with the line and being wound so that their magnetic fields are in opposite directions—i.e. one upwards and one downwards.

When no line current is flowing, the two pendulums, which are of exactly the same length, swing at the same rate; but if a current flows through C_1 and C_2 one of these coils exerts a force upon its adjacent pendulum which reduces the speed of the latter and the other coil causes the speed of its adjacent pendulum to increase. This can be seen from the following.

The time of swing of a pendulum, whether simple or compound, may be written

$$T = \frac{k}{\sqrt{g}} \quad . \quad . \quad . \quad . \quad (21.4)$$

where k is a constant depending upon the dimensions of the pendulum and g is the acceleration due to gravity.

Thus, the number of swings per second, N, for each of the pendulums when swinging freely is given by

$$N = K\sqrt{g} \text{ where } K = \frac{1}{k}$$

The effects of the magnetic fields of coils C_1 and C_2 are to produce forces + F and − F acting, one up and one down, upon the pendulums adjacent to the coils C_1,C_2. Let $+a$ and $-a$ be the corresponding accelerations. Then the rates of swing of the two pendulums will be

$$N_1 = K\sqrt{g+a} = K\sqrt{g}\sqrt{1+\frac{a}{g}}$$

and

$$N_2 = K\sqrt{g-a} = K\sqrt{g}\sqrt{1-\frac{a}{g}}$$

It can be shown that

$$N_1 - N_2 = K\sqrt{g} \cdot \frac{a}{g} = \frac{Ka}{\sqrt{g}} \cdot \quad . \quad . \quad (21.5)$$

Now, the acceleration a is dependent upon the ampere-turns on the coils P_1,P_2 and C_1,C_2, and upon the dimensions and relative positions of the coils, so that we may write

$$a = k' \cdot \bar{A}_p \cdot \bar{A}_c$$



(*Associated Electrical Industries, Ltd.*)

FIG. 21.11. A.E.I. POLYPHASE METER TYPE NF3

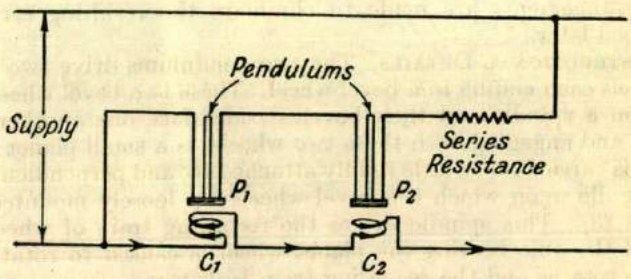


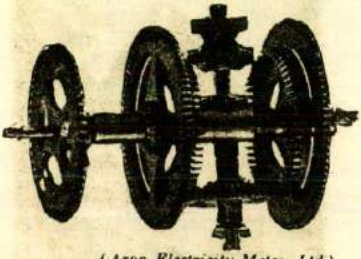
FIG. 21.12 CONNECTIONS OF ARON CLOCK METER

where k' is a constant and $\bar{A}_p$ and $\bar{A}_c$ are the ampere-turns on the voltage coils and upon the current coils respectively.

Thus,

$$N_1 - N_2 = \frac{Kk'}{\sqrt{g}} \bar{A}_p \bar{A}_c = K' \bar{A}_p \bar{A}_c = K'' i_p i_c \quad . \quad (21.6)$$

where K' and K'' are constants, the latter including the product of the number of turns in a voltage coil and in a current coil; i_p and i_c, are the currents in the voltage and current coils respectively. The product $i_p i_c$ is proportional to the instantaneous power in the circuit, if these currents are instantaneous, the assumption being made that i_p is directly proportional to the voltage of the circuit.



(*Aron Electricity Meter, Ltd.*)

FIG. 21.13. DIFFERENTIAL GEAR OF ARON CLOCK METER

The *difference* between the rates of swing of the two pendulums is therefore proportional to the power in the circuit. By means of a differential gear this difference in rate is integrated so that the energy supplied to the circuit is recorded by a train of wheels and dials in the usual way. It should be noted that in the above discussion the natural periods of swing of the two pendulums are assumed to be exactly equal. Actually such exact equality is difficult to obtain, and arrangements are made to eliminate the resulting error, as described later.

CONSTRUCTIONAL DETAILS. The two pendulums drive two trains of wheels each ending in a bevel wheel. These two bevel wheels run loose on a spindle and their bevelled sides face one another. Between, and engaging with these two wheels, is a small planet wheel which is carried on an axle rigidly attached to, and perpendicular to, the spindle upon which the bevel wheels are loosely mounted (see Fig. 21.13). This spindle drives the recording train of wheels, so that, if the axle bearing the planet wheel is caused to rotate, the spindle rotates and the recording train is driven.

If the two pendulums swing at the same speed the two bevel wheels rotate in opposite directions at the same speed; the planet wheel rotates on its axle, but the axle itself remains stationary.

If, however, the pendulums swing at different rates, the bevel wheels rotate at different rates and the axle of the planet wheel is driven round, producing a rotation of the spindle and a movement of the recording train of wheels. The speed of rotation of this spindle—and hence the speed of the recording wheels—is proportional to the *difference* in speed of the bevel wheels, i.e. to the difference in the rates of swing of the pendulums and, therefore, to the power in the circuit.

Because of the difficulty of obtaining exactly equal natural periods of swing of the two pendulums the direction of current in the pendulum coils is automatically reversed every 10 min, there being two alternative trains of driving wheels so that the drive from the differential gear can be thrown over at the same time (otherwise the meter would read backwards when the current direction was reversed).

The mainspring which drives the pendulums is wound electrically twice a minute by an automatic arrangement. This is accomplished by an electromagnet which attracts a small armature attached to the end of the spring. This electromagnet only comes into operation, for an instant, when the spring is run down, when a contact is made which energizes the magnet. The use of such an arrangement avoids a continuous waste of power.

Remote Indication. The complexity of present-day electricity supply systems, as well as the extensive use of electrical methods for the measurement of non-electrical quantities such, for example, as the pressure and rate of flow of gas, water or steam, the water level in reservoirs, etc., have encouraged the development of systems for the transmission of information, concerning various measured values, to a distance. Some examples of these follow.

The Electronic Repeater (*Evershed and Vignoles*). The principle of this apparatus is shown in Fig. 21.14 (which shows a wattmeter transmitter).

The pointer of the instrument, the indication of which is to be transmitted by pilot lines carrying direct current, carries a coil working inside a pot magnet and also a disc contact which is connected to the grid of a triode valve. The coil is in the cathode circuit of the valve with the circuit completed through distant indicators or recorders. The disc lies between two fixed contacts carrying positive and negative biasing voltages respectively. The voltages alter the anode current until the torque exerted by the coil balances that due to the instrument originating the movement. The capacitor connected across the grid circuit of the valve provides storage of the required biasing voltage.

Variations in line resistance or in mains voltage are automatically compensated so that the current transmitted remains proportional to the quantity being measured. The instrument operates from a.c. mains.

Elliott Transmission System (*Elliott Brothers* (*London*), *Ltd.*). In this sytem, also operated from a.c. mains, the transmitter has an excitation field system, a moving element, and an output field system. The arrangement is shown in Fig. 21.15. The exciting field system has a laminated iron core and a magnetizing coil, and it has a narrow,

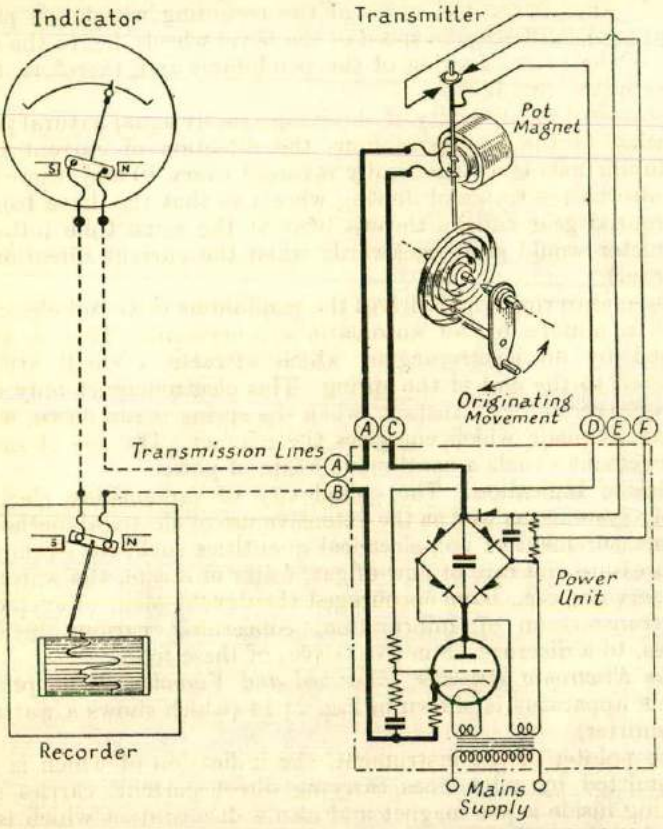


FIG. 21.14. MECHANICAL TRANSMITTER OPERATING INDICATOR AND RECORDER

specially-shaped, air gap. A static voltage regulator maintains a constant voltage across the coil. The moving element is a metal loop which is caused to rotate by the originating instrument movement. When the loop enters the air gap of the mains-excited magnet an e.m.f. is induced in it, and this is proportional to the amount of gap flux embraced by the loop. The loop is so shaped that the induced current in it is closely proportional to its angular movement.

The output field system is a closed, laminated-iron core with a single coil on it. The rotated loop is inductively coupled with this coil so that the latter has, induced in it by the loop current, an

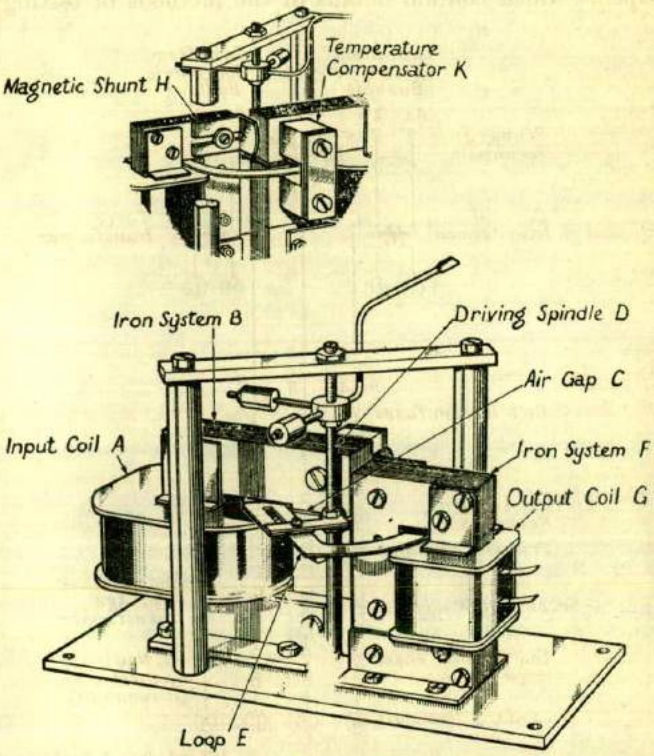


FIG. 21.15. ELLIOTT–SHOTTER TRANSMITTING UNIT

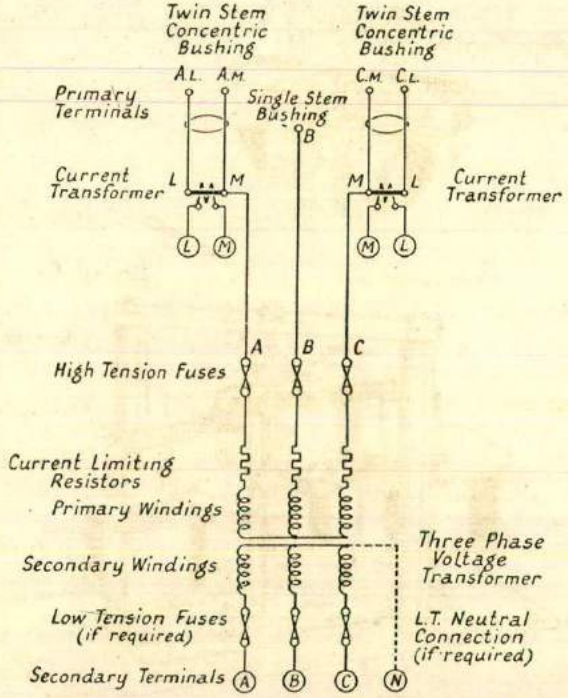
e.m.f. proportional to this current and so proportional to the angular movement of the loop and of the moving system of the originating instrument. The loop and output system of the transmitter act as a current transformer, and thus the output current is, within limits, independent of the impedance of the output circuit.

Combined Instrument-transformer Metering Units (*Chamberlain and Hookham, Ltd.*). These units have been developed, for pole or platform mounting, to provide accurate remote metering at medium and high voltages particularly for overhead branch lines in rural areas.

The units consist of a three-phase voltage transformer mounted, with the necessary current transformers (5 VA, Class CM), all oil

immersed, in a welded sheet steel tank. The arrangement is shown in Fig. 21.16.

Meter Testing. The testing of meters for commercial work has been standardized by the provisions of the Electricity Supply (Meters) Act, 1936. The Electricity Commissioners issued a series of papers* which contain details of the methods of testing which



(*Chamberlain & Hookham, Ltd.*)

FIG. 21.16. CONNECTIONS OF 3-PHASE, 3-WIRE COMBINED METERING UNIT

should be employed and the approved apparatus which should be installed in meter-testing stations for the purpose of carrying out such tests. These papers should be studied by the reader who is

* E.g. Electricity Supply (Meters) Act, 1936: Approved Apparatus for Testing Stations, etc.; Explanatory Memorandum concerning the Testing of Electricity Meters; The Apparatus Approved for Use in Meter-testing Stations, etc.; Further Explanatory Notes concerning the Testing and Certification of Electricity Meters, etc.; Supplementary Notes in Amplification of the Explanatory Memorandum, etc. (Published by H.M. Stationery Office.)

interested in the subject of meter testing. Only a few of the more important provisions can be given here.

Dealing with the approved apparatus first, this is listed as follows—

Standard Apparatus

D.C. potentiometer.

Substandard Apparatus

(*a*) Indicating wattmeter
(*b*) Rotating meter.
(*c*) Ammeter.
(*d*) Voltmeter.
(*e*) Electrolytic ampere-hour meter.

Time Standard

Ship's chronometer or pendulum-type clock.

Time Substandard

Stop-watch or other suitable timing device.

It is laid down that the potentiometer shall be tested at intervals at the National Physical Laboratory, and that the other apparatus shall be tested against a standard d.c. potentiometer or against substandard indicating instruments at stated intervals.

Specifications are given for the approved apparatus, and these include constructional requirements and allowable accuracy limits.

Methods of Test. For motor meters, three methods of test are given—

Method A

Long-period dial tests, using substandard rotating meters.

Method B

(*a*) Tests (other than long-period dial tests) using substandard rotating meters.
(*b*) One long-period dial test.

Method C

(*a*) Tests by substandard indicating instruments and stop-watch.
(*b*) One long-period dial test.

It is laid down that "Method C alone is to be used for testing direct-current motor meters."

All meters are to be tested (*a*) at the lowest percentage of their marked current specified in the limits of error for meters of their class under the Electricity (Supply) Acts; (*b*) at one intermediate load; and (*c*) at the highest percentage of marked current specified in the limits of error.

In the case of a.c. meters, they are also to be tested at marked current and marked voltage at 0·5 power factor lagging.

Watt-hour meters must also be tested for "creep" by applying 10 per cent overvoltage to the voltage coil, the main circuit (current-coil circuit) being open. The meter must not run under these conditions.

It will be observed that the testing engineer has the liberty of choice between the above three methods of test (in a.c. meters), but all three methods involve at least one dial test.

D.C. METERS. All supply meters of the motor type have a meter-constant marked on them. This constant is expressed in ampere-seconds per revolution or in revolutions per kilowatt-hour. The full-load current and line voltage for which the meter is intended are also stated. From these data the number of revolutions per minute which the meter *should make* when tested with a certain fraction of its full load can be calculated. The number of revolutions per minute which it *actually does make* when tested at this load is then observed and the error (if any) deduced therefrom.

Thus, if K is the constant of the meter, the number of revolutions per minute which it should make when tested at some fraction a of its full load is given by

$$\frac{K \cdot aW}{60}$$

where W is its full load in kilowatts.

An example illustrating such a test is given on page 823.

As an alternative to the above method of testing, a standard watt-hour meter is often used instead of timing a definite number of revolutions of the meter under test with a stop-watch. In such cases the current coil of the standard instrument is connected in series with that of the meter under test, and their voltage coils are connected in parallel. This ensures that the power operating the two meters is exactly the same. The standard meter records single revolutions and is readable to a small fraction of a revolution. An arrangement is also fitted so that the register of the standard can be started and stopped instantaneously.

Specifications for such rotating standards are given in the Electricity Commission's paper on "Approved Apparatus."

The procedure, after adjusting the load to the required value, is to allow the meter under test to make a certain number of revolutions, and to observe the number of revolutions made by the standard in the same time.

If the constants of both meters are the same, the error in the meter under test can then be obtained directly.

If the constants are unequal,

Let K_x = No. of revolutions per kilowatt-hour for the meter under test (nominal value)

K_s = No. of revolutions per kilowatt-hour for the standard

N_x = No. of revolutions made in a certain time during the test by the meter under test
N_s = No. of revolutions made by the standard in the same time

Then, the energy indicated by the test meter in the time of the test is

$$\frac{N_x}{K_x} \text{ (kWh)}$$

and that indicated by the standard is

$$\frac{N_s}{K_s} \text{ (kWh)}$$

These should obviously be equal if the meter under test is correct. If not, the error is given by

$$\left(\frac{\frac{N_x}{K_x} - \frac{N_s}{K_s}}{\frac{N_s}{K_s}}\right) \times 100 \text{ per cent}$$

or

$$\left(\frac{N_x K_s}{K_x N_s} - 1\right) \times 100 \text{ per cent}$$

In order to ensure that the meters have attained a steady temperature by the time the test is made, it is necessary that they shall be connected in circuit, with the load on, for some 15 to 30 min before making the test.

The advantages of this method of test, as compared with the method using a stop-watch, are that errors introduced by the latter are avoided and also that variations in the load during the time of test affect both meters equally and are therefore unimportant.

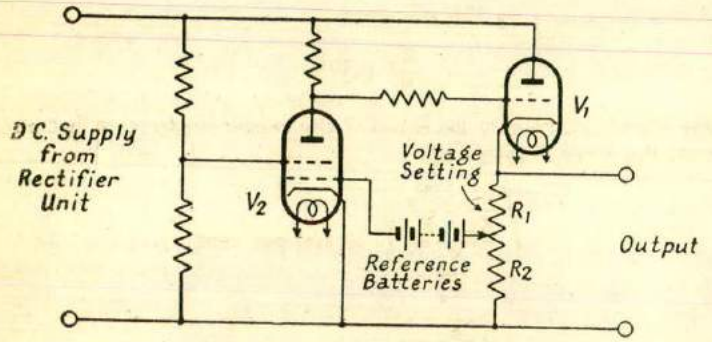
Dial Tests. In addition to the short-time tests described above, it is usual (and is required by the Minister of Power in commercial testing) to carry out a "dial" test extending over an hour or more with the load maintained constant at its full value, in order to check the gearing and ensure that the readings given by the recording train are correct.

This test gives an opportunity of discovering errors due to self-heating, and it is usual, at the end of such a test, to repeat the short-time test at full load to investigate the effect of this heating upon the rate of revolution of the disc.

Other Tests. The British Standard Specification states that a meter should start at one-hundredth full load and also that it should *not* run when a voltage of 10 per cent over the normal is applied,

the load current being zero. Tests must be made, extending for at least ten minutes, to ensure that a meter complies with these requirements in addition to those regarding errors for which the foregoing tests are carried out.

"Phantom" or "Fictitious" Loads. When the capacity of a meter under test is high, a test with the ordinary loading arrangements would involve a considerable waste of power. To avoid this, "phantom," or "fictitious," loading is employed which consists in supplying



(*H. Tinsley & Co., Ltd.*)

FIG. 21.17. GLYNNE ELECTRONIC STABILIZER

the voltage circuit with the required (normal) voltage, and the current circuit from a separate low-voltage supply. This means that the total power supplied for the test is that due to the small voltage-coil current at normal voltage, plus that due to the load current at a low voltage; and the total power supplied is therefore only small.

If a high-capacity meter is to be tested whilst in service, the voltage coil is supplied from the line in the normal way, but the current coil is removed from the consumer's load circuit and replaced by a short-circuiting connection. The current coil is then supplied from a battery or other low-voltage source for purposes of testing.

The Glynne Electronic Stabilizer. H. Tinsley and Co. make this instrument for use in meter stations where a fairly high-voltage d.c. supply, of good stability, is needed; it replaces high-voltage accumulators. The voltage supplied can be of any value between 25 and 600 V at up to 120 mA, with a short-term stability of 1 in 50,000 and a long-term one of 1 in 10,000.

A schematic diagram is shown in Fig. 21.17 in which V_1 is a low-impedance valve. A second valve V_2 (a tetrode) provides the control grid of V_1 with a potential such as to maintain the voltage across the load constant. The unstabilized input is derived from a full-wave mercury-vapour rectifier circuit with choke input filter fed

from an h.v. transformer with a tapped secondary. The instrument operates from a 200 to 250 V a.c., 50 c/s, supply.

It is possible to use a neon gas discharge tube to develop the reference voltage, as an alternative to the h.t. battery used in the Glynne stabilizer. The voltage drop across these tubes is constant for a wide range of current through them. A variety of d.c. stabilizers have been developed using such discharge tubes as the reference element, and these are described in Refs. (46), (47).

A. P. T. Electronic Industries, Ltd. make a range of a.c. mains-operated d.c. power-supply units having output voltages variable up to 500 V. The voltage stability is 1 part in 50,000 with a ripple of less than 1 mV r.m.s. Output currents up to 500 mA can be provided.

A.C. Meter Tests. In testing a.c. meters, it is not only necessary to vary the voltage and load current, but, in addition, the relative phases of the two quantities must be altered; in other words the power factor of the load must be variable also. Variation of power factor may be brought about by making up the load circuit with variable resistors, inductors, and capacitors, and adjusting these as required. This method is, however, inconvenient in most cases, since it involves the waste of the power in the load and necessitates several fairly expensive pieces of apparatus, these objections being particularly apparent if the capacity of the meters is high.

For this reason the load employed for testing purposes is usually a fictitious one, as in the case of d.c. meter testing. Provision must, of course, be made for varying the phase of the current in the current circuit relative to that of the voltage circuit in addition to the variation of the magnitude of the current.

This phase of the current is usually altered in one of two ways: either by using, as supplies, two similar alternators coupled together and driven by a motor, with the stator of one of them capable of rotation through any desired angle relative to the position of the stator of the other, or by using a phase-shifting transformer. In the former case one alternator supplies the voltage-coil circuits of the meters under test, and the other supplies the current-coil circuits, a step-down transformer being used to obtain the (possibly) heavy currents required for the current coils. By rotating the stator of the alternator supplying the current, using a worm-wheel and a graduated circular scale, the phase can be altered relative to that of the voltage. The angle through which the alternator stator is moved gives a measure of the phase angle between current and voltage. The frequencies of both current and voltage are, of course, identical, since the two alternators are direct coupled.

When a phase-shifting transformer is used, instead of a motor-alternator set, the stator must be supplied with two- or three-phase current in order to produce a rotating field, or, if the supply is single-phase, a phase-splitting device must be used. The phase of

the current induced in the rotor of the phase-shifter can then be altered as required by rotating the latter through the appropriate angle.

The magnitude and power-factor of the load having been adjusted according to the requirements of the test, the errors of the meters under test can be obtained either by timing a certain number of

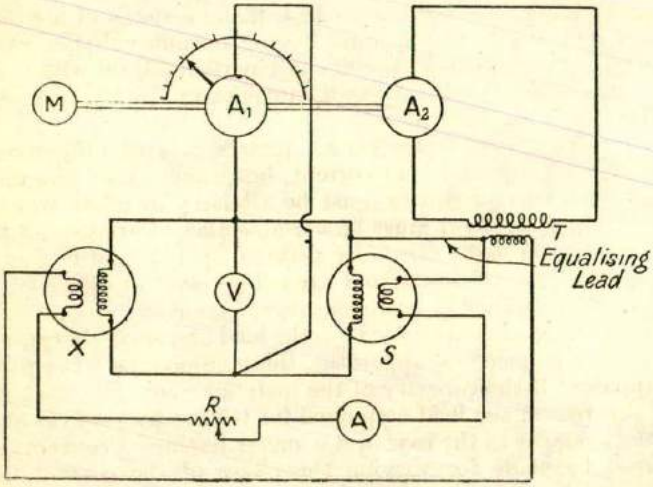


FIG. 21.18. CONNECTIONS FOR THE TESTING OF SINGLE-PHASE METERS

revolutions—the load being measured by an indicating wattmeter—or by comparing their rates of revolution with that of a standard watt-hour meter connected so as to measure the same load.

