Chapter -6

Credit-Granting Decisions

This chapter concerns the *credit-granting decision*: which of the selling firm's credit applicants will be allowed to purchase goods and services on credit, and which will be required to pay cash. This is a critical decision for the seller. Since credit granting has economic value to buyers, the decision concerning which potential buyers can purchase on credit determines, to a large extent, the base of customers to which sales can be made. The size of the firm's customer base is in turn an important determinant of the firm's sales, cash flows, and value to shareholders.

The outcomes of the firm's credit-granting decisions are sometimes called its *credit standards policy*. The methods used to make these decisions are quite different from those used to make terms of sale decisions. This difference stems from a divergence in the law that governs terms of sale and credit-granting decisions. Under U.S. antitrust statutes, *the selling firm must grant the same terms of sale to all buying firms, if they grant credit to these firms*. This means that some buyers will receive longer terms than are necessary to make the sale. For example, if the seller's terms of sale are net 30 days, it will be able to sell to all buyers who require 30 days *or less* to pay for goods. Buyers that require only 20 days, for example, get an additional 10 days in which to pay that the seller would not have to grant if it could negotiate with each buyer individually.

But in credit-granting decisions, the firm is not constrained to offer the same conditions of sale to all possible buyers so some buyers gain a windfall. *It may instead analyze each potential buyer and grant credit or not based on the attributes of that particular buyer*. Thus, while the seller makes *one* terms of sale decision for *all* the customers purchasing a particular good or service, the firm may elect to make credit-granting decisions *individually* for *each* of the customers purchasing that good or service.[1] While the level of analysis

[1]Whether it is optimal for the seller to consider each applicant individually or to group applicants according to common characteristics is a question we will consider later in this chapter.

for the firm's terms of sale policy is *in aggregate across customers*, the leve of analysis for credit-granting policy can be *at the level of the individua customer*. Because of the expense involved in changing the firm's terms o sale to *all* customers, terms-of-sale decisions are made *infrequently* (once a year or less).[2] Credit-granting decisions, on the other hand, *occur constantl* as new customers come to the firm, and old customers are reviewed.

It is easy to see that credit-granting decisions are a major task in financial management of the firm. The policies necessary for the proper management of credit granting are extremely important and need to be carefully formulated, as the questions involved in credit granting are complex and their impact is substantial. Among the policy questions that must be addressed are:

- ☐ How much information should the firm collect on each credit applicant?
- ☐ What method of analysis should the seller use to determine which applicants should be granted credit?
- ☐ How many periods should be considered in evaluating the expected cash flows from selling to an applicant?
- ☐ How should the credit-related parameters of credit applicants be estimated?

We examine these questions in this chapter. The first of these, which concerns the amount of information that should be gathered on each applicant, is called the *credit investigation problem*. Following discussion of this, we present the traditional and net present value methods for determining which applicants should receive credit. The remaining questions, which concern the number of periods to model and the estimation of credit-related parameters regarding applicants, are discussed with reference to this net present value approach.

INFORMATION COSTS AND THE CREDIT-GRANTING DECISION

In a wealth-maximizing approach to the credit-granting decision, the selling firm evaluates the cash flows that would result from granting credit to a credit applicant versus those that would result if credit were not granted to that applicant. As in terms-of-sale analysis, these cash flows result from *the cost and revenue effects of the decision*. These are the changes in sales and collections, the costs of production, bad debt expense, and so forth that are

[2]The costs of changing the firm's terms of sale include the costs of printing new invoices, manuals, and price specifications, as well as the costs of notifying all the firm's customers and salespeople of the new terms.

contingent on the granting or not granting of credit. The analyst must estimate each of these effects in making the decision. Estimates of some of these costs and revenues are easy to obtain, while others are difficult and costly to estimate. Estimates of potential sales to a particular customer are usually obtained from the firm's sales or marketing departments or directly from the customer. Factors such as costs of production and service are obtained from the selling firm's accounting records. However, the remaining costs (such as bad debt expense) must be estimated via the available *sources of credit information* about potential customers. These sources vary in their cost and the type of information they provide. The following paragraphs list and discuss several sources of credit information, proceeding from the least costly to the most costly.

The Seller's Prior Experience with the Customer One of the cheapest and most reliable sources of information about expected future payment patterns is that customer's history of dealings with the seller. If the customer has paid promptly for the last 20 orders, it is very likely that the customer will pay promptly for the twenty-first. To obtain information on the customer's history of payments to the seller, the credit analyst queries the seller's data base of prior customer payments.

Credit Agency Ratings and Reports Several firms are in the business of collecting and selling credit-related information to sellers. Some of these investigation firms report on wide varieties of buyers (such as Dun and Bradstreet) while others report only on buyers in a single industry. Most of these agencies provide several types of service. They issue ratings based on buyers' financial strength and payments to sellers. They provide reports which may include the buyer's financial statements and the results of an investigation into legal records regarding the buyer's business dealings. They collect and distribute *trade clearances*, which are lists of amounts owing, amounts past due, and histories of payments by buyers to their trade creditors.

Personal Contact with the Applicant's Bank and Other Creditors A more expensive (but very reliable) source of information is the applicant's current creditors and its bank. To obtain such information, the analyst may personally contact these sources to discuss their experiences with the applicant. In this way, the analyst can verify the information in credit agency reports and gain other insights that are not available from these reports. Unfortunately, such an investigation is usually quite time consuming.

Analysis of the Applicant's Financial Statements Any credit applicant's ability to pay is in great part dependent on its financial condition. Analysis of this financial condition can thus considerably aid in credit evaluation. Financial statements must first be obtained. This may entail significant effort

if the applicant firm is not publicly traded. These statements must then be examined and analyzed. This analysis often begins by making adjustments to these statements to utilize accounting conventions other than those chosen by the applicant's accountants. Such adjustments are made if the analyst believes that the conventions used did not do the best job of portraying the economic health of the firm. Once these adjustments are made, ratio analysis and/or statistical analysis is then applied to these statements to assess the financial health of the applicant.

Customer Visit One very expensive option in the collection of information is for the credit analyst to visit the applicant. An on-site viewing of the applicant's facilities and discussions with the applicant's management can provide insights and information that cannot be obtained in other ways. For example, the credit analyst can obtain impressions on the competence of the applicant's management and on the physical condition of the applicant's plant, service facilities, and equipment.

One important question in the formulation of policies for credit granting asks: Which source of information should be used and how much information should be collected on credit applicants? This is an important consideration in credit management. The proper strategy for the collection of information in any decision situation revolves around the relative *cost of the information* versus the resulting *cost of errors in decisions*. In many other financial decisions, such as the purchase of a new manufacturing facility, the expected effects on the firm's cash flows are fairly large, often on the order of hundreds of thousands of dollars. In such situations, the firm may spend considerable time and money estimating the resulting cash flows, evaluating their likely effects on the firm, and investigating the risks associated with the decision. The money and time spent on the decision is very small relative to the effects of the decision on the firm. However, in many credit-granting decisions, the revenues and costs associated with selling to some individual buyers may be quite small. In such cases, *it may not make sense to spend much management time or cost on the investigation of these buyers*. The cost of information and analysis may outweigh the costs of any errors that might be made.

To see the effects of the cost of information in credit-granting decisions, assume that the seller is making a credit decision with respect to a particular credit applicant firm. The seller has already spent some money on investigation and, given the information already in hand, the expected cost of an error (either granting credit to a firm which will eventually default or not granting credit to a firm which will not default) is $15. The next step in the investigation process, however, costs $20. In this case, it is clear that *the cost of information is greater than the cost of error without the information*. Therefore, the firm should not spend the additional $20; it should forego the collection of this information since the cost of the error is less than the information cost.

To see how the principles of information cost versus error cost apply to the collection of credit information, let us present an extensive and reasonably realistic example problem. It will pay the student to read the statement of this problem several times for comprehension before proceeding.

AN EXAMPLE PROBLEM IN THE COSTS OF CREDIT INVESTIGATION

A firm receives orders in the amounts of $50, $1,000, and $10,000. Out-of-pocket costs of the product are 80 percent of sales, and are paid when the order is shipped. Collection costs are trivial. The seller's risk-adjusted discount rate for credit decisions is 12 percent. For purposes of this example, we will assume that there is no recovery if the applicant defaults, and we will ignore taxes. When an order is received, the firm may investigate via one or more sources of credit information. These sources, in sequence of cost, are:

Source of Information	*Cost per Query*
Record of past payments	$ 2
Credit agency rating and report	$ 15
Comprehensive analysis (includes personal contact with banks and other creditors, analysis of financial statements, and so forth)	$300

If the firm performs no investigation (and incurs no investigation costs), it must make a credit decision with no information regarding the applicant. For the average applicant, there is a 3 percent chance that the applicant will default (that is, the *expected default rate* is 3 percent); if the applicant pays, the average time taken to pay will be 45 days. (These estimates could be taken from publicly available statistics of default rates and payments.) At this point, the analyst may choose to grant credit, reject the applicant, or investigate further. If the firm chooses to investigate the applicant, it will first query its data base of past payments by customers (since this is the cheapest source of information available). These are four possible outcomes from this query. First, the customer may have paid promptly (net 30 days are the terms of sale). Second, the customer may have paid 30 days beyond terms. Third, the firm may have no prior experience selling to the customer. Finally, the firm may have previously sold to the customer and the customer may have defaulted. The probabilities of these four outcomes in querying the data base of past customer payments are 0.50, 0.29, 0.20, and 0.01. If the applicant has previously purchased from the seller, it is assumed that the applicant will pay as it has in the past.

If the query to the data base shows no past experience, the analyst again faces the choice of granting credit, rejecting the applicant, or investigating

further. If the analyst chooses to grant credit, the expected default rate and days to pay are 3 percent and 45 days as before, since the query to the data base provided no new information. If the analyst chooses to investigate, a credit agency report and rating is ordered at a cost of $15. This report may show ratings designated 1, 2, or 3. A rating of 1 indicates that the applicant pays promptly and has no chance of default. A rating of 2 indicates that the applicant has a 15 percent probability of default but is expected to pay in 60 days if payment is received. A rating of 3 indicates that the applicant has a 25 percent probability of default, but is expected to pay in 90 days if payment is received. Overall, 84 percent of firms are rated 1 by the credit agency, 10 percent are rated 2, and 6 percent are rated 3. The ratings of the credit agency for applicants rated 2 or 3 are, however, open to some degree of error and represent only average default rates and payment periods.

At this point, the analyst may grant credit, reject the applicant, or perform an extensive credit analysis. Because applicants rated 1 are granted credit (since they have no chance of default) this procedure is used only for applicants with ratings of 2 or 3. The outcome of this analysis will show the applicant's financial position to be strong or weak relative to the average firm with that credit rating. The probability of each outcome is 50 percent. If the applicant is rated 2 and the analysis shows a relatively strong financial position, the expected default rate is 1 percent and the expected time to pay is 30 days. If the applicant is rated 2 and is financially weak, the default rate is 29 percent and the expected time to pay is 90 days. If the applicant is rated 3 and shows a strong financial position, the expected default rate is 15 percent and the expected days to pay is 60. If the applicant is rated 3 and shows a weak financial position, these statistics are 35 percent and 120 days respectively. All the information on probabilities, expected default rates, and days to pay if default does not occur are summarized in Table 6-1.

This credit investigation problem involves *discrete probabilities and events.* When a problem in finance involves discrete, sequential events and decisions, one method of analysis is the *decision-tree approach.*[3] The decision tree for the example credit investigation problem for an order size of $10,000 is presented in Figure 6-1. This tree is quite complex, but we will review it on an item-by-item basis. In this tree, later time is to the right and earlier time to the left. In a decision tree, a *square* represents a *decision node* and a *circle* represents a *chance-event node.* All the nodes in this decision tree have been numbered, and we will be referring to them by these numbers. Most of the

[3]This approach to the credit investigation problem is found in D. Mehta, "The Formulation of Credit Policy Models," *Management Science*, Vol. 15, No. 1 (October 1968), pp. B30–B50. Problems of this sort can also be solved via integer programming; see J. D. Stowe, "An Integer Programming Solution for the Optimal Investigation/Credit Granting Sequence," *Financial Management* (Summer 1985), pp. 66–76.

Table 6-1. *Summary of Outcomes in Example Problem in Credit Investigation—4 Stages.*

Stage 1: No Investigation

Expected default rate:	3%
Expected time to pay:	45 days
Probability:	1.00

Stage 2: Check of History of Applicant's Payment to Firm

Outcome:	Prompt in Past	Slow in Past	No Data	Default in Past
Expected default rate:	0%	0%	3%	100%
Expected time to pay:	30 days	60 days	45 days	—
Probability:	.50	.29	.20	.01

Stage 3: Credit Agency Report and Rating

Credit Rating:	1	2	3
Expected default rate:	0%	15%	25%
Expected time to pay:	30 days	60 days	90 days
Probability:	.84	.10	.06

Stage 4: Comprehensive Credit Investigation

Credit Rating:	2	2	3	3
Financial Position:	Strong	Weak	Strong	Weak
Expected default rate:	1%	29%	15%	35%
Expected time to pay:	30 days	90 days	60 days	120 days
Probability:	.50	.50	.50	.50

branches of this decision tree are labeled with the capital letters "A," "R," "I," "P," "D," "S," or "W." An "A" indicates that the applicant is *accepted* (granted credit), an "R" indicates that the applicant is *rejected* for credit, an "I" indicates that further *investigation* is undertaken, a "D" indicates that the applicant has *defaulted*, a "P" indicates that the applicant has *paid*, an "S" indicates that the credit applicant's financial position is relatively *strong*, and a "W" indicates that this position is relatively *weak*. The decimal numbers near branches after a chance-event node represent the probabilities of events occurring at that node. At decision nodes, the firm chooses one of the possible paths leading from that node. Dollar amounts beneath "I" symbols indicate that investigation costs have been incurred; dollar amounts at the ends of branches indicate the net present value for that outcome. The cash flows and

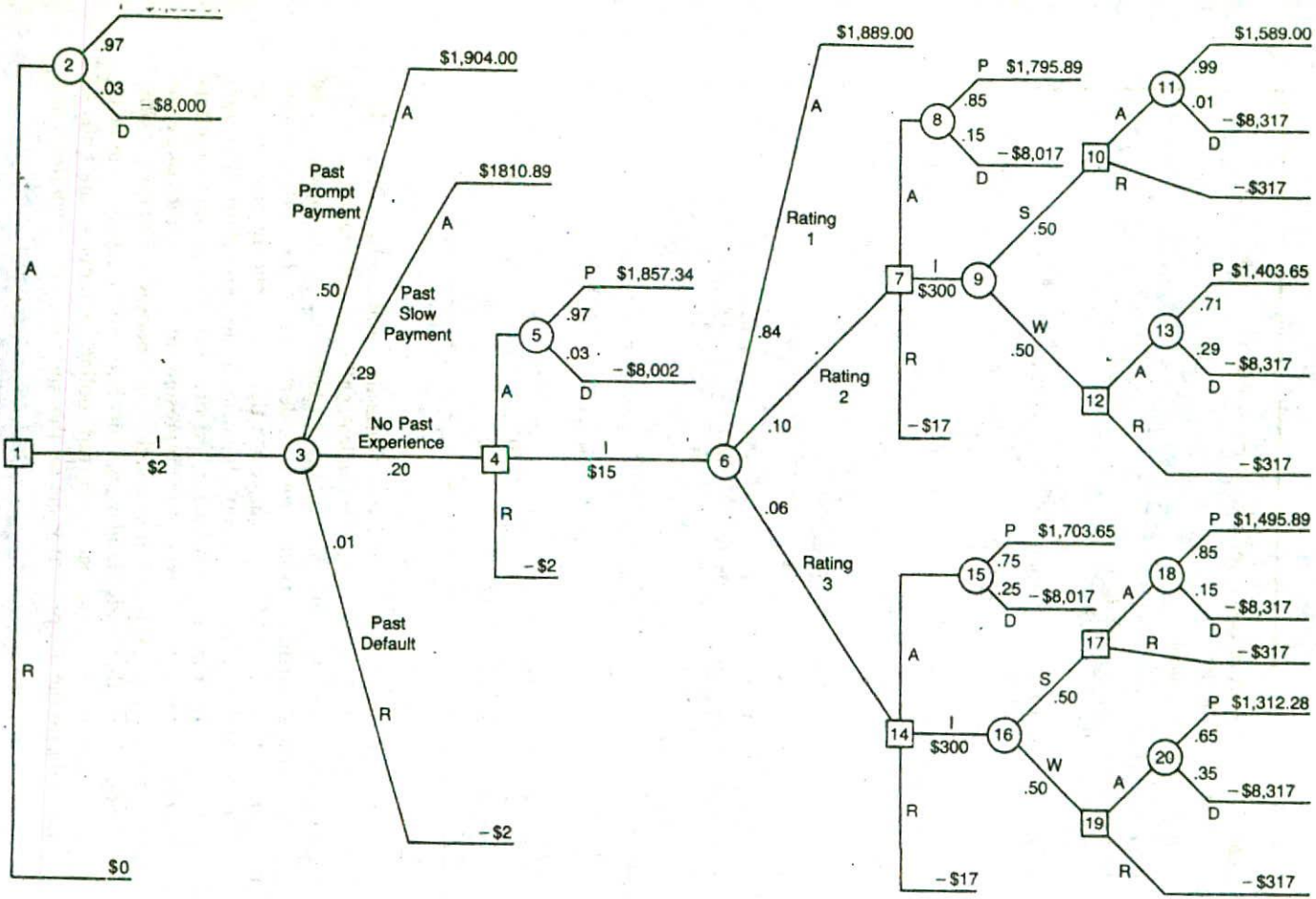


Figure 6-1. Decision Tree for Example Credit Investigation Problem (order = $10,000)

net present values for each possible outcome are presented in Table 6-2.[4] For example, for outcome 5P (the "applicant pays" outcome from node 5), the net present value of the cash flows is calculated as $-\$8{,}000 - \$2 + (10{,}000)(1.12^{-45/360})$, which equals the $1,857.34 shown.

We now review how this tree represents the events of the example problem. We start with the firm's *initial decision* at node 1: it may grant credit without investigation, investigate, or reject the applicant. If the analyst grants credit at this stage, the expected default rate is 0.03 and the expected time to pay is 45 days for all applicants. If credit is granted here, the process moves to node 2, the chance event which represents the uncertain process of payment or default for the applicant given the information available. Before granting credit, the selling firm must pay $8,000 for the goods to service the order. This payment occurs regardless of whether the applicant pays for the order or not. If the applicant pays, the firm receives $10,000 in 45 days, and the net present value is $1,859.34. If the applicant does not pay, the net present value is $-\$8{,}000$. The expected value of granting credit, given that no investigation has been undertaken, is $1,563.56, equal to $(0.97)(1{,}859.34) + (0.03)(-\$8{,}000)$. At node 1, if the firm rejects the credit applicant without having spent any funds on investigation, a net present value of zero results. The third alternative is to investigate further, the net present value of which is unknown at this stage of the analysis.

The collection of more data, even though this data collection is costly, *may result in strategies with higher expected net present values* than the $1,563.56 obtained from granting credit with no investigation. The possibility of further investigation therefore must be evaluated. If the firm spends $2 to check its prior experience with the applicant (moding to node 3), it can gain some important information regarding a majority of the applicants. Specifically, *it will be able to better evaluate the net present values from selling to 80 percent of applicants*: those that paid promptly, those that paid slowly (but eventually paid), and those that defaulted. Since the net present values are positive for the "prompt payment" and "slow payment" groups, they would be granted credit at this point, while credit to the "default" group should not be granted.

If the firm has no past experience with the applicant (which occurs 20 percent of the time in this problem), it must decide whether to grant credit, reject the applicant, or investigate further via the credit agency rating and report. If it grants credit, the expected net present value is $1,561.56 at node 5 $(0.03)(-8{,}002) + (0.97)(1{,}857.34)$. If the applicant is rejected, the net present value is $-\$2$, representing the costs of credit investigation spent to this point. If the seller seeks more information, $15 is spent for the credit report, the firm moves to node 6. The result will be a rating of 1, 2, or 3 for

[4]The information in Table 6-2 and in Tables 6-3, 6-4, and 6-5 (to be presented later) can be generated via a spreadsheet. However, because the main analytic thrust here is the decision tree, and because the spreadsheets involved are very simple, we suppress discussion of spreadsheet aspects in this analysis.

Table 6-2. Net Present Values for Decision-Tree Credit Investigation Problem (order = $10,000).

Outcome	Costs of Product ($)	Stage 2 Invest. Costs ($)	Stage 3 Invest. Costs ($)	Stage 4 Invest. Costs ($)	Inflow from Payment ($)	Days to Pay	Net Present Value ($)
1 R							0.00
2 P	8,000				10,000	45	1,859.34
2 D	8,000						−8,000.00
3 Prompt	8,000	2			10,000	30	1,904.00
3 Slow	8,000	2			10,000	60	1,810.89
3 Default		2					−2.00
4 R		2					−2.00
5 P	8,000	2			10,000	45	1,857.34
5 D	8,000	2					−8,002.00
6 Rtg. 1	8,000	2	15		10,000	30	1,889.00
7 R		2	15				−17.00
8 P	8,000	2	15		10,000	60	1,795.89
8 D	8,000	2	15				−8,017.00
10 R		2	15	300			−317.00
11 P	8,000	2	15	300	10,000	30	1,589.00
11 D	8,000	2	15	300			−8,317.00
12 R		2	15	300			−317.00
13 P	8,000	2	15	300	10,000	90	1,403.65
13 D	8,000	2	15	300			−8,317.00
14 R		2	15				−17.00
15 P	8,000	2	15		10,000	90	1,703.65
15 D	8,000	2	15				−8,017.00
17 R		2	15	300			−317.00
18 P	8,000	2	15	300	10,000	60	1,495.89
18 D	8,000	2	15	300			−8,317.00
19 R		2	15	300			−317.00
20 P	8,000	2	15	300	10,000	120	1,312.28
20 D	8,000	2	15	300			−8,317.00

the applicant. If the applicant is rated 1, the net present value of granting credit is $1,889.00 for certain, since applicants with this rating have a zero probability of default, and credit should be granted.