If the power-factor adjustment of a meter has been made at some high value of the power factor, so that the error of the meter is zero, no error, due to power-factor variation, will be observed at the lower power factor. If, however, the adjustment has been made so that the error is (say) 0·5 per cent at 0·8 power factor, it will be found that the error due to this cause at 0·1 power factor is more than 6 per cent. This will be explained by a reference to the correction curves given in Fig. 20.2 in the preceding chapter.

SINGLE-PHASE METERS. Fig. 21.18 shows the connections for the testing of a single-phase meter X, using two coupled alternators A_1 and A_2 for the supply to the voltage- and current-coil circuits respectively. The stator of A_1 is capable of rotation for phase variation. A_2 is connected first of all to a step-down transformer T whose secondary then supplies the current coils of the two meters X and S, the latter being either a substandard indicating wattmeter or a standard watt-hour meter, according to the method of

test employed. The variable resistor R is for adjustment of the current in the current-coil circuit. V and A are a voltmeter and ammeter for the measurement of the load voltage and current. A phase-shifting transformer may, of course, be used instead of the two alternators. The "equalizing lead" shown is necessary when a

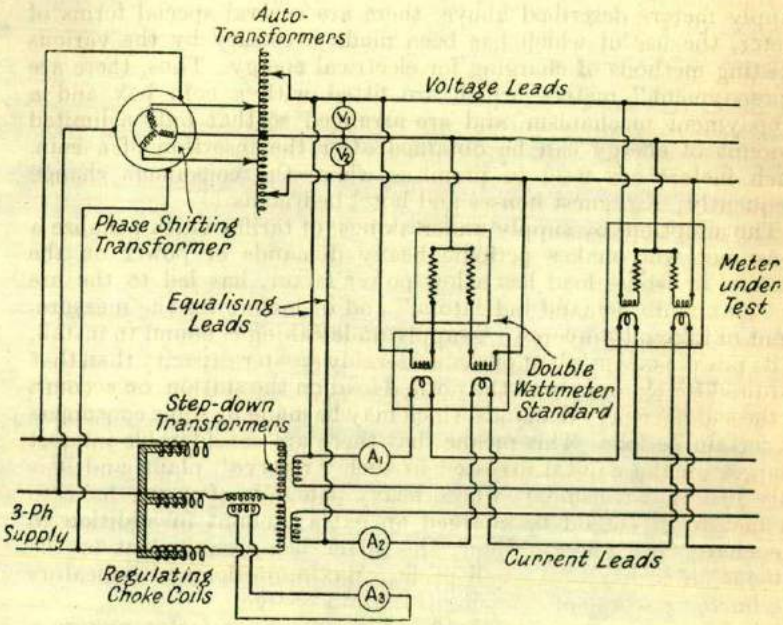


FIG. 21.19. CONNECTIONS FOR THE TESTING OF THREE-PHASE, THREE-WIRE METERS

fictitious load is used, in order to ensure that the potential difference between the current and voltage coils of the meters is zero, as it is when they are in service.

As implied in Fig. 21.18, the link between the current and voltage coils in each meter must be disconnected for this test.

POLYPHASE METERS. Fig. 21.19 gives a diagram of connections for testing three-phase three-wire meters. In this case there is a three-phase supply. This is used for two purposes: (*a*) to supply the stator of a phase-shifting transformer, (*b*) to supply three step-down transformers, two of whose secondaries supply the current coils of the meter. A regulating choke coil is connected in the primary circuits of these three transformers. The rotor of the phase-shifting transformer supplies two auto-transformers having variable secondary tappings. These supply the voltage coils of the meters, the

latter consisting of a double standard wattmeter and one or more meters to be tested. These are connected as shown. A_1, A_2, and A_3 are ammeters and V_1 and V_2 voltmeters, for adjustment of the load.

Meters used for Special Purposes. In addition to the forms of supply meters described above, there are several special forms of meter, the use of which has been made necessary by the various existing methods of charging for electrical energy. Thus, there are "prepayment" meters, which are fitted with a coin box and a prepayment mechanism, and are arranged so that only a limited amount of energy can be obtained after the insertion of a coin. Such meters are used in premises where the consumers change frequently; e.g. guest houses and hotel bedrooms.

The adoption, by supply undertakings, of tariffs which penalize a consumer who makes periodic heavy demands of power on the system, or whose load has a low power factor, has led to the use of "maximum demand indicators," and of meters for the measurement of kilovolt-amperes. A supply undertaking is bound to install, in its power station, plant of a considerably greater capacity than that required for the supply of the normal load on the station, on account of the sudden heavy demands which may be made by some consumers at certain periods. This means that there are considerable interest charges on the capital invested in such "reserve" plant, and it is only just that consumers whose heavy demands of power have to be met by it should be charged an extra amount in addition to the charge per kilowatt-hour, the latter being equivalent to the generation cost, plus a small profit. Maximum demand indicators are for the purpose of assessing this extra charge.

Again, consumers whose load has a low power factor require a much larger current, for a given amount of active power required, than would be necessary if their power factor were high. These increased current requirements necessitate the use, in the power station, of generators of a correspondingly high current-capacity, and also cause the copper losses in machines and distribution lines to be high. Thus, low power factor results in increased running costs to the supply undertaking, and again it is reasonable that a consumer should be charged extra to compensate for the increase in costs for which his low power factor is responsible.

Reactive-power meters and kilovolt-ampere meters are both used in connection with tariffs which take the consumer's power factor into account. In the first case a measure of the consumer's departure from unity power factor is obtained directly. In the case of kilovolt-ampere meters, a record is obtained of the product of volts and amperes in the circuit, and the charge in the maximum-demand system of charging is often based upon this product rather than upon the actual maximum power demand. In such cases a consumer's bill is increased if his power factor is low, since the

volt-amperes taken by his load will be higher for a given amount of power than if his power factor had been high.

PREPAYMENT METERS. Essentially, these are not a special type of meter from the point of view of principle of operation, but consist of meters of one of the types described above, in which is incorporated a device which causes the load and meter circuit to remain open until a coin is inserted in the mechanism, thus closing a switch in the meter, which automatically opens the circuit again after a certain definite amount of energy has been supplied subsequent to this insertion.

Many different types of mechanism are employed by the various manufacturers in order to produce the operation required. These do not, however, merit individual description here. The principal requirements of such meters are—

(*a*) The switch regulating the opening and closing of the supply circuit must make good contact and must operate correctly after an amount of energy corresponding to that prepaid for has been supplied.

(*b*) The meter must be proof against interference by any but authorized persons.

(*c*) The coins inserted must not jam.

(*d*) The meter must be capable of easy adjustment in the case of alteration of the price per unit.

(*e*) The manipulation by the consumer must be of a simple nature.

(*f*) Allowance must be made in the operating mechanism for the fact that all the coins inserted will not be of the same weight and thickness.

(*g*) The number of coins which have been inserted should be indicated.

Fig. 21.20 shows the internal connections of one type of prepayment meter.

MAXIMUM-DEMAND INDICATORS. The chief requirement of these indicators is that they shall record the maximum power, or rate of using energy, taken by a consumer during any particular quarter, the charge for the quarter then being based upon this maximum demand, as well as upon the total number of units consumed. In order that momentary heavy demands shall not be taken into account, and the consumer penalized, it is also necessary that the indicator shall only record the average power over successive predetermined periods, such as half-hours. Such indicators may either take the form of a separate instrument, or may be in the form of an attachment to a motor- or clock-type meter of the ordinary kind.

Wright Maximum-demand Indicator. This was one of the earliest demand indicators to be introduced. It indicates the maximum *current* taken by a consumer, the voltage (and power factor, if used

on a.c.) being neglected. The principle of this instrument is illustrated by the line drawing of Fig. 21.21.

The indicator is essentially a differential air thermometer. It consists of a glass U-tube whose upper ends are closed by two bulbs of similar capacity, as shown. One of these bulbs is surrounded by a metal heater strip through which the current in the supply circuit flows, heating the air inside this bulb as it does so. Near the upper end of the other limb of the tube, and just below the bulb closing it,

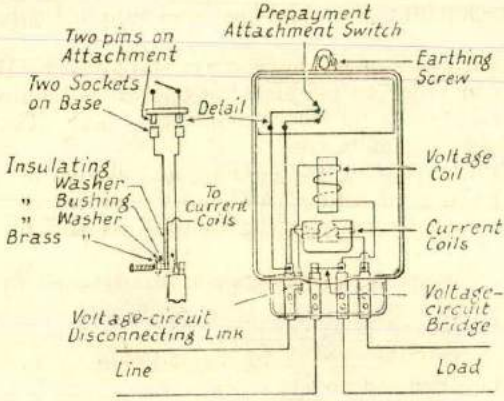


FIG. 21.20. DIAGRAM OF CONNECTIONS OF SINGLE-PHASE PREPAYMENT METER

is a side tube leading to a narrow-bore index tube alongside which is a scale. The U-tube contains a liquid with a small coefficient of expansion—which acts as an indicator. When the indicator is set, initially, the right-hand limb of the tube (Fig. 21.21) is filled with sulphuric acid up to the level of the branch tube. Acid is then added until its level in the index tube is at the zero mark on the scale.

Now, when current flows through the heater, the air in the bulb which the heater surrounds expands and causes the sulphuric acid in the right-hand limb of the U-tube to overflow into the index tube. The greater the current, the greater is the expansion of the liquid, and, therefore, the greater the overflow to the index tube. After the heater has cooled, the current in it must subsequently reach a higher value than any previous one if any further overflow into the index tube is to be obtained. The level of the liquid in this index tube gives, therefore, a true record of the maximum current which has passed through the instrument during any given period. The constrictions and traps in the limbs of the tube are for the purpose of preventing the passage of air from one bulb to the other when the indicator is reset, the latter operation being carried out by tilting

the instrument and allowing the sulphuric acid from the index tube to drain back into the main tube.

Momentary heavy currents do not cause the indicator to register, owing to the length of time taken for the temperature of the heater to rise and also on account of the low thermal conductivity of the tube which it surrounds.

Fig. 21.22 shows the construction of the Price and Belsham maximum-demand indicator. The instrument is thermally operated,

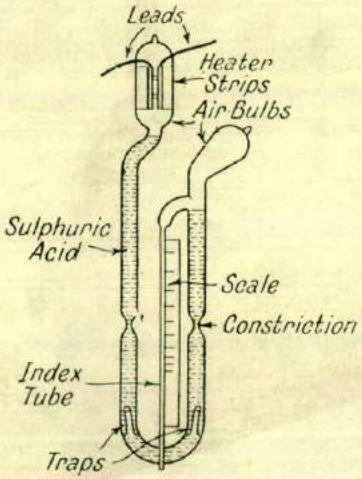


FIG. 21.21. PRINCIPLE OF WRIGHT MAXIMUM-DEMAND INDICATOR

its movement consisting of two matched, flat coils of bi-metal strip —the actuating coil and compensating coil—the former being surrounded by a heater element carrying the load current (or a definite fraction of it, obtained by the use of a shunt or current transformer). The two coils are coupled together at their outer ends, the inner end of the actuating coil being fixed and the inner end of the compensating coil carrying the needle. The movement of the needle depends upon the heat produced in the heater, and hence upon the load current, and the needle carries forward with it a maximum demand pointer which remains fixed, by a special friction arrangement, at the maximum reading.

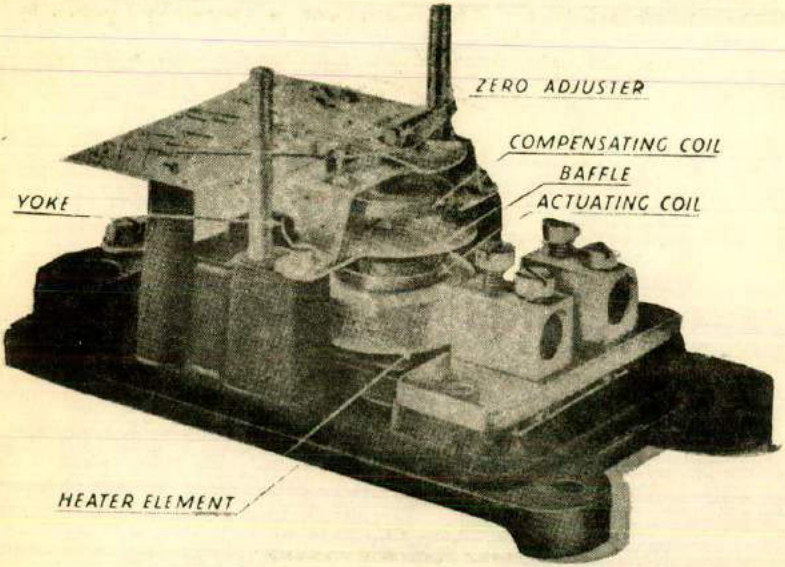
Through the use of the two matched coils the readings are unaffected by changes of ambient temperature

G. W. Stubbings (Ref. (27)) has discussed the theory of this type of maximum-demand indicator.

Merz-Price Demand Indicator. This indicator is not in the form of a separate instrument, but is a fitting which can be used with any

type of motor or clock meter to indicate the maximum consumption during a half-hour or other period throughout a quarter of a year (or other period between consecutive resettings of the instrument). Unlike the two preceding instruments, this indicator can be used to record either maximum current or maximum power.

The principle of operation is as follows. A separate dial is fitted, inside the instrument, the pointer of which is driven by the spindle



(Price and Belsham, Ltd.)

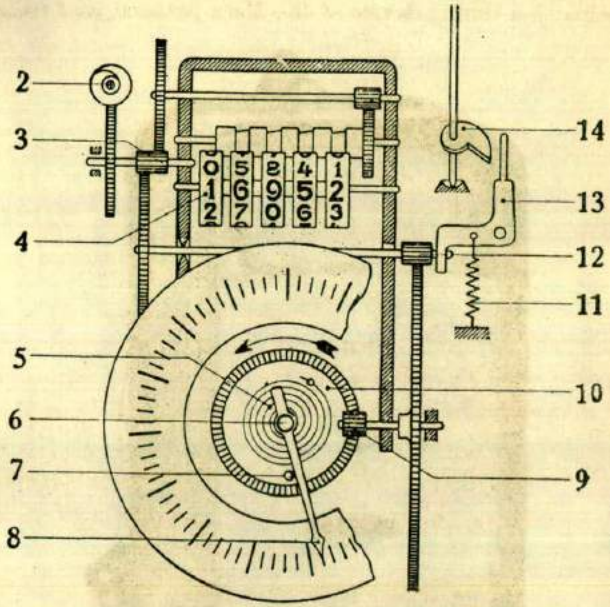
FIG. 21.22 PRICE AND BELSHAM MAXIMUM-DEMAND INDICATOR

of the moving system of the meter in the ordinary way. After this dial has been in gear for a certain period—usually half an hour—a device comes into operation which brings the mechanism of the disc back to the zero position. The pointer, however, does not return to zero but is lightly held by a special friction device and continues to indicate the number of units consumed in the previous half-hour period. The position of this pointer will not be moved forward unless the number of units consumed in some subsequent period exceeds the number recorded by the pointer. In this way the maximum demand, in units per half-hour, for any given period of time, is obtained.

The device for returning the driving mechanism of the demand dial to zero at the end of each half-hour period is operated by clockwork inside the meter or by a separate "time switch" external to the meter. These "time switches" consist of a clock, either electrically or hand wound, which makes and breaks certain contacts, at predetermined

intervals of time. These intervals depend upon the setting of the switch, which can be altered as required.

An advantage of this demand indicator is that a heavy demand must be existent for an appreciable time for it seriously to influence the number of units consumed in so long a period as half an hour,



(*Landis & Gyr, Ltd.*)

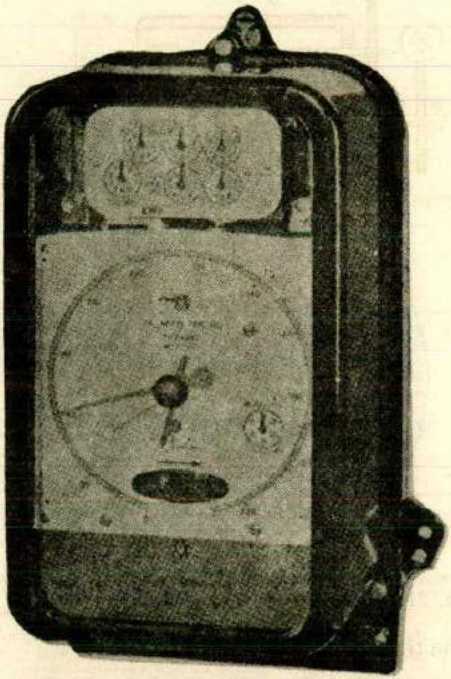
FIG. 21.23. MERZ MAXIMUM DEMAND INDICATOR

and, at the same time, short-period heavy demands are not neglected by the device.

The construction of a Merz pattern maximum-demand indicator, as manufactured by Messrs. Landis & Gyr, is shown in Fig. 21.23. Normally, the pin (7) drives forward the pointer (8) for (say) a half-hour period, and the energy consumed during this time is indicated on the dial. At the end of the period the cam (14) momentarily disengages the pinion (12) by means of the bell crank (13). This allows the pin (7), with its driving mechanism, to return to its zero position under the action of the spring (5). The pointer is, however, left stationary and continues to record the energy consumed during the previous half-hour period.

During the next half-hour period the pin (7) is again driven forward, but the pointer is only moved forward if the energy consumed in a subsequent period exceeds that consumed in all preceding periods.

Another instrument which is now quite commonly used in connection with power-factor tariffs is the Hill-Shotter maximum-demand kVA indicator, which is manufactured by Messrs. Aron Electricity Meter, Ltd. One type of this instrument is really an a.c. ampere-hour meter, and is of the induction type, having a shaded pole and a continuously driven disc. A permanent magnet is used for braking. It is used with a timing device of the Merz pattern, and records the



(*Associated Electrical Industries, Ltd.*)

FIG. 21.24. A.E.I. POLYPHASE METER WITH MAXIMUM-DEMAND INDICATOR

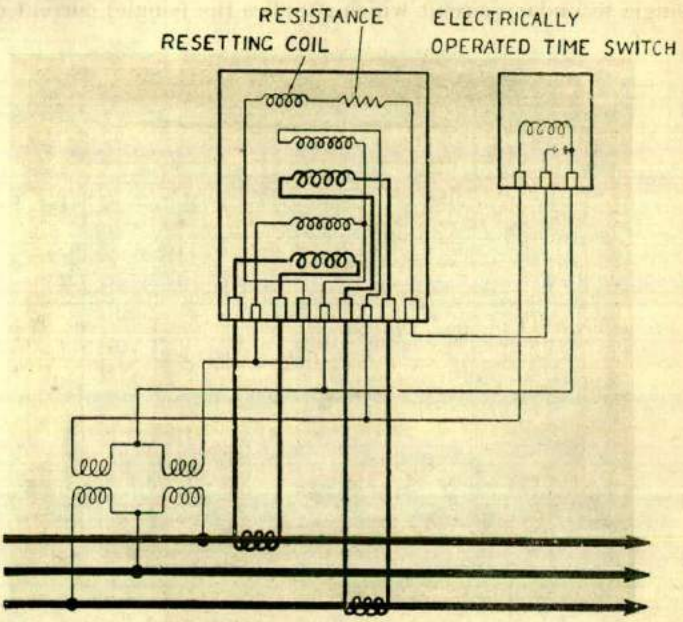
maximum ampere-hours consumed during successive time intervals of 15 min. The dial is marked in kVA corresponding to the voltage of the circuit on which the indicator is to be used. The instrument reading is independent of power factor, and of frequency, within such variations of the latter as might be expected in the normal supply system.

The theory of this indicator is fully discussed by G. W. Stubbings (Ref. (4)).

Figs. 21.24 and 21.25 show an Associated Electrical Industries

polyphase meter with maximum-demand indicator, and the diagram of connections of such a meter, respectively.

Summation Metering. When a number of circuits, on an electricity supply system, work in parallel the total energy supplied to them is often required. This total can be obtained by addition of the individual totals for the circuits so that special metering for the



(*Associated Electrical Industries, Ltd.*)

FIG. 21.25. CONNECTIONS OF A.E.I. POLYPHASE METER WITH MAXIMUM-DEMAND INDICATOR

purpose is not essential. The measurement of the total maximum demand is, however, a different matter; it cannot, in general, be obtained by arithmatical addition of the individual maxima because these are very unlikely to occur at the same time. Arithmetical addition will give too high a value. Summation metering is commonly used, therefore, to determine the simultaneous maximum demand for a number of circuits.

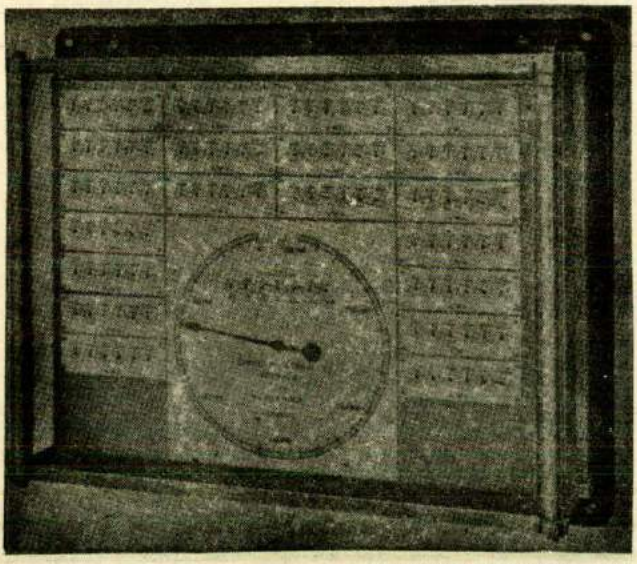
There is not space here for full discussion of this subject, which is a large one, and the references at the end of the chapter should be consulted for a detailed study.

The most important of the methods used are—

(i) A meter with several current coils: this method can be used satisfactorily for summation in the case of two or three parallel circuits.

(ii) Paralleled current transformers: this method is similar to (i) except that only one current coil is needed per meter element (see Chapter XX, p. 803).

(iii) Meter with a summation current transformer: if the number of circuits to be summated is greater than can be handled by methods (i) and (ii) a transformer having a number of primary windings—one for each summated circuit—can be used. The transformer has a single secondary circuit which supplies the (single) current coil of



(Chamberlain & Hookham, Ltd.)

FIG. 21.26. SWITCHBOARD PATTERN SUMMATOR FOR 20 CIRCUITS

the meter element. The primary windings are supplied from current transformers in the various load circuits.

(iv) Summation meter with multiple elements: this has a single rotor operated by a number of driving elements—one from each circuit. There is obvious difficulty in constructing such a rotor for more than a small number of circuits.

(v) Summation by impulsing meters: this method can be used for a large number of circuits. The energy meter in each circuit is fitted with a contact-making device which, for a pre-determined amount of energy, sends an impulse to a mechanism in the summator which records the total number of such impulses received. This system was introduced by Chamberlain and Hookham, Ltd. in 1932 and they now make a summator for 22 circuits (maximum). This instrument, shown in Fig. 21.26 for 20 circuits, registers the

consumption for each circuit, gives the total consumption and also the simultaneous maximum demand.

TWO-RATE METERS. These meters are required in connection with a two-rate or double tariff method of charging, which differs from the maximum-demand system in that two different rates of charge per unit are made, according to the time of the day at which the energy is consumed. By charging at a lower rate for energy consumed during periods of light load on the generating station, the supply undertaking endeavours to distribute the demand more uniformly over the day and so improve its "load factor," this being the ratio $\frac{\text{Average load}}{\text{Maximum load}}$. Assuming the average load to remain constant, the load factor is obviously increased by reducing the

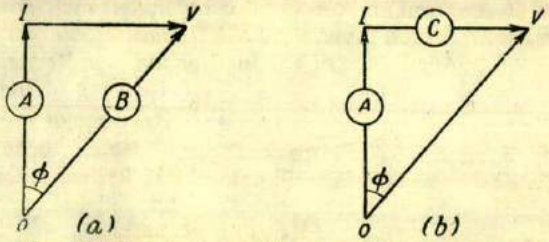


FIG. 21.27. METERING METHODS (DIAGRAMMATIC)

maximum load; and this can be achieved by causing some of the load to be taken at times of light load rather than when the load on the station is heavy.

A two-rate meter is of the ordinary type, but has two registering trains of wheels and dials, both of which are operated—but not at the same time—by the moving system of the meter. These registers are put into gear with the driving spindle alternately and are used at such times, and for such periods, as are desired by the supply undertaking for the purpose of the double tariff method of charging. The times of operation of the high-rate and low-rate trains are usually governed by a time switch such as was mentioned in connection with the maximum demand system.

The time switches are set so that, in general, the high-rate register is in operation during the evening, when the load is heavy, the low-rate register operating during the remaining portion of the 24 hours when the load is lighter.

Measurement of Kilovolt-amperes. As has been mentioned previously, the systems of charging for energy which penalize low-power factor require the use of another meter in addition to that measuring the total real energy supplied (i.e. a watt-hour meter). Dorey (Ref. (19)) illustrates the two alternative methods of metering for such purposes by vector diagrams as in Fig. 21.27. In these

diagrams *OI* represents the active power, while *OV* represents the kVA. In diagram (*a*), meter *A* is a watt-hour or "cosine" meter, and measures the real energy supplied, while meter *B* records the kilovolt-ampere-hours. In other words, *A* records $\int_0^t \text{kVA} \cos\phi \, dt$, while *B* records $\int_0^t \text{kVA} \, dt$. In diagram (*b*), meter *A* records the true energy as before, and meter *C* records the reactive energy, or $\int_0^t \text{kVA} \sin\phi \, dt$. The angle ϕ is the phase angle of the circuit. Obviously,

$$(\text{kVA} \sin\phi)^2 + (\text{kVA} \cos\phi)^2 = (\text{kVA})^2$$

In the first diagram, meter *B* may be fitted with a maximum-demand indicator which gives the kVA demand.

The average power factor of the load or average "energy factor" may be obtained in case (*a*) by the ratio $\frac{\text{Reading of meter } A}{\text{Reading of meter } B}$, the reading being for corresponding periods of time. In case (*b*) the average energy factor is given approximately by

$$\cos\phi = \frac{\text{Reading of meter } A}{\sqrt{(\text{Reading of meter } A)^2 + (\text{Reading of meter } C)^2}}$$

This expression is not, however, exact if the power factor is subject to considerable variations during the period of time to which the readings refer (*see* E. W. Hill's remarks in the discussion on the paper mentioned in Ref. (16)).

In one method of charging the total charge is made up of two parts; one at so much per unit of energy, and the other based on the quantity $\frac{\text{Kilowatt-hours} \times P}{\cos\phi}$, where $\cos\phi$ is the average energy factor and *P* is the "lowest permissible power factor," and is laid down by the supply undertaking.

Obviously, if $\cos\phi$ is less than *P*, the consumer is charged, in the second part of the tariff, for a greater number of units than those actually consumed, whereas if his power factor is higher than *P*, the charge is for less units than the number actually consumed.

An induction watt-hour meter can be compensated to read kVAh within a limit of 2 or 3 per cent by adjusting the flux of the voltage circuit, so that it lags by $90° + \phi_a$ behind the applied voltage instead of by merely 90°, ϕ_a being the average phase angle of the load circuit. Thus, if the phase angle of the load circuit is, at any time, exactly ϕ_a, the meter behaves as though the total applied voltage were exactly in phase with the current; i.e. it indicates

volt-ampere-hours. For other power factors, the speed of the meter will not be exactly proportional to the volt-amperes, but to volt-amperes $\times \cos(\phi - \phi_a)$, where ϕ is the actual phase angle.

The power in an a.c. circuit—volts $\times$ amperes $\times \cos\phi$—is very nearly the same as the volt-amperes if the angle ϕ is small, when $\cos\phi$ is nearly unity. As an example, if ϕ is 20°, $\cos\phi$ is 0·9397; the power measured, in this case, is thus less—by 6·03 per cent—than the volt-amperes in the circuit. This will apply whether the power factor is lagging or leading. If, then, the meter is adjusted

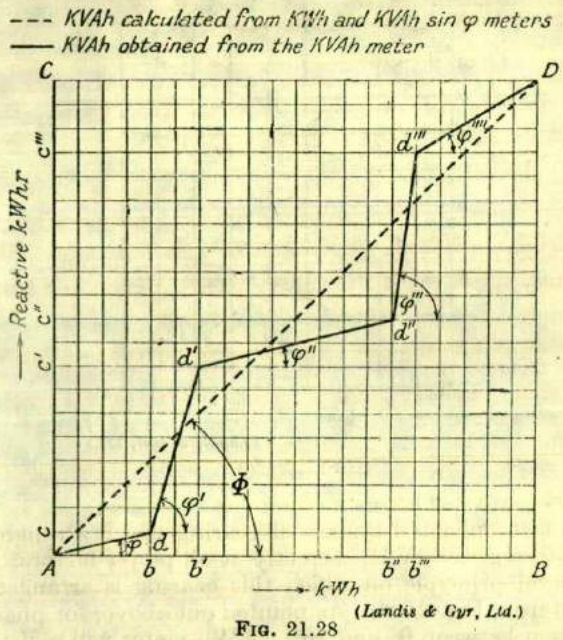


FIG. 21.28

so that it runs 3 per cent fast at unity power factor, it will measure the volt-amperes in the circuit correct to within $\pm$ 3 per cent, provided the phase angle ϕ of the load circuit does not exceed 20° (lagging or leading).

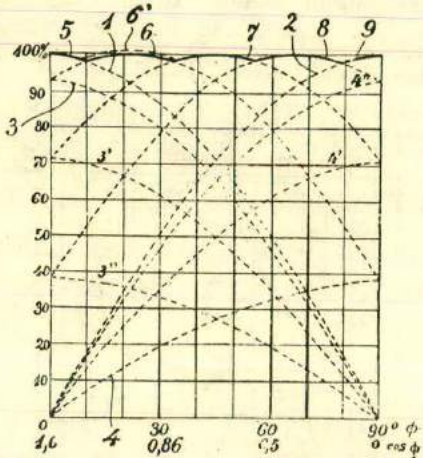
The required compensation of a watt-hour meter in order to cause it to indicate as above may be obtained by using quadrature loops of abnormal thickness on the shunt magnet.

Landis and Gyr Trivector Meter. This meter measures kVAh and also kVA of maximum demand.

Referring to Fig. 21.28, it can be seen that, if the power factor of the circuit changes during a period of time from $\cos\phi$ to $\cos\phi'$, $\cos\phi''$, etc., the true total of kVAh is given by the sum of Ad,

dd', $d'd''$, etc. If an attempt is made to measure this total simply by means of a kWh meter together with a reactive kVAh meter, the value obtained will be $\sqrt{AB^2 + BD^2} = AD$. This value is obviously, in general, less than the true value.

The trivector meter registers the true value of kVAh correct to within $\pm$ 1 per cent. The meter consists of a kWh meter and a reactive kVAh meter in the same case with a special summator mounted between them. Both meters drive the summator through



(*Landis & Gyr, Ltd.*)

FIG. 21.29

a somewhat complicated system of gearing which arranges for the summator to register kVAh correctly at all power factors.

The general principle on which this gearing is arranged can be understood from Fig. 21.29. As pointed out above, for phase angles of the system between 0° and 10°, a kWh meter will register kVAh correctly within narrow limits, just as a reactive kVAh meter will register total kVAh very closely for phase angles of 80° to 90°. The gearing arranges, therefore, that the summator shall be driven principally by the kWh meter when the phase angle is small (curve 5, Fig. 21.29), and principally by the reactive kVAh meter when the phase angle is nearly 90° (curve 9, Fig. 21.29). At intermediate phase angles, both meters are responsible for the drive of the summator through different combinations of gears. The result is that the kVAh, measured at various power factors, is almost constant, as shown by the full-line curve at the top of the graph.

For the fuller discussion of the question of kVA measurement, the reader is referred to Refs. (4), (7), (8), (11), (19), (24).

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CHAPTER XXII

MEASUREMENT OF SPEED, FREQUENCY AND PHASE DIFFERENCE

Rotational Speed Measurements. Electrical methods, or methods involving electrically-operated measuring equipment, can be used for the measurement of the rotational speed of a machine. If this machine is an alternator the speed is a measure of the frequency of the alternating voltage generated, the number of poles of the alternator being known.

One simple and direct method is to use a small generator with a permanent-magnet field, and to measure, with a voltmeter, the generated voltage. This is proportional to the speed, and the moving-coil voltmeter, with a uniform scale, can be calibrated directly in rotational speed (r.p.m. or r.p.s.).

Fig. 22.1 shows the construction of a small generator for this purpose, manufactured by the Record Electrical Co., Ltd., and used with Record "Cirscale" Voltmeters. This generator has cobalt-steel magnets; the armature runs on ball bearings and is wound to give 100 volts at 2,000 r.p.m. A magnetic shunt is fitted for calibration purposes. The armature and field magnets are enclosed within a cast-iron body which both provides screening from external magnetic fields, and also protects the generator from dust and moisture—an important point if the device is to be capable of being used universally for speed measurements.

The armature resistance of the generator is low in order that the voltage drop shall be small, even when the generator is used to supply several indicators in parallel. A resistor is provided for the purpose of limiting the current from the generator in the event of a short-circuit on the wiring to the indicators.

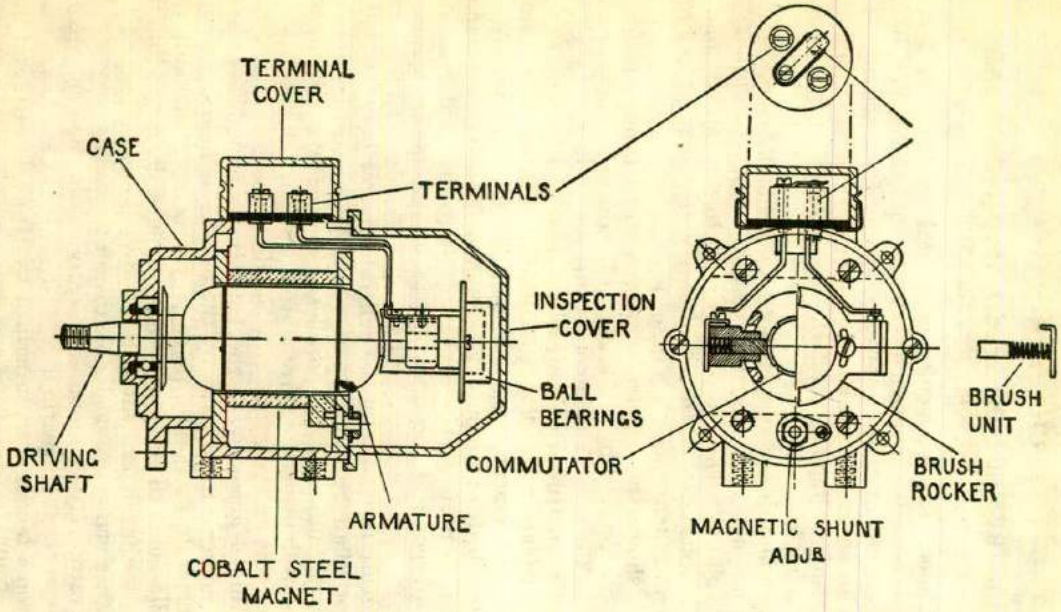
The combined accuracy of the generator and indicator at the calibrated temperature is given by the makers as follows.

From full scale to middle point, 1 per cent of the indication. From middle point to the lower limit of the scale, a possible fixed error not exceeding 1 per cent of the middle point reading.

Temperature errors are negligible over a wide range.

The generator may be mounted directly on the machine shaft, or may be driven through gearing.

An electronic tachometer, described by H. G. Jerrard and S. W. Punnett (Ref. (9)), is capable of measuring speeds of revolution of a shaft to an accuracy of $\pm$ 0·05 per cent. The rotation of the shaft is made to generate an alternating voltage in one of two ways—by an electromagnetic generator or a photo-electric generator. This voltage is amplified and then connected to the Y plates of a



(Record Electrical Co., Ltd.)

FIG. 22.1. GENERATOR FOR USE WITH A VOLTMETER FOR SPEED MEASUREMENTS

cathode-ray tube. To the *X* plates is applied the output of a variable-frequency oscillator calibrated to ± 0·05 per cent. The trace on the tube is a Lissajous figure and, if the frequency from the oscillator is adjusted to produce a stationary and easily identifiable Lissajous figure—most conveniently an ellipse—the frequency from the shaft generator, and therefore the speed of the shaft, become known to the same accuracy as that of the oscillator. Any change in speed is immediately detected because it produces a precession of the Lissajous figure.

For the electromagnetic generation of the alternating voltage from the rotating shaft a brass disc, carrying sixty soft-iron pegs equally

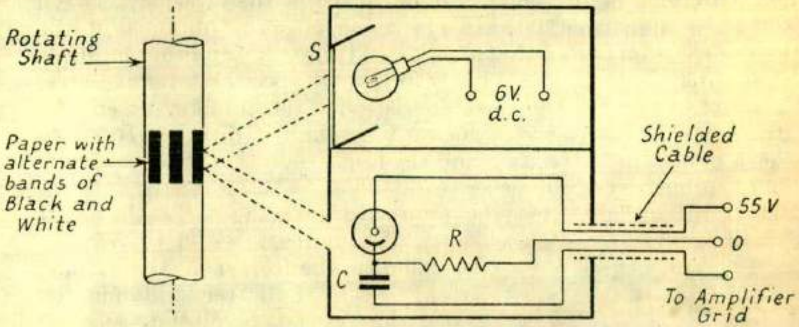


FIG. 22.2. ELECTRONIC TACHOMETER WITH PHOTO-ELECTRIC GENERATOR

spaced round its circumference, is mounted on the end of the shaft. This runs in the air gap of a pick-up on the core of which are two coils (actually the earpiece of a headphone was used to provide the coils and main part of the core of the pick-up). The change in magnetic flux caused by the passage of the soft-iron pegs induces an e.m.f. in the coils and this is passed on to the amplifier.

The arrangement of the photo-electric generator is shown in Fig. 22.2. There is a small light source in the upper compartment and a midget photo-cell in the lower one. The resistance R and capacitance C are 2 MΩ and 0·01 μF respectively. As the shaft rotates the light is reflected from a strip of paper, 1 in. wide and having alternate equal bands of black and white, pasted on the shaft. An alternating e.m.f. is generated in the photo-cell circuit. A diffusing screen S is used because the light is not focused on the photo-cell. If it were the wave-form of the output would be square topped, not sinusoidal.

A calibration unit consisting of a standard frequency generator (having a crystal-controlled oscillator) and a selective amplifier is used with the equipment.

Stroboscopic Methods. A stroboscopic device consists essentially of a disc carrying a geometrical pattern, the disc being illuminated by a series of rapidly-recurring flashes, each of which is of very

short duration compared with the time interval between successive flashes.

Fig. 22.3 shows a disc on which the pattern is a twelve-point star. If this disc is rigidly attached to the shaft of a rotating machine, of which the speed is such that each point of the star moves forward a distance of one point-pitch p during the time interval between successive illuminating flashes, the pattern will appear to be stationary. This effect is caused by the fact that the point adjacent to that viewed in the preceding flash, or glimpse, has now taken up the exact position of the latter, and, since all the points are identical, the eye cannot distinguish between them. Thus the impression is formed that no movement has taken place in the interval between the two illuminating flashes. The same effect is obtained if the speed of rotation is exactly twice, three times, or any other multiple of this speed—which is called the "primary" speed. If, for example, the speed of the rotating machine is twice the primary speed, the pattern will actually move forward through a distance equal to twice a point-pitch—i.e. $2p$—and the impression that it is stationary will be formed by an observer.

The series of flashes may be obtained by placing a disc, driven at a steady speed, between a source of illumination and the stroboscopic disc, the former disc containing a slit so that the latter is illuminated once for every revolution of the former. Another method of using the device is to employ an electrically-driven tuning-fork, the prongs of which carry thin aluminium shutters, each with a slit in it. These shutters overlap, and give two "glimpses" of the stroboscopic disc for every complete vibration of the tuning fork, if the observer's eye is placed behind them. Alternatively, the stroboscopic disc may be illuminated through the slits, when a series of flashes will produce the same effect as that obtained with glimpses.

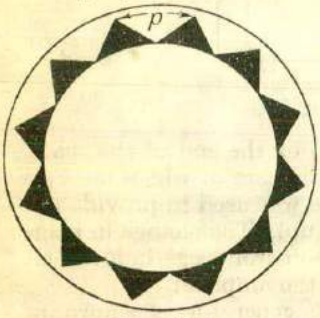


FIG. 22.3

The advantage of such a tuning-fork is its remarkably constant frequency of vibration.

Theory of the Method. When the pattern of the disc appears to be stationary, the speed of the rotating machine is given by

$$N = \frac{F}{n} \quad \text{revolutions per minute} \quad . \quad . \quad (22.1)$$

where F = No. of flashes, or glimpses, per minute

and n = No. of points on the pattern

In this case, N is the "primary" speed. The fact that the pattern will be stationary for speeds which are whole multiples of N will, in practice, cause no confusion, since the speed can easily be measured by a simple, and less accurate, method, with sufficient accuracy to distinguish between N and its multiples.

If the machine rotates at a speed slightly in excess of the "primary" speed, the pattern will appear to rotate slowly forwards; if the speed is slightly less than the primary speed, the apparent rotation of the pattern is backwards. The number of apparent revolutions per minute of the pattern gives the difference—above or below—between the actual speed of the machine and the primary speed.

Again, if the speed of the machine is exactly half the primary speed, the pattern will still appear stationary; but the number of points will appear to be twice the actual number; if the speed is one-third of the primary, the apparent number of points is three times the actual number, and so on.

Let N = the primary speed in revolutions per minute
n = No. of points on the pattern
F = No. of flashes, or glimpses, per minute
p = the pitch of the points in inches
d = diameter of circular pattern in inches

Then,
$$n = \frac{\pi d}{p}$$

The distance moved forward by the pattern, when the speed is N, during the interval between glimpses is

$$\frac{\pi d \,.\, N}{F} \text{ inches}$$

But this is equal to the pitch p,

$$\therefore p = \frac{\pi d N}{F}$$

Also
$$p = \frac{\pi d}{n}$$

$$\therefore n = \frac{F}{N} \text{ or } N = \frac{F}{n} \text{ as stated above.}$$

Example. If the speed of a machine is about 500 r.p.m., and its variation from this speed is to be measured by stroboscopic means, the number of points on the pattern should be twelve (as in Fig. 22.3) if the tuning fork vibrates at 50 cycles per second—giving 6,000 glimpses per minute.

This number is obtained from

$$n = \frac{6{,}000}{500} = 12$$

Again, if instead of being exactly 500 r.p.m., the speed of the machine is 510 r.p.m., the pattern actually moves forward a distance $\frac{\pi d \times 510}{6{,}000}$ in.,

between successive glimpses, and thus the *apparent* movement forward in the interval between glimpses is

$$\frac{\pi d \times 510}{6{,}000} - \frac{\pi d \times 500}{6{,}000} = \frac{\pi d \times 10}{6{,}000} \text{ inches}$$

Now, this interval between glimpses is $\frac{1}{6{,}000}$ min, so that in one minute the apparent movement forward is $10\pi d$ in., which corresponds to a rotation of the pattern of 10 r.p.m., thus giving the excess of the actual speed of the machine over the primary speed (500 r.p.m. in the case considered). If the apparent motion of the pattern had been backwards, the actual speed of the machine would have been 490 r.p.m.

The general expression for the *apparent* speed of rotation of the pattern is

$$R = \pm (a - 1)\ N \qquad \text{revolutions per minute} \quad . \quad (22.2$$

the positive sign applying when the apparent motion is forwards and the negative when backwards. a represents the ratio

$$\frac{\text{Actual speed of machine}}{\text{Primary speed}}$$

It is obvious from the above that the speed of any machine can be obtained by such means in terms of its difference from some fixed speed, the actual standard used being the vibrating tuning-fork whose frequency is known and is constant. Since the fixing of the stroboscopic disc to the shaft of the machine produces no appreciable increase in the load, the speed is not affected by the device used for its measurement, and in this fact lies one of the principal advantages of stroboscopic methods.

The disc used, instead of bearing only one pattern, as in Fig. 22.3. usually has several patterns with different numbers of points, the patterns being drawn concentrically. Several primary speeds can then be dealt with by using the different patterns.*

STROBOSCOPIC OBSERVATIONS. Stroboscopic methods are now very generally used for studying the behaviour of moving objects. Special discharge lamps, developed for this purpose, give very high intensity flashes of short duration—usually much less than 40 microseconds. In use, the lamp is connected in parallel with a capacitor across a high-voltage d.c. supply. The lamp has an auxiliary trigger electrode, and when a pulse is applied to this electrode, the energy stored in the capacitor is dissipated in a short-duration discharge between the anode and cathode of the lamp. The trigger pulse is obtained from a variable-frequency oscillator, and flash repetition rates in the region of 300 c/s are readily obtainable.

Mullard Electronic Products, Ltd. make a stroboscope which produces light flashes of duration between 3 and 10 microseconds,

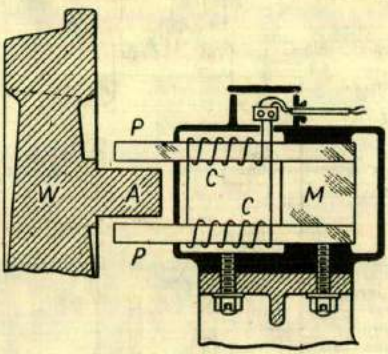
* For further information regarding stroboscopic methods, the reader is referred to the works mentioned in Refs. (1) and (6).

over a frequency range of 0·5 to 250 c/s. It includes a frequency meter to measure directly the flashes per second.

Dawe Instruments, Ltd. manufacture a wide variety of stroboscopic instruments for different purposes, which are fully described in the maker's catalogue.

The Measurement of Linear Speed. The measurement of the speed of railway trains is important in maintaining precise running schedules and several electrical instruments have been developed for this measurement.

Elliott Brothers (London), Ltd. have developed an equipment consisting of an a.c. generator, directly coupled to the axle, and a speed indicating or recording instrument. The generator is designed



(*Everett Edgcumbe & Co., Ltd.*)

FIG. 22.4 INDUCTOR RAILWAY SPEEDMETER: SECTION OF TRANSMITTER

to withstand the very severe vibration and shocks which must occur with such direct mounting. It is a 12-pole inductor-type machine with a rotor of 35 per cent cobalt magnet steel.

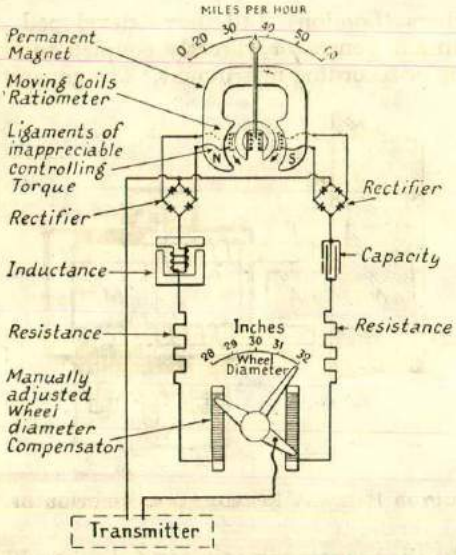
Temperature compensation is provided by the use of a magnetic shunt, with a negative coefficient of permeability, fitted to the rotor. There is also a compensator unit to take into account reduction of the wheel diameter through wear.

Co-operation between Everett Edgcumbe and Co., Ltd., Evershed and Vignoles, Ltd. and the London Passenger Transport Board led to the development of an inductor-pattern Railway Speedmeter which dispenses with a mechanical drive. It consists of a transmitter (shown in Fig. 22.4) and an indicator which is, in fact, a frequency meter of the moving-coil ratiometer type with two coils. One coil has an inductor in series with it and the other a capacitor. Each has a rectifier in series with it as shown in Fig. 22.5.

The transmitter has a permanent magnet *M* and two coils *CC* the flux threading which varies as the armature *A* (of which there are 8 or 10 on the wheel concerned in the measurement) passes

through the air gap of the magnet system. An e.m.f. is induced in the coils. The frequency of this e.m.f. is proportional to the speed of the train. Although the induced e.m.f. also varies with train speed the readings of the frequency meter are independent of this variation.

The principle of operation of the indicator is that, as the frequency varies, the relative values of the two opposing torques produced by currents in the two coils change, e.g. an increase in



(*Everett Edgcumbe & Co., Ltd.*)

FIG. 22.5. PRINCIPLE OF OPERATION OF RAILWAY SPEEDMETER

frequency reduces the current in the inductive circuit and increases that in the capacitive branch.

The Measurement of Frequency. We consider, first, frequency standards: for calibration these must be referred ultimately to the fundamental frequency, which is that of the rotation of the earth. This can be very accurately measured by astronomical methods.

Thus, a primary standard of frequency takes the form of an oscillator generating a frequency which is constant over long periods and which is checked against that of the earth's rotation.

Very stable oscillators used as secondary standards are checked against the primary standards.