If the applicant is rated 2 or 3, the analyst again may elect to grant credit, reject the applicant, or investigate further. These decisions occur at nodes 7 and 14 respectively. At node 7, if the applicant is granted credit without further investigation, the default rate is 0.15 and the expected days to pay is 60 (node 8). The decision to grant credit will result in an expected net present value of $323.96 ((0.85)(1,795.89) + (0.15)(−8,017)). If the firm elects not to grant credit at node 7, the net present value is −$17, since investigation costs in this amount have been spent. If the firm elects to investigate further, it must spend $300 (node 9) to find out whether the firm is relatively strong (node 10) or relatively weak (node 12). At nodes 10 and 12, the firm must decide whether to grant credit (nodes 11 and 13) or to reject the applicant (these are the only two alternatives, since no further investigation is possible). The firm is faced with parallel decisions and chance events if the credit rating of the applicant is 3; this process starts at node 14.

With this model of the credit investigation process in hand, we can use *inductive reasoning* to develop a set of strategies regarding credit investigation and credit granting. In this inductive process, we start at the furthest point in time and work backward, calculating expected values and making choices along the way, *given the circumstances implied by our location on the decision tree.* Each point on the decision tree implies a set of circumstances. For example, at node 10 the circumstances are: "A decision on credit granting has not yet been made. The seller has no past experience with the applicant, the applicant's credit rating is 2, and the applicant has a strong financial position relative to those with this rating. The seller may reject the applicant (with a net present value of −$317) or may accept the applicant. If the applicant is accepted, a net present value of $1,589.00 will result with a probability of 0.99 and a net present value of −$8,317 will result with a probability of 0.01."

The use of inductive reasoning to assess strategies is called *rolling back the decision tree.* Two rules are used in this process:

1. At any chance-event node, calculate the value of the node by taking the expected value of all the future decisions and events leading forward in time from this node.
2. At any decision node, assume that the firm will choose the branch that results in the highest expected value. This highest expected value is then the value of that node.

The roll-back inductive reasoning procedure as applied to the decision tree in Figure 6-1 is presented in Figure 6-2. The supporting calculations of the expected net present values for chance event nodes are presented in Table

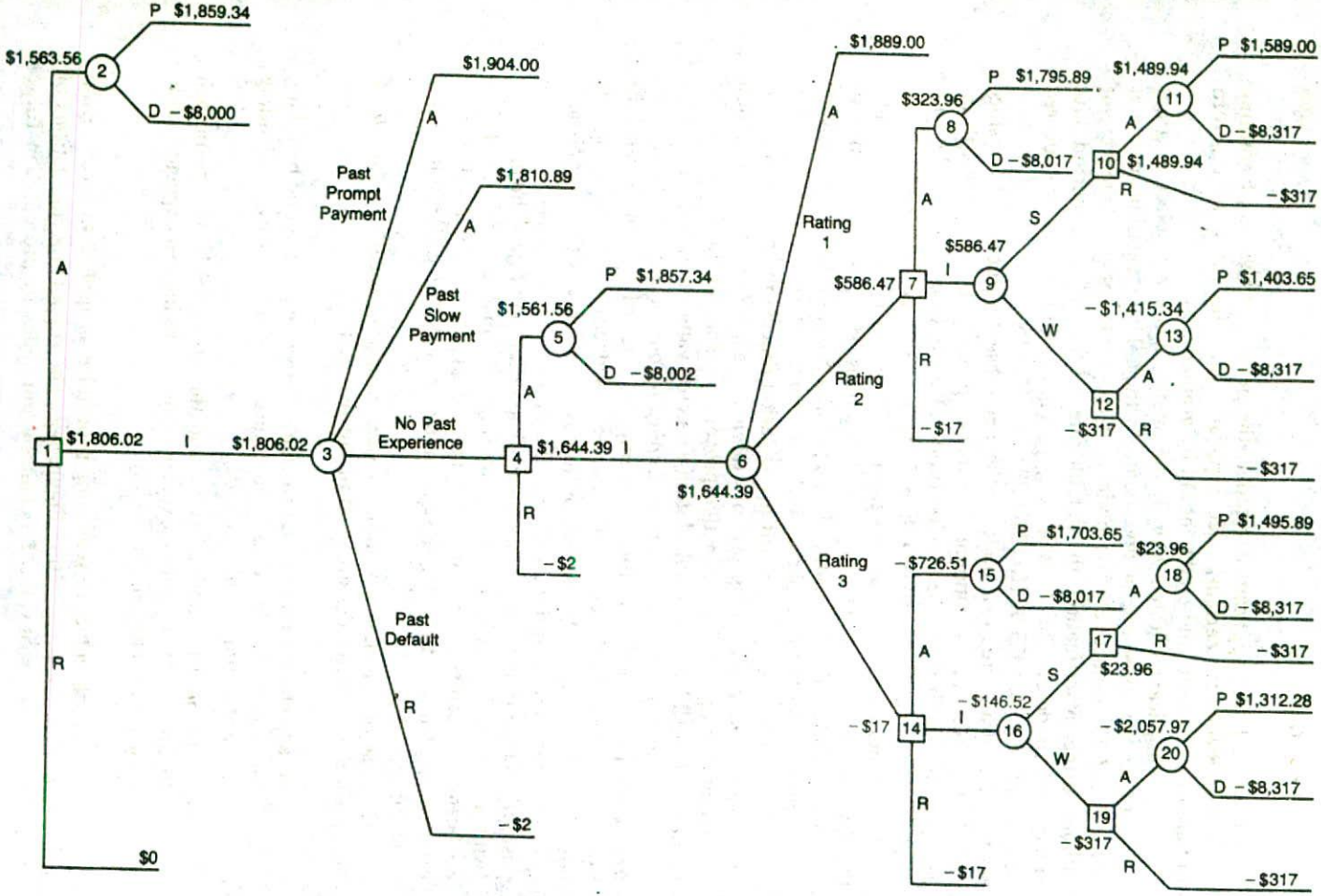


Figure 6-2. Roll-Back of Decision Tree for Example Problem (order = $10,000)

Table 6-3. Computation of Expected Net Present Values for Roll-Back of Decision Tree (order = $10,000).

Node	NPV 1	Prob. 1	NPV 2	Prob. 2	NPV 3	Prob. 3	NPV 4	Prob. 4	E(NPV)
11	1,589.00	.99	−8,317	.01					1,489.94
13	1,403.65	.71	−8,317	.29					−1,415.34
18	1,495.89	.85	−8,317	.15					23.96
20	1,312.28	.65	−8,317	.35					−2,057.97
9	1,489.94	.50	−317	.50					586.47
16	23.96	.50	−317	.50					−146.52
8	1,795.89	.85	−8,017	.15					323.96
15	1,703.65	.75	−8,017	.25					−726.51
6	1,889.00	.84	586.47	.10	−17	.06			1,644.39
5	1,857.34	.97	−8,002	.03					1,561.56
3	1,904.00	.50	1,810.89	.29	1,644.39	.20	−2	.01	1,806.02
2	1,859.34	.97	−8,000	.03					1,563.56

6-3. In the interests of simplicity, the costs of investigation and the probabilities of the uncertain events are not shown on Figure 6-2. The dollar figures proximate to the nodes are the optimum expected net present values at these nodes after the application of the two rules given above. To see the strategies implied by the roll-back procedure, let us review its application to see how the various node values were developed.

We start at the furthest point in time, which is the right of the decision tree. Moving left, the first nodes encountered are nodes 11, 13, 18, and 20. These are the chance events associated with granting credit to strong and weak applicants with ratings 2 and 3. Using the first of the prior rules, the statistics summarizing the worth of these nodes are their expected net present values. At nodes 10, 12, 17, and 19, the firm must choose between these expected values of granting credit and the net present value of rejecting the applicant (which is −$317 in each case). If the applicant has a rating of 2 and is financially strong, or has a rating of 3 and is financially strong, the expected net present value of granting credit is greater than the expected value of refusing credit, and the applicant should be granted credit. The net present values of $1,489.94 at node 10 and $23.96 at node 17 are then the expected net present values of accepting the credit applicant. If, however, the credit applicant is relatively weak, the alternative of rejecting the applicant produces the highest expected net present value, and is the proper choice at nodes 12 and 19 by the second rule. The values at these nodes are the −$317.

If the applicant is rated 2 and the selling firm investigates further, there is a 50 percent chance that the applicant will be weak and a 50 percent chance

that the applicant will be strong. At node 9, *given the optimal future decision to reject the applicant if a weak position is found*, there is a 50 percent chance of an expected net present value of $1,489.94 and a 50 percent chance of an expected net present value of −$317.00. The expected net present value at this node is $586.47; by similar logic, it is −$146.52 at node 16.

At node 7, the selling firm has a choice of three expected net present values: $323.96 (from granting credit without further investigation), $586.47 (from investigating further), and −$17 (from rejecting the applicant). The highest net present value results from investigating further. The spending of $300 in additional investigation expenses enables the firm to economically assess the financial strength of applicants with a rating of 2, after which it will reject the applicants with relatively weak financial positions and grant credit to stronger applicants.

The decision faced by the firm at node 14, where the applicant firm has a rating of 3, is quite different. Here, the highest net present value (of −$17) is achieved by *rejecting all applicants with this rating*. The number of applicants with relatively strong financial positions and the expected value of granting credit to these applicants is sufficiently small that *it makes more sense to reject all applicants with this rating and save the $300 in investigation expenses*. Note that by this strategy the selling firm will reject the 50 percent of the applicants *that in fact have positive expected net present values* of $23.96. However, since it costs the firm $300 to find out which half of the applicants possess this quality, *it is better to reject them all.*

The remainder of the roll-back procedure is straightforward. The expected value at node 6 is $1,644.39. At node 4, the firm faces the choice of investigating further (with an expected net present value of $1,644.39), granting credit to all firms with whom the firm has had no prior credit experience (node 5, with an expected net present value of $1,561.56), or rejecting such applicants (with an expected net present value of −$2). Investigating by checking the firm's credit rating and credit report is the optimum choice. At node 3, the four possible outcomes give the firm an expected net present value of $1,806.02, which is the optimum choice at node 1.

It is useful at this point to summarize the credit investigation rules that we have developed for the example problem and an order size of $10,000. These rules are:

1. When an order of size $10,000 is received, always check the past experience in selling the firm. If the firm has paid promptly or slowly, grant credit without further investigation. If the firm has defaulted, reject credit without further investigation.
2. If the seller has no past experience with the firm, always obtain a credit rating and report. If the applicant's rating is 1, grant credit without further investigation. If it is 3, reject credit without further investigation.

3. If the applicant's rating is 2, investigate further via a comprehensive credit analysis. If the applicant's financial position is strong relative to other firms with ratings of 2, grant credit. If this position is weak, refuse credit.

These rules balance the costs of information and the expected returns from credit granting for orders of $10,000. Using this strategy, the expected net present value of evaluating applicants for a purchase of $10,000 is $1,806.02. However, *for smaller orders, different rules may be optimal* because the costs of investigation bear more heavily on the value of credit granting. Table 6-4 presents the net present values for the various outcomes of the example decision tree problem for an order size of $1,000, Figure 6-3 presents the roll-back of the decision tree for orders of this size, and Table 6-5 presents the expected net present value calculations for this tree. For an order size of $1,000, the optimal credit investigation strategy is:

1. When an order of size $1,000 is received, always check the past experience in selling to the firm. If the firm has paid promptly or slowly, or if the applicant has no previous experience with the selling firm, grant credit without further investigation. If the firm has defaulted, reject credit without further investigation.
2. For orders of size $1,000, never obtain a credit agency report and rating for any applicant.

The expected value of this strategy is $177.64 for order sizes of $1,000. Because of the costs of investigation, for orders of this size the selling firm will grant credit to firms with credit ratings of 3 and also to firms with ratings of 2 whose financial positions are relatively weak. *These two groups would be rejected if this information were known.* However, given the proportion of such firms in the population of firms with whom the seller has no previous credit experience, and given the expected costs of investigation, *it is more economical to grant credit to these firms than to find out who they are.* The student may verify that for orders of $50 the optimum policy is to grant credit to all applicants without checking on the seller's prior experience with them. The expected value of this strategy is $7.82 per order.[5]

Some Lessons from the Decision-Tree Credit-Investigation System The decision-tree approach balances the costs of investigation of credit applicants with the costs of error in granting credit to them. Costly errors may occur in

[5]Both Mehta, "The Formulation of Credit Policy Models," and Stowe, "An Integer Programming Solution," provide general procedures for the formulation of credit investigation policies for orders of any size, not just for the discrete sizes used in this example.

Table 6-4. *Net Present Values for Decision-Tree Credit Investigation Problem (order = $1,000).*

Outcome	Costs of Product ($)	Stage 2 Invest. Costs ($)	Stage 3 Invest. Costs ($)	Stage 4 Invest. Costs ($)	Inflow from Payment ($)	Days to Pay	Net Present Value ($)
1 R							0.00
2 P	800				1000	45	185.93
2 D	800						−800.00
3 Prompt	800	2			1000	30	188.60
3 Slow	800	2			1000	60	179.29
3 Default		2					−2.00
4 R		2					−2.00
5 P	800	2			1000	45	183.93
5 D	800	2					−802.00
6 Rtg. 1	800	2	15		1000	30	173.60
7 R		2	15				−17.00
8 P	800	2	15		1000	60	164.29
8 D	800	2	15				−817.00
10 R		2	15	300			−317.00
11 P	800	2	15	300	1000	30	−126.40
11 D	800	2	15	300			−1,117.00
12 R		2	15	300			−317.00
13 P	800	2	15	300	1000	90	−144.93
13 D	800	2	15	300			−1,117.00
14 R		2	15				−17.00
15 P	800	2	15		1000	90	155.07
15 D	800	2	15				−817.00
17 R		2	15	300			−317.00
18 P	800	2	15	300	1000	60	−135.71
18 D	800	2	15	300			−1,117.00
19 R		2	15	300			−317.00
20 P	800	2	15	300	1000	120	−154.07
20 D	800	2	15	300			−1,117.00

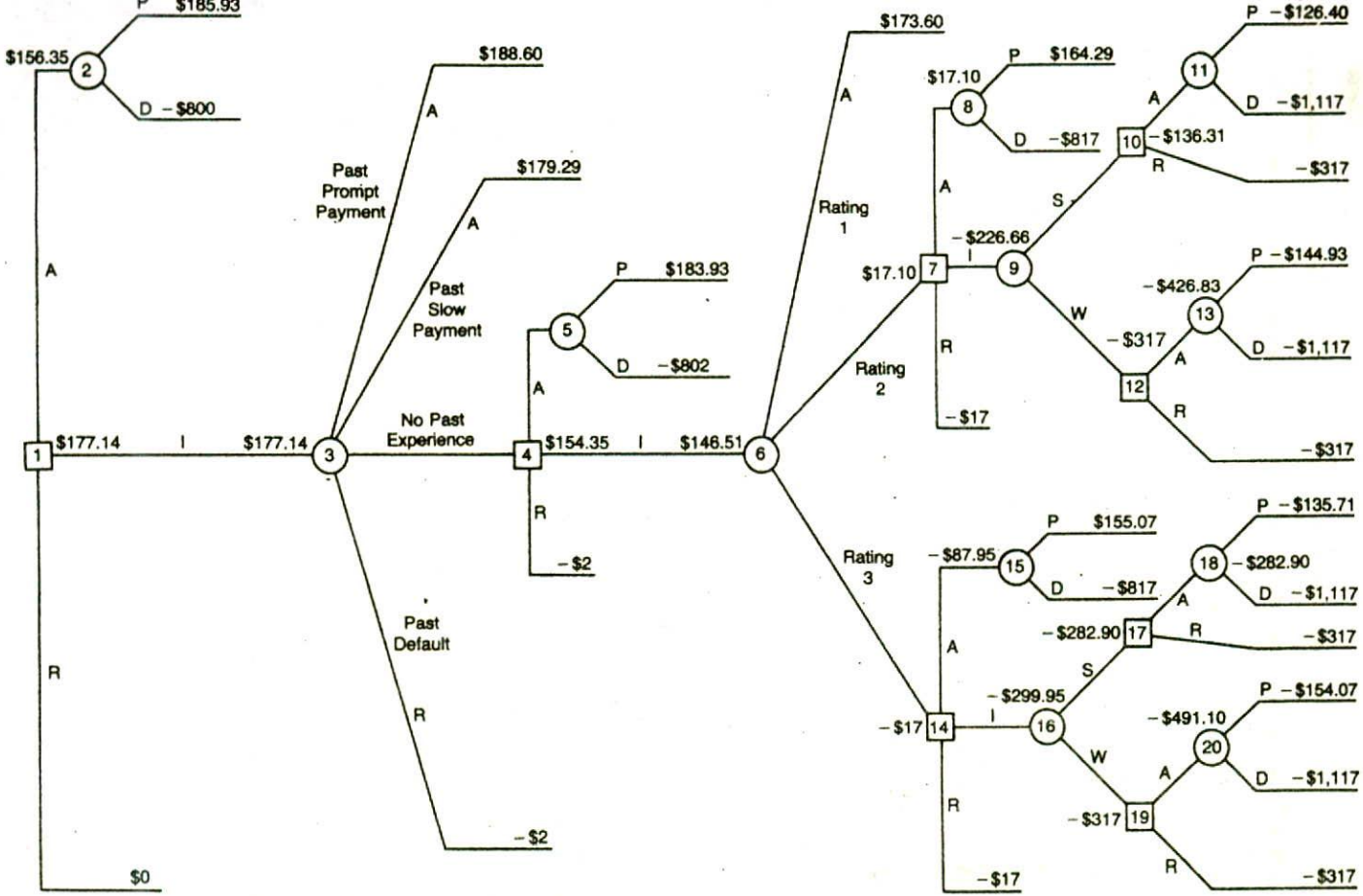


Figure 6-3. Roll-Back of Decision Tree for Example Problem (order = $1,000).

Table 6-5. Computation of Expected Net Present Values for Roll-Back of Decision Tree (order = $1,000).

Node	NPV 1	Prob. 1	NPV 2	Prob. 2	NPV 3	Prob. 3	NPV 4	Prob. 4	E(NPV)
11	−126.40	.99	−1117	.01					−136.31
13	−144.93	.71	−1117	.29					−426.83
18	−135.71	.85	−1117	.15					−282.90
20	−154.07	.65	−1117	.35					−491.10
9	−136.31	.50	−317	.50					−226.66
16	−282.90	.50	−317	.50					−299.95
8	164.29	.85	−817	.15					17.10
15	155.07	.75	−817	.25					−87.95
6	173.60	.84	17.10	.10	−17	.06			146.51
5	183.93	.97	−802	.03					154.35
3	188.60	.50	179.29	.29	154.35	.20	−2	.01	177.14
2	185.93	.97	−800	.03					156.35

(1) granting credit to negative-NPV applicants, and (2) not granting credit to positive-NPV applicants. Besides providing a general method for formulating strategy for credit investigation procedures, there are some lessons to be learned from the logic of this procedure that are not generally obvious.

First, as long as credit investigation is costly, there will be some firms granted credit who will default. This is in large part due to the cost of obtaining the information necessary to separate these firms from other firms whose probability of default is lower. The goal of having a minimum bad debt expense (which is occasionally voiced by some managers) thus entails excessive credit expenses, and is not an optimal policy.[6]

Second, as long as credit investigation is costly, the amount of time and money spent on the investigation of individual applicants will vary with the size of the order. For smaller orders, less time and money will be spent, and rather crude but cheap methods (such as credit ratings) will be the economical approach to investigation. Because these methods entail significant errors in the classification of firms (since firms within ratings classes are not homogeneous), applicants will be refused credit who would receive credit if more was spent on investigation. The opportunity cost of these lost profits is a result of the costliness of credit investigation.