The primary standards have as a basis a quartz oscillator operating at 100 kc/s and having a low temperature coefficient of frequency with thermostatic control of temperature. It has been found possible with such methods (Ref. (12)), by careful design, to

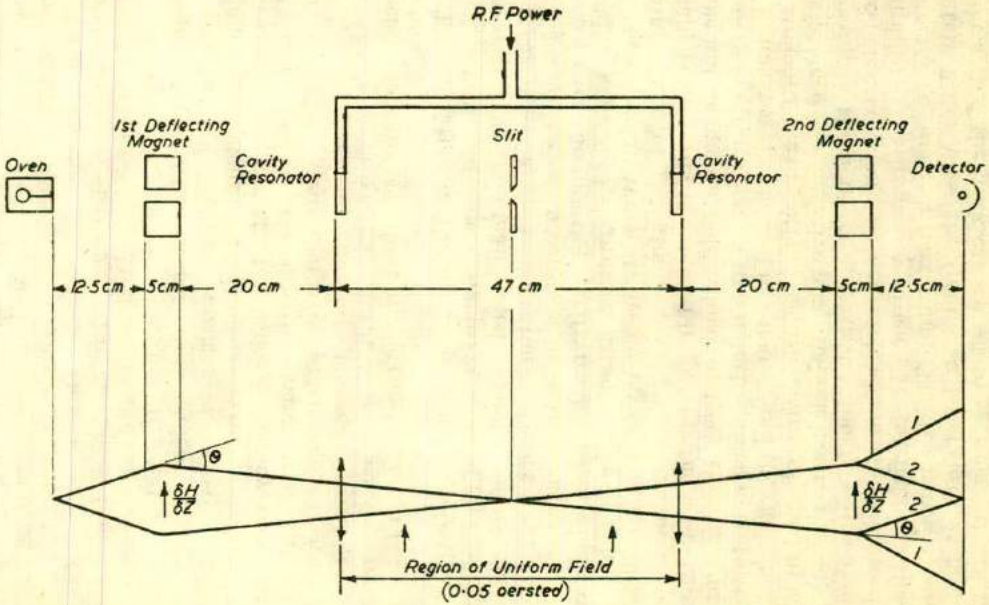
obtain an accuracy approaching 1 part in 10^8 over a period of several months, and an even greater accuracy for shorter periods.

THE CAESIUM RESONATOR. In recent years L. Essen and his associates, at the National Physical Laboratory, have developed the "Caesium Resonator" as a fundamental frequency standard (Refs. (23), (24), (25)). This is the most precise frequency standard yet made and has a reproducibility better than $\pm$ 1 part in 10^{10}. It is used to calibrate quartz oscillators, and compared with astronomical methods, it is believed to give an improvement in accuracy of over 100 times in such calibrations. Once the exact frequency of this device in terms of the second of ephemeris time has been decided, it will provide a standard independent of astronomical measurements.

The operation of the Caesium Resonator depends upon the magnetic properties of the atom which are due to the angular momentum of the extra-nuclear electrons, their spin, and the magnetic moment of the nucleus. When atoms are passed through an alternating field at a particular frequency, there may be a change of energy state associated with a change in polarity of the magnetic dipoles. The frequencies at which such transitions occur are critical and unique. Caesium is used because the frequency of transition, approximately 9,192 Mc/s, is convenient for waveguide techniques, and, in addition, the caesium atom is relatively easy to detect.

The operation of the resonator will be understood by reference to Fig. 22.6. The new standard consists of an evacuated tube with an oven at one end maintained at 200° C and containing a caesium source. A beam of caesium atoms emerges from a slit in the oven and passes through a constant deflecting magnetic field. The beam then passes through two cavity resonators in which an exciting r.f. field is established. A weak constant magnetic field is superimposed on the r.f. field. When the frequency of the r.f. field is at its critical value, some of the atoms change their magnetic polarity. This change of polarity is detected by passing the beam through a second constant magnetic field which deflects those atoms which have undergone transition on to a detector. The detector takes the form of a tungsten wire heated to 1,000°C, and the caesium atoms striking it are emitted as positive ions, which are collected by a curved plate maintained at a negative potential relative to the wire. The current flowing in the detector is passed through a 10^{10}-ohm resistor and the potential difference measured with a vibrating-reed electrometer amplifier. The frequency of the exciting field is adjusted until the detector current is a maximum. The resonance occurs at approximately 9,192 Mc/s, the frequency bandwidth at half deflection is 350 c/s, and the peak can be set to $\pm$ 1 c/s. Hence the frequency can be set to $\pm$ 1 part in 10^{10}.

PRECISE FREQUENCY MEASUREMENT. A quartz crystal oscillator will give a single frequency with a very high precision, but, for



(Courtesy of The Royal Society)

FIG. 22.6. CAESIUM RESONATOR

frequency measurement, a wide variety of accurately known frequencies must be available. Such a range of frequencies can be obtained from a quartz oscillator by means of harmonic generators and frequency dividers. Fig. 22.7 gives a block diagram of a typical arrangement. Simple multiples of the oscillator frequency are usually obtained by means of Class C harmonic generators. These are simple triode amplifiers, biased beyond cut-off, and having anode circuits tuned to a harmonic of the input signal. A single Class C stage will give satisfactory frequency multiplication up to

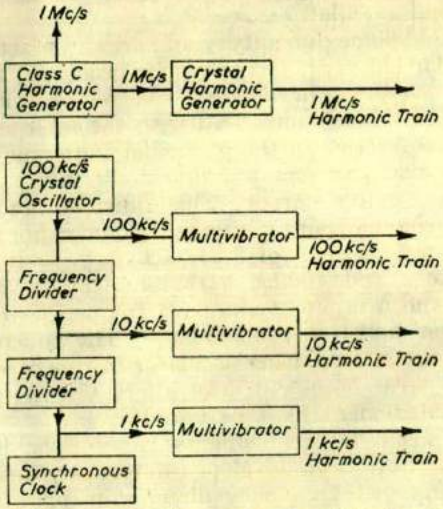


FIG. 22.7. FREQUENCY STANDARD WITH HARMONIC TRAINS

$\times$ 5, so that two stages are normally required for $\times$ 10 multiplication (Ref. (26)).

Any non-linear device will give a continuous chain of harmonics from a single-frequency signal, and a silicon or germanium crystal rectifier is adequate for this purpose above 1 Mc/s. Below 1 Mc/s it is usual to employ an electronic mutivibrator for this purpose. The action of the multivibrator is described in most of the standard works on electronics (Ref. (26)). It is clear that each harmonic in a harmonic train will have the same frequency stability as the fundamental, and, if a multivibrator is driven from a quartz oscillator, its output will consist of a train of accurately known multiples of the oscillator frequency.

A frequency divider will give an exact sub-multiple of the input frequency, a scale of 10 being usually adopted. A variety of circuit arrangements have been devised for frequency division, the most common being the regenerative frequency divider (Ref. (26)) and the so-called "Phantastron" circuit (Ref. (27)).

Referring again to Fig. 22.7, the 1-kc/s output from the last frequency divider drives a synchronous clock which is used to check the oscillator frequency against standard time signals.

The frequency-measuring set gives a large number of accurately known frequencies, and, when measuring an unknown frequency, the standard frequency nearest to the unknown frequency is mixed with it in a modulator which gives both the sum and difference frequencies. The difference frequency is then measured by mixing it in another modulator with the output of a continuously variable interpolating oscillator whose frequency is adjusted until the difference frequency from the second modulator is zero.

It is clear that some difficulty would arise in practice in indentifying a particular harmonic of the quartz oscillator, and it is usually necessary to select the harmonic by means of switched filters. If the frequency difference measured with the interpolating oscillator is a substantial fraction of the particular harmonic used, there will be a significant loss in accuracy unless the interpolating oscillator itself has a very high accuracy. This difficulty can be overcome by mixing the harmonic trains giving additional calibrating points. If, for example, a frequency of 3,247·3 kc/s is to be measured, the 3 Mc/s, 200 kc/s, and 40 kc/s harmonics can be mixed with the 1-kc/s train, and a range of standard frequencies obtained spaced 1 kc/s between 3,240 and 3,250 kc/s. The difference frequency measured on the interpolating oscillator in this case is 300 c/s, and a 1 per cent error here will only affect the overall accuracy of frequency measurement by 1 part in 10^6.

Frequency-measuring instruments based upon high-speed electronic counters have been developed in recent years. These devices display the unknown frequency directly in digital form, and can also be used for the precise measurement of time intervals: they are described in Chapter XXIII.

H. W. Sullivan, Ltd. make several valve oscillators which are sufficiently stable in frequency for them to be used as frequency standards.

A special Sullivan-Griffiths oscillator for 5 to 220 kc/s, with a direct-reading frequency scale of 0·03 per cent accuracy and a frequency stability of 0·003 per cent, has been developed especially for telecommunication testing within the carrier-frequency band.

The same makers also produce two heterodyne oscillators of such stability that they are useful also as frequency standards of 0·2 per cent accuracy. They cover a very wide frequency range and are continuously variable in a single sweep of a dial which is direct reading in frequency. Two frequency ranges are made: 0–16,000 c/s and 50–170,000 c/s.

For most laboratory purposes a tuning-fork which is capable of giving an accuracy of one part in a million is adequate.

An electrically-maintained tuning-fork, made by H. Tinsley and

Co. as a standard frequency source, is run from a 2-V accumulator. The fork has two electrical contacts; one is used for maintaining the fork in vibration and the other can be used in the drive of a chronoscope or phonic motor, or for flashing a stroboscopic lamp. The fork is usually of frequency 50 or 25 c/s but can be made for frequencies up to 500 c/s. It is adjusted to within 2 parts in 10,000 at 20° C with an amplitude of 2 mm, and has a temperature coefficient of 1 part in 10,000 per 1° C rise in temperature.

Frequency Bridges. These are bridge networks similar to those discussed in Chapter VI; indeed, in some cases they are exactly the same networks. The same combination of impedances, and other

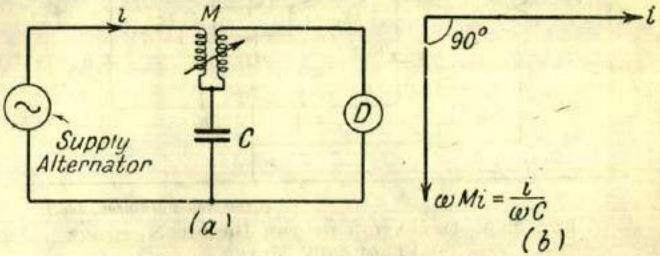


FIG. 22.8. CAMPBELL FREQUENCY BRIDGE

apparatus constituting the network, can be used for more than one purpose; e.g. the same bridge can be used for the measurement of inductance in terms of impedances and the frequency, or alternatively it can be used to measure the frequency in terms of impedances.

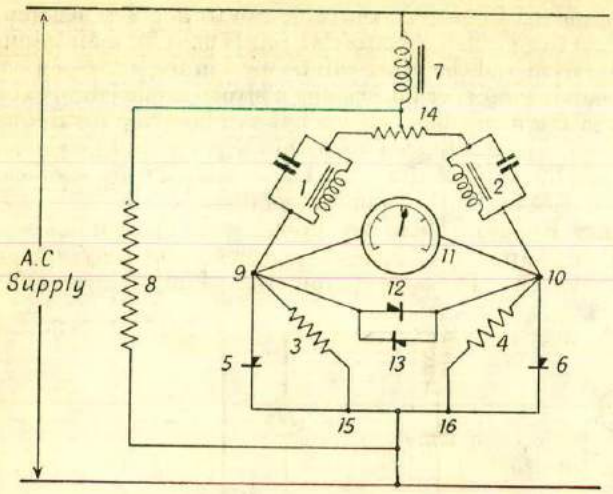
The primary requirement of such bridge networks is that the balancing of the network shall be dependent upon the supply frequency which, when the bridge is so used, is the frequency to be measured.

The choice of a frequency bridge is influenced by the magnitude of the frequency to be measured, upon the apparatus available, and upon the ease with which a complete balance can be obtained.

CAMPBELL'S FREQUENCY BRIDGE. This bridge, which is commonly used, has the advantages of simplicity and of a fairly large range of frequency, provided the capacitor employed is loss-free, and the mutual inductor is free from impurity.

The connections are as shown in Fig. 22.8 (*a*), in which the alternator whose frequency is to be measured is connected to the primary of a mutual inductor M, in series with a loss-free capacitor C. A detector D, which may be either telephones or a vibration galvanometer, according to the frequency under test, is connected in series with the secondary of M, as shown.

Then, neglecting impurities in both M and C, we can say that, when a current i flows in the primary of M, and when M has been



(*Crompton Parkinson, Ltd.*)

FIG. 22.9. OPERATION OF THE BRIDGE NETWORK FREQUENCY METER

With normal frequency the magnitudes of the impedances 1 and 2 are equal, and since the impedances of the resistor/rectifier circuits 3/5 and 4/6 are also equal the bridge network is balanced and no effective current will flow between points 9 and 10.

The permanent-magnet moving-coil microammeter will thus indicate zero microamperes, which point on the scale is figured to correspond with the normal frequency.

With frequency lower than normal, the magnitude of the impedance of the reactive bridge element 1 increases, and that of the reactive bridge element 2 decreases and, since the impedance of the resistor/rectifier elements 3/5 and 4/6 remains constant, the bridge is unbalanced.

The rectifiers 5 and 6 offer a path of very low impedance to one half of the a.c. current wave, so that during this half wave the points 9 and 10 of the bridge network are brought substantially to the same potential as points 15 and 16 which, being solidly connected together, are at the same potential.

Under these circumstances a difference of potential only exists between points 9 and 10 during the other half of the wave, and the pulsating current due to this half wave flows through the microammeter from point 10 to point 9 causing it to deflect in a counter-clockwise direction to a position corresponding to the magnitude of the unbalance of the bridge network, and consequently to the frequency of the supply.

Conversely, with a frequency higher than normal the magnitude of the impedance of the reactive bridge element 1 decreases and that of the reactive bridge element 2 increases, and the bridge network is unbalanced. A pulsating current then flows from point 9 to point 10 of the bridge network.

This pulsating current passes through the microammeter and causes it to deflect in a clockwise direction to a position corresponding to the magnitude of the unbalance of the bridge network, and consequently to the frequency of the supply.

In a microammeter indicating the degree of unbalance of a bridge network, the magnitude of its deflection, in addition to being a function of the degree of unbalance of the bridge, is also dependent on the voltage applied to the bridge network.

In order to render the indication of the microammeter relatively insensitive to variations of the supply voltage within the limits of ± 20 per cent departure from normal, the half-wave rectifiers 12 and 13 are connected to the instrument to form shunts of non-linear impedance.

adjusted so that no current flows through the detector, the voltage induced in the secondary of the mutual inductor is both equal to, and in phase with, the voltage drop across C. This is shown in the simple vector diagram, Fig. 2.28 (b). Thus, the induced voltage in the secondary of M lags 90° behind the primary current i, and is equal to ωMi, where $\omega = 2\pi \times$ frequency.

Again, the voltage drop across C lags 90° behind i, and equals $\frac{i}{\omega C}$.

Hence, at balance, $\omega Mi = \frac{i}{\omega C}$

or $$\omega^2 = \frac{1}{MC}$$

from which the frequency is

$$f = \frac{1}{2\pi\sqrt{MC}} \quad . \quad . \quad . \quad . \quad (22.3)$$

If M and C are not free from impurity, either the impurities must be known and taken into account, or modifications to the simple circuit shown in the figure must be made, in order to render their effect small (see Refs. (5) and (7)). It is also necessary, if a perfect balance is to be obtained, that the frequency shall be constant and that the wave-form shall be free from harmonics.

To overcome the difficulty inherent in the use of the Campbell bridge at low frequencies (below 250 c/s), namely that large values of M and C are required, the Kennelly and Velander bridge (see Ref. (5), 4th Edition, p. 476) can be used.

Frequency Meters. Such meters indicate the supply frequency of a circuit directly, and are thus more convenient for most practical purposes than the foregoing, more precise, methods.

Two main classes of frequency meters are of importance—

(a) Instruments employing electrical resonance.

(b) Instruments whose action depends upon the variation of impedance of an inductive circuit with frequency.

One example of each of these types will be described.

Electrical Resonance Frequency Meter. A very simple form of indicating frequency meter, whose action is dependent upon electrical resonance is illustrated in Fig. 22.10 (a). A magnetizing coil, which is connected across the supply circuit, and therefore carries a current of the frequency to be measured, is wound on one end of a laminated iron core. Over this core is a moving coil, pivoted as shown, and having a pointer attached to it. The terminals of this coil are connected to a suitable capacitor C.

The principle of operation can be understood from the three vector diagrams, Fig. 22.10, (b), (c), (d), in which I is the current in the magnetizing coil and ϕ the flux in the iron core, this being assumed to be in phase with I. The voltage induced in the pivoted coil will lag 90° in phase behind ϕ in all cases. i is the current in the pivoted coil.

Now, in diagram (*b*), the circuit of the pivoted coil is assumed to be largely inductive: the current in it thus lags behind the induced voltage *e*, and the torque acting upon the coil is proportional to $Ii \cos (90° + \alpha)$. In diagram (*c*), the circuit is supposed to be largely capacitive, so that the current *i* leads *e* in phase. The torque is now proportional to $Ii \cos (90° - \beta)$, and is obviously in the opposite direction to what it was in the case represented by diagram (*b*). In diagram (*d*), the inductive reactance is equal to the capacitive

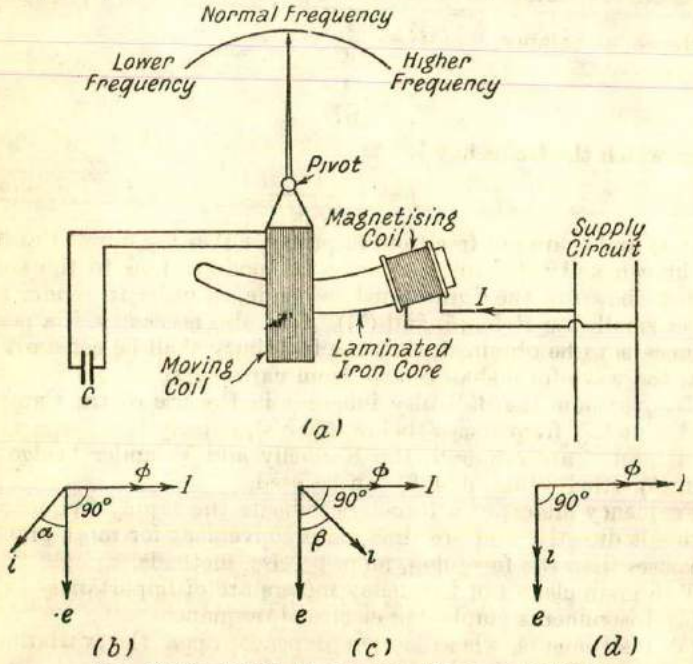


FIG. 22.10. A.E.I. RESONANCE FREQUENCY METER

reactance, so that *i* is in phase with *e*, and the torque upon the pivoted coil is proportional to $Ii \cos 90°$, which is zero.

Operation. Referring now to the actual operation of the instrument, the capacitive reactance $\frac{1}{\omega C}$ $\left(\text{or } \frac{1}{2\pi fC}\right)$ is constant for any given frequency The inductive reactance ωL depends upon the position of the pivoted coil on the core. The nearer this coil approaches the magnetizing coil the greater is its inductance. The pivoted coil is pulled towards the magnetizing coil, therefore, until $\omega L = \frac{1}{\omega C}$, when, as already seen, the torque is zero. Obviously the circuit of the moving coil is in resonance when the torque is zero.

The value of the capacitor C is so chosen that the pivoted coil takes up a convenient mean position when the frequency is at its normal value. If the frequency increases, the capacitive reactance $\frac{1}{\omega C}$ decreases, and the coil moves *off* the core so that its inductance is reduced to the point where ωL again equals $\frac{1}{\omega C}$. The coil moves farther *on* to the core if the frequency falls.

An advantage of this type of instrument is that, if the inductance

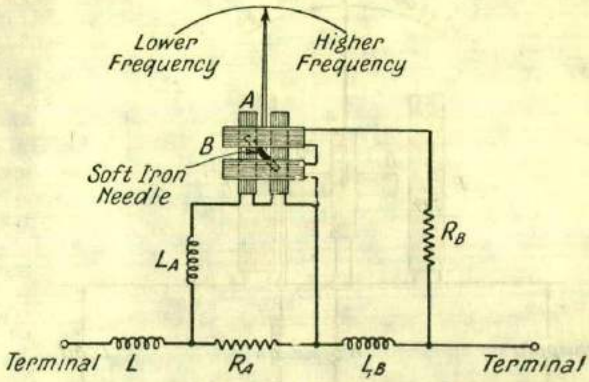


FIG. 22.11. WESTON FREQUENCY METER

of the moving coil changes slowly with variation of its position on the core, great sensitivity can be obtained.

WESTON FREQUENCY METER. This is a moving-iron instrument whose action depends on the variation in current distribution between two parallel circuits—one inductive and the other non-inductive—when the frequency changes.

The construction and internal connections are shown in Fig. 22 11. There are two fixed coils, A and B, each in two equal parts. These coils are fixed so that their magnetic axes are perpendicular, and at their centre is a pivoted soft-iron needle which is long and thin. The spindle bearing the needle also carries a pointer and damping vanes, but there is *no controlling device.*

Coil A is connected, in series with an inductor L_A, across a non-inductive resistor R_A, and coil B is connected, in series with a resistor R_B, across an inductor L_B, as shown. The inductor L is for the purpose of damping out harmonics in the wave-form of the current through the instrument, and so eliminating errors due to such harmonics.

The soft-iron needle takes up a definite position which is dependent upon the currents through the coils A and B. If the frequency is increased the current through coil A is reduced, since the reactance of L_A increases while the voltage drop across R_A remains the same.

On the other hand, the current in coil B increases. The total effect is therefore that of turning the pivoted needle so that it lies more nearly parallel to the axis of coil B.

Decrease of frequency obviously has the opposite effect, and these variations of frequency are followed by the pointer, as shown.

Phase, or Power-factor, Meters. These instruments indicate the power factor of a circuit directly, as distinct from instruments from

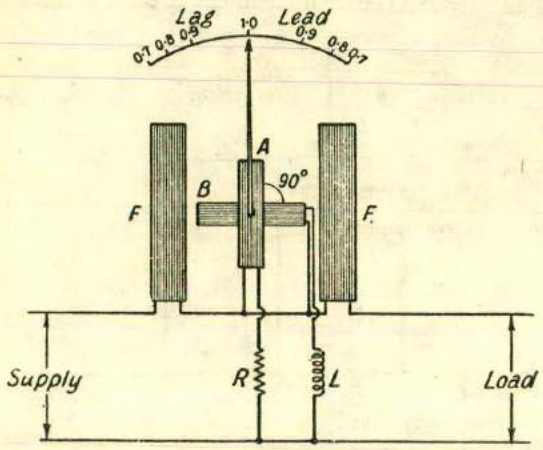


FIG. 22.12 SINGLE-PHASE POWER-FACTOR METER, DYNAMOMETER TYPE

whose readings the power factor may be obtained by dividing the watts supplied, by the volt-amperes in the circuit.

Power-factor meters—like wattmeters—have a current circuit and a voltage circuit. The former carries the current in the circuit whose power-factor is to be measured, or a definite fraction of this current. The voltage circuit is usually split into two parallel paths —one inductive and one non-inductive—and the deflection of the instrument depends upon the phase differences between the main current and the currents in the two branches of the voltage circuit, i.e. upon the power factor of the circuit.

DYNAMOMETER-TYPE SINGLE-PHASE POWER-FACTOR METER. The principle of this instrument is illustrated in Fig. 22.12. The two fixed coils, FF, carry the current in the circuit under test, and the magnetic field of these coils is thus proportional to the main current. Pivoted within this field, between the fixed coils, are two coils A and B, rigidly fixed at an angle of 90° apart. These coils move together and carry a pointer which indicates the power factor of the circuit directly, on the scale above. The numbers of turns on these two coils, as well as their dimensions, are the same, so that they

produce equally strong magnetic fields (90° displaced in space and also in time phase) when equal currents, having a time-phase difference of 90°, are passed through them. The resistor R and inductor L, in series with coils A and B respectively, are adjusted so that these currents are equal when the frequency is at its normal value. At other values of frequency the two currents will be different, since that in coil B is dependent upon frequency—owing to the reactance of L being so dependent—while the current in A is practically independent of frequency.

In discussing the action of the instrument it will be assumed that the time-phase difference between the two currents in A and B is

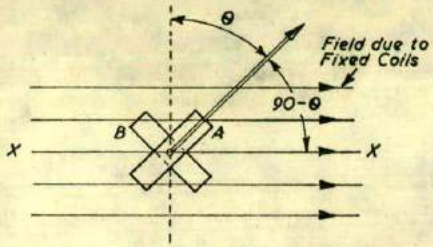


FIG. 22.13

exactly 90°. Actually this is not quite correct, and the angle between the two coils is made rather less than 90°—i.e. equal to the actual time-phase displacement between the currents.

A capacitor may be used instead of the inductor L, when a phase displacement of more nearly 90° may be obtained.

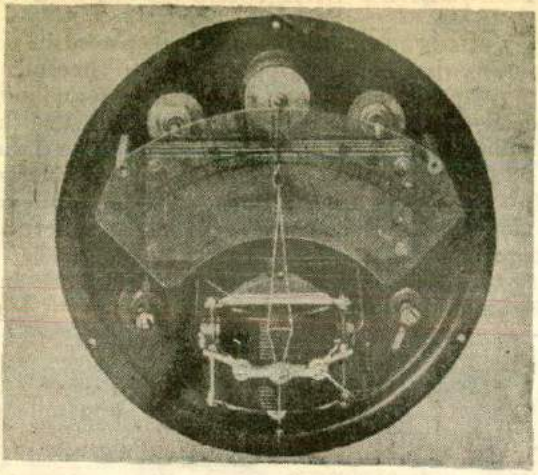
There is no controlling torque acting upon the moving system, the currents being led into the coils by fine ligaments which exert no control.

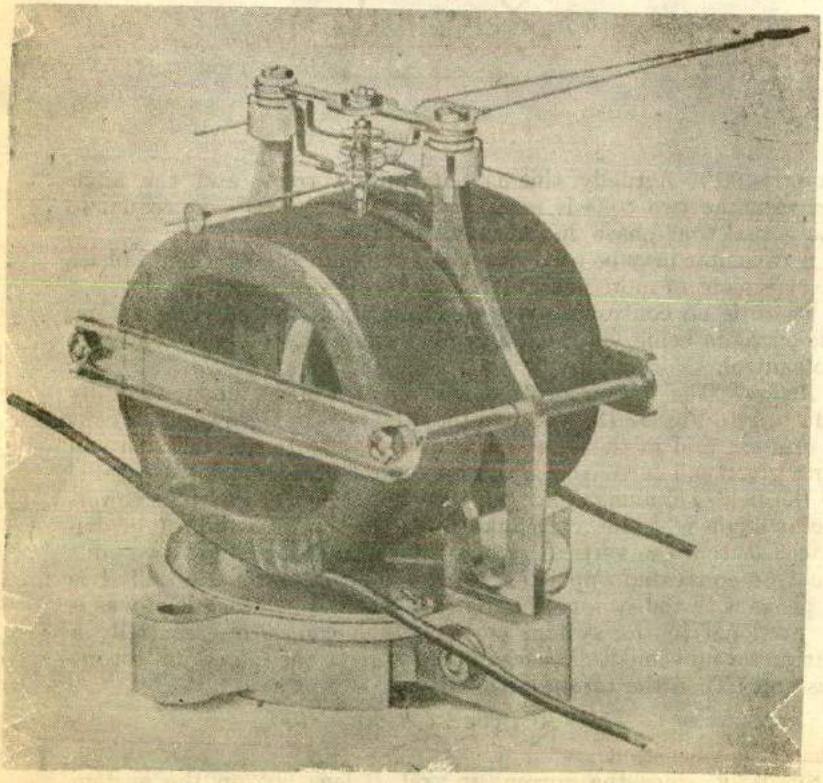
Action. The action of the instrument can be understood by referring to Fig. 22.13. Assume that the field of the two fixed coils is uniform and in the direction arrowed. The torque on each coil for a given coil current will be a maximum when the coil is parallel to the field, i.e. along XX. Suppose that, when the system power-factor angle is ϕ, the coils take up a position of equilibrium displaced θ from the vertical. Then the torques due to the two coils must be equal and opposite. Now, since the current in coil A is in phase with the system voltage, and the field in which it moves is proportional to the system current, then coil A is essentially a wattmeter movement displaced $90° - \theta$ from the maximum torque position. Then the torque of A is

$$T_A = kVI \cos\phi \cos(90° - \theta)$$

where k is a constant.

Similarly, since the current in coil B lags 90° on the system





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FIG. 22.14. WESTON POWER-FACTOR METER

voltage, coil B is a sine-meter movement displaced θ from the maximum torque position. The torque on B is

$$T_B = kVI \sin\phi \cos\theta$$

When the movement of the instrument is in equilibrium,

$$T_A = T_B$$

i.e. $$\cos\phi \cos(90° - \theta) = \sin\phi \cos\theta$$

$$\tan\theta = \tan\phi$$

or $$\theta = \phi$$

i.e. the angular position taken up by the coils is equal to the system phase angle.

The full theory of the instrument is given by Dover (Ref. (3)) and

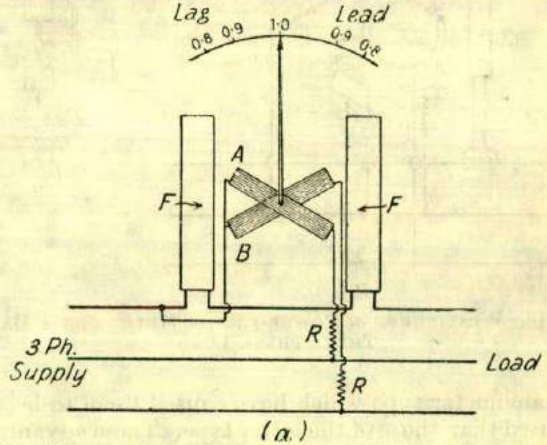


FIG. 22.15. DYNAMOMETER POWER-FACTOR METER FOR A BALANCED THREE-PHASE LOAD

by Laws (Ref. (2)). It has the advantage of being unaffected by the values of current and voltage unless these are very small, but its readings are affected by frequency and wave-form.

Fig. 22.14 shows the construction of an instrument of this type.

DYNAMOMETER POWER-FACTOR METER FOR BALANCED THREE-PHASE LOAD. Fig. 22.15 shows the connections of a three-phase power-factor meter, the readings of which are correct only when the load is balanced.

In this instrument the two moving coils are fixed with their planes 120° apart, and are connected across two different phases of the supply circuit, the fixed coils being connected in the third phase and carrying the current in the line. There is now no necessity for phase-splitting by artificial means, since the required phase displacement between the currents in the two moving coils can be obtained from the supply itself, as shown.

Provided the two moving coils are 120° apart, the angle through which the pointer is deflected from the unity-power-factor position is equal to the phase angle of the circuit.

The three-phase instrument gives indications which are independent of frequency and wave-form, since the currents in the two moving coils are both affected in the same way by any change of frequency.

MOVING-IRON POWER-FACTOR METERS. Instruments of the moving-iron type have certain advantages over power-factor meters.

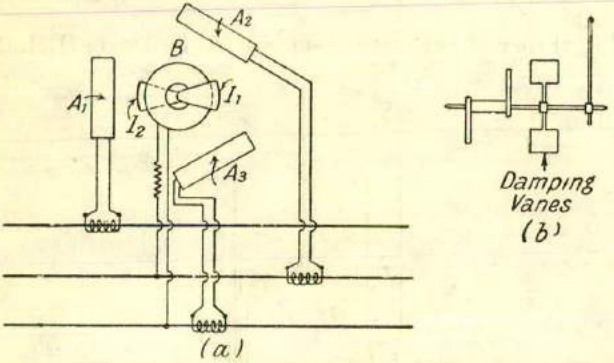


FIG. 22.16. WESTINGHOUSE POWER-FACTOR METER FOR A BALANCED THREE-PHASE LOAD

of the dynamometer type which have caused them to become more generally used than those of the latter type. These advantages are—

(*a*) Larger working forces—these forces being rather small in the dynamometer type.

(*b*) The absence of ligaments to lead in current to moving coils, all the coils in the moving-iron types being fixed.

(*c*) A scale which extends over 360°.

Errors are, however, introduced owing to losses in the iron parts of the instrument, these losses being dependent upon the load and frequency.

In general, moving-iron power-factor meters are less accurate than those of the dynamometer type.

ROTATING-FIELD TYPE FOR BALANCED THREE-PHASE LOADS. Fig. 22.16 illustrates the principle of the Westinghouse instrument of this type. A rotating field is set up by the three coils A_1, A_2, and A_3, which are supplied from the three-phase mains through current transformers. The coil B is placed at the centre of this system of coils, and is connected in series with a resistance, across two lines of the supply. Inside coil B is a short pivoted iron rod, with sector-shaped iron pieces I_1 and I_2, at its ends. Damping vanes and a

pointer are also carried on the same spindle. The moving system is drawn separately in Fig. 22.16 (*b*). There are no controlling forces.

The moving-iron system carries an alternating flux. It does not tend to rotate continuously, but sets in a definite position which depends upon the relative phases of the current in coil *B*—which is practically in phase with the voltage—and of the currents in coil *A*. The deflection of the moving system is approximately equal to the angle of phase displacement between current and voltage in the three-phase circuit.

The presence of iron in the instrument and the dependence of the reactance of coil *B* upon frequency render calibration of the instrument at the normal frequency necessary.

NALDER-LIPMAN POWER-FACTOR METER. This instrument, due to Lipman and manufactured by Nalder Bros. & Thompson, Ltd., has three moving irons like those used in the instrument described above. These are all mounted on a single spindle, one above the other as shown in Fig. 22.17. They are separated from one another by non-magnetic distance pieces, and the three pairs of sectors are displaced in space by 120° relative to one another. Each iron is magnetized by a voltage coil, these coils being connected across the three phases of the supply. There is one current coil, divided into two equal parts, parallel to one another, one on each side of the moving system, this coil being connected in one line of the supply.

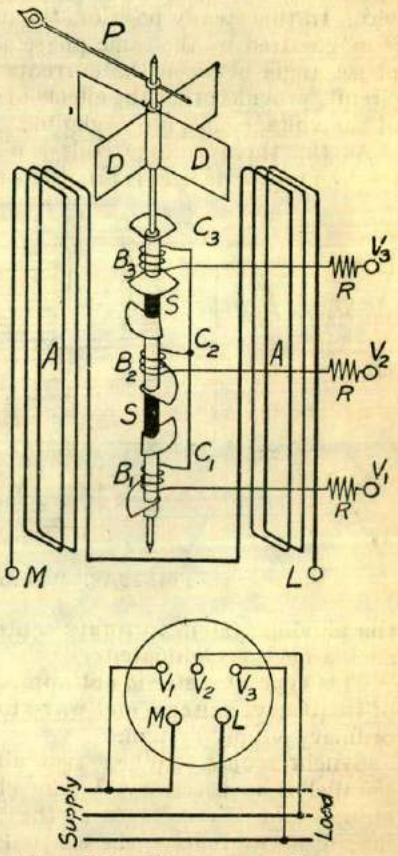


FIG. 22.17. MOVING-IRON POWER-FACTOR METER

(From *Theory and Practice of Alternating Currents*, Dover)

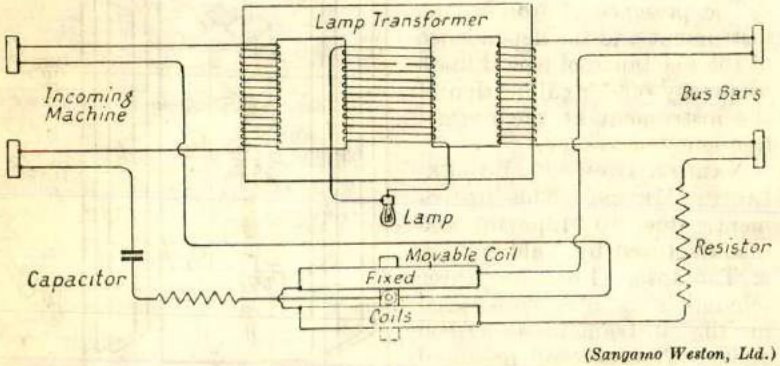
The spindle of the moving system also carries damping vanes and a pointer, but there is no controlling torque.

As arranged in the figure, the instrument is for use on a balanced three-phase load, but it can also be modified for use on an

unbalanced three-phase circuit and for two-phase and single-phase circuits.

When in action, the moving system of the instrument turns into such a position that the mean torque upon one of the iron pieces is neutralized by the two other torques, so that the resultant torque is zero. In this steady position, the deflection of that iron piece which is magnetized by the same phase as the current coil is equal to the phase angle between the currents and voltages of the three-phase circuit, provided that the effects of iron losses and of the inductance of the voltage coils are negligible.

As the three voltage coils are at different levels, no resultant *rotating* magnetic field is produced by them, so that any tendency for



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FIG. 22.18. WESTON SYNCHROSCOPE

the moving system to rotate continuously, following the "drag" of such a field, is eliminated.

This type of meter is not appreciably affected by such variations of frequency, voltage, and wave-form, as might be expected in the ordinary supply system.

Synchroscopes. When two alternators are to be operated in parallel it is necessary, before closing the paralleling switch, to ensure that the voltages of the two machines are equal in both magnitude and phase, the continuous fulfilment of the latter condition implying also that the two machines are of exactly the same frequency.

Synchroscopes are used to indicate when these conditions have been fulfilled. These instruments may be either of the dynamometer or moving-iron type, the latter being the commoner, owing to its 360° scale. Both types are really special forms of phase, or power-factor, meters.

WESTON SYNCHROSCOPE. This is an instrument of the dynamometer type, and is spring-controlled. It is of similar construction to a single-phase wattmeter of the dynamometer type, except that

the fixed coil, as well as the moving coil, is designed for small currents, and is connected, in series with a resistor, across the terminals of the incoming alternator. The moving coil is connected, in series with a capacitor, across the supply with which the alternator is to be paralleled. It may be necessary also to include a small inductance in the fixed-coil circuit, so that exact quadrature between the currents in the two coils may be obtained when the two supplies to be paralleled are in exact synchronism.

When these two currents are in quadrature, there is no deflecting torque acting on the moving system, and the instrument indicates that synchronism has been attained. Since the currents in the two coils will also be in quadrature—giving zero torque—when the two supplies are 180° out of synchronism, a synchronizing lamp is also included in the instrument, and this lamp is at its maximum brightness when synchronism has been obtained. If the supplies are 180° out of synchronism the lamp does not glow at all. This lamp is fed from the secondary winding of a special transformer, which has two primary windings—one supplied from the incoming alternator and the other from the supply with which the machine is to run in parallel. The connections of both the lamp and the coils of the instrument are shown in Fig. 22.18.

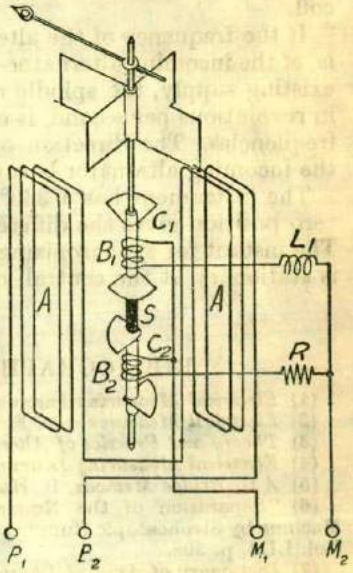


FIG. 22.19. PRINCIPLE OF MOVING-IRON ALTERNATING-FIELD SYNCHROSCOPE

(From *Theory and Practice of Alternating Currents*, Dover.)

The scale of the instrument is of opal glass, and is marked "Slow" and "Fast" on either side of the centre position. The lamp is placed behind the pointer and scale, so that there can be no confusion regarding the correct instant for synchronizing, this instant being when the pointer is steady at the centre-zero position on the scale and the lamp is at full brightness.

If the frequencies of two supplies are not the same, the pointer will oscillate about the centre position.

MOVING-IRON TYPE. A common form of moving-iron synchroscope is the Lipman instrument, which is of similar construction to the single-phase Lipman power-factor meter.

It has two iron pieces mounted one above the other on a common spindle, as shown in Fig. 22.19. The coil which, in the power-factor

instrument, was the current coil, and was divided into two equal parts on opposite sides of the spindle, is now connected, in series with a non-inductive resistor, across the existing supply. The two voltage coils are connected across the terminals of the incoming alternator, one of the coils having a non-inductive resistor in series with it, while the other is connected in series with an inductive coil.

If the frequency of the alternating field of the moving irons—that is, of the incoming alternator—is not exactly the same as that of the existing supply, the spindle rotates continuously at a speed which, in revolutions per second, is equal to the difference between the two frequencies. The direction of the rotation depends upon whether the incoming alternator is running fast or slow.

The instrument has a 360° scale, and the displacement from the zero position gives the difference in phase between the two e.m.f.s. The instant for synchronizing is when the pointer of the instrument is stationary at the central, or zero, position.

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CHAPTER XXIII

ELECTRONICS AND ELECTRONIC MEASURING DEVICES

DURING the last few years electronic devices of various kinds have been used increasingly for measurement purposes. Some of the most important of the methods employed and their application to the measurement of certain non-electrical quantities are described in this chapter, which must, however, begin by giving a brief resume of the underlying physical principles.

The Physics of the Thermionic Valve. The behaviour of the valve can be explained by means of the model of the atom given in 1913 by Bohr (Ref. (1)). Modern physics requires a rather more complicated model, but Bohr's atom is sufficient for the present purpose.

According to this the atom consists of a heavy positive *nucleus* surrounded by a number of unit negative charges or *electrons*. In the normal state of the atom the total negative charge of the electrons is equal to the total positive charge of the nucleus. The number of electrons determines the chemical properties of the atom, and is called the *atomic number*. The mass of the atom is mainly concentrated in the nucleus, and the relative mass (with oxygen as 16) is called the *atomic weight*. Two atoms may have the same atomic number, and therefore the same chemical properties, but different atomic weights: they are then called *isotopes*.

The electrons rotate around the nucleus in orbits which are not arbitrary but must have one of a given range of sizes. The orbits fall into shells, each of which can accommodate only a certain number of electrons, e.g. the first can accommodate 2, the second 8, the third 18, etc. The first 30 elements are listed, with the arrangement of their electrons, in Table XXIV (Ref. (2)). The atomic weights given are those of the element as it is found in nature. In many cases the element so found is a mixture of isotopes: each isotope has an atomic weight nearly equal to a whole number, e.g. ordinary chlorine consists of 75·4 per cent of an isotope having atomic weight 34·9803, and 24·6 per cent of another having atomic weight 36·9779.

In a solid the atoms are arranged in some sort of pattern. If the outer shell of each atom has only a few electrons, these will be only slightly attracted by the nucleus, and will be almost equally attracted by adjacent nuclei. They can thus move freely through the solid, which is therefore a conductor. This is the state of affairs in metals. In an insulator the electrons are all bound to their own nucleus, and considerable energy is needed to tear them away.

The electrons in a metal move about continually at random, somewhat like the atoms of a gas. Modern theory (Ref. (1)) shows that the energy will be distributed among the electrons as shown in Fig. 23.1. Even at a temperature of absolute zero the electrons

TABLE XXIV

Element	Atomic Number	Atomic Weight	No. of electrons in shell			
			1st	2nd	3rd	4th
Hydrogen	1	1·008	1			
Helium	2	4·003	2			
Lithium	3	6·940	2	1		
Beryllium	4	9·02	2	2		
Boron	5	10·82	2	3		
Carbon	6	12·010	2	4		
Nitrogen	7	14·008	2	5		
Oxygen	8	16·000*	2	6		
Fluorine	9	19·00	2	7		
Neon	10	20·183	2	8		
Sodium	11	22·997	2	8	1	
Magnesium	12	24·32	2	8	2	
Aluminium	13	26·97	2	8	3	
Silicon	14	28·06	2	8	4	
Phosphorus	15	30·98	2	8	5	
Sulphur	16	32·06	2	8	6	
Chlorine	17	35·457	2	8	7	
Argon	18	39·944	2	8	8	
Potassium	19	39·096	2	8	8	1
Calcium	20	40·08	2	8	8	2
Scandium	21	45·10	2	8	9	2
Titanium	22	47·90	2	8	10	2
Vanadium	23	50·95	2	8	11	2
Chromium	24	52·01	2	8	13	1
Manganese	25	54·93	2	8	13	2
Iron	26	55·85	2	8	14	2
Cobalt	27	58·94	2	8	15	2
Nickel	28	58·69	2	8	16	2
Copper	29	63·57	2	8	18	1
Zinc	30	65·38	2	8	18	2

* By definition.

have considerable energy. This is a consequence of the quantum theory (Ref. (3)), according to which the energy of an electron can only change by steps, and not continuously.

An electron can move freely inside a metal because as it passes out of the attracting field of one nucleus it comes within that of another. If an electron tries to leave the metal by passing through the surface, however, it is no longer subject to nearly equal attractions on all sides, and work must be done on it in order to remove it from the metal. The amount of this work depends upon the metal. It is measured by the least amount of energy which must be used

to remove an electron from the metal at 0° K. This is called the *work-function*. The state of affairs is shown diagrammatically in Fig. 23.2.

It is evident from the figure showing the energy distribution among

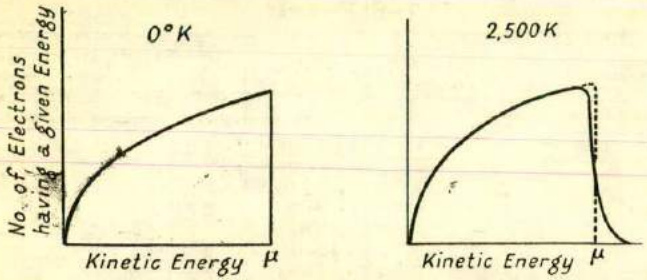


FIG. 23.1. DISTRIBUTION OF ENERGY AMONG ELECTRONS IN A METAL

electrons that, if the temperature is sufficiently high, some electrons will have sufficient energy to escape. If such an electron approaches the surface it will then pass through it. (Some electrons are in fact reflected back into the metal again although they have sufficient

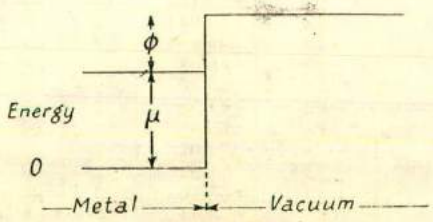


FIG. 23.2

energy to escape.) There will thus be an electron current flowing out of the metal, i.e. a conventional current flowing into the metal, and by *Richardson's equation* this current is proportional to

$$T^2 e^{-a\frac{\phi}{T}}$$

where T is the absolute temperature and a is a constant.

$$a = \frac{e}{k}$$

where e is the charge of an electron and k is Boltzmann's constant. The value of ϕ depends on the metal, and it can be changed very considerably by covering the surface with a layer of another metal only a few molecules thick. The cathodes of many valves are coated with barium and strontium oxides which, after suitable treatment,

give an emitting surface having a small value of ϕ. These are the "oxide-coated cathodes."

Electrons inside a metal can acquire sufficient energy to escape by absorbing energy from incident radiation. The result is known as *photo-electric emission*. The energy of the emitted electrons depends on the wave-length of the incident radiation, while their number depends on the intensity of the radiation. This effect is used in *photo-emissive cells*.

It has been said that the electrons in an insulator require a large amount of energy before they can leave the nucleus with which they are associated. In some solids, called *semi-conductors*, the electrons cannot move freely, but require only a moderate amount of energy to enable them to move. This energy can be acquired thermally—semi-conductors have a lower resistance when hot—or through the absorption of radiation. The *photo-conductive cell* makes use of a semi-conductor which has a lower resistance when illuminated than when in the dark.

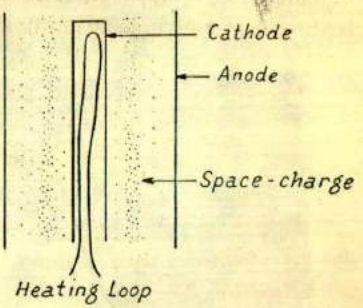


FIG. 23.3. DIAGRAMMATIC SECTION OF A DIODE VALVE

The Diode. The simplest type of thermionic valve is the *diode*. It consists of an electrically-heated *cathode* from which electrons are emitted, and an *anode* to which the electrons can pass, and the whole is enclosed in an evacuated envelope, usually of glass. The cathode of battery-operated valves is generally *directly-heated*, i.e. it consists of a loop of wire through which the heating current is passed, while the surface (which is specially treated) is the actual emitting surface. This design is economical of heating power but does not allow the heating circuit to be electrically isolated from the cathode. In valves designed for mains-operation the cathode is *indirectly-heated*. A loop of wire carries the (alternating) heating current, but the actual cathode is a cylinder surrounding this loop and insulated from it electrically. The cathode surface is specially treated and when heated by the heating-loop it emits electrons.

When the cathode is first heated, some of the electrons inside the metal acquire sufficient energy to escape. If the escaped electrons are not removed they will accumulate in a cloud around the cathode and form the *space-charge* (see Fig. 23.3). Equilibrium will be reached when the number of electrons leaving the cathode is equal to the number which return from the space-charge. (The latter affects the number of electrons emitted, but only to a small extent, by setting up an adverse field near the cathode.) If the anode of the diode is made positive, all the electrons between the anode and

cathode will have an acceleration superimposed on their previous motion, and a current of electrons will flow from cathode to anode. The space-charge will therefore be modified, becoming less dense where the electrons are moving faster, i.e. near the anode. The field produced by the anode has little effect at the surface of the cathode, which is screened by the space-charge.