Finally, if this approach to optimal credit investigation is adopted by the seller, it prescribes simple and low-cost strategies for the handling of routine orders. We have taken care to make the example problem repre-

[6]Also, such a goal would require that the firm refuse credit to many firms with a very low probability of default (and thus a positive expected net present value) because of the small chance that they might default.

sentative of typical industrial situations, and it illustrates strategies of these sorts. Orders of $50 are serviced without any investigation, and orders of $1,000 require only a query to the selling firm's data base of past experience with customers. For orders of $10,000, routine transactions with existing customers (80 percent of the total number of such orders) are also handled after the data base query. Of the remaining 20 percent, all those orders from very healthy firms (rated 1) and very unhealthy firms (rated 3) are approved or rejected via a check of the applicant's credit rating (from a credit agency report). The only firms that require extensive and comprehensive investigation are those where the *order amounts are large and the applicant's financial health is neither excellent nor awful.* Because credit-granting decisions for *all other orders* are handled in a quick and low-cost fashion, the credit analyst is free to devote time and effort to this important class of credit applicants. For these applicants, the amount of the order is sufficiently large and the applicant's financial situation sufficiently uncertain to justify spending a substantial amount of time, money, and effort in the analysis of the applicant. In credit parlance, such applicants are called *marginal accounts.*

The analysis of marginal accounts requires a *different approach* than that used for other customers. Rules for the acceptance or rejection of these *other* customers come from the decision tree of credit investigation. The probabilities and days-to-pay statistics that appear on the decision tree are *aggregate probabilities* for *classes* of applicants. These statistics can be developed from survey evidence. For example, by examining a history of applicants and defaults, the firm might find that 100 of the 2,000 applicants with a particular credit rating have defaulted. For any new applicant with that rating, it may then estimate the probability of default at 5 percent (100/2,000).

However, for marginal accounts, the size of the order is large enough that the selling firm should not use inexpensive but inherently inaccurate estimation methodologies to evaluate the value of granting credit. Instead, the seller should *examine the credit-related attributes of the individual applicant* rather than rely on proxies for these attributes (such as credit rating). The intent is to directly estimate the value of granting credit *to that particular applicant,* with all the eccentricities of that applicant incorporated in the investigation. Discussion of procedures for the evaluation of such marginal accounts constitutes the balance of this chapter.

CREDIT GRANTING TO MARGINAL ACCOUNTS: TRADITIONAL APPROACH

In the traditional approach to the credit-granting decision on marginal accounts, the credit analyst tries to assess the *creditworthiness* of the applicant. Creditworthiness is a concept related to the positive and negative aspects of granting credit to the applicant. These aspects include the business history of

the applicant, the manner in which the applicant makes payments to other trade suppliers, the profitability of the products that the applicant wants to purchase, the applicant's financial position, and so forth. Much of this information is gathered via the sources of credit information previously discussed. In the traditional approach to the credit-granting decision, it is the credit analyst's job to *synthesize all information that has been collected and reach a judgment* regarding the applicant's creditworthiness.

To perform this synthesis, it is useful to have some mechanism for organizing the information that has been collected. One traditional way of organizing this information is by characterizing the applicant along five dimensions. These dimensions are called the *Five Cs of Credit*—capital, character, collateral, capacity, and conditions. These are discussed in the following paragraphs.

Capital The evaluation of the applicant's capital refers to an analysis of the applicant firm's *financial position*. What are the applicant firm's financial strengths? What are its weaknesses? Overall, is it stronger or weaker than other firms that the seller believes are creditworthy?

To assess the capital dimension, the credit analyst considers the data obtained from the applicant's financial statements. The usual procedure is to perform an extensive ratio analysis, comparing the applicant's financial ratios to ratios for the applicant's industry and performing trend analysis of the applicant's ratios over time. This ratio analysis can be quite sophisticated, using graphical presentations of ratio trends and statistical comparison of the applicant's ratios with industry standards.[7] Spreadsheet analysis can aid in this ratio analysis process. Indeed, the ratio analysis of firms is a typical introductory exercise in spreadsheet application and is found in most textbooks on the financial applications of spreadsheets.[8]

The assessment of the financial position of an applicant from a morass of ratio data is not an easy task. To aid in this process, some credit analysts choose to emphasize a few of the financial aspects of the applicant and deemphasize others. Frequently, analysts will focus on the applicant firm's *aggregate liquidity position* and its *total debt position*. The first is measured by ratios such as the current ratio and the quick ratio, while the latter is measured by the total debt to total equity and total debt to total assets ratios. Analysts emphasize these ratios because the seller wishes to know the likelihood of payment from the applicant, how long it will take to receive the payment, and what is likely to happen if the applicant defaults. Ratios that measure

[7]For statistical procedures in the comparison of ratios with industry standards, see F. Scherr, "Ratio Analysis for Credit Decisions on Marginal Accounts: A Statistical Approach," *Akron Business and Economic Review*, Vol. 11, No. 3 (Fall 1980), pp. 12–16.

[8]See, for example, N. Cohen and L. Graff, *Financial Analysis with Lotus® 1-2-3®* (Bowie, Md.: Brady Communications, 1984), Chapter 5.

aggregate liquidity assess the relative amounts of current assets and current debt, and thus the funds received and disbursed as part of the applicant's working capital cycle. As discussed in Chapter 1, problems within the working capital cycle are a major cause of default. Measures of aggregate liquidity are thus related to the applicant's ability to pay.

Total debt position tells the relationship between the applicant's assets, equity, and debt. Debt position is also related to the possibility of default, since more debt means a greater probability of default. However, in conjunction with the analysis of collateral (to be discussed later), ratios measuring debt position tell something about the likely recovery of trade creditors if the applicant defaults and is forced to liquidate. The more debt and the fewer assets, the lower will be the recoveries of individual creditors in such liquidations.

Character In order to make payments to trade suppliers, the applicant must have both the *funds to pay the debts* and the *willingness to pay the debts*. The capital dimension involves the former, while the character dimension involves the latter. In assessing character, the credit analyst considers all the information that relates to willingness to pay by the applicant's management. What is the applicant's history of payments to the trade? Has the firm defaulted to other trade suppliers? Does the applicant's management make a good-faith effort to honor debts as they come due? Information in these areas bears on the analyst's assessment of the applicant's character.

Collateral If the applicant experiences financial difficulty, it may be forced to liquidate. In such a situation, the recoveries to trade creditors will depend on (1) the recoveries on assets sold, (2) the amount of debt owed by the firm, and (3) the extent to which these debts are secured. If the firm liquidates, the recoveries on assets that are security for debt will go to the holders of that secured debt. That is, the secured creditors get paid first from the revenues of selling the assets that have been granted to them as security. Since, in general, it is very difficult for trade creditors to obtain secured positions, this means that the recoveries to trade creditors are significantly lower when the applicant has financed by using secured borrowings.[9] Information on secured borrowings is gleaned from the applicant's financial statements, from the applicant's bank, from credit reports on the applicant, or directly from conversations with the applicant. More existing secured financing means lower creditworthiness from the trade creditor's standpoint.

Capacity This dimension has two aspects: *management's capacity to run the business* and the applicant firm's *plant capacity*. Management's capacity

[9]For more discussion on the recoveries to creditors in bankruptcy, see the "Bankruptcy and Reorganization" chapter in any good corporate finance text.

to run the business relates to the competency (ability) of the management personnel in the applicant's operations. Any information relevant to this capacity is assessed, including personal impressions, the history of success or failure by the managers running the applicant business, the number of years that the applicant has been in business, and so forth. The better is management's capacity to run the firm, the lower is the chance of default.

Physical capacity refers to the value and technology of the applicant's production or service facilities. Accounting conventions can paint an unrealistic picture of the value of such assets, particularly if the technology of the applicant's industry is subject to rapid change. The more up-to-date and well-maintained are the applicant's facilities, the more likely that the applicant will be able to stay in business and to take advantage of business upturns.

Conditions These are the economic conditions in the applicant's industry and in the economy in general. If there is a good deal of foreign and domestic competition in the applicant's industry, the possibility of failure and default to trade creditors is larger, since profit margins are likely to be lower. If the economy in general is undergoing a contraction, failures are more likely to occur than during an expansionary period.

Once the credit analyst has gathered information on these dimensions of the marginal credit applicant and information on the profitability of the product to be purchased, the traditional approach requires that all this information be *analyzed* and *synthesized*. By this process, the analyst is to make an *informed judgment* on the overall creditworthiness of the applicant. Unfortunately, there are some problems endemic to this traditional analysis methodology which call its usefulness into substantial question. These problems are discussed in the next section.

PROBLEMS WITH THE TRADITIONAL APPROACH

The traditional judgmental approach to credit-granting decisions on marginal accounts is very flexible. In the process of synthesis, the analyst can take into account any and all of the special features that may affect the desirability of the applicant as a customer. Set against this flexibility are several substantial disadvantages inherent in this decision methodology.

No Analytic Framework Most financial decision methods start with a basic analysis technique, such as net present value. It is then up to the financial analyst to make the necessary estimations to execute the technique. However, there is no such analytic framework in the traditional credit analysis methodology. The analyst is asked to make a *judgment*, with few guidelines on what the appropriate criteria for that judgment should be. Given the large amount of information encompassed in the five Cs and the lack of specific

evaluation criteria, this judgment is not easy to make. Consequently, the traditional analysis method is very difficult to execute effectively.

No Link to Shareholder Wealth Maximization In most financial decision methods, the analyst starts with a technique that is theoretically and empirically linked to shareholder wealth maximization. For example, it is known that shareholders value cash flows, so techniques such as net present value concentrate on the evaluation of these flows. The traditional credit-granting decision technique does not contain such an explicit link to the creation of shareholder wealth, so there is no necessity that decisions based on this technique will be consistent with that goal.

No Consistency of Analysis Because the synthesis process is almost totally judgmental, the results it produces can be inconsistent. The same analyst considering the same applicant on different days can reach different decisions, as can two different analysts considering the same applicant. This inconsistency gives the impression to those outside of the credit department that decisions are being made in an arbitrary fashion. This seeming arbitrariness can cause friction between the credit and other departments of the firm.

Difficult for the Inexperienced Analyst to Execute Over time, a credit analyst, through trial and error, will become experienced at assessing the strengths and weaknesses of applicants, but any judgments will be biased by the analyst's experience. For example, if the analyst grants credit to several applicants who are below-average with respect to a particular ratio and these applicants then default, the analyst will tend to be biased against applicants with such ratios in the future even if the default of these applicants was an aberration, and the particular ratio involved was not truly related to the default. The longer an analyst continues with the traditional method, the larger will be the sample of applicants analyzed, and the less will be the likelihood of such bias.

However, the generation of experience through years of wrong decisions is costly to the selling firm. Inexperienced analysts will have great difficulty with the synthesis process and will make numerous costly errors. Credit will be granted to applicants that are not of advantage to the firm, and it will not be granted to many firms that would be advantageous customers.

In an attempt to overcome some of these substantial problems with the traditional analysis method, firms and authors have developed *judgmental scoring systems* for the evaluation of marginal credit applicants. There are two basic versions of these systems. In *checklist systems*, a series of questions is asked about the applicant, such as: "Is the applicant's current ratio above 2.0?" and "Is the applicant's total debt to total assets ratio below 0.5?" If the applicant gets positive responses on a predetermined portion of the ques-

tions, the applicant is granted credit. In checklist systems, all the questions receive the same weight in decision making. In *weighted scoring systems*, however, the weights can vary from question to question, reflecting the importance of a particular dimension in the evaluation of the applicant.[10] Such weighted evaluation methods often use *point scores* to assess the overall creditworthiness of the applicant. For example, an applicant may receive no points if the current ratio is below 1.5, 2 points if it is between 1.5 and 2.0, 3 points if it is between 2.0 and 2.5, and 4 points if it is above 2.5. The sum of the points scored on various measures of creditworthiness is then compared to some preset number of points when making the credit-granting decision.

Judgmental scoring systems are developed by consultation among experienced credit analysts. These are *expert systems* in that they simulate the decisions an expert (the experienced credit manager) would make when considering the applicant.[11] They have advantages over the traditional judgmental approach in that they provide an analytic framework for analysis, they produce consistent results, and they can be used by a novice credit analyst to make decisions.

Despite their advantages over the traditional judgmental approach, judgmental scoring systems still have one substantial problem. Like the traditional approach, judgmental scoring systems *are not developed from the benchmark of shareholder wealth maximization*. Since they are not developed from this benchmark, *there is little certainty that the decision systems they use will produce a wealth-maximizing result*. A more useful system, from the standpoint of shareholders, would *start with the idea of maximizing shareholder wealth*, and develop decision rules based on this precept. Such a system for the evaluation of marginal credit applicants is developed in the remainder of this chapter.

A ONE-PERIOD NET PRESENT-VALUE MODEL WITH NONZERO RECOVERIES

Since net present value is based on shareholder wealth maximization, the net present-value evaluation method is a logical approach to the credit-granting decision problem. This was the approach taken in the prior discussion

[10]When we talk about judgmental scoring systems here, we are referring to systems based on the opinions of credit analysts (their judgments) rather than to systems based on the statistical analysis of defaulted and nondefaulted firms. See, for example, the systems described in "Credit Limits Established by Formula and Computer," Occasional Paper, Credit Research Foundation, and J. E. Kristy, *Analyzing Financial Statements: Quick and Clean* (Buena Park, Cal.: Books on Business).

[11]For an interesting attempt to replicate the decisions of experienced credit managers via statistical analysis, see V. Srinivasan and Y. Kim, "Credit Granting: A Comparative Analysis of Classification Procedures," *Journal of Finance* (July 1987), pp. 665–83.

of the credit investigation decision. In the balance of this chapter, we apply analysis based on net present value to the credit-granting decision.[12]

In the prior analysis of the credit investigation decision, it was assumed that if the applicant was granted credit, either payment was made in the expected number of days (outcomes designated P in prior decision trees) or the applicant defaulted and the seller recovered nothing. It was assumed that only the cash flows from the current order (the $50, $1,000 and $10,000 order amounts) were relevant to the analysis of the credit applicant. Also, the effects of taxes on decisions were ignored. Since we are now centering on the analysis of credit granting to large customers, we can afford to build models of the cash flows from credit granting that are a bit more sophisticated than those based on these assumptions but require more information and cost to execute. In particular, it is useful to consider situations in which: (1) the recoveries if the applicant defaults are nonzero; (2) there may be other future cash flows that are dependent on the firm's credit-granting decision; and (3) taxes play an important part in the analysis. A model for the first of these is developed in this section. The other two situations are considered in future sections.

To develop a model of the cash flows from credit granting that incorporates recoveries in default, it is first necessary to understand the potential events within the default process. When an applicant defaults on a debt to a trade creditor, it is sometimes because the applicant has adopted the policy of default to that particular creditor. That is, the applicant may be able to pay but may be unwilling to do so. The applicant will then usually pay after the creditor sues. However, in many cases, the applicant may be willing to pay but simply does not have the funds to pay. Several courses of action are available to the applicant firm in this situation. The firm may try to reach an accommodation with creditors, may seek the protection of the courts by filing for bankruptcy, or may liquidate immediately. In the case where the applicant files for bankruptcy and the applicant's attempts at reorganization are unsuccessful, an eventual liquidation also results.[13]

The important point to recognize is that *when the buyer defaults, future cash flows to the seller are delayed, and their amount may be reduced* relative to payment in nondefault. Both of these effects occur in the liquidation of the buyer. Also, liquidation is one of the most common outcomes of the default process. Therefore, we will phrase the discussion of recoveries from defaults in terms of liquidation.

In a liquidation, the amount of the recovery depends on the market value of the applicant's assets, the security arrangements that are in force

[12]For an alternate approach using the option pricing model, see J. A. Miles and R. Varma, "Using Financial Market Data to Make Trade Credit Decisions," presented at the Eastern Finance Association's 1984 annual meeting, and available from the authors at Pennsylvania State University.

[13]For a detailed discussion of the court procedures and negotiations which the defaulted firm faces, see F. Scherr, "The Bankruptcy Cost Puzzle," *Quarterly Journal of Economics and Business*, Summer 1988.

regarding these assets, the size of priority claims (which are paid before any funds are disbursed to creditors), and the total amount of debt owed by the applicant. While recoveries in some defaults will be zero, recovery in others may be substantial. This model developed in this section, which incorporates such recoveries, serves as a basis for our discussion of multiperiod credit-granting models. We will also use this model later in the chapter as a basis for modeling the effect of taxes on credit-granting decisions.

Let us develop a mathematical expression for the net present value of granting credit with recoveries. The decision tree for granting credit is in Panel A of Figure 6-4. Investigation and collection costs can safely be ignored here: we are dealing with marginal applicants of sufficient size that these costs are small relative to the other revenues and costs that relate to the sale. With no investigation costs, the net present value of rejection is zero. Let S be the amount of the sale and c be the expected time from the sale to the payment if the applicant pays. Let V be the costs of making the sale, payable immediately. Let X be the probability of default (thus, $1 - X$ is the probability of payment). The net present value of the sale, should outcome $2P$ occur, is then:

$$\text{NPV}_{2P} = -V + S(1 + k)^{-c} \tag{6-1}$$

where k is the required rate of return on the expected value of the future cash inflow. Let the recovery rate in default be R (as a proportion of the sale) and let the time until this recovery is received be d. The net present value of the sale, should outcome $2D$ occur, is then:

$$\text{NPV}_{2D} = -V + RS(1 + k)^{-d} \tag{6-2}$$

Rolling back the decision tree, the value of node 2 is then:

$$E(\text{NPV})_2 = (1 - X)(-V + S(1 + k)^{-c} + X(-V + RS(1 + k)^{-d})$$

$$= -V + (1 - X)S(1 + k)^{-c} + XRS(1 + k)^{-d} \tag{6-3}$$

This is the decision statistic for the two-state, one-period, no-tax decision-tree model of the credit-granting decision. The alternative of rejecting the applicant has a net present value of zero, so if equation (6-3) is positive, the applicant should be granted credit. This is a two-state model because it has been assumed that only two outcomes can result: payment in the expected time and default.[14] To see how this model would be used to make credit-granting decisions, assume that an applicant has placed an order with the seller for $75,000 worth of goods or services. These goods or services will

[14]If the additional complication is warranted, no-tax, single-period models can be made more comprehensive than this. W. Beranek and W. Taylor ("Credit-Scoring Models and the Cut-Off Point—A Simplification," *Decision Sciences* (July 1976), pp. 394–404) present a model with a third branch: collection beyond the expected time to payment in nondefault.

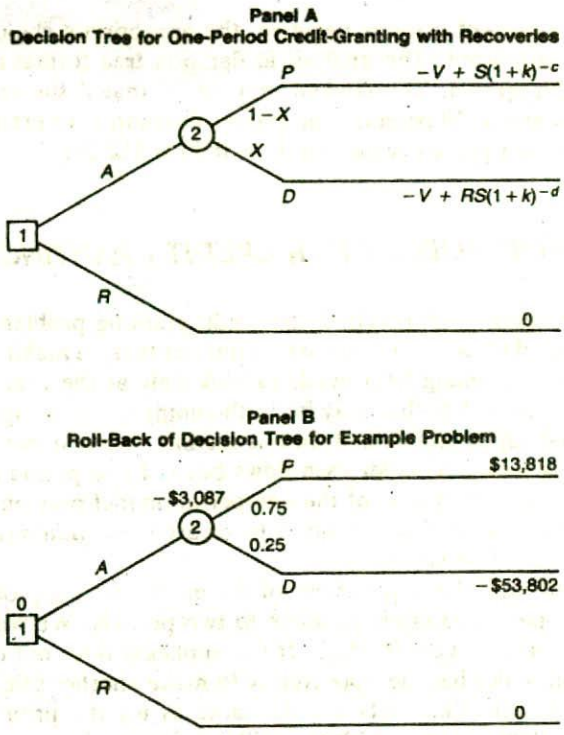


Figure 6-4. Decision Tree for One-Period Credit Granting with Recoveries.

cost $60,000 to provide. If the applicant pays, it is expected that payment will be received in 60 days. If the applicant defaults, a 10 percent recovery is expected, and it is expected that the courts will take two years to liquidate the firm and disburse the proceeds. The estimated probability of default is 25 percent and the required return is 10 percent per year.[15] Should credit be granted?

To answer this question, we insert the estimates of the variables into equation (6-3).

$$E(\text{NPV})_2 = -V + (1 - X)S(1 + k)^{-c} + XRS(1 + k)^{-d} \qquad (6\text{-}3)$$

$$= -60{,}000 + (1 - .25)75{,}000(1.10)^{-60/360} + (.25)(.10)75{,}000(1.10)^{-2}$$

$$= -60{,}000 + 55{,}363.53 + 1{,}549.59$$

$$= -\$3{,}086.88 \qquad (6\text{-}4)$$

[15]Methods for the estimation of these parameters will be discussed later in the chapter.

Since the net present value is negative, the appropriate choice at node 1 is to reject the applicant. The analysis in decision tree format is presented in Panel B of Figure 6-4. The student may verify that if the recovery rate in liquidation is above 30 percent, the proper decision is to grant credit to the applicant (the net present value for $R = 0.30$ is \$12.29).

MULTIPERIOD MODELS FOR CREDIT-GRANTING DECISIONS

The one-period approach to the credit-granting problem should leave the sophisticated student a bit uneasy. It implies that, in making credit-granting decisions, the selling firm needs to look only at the order in hand and not at any future orders that may be forthcoming from the applicant once a selling relationship is established. In this section, decision methodologies for credit granting which consider cash flows beyond one period are discussed. These models are extensions of the one-period model previously presented. The *main objective* of this section is to *assess the conditions under which multiperiod analysis is necessary*.

To understand the implications of the multiperiod approach, let us expand our one-period example problem to two periods. We will assume that the applicant orders every 90 days. If the applicant does not default on the initial sale, the seller has the opportunity to make another sale in the second 90-day period. All other facts are the same as for the prior single-period problem. The decision tree, with the rollback figures, is presented in Figure 6-5; the supporting calculations are presented in Table 6-6. Despite the possibility of a future sale in the second period, the rollback of the decision tree shows that *at node 1 the firm faces exactly the same decision that it faced when only one sale was considered*: the firm has the choice of an expected net

Figure 6-5. Two-Period Decision Tree Example Problem.