It is evident that the electron current flowing from the space-charge to the anode will increase as the anode potential is raised. The current under these conditions is much less than that which the cathode is continually emitting, since many electrons return from

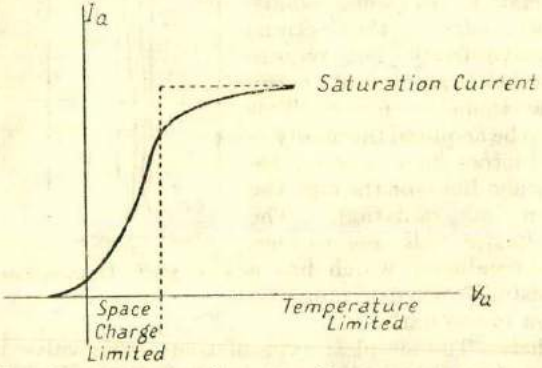


FIG. 23.4. CURRENT/VOLTAGE CHARACTERISTIC FOR A DIODE VALVE

the space-charge to the cathode. The current is said to be *space-charge limited*. If the anode is made very positive, however, all the emitted electrons will go to it, but the field of the anode can only slightly increase the emission of electrons from the cathode. The current will therefore become nearly constant and is said to be *temperature limited*, or to reach *saturation*. If the anode is made negative, electrons will be repelled from it and no current will flow. (Actually a small current flows if the anode is slightly negative, as some electrons are moving fast enough to reach it against the adverse field.)

Thus the graph of current against anode voltage is as shown in Fig. 23.4. The cold anode does not emit electrons, so that no current can flow from cathode to anode. It does, however, emit some secondary electrons (see under Tetrode) but only when anode current is flowing. The secondary electrons are then recaptured by the anode and have no effect on the operation of the valve. The diode valve can evidently be used as a rectifier. It is operated within the space-charge-limited region.

The Triode. Between the cathode and anode of a triode valve is a third electrode, the *grid*. This consists of an open coil of thin wire

surrounding the cathode (see Fig. 23.5). As in the diode, a space-charge is produced near the cathode, but it is now subject to the combined field of the anode and the grid. The grid, being nearer to the space-charge, produces a greater change of current for a given change of voltage than the anode. The characteristics of a triode valve are expressed by two sets of graphs as in Fig. 23.6. It will be noticed that the curves are given for negative values of V_g. Under these conditions the grid does not take any current as the electrons are repelled from it. (Minute currents due to secondary effects are found in actual valves.) Thus it serves to alter the anode current but requires no power to do so. Triodes are normally used with the grid negative, and the grid in most triodes will be damaged if made to carry any appreciable current.

The anode current can be stopped, for any value of V_a, by a sufficiently negative value of V_g. The valve then is said to be *cut off*.

Notice that the current is space-charge limited. If V_a and V_g are sufficiently increased, the current becomes constant, as in the diode. Few valves can be operated in the temperature-limited range without damage.

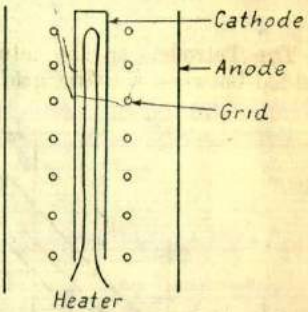


FIG. 23.5. DIAGRAMMATIC SECTION OF A TRIODE VALVE

The two sets of graphs for a valve are not independent, and one can be derived from the other as shown in Fig. 23.7.

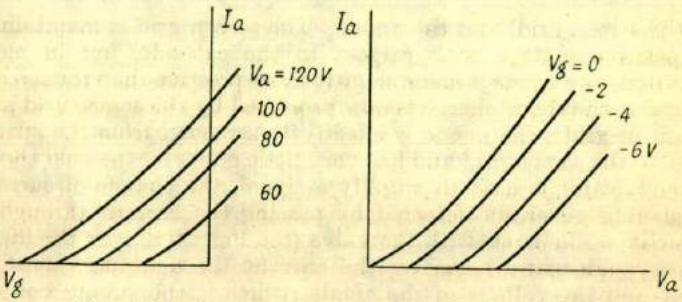


FIG. 23.6. CHARACTERISTICS OF A TRIODE VALVE

A triode valve is generally required to operate with small alternating voltages superimposed on the steady anode and grid voltages. Three quantities (Ref. (1)) are used to define its performance under these conditions—

The anode resistance $$R_a = \left(\frac{\partial V_a}{\partial I_a}\right)_{V_g \text{ constant}}$$

The mutual conductance $g_m = \left(\frac{\partial I_a}{\partial V_g}\right)_{V_a \text{ constant}}$

The amplification factor $\mu = \left(\frac{-\partial V_a}{\partial V_g}\right)_{I_a \text{ constant}}$

It is evident that R_a and g_m are, respectively, the reciprocal of the slope of the right-hand graph and of the slope of the left-hand graph in Fig. 23.6. It can be shown that

$$\mu = g_m \cdot R_a$$

The Tetrode. In the tetrode a second grid—the *screen grid*—is added between the first grid (called the *control grid* to distinguish it

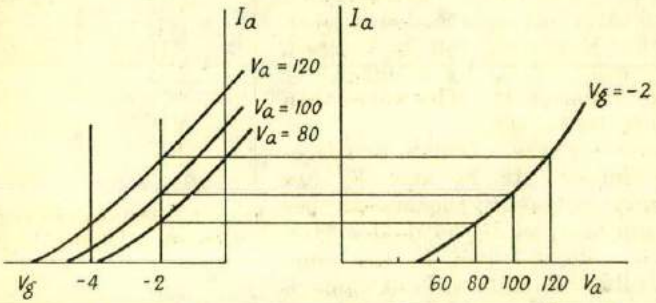


FIG. 23.7. RELATION BETWEEN CHARACTERISTIC CURVES OF A TRIODE VALVE

from the screen grid) and the anode. The screen grid is maintained at a positive voltage with respect to the cathode, but in most applications the anode is made slightly more positive than the screen The field at the space-charge is now produced by the screen grid and the control grid; the anode is effectively screened from the space-charge by the screen grid and has very little effect on the field there.

When a valve is used to amplify a signal, the change of current through it is generally detected by passing the current through a *load resistance* in series with the valve (see Fig. 23.8). As the input signal is made more negative, the current through the valve decreases, and the voltage of the anode (which is the supply voltage less $R_L I_a$) rises. In the triode amplifier shown the increase of the anode voltage tends to increase the current flowing through the valve by attracting more electrons from the space-charge. The screen-grid in a tetrode prevents this effect, and so increases the amplification which can be obtained. It also reduces capacitive coupling between anode and grid.

A tetrode amplifier is shown in Fig. 23.9 although the simple tetrode is not often so used owing to a secondary effect which occurs. Since the screen grid is positive some electrons are attracted

to it and current flows in the screen-grid circuit. Most of the electrons, however, acquire sufficient speed to take them through the meshes of the screen grid and into the attracting field of the anode. When they strike the anode they impart some of their energy to the electrons in the metal and some of these electrons may have sufficient energy to escape from the anode. They are termed *secondary*

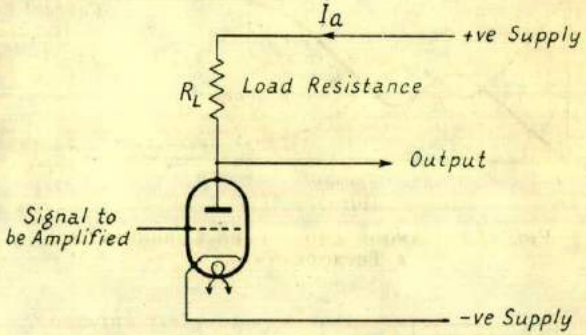


FIG. 23.8. SIMPLE TRIODE AMPLIFIER

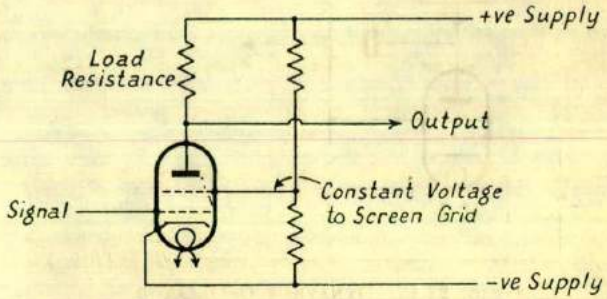


FIG. 23.9. TETRODE VALVE IN AN AMPLIFIER CIRCUIT

electrons to distinguish them from the *primary electrons* arriving from the cathode.

The path of the secondary electrons is determined by the relative potentials of anode and screen. If the anode is more positive than the screen, the majority will fall back into the anode and will not affect the operation of the valve. If, however, the screen is more positive than the anode many of the secondary electrons may be captured by the screen and they constitute a current between screen-grid and anode. The variation of anode current with anode voltage is as shown (Ref. (1)) in Fig. 23.10. In range *A* the primary electrons do not have sufficient energy when they reach the anode to produce a large number of secondary electrons. In range *B*, each

primary electron is producing several secondary electrons, and many of the secondary electrons are captured by the screen which is more positive than the anode. The anode current falls as its voltage rises, so that the valve presents a negative resistance at its anode.

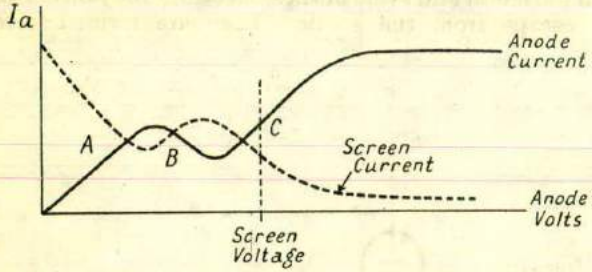


FIG. 23.10. ANODE AND SCREEN CURRENTS IN A TETRODE VALVE

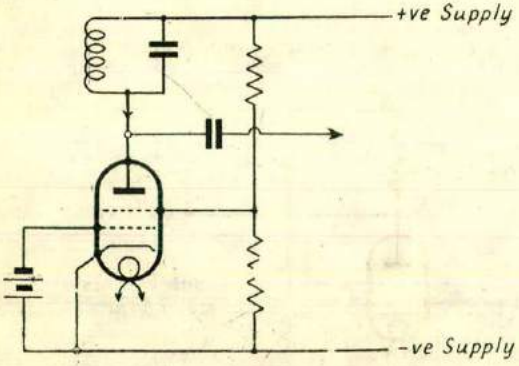


FIG. 23.11. DYNATRON OSCILLATOR
(Note that the screen must be positive with respect to the anode.)

In range *C* secondary electrons are still being produced but most of them are recaptured by the anode.

The negative resistance or *dynatron* range of a tetrode can be used to produce an oscillator. The valve is connected to a tuned circuit, and its negative resistance makes good the losses of this circuit (Fig. 23.11).

A special form of tetrode, known as the *beam tetrode*, avoids the negative-resistance region by a special construction using auxiliary plates to concentrate the primary electrons into a beam.

The Pentode. If a further grid is introduced between the screen and the anode, and is connected to the cathode, all the secondary electrons will return to the anode, since they are protected from

the field of the screen by the intervening *suppressor grid*. The anode characteristics of such a pentode are shown in Fig 23.12.

The Pentode Voltage-amplifier. A very common circuit element is the *voltage-amplifier*, i.e. a valve which gives an output voltage much

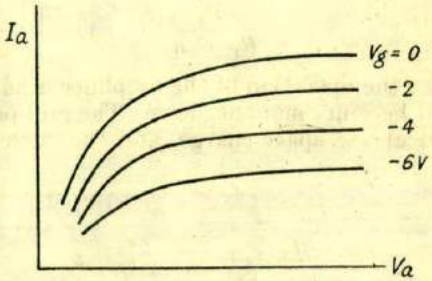


FIG. 23.12. ANODE CHARACTERISTICS OF A PENTODE VALVE

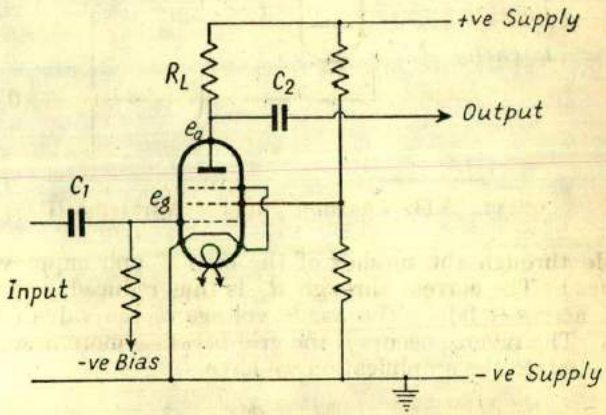


FIG. 23.13. PENTODE VOLTAGE AMPLIFIER (A.C.)

greater than the input voltage, and nearly proportional to it. The properties of such an amplifier will change with time, but can be stabilized by negative feed-back (see below) which, however, reduces the amplification that can be obtained.

Amplifiers are classified as d.c. or a.c. according as they can amplify a direct voltage, or only alternating voltages above a certain frequency.

Fig. 23.13. shows a pentode used as an a.c. amplifier. The lowest frequency which can be amplified depends on the values of C_1 and C_2.

Fig. 23.14. shows a d.c. amplifier. The output is generally required to be near earth potential, while the anode of the valve is at about 150 V positive to earth. A negative supply is therefore provided, and R_1 and R_2 provide a potential divider between the anode and the negative line. The amplification is, of course, reduced in the ratio

$$\frac{R_1}{R_1 + R_2}$$

In each case the operation of the amplifier is as follows. Suppose that the input becomes more negative. The grid produces a stronger negative field at the space-charge, and the current flowing to the

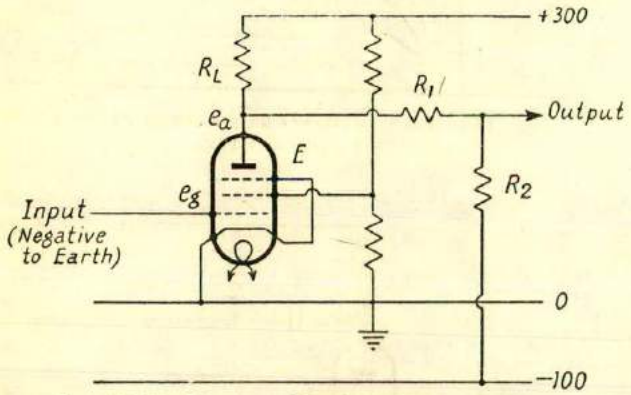


FIG. 23.14. PENTODE VOLTAGE AMPLIFIER (D.C.)

anode through the meshes of the screen- and suppressor-grids is reduced The current through R_L is thus reduced, and the voltage drop across it falls. The anode voltage of the valve consequently rises. The reverse occurs if the grid becomes more positive.

To obtain the amplification we have

$$g_m = \frac{\partial I_a}{\partial e_g} \qquad R_a = \frac{\partial e_a}{\partial I_a}$$

$$\delta e_a = -R_L \delta I_a$$

Now, when e_g and e_a are both changed,

$$\delta I_a = \frac{\partial I_a}{\partial e_g} \cdot \delta e_g + \frac{\partial I_a}{\partial e_a} \cdot \delta e_a$$

$$= g_m \delta e_g + \frac{\delta e_a}{R_a}$$

so $$\delta e_a = -R_L \left[g_m \delta e_g + \frac{\delta e_a}{R_a} \right]$$

$$\delta e_a\left(1 + \frac{R_L}{R_a}\right) = -g_m R_L \delta e_g$$

$$A = \frac{\delta e_a}{\delta e_g} = -\frac{g_m R_L}{1 + \frac{R_L}{R_a}} = -\frac{g_m R_a R_L}{R_a + R_L} = -\frac{\mu R_L}{R_a + R_L}$$

since $\mu = g_m R_a$.

Pentodes are generally used in voltage-amplifier stages, since they have a considerably higher value of μ than triodes, and a

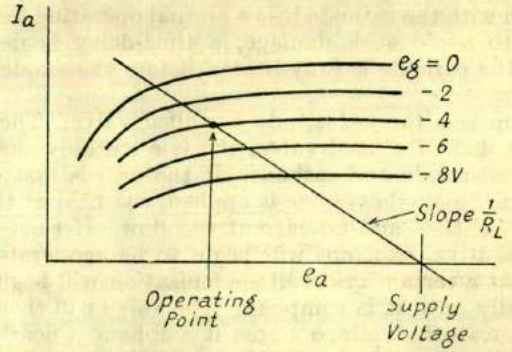


FIG. 23.15. OPERATION OF A PENTODE VOLTAGE AMPLIFIER

greater amplification can be obtained with them. This is, however, not greater in the same proportion as μ, since $\frac{R_L}{R_a + R_L}$ must be less for a pentode than for a triode, to avoid the distortion which is otherwise found.

The operation of a pentode amplifier can be readily followed from the I_a/e_a curves given in Fig. 23.15. The operating point moves on the *load line* of slope $\frac{1}{R_L}$. The change in e_a for a given change of e_g can easily be read off, and the distortion produced when the change in e_g is large is also apparent. (*Distortion* exists when equal changes of e_g do not always produce the same change of e_a.)

The Thyratron. All the valves so far described have been *hard valves*, i.e. the envelope is exhausted to a high vacuum. In the thyratron, gas under low pressure is purposely left within the envelope.

If an electron moving fairly fast within such a valve strikes one of the gas molecules, it imparts energy to the electrons of its constituent atoms, and one or more of these electrons may obtain sufficient energy to escape from the atomic forces. The remaining part of the molecule will have a positive charge, and is termed an *ion*.

The ions produced in this way will be accelerated by the electric field within the valve, and will move in the opposite direction to the electrons, but much more slowly owing to their greater mass. The electrons lost by the ions will join the stream of primary electrons, and may in turn ionize other molecules. The whole process will therefore be cumulative, and once started by a sufficiently strong field, will proceed until the whole of the gas is ionized.

The cathode of a thyratron has to be capable of withstanding the bombardment of the heavy ions. It is generally made of tungsten, sometimes thoriated, i.e. coated with rare-earth oxides. Thyratrons are operated at relatively high currents, and can easily be damaged by running them with the cathode below normal operating temperature. Sometimes, to avoid such damage, a time-delay is necessary to ensure that the cathode is fully-heated before the anode voltage is applied.

A thyratron is a three-electrode gas-filled valve. The grid does not resemble that of a hard valve, but is a suitably shaped metal screen between anode and cathode. If the grid is first made very negative, and the anode voltage is applied, the field at the cathode will remain negative, and no current will flow. If then the grid is made less negative, electrons will begin to be accelerated towards the anode. At a certain grid voltage ionization will begin, and will proceed rapidly until it is complete. The valve will then represent a low resistance—the voltage across it will have a nearly constant value depending on the gas (15 to 20 V with mercury) and independent of the anode current. Once the gas is ionized the grid has no more control. If it is made negative it merely attracts the ions, and these, forming a cloud round it, neutralize its field. The discharge through the valve can only be stopped by removing the anode voltage. Thus the grid in a thyratron acts as a trigger which controls the time at which the valve becomes conducting, but thereafter has no effect.

Small thyratrons can easily carry 0·5–1 A, whereas hard valves of the same size will carry only 10–30 mA. Thyratrons are therefore useful for operating relays, etc. They can also be used as d.c. amplifiers when fed from an a.c. supply, the voltage applied to the grid determining the moment during the cycle of alternating voltage at which the valve begins to conduct, and so altering the current flowing.

Negative Feed-back. We are concerned here with the applications of valves to measurements. In most cases the purpose of the valve will be to give a quantity (current or voltage) strictly proportional to the quantity to be measured but at a much higher power level. We thus extend the range of conventional methods to powers insufficient to operate by themselves, without amplification, the conventional instruments.

The second part of our requirement—to increase the power level

—is easily fulfilled: it is provided in every radio set or gramophone amplifier. The chief difficulty is in the first requirement, that the amplified signal shall be strictly proportional to the input signal. The valve is not in itself a stable or reproducible component, and the amplification of (say) a radio set alters with time and is affected by replacing the valves. In addition, the characteristics of a valve are only approximately linear and only over a small range.

We avoid these difficulties by *negative feed-back*. Consider an amplifier which gives an output which is $-n$ times the input and

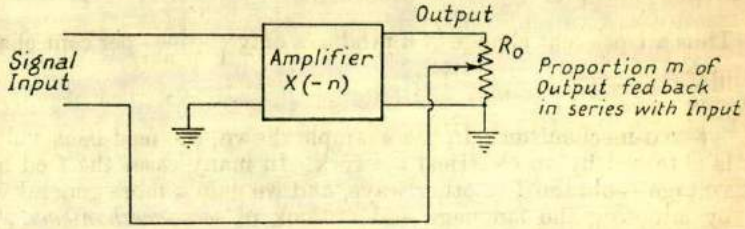


FIG. 23.16. ILLUSTRATION OF NEGATIVE FEED-BACK

suppose that a proportion m of the output is fed back to the input, (see Fig. 23.16).

Let the input signal be e and the output signal E. Then, the input to the amplifier is

$$e + mE$$

But the output is $-n$ times this, i.e.

$$E = -n(e + mE)$$

$$E(1 + nm) = -ne$$

$$E = -e\frac{n}{1 + nm}$$

Now suppose n is large—quite a simple amplifier will give $n = 1{,}000$ or 10,000—and that m is not very small. Then, very nearly,

$$E = \frac{-e}{m}$$

The value of m is fixed by circuit elements, i.e. resistances in the above example, and is quite stable. The effect of n, which is unstable and not independent of the amplitude of the input signal, has been eliminated. Thus if the characteristics of the valves change with time, or with changes of the supply voltages, or if a valve is replaced, the performance of the amplifier remains unchanged so long as n remains large enough.

Differentiating,

$$\frac{d}{dn}\left(\frac{E}{e}\right) = \frac{d}{dn}\left(\frac{-n}{1+nm}\right)$$

$$= -\frac{1+nm-nm}{(1+nm)^2} = \frac{-1}{(1+nm)^2}$$

$$\frac{\delta\left(\frac{E}{e}\right)}{\frac{E}{e}} = \frac{\delta n}{(1+nm)^2}\,\frac{1+nm}{n} = \frac{\delta n}{n}\cdot\frac{1}{1+nm}$$

Thus a 1 per cent change of n produces only $\frac{1}{1+nm}$ per cent change in $\frac{E}{e}$.

Servo-mechanisms. In the example shown, the feed-back voltage is obtained by an electrical network. In many cases the feed-back voltage is obtained in other ways, and we gain a more general view by adopting the language and outlook of *servo-mechanisms*. The elements of a *closed-loop* control system are shown in Fig. 23.17

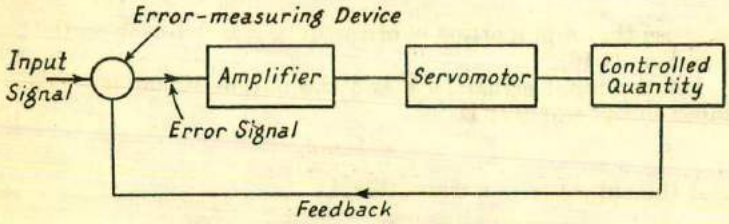


FIG. 23.17. CLOSED-LOOP CONTROL SYSTEM

(Ref. (4)). The input signal is compared with a feed-back signal obtained from the controlled quantity, and the difference should be zero. If it is not zero, it is amplified and applied to a servo motor which resets the controlled quantity in such a way as to reduce the error.

Returning to our previous example of the feed-back amplifier (Fig. 23.16), we can identify each of the elements in the last example. The error-measuring device is simply the series-connected circuit at the input of the amplifier which subtracts a proportion of the output voltage from the input signal. The error signal is amplified, and the output stage of the amplifier is the "servomotor" which supplies power to the resistor R_o and develops a voltage across it. This voltage is the controlled quantity, and a constant proportion m of it is fed back to the error-measuring device.

This identification of elements may seem rather artificial, but a

second example in which some of the elements are mechanical will show how the scheme of a servo-mechanism helps in bringing very different systems under a common theory.

Fig. 23.18 shows a servo-system for setting a shaft to an angle proportional to an input voltage. A linear potentiometer driven by the shaft gives a voltage proportional to the angular setting of the shaft. This voltage is subtracted from the input, and any difference is amplified and drives a motor in such a direction as to reduce the error. Thus the feed-back voltage is always kept equal to the input signal, and to each value of the latter therefore corresponds a given angular setting. The potentiometer and input circuit of the amplifier here form the error-detector, and the feed-back path is the mechanical one via the shaft.

If we regard the voltage picked off from the potentiometer in the last example as the controlled quantity it is evident that the system forms a self-resetting potentiometer. The potentiometer then must

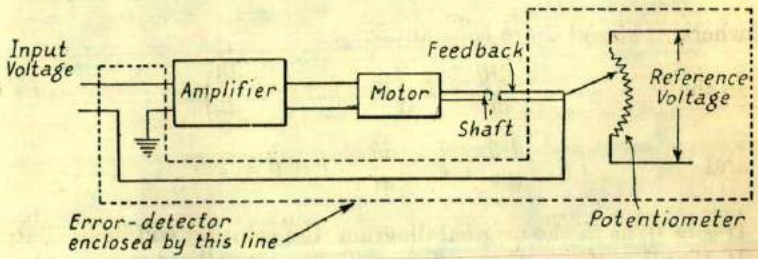


FIG. 23.18. DIAGRAM OF A CONTROL SYSTEM

be considered as part of the "servomotor," and the feed-back path is the electrical one.