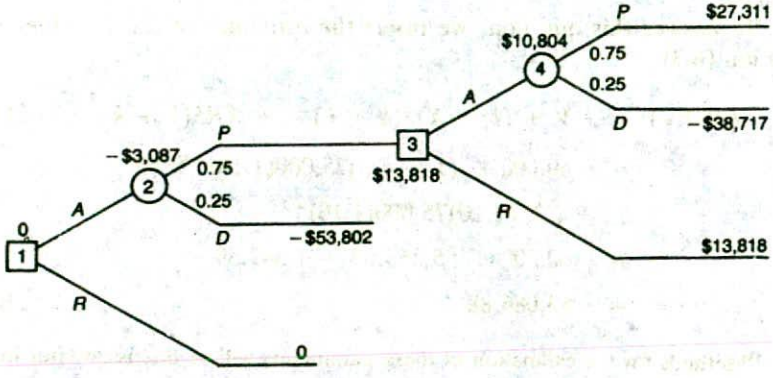


Table 6-6. *Supporting Figures for Roll-Back of Two-Period Decision Tree.*

Day	0	60	90	150	720	810	
Outcome		Cash Flow at Day					NPV
4P	−$60,000	$75,000	−$60,000	$75,000			$27,311
4D	−$60,000	$75,000	−$60,000			$7,500	−$38,717
3R	−$60,000	$75,000					$13,818
2D	−$60,000				$7,500		−$53,802
1R							0

Node	Outcome 1	Prob. 1	Outcome 2	Prob. 2	E(NPV)
4	$27,311	0.75	−$38,717	0.25	$10,804
2	$13,818	0.75	−$53,802	0.25	−$3,087

present value of −$3,087 from granting credit or a net present value of $0 by refusing it. What is going on here?

The answer is fairly simple. For the same problem parameters (default rate, cost of the sale, and so forth), *if it is not of advantage to grant credit to the applicant firm for the initial sale, it will not be of advantage to grant credit for any future sales either.* This is true because, for any number of periods, it will not be advantageous to grant credit in the *last* sales period of the analysis. If credit is refused for this last sale, there will be no contribution of that *last* sale to the net present value of the *next-to-last* period's decision, and thus the period before that, and so forth. Since there are no contributions to the net present value from selling in any future periods, *in the initial period the firm is faced with the same decision as if only one period were analyzed.* That is, adding the opportunity to make negative-NPV credit sales in the future does not affect the decision as to whether to make a credit sale initially.[16]

To make multiperiod analysis necessary, there has to be some expectation that *profitable future sales opportunities will be missed* if the applicant is not granted credit for the unprofitable current sale. For future profitable sales to occur when current sales are unprofitable, *some parameter of the credit-granting decision must change over time.* For example, let us alter the previous problem by assuming that, if the applicant pays for the first order, there is only a 5 percent (rather than a 20 percent) chance that the applicant will default on the second order. The required rolled-back decision tree and

[16]This conclusion is not affected by the size of the future sale. As long as the ratio of V/S and the other parameters of the credit-granting decision (X, c, and so forth) remain the same, the size of the order will not affect the decision as to whether credit granting for that order has a positive expected net present value.

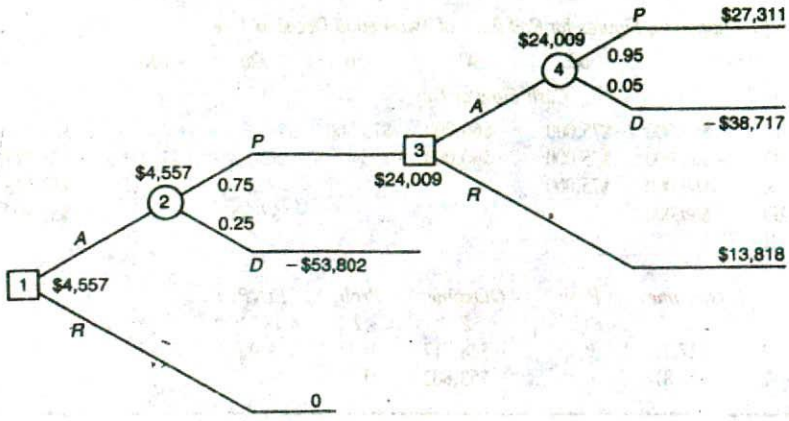


Figure 6-6. *Two-Period Decision Tree Example Problem with Changing Default Probabilities.*

supporting calculations are presented in Figure 6-6 and Table 6-7. The optimal decision at node 1 is now to *grant credit* for the initial sale, with an expected net present value of $4,557.

The multiperiod analysis in Figure 6-6 assumes that the firm must make the first sale in order to make the second one. That is, it assumes that the applicant is *brand loyal* and tends to purchase from the same supplier for repeat sales. If the applicant is brand loyal, *refusing credit at node 1 prohibits the seller from granting credit at a future time when the parameters of the credit-granting decision have changed.* An alternate analysis where the applicant is *not* brand loyal is presented in Figure 6-7, with supporting calculations in

Table 6-7. Supporting Figures for Roll-Back of Two-Period Decision Tree with Changing Default Probabilities.

Day	*0*	*60*	*90*	*150*	*720*	*810*	
Outcome			*Cash Flow at Day*				*NPV*
4P	−$60,000	$75,000	−$60,000	$75,000			$27,311
4D	−$60,000	$75,000	−$60,000			$7,500	−$38,717
3R	−$60,000	$75,000					$13,818
2D	−$60,000				$7,500		−$53,802
1R							0

Node	*Outcome 1*	*Prob. 1*	*Outcome 2*	*Prob. 2*	*E(NPV)*
4	$27,311	0.95	−$38,717	0.05	$24,009
2	$24,009	0.75	−$53,802	0.25	$4,557

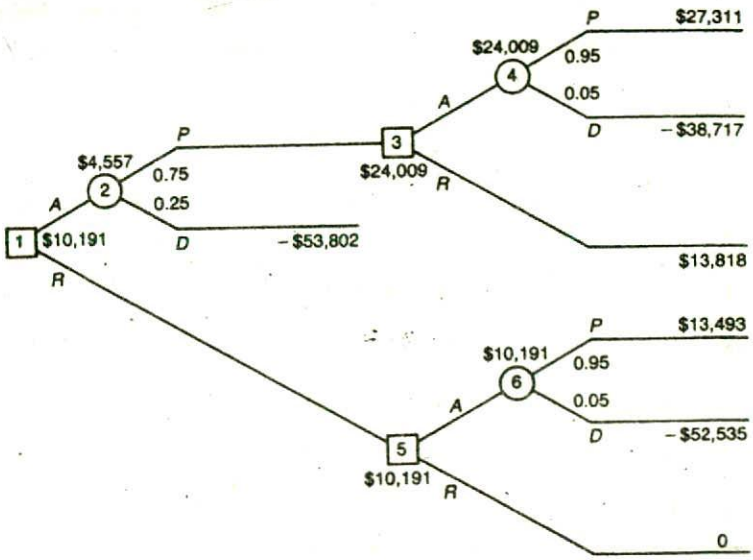


Figure 6-7. Two-Period Decision-Tree Example Problem with Changing Default Probabilities and No Brand Loyalty.

Table 6-8. Since the customer is not brand loyal, the refusal of credit at node 1 does not preclude the seller from receiving orders and granting credit in the future (at node 5). Further, the analysis shows that *this is the optimal strategy*. If the applicant is not brand loyal, the seller should refuse credit for the order in the current period. If the applicant has not defaulted to other trade suppliers by the end of the *first* period, the seller should grant credit for orders in the *second* period.

Let us summarize this discussion of the need for multiperiod analysis in credit-granting decisions on marginal accounts. It is assumed that the credit analyst has performed a single-period analysis of a credit applicant. If the expected net present value is positive, credit is granted; when future orders are received from this customer, these can be analyzed as needed. If the one-period expected net present value is negative, however, the analyst may be concerned that refusing credit to the applicant may result in the loss of future positive-NPV orders. In the latter situation, the analyst should perform multi-period analysis *only if both of the following are true*:

1. The parameters of the credit-granting situation (such as the expected recovery rate or the expected default rate) are expected to change over time.
2. The applicant is brand loyal.

Table 6-8. Supporting Figures for Roll-Back of Two-Period Decision Tree with Changing Default Probabilities and No Brand Loyalty.

Day	0	60	90	150	720	810	
Outcome			Cash Flow at Day				NPV
4P	−$60,000	$75,000	−$60,000	$75,000			$27,311
4D	−$60,000	$75,000	−$60,000			$7,500	−$38,717
3R	−$60,000	$75,000					$13,818
2D	−$60,000				$7,500		−$53,802
6P			−$60,000	$75,000			$13,493
6D			−$60,000			$7,500	−$52,535
5R							0

Node	Outcome 1	Prob. 1	Outcome 2	Prob. 2	E(NPV)
4	$27,311	0.95	−$38,717	0.05	$24,009
2	$24,009	0.75	−$53,802	0.25	$ 4,557
6	$13,493	0.95	−$52,535	0.05	$10,191

As previously illustrated, if the first is not true, future opportunities for unprofitable sales will not change the initial decision. If the second is not true, the seller can wait and see if the applicant defaults, granting credit in the future if default does not occur. Only in the case where both are true is a multiperiod approach to credit-granting decisions required.

The application of a complete multiperiod analysis to credit-granting decisions requires some rather complex calculations and programming methodologies; the methodology is presented in an article by Bierman and Hausman, and is not reviewed here.[17] However, the two conditions listed above that require multiperiod analysis are relatively restrictive. For the parameters of the credit-granting decision to change over time, the applicant must be in a period of changing financial structure or policy. Also, the applicant must be brand loyal. The likelihood of these two conditions occurring simultaneously is rather small. Therefore, for most circumstances a single-period anal-

[17]See H. Bierman, Jr., and W. H. Hausman, "The Credit-Granting Decision," *Management Science* (April 1970), pp. B591–B532. Note however that the techniques in this article are primarily directed toward situations where the credit-granting firm has incomplete information on applicants (and thus the experience of payment or nonpayment affects the estimation of the applicant's parameters for future sales) rather than to marginal accounts with changing financial positions (and therefore changes in the expected parameters of credit granting over time). Extensions of Bierman and Hausman's analysis may be found in Y. M. I. Dirickx and L. Wakeman, "An Extension of the Bierman-Hausman Model for Credit-Granting," *Management Science*, Vol. 22, No. 11 (July 1976), pp. 1229–1237.

ysis is adequate, although the analyst should be aware of the limitations of such an approach.

A ONE-PERIOD MODEL WITH NONZERO RECOVERIES AND TAXE

For the balance of this chapter, we assume that multiperiod analysis is not necessary and continue discussion using models that consider only the next sale to the customer. In a prior section, we developed a one-period model that allowed for recoveries in the event of default. In this section, we extend this one-period model with recoveries to the evaluation of credit-granting decisions where the seller is taxable.[18] This one-period model with taxes and recoveries is the one we recommend as a practical and realistic approach to credit-granting decisions on marginal accounts. Later sections will discuss the estimation of the parameters and the assessment of the risk inherent within this model.

In the prior analysis, three types of cash flows were considered: the initial cost of producing the goods or services for sale (which occurs whether or not the applicant pays), the future cash flow from this payment (occurring with probability $X - 1$) and the future cash flow from recovery in liquidation (occurring with probability X). If the selling firm is taxable, there is a fourth type of cash flow from the sale: the firm's tax payments and refunds. *The timing and amount of these tax flows will affect the value of granting credit to a particular applicant.* Further, the timing and amount of these cash flows depends on the treatment of bad debts that the selling firm elects for tax purposes. Two treatments are common: recognizing bad debts as they occur, and accruing bad debt reserves based on average loss rates. We will develop net present value rules based on expected net present value for each of these two popular tax treatments.

Bad Debts Recognized as They Occur In this tax treatment, the firm does not keep a reserve for bad debt. Instead, it recognizes and deducts bad debt expenses when buyers default. This alternative is simple from an accounting standpoint, but it requires that the firm defer the tax reduction that comes with bad debt expense until defaults actually occur. Let V_T be the firm's cost of goods sold for tax purposes. Due to tax accounting rules, this may not be equal to V, the cash cost of such goods. Since no reserve for bad debt is accrued, the firm will face an initial tax bill on the entire profit from the sale. The amount of this tax bill will be $(S - V_T)T$, where T is the seller's corporate tax rate. Let us assume that this tax bill is payable at the time of

[18]Much of the discussion in this section parallels F. Scherr, "The Industrial Credit-Granting Decision: Two-State Models Including Tax Effects," *Akron Business and Economic Review*, Vol. 16, No. 4 (Winter 1985), pp. 7–11.

the sale.[19] The initial cash flow will then be $-V - (S - V_T)T$ instead of $-V$. If the applicant pays, there is no tax loss, and the net cash flow in the event of payment is the same as in prior models. However, if the applicant defaults, the uncollected balance is a tax deduction that results in a tax refund or a tax reduction. Assume that the firm takes this deduction at the time the recovery occurs. The net present value of the tax reduction is then $T(1 - R)S$, and the net cash flow at the time of recovery will be $RS + T(1 - R)S$. Applying the probability of X to the cash flows from default and the probability $1 - X$ to the cash flows in nondefault and discounting these expected cash flows gives the expected net present value of the credit-granting decision where the firm does not accrue a bad debt reserve:

$$E(\text{NPV}) = -V - (S - V_T)T + (1 - X)S(1 + k)^{-c} + X[RS + T(1 - R)S](1 + k)^{-d} \quad (6\text{-}5)$$

Let us address an example problem. A seller facing a tax rate of 33 percent has received an order for \$100,000. The cost of materials for servicing this order will be \$85,000; the cost of these goods for tax purposes is the same as the cash cost. The estimated probability of default for the applicant is 10 percent and the estimated recovery rate, should the applicant default, is 20 percent. If the applicant does not default, payment is expected in 75 days; if the applicant does default, it is expected that disbursements from the court will be received in three years. The required rate of return is 13 percent per year. The initial tax bill will be $(100{,}000 - 85{,}000)(0.33) = \$4{,}950$. If the applicant defaults, \$20,000 will be recovered (that is, $RS = (0.20)(100{,}000) = \$20{,}000$), and the remaining \$80,000 will be written off, resulting in a tax reduction of \$26,400 ($T(1 - R)S = (0.33)(1 - 0.2)\$100{,}000$). The expected net present value is then:

$$E(\text{NPV}) = -V - (S - V_T)T + (1 - X)S(1 + k)^{-c} + X[RS + T(1 - R)S](1 + k)^{-d} \quad (6\text{-}5)$$

$$= -85{,}000 - 4{,}950 + (1 - 0.9)100{,}000(1.13)^{-75/360} + (0.10)(20{,}000 + 26{,}400)(1.13)^{-3}$$

$$= -85{,}000 - 4{,}950 + 87{,}737 + 3{,}216 = \$1{,}003 \quad (6\text{-}6)$$

Under this tax election decision, the expected net present value granting credit for the sale is positive; the sale is of advantage to the firm, and credit should be granted.

[19]We assume that any tax flows occur concurrently with other events for simplicity of presentation. If, however, the tax flows occur at other times, they can be separately discounted to obtain their values without affecting the basic structure of the model. The same procedure would apply if the cash flow for the cost of the order (V) does not occur at time zero.

Reserve for Bad Debt is Accrued Another tax option is to accrue a reserve for bad debts. Firms accrue this reserve based on average loss rates and average recovery rates for particular types of customers, such as all the customers purchasing from the unit of the firm that is considering the applicant. This reserve is accrued as a deduction from the profit on each sale by that unit. The reserve builds up over time, and when customers default, the amounts of the losses are offset against the reserve. This offset has no tax effect. In this system of tax accounting for bad debts, there are no tax effects after the initial reduction of taxable profit to increase the reserve.

Let P_a be the average default rate for the customers of a particular unit of the selling firm, and let R_a be the average recovery rate on these defaults. The tax-deductible addition to the reserve for bad debt from a sale will then be $P_a\ (1 - R_a)S$, and the initial tax bill on the profit from this sale will be $(S - V_T - P_a(1 - R_a)S)T$. In this tax option, there will be no tax effects beyond this initial tax bill, so the remaining terms in the net present-value equation are the same as in those of the no-tax model portrayed in equation (6-3). The expected net present value will be:

$$E(\text{NPV}) = -V - (S - V_T - P_a(1 - R_a)S)T + (1 - X)S(1 + k)^{-c} + XRS(1 + k)^{-d} \qquad (6\text{-}7)$$

To illustrate this tax option, we use the same facts as in the last prior example problem, but also assume that the average default rate of customers of this unit of the selling firm is 1 percent and that the average recovery rate on defaults is 15 percent. The addition to the bad debt reserve is \$850 $(P_a(1 - R_a)S = (0.01)(1 - 0.15)100{,}000)$, so the initial tax bill is \$4,670 $(S - V_T - P_a(1 - R_a)S)T = (100{,}000 - 85{,}000 - 850)\,(0.33))$. The expected net present value is then:

$$\begin{aligned} E(\text{NPV}) &= -V - (S - V_T - P_a(1 - R_a)S)T \\ &\quad + (1 - X)S(1 + k)^{-c} + XRS(1 + k)^{-d} \qquad (6\text{-}7) \\ &= -85{,}000 - 4{,}670 + 87{,}737 + (0.10)(0.20)(100{,}000)(1.13)^{-3} \\ &= -\$546 \qquad (6\text{-}8) \end{aligned}$$

Under *this* tax election option, granting credit is not advantageous to the firm.[20]

[20]The difference in expected net present value between the two tax elections for this particular problem is due mainly to the difference in default rates. In the second tax treatment, the firm may accrue a reserve at only average default rates, and the probability of default for this applicant is higher than average, so some tax benefit of this applicant is lost. In this problem, this lost benefit more than compensates for the fact that in the second election option, the benefit is received sooner (immediately, rather than when the loss is realized).

The important point made by this analysis is that *the existence of taxes and the different tax treatments of bad debts elected by the firm may affect the credit-granting decision for some applicants*. If taxes did not exist, equation (6-3) applies, and the expected net present value is:

$$E(\text{NPV}) = -V + (1 - X)S(1 + k)^{-c} + XRS(1 + k)^{-d} \qquad (6\text{-}3)$$

$$= -85{,}000 + (1 - 0.10)100{,}000(1.13)^{-75/360}$$

$$+ (0.10)(0.20)100{,}000(1.13)^{-3}$$

$$= -85{,}000 + 87{,}737 + 1{,}386 = \$4{,}123 \qquad (6\text{-}9)$$

This result suggests that the applicant should be granted credit. Yet, we know that for firms accruing a bad debt reserve at average loss rates, granting credit is not of advantage. Thus, *it is important to incorporate tax flows into the analysis of marginal credit applicants*. Since the models portrayed in equations (6-5) and (6-7) account for taxes (and for possible recoveries in default), they are our preferred models for credit analysis.

ESTIMATING PARAMETERS FOR MARGINAL CREDIT APPLICANTS

To use the after-tax models for credit-granting decisions that are presented in equations (6-5) and (6-7), the analyst must make estimates of the values for each of the variables which appear in these equations. A recap of these variables is presented in Table 6-9. One variable, the time to recovery if the applicant defaults and is liquidated (d), is a function of efficiency of the court system in the area where the applicant resides, and must be estimated by the seller based on the seller's knowledge of the typical delays that occur in the required legal proceedings. The remaining variables can be divided into two classes: variables that originate with the *seller* and variables that originate with the *applicant*. Estimation of many of the variables that originate with the seller requires only a reference to the seller's accounting system aided by basic financial analysis. Most selling firms have no trouble constructing data on the out-of-pocket costs of servicing the order (V), the tax cost of this service (V_T), the selling firm's tax rate (T), the average default rate (P_a), and the average recovery rate on defaults (R_a). The size of the order (S) is usually available, since it is often an order which precipitates the analysis. If the impetus for evaluation is an anticipated order, information on the likely size of orders from the applicant can often be provided by the seller's sales force.

The one parameter of the decision that originates with the selling firm and that is a bit more difficult to estimate is the appropriate discount rate for the expected future cash flows (k). This is a risk-adjusted rate that reflects both the time value of money and the risk of the cash flows. However, these

Table 6-9. *Variables That Must Be Estimated for Equations (6-5) and (6-7).*

For Equation (6-5):

V = Out-of-pocket cost of materials to service order from applicant, in dollars.

S = Size of order from applicant, in dollars.

V_T = Tax cost of materials to service order from applicant, in dollars.

T = Seller's tax rate on profits.

X_a = Probability of applicant's default.

k = Required rate of return on cash flows (a risk-adjusted discount rate).

c = Expected time for payment to be received from the applicant if the applicant does not default.

R = Recovery rate if applicant defaults (a proportion).

d = Time to recovery inflow if applicant defaults.

Additional Variables for Equation (6-7):

P_a = Average default rate for customers of this unit of the selling firm.

R_a = Average recovery rate for customers of this unit of the selling firm.

cash flows are from the additional sales of an ongoing product, and it is thus appropriate to use the risk-adjusted discount rate that is relevant to that product line. For most firms, this is the firm's cost of capital adjusted for differences in risk among the lines of business in which they deal.