Velocity Feed-back. There is a considerable and rapidly-growing literature of servo-mechanisms, in which their dynamical stability and response are discussed. We need not enter deeply into the subject here, but one device is so commonly used that it must be mentioned.

In the last example, suppose that the system is at rest in an equilibrium position, and that the input voltage is then suddenly changed. The amplifier drives the motor in a direction which reduces the error, and if the motor had no inertia (and the amplifier no delay) the speed of the motor would fall off as the shaft approached its final position and become zero when it reached it. The motor, however, will have inertia, and while the shaft is moving for the first time towards its final position, energy is being stored in the motor. Unless arrangements are made to remove this energy the shaft will overshoot its final position, and oscillate about it. One way of dealing with this difficulty would obviously be to fit a mechanical damping device on the shaft. A better method, however, is to make

the feed-back proportional partly to position, but also partly to the rate of change of position.

Let e = the input voltage

θ = the position of the shaft

E = the output voltage of the amplifier

Then, if the inertia of the motor is large, we can neglect its back-e.m.f. and write

$$\frac{d^2\theta}{dt^2} = kE$$

where k is a constant.

But
$$E = n\left(e - m\theta - \alpha\frac{d\theta}{dt}\right)$$

where n, m and α are constants.

so that
$$\frac{d^2\theta}{dt^2} = kn\left(e - m\theta - \alpha\frac{d\theta}{dt}\right)$$

and
$$\frac{d^2\theta}{dt^2} + kn\alpha\frac{d\theta}{dt} + knm\theta = kne$$

If $\alpha = 0$, as in the original diagram, the solution will be oscillatory. If $(kn\alpha)^2 = 4knm$ the system will be critically damped, while if

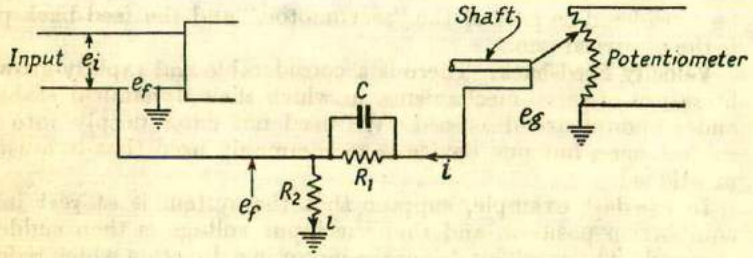


FIG. 23.19. CIRCUIT GIVING VELOCITY FEED-BACK

$(kn\alpha)^2 > 4knm$ the system will be over-damped. Thus the effect of velocity feed-back is the same as that of mechanical damping. But the result is achieved without a great loss of energy, and without increasing the size of the motor in order to supply the damping-forces.

The required velocity feed-back can be achieved very simply by electrical means as in Fig. 23.19, which shows a slight modification of

the arrangement of Fig. 23.18. We have, if R_2 is small, so that we may neglect the voltage across it in comparison with e_g,

$$i = \frac{e_g}{R_1} + \frac{dq}{dt}$$

$$q = Ce_g$$

so that

$$e_f = R_2 i = R_2\left[\frac{e_g}{R_1} + C\frac{de_g}{dt}\right]$$

$$= \frac{R_2}{R_1}e_g + R_2C\frac{de_g}{dt}$$

It should be noted that the above diagrams are given for example only, and are not necessarily the best means for achieving the desired performance (Ref. (8)). The actual design of such systems is difficult, and partly a matter of experiment—the diagrams are only intended to explain the operation of simple systems.

Considering the feed-back amplifier of page 895 as a servo-system it is easily seen that the amplifier resets the controlled voltage by an amount proportional to the error. We found that

$$E = -e\left(\frac{n}{1+nm}\right)$$

so that the input to the amplifier is

$$e + mE = e\left(1 - \frac{mn}{1+mn}\right)$$

$$= \frac{e}{1+mn}$$

Thus, however large mn may be, there is a limit to the value of e which can be applied without over-loading the amplifier.

In Fig. 23.18, however, if the inertia of the motor is large, its acceleration will be proportional to the amplifier output: i.e. the amplifier resets not the controlled quantity but its acceleration. It is evident that when the system is at rest the input to the amplifier is zero, and that over-loading of the amplifier sets no limit to the input signal which can be accommodated. Furthermore, since the amplifier input is always zero when the system is at rest, the amplifier characteristics do not affect the accuracy and linearity of the servo-system, but only the behaviour while it is seeking the rest position.

Examples of Servo-systems using Velocity Feed-back. (*a*) An interesting example is given in Ref. (20). The problem is to make a frost-point hygrometer automatic in operation. Light scattered by the deposit on a cooled thimble is picked up by a photo-cell, P_1 (Fig. 23.20), and the resulting voltage is amplified. As the light reaching

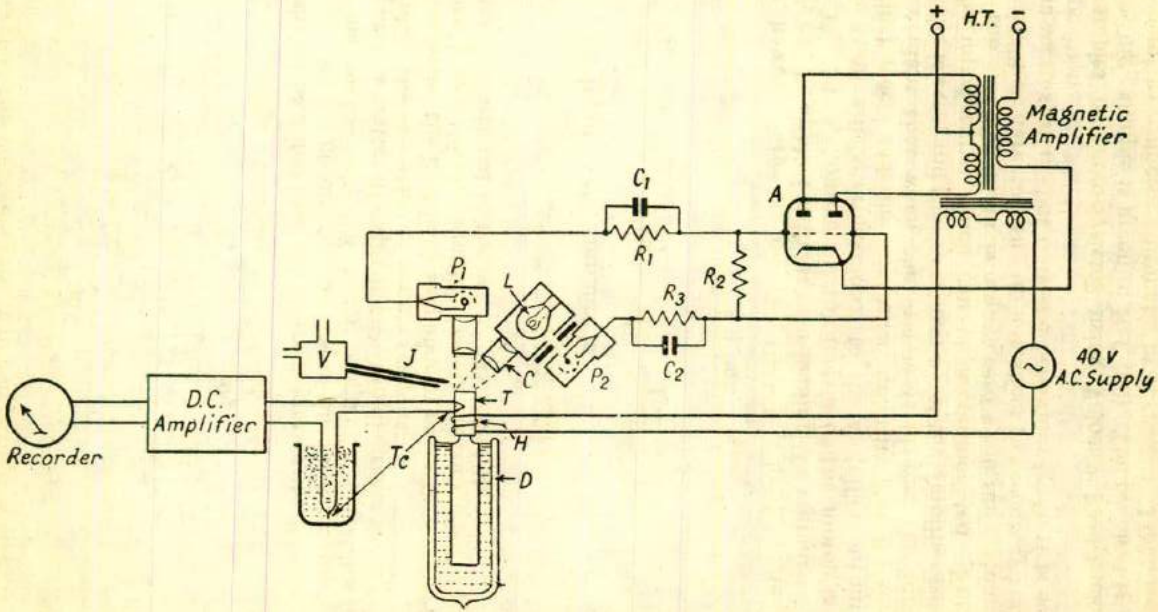


FIG. 23.20. AUTOMATIC FROST-POINT HYGROMETER

the photo-cell increases, the thimble is heated, and conversely. For equilibrium the frost deposit must be neither increasing nor decreasing.

The authors (Brewer and Dobson) state that the frost deposit has the characteristics of a large mass. The servo-system would be unstable unless precautions were taken. Velocity feed-back is used to stabilize the system and is obtained by an electrical circuit as

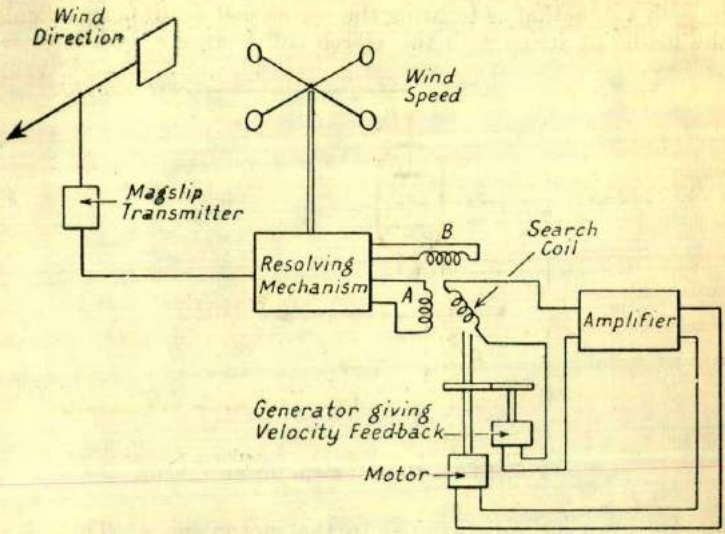


FIG. 23.21. WIND-DIRECTION INDICATOR

shown. The second photo-cell, P_2, is used to eliminate the effect of changes in brightness of the lamp.

(*b*) Another example is given in Ref. (15). The direction of a wind-vane is to be indicated at a distant position, and only a mean position is required, i.e. the faster motions of the vane are to be smoothed out. This cannot easily be done by damping the vane, since the wind-forces are proportional to the square of the wind-speed; damping which is sufficient at one wind-speed will be much too great at lower speeds, and too little at higher speeds.

The information is passed on by magslip transmitter and the damping is provided by heavy velocity feed-back. As we have seen, this is equivalent to mechanical damping, and it is of course independent of wind-speed. Thus the required smoothing is obtained.

The circuit is shown diagrammatically in Fig. 23.21. This is very much simplified from the actual arrangement, which, besides the function mentioned, also resolves the wind, and is intended to show

the true wind direction on board ship by subtracting the ship's vector velocity from the wind velocity.

The resolving mechanism shown in the diagram gives a current in A proportional to one component of the wind velocity, and in B proportional to the component at right angles to the first. When the search coil is perpendicular to the resultant field no voltage will be induced in it, and the motor will remain at rest. If a voltage is induced in the search coil it is amplified and used to drive the motor in such a direction as to bring the search coil again perpendicular to the field. In series with the search coil is an a.c. generator which

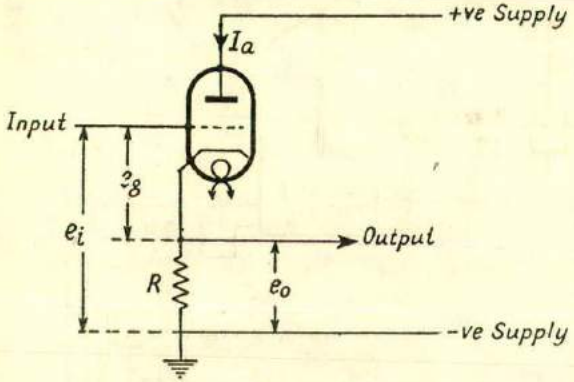


FIG. 23.22. CATHODE-FOLLOWER CIRCUIT

gives an output proportional to the motor speed. This provides velocity feed-back and damps the movement of the motor as already described. The position of the motor shaft is indicated by a further magslip transmitter and receiver (not shown) and shows the mean wind direction.

The Cathode-follower. A common and very simple example of a feed-back amplifier is the cathode-follower. It is used where we wish to increase the current available at a given signal voltage and is thus a device with a high input impedance and low output impedance, giving an output voltage nearly equal to the input voltage (see Fig. 23.22.).

We have $$e_g = e_i - e_o$$

and $$g_m = \frac{\partial I_a}{\partial e_g}$$

(neglecting the small voltage e_o compared with the supply voltage),

while $$e_o = RI_a$$

so $$\delta e_o = R\delta I_a = g_m R \delta e_g$$

$$= g_m R\delta(e_i - e_o)$$

$$\delta e_o(1 + g_m R) = g_m R\delta e_i$$

$$\delta e_o = \delta e_i \frac{g_m R}{1 + g_m R}$$

or, if $g_m R \gg 1$, $\quad \delta e_o = \delta e_i$

Thus a change in e_i produces an equal change in e_o.

To find the output impedance of the cathode-follower, suppose that a current I_o is drawn from the output. Then

$$e_g = e_i - e_o$$

$$g_m = \frac{\partial I_a}{\partial e_g}$$

$$e_o = R(I_a - I_o)$$

$$\delta e_o = R\delta I_a - R\delta I_o$$

$$= g_m R\delta e_g - R\delta I_o$$

$$= g_m R\delta(e_i - e_o) - R\delta I_o$$

If e_i is kept constant, $\delta e_i = 0$, and

$$\delta e_o = - g_m R\delta e_o - R\delta I_o$$

$$\delta e_o(1 + g_m R) = - R\delta I_o$$

so $$R_o = - \frac{\delta e_o}{\delta I_o} = \frac{R}{1 + g_m R}$$

where R_o is the output impedance, and if $g_m R \gg 1$

$$R_o = \frac{1}{g_m}$$

For most amplifying valves (Ref. (5)) g_m will lie between 1 and 10 mA/volt, so that R_o will be between 1,000 and 100 Ω. The input impedance is extremely high, the only limitation being that the grid in most valves either collects or emits electrons, and a path to earth must be provided for this small current. Some ordinary valves (e.g. EF 37A) will operate successfully with 50–100 MΩ between grid and earth.

An Application of the Cathode-follower. *Reducing the Effect of Earth Admittances in an A.C. Bridge.* The Wagner earth, which is fitted to precision a.c. bridges at power frequencies, and which is essential at higher frequencies, has to be adjusted continually as balance is approached. Each change of the Wagner earth controls alters the balance point of the bridge, and the necessary procedure is often slow and laborious.

G. H. Rayner and R. W. Willmer (Ref. (9)) describe a method for increasing the rapidity with which balance can be reached by

reducing the effect of the Wagner earth setting upon the bridge setting (see Fig. 23.23).

The shield surrounding the critical parts of the bridge (C, D, etc.) is driven by a cathode-follower through the capacitor C_3. The grid of the cathode-follower is connected through C_1 to point D, and if the a.c. output of the cathode-follower were exactly equal to the input there would be no resultant a.c. potential between D and the screen. In fact there will be a slight a.c. voltage between them since, as we have seen, e_o does not exactly equal δe_i, but is slightly less—

$$\delta e_o = \frac{\delta e_i \, g_m R}{1 + g_m R}$$

There will also be a p.d. between C and the screen, apart from the

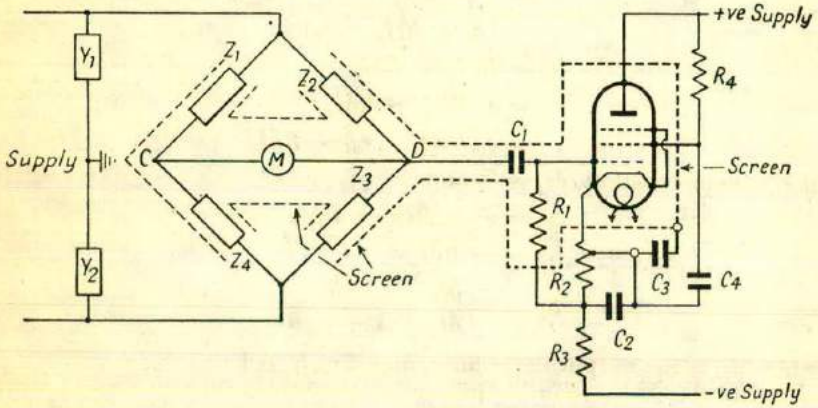


FIG. 23.23. CATHODE-FOLLOWER DRIVING THE SCREEN OF A BRIDGE

above effect, if the bridge is not balanced. The authors state that the cathode-follower reduces the effect of a given unbalance of the Wagner arms Y_1, Y_2 to about 1/50th of what it would be with the screen earthed.

Several points in the circuit deserve notice. C_1 and R_1 form a convenient means of applying the a.c. signal to the grid while keeping the mean grid potential at a negative value determined by R_2 and R_3. R_1 is not returned to the negative supply, but to a point where the a.c. voltage is nearly equal to that of the cathode, and hence that of the grid. This keeps the a.c. input impedance high. (C_2 ensures that the a.c. voltage across R_2 is very small.) The screen-grid of the valve is similarly connected by C_4 to the cathode, and the effect of capacitance between screen-grid and control-grid is thus reduced. Finally, the grid leads of the cathode-follower are enclosed in the driven screen. The effect of all these precautions is that the shunting effect of the cathode-follower connected to D is only

50 MΩ and 1 $\mu\mu$F in parallel. Since D is at earth potential when the bridge is balanced, even this small shunting effect does not affect the balance point.

The other parts of the bridge and cathode-follower, not enclosed in the screen shown, are enclosed in an earthed screen which is omitted from the diagram.

The Differential Cathode-follower. The cathode-follower described on page 902 is essentially a device for providing a voltage output. Often it is required to operate a meter from a signal which has insufficient power to drive it directly. In this case a current output is generally required. The cathode-follower can be used, but there is a standing current when the input voltage is zero, and this is often inconvenient. The standing current can be backed-off with a separate battery but a neater solution is to use a differential cathode-

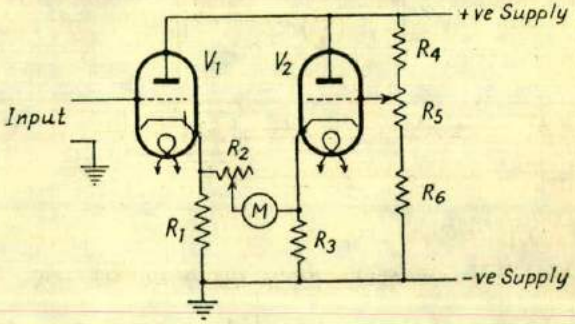


FIG. 23.24. DIFFERENTIAL CATHODE-FOLLOWER

follower as in Fig. 23.24. V_1 and V_2 can conveniently be the two halves of a double-triode containing two triodes in a single envelope. The operation of the circuit will be self-evident. R_5 sets the zero on M, while R_2 sets the sensitivity. The adjustments are independent. The circuit is a convenient and stable device for using an ordinary moving-coil meter to measure voltages at a very high impedance level—it is in fact a form of valve-voltmeter.

The Auto-repeater. The auto-repeater (Everett Edgecumbe, see Fig. 23.25) is a device for obtaining a direct current proportional to the power in an a.c. circuit. It provides an illustration of feedback in a servo-mechanism using a d.c. amplifier.

The instrument consists of a standard three-element a.c. wattmeter movement mounted on the same shaft with a moving-coil d.c. milliammeter movement. The common shaft carries a small mirror which, as it turns, reflects the light from a lamp on to the emission photo-cell V_1.

The amplifier gives an output current which increases as the illumination of V_1 increases. The current from the amplifier passes

through the moving-coil movement, producing a turning moment which resists that of the wattmeter movement, and tends to reduce the illumination of V_1.

For any given a.c. power, the instrument will come to rest when the moving-coil movement produces an equal and opposite torque to that of the wattmeter movement. The latter torque is proportional to the power, while the former is proportional to the current flowing through the moving coil. It follows that this current is proportional to the a.c. power, and it is passed through one or more external milliammeters which thus indicate or record the power.

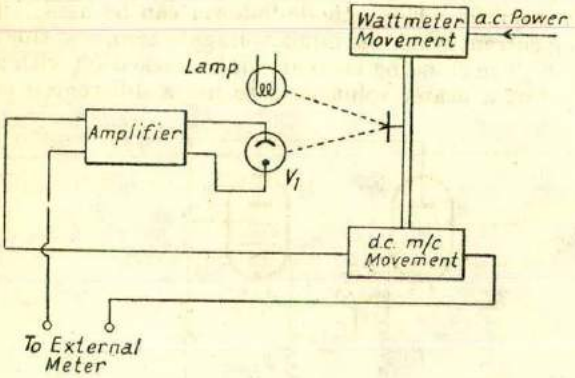


FIG. 23.25. ARRANGEMENT OF THE AUTO-REPEATER

So long as the gain of the amplifier is large, as we have seen (see page 895), changes in the gain (i.e. amplification) have little effect on the accuracy of the instrument.

The auto-repeater enables the indicator or recorder to be placed in a distant position since only two wires need to be taken to it, and the resistance of these wires does not affect the accuracy of the indication. Also, since the movement of the shaft carrying the mirror is very small, the response of the instrument is relatively fast, and it can be used to record rapid fluctuations of power.

The amplifier used in the instrument is a Tinsley d.c. amplifier using a thyratron (Fig. 23.26). When the photo-electric cell is not illuminated, the alternating voltage applied to the grid of the thyratron is in such a phase that it prevents the valve from conducting. Illumination of the cell changes the voltage of the grid, and the thyratron begins to conduct at some point in the a.c. cycle. The amount of illumination thus controls the proportion of each cycle during which the thyratron conducts, and so varies the mean value of the output current.

A Counting-rate Meter. Another example of the use of feed-back is provided by the A.E.R.E. counting-rate meter (Ref. (7)). The

problem is to measure the average rate of pulses occurring at random—the instrument is in effect a frequency meter for measuring the mean value of a rapidly-fluctuating frequency.

The fundamental circuit which accomplishes this is shown in Fig. 23.27 (*a*). Each time a pulse is received it is arranged that the

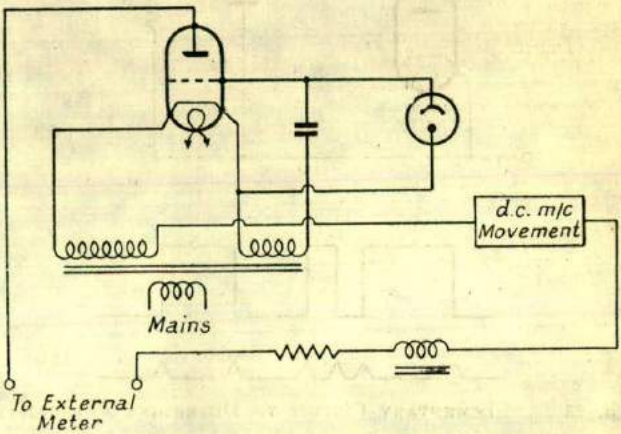


FIG. 23.26. AMPLIFIER IN THE AUTO-REPEATER

voltage applied to the input is changed by a constant amount (Fig. 23.27 (*b*)). The means by which this is done is explained later (*see* "Electronic Counters"). As the input goes positive, V_1 conducts and C_f is charged to the voltage V. As the input goes negative, V_2 conducts, and the charge on C_f is fed into C_t. It is readily shown that, if there are on the average $2n$ pulses per second, the mean voltage across R is

$$v = \frac{VnC_fR}{1 + nC_fR}$$

The quantity nC_fR in the denominator is due to the back-voltage developed across R. We require v to be proportional to n, and with the simple circuit shown this requires C_fR to be small. Unless V is very large, v itself is then small. The difficulty is due to the voltage developed across R, which reduces the charge fed into C_t. The back-voltage is avoided by making C_t and R the feed-back elements of a d.c. amplifier as in Fig. 23.28.

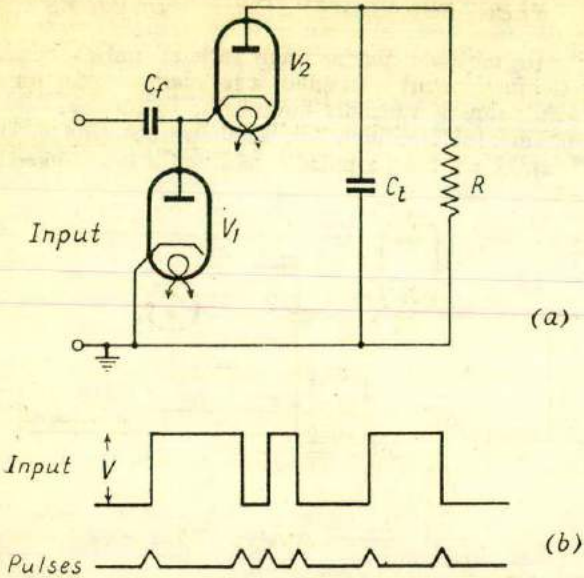
We have

$$E = -ae$$
$$v = -E + e = (1 + a)e$$

so that

$$e = \frac{v}{1 + a}$$

Thus the back-voltage is reduced in the ratio $\frac{1}{1 + a} : 1$, and the



F.G. 23.27. Elementary Circuit to Determine Pulse Rate

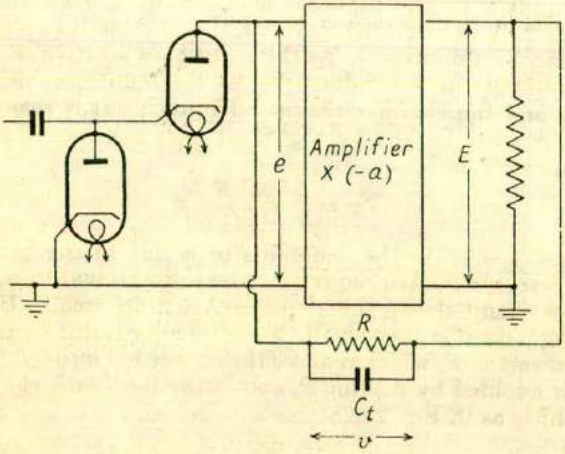


Fig. 23.28. Circuit for Pulse-rate Determination using Feed-back Amplifier

charge fed into C_t is therefore nearly independent of v, so that

$$v = VnC_tR \text{ very nearly.}$$

The actual circuit is shown in a simplified form in Fig. 23.29. The amplifier consists of a pentode to give a high amplification of the voltage, followed by a cathode-follower to provide the necessary current.