This leaves three variables that must be estimated in order to compute equations (6-5) and (6-7). Each of these variables originates not with the selling firm but with the credit applicant. The estimation of each requires some sophistication and insight. These three variables are the expected time that payment is to be received if the applicant does not default (c), the recovery rate in default (R), and the probability of the default (X).

Estimating the Time to Pay in Nondefault As previously discussed, the pattern of payments to trade creditors by a debtor firm depends on both the debtor's *financial position* (ability to pay) and *management policies* (inclination to pay). Measures of ability to pay, such as the applicant's current and quick ratios, thus have some bearing on the estimation of the time to pay in nondefault. However, these are not perfect measures. Debtors with seemingly few available funds sometimes show a startling ability to pay trade debts as they come due, and debtors with strong financial positions sometimes leave creditors unpaid for extended periods.

A pragmatic approach to the required estimation is to obtain the applicant's historical pattern of payments to the trade and assume that this pattern will continue into the future. This information can be gleaned from other suppliers of the applicant. For example, if conversations with other suppliers or the examination of the applicant's current trade clearance shows a general pattern of paying promptly, it may be expected that the applicant will be most likely to pay an additional seller on this basis.

Another pragmatic approach is to compute the applicant's *accounts payable turnover ratio* based on the latest data that is available. The accounts payable turnover ratio is:

Annual Cost of Materials and Services Purchased/Payables to Trade (6-10)

This ratio measures the average turnover of trade debts, and therefore the average time that the creditor has taken to pay these debts. For example, assume that an applicant is being considered and that the analyst has a copy of the applicant's financial statement for the fiscal year just passed. By examining the income statement, the analyst finds that purchases from the trade of goods and services (which are part of cost of goods sold) were $12 million and that year-end accounts payable were $3 million.[21] The accounts payable turnover ratio is 12/3 = 4.0 times per year. On average, the applicant paid creditors in 360/4.0 = 90 days. While some types of creditors may have been paid more rapidly and some more slowly than this average indicates, this ratio is a good indicator of the applicant's policies and its ability to pay trade creditors.

Estimating the Recovery in Default The model we are using assumes that the applicant will be liquidated if default occurs. The challenge is to estimate the recoveries from this liquidation. While ratios such as debt to assets are useful in this context, they are relatively crude proxies for expected recovery. One method for estimating this recovery is the *simulated asset sale approach*. In this approach, the analyst uses the applicant's latest financial statement, estimated recovery rates based on the type and quality of assets, and knowledge about any security interests that the applicant has granted. With this information, the analyst simulates the recoveries and transactions that would occur in liquidation. The calculated recovery rate is used as an estimate of R.

Estimating the Probability of Default Of all the estimates required in the analysis of credit applicants, this one is the most difficult to address. What is the chance that an applicant will not pay? Much of the traditional approach to the credit-granting decision centers on this question. One possible method

[21]The use of an average accounts payable figure rather than year-end accounts payable in this calculation would be more descriptive, but requires that both beginning and ending figures are available.

of estimating this probability is to start with the industry failure rate, which represents the average chance that the applicant will default, and adjust this rate up or down to reflect the applicant's strength relative to the average firm in the industry. This approach is very flexible in that any information on the applicant, quantitative or nonquantitative, can be incorporated in the estimation process. For example, the analyst's opinion of the ability of the applicant's management team can be used in the analysis. However, an accurate estimate using such a methodology clearly requires very extensive insight by the analyst.

Another approach is to develop a *failure forecasting function* and use this function to estimate the probability of default by the applicant. There has been very substantial academic research on the estimation of these functions, and this research has shown that such functions can produce excellent results in the classification of firms as future failures or nonfailures, if the required data are available.[22] To estimate such a function, five steps are required.

First, the firm must gather a sample of failed and nonfailed firms. The population from which this sample is drawn will determine the applicability of the estimated failure-forecasting function. If the firm gathers a sample of defaulted and nondefaulted firms from the general population of all the firms in an industry, then the estimated failure forecasting function is applicable to any applicant from that industry. If the sample is limited to those firms that have passed the selling firm's initial credit screening criteria, then the function should be applied only to such applicants.

One problem that can arise at this point is that of sample size. To estimate a reliable failure-forecasting function, the sample should contain at least 20 default firms and 20 nondefaulted firms and preferably more. Since for statistical purposes the size of the smaller group acts as a constraint on the effectiveness of the technique, the usual procedure is to use an industry's entire population of defaulted firms and a randomly selected, equal-sized sample of nondefaulted firms. Even then, the size of the defaulted group of firms in a particular industry may be insufficient, and the selling firm may be forced to combine defaulted firms from several industries to obtain a sufficient sample size. This mixing of industries in estimating the function is an empirical trade-off; there are substantial disadvantages to this procedure if there are divergent causes of default among the industries that are mixed.[23]

Second, the analyst must select variables that are believed to be related

[22]The literature on failure forecasting is extensive. Most of the seminal studies were published between 1969 and 1980; for a review of this literature, see E. I. Altman, R. B. Avery, R. A. Eisenbeis, and J. F. Sinkey, Jr., *Application of Classification Techniques in Business, Banking and Finance* (Greenwich, Conn.: JAI Press, 1981), pp. 255–306.

[23]See F. Scherr, "Estimating and Using Failure-Forecasting Functions: Some Problems and Some Proposed Solutions," *Baylor Business Studies* (January 1982), pp. 23–27.

to default and obtain measures of these for each defaulted and nondefaulted firm in the sample. Financial ratios are typically selected; these ratios have to be good predictors of default in numerous studies (although other variables capturing firm age, firm size, and the state of the economy can also be incorporated). Ratios measuring liquidity and debt are often included, since these reflect potential causes of default.[24]

Third, the analyst must select a statistical technique to relate these variables to the default/nondefault of the firms in the sample. There are several alternative statistical techniques available, including linear probability models, probit models, logit models, discriminant analysis models, and recursive partitioning models. These models have different statistical requirements and require different interpretations.[25]

Fourth, the function must be estimated and adjusted. Estimation is done via a computerized statistical package. Once the initial output is received, the analyst may add or drop variables in an attempt to achieve a superior formulation, then reestimate the function. The goal in the estimation of this function is accurate prediction, and this goal should be kept in mind in the development process.

Finally, the developed failure-forecasting function must be tested on data outside that on which it was developed. This process is called *validation* and is critical in the assessment of the accuracy of the function. The ability of the function to assess the default potential of a *new* applicant is, of course, the proper index of its usefulness.

Once the failure-forecasting function has been estimated and validated, it can then be used to calculate the estimated probability of default for new applicants. This calculation requires two steps. First, data on the applicant is inserted into the function, and a probability of default is calculated. However, *the probability of default calculated in this way assumes that the proportion of defaults among applicants is the same as that in the sample* that was used to estimate the function. That is, this unadjusted probability assumes that the chance of default is 50 percent for any applicant if this is the proportion of defaults in the sample. On average, a much smaller proportion of applicants than this will actually default. To adjust for this difference in probability between that of the sample and that of the population of applicants, Bayes' rule can be used. Bayes' rule states:

$$\text{Posterior odds} = (\text{prior odds})\,(\text{information odds}) \qquad (6\text{-}11)$$

For example, suppose a failure-forecasting function is estimated using an equa'

[24]See Scherr, "Failure-Forecasting Functions," pp. 20–23 and 27–29.

[25]Discussion of the linear probability, probit, logit, and discriminant analysis models can be found in Altman et al., *Classification Techniques*, pp. 1–58. Discussion of the recursive partitioning technique can be found in H. Frydman, E. Altman, and D. Kao, "Introducing Recursive Partitioning for Financial Classification: The Case of Financial Distress," *Journal of Finance* (March 1985), pp. 269–91.

proportion of defaulted and nondefaulted firms. This function is then used to calculate the probability of default for a credit applicant. Assume that the calculated probability of default for the applicant is 60 percent. Thus, the *information odds* given by the failure-forecasting function are 60/40 (60 defaults and 40 nondefaults in 100 applicants with these characteristics). However, suppose that in the overall population of credit applicants, only 2 firms in 100 default. The *prior odds* are then 2/98. The applicant's true default odds are:

$$(2/98)\ (60/40) = 0.0306 \tag{6-12}$$

The proper estimate of X for equation (6-5) or (6-6) is then 3.06 percent.

Failure-Forecasting Functions in Perspective The estimation of the probability of failure via statistical techniques is a relatively new technology and deserves additional discussion. The use of these techniques to estimate the probability of default avoids any personal biases of credit analysts, provides a probability estimate of default, and has been shown, in several studies, to be quite accurate. An additional advantage is that most statistical techniques used to estimate failure-forecasting functions also provide estimates of the error inherent in the forecast. However, failure-forecasting functions require considerable data collection and statistical sophistication to develop. In particular, the selection and adjustment of the appropriate statistical model requires expertise that most businesspersons do not possess. To successfully develop a reliable failure-forecasting function, this expertise must be available within the selling firm, or it must be purchased from a consultant. Further, the resulting failure-forecasting function can be used most advantageously to estimate the default probability only when data on the applicant is available for each of the variables in the failure-forecasting function. Difficulties such as these do not invalidate the failure-forecasting function as a way to estimate default probability, but they do make this technique more difficult to execute.

UNCERTAINTY IN CREDIT-GRANTING DECISIONS

The risk of the applicant's default in the credit-granting decision is captured by the expected value process and the use of a risk-adjusted discount rate. What has not yet been addressed is the *uncertainty in the estimation of parameters* for the evaluation of the applicant. What if the estimates of the parameters used in making the credit-granting decision are subject to error? How does this affect the decision process?

To address this question, we must look at the estimates made in order to calculate equations (6-5) and (6-7) and assess the effect of possible variations in these estimates on the decision to grant or not to grant credit. In examining the variables in Table 6-9, there is likely to be considerable un-

certainty about the values of some of these, while the values of others are relatively certain. For example, the selling firm is likely to know its near-term tax rate (T) with considerable certainty. However, the estimations of three important variables are likely to be very uncertain: the expected time to payment in nondefault (c), the recovery rate in default (R), and the probability of default (X).

The estimated time to pay in nondefault is subject to error in part because of the different policies that debtors use in paying various suppliers. For example, some debtors follow the policy of paying continuing suppliers more promptly than they pay suppliers from which they buy only occasionally. In such a case, the *average* payment time to suppliers may misrepresent the expected payment time to a particular supplier. Also, the use of past data (past payment patterns in this case) to estimate future behavior always carries with it the risk that the applicant will change policies.

The estimated recovery rate in default is uncertain for two reasons. First, this recovery rate is usually estimated based on the applicant's financial structure *at the time of the sale*. Between that time and the time of liquidation, however, this structure may have changed considerably. Second, in the simulated asset sale approach, estimated recovery rates are applied to the applicant's assets. However, the actual recovery rates in the liquidation of these assets may be much higher or lower, depending on the condition of the assets at the time of liquidation.

Finally, the estimated default probability is uncertain, even if the most careful estimation techniques are used. No statistical or other estimation technique can capture all the peculiarities that are inherent in the survival or failure of firms. For example, an intelligent and skillful manager can keep a financially strapped business going for a surprising amount of time, while inept management can run a strong firm out of business in months. Because of such peculiarities, any failure-forecasting methodology will always have error in the estimate of default probability.

Because there is a distinct cutoff between acceptance and rejection (based on the expected net present value of granting credit for the sale to the applicant), a *simulation* of the effect of these uncertainties on this net present value can go a long way to investigating risk in the credit-granting decision. In such a simulation methodology, an estimated probability distribution of the applicant's payment pattern to trade suppliers and of the recovery rate, along with the standard error of the estimate of the failure-forecasting function, can be used to develop the probability distribution of the expected net present value from granting credit to the applicant. This probability distribution is then examined relative to the cutoff of zero expected net present value. Two examples are presented in Figure 6-8. In both of these, the expected net present value is positive, so the applicant will be granted credit in the absence of uncertainty considerations. However, in Panel A, there is very little chance that the errors in the estimation of the parameters

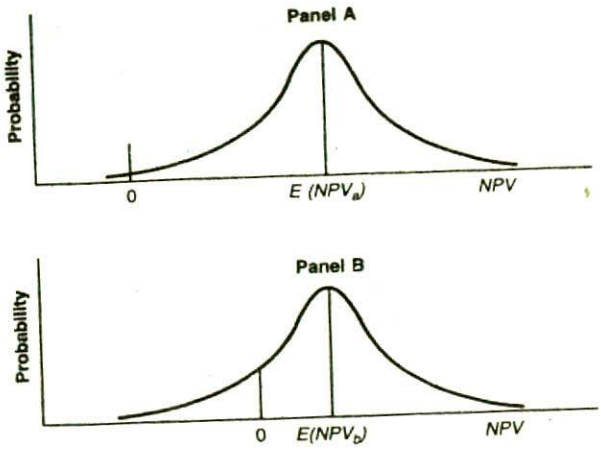


Figure 6-8. Two Probability Distributions of Net Present Value from Granting Credit.

are such that the true expected net present value is negative; the seller can have considerable confidence that granting credit to this applicant is an appropriate decision. In Panel B, however, there is a very substantial chance that the true expected net present value is negative, and the seller should carefully examine the estimates made in assessing this applicant before proceeding with the granting of credit.

CREDIT LIMITS: AN UNANSWERED QUESTION

Up to this point, we have treated the credit-granting decision as dichotomous: the selling firm either granted credit to the applicant for the amount of the order or it did not. However, there is a third option: the seller may grant a *limited amount* of credit, called a *credit line* or a *credit limit*. In such a credit-granting system, the seller will specify a maximum amount of outstanding debt from each customer. If an additional order is placed so that the credit limit is exceeded, the order may be refused or the customer may be required to pay for prior orders so that this maximum is not violated. We will see in the next chapter that many selling firms limit credit granting in this way.

Except in special cases, the credit limit approach does not seem to agree with the maximization of shareholder wealth as expressed by equations (6-5) and (6-7). For the credit limit approach to be in line with the maximization of shareholder wealth, the net present value in these equations *must go through a maximum in sales* (S) then fall with additional sales. In such a situation, it would be optimal for the seller to limit sales to that particular customer to

the amount that results in this maximum NPV.[26] The prevalent use of credit limits by industrial sellers has left researchers in accounts receivable management searching for some cost or revenue that may be a function of sales volume and may produce the NPV optimum, explaining the behavior of firms. Several such costs and revenues have been suggested. We discuss them and other possible reasons for limiting credit in the following paragraphs.

Overbuying One possibility is that industrial purchasers will "overbuy" if granted credit beyond their needs.[27] In such a case, the purchaser will take on trade debt beyond that needed for purchases; the purchaser will also become less liquid, as its inventory turnover will fall. This overbuying will then increase the probability of the purchaser's default (X) for some levels of additional sales and, at some point, produce a maximum in NPV.

Increases in Production Costs Another possibility is that the cost of goods produced to service the order (V) may rise disproportionately with sales. In this case, net present value may rise initially with sales volume, then decline with increases in sales to the customer as unit production costs rise.

Funds Constraints Another circumstance that can lead to credit limits as an optimal policy occurs when the seller faces funds constraints in the firm's investment in accounts receivable. In such circumstances, the selling firm must optimize net present value given this constraint, and limiting sales volumes to customers is one possible response. Note that this situation implies the rejection of sales that have positive net present values.[28]

Changes in the Firm's Systematic Risk Granting credit can raise the systematic risk of the firm and therefore alter its required returns via the CAPM.[29] Recall from Chapter 1 that in the CAPM the required return on an asset is:

$$R_i = R_f + b_i(R_m - R_f) \tag{1-5}$$

[26]For discussion of this issue, see F. Scherr, "Marginal Cost and Revenue Industrial Credit-Granting Models: Decision Implications and an Example Application," *Akron Business and Economic Review*, Vol. 13, No. 4 (Winter 1982), pp. 38–42.

[27]See Jess H. Chua, "A Trade Credit Granting Model with Order-Size Dependent Credit Risks," available from the author at the University of Calgary and S. Besley and J. S. Osteryoung, "Accounts Receivable Management: The Development of a General Credit-Granting Algorithm for Determining Credit Limits Under Funds Constraints," presented at the Financial Management's 1984 annual meeting and available from Besley at the University of South Florida. This overbuying phenomenon is almost certainly the reason that credit limits are commonly used in the management of consumer credit.

[28]See Besley and Osteryoung, "Accounts Receivable Management."

[29]See T. E. Copeland and N. T. Khoury, "Analysis of Credit Extensions in a World of Uncertainty," in K. V. Smith, ed. *Readings in the Management of Working Capital*, Second Edition (St. Paul: West Publishing, 1980), pp. 323–30.

In this equation, the beta for the asset (b_i) is the comovement between the asset's returns and the returns on the market portfolio. Beta is directly proportional to the covariance between the asset's returns and the returns on the market portfolio. This covariance is:

$$\text{Cov}(R_i,R_m) = SD_{Ri}SD_{Rm}\text{Cor}_{RiRm} \qquad (6\text{-}13)$$

where SD_{Ri} is the standard deviation of the returns on the asset, SD_{Rm} is the standard deviation of the returns on the market portfolio, and Cor_{RiRm} is the correlation coefficient between these returns. If granting credit to an applicant adds to the dispersion of returns on the firm's assets (SD_{Ri}), it will increase the firm's beta, and thus its overall level of risk in a CAPM context. If the cash flows from the applicant are correlated with returns in the economy (Cor_{RiRm}), this can also increase risk. In either case, the required returns on the firm's assets will increase. The resulting decrease in the value of the firm's assets because of this increase in required return may offset the increased expected cash flows from credit granting, resulting in an optimum in NPV.

Changes in the Variance of the Receivables Portfolio As additional credit is granted to a buyer, the variance of the cash flows from the seller's portfolio of accounts receivable is altered. Depending on the correlation between the cash flows from additional receivables and the amount of such receivables, the variance of the cash flows from the receivables portfolio may increase. If the seller is averse to this variance, the seller will price it by increasing the required return on new receivables (k) with the amount of credit granted. This can cause a maximum in the NPV of additional credit sales.[30]

Agency Problems Another possibility is that managers are not acting in the shareholders' interests; that is, they are not maximizing net present value. In such a case, they may not use decision processes that parallel equations (6-5) and (6-7); the decision processes they do use may require credit limits. Situations where managers do not act in the shareholders' best interests (which are called *agency problems*) may be caused by an unintentionally faulty implementation of the rules of shareholder wealth maximization in the development of policies for the selling firm.[31] In the context of setting policies for the evaluation of credit management personnel, firms may have chosen statistics such as accounts receivable turnover or bad debt expense as their measures of credit-manager effectiveness, with the implication that higher turnover and lower bad debts indicate better management. In such a case, credit managers may tend to be more conservative in their credit-granting

[30]See F. Scherr, "A Portfolio Approach to Credit-Granting Decisions" available from the author at West Virginia University.

[31]See A. Barnea, R. Haugen, and L. Senbet, *Agency Problems in Financial Contracting* (Englewood Cliffs, N.J.: Prentice Hall, 1985) for a discussion of agency problems in finance.

decisions than the net present value rules would dictate, since they are penalized for defaults and slowness but are not credited when positive-NPV sales are made.[32] They may use credit limits to reduce bad debt expense, even though the orders refused have positive expected net present values.

Credit Limits and Credit Investigation Another possibility is that credit limits are used within firms as *monitoring tools*, but these limits are not as enforced as limits on the amounts that customers can purchase. Credit managers are often charged with making the proper credit investigation decisions, in addition to credit-granting decisions. If they assign credit limits to customers based on the amount of credit information that has been collected on that customer, the violation of the credit limit can be used as a trigger for additional credit investigation. When the credit limit set for this purpose is violated, this violation provides a signal to the credit analyst to collect additional credit information and to reexamine the applicant based on this information. Based on this additional information, the credit analyst may approve the order by raising the applicant's credit line or may withdraw credit altogether.

Financial research has not yet provided an answer to why firms use credit limits. The link between shareholder wealth and the determination of credit limits is not understood. There may be several reasons for this use, or the reasons may vary from firm to firm. Given the current state of research, the most that can be said is that this use is widespread but the general explanation for it is unknown.

SUMMARY

This chapter dealt with the credit-granting decision: to whom should the firm grant trade credit? There are numerous sources of information that give the selling firm insight into the costs and revenues from credit granting. However, the collection of information from these sources is costly. These costs of information play an important part in developing strategies for credit granting. For small order sizes, it is not economical for the selling firm to spend large amounts on the investigation of the credit applicant. The decision-tree approach to the credit investigation gives optimal rules for investigation based on order size. This approach was illustrated for three order sizes in an extensive example problem.

For large orders from customers of uncertain financial strength, it is optimal for the selling firm to collect a great deal of information on the credit applicant. With this information in hand, some method must be used to

[32]We will see in the next chapter that the proper way to use measures of accounts receivable turnover and bad debt is with reference to budgets, with these budgets being set up based on the terms of sale and credit-granting decisions of the selling firm.

integrate the facts regarding these marginal applicants and decide whether or not to grant credit. In the traditional approach to this credit-granting decision on marginal accounts, the credit analyst uses the Five Cs of Credit as a framework to assess the creditworthiness of the applicant, and makes a judgment on credit granting. Unfortunately, there are several problems with this method, the most serious of which is that there is no direct link to shareholder wealth maximization.

To directly link the credit-granting decision on marginal accounts to the desires of shareholders, a net present-value model for credit-granting decisions was developed. This model allows for taxes and recoveries in the default of the credit applicant. The model should be used in its one-period form unless the applicant is brand loyal and the estimated parameters of the credit applicant are expected to change over time. (If the applicant is brand loyal and estimated parameters are expected to change, a multiperiod approach is required.) Methods of estimating the parameters necessary to execute this model were discussed. The risk associated with this model originates with the uncertainty in the estimation of these parameters, and can be assessed via simulation analysis.

The terms of sale and credit-granting policies that the firm adopts will imply some level of expected revenues and costs. Unfortunately, comparing the actual levels of some of these revenues and costs with the expected levels is a relatively tricky business. Yet such a comparison is necessary to enable the firm to monitor the effectiveness of its policy decisions. In the next chapter, we discuss the monitoring of these revenues and costs for accounts receivable. We also review the literature on the practices currently used in credit management.

Problems

6-1. A firm is trying to formulate a credit investigation policy for order amounts of $500. When an order is received, the firm can accept the order, reject the order, or investigate further. If the order is accepted without further investigation, the probability of default is 5 percent and it is expected that payment will be received in 45 days (terms of sale are 30 days). If the firm elects to investigate, the costs of this investigation are $5, and three results are possible: the applicant may have paid promptly in the past, the applicant may have defaulted in the past, or the firm may have no experience with the applicant. The probabilities of these outcomes are 0.75, 0.05, and 0.20. If the seller has experience with the applicant, it is assumed that future credit granting will have the same result as past credit granting. At this stage, the seller may grant credit to the applicant without further investigation, reject the applicant, or

investigate further. If the seller has no experience with the applicant and credit is granted without further investigation, the probability of default is 5 percent and the expected time to pay is 45 days. If additional credit investigation is undertaken, investigation costs will be $25 and two outcomes are possible: the applicant may have a strong or a weak financial position. Financially strong firms pay promptly; there is a 70 percent chance the applicant will be financially strong. If the applicant is found to be financially weak, the probability of its default is 15 percent. If the financially weak firm does not default, it is expected that payment will be received in 80 days. Construct the necessary decision tree and develop the appropriate credit investigation strategy. The firm's cost of capital is 10 percent and costs of materials are 85 percent of sales (payable immediately). Ignore taxes and recoveries in default in your calculations.

6-2. A credit applicant has placed three orders with a seller, and the seller must make a credit-granting decision regarding the applicant. The first order is to be delivered immediately, the second in 120 days, and the third in 240 days. Each order is for $10,000. Costs of goods sold are 85 percent of sales, payable at the time the order is delivered. If the applicant defaults, no recoveries are expected. The applicant is undergoing a period of expansion and is brand loyal. The seller has estimated that the probability of default for the initial order is 20 percent, and that payment for this order is expected in 75 days if the applicant does not default. If the applicant does not default in paying for the initial order, it is expected that the probability of default for the second order will be 10 percent and the expected time to pay for that order will be 60 days. If the applicant does not default on the second order, the probability of default on the third order is 5 percent, and the expected time to pay for that order is 45 days. Indicate whether credit should be granted for the initial order. The seller's cost of capital is 13 percent; ignore taxes in your calculations and assume that all credit investigation costs have been spent.

6-3. A selling firm is taxable and wants to evaluate a credit applicant via the one-period, after-tax, net present-value approach. The appropriate discount rate for expected future flows is 10 percent per year. If the applicant does not pay in the normal course of business, it is expected that the applicant will be liquidated. The liquidation process will take two years, and trade creditors are expected to experience a 50 percent recovery rate, though the average recovery by the seller in such cases is 20 percent. If the applicant does not default, it is expected that payment will be received in 90 days. Based on a failure-forecasting function estimated with equal sample

sizes of defaulted and nondefaulted firms, a probability of default of 80 percent for the applicant has been computed. However, only 1 percent of applicants actually default. The seller is in the 35 percent tax bracket, and the order amount is $30,000. The tax cost of the inventory to service the order will be $27,000, although the actual out-of-pocket cost to replenish inventory when the order is shipped will be only $26,000. Use a 360-day year.

a. Calculate the expected net present value of the sale if the firm accrues a reserve for bad debt based on average loss rates.

b. Calculate the expected net present value of the sale if the firm does not accrue a reserve, but instead writes off bad debts as they occur.

Selected Readings

Altman, E. I., R. B. Avery, R. A. Eisenbeis, and J. F. Sinkey, Jr., *Application of Classification Techniques in Business, Banking and Finance* (Greenwich, Conn.: JAI Press, 1981), pp. 255–306.

Beranek, W., and W. Taylor, "Credit-Scoring Models and the Cut-off Point—A Simplification," *Decision Sciences* (July 1976), pp. 394–404.

Bierman, H., Jr., and W. H. Hausman, "The Credit-Granting Decision," *Management Science* (April 1970), pp. B591–B532.

Mehta, D., "The Formulation of Credit Policy Models," *Management Science*, Vol. 15, No. 1 (October 1968), pp. B30–B50.

Scherr, F., "Estimating and Using Failure-Forecasting Functions: Some Problems and Some Proposed Solutions," *Baylor Business Studies* (January 1982), pp. 23–27.

Scherr, F., "Marginal Cost and Revenue Industrial Credit-Granting Models: Decision Implications and an Example Application," *Akron Business and Economic Review*, Vol. 13, No. 4 (Winter 1982), pp. 38–42.

Scherr, F., "The Industrial Credit-Granting Decision: Two-State Models Including Tax Effects," *Akron Business and Economic Review*, Vol. 16, No. 4 (Winter 1985), pp. 7–11.

Stowe, J. D., "An Integer Programming Solution for the Optimal Investigation/Credit Granting Sequence," *Financial Management* (Summer 1985), pp. 66–76.

case

Vopal Plastics[1] *Credit-Granting Decisions*

On January 7, 1976, Dick Smith (the assistant credit manager of Gymbal Plastic Resins) had just finished a meeting with his boss, Bob Little. Mr. Little had received a call the previous day from Gymbal's salesman for Michigan regarding Vopal Plastics, a firm located in Muskegon. Vopal had been a customer in 1971 and 1972, but had not purchased anything from Gymbal for the past three years.

"I haven't looked at the Vopal file, Dick," Mr. Little had said. "You weren't here then, but when we were selling to them in 1972, they were slow payers. The salesman tells me that they lost money in 1974 and 1975; things are probably pretty grim there now from a financial standpoint. However, the salesman says that they are buying a lot of materials from our competitors. We could make some money if we could sell some plastic there, payments aren't too far beyond our terms of sale, and the risk of default isn't too great. The salesman says we might sell them as much as $50,000 per month, though the order we have in hand is for less than that. Check it out and get back to me."

[1]This case has been adopted form Iqbal Mathur and Frederick C. Scherr, *Cases in Financial Management* (New York: Macmillan, 1979), pp. 7–11.

Gymbal had maintained a running credit file on Vopal Plastics. This file contained credit agency reports and trade clearances (lists of amounts owing, amounts past due, terms of sale, and payment patterns to suppliers). Stock in Vopal was traded over-the-counter; audited financial statements from annual reports had been reproduced in credit agency presentations. The auditor was a major accounting firm, and the auditor's opinion contained no disclaimer.

From reading the credit agency reports, Dick ascertained that Vopal's recent history had contained some peaks and valleys. The firm had been founded in the late 1940s and was one of the oldest plastic-molding firms in the Muskegon area. Sales volumes had grown steadily in the 1969–1974 period, but had suffered a severe decline in 1975. In a mid-1975 message to stockholders, Vopal's president had attributed this decline to customers' caution in responding to the economic upturn underway at the time.

Profits, too, had recently declined. The years 1971 to 1973 had been profitable, but 1974 and 1975 had been loss years. The losses in these years had been attributed by Vopal's management to several factors. Profits in 1974 were good through three quarters, but signif-

icant losses were incurred in the fourth quarter due to diminished demand, rising raw material costs, customer resistance to price increases, rises in indirect costs, and a write-down of some assets. The latter factor alone had increased the net loss by $130,000. In 1975, the firm had attempted to return to profitability by reducing its selling and administrative expenses. An unprofitable operation was also closed. This closing resulted in a nonrecurring item that reduced income by $126,000. However, operations still resulted in a small net loss due to decreased sales volumes. Vopal's financial statements from 1971 through 1975 are presented in Exhibit 1.

According to their banker's comments, Vopal's relationships with their banks had always been satisfactory. In 1969, Vopal had consolidated a number of term loans from their bank into a medium-term secured loan. In 1973, this loan had been renegotiated. The firm's current banking arrangements were comprised of a long-term secured loan and a revolving credit arrangement in which three local banks participated. The covenants of this loan agreement were given in Vopal's annual report, and included restrictions on net worth, working capital, cash dividends, and capital spending. These loans were secured by all the firm's assets.

Dick Smith noted that the raw materials purchased by Vopal were not sold on cash discount terms. Vopal had always had a reputation in the trade as a slow-paying account, but Vopal's management had always made some excuse for this condition. Dick Smith used his credit file's trade clearances to make a table of Vopal's payment pattern to the trade over time and the reasons given for slowness (see Exhibit 2).

The usual credit-granting decision method used by Gymbal was the traditional approach. After analyzing Vopal Plastics via this judgmental analysis procedure and coming to a conclusion regarding the granting of credit to the firm, Dick Smith was quite unsatisfied. It seemed to him that his analysis had been unnecessarily lengthy and time consuming. He also felt that his conclusion regarding the granting of credit to Vopal, while based on his best judgment regarding the positive and negative aspects of the applicant, was to some extent arbitrary and thus could easily be incorrect relative to the goals of his firm. These difficulties, which he felt made the judgmental analysis of credit applicants more of an art than a science, had troubled him for some time. Therefore, he decided to attempt a net present value analysis of the initial monthly order from Vopal, which was for 200,000 pounds of high-density polyethylene plastic pellets, to be sold to Vopal for $0.13 per pound.[2]

To obtain an estimate of Gymbal's cost for this material, Mr. Smith consulted with his firm's cost accounting department. After a period of discussion and analysis, Mr. Smith determined that the out-of-pocket costs of replacing the inventory sold to Vopal Plastics would be about $0.10 per pound for materials (chiefly ethylene gas and catalysts) and direct labor. However, because of the method used by Gymbal in valuing inventory for tax purposes, the tax bill for the sale would be based on a unit cost of $0.105 per pound. The sale to Vopal, were it made, would not affect Gymbal's present or future levels of capital expenditure; Gymbal's polyethylene production facility had substantial excess production capacity. Gymbal's terms of sale for polyethylene were net 30 days.

[2] While Gymbal's salesman for Michigan had estimated the potential sales volume to Vopal as ". . . as much as $50,000 per month," this estimate assumed that Gymbal would be the sole supplier of polyethylene to Vopal. Vopal's actual order represented only part of this total material use. Vopal usually bought product simultaneously from several suppliers of each type of plastic, placing a particular order with whichever supplier offered the best unit price and delivery terms at that time.

Exhibit 1

Vopal Plastics
Financial Statements for the Fiscal Years Ending 12/31/71 to 12/31/75
(rounded thousands of dollars)

Year End	*12/31/71*	*12/31/72*	*12/31/73*	*12/31/74*	*12/31/75*
			Statements of Income		
Sales	$4,639	$5,850	$7,184	$8,282	$6,207
Cost of Goods Sold	3,841	4,899	6,084	7,148	5,115
Gross Margin on Sales	798	951	1,100	1,134	1,092
Selling and Admin. Expenses	739	724	970	1,373	1,105
Operating Income	59	227	130	−239	−13
Extraordinary Profit and Loss	−5	−28	84	0	−126
Earnings Before Tax	54	199	214	−239	−139
Income Taxes	−12	98	86	−98	0
Earnings After Taxes	$ 66	$ 101	$ 128	$−141	$−139
			Balance Sheets		
Cash	$ 48	$ 90	$ 90	$ 184	$ 319
Accounts Receivable	630	766	939	1,090	1,015
Inventories	888	725	958	1,261	785
Prepaid Expenses	40	52	61	60	47
Other Current Assets	19	19	21	114	17
Total Current Assets	$1,625	$1,652	$2,069	$2,709	$2,183
Fixed Assets less Depreciation	$1,288	$1,311	$1,298	$1,796	$1,791
Other Assets	259	121	78	43	22
Total Assets	$3,172	$3,084	$3,445	$4,548	$3,996
Accounts Payable	$ 722	$ 390	$ 510	$ 706	$ 685
Due on Credit Line			100	325	312
Current Long-Term Debt	218	210	404	373	352
Accruals	80	110	124	229	248
Other Current Liabilities	73	156	99	52	22
Total Current Liabilities	$1,093	$ 866	$1,237	$1,685	$1,619
Long-Term Debt	$ 596	$ 634	$ 499	$1,295	$ 951
Common Stock	96	96	93	93	90
Paid-In Capital	440	440	440	440	440
Retained Earnings	947	1,048	1,176	1,035	896
Total Equity	$1,483	$1,584	$1,709	$1,568	$1,426
Total Liabilities and Equity	$3,172	$3,084	$3,445	$4,548	$3,996

Exhibit 2

Vopal Plastics
Summary of Trade Payments and Management Comments, 1971 to 1975.

Year	*Trade Payment Habit*	*Reasons Given for Slowness*
1971	Generally pays 30 days beyond terms. Some supplies paid promptly.	"Surplus" cash was invested in Treasury bills.
1972	Generally pays 60 days beyond terms.	Lack of working capital.
1973	Generally pays promptly; some minor slowness of up to 20 days beyond terms.	Funds being used for expansion.
1974	Generally pays promptly; some minor slowness up to 20 days beyond terms.	Increases in cost of raw materials.
1975	Some prompt payment; some slowness up to 10 days beyond terms.	Slow-moving inventory.

In estimating a probability of default, Mr. Smith considered several approaches. One method was to start with the average default rate for firms in Vopal's industry and then judgmentally adjust this average rate based on the particular characteristics of Vopal Plastics. For example, if he decided that Vopal was less prone to default than the average firm in industry, the default rate would be adjusted downward.[3] Another method was to statistically analyze the characteristics of defaulted versus nondefaulted firms. To execute this last procedure, Mr. Smith first collected data on a sample of 10 recently defaulted firms and 10 randomly selected nondefaulted firms in Vopal's industry.[4] He decided to limit his analysis to the firms' liquidity and debt positions, which he considered to be the most important affectors of default from a financial standpoint. He then selected the current ratio and the ratio of total debt to total assets as measures of these financial aspects of firms. Exhibit 3 presents the data collected by Mr. Smith.

[3]On average, 30 of every 1,000 firms in Vopal's industry defaulted on trade debts. The average recovery by Gymbal on defaults was 10 percent and the average time from credit granting to default was three years. These later averages were very similar to those experienced by other firms in Gymbal's industry.

[4]This is too few data points for a complete statistical investigation, but Mr. Smith was experimenting with the technique.

Exhibit 3

Vopal Plastics
Ratio Data on a Sample of Firms.

Case No.	CA/CL	TD/TA
Defaulted Group:		
1	1.15	0.72
2	0.48	0.60
3	0.38	0.73
4	0.89	0.98
5	0.55	1.06
6	0.78	0.88
7	0.65	0.71
8	0.49	1.16
9	0.88	1.20
10	0.55	1.32
Mean	0.68	0.94
Nondefaulted Group:		
11	2.12	0.50
12	1.09	0.62
13	1.32	0.27
14	1.81	0.49
15	0.73	0.61
16	2.05	0.40
17	0.56	0.85
18	1.03	0.73
19	1.78	0.29
20	1.16	0.80
Mean	1.37	0.56

Exhibit 3 presents the data collected by Mr. Smith.

To estimate recovery rates to unsecured trade creditors if Vopal should default, Mr. Smith decided to assume that default would result in the liquidation of the firm. It was his experience that this outcome was the result of default on debts in the majority of cases in Vopal's industry. To estimate the recovery to creditors, Mr. Smith decided to use a "simulated asset sale" procedure. In this procedure, the most recent book values of the assets of the applicant were multiplied by estimated recovery rates to obtain estimated dollar recoveries. The estimated liquidation recovery rates used by Mr. Smith, which were based on his evaluation of the quality of Vopal's assets, are presented in Exhibit 4. The liquidation of Vopal Plastics, should it occur, would be a relatively simple procedure. As soon as the firm defaulted, the banks participating in Vopal's bank borrowings would "offset" Vopal's cash balances against the debt owed them by the firm. This offset procedure would reduce their claims by the amount of the cash offset. Then, from the recovery on the sale of the remaining assets, "priority claims" would be paid first. Included here would be court and attorney fees, accrued wages, and accrued taxes.[5] Vopal owed no accrued taxes. Accrued wages were estimated by Mr. Smith at 75 percent of "Accruals"; he also estimated court and attorney fees at 15 percent of the total estimated recovery on assets. After priority claims were paid, recoveries on assets that had been pledged as security for Vopal's bank debts would then be assigned to secured creditors. If these recoveries were insufficient to satisfy these creditors, then the excess claims of these creditors would be classified with the claims of unsecured creditors. If there were more recoveries from these pledged assets than needed to satisfy the claims of secured creditors, then this excess would be available to satisfy the claims of the unsecured creditors. Unsecured creditors would then be paid on a pro rata basis (that is, if there was not enough money available to pay unsecured creditors in full, the same percentage of each claim would be paid). If there were any funds left over after priority claims and unsecured creditors were paid, these funds would go to equity holders.

[5]The priority portion of accrued wages is limited to a certain figure per employee, but this limitation would not be surpassed for any of the employees of Vopal, given the firm's current salary schedule and pay policies.

Exhibit 4

Vopal Plastics
Estimated Recoveries, Percent of Book Value

Account	*Estimated Recovery*
Accounts Receivable	85%
Inventories	65%
Prepaid Expenses	40%
Other Current Assets	0%
Fixed Assets less Depreciation	40%
Other Assets	0%

In discounting cash flows, Mr. Smith decided to use Gymbal's after-tax, weighted-average cost of capital. As of 1976, Gymbal's capital structure was composed of 46 percent debt and 54 percent equity. New bonds with covenants similar to those of the firm had recently been sold near par with promised coupon rates of 8.1 percent. Gymbal's marginal tax rate was 48 percent. Gymbal's stock was selling at $53 per share, and during 1975 had paid a dividend of $2.40 per share. Gymbal's dividends had historically grown at 6.1 percent per year.

Chapter -7

Monitoring Accounts Receivable

In this, the final chapter in the management of accounts receivable, we discuss the *monitoring* of this asset. Accounts receivable is a major investment, and the continuous assessment of the state of this asset is a necessary function in the financial management of the firm. Monitoring is intended to fulfill this assessment function. We also review survey evidence on current practice in the management of accounts receivable.

WHY MONITOR ACCOUNTS RECEIVABLE?

In the prior two chapters, we discussed the formulation of the firm's terms-of-sale and credit-granting policies. These policies are made and implemented based on expectations regarding the behavior of the accounts receivable asset. In particular, the firm has expectations regarding the *turnover* of this asset and the amount of *bad debt* resulting from it. If these expectations are not fulfilled, this is a signal to management that the assumptions used in making the firm's terms-of-sale and credit-granting policies may be faulty, or that these policies are not being implemented properly. Deviations from the expected levels of turnover and of bad debt can signal several different problems. Three of these problems are discussed in the following paragraphs.

Changing Customer Payment Characteristics In contrast to longer-term assets and liabilities, working capital accounts turn over rapidly. If the average customer pays in 45 days, then in the course of a year, the accounts receivable asset will turn over eight times. The accounts receivable manager is dealing with *a constantly changing portfolio of receivables* as customers pay invoices, and new sales are made. Because of the rapid maturation of the assets in this portfolio, changes in the firm's customers or in economic and competitive conditions can impact receivables with remarkable suddenness. For example,

economic slowdowns first manifest themselves in slowing sales *and in slowing collections on these sales*. Since turning receivables into cash is a critical part of the selling firm's working capital cycle, changes in the pattern of cash flows from the receivables assets have important implications for management.

Inaccurate Policy Forecasts In the interests of shareholders, managers formulate terms-of-sale and credit-granting policies based on net present value rules. To calculate these net present values, management must estimate the effects of these policies on bad debt, turnover, and other relevant variables. For example, in deciding whether to change its terms of sale, the firm will make estimates of the new turnover of receivables, bad debt levels, and sales effects. There is substantial uncertainty in this estimation process. If there are deviations from these expectations, the policies need to be reconsidered and reanalyzed so that shareholder wealth is maximized. Therefore, firms need to compare the outcomes of policy with what was estimated in their formulation.

Improper Policy Implementation When the managers of a firm set policies, these policies must be implemented. The details of the policies, the rationales for them, and the means to carry them out must be given to the employees who will execute them. If the implementation is faulty, shareholder wealth will not be enhanced. Difficulties in the implementation of policies may stem from problems in communicating the policies to employees, from inappropriate evaluations of employees relative to these policies, or from any number of other sources.

Deviations from the expected levels of turnover and bad debt expense can signal such implementation problems. For example, suppose that a firm sells on net 30-day terms. The firm's top marketing and credit managers have analyzed the firm's credit-granting and terms-of-sale policies and determined that the most advantageous level of accounts receivable turnover is 40 days and that bad debt expense should be 1 percent of sales. Suppose further that the individuals who manage credit relations with the firm's customers do not understand this, but believe that the lower these statistics, the better for the firm. Alternatively, the evaluation systems for these employees could be such that they are penalized for bad debts and payments beyond terms, despite top management's beliefs about the most advantageous levels of these variables. In such circumstances, the employees may make decisions in such a way as to produce turnover of 35 days and bad debt expense of 0.5 percent of sales. To achieve these results, the employees may be turning away profitable but risky customers. By monitoring these statistics, the firm's management can detect and correct deviations from the intended policy that result from improper policy implementation.