Evershed and Vignoles Quick-Response Recorder. This is an example of a servo-driven d.c. voltage recorder using a valve-

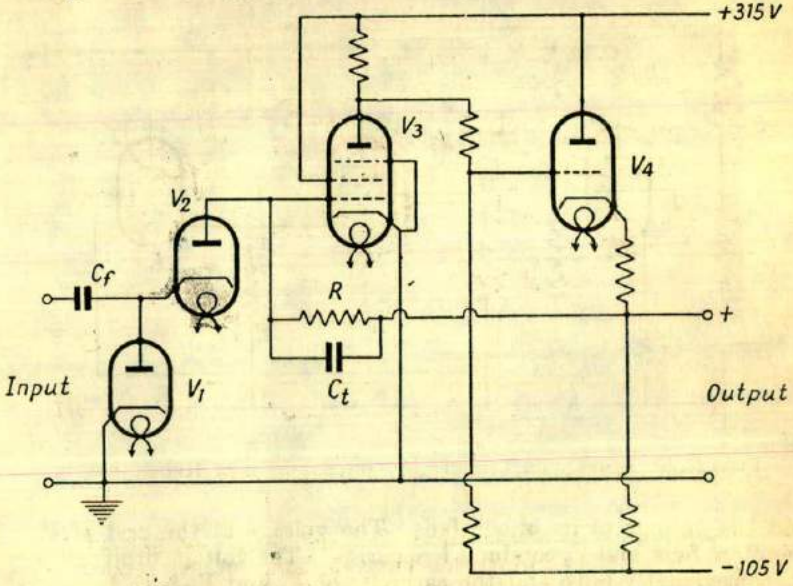


FIG. 23.29. SIMPLIFIED CIRCUIT OF PULSE-RATE METER

amplifier as in Fig. 23.30. R_1 and R_2 are each 500,000 Ω, and the voltage at the control grid of V_1 is hence the mean of the input voltage and the voltage obtained from R_{10}. The latter is a finely-wound wire potential divider supplied with a reference voltage, and the slider is fixed to the pen-arm of the recorder. V_1 is a pentode amplifier direct-coupled through R_7 and R_8 to V_2. R_3 provides some negative feed-back to V_1, so stabilizing its gain, and also ensures that the cathode is positive with respect to the grid. In the same way R_9 has a voltage across it, and it is not necessary to return R_8 to a negative voltage in order to make the grid of V_2 negative with respect to its cathode. V_2 and V_3 have a common grid resistor R_9. They are connected as a "long-tailed pair," their anodes being fed through the two halves M, M of the centre-tapped moving coil of the recorder.

When the voltages fed to R_1 and R_2 are equal and opposite, the grid of V_1 is at earth potential. Under these conditions V_2 and V_3 are arranged to pass equal currents, and these flowing through the two halves of the movement produce equal and opposite torques. The pen therefore remains at rest. If now the input to R_1 is made more positive the grid of V_1 becomes positive with respect to earth,

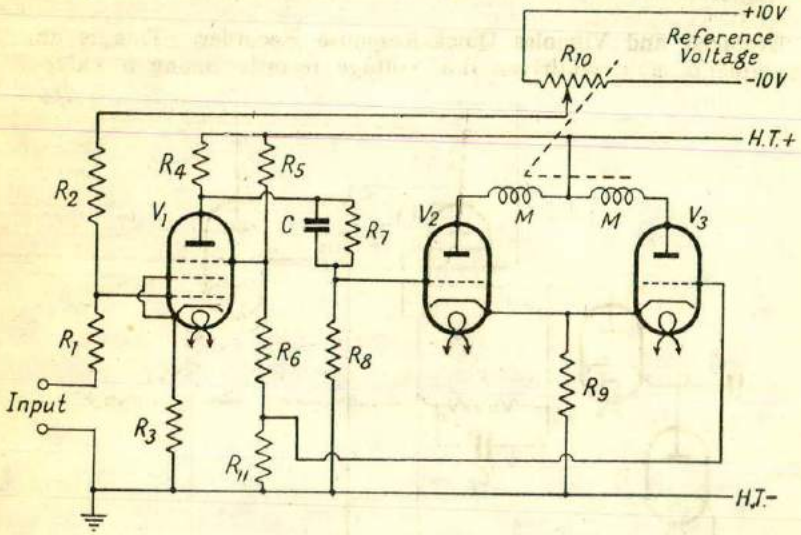


FIG. 23.30. SIMPLIFIED CIRCUIT OF QUICK-RESPONSE RECORDER

and the voltage of its anode falls. The voltage at the grid of V_2 therefore falls, and V_2 conducts less current. The voltage drop across R_9 consequently falls, and the cathodes of V_2 and V_3 both become more negative. Since the grid of V_3 is fed from a nearly constant voltage, V_3 will conduct more current. The sum of the currents through V_2 and V_3 remains nearly constant, but more is now flowing through V_3 than V_2. The torques produced by the two halves of the moving coil are now unequal, and the pen begins to move. As it does so it changes the voltage fed from R_{10} to R_2, making it more negative. The voltage fed to the grid of V_1 falls, and equilibrium is again reached when the voltage picked up from R_{10} is equal and opposite to the input voltage. R_{10} is a linear potential divider, and it follows that the movement of the pen is directly proportional to the input voltage. Also, the accuracy of the recorder depends only on the accuracy of the reference voltage and of R_{10}, and not on the characteristics of the amplifier.

The pen-arm of the recorder has inertia, and the servo-system would be unstable unless arrangements were made to stabilize it.

This is done by adding the capacitor C. The action of C will be clear from page 899, though it will be noticed that it is not in the same part of the servo-loop as that previously described. A further capacitor in parallel with R_2 is found in practice to help in preventing the pen from overshooting.

It will be noticed that this recording voltmeter is in fact a self-balancing potentiometer; R_{10} is the potentiometer, and the voltage from it is automatically kept equal and opposite to that at the input terminals.

"Quality Factor" Q and its Measurement. Consider the circuit shown in Fig. 23.31, in which R and L represent the resistance and inductance of a coil. Then, for the total parallel impedance Z we have,

$$\frac{1}{Z} = j\omega C + \frac{1}{R + j\omega L} = \frac{j\omega C(R + j\omega L) + 1}{R + j\omega L}$$

$$Z = \frac{R + j\omega L}{(1 - \omega^2 LC) + j\omega CR}$$

$$= \frac{(R + j\omega L)(1 - \omega^2 LC - j\omega CR)}{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2}$$

$$= \frac{R + j(\omega L - \omega^3 L^2 C - \omega CR^2)}{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2}$$

If we put $\omega_0^2 = \frac{1}{LC}$, $Q = \frac{\omega L}{R}$, $\omega = h\omega_0$

and substitute $C = \frac{1}{\omega_0^2 L}$, $R = \frac{\omega L}{Q} = \frac{h\omega_0 L}{Q}$

we obtain

$$Z = Q\omega L \left[\frac{1 + j\left(Q - h^2 Q - \frac{h^2}{Q}\right)}{Q^2(1 - h^2)^2 + h^4}\right]$$

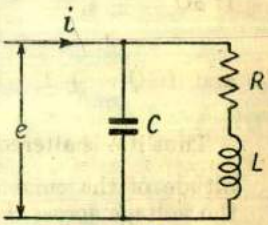


FIG. 23.31

Thus Z can be expressed in terms of ωL—the reactance of L—and of Q and h. Q is termed the "quality-factor" of the coil. The resistance R of the coil varies with frequency owing to skin effect, dielectric losses, etc., and at high frequencies Q is often more constant with frequency than R. It is also generally more useful to know Q than R.

When the total reactance of the circuit is zero,

$$Q - h^2\left(Q + \frac{1}{Q}\right) = 0$$

$$h^2 = \frac{Q}{Q + \frac{1}{Q}} = 1 \text{ very nearly if } Q \text{ is large.}$$

The impedance of the circuit when $h = 1$ is

$$Q\omega_0 L \frac{\sqrt{1 + \frac{1}{Q^2}}}{1} = Q\omega_0 L$$

very nearly, when Q is large. This, also, when Q is large, is very nearly the maximum value of the impedance Z (expressed as an actual value and not in symbolic j notation), and the value when the reactance is zero.

Suppose that h differs from unity by an amount α. Then

$$h = 1 \pm \alpha$$

$$Z = Q\omega L \frac{\sqrt{1 + Q^2\left(1 - h^2 - \frac{h^2}{Q^2}\right)^2}}{Q^2(1 - h^2)^2 + h^4}$$

$$\fallingdotseq Q\omega_0 Lh \frac{\sqrt{1 + Q^2(1 - h^2)^2}}{Q^2(1 - h^2)^2 + h^4}$$

and $1 - h^2 = 1 - (1 \pm \alpha)^2 = \pm 2\alpha - \alpha^2 = \pm 2\alpha$ if α is small

so $$Z = Q\omega_0 L \frac{\sqrt{1 + 4\alpha^2 Q^2}}{4\alpha^2 Q^2 + 1} = \frac{Q\omega_0 L}{\sqrt{1 + 4\alpha^2 Q^2}}$$

If $\alpha Q = \pm \frac{1}{2}$, $$Z = \frac{Q\omega_0 L}{\sqrt{2}}$$

and if $\alpha Q = \pm 1$, $$Z = \frac{Q\omega_0 L}{\sqrt{5}}$$

Thus if h is altered by an amount $\pm \frac{1}{2Q}$ from resonance, the magnitude of the current remaining constant, then the magnitude of the voltage across the circuit falls to 0·707 of its value at resonance. If h is altered by $\pm \frac{1}{Q}$, the voltage falls to 0·447 of its value at resonance.

Three methods are now available for obtaining Q—

(*a*) We may make $h = 1$, and measure e and i. Then $\frac{e}{i} = Q\omega_0 L$, and, knowing ω_0 and L, we can calculate Q. This method is not very accurate, since e and i are difficult to measure at high frequencies.

(*b*) We may keep i constant, and vary h, noting the maximum value of e. We then change h by such an amount that e falls to 0·707 of its maximum value. We repeat the reading on the other side of resonance (α negative) (see Fig. 23.32. (*a*)). Then $\frac{1}{2\alpha} = Q$.

The variation of h can be achieved by varying the frequency of i,

or by changing ω_0 by altering C. There are thus two variants of this method.

(*c*) In the same way we may reduce e to 0·447 e_0 (see Fig. 23.32 (*b*)). There are again two ways of altering h—by varying ω or ω_0.

Methods (*b*) and (*c*) do not require measurements of the actual values of i and e; the current i has merely to be kept constant, and

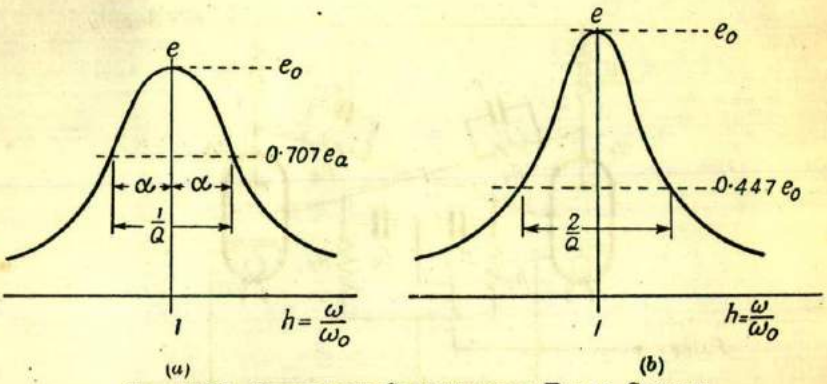


FIG. 23.32. RESONANCE CURVES FOR A TUNED CIRCUIT

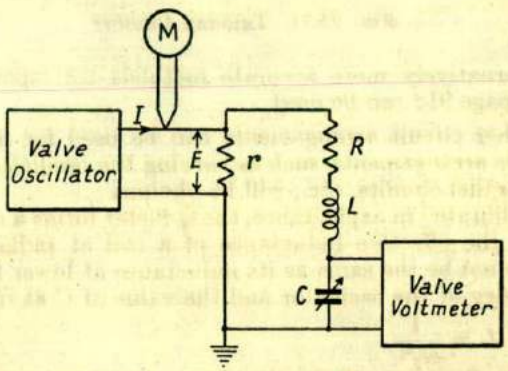


FIG. 23.33. DIAGRAM OF Q-METER

only a ratio of voltages ($e_0 = 0{\cdot}707e_a$, etc.) is required. These methods are consequently preferable to (*a*).

A similar analysis can be made of series resonant circuits. The analysis is simpler, and leads to similar results; it is therefore omitted.

The Q-Meter. The principle of a common device for measuring Q, the so-called "Q-meter" (Ref. (10)), is illustrated in Fig. 23.33. The

current I is set to a standard value by the thermocouple meter M. It produces a standard voltage E, which is applied to the coil (R, L) and a variable capacitor C in series. The voltage across C is measured by a valve voltmeter. It is easily shown that the maximum voltage across C is $E\sqrt{1+Q^2}$, which is very nearly EQ when Q is large, and the valve voltmeter can be calibrated directly in values

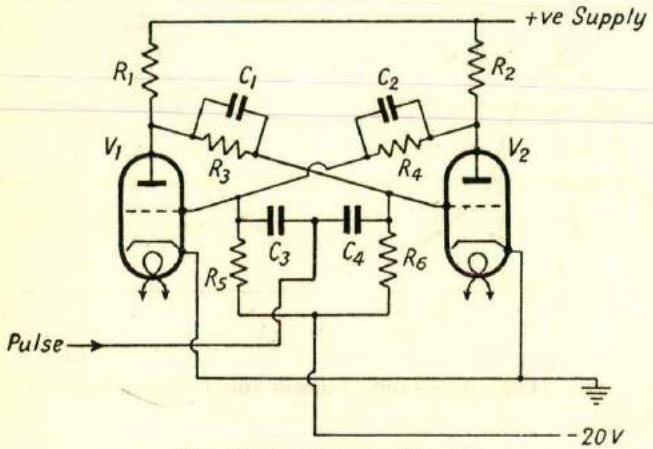


FIG. 23.34. TRIGGER CIRCUIT

of Q. Alternatively more accurate methods corresponding to (*b*) and (*c*) on page 912 can be used.

Many other circuit arrangements can be used for measuring Q. The possible arrangements, such as varying the oscillator frequency, or using parallel circuits, etc., will be obvious.

If C is calibrated in capacitance, the Q-meter forms a useful means for finding the effective inductance of a coil at radio frequencies (which may not be the same as its inductance at lower frequencies). The frequency of the oscillator and the value of C at resonance are found, and $L = \dfrac{1}{\omega_0^2 C}$.

Electronic Counters. (Ref. (18)). Consider the circuit given in Fig. 23.34 (see Refs. (19) and (23)). This is the Eccles-Jordan Trigger Circuit.

Suppose V_1 is conducting. Then the voltage at its anode will be relatively low, and R_3R_6 form a potential divider such that the grid of V_2 is very negative—so negative that V_2 cannot conduct. The anode of V_2 is therefore at a high voltage, and R_4R_5 ensure that the grid of V_1 is at about earth potential, so that V_1 conducts. The arrangement is obviously stable for small disturbances.

If now a large positive pulse is applied through C_3 and C_4 to both

grids, it will have no appreciable effect on V_1. V_1 is already conducting, and its grid cannot go positive, as any attempt to drive it positive will result in the flow of grid current. There will, however, be a considerable effect on V_2 which was cut off. The pulse will drive the grid positive until V_2 begins to conduct. The anode voltage of V_2 will then begin to fall, and this, through R_4R_5, reduces the voltage at the grid of V_1. This in turn increases the voltage at

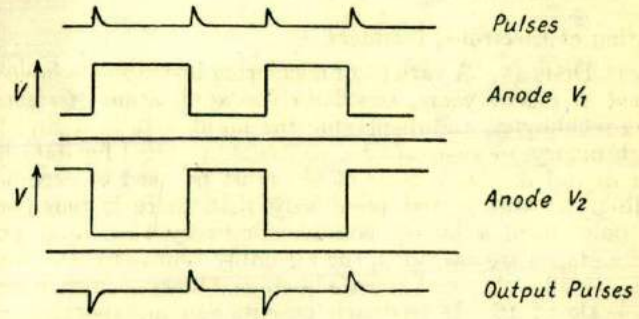


FIG. 23.35. VOLTAGES IN THE CIRCUIT OF FIG. 23.34

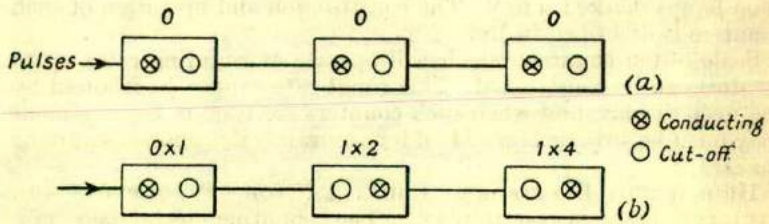


FIG. 23.36. PRINCIPLE OF SCALE-OF-TWO COUNTER

the anode of V_1 and hence at the grid of V_2. The process is cumulative, and it stops only when V_1 is cut off and V_2 conducting. C_1 and C_2 assist the process of changing-over.

When a second positive pulse is applied, a similar effect occurs, and V_1 again becomes conducting while V_2 is cut off. The voltages at the anodes of V_1 and V_2 are thus as shown in Fig. 23.35.

The lowest line shows the pulses which would be obtained by connecting a capacitor to the anode of V_2. It is easy to cut off the negative pulses, and it will be seen that there is one positive output pulse for every two input pulses.

If a number of such circuits are arranged one after the other, we can divide successively by two. If there is some device to show which of the two valves in each pair is conducting, we have a simple "scale-of-two counter" as in Fig. 23.36 (*a*). We count each stage as "0" if

the left-hand valve is conducting, "1" if the right-hand valve is conducting. Suppose six pulses are fed in. The final result will be as in Fig. 23.36 (*b*). We count this (taking the stages from right to left) as 1, 1, and 0 so that the count is

$$1 \times 4 + 1 \times 2 + 0 \times 1 = 6$$

Most electronic calculators work on a scale-of-two using circuits similar to the one described.

Application of Electronic Counters

DIGITAL DISPLAY. A variety of measuring instruments have been developed in recent years, based on the application of electronic counting techniques, and displaying the input data in digital form. Although binary, or scale-of-two, counters are ideal for data handling, for digital display a scale of ten must be used to permit ease in reading. It was stated previously that there is one positive output pulse from a binary counter for every two input pulses; hence if n stages are cascaded, the assembly counts by 2^n. A scale-of-ten is usually obtained from a 4-stage binary, which normally gives a scale of 16. If feedback circuits are incorporated which add in extra pulses, the 4 stages run through a full cycle for ten input pulses. The stored count is indicated by one of ten miniature neon lamps marked 0 to 9. The construction and operation of such counters is described in Ref. (23).

Scale-of-ten counters which will operate at counting rates up to 10 Mc/s can be constructed. This count rate cannot be followed by the neon display, and when such counters are used in measurement they must be switched on and off for accurately determined sampling times.

HIGH-SPEED ELECTRONIC COUNTERS FOR FREQUENCY AND TIME-INTERVAL MEASUREMENT. The counting techniques previously described have led to the development of high-speed counters which can also be used both for accurate frequency measurement and for the precise measurement of small time intervals. A typical example of instruments of this type is the Marconi Electronic Counter illustrated in Fig. 23.37. The counting section of this instrument comprises eight decades connected in cascade, giving a one-cycle storage of 99,999,999. Three forms of decade are used, having maximum count rates of 10 Mc/s, 1 Mc/s and 100 kc/s, all working on the principle of a four-binary scale of sixteen having feedback arrangements to give the scale of ten. A much simplified block diagram is given in Fig. 23.38.

The instrument incorporates a 100 kc/s quartz crystal oscillator, the crystal being mounted in a temperature-controlled oven to ensure frequency stability. This oscillator drives a chain of phantastron decade frequency dividers, providing frequencies between 0·1 c/s and 1 kc/s in multiples of ten. These frequencies

are used to actuate an electronic switch or "gate" which is opened and closed by successive cycles. The gating times available are 0·001, 0·01, 0·1, 1 and 10 seconds.

An unknown frequency being measured is first amplified in the instrument and shaped into pulses; these pulses pass to the

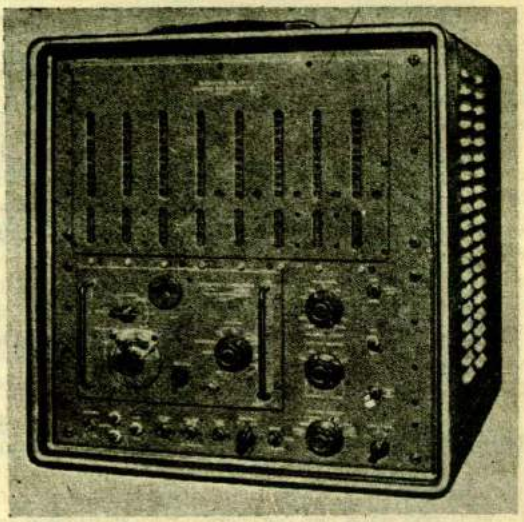


FIG. 23.37. THE MARCONI INSTRUMENTS 10.MC/S ELECTRONIC COUNTER, TYPE TF 1345

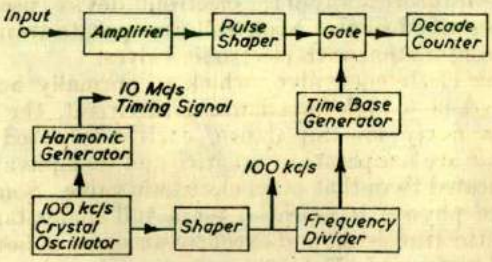


FIG. 23.38. BLOCK DIAGRAM OF FREQUENCY COUNTER

counter when the gate is open, and thus the counter displays the number of pulses passing during the gating time, and the display is a direct measure of the frequency of the unknown.

The accuracy of measurement is limited to one count and at low frequencies it is better to measure the period of the unknown signal. In this case the counter is driven from the crystal oscillator, either at 10 Mc/s (obtained as a harmonic of the crystal frequency), 100 kc/s,

1 kc/s or 10 c/s, and the counter gate is opened by the unknown signal for one cycle. If, for example, a 50 c/s signal is being measured, and a timing frequency of 100 kc/s is employed, when the gate is opened for 1 cycle of the unknown the count will be 2,000 $\pm$ 1 count, each count corresponding to 10 microseconds. It is clear that this arrangement can also be used for the accurate measurement of time intervals between successive events, and accessories are available which enable the gate to be opened and closed by signals from external sources.

The upper frequency-measuring limit of this instrument can be extended to 220 Mc/s by the use of converter units. In these converters, an appropriate harmonic of the 10 Mc/s counter signal (which is itself derived from the 100 kc/s crystal oscillator) is selected and mixed with the unknown frequency, so that the difference frequency is within the counting rate of the counter.

The operation of this instrument is fully discussed in Ref. (25).

The signal gate normally used in these devices is a special pentode valve in which the suppressor grid acts as a second control grid, switching the tube on and off with a relatively small change in voltage. These circuits are discussed in Ref. (23). Details of the phantastron circuit will be found in Ref. (24).

The Transistor. The transistor, which was invented in 1948, is now replacing the electronic valve in many applications and is being rapidly introduced into the field of electronic instrumentation. The transistor has the particular advantages of small size and very low power consumption, so that transistorized devices are physically very small compared with the corresponding devices using electronic valves, and their power supplies can conveniently and economically be provided by a single small dry battery. We may anticipate that in the very near future many of the electronic devices used in measurement and described in this book will incorporate transistors either alone or in conjunction with electronic valves.

Unlike the electronic valve, which is normally actuated by a voltage signal in a high-impedance grid circuit, the transistor is essentially a fairly low-impedance, current-actuated device; its characteristics are temperature sensitive and its equivalent circuit is more complicated than that of an electronic valve. Some knowledge of solid-state physics is essential for a full understanding of the operation of the transistor, and excellent accounts of both theory and practice will be found in Ref. (26) and (27). Counting circuits using valves and transistors are described in Ref. (28).

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