Monitoring *provides signals* of deviations from expectations. When a signal is detected, *it is up to managers to investigate and to assess the reason for deviation*. The managers must then take the necessary corrective action,

and the type of necessary action will vary with the cause of the deviation. If the problem is changing customer payment patterns or inaccurate prior estimations, the credit-granting and terms-of-sale decisions of the firm need to be reanalyzed based on this new data. However, if the problem is in implementation, the firm's managers need to assess why the firm's policies are not being executed. In any case, *both positive and negative deviations* in accounts receivable statistics need to be investigated, since deviations in either direction signal differences from the results that management believe to be the most advantageous for the firm.

The monitoring process is a comparison of expectations and outcomes. Within the firm, these expectations are captured in the form of *budgets*. These budgets are part of the accounting control system of the firm, a system intended to detect deviations from policy. Some of the deviations from budgeted figures will represent one or more of the problems outlined previously, while others will represent random events. For example, a delay in the receipt of some customer checks (and thus a slowing of accounts receivable turnover) might be due to a strike of postal employees (a random event), and not due to any factors controllable by the selling firm. Since the investigation of deviations from expectations is a costly and time-consuming process, managers do not want to expend effort on this process when the deviations from expectations occur because of such random processes. The task of monitoring receivables, then, is to signal *nonrandom deviations from the budgeted receivables statistics*.

The statistics that have a primary bearing on credit decisions are levels of sales, collection expense, receivables turnover, and bad debt expense. The monitoring of sales levels and collection expense can be addressed in a straightforward fashion via the firm's systems of accounting controls.[1] However, *the monitoring of accounts receivable turnover and bad debt expense is a relatively tricky process when firms experience fluctuations in sales over time.* Unfortunately, fluctuations in the sales of firms due to economic, seasonal, and product life-cycle effects are more the rule than the exception. In the following sections, biases in the simpler monitoring tools when sales fluctuate are discussed, and better tools for this monitoring process are developed.

BIAS IN SIMPLE MEASURES OF RECEIVABLES TURNOVER

The sales volumes of firms fluctuate, and therefore so do dollar investments in receivables. In monitoring the collection (turnover) of accounts receivable, management wants to use a methodology that measures the re-

[1]For an approach to the assessment of sales effects in accounts receivable management decisions, see G. W. Gallinger and A. J. Ifflander, "Monitoring Accounts Receivable Using Variance Analysis," *Financial Management* (Winter 1986), pp. 69–76 and particularly pp. 74–76. As discussed in prior chapters, collection costs are usually very small relative to other credit-related costs; we do not discuss the monitoring of collection costs in this text.

lationship between the firm's receivables and its sales, *regardless of fluctuations in sales level*. Ideally, the methodology meets at least three criteria: (1) it *should not* signal a deviation from expectations in the payment behavior of the firm's customers when *none* has occurred; (2) it *should* signal a deviation when one *has* occurred; and (3) it should be simple to implement and to understand.[2] Two common methods of receivables' monitoring are used for this purpose: the *days' sales outstanding* statistics (also known as average collection period) and the *aging fraction* statistics.[3] Unfortunately, both of these approaches to the accounts receivable monitoring problem are seriously flawed with regard to the first two of these three criteria. When sales vary over time, both days' sales outstanding and aging fractions will give inappropriate signals. To see this and to understand how these statistics are computed, let us calculate them for an example situation.

Assume that a firm makes 5 percent of sales for cash. Sixty-five percent of a month's sales are paid in the next month, 25 percent in the second month following, and the remaining 5 percent in the third month following the sale. This pattern of customer payment does not vary over the year, but monthly sales volume is seasonal. The firm experiences constant sales volumes of $5 million per month from November through March; sales then increase through April to a seasonal peak of $9 million per month in May, then decrease to a low of $1 million per month in September, finally climbing back to $5 million a month in November. The important thing to remember is that, by assumption, the payment habits of customers are the same throughout the year: customers pay in an average 39.0 days.[4] Sales average $5 million per month, and there is no overall trend in sales except for seasonality.

This situation is presented in Table 7-1. The accounts receivable balances are computed using the collection rates in the upper left-hand corner of this table. For example, let us examine the $10.9 million receivables balance for May. May's sales are $9 million. Of these, 5 percent are for cash, so 95 percent ($8.55 million) are outstanding as of the end of May. Also outstanding at the end of May are receivables from sales in April and March. April's sales were $7 million. Of these, 5 percent were for cash, and 65 percent were collected during May. As of the end of May, the remaining 30 percent ($2.1 million) are outstanding. March's sales were $5 million; by the end of May, all but 5 percent have been collected (these will be collected in June); this 5 percent is $0.25 million. The firm's end-of-May receivables are thus $8.55 million plus $2.1 million plus $0.25 million, or $10.9 million.

[2]For an excellent discussion of these points, see W. G. Lewellen and R. W. Johnson, "Better Way to Monitor Accounts Receivable," *Harvard Business Review* (May–June 1972), pp. 101–109.

[3]See B. K. Stone, "The Payment-Patterns Approach to Forecasting and Control of Accounts Receivable," *Financial Management* (Autumn 1976), pp. 65–82.

[4]This is the weighted average based on the assumed payment pattern, and equal to 5 percent times zero days to pay, plus 65 percent times 30 days to pay, plus 25 percent times 60 days to pay, plus 5 percent times 90 days to pay.

Table 7-1. Days' Sales Outstanding and Aging Fractions Statistics for Example (dollar figures in rounded thousands).

Cash Sales	5%
Coll., First Prior Mo.	65%
Coll., Second Prior Mo.	25%
Coll., Third Prior Mo.	5%

Month	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.	Jan.
Sales	$5,000	$5,000	$5,000	$7,000	$ 9,000	$7,000	$5,000	$3,000	$1,000	$3,000	$5,000	$5,000	$5,000
Out., Current Month	4,750	4,750	4,750	6,650	8,550	6,650	4,750	2,850	950	2,850	4,750	4,750	4,750
Out., First Prior Mo.		1,500	1,500	1,500	2,100	2,700	2,100	1,500	900	300	900	1,500	1,500
Out., Second Prior Mo.			250	250	250	350	450	350	250	150	50	150	250
Receivables Outstanding			$6,500	$8,400	$10,900	$9,700	$7,300	$4,700	$2,100	$3,300	$5,700	$6,400	$6,500
DSO (2-Month Average)			39.00	42.00	40.88	36.38	36.50	35.25	31.50	49.50	42.75	38.40	39.00
Aging Fractions:													
0 to 30 days old			0.73	0.79	0.78	0.69	0.65	0.61	0.45	0.86	0.83	0.74	0.73
31 to 60 days old			0.23	0.18	0.19	0.28	0.29	0.32	0.43	0.09	0.16	0.23	0.23
61 to 90 days old			0.04	0.03	0.02	0.04	0.06	0.07	0.12	0.05	0.01	0.02	0.04
Total			1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

The days' sales outstanding (DSO) statistic is interpreted as the average number of days of sales that are still owed at the end of the month being evaluated. It is computed as:

$$\text{DSO} = (\text{month-end receivables/average sales})\ (30\ \text{days/month}) \qquad (7\text{-}1)$$

The average sales figure may use any of several averaging periods; in Table 7-1, it is computed as the average of the last two months' sales. Thus, for May, average sales are \$8 million (May's sales are \$9 million and April's sales are \$7 million) and the DSO is:

$$\text{DSO} = (10.9\ \text{million}/8\ \text{million})\ (30\ \text{days/month}) = 40.88\ \text{days} \qquad (7\text{-}2)$$

The aging fractions are computed by dividing the outstanding receivables of a particular age by the total receivables balance at the end of that month. For example, at the end of May, \$8.55 million in receivables are one month old or less. Since there are \$10.9 million in total receivables, the aging fraction for receivables less than one month old (between zero and 30 days) is 8.55/10.9 = 0.78. The remaining figures for May are similarly computed.

An examination of the monthly DSO and aging fractions figures in Table 7-1 shows substantial variation, to say the least. DSO reaches a minimum of 31.5 days in September and a maximum of 49.5 days in October. The aging fraction for zero to 30 days, for example, ranges from 0.45 to 0.86. *This should seem distressing since by construction, customer payments followed exactly the same pattern in every single month.* What has happened relates to *the comparison of receivables and sales from different months.* It is a "comparing-apples-and-oranges" problem. When we equally weight receivables resulting from varying sales, we get distorted results. Both of these measures of receivables do this, and therefore both give inappropriate signals. The manager of a credit department using DSO to monitor receivables behavior and who observes the set of measures in Table 7-1 may believe that payment patterns had varied widely over the year, when in fact they were uniform. Further, this example is for a firm with only one variation in sales pattern, that of seasonality. In other situations, there may be an overall trend in sales and changes in the proportion of sales made under various terms of sale. These fluctuations, because of measurement errors, can mask true differences between actual and expected payment behavior. Because of the way they compare sales and receivables, *DSO and aging fractions give the proper signals of receivables movement only in the special case where sales exhibit no fluctuations over time.*

Adherents of these methods may counter that some simple modification may negate this measurement problem. In particular, it can be contended that the judicious selection of a longer or shorter averaging period for sales in computing DSO may result in a more accurate representation of true

payment behavior. Unfortunately, this is not correct. As Lewellen and Johnson put it:

> [N]o single averaging period will consistently yield a correct appraisal where there are fluctuations in sales. In fact, within any given sales interval, even the *direction* of the signal depends on the averaging period chosen.[5]

To see this, examine Table 7-2, in which the DSO figures for the example are presented for sales averaging periods of one month, two months, and three months. The errors in these assessments, relative to the known payment pattern, are also given. These errors vary widely among the averaging periods, with none of the periods consistently producing an accurate measure. In fact, the two-month averaging period turns out to be the most accurate for this example problem in the sense that the greatest absolute error for this method is "only" 10.5 days, as opposed to 24.0 days for the one-month averaging period and 18.0 days for the three-month averaging period.[6]

Better Tools for Monitoring Accounts Receivable While knowledgeable managers might try to judgmentally adjust DSO or aging fraction figures to reflect these inaccuracies, it would be very hard for them to know how much adjustment is necessary. A more appropriate approach is to use a tool that is not subject to distortions caused by sales fluctuations. In this section, three such tools are examined: the ratios of receivables outstanding to original sales,[7] customer's payment proportions,[8] and the sales-weighted DSO.[9,10]

[5]Lewellen and Johnson, "Better Way to Monitor."

[6]The conclusion that the two-month period is the least inaccurate is a product of the particular data used in this example and does not hold for all receivables monitoring situations.

[7]The measure is recommended in Lewellen and Johnson, "Better Way to Monitor." Also see W. G. Lewellen and R. O. Edminster, "A General Model of Accounts Receivable Analysis and Control," *Journal of Financial and Quantitative Analysis* (March 1973), pp. 195–206.

[8]See Stone, "The Payment-Patterns Approach."

[9]See M. D. Carpenter and J. E. Miller, "A Reliable Framework for Monitoring Accounts Receivable," *Financial Management* (Winter 1979), pp. 37–40.

[10]One method not discussed in this chapter is the Markov chain approach to monitoring payment behavior. The Markov chain approach is primarily oriented toward monitoring payments on consumer installment credit where payment patterns may not exhibit seasonal or cyclical fluctuation, rather than patterns of trade credit which may fluctuate with sales. For an example in the application of this approach, see J. G. Kallberg and K. Parkinson, *Current Asset Management* (New York: Wiley, 1984), pp. 182–84.

Table 7-2. Days' Sales Outstanding for Various Sales Averaging Periods (dollar figures in rounded thousands).

Cash Sales	5%												
Coll., First Prior Mo.	65%												
Coll., Second Prior Mo.	25%												
Coll., Third Prior Mo.	5%												
DSO Averaging Per.	60 days												
Month	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.	Jan.
Sales	$5,000	$5,000	$5,000	$7,000	$ 9,000	$7,000	$5,000	$3,000	$1,000	$3,000	$5,000	$5,000	$5,000
Out., Current Month	4,750	4,750	4,750	6,650	8,550	6,650	4,750	2,850	950	2,850	4,750	4,750	4,750
Out., First Prior Mo.		1,500	1,500	1,500	2,100	2,700	2,100	1,500	900	300	900	1,500	1,500
Out., Second Prior Mo.			250	250	250	350	450	350	250	150	50	150	250
Receivables Outstanding			$6,500	$8,400	$10,900	$9,700	$7,300	$4,700	$2,100	$3,300	$5,700	$6,400	$6,500
DSO (1-Month Average)			39.00	36.00	36.33	41.57	43.80	47.00	63.00	33.00	34.20	38.40	39.00
Actual Collections Rate			39.00	39.00	39.00	39.00	39.00	39.00	39.00	39.00	39.00	39.00	39.00
Over or (Under)			0.00	−3.00	−2.67	2.57	4.80	8.00	24.00	−6.00	−4.80	−.60	0.00
DSO (2-Month Average)			39.00	42.00	40.88	36.38	36.50	35.25	31.50	49.50	42.75	38.40	39.00
Actual Collections Rate			39.00	39.00	39.00	39.00	39.00	39.00	39.00	39.00	39.00	39.00	39.00
Over or (Under)			0.00	3.00	1.88	−2.63	−2.50	−3.75	−7.50	10.50	3.75	−.60	0.00
DSO (3-Month Average)			39.00	44.47	46.71	37.96	31.29	28.20	21.00	42.43	57.00	44.31	39.00
Actual Collections Rate			39.00	39.00	39.00	39.00	39.00	39.00	39.00	39.00	39.00	39.00	39.00
Over or (Under)			0.00	5.47	7.71	−1.04	−7.71	−10.80	−18.00	3.43	18.00	5.31	0.00

Table 7-3. Example of Ratios of Receivables Outstanding, Payment Proportions, and Sales-Weighted DSO.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Cash Sales	5%							5%					
2	Coll., First Prior Mo.	65%							55%					
3	Coll., Second Prior Mo.	25%							20%					
4	Coll., Third Prior Mo.	5%							20%					
5														
6	Month	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.	Jan.
7	Sales	5000	5000	5000	7000	9000	7000	5000	3000	1000	3000	5000	5000	5000
8														
9	Out., Current Month	4750	4750	4750	6650	8550	6650	4750	2850	950	2850	4750	4750	4750
10	Out., First Prior Mo.		1500	1500	1500	2100	2700	2100	2000	1200	400	1200	2000	2000
11	Out., Second Prior Mo.			250	250	250	350	450	1400	1000	600	200	600	1000
12														
13	Receivables Outstanding			6500	8400	10900	9700	7300	6250	3150	3850	6150	7350	7750
14														
15	Ratios of Receivables Outstanding:													
16														
17	Current Month			.95	95	.95	.95	.95	.95	.95	.95	.95	.95	.95
18	First Prior Month			.30	.30	.30	.30	.30	.40	.40	.40	.40	.40	.40
19	Second Prior Month			.05	.05	.05	.05	.05	.20	.20	.20	.20	.20	.20
20														
21	Payment Proportions:													
22														
23	Current Month			.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05
24	First Prior Month			.65	.65	.65	.65	.65	.55	.55	.55	.55	.55	.55
25	Second Prior Month			.25	.25	.25	.25	.25	.10	.20	.20	.20	.20	.20
26														
27	Sales-Weighted DSO:													
28														
29	Contribution, Current Month's Sales			28.50	28.50	28.50	28.50	28.50	28.50	28.50	28.50	28.50	28.50	28.50
30	Contribution, Fst Prior Month's Sales			9.00	9.00	9.00	9.00	9.00	12.00	12.00	12.00	12.00	12.00	12.00
31	Contribution, Snd Prior Month's Sales			1.50	1.50	1.50	1.50	1.50	6.00	6.00	6.00	6.00	6.00	6.00
32														
33				39.00	39.00	39.00	39.00	39.00	46.50	46.50	46.50	46.50	46.50	46.50

An example spreadsheet for the computation of these three measures is presented in Table 7-3. In this spreadsheet, the sales figures for all months and the receivables balances for March through July are the same as in Tables 7-1 and 7-2. We have assumed that a change in payment patterns occurs in August and that this change is in effect through the remainder of the table. The outstanding receivables in this table have been calculated by applying the collection rates in cells B1 to B4 and I1 to I4 to the sales in row 7. For example, cell D10 contains "(1−((B1−B3)/100))*C7"; since 5 percent of sales are for cash (B1) and 65 percent of sales are collected during the first month (B2), the receivables balance after one month will be (1 − (0.05 + 0.65)), or 30 percent. Applying this to sales for February (cell C7) gives the $1.5 million balance outstanding from February at the end of March. This spreadsheet is intended to show that these three tools *do not* provide improper signals when *no* deviation from expected payment patterns has occurred and *do* provide signals when a change *has* in fact occurred.

The *ratios of receivables outstanding to original sales* remove the confusion caused by fluctuating sales levels by comparing only "oranges and oranges." That is, receivables outstanding at any particular time are compared *only to sales in the period during which the receivables were generated.* This gives a true picture of receivables payment behavior. The ratios of receivables outstanding in Table 7-1 appear in rows 17 through 19 and are computed by dividing the receivables outstanding from a particular month by the sales from that month. For example, in May, the ratios of receivables outstanding are 0.95, 0.30, and 0.05. These are computed in cells F17 through F19. For example, F18 contains "F10/E7" since cell F10 contains the balance of receivables which are one month old and cell E7 contains last month's sales (that is, sales for April). The ratio of 0.30 is receivables of $2.1 million (the receivables still outstanding from April as of the end of May) divided by $7 million (sales for April).

The *payment proportions* figures displayed in rows 23 through 25 of the table are computed as the proportion of a month's sales that are collected in the month being analyzed. Looking again at the figures for May, the payment proportions are 0.05, 0.65, and 0.25. These are calculated by comparing changes in receivables balances with sales for the month during which the receivables originated. For example, during May, the receivables balance from sales in April was reduced from $6.65 million (the balance as of the end of April from April's sales, in cell E9) to $2.1 million (in cell F10); therefore during May, $4.55 million of receivables from April's sales must have been collected. This figure divided by April's sales of $7 million gives the 65 percent payment proportion. Thus, cell F24 contains "(E9-F10)/E7" since sales for April are found in cell E7.

Sales-weighted DSO allows for fluctuations in sales by weighting the receivables balances by the sales incurred in the month during which the

receivables were generated. The sales-weighted DSO (SWDSO) can be calculated as:

$$\text{SWDSO} = \sum_{t=0}^{n} \left(\frac{AR_t}{S_t} \right) (30 \text{ days/month}) \tag{7-3}$$

where t is the month of sale, n is the number of months for which receivables are outstanding, AR_t is the outstanding accounts receivable balance from month t, and S_t is the sales in month t. However, the sales-weighted DSO is actually a good deal simpler than this. Recognizing that AR_t/S_t is the ratio of receivables outstanding from month t (call this RRO_t) then equation (7-3) becomes:

$$\text{SWDSO} = \sum_{t=0}^{n} (RRO_t) (30 \text{ days/month}) \tag{7-4}$$

and factoring out the 30 days/month:

$$\text{SWDSO} = (30 \text{ days/month}) \sum_{t=0}^{n} (RRO_t) \tag{7-5}$$

The sales-weighted DSO is therefore the sum of the ratios of receivables outstanding times 30 days/month.

To calculate the SWDSOs in Table 7-3, equation (7-4) was used. The contributions to the SWDSO for each month were computed by multiplying each of the ratios of receivables outstanding in rows 17 through 19 by 30 days per month. These figures appear in rows 29 through 31 of the spreadsheet. For example, the contribution of April's receivables outstanding based on April's sales to the SWDSO for May can be found in cell F30, which contains "F18*30". The SWDSO for May is the sum of the contributions to this statistic for the three months from which receivables are outstanding, and appears in cell F33. This cell contains "@SUM(F29 . . . F31)".

All these three methods do a much better job than aging fractions or traditional DSO in monitoring payment behavior since they do not give false signals when no deviations from the expected payment pattern have occurred and do give proper signals when such deviations have actually occurred. While aging fractions and DSO fluctuate considerably during the period from January through July (see Table 7-1), when there are really no deviations from historic payment patterns, all three new measures do not vary over this period. However, when payment patterns actually *do* change in August, all three new methods accurately show this change.

Using These Tools to Monitor Payment Patterns The proper use of these measures as management-by-exception tools requires the construction of confidence intervals around their expected values. In this way, the chance that

Table 7-4. Example Historic Data on Ratios of Receivables Outstanding.

	Mean	*Standard Deviation*
Current Month	0.950	0.015
First Prior Month	0.300	0.020
Second Prior Month	0.050	0.003

costly management investigation will be triggered by random events is reduced. To construct these confidence intervals, the analyst may obtain a sample of data on these measures from the period of time during which fluctuations in the measures are thought to have been random. The frequency distribution of these measures can then be compared to various probability distributions based on goodness-of-fit tests, or it can be assumed that the data fits some known probability distribution.[11] Given the probability distribution, the appropriate confidence intervals are then constructed based on statistics calculated from the sample.

For example, suppose that a firm had decided that there were no variations, except for random ones, in its collection patterns over the last two years. These patterns were, on average, in keeping with those expected from the firm's accounts receivable management strategies. The firm's managers monitor their receivables via ratios of receivables outstanding. Using the last two years of monthly data, the standard deviations for these ratios have been computed; these are presented in Table 7-4. The firm believes that the data are distributed as Student's t.[12] The managers of the firm want to investigate if there is a 90 percent chance that deviations from the expected payment patterns have occurred. Deviations either above or below the expected levels are to be investigated.

To signal the managers of such deviations, a two-tailed test is needed with 5 percent of the distribution in each tail. Since there are 24 observations, the sample statistics in Table 7-4 have 23 degrees of freedom. From a table of the Student's t distribution, for a one-tailed test, the appropriate t-statistic is 1.714. That is, with 23 degrees of freedom, 90 percent of the Student's t distribution occurs between t-statistics of -1.714 and $+1.714$. For the ratio of receivables outstanding from the first prior month, the confidence interval is then 0.300 plus and minus 1.714 times the standard deviation of 0.020, or between 0.265 and 0.334. If, in the future, the ratio of receivables outstanding

[11]If lagged regression has been used to estimate payment proportions, then the estimated standard errors of the payment proportions can be used in construction of confidence intervals.

[12]We use the Student's t-test here for simplicity. However, students with some sophistication in statistics should note that the distributions of ratios of receivables outstanding are bounded (they cannot take on values above one or below zero).

Table 7-5. Example Confidence Intervals for Ratios of Receivables Outstanding.

	Mean	*Standard Deviation*	*LCL*	*UCL*
Current Month	0.950	0.015	0.924	0.976
First Prior Month	0.300	0.020	0.266	0.334
Second Prior Month	0.050	0.003	0.045	0.055

The lower confidence limit (LCL) and the upper confidence limit (UCL) are based on a 90 percent confidence interval, 23 degrees of freedom, and a Student's *t*-distribution; see text.

from the first prior month falls outside this range, management would investigate payment behavior for payments from this month. Otherwise, no investigation of these payments would be undertaken. Similarly computed control limits for the other two ratios of receivables outstanding in Table 7-4 are presented in Table 7-5.

In examining these tables, readers will notice that the example standard deviations are much smaller than the expected values of these ratios of receivables outstanding. The coefficients of variation are less than 10 percent. The small standard deviations relative to the means in this example are not atypical. Indeed, random shifts in payment patterns are usually relatively minor for firms of substantial size. Large- and medium-sized firms receive hundreds of checks per month; this large sample size tends to negate deviations in payment patterns caused by the random receipt or nonreceipt of checks from individual customers or groups of customers.[13] This is why the choice of an accounts receivable monitoring tool that accurately measures payment patterns is so useful. Since random movements are minor, small deviations in payment patterns can be quickly detected and acted upon. Monitoring instruments that allow the measurement of nonrandom changes in payment patterns to be overwhelmed by measurement errors caused by fluctuations in sales are therefore a good deal less effective than are more accurate measures.

Deciding Which Method to Use in Monitoring Collections In prior sections, we had discussed five monitoring methods: traditional DSO, aging fractions, ratios of receivables outstanding, payment proportions, and the sales-weighted DSO (SWDSO). The practicing manager must choose one or more methods of monitoring the firm's receivables portfolio. While the first two methods can be inaccurate in their measurement of payment behavior,

[13]However, for smaller firms who receive fewer checks per month, the standard deviations of collection patterns may be larger relative to their expected values.

other arguments are sometimes made in their favor. Typical arguments for these methods include: (1) they are simple to compute, and (2) industry figures are reported—particularly with regard to DSO—giving management some standard of comparison in assessing trends in the payment behavior of customers. However, none of these arguments holds much weight. Regarding the first, the additional effort necessary to compute performance statistics for more accurate methods is very small. The second argument would be important only if the firm was limited to the computation of one measure of payment behavior. However, there is no reason that the firm cannot use an accurate measure of payment behavior to assess its *own* patterns of receipts and *another* measure to compare this behavior to published figures.

The choice of receivables monitoring methods for internal control purposes is then among the more accurate methods: ratios of receivables outstanding, payment proportions, SWDSO, and other similar measures.[14] Each of these measures has advantages and disadvantages. Measures such as ratios of receivables outstanding and payment proportions indicate payment performance via a series of numbers, while SWDSO is a single-number measure. Therefore, SWDSO is simpler to interpret. However, this single figure (while an accurate aggregate measure) may hide deviations from expectations among the collection rates from the various sales periods. One possible system is to utilize SWDSO as a screen for the examination of payment behaviors. In this system, confidence limits would be computed for SWDSO based on the firm's expected payment patterns and an estimate of the random variation around these expectations. If the firm's actual SWDSO fell outside these confidence limits, investigation would be triggered. The first step in this investigation might be the examination of some measure of collection rates or receivables balances from individual periods. Such an investigation process is described in the next section.

INVESTIGATING PAYMENT PATTERNS USING "VARIANCE ANALYSIS"

Let us assume that some event (such as the SWDSO falling outside its confidence limits) has triggered an analysis of the payment behaviors of the firm's customers. Investigation of the deviations from expectations is then

[14]The accuracy of these methods should not be overstated. They will not detect changes in payment behavior that occur *within* the month. For example, if customers who had paid near the middle of the month choose instead to pay near the end of the month, this would go undetected by all the measures discussed herein. More microscopic measures of receivables performance are necessary to detect changes of this sort; see G. N. Engler and D. V. Considine, "Excess Collection Days: A New Concept in Monitoring Accounts Receivable," paper presented at the Eastern Finance Association's 1985 Annual Meeting, available from Considine at California State University, Los Angeles.

required. One possible tool in this investigation to determine the sources of the problem and the amounts involved is "variance analysis."[15] This tool will determine how much of the total fluctuation of the balance in the accounts receivable account has been caused by sales discrepancies and how much has been caused by collection discrepancies.[16]

Variance analysis is a very simple tool. The differences between the *actual* and *budgeted* accounts receivable balances are affected by two variables: the level of sales and the ratios of receivables outstanding. In variance analysis, the effects of sales and collections on balances are separated by generating a third set of balances: the accounts receivable balances given the *actual* sales figures but *as if the expected ratios of receivables outstanding had occurred*. In this system, the *total dollar variance* is the difference between the actual and budgeted receivables balances. The portion of this variance caused *by collections* is the difference between the actual receivables and the third set of balances (actual sales times expected ratios of receivables outstanding). The remaining variance, which is equal to this third set of balances minus the budgeted accounts receivable balance, is due to sales effects.[17]

To see how this system works and the insights it can lend, let us address an example problem. A firm has made a three-month budget assuming sales of $100,000 for the first month, $200,000 for the second month, and $300,000 for the final month. Ratios of receivables outstanding are projected as 0.90 for sales from the month just completed, 0.40 for sales two months ago, and 0.10 for sales three months ago. Therefore, the expected SWDSO statistic is 42.00 ((0.90 + 0.40 + 0.10) times 30 via equation (7-5)). The expected

[15]The basic published references on this technique are Gallinger and Ifflander, "Monitoring Accounts Receivable," and J. A. Gentry and J. M. De La Garza, "A Generalized Model for Monitoring Accounts Receivable," *Financial Management* (Winter 1985), pp. 28–38. Here, "variance analysis" takes the accounting meaning of variation from budget, rather than the statistical meaning of variation around the mean.

[16]Some firms monitor receivables balances rather than payment behavior. When these balances are larger or smaller than budgeted balances, these firms want to know how much of this discrepancy is caused by sales effects and how much by collection effects. Also, while this discussion concentrates on the firm's expectations regarding collection rates, the variance analysis technique can also be used to assess the changes in sales from a change in credit policy. See Gallinger and Ifflander, "Monitoring Accounts Receivable," pp. 74–76.

[17]There is actually another type of variance, called the "joint effect" variance, which occurs because of the interaction between the sales and collection effects. Following Gallinger and Ifflander, "Monitoring Accounts Receivable," we have included joint effect variance with the variance caused by differences in collection rates. For more discussion of joint effect variance and its implications in monitoring accounts receivable, see V. Srinivasan and Y. Kim, "Credit Management: A Variance Framework for Monitoring Accounts Receivable," paper presented at the Financial Management Association's 1987 Annual Meeting and available from Srinivasan at Northeastern University.

accounts receivable balance at the end of the third month is $360,000 ((0.90)(300,000) + (0.40)(200,000) + (0.10)(100,000)). At the end of the third month, the actual SWDSO statistic is 46.80, which is outside the confidence limits set by management, and the accounts receivable balance is $513,000. Since the SWDSO statistic is outside the confidence levels, investigation of differences from expectations is undertaken. Actual sales for the first month were $120,000, for the second month were $250,000, and for the third month were $400,000. At the end of the third month, there is $380,000 outstanding from the third month's sales, $115,000 outstanding from the second month's sales, and $18,000 outstanding from the first month's sales.

We want to assess the differences between expected and actual collections and the effect of these differences on the firm's accounts receivable balances. Note first that since the SWDSO statistic is corrected for sales trends, the difference from the expected value of this statistic *cannot be merely a sales effect; it must arise from collections*. Variance analysis helps highlight the source of this deviation in collections and assess its effect on accounts receivable investment.

The required analysis is presented in Table 7-6. Columns B and C contain the budgeted sales for the three-month period (starting with the last month) and the budgeted accounts receivable balances. Column D contains the budgeted ratios of receivables outstanding. Columns E, F, and G contain the actual sales, actual ending accounts receivable, and actual ratios of receivables outstanding, with these ratios computed as the receivables from a month divided by the actual sales for that month. The total variance is the difference between the actual ending receivables balance of $513,000 and the projected balance of $360,000, and appears in cell H12 (which contains "F9-C9").

However, not all of this variance of $153,000 is due to differences from expectations in the collection rates from customers. Column H contains accounts receivable balances projected by using *actual* sales figures but *projected* ratios of receivables outstanding. For example, the budgeted ratio of receivables outstanding from the first prior month is 0.90. If 90 percent of the prior month's actual sales were outstanding as of the end of the third month, the accounts receivable balance for that month would be $360,000 (0.90 times $400,000). Thus, cell H5 contains "D5*E5", and cells H6 and H7 contain parallel entries. The projected accounts receivable balance, *using budgeted collection rates*, is $472,000, which appears in cell H9. Of the $153,000 variance, $41,000 ($513,000 minus $472,000) is caused by the unexpectedly slow collection rates from customers. The remaining $112,000 ($472,000 minus $360,000) was caused by unexpectedly large sales. These two parts of the total variance appear in cells H13 and H14, which contain "F9-H9" and "H9-C9" respectively.

An examination of the ratios of receivables outstanding in columns D and G shows that each is larger than original expectations. Therefore, col-

Table 7-6. Computation of Sales and Collection Variances for Example Problem.

	A	B	C	D	E	F	G	H	I
1	Month	Proj. Sales	Proj. A. R. Balance	Proj. RRO	Actual Sales	Actual A. R. Balance	Actual RRO	Actual Sales, Proj. RRO	Collection Variance by Month
2									
3									
4									
5	3	300	270	.90	400	380	.95	360	20
6	2	200	80	.40	250	115	.46	100	15
7	1	100	10	.10	120	18	.15	12	6
8									
9	Totals	600	360	1.40	770	513	1.56	472	41
10			SWDSO	42.00			46.80		
11									
12									
13							Total Variance	153	
14							Collection Var.	41	
							Sales Variance	112	

lections have been unexpectedly slow from sales made in *all months* of the forecast. However, since sales have been rising over time, the major collection effect that has caused an increase in receivables balances *has come from the final month's sales*. This is seen in column I, where the variance in the receivables balances is computed for receivables from each month. Cells I5 through I7 contain formulas for the difference in the corresponding cells in columns F and G (for example, I5 contains "F5-G5"). Slower receipts from sales in the third month account for $20,000 of the $41,000 variance which is due to collections.

Now that the patterns and amounts of the differences between the expected collection rates and accounts receivable balances are known, the management of the firms must assess why this difference has occurred. Are customers paying more slowly than expected because of economic or competitive effects? Is there something wrong with the forecasting procedures used to estimate collection rates? Are the firm's policies being improperly implemented? While the across-the-board decrease in collection rates from those that were expected suggests an economic or competitive effect, the other explanations remain possible, and management must assess the cause of these deviations in reexamining its strategies.

MONITORING BAD DEBT LOSSES

The usual measure of bad debt losses is the ratio of bad debts to sales. The basic approach to comparing actual bad debt losses to those that are expected is the same as that for the monitoring of payment patterns. Confidence limits are determined based on the expected value of this ratio (given the firm's credit policies) and the historic random variation in this ratio. If the firm's actual ratio of bad debt expense to sales falls outside these confidence limits, investigation is undertaken. As with collection experience, either a positive or negative deviation from expectations is cause for concern.[18]

However, there are some difficulties inherent in this comparison process that must be recognized and addressed. These difficulties are illustrated in the spreadsheet presented in Table 7-7. The first difficulty concerns the matching of debt expense *with its proper period of origin*. Before recognizing that a sale has resulted in a bad debt, the seller must make every effort to collect from the buyer. This usually takes some period of time. Thus, bad debts are *recognized* in a period *far after the sale occurs*. In this spreadsheet, we have assumed that bad debts are recognized after sales are 120 days old. Thus, the bad debts of $180 recognized in *May* (cell F5) occurred because of sales in

[18]As with payment patterns, unexpectedly low bad-debt loss is just as much a concern as is unexpectedly high bad-debt loss. For example, unexpectedly low bad-debt loss might signal overestimation of the default risks of applicants in the firm's credit granting process, causing credit-granting decisions to be overly restrictive.

Table 7-7. Example Problem in Monitoring Bad Debt Expense.

	A	B	C	D	E	F
1	Month	Jan.	Feb.	March	April	May
2						
3	Sales for Month	10000	15000	20000	25000	30000
4						
5	Bad Debts Recognized	50	105	0	5	180
6						
7	Percent of Sales	.50	.70	0.00	.02	.60
8						
9	Bad Debts Originated	180	0	0	150	100
10						
11	Percent of Sales	1.80	0.00	0.00	.60	.33
12						
13						
14						
15						
16	Qly. Percent of Sales			.40		
17						
18						
19						

January. The comparison in row 7 of bad debts recognized as a percent of the *current* month's sales is misleading. Sales from January resulted in bad debts of $180 (recognized in May), which is 1.80 percent of January's sales, rather than the 0.5 percent that appears in cell B7. The correct calculation of bad debt expense ratios is presented in row 11, where the dollar bad debt expenses are compared with sales in the month in which the sales occurred. For example, cell B11 (the ratio of bad debts from January to sales for January) contains "(B9/B3)*100" since the bad debt expense originating in January appears in cell B9 and sales for January appear in cell B3.

The second difficulty in monitoring bad debt expense has to do with the "lumpy" pattern by which bad debt expenses occur to the firm and the effect of this lumpiness on estimates of dispersion of bad debt expense. In general, the credit policies of most firms and the failure rates of trade customers are such that *a few defaults account for most of the firm's bad debt expense*. Most firms experience *very few* defaults by trade debtors, but when a debtor does default, a significant amount of money is often lost. This lumpiness in defaults means that *the dispersion of bad debt losses over time is substantial*, particularly for short time periods. An industrial seller can go for months without a major default, then several defaults may occur over a short period. This dispersion of loss is portrayed in row 11 of Table 7-7; losses range from 1.80 percent of sales (for January) to 0.0 percent of sales (for February, March, and November). The mean loss as a percent of the originating month's sales is 0.47 percent (cell M13), with a standard deviation of 0.95 percent.

!	G !!	H !!	I !!	J !!	K !!	L !!	M !
	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
	35000	35000	30000	25000	20000	15000	10000
	0	0	150	100	20	75	100
	0.00	0.00	.50	.40	.10	.50	1.00
	20	75	100	250	20	0	120
	.06	.21	.33	1.00	.10	0.00	1.20
						Mean	.47
						S.D.	.95
	.30			.47			.31
						Mean	.37
						S.D.	.08

Because of this large standard deviation relative to the mean, the confidence limits for bad debt loss as a percent of monthly sales are too wide to be of practical use; the month-to-month random variation overwhelms any nonrandom effects. To monitor bad debt expense, most firms must use a longer averaging period than a month so that there is a sufficient number of bad-debt events to obtain a reasonably sized sample of bad-debt experience. Quarterly figures are presented in row 16, with the mean and standard deviation of these figures in cells F18 and F19.[19] These quarterly figures are computed as the sum of the bad debts originating in the quarter divided by the sum of the sales over the quarter. Thus, cell D16, which shows the bad debt experience for the first quarter, contains "(@SUM(B11...D11)/@SUM(B3 . . . D3))*100". Note that, in this quarterly averaging procedure, the standard deviation of bad debt expense is much smaller relative to its mean than for the monthly averaging procedure. Because of this, reasonable confidence intervals for quarterly bad debt expense can be generated.

However, because of the delay in the recognition of bad debt expense and the necessity of averaging bad debt expense over several periods, there will always be a substantial time lag in the detection of differences between expected and actual bad debt losses. This makes the current level of bad debt

[19]The difference in the mean losses between cells M13 and M18 occurs because these are averages of averages for fluctuating sales and are not weighted by sales volume.

expense a good deal harder to assess than current payment patterns from customers. To monitor the current payment patterns from customers, the firm merely computes the appropriate statistics at the end of a period and compares these to their confidence limits. This calculation and comparison can be made almost immediately as periods end, and quick action can be taken to investigate and correct any collection problems. For bad debt expense, however, a significant waiting period is necessary before such a comparison and investigation can proceed. The firm must wait until the bad debts associated with a sale have been recognized, a process of several months' duration. Further, because of the necessary averaging of bad debt losses over several months, differences from historic patterns can have occurred for several prior periods without detection. These lags between occurrence and detection make the management of current bad debt expense a difficult business.

THE MANAGEMENT OF ACCOUNTS RECEIVABLE IN PRACTICE

Since this is the last chapter on the management of accounts receivable, it is appropriate to contrast the techniques described in this text with current practice in the management of this asset. This discussion of current practice is organized according to the major topics discussed in the last three chapters. As in our discussion of practice in the management of cash and marketable securities, in the interests of brevity the surveys will be referred to by abbreviation; details of the surveys are given in Table 7-8.

Interestingly, this assessment of current practice will find a substantial divergence between the techniques displayed in the academic literature and those commonly used in credit management. Some of the techniques discussed in these chapters are extensively used, while others have not yet been widely adopted.

The Prevalence of Trade Credit in the Sale of Goods The vast majority of manufacturing output is sold via trade credit; 87 percent of manufacturing firms report that between 91 and 100 percent of their sales are made on a credit basis (BO, p. 73). The availability of trade credit does not depend on economic conditions or interest rates. This is so because, in general, firms do not adjust their credit policies to reflect changes in these conditions (HWS; Walker, p. 20).

Terms-of-Sale Decisions Since credit terms are an aspect of price but have implications for the financial management of selling firms, it is logical that terms-of-sale decisions should be made jointly by the finance and marketing departments. Survey evidence confirms this logic (HWS). In evaluating the terms-of-sale policy, firms consider the effect of terms changes on sales,

investments, bad debt losses, and returns (SS, pp. 56 and 70). In general, these considerations are in keeping with the terms-of-sale evaluation methodologies portrayed in this text.

However, once set, firms rarely consider changes in their terms of sale (HWS). Most firms simply utilize the traditional terms of sale in their markets (HWS; Scherr, p. 14). When competitors change terms, other sellers typically will "follow the leader" (HWS). This is in sharp contrast to the aggressive marketing policies followed by many firms, where other price-related variables are manipulated to influence sales volume. The management of terms-of-sale policies to obtain desired patterns of sale is certainly an area where current practice can be reconsidered.

Credit Investigation Policies There is considerable evidence that firms follow cost-of-information principles in setting their credit investigation policies. Small firms, who on average receive orders for small dollar amounts, tend to use low-cost sources of credit information, such as past experiences with the applicant and credit ratings from credit agencies (Walker, pp. 17–19, 24). Larger firms tailor the amount of information collected to the amount of the order, with more information being collected in making credit-granting decisions on larger orders (Scherr, p. 15).

Credit-Granting Decisions on Marginal Accounts Most firms find the credit-granting decision to be more difficult to make than decisions regarding terms of sale or credit investigation policy (Scherr, p. 15). There appears to be substantial disagreement within surveys regarding these credit-granting decisions. Several surveys have shown that firms use the traditional "Cs of Credit" to make judgmental decisions on credit applicants, though a substantial fraction use some type of credit-scoring approach (Scherr, p. 16; SS, pp. 55 and 69). However, there is also survey evidence that firms can and do make reasonably accurate estimates of default probability, delinquency, and discount rates (BO, p. 79; Scherr, p. 16). If the seller can make an accurate estimate of these statistics, why should firms bother with a judgmental approach to credit-granting decisions? Why not just compute the net present value of expected cash flows, since such a technique is straightforward and is consistent with shareholder wealth maximization? It would seem that knowledge about current practice in credit-granting decisions is incomplete.

Monitoring Accounts Receivable Unfortunately, most of the published survey information on practice in the monitoring of accounts receivable is fairly old. Published survey results from the mid-1970s showed aging fractions to be the most popular method of monitoring customer payment patterns at the time (SS, pp. 55–56 and 69; Stone, pp. 70–71). The use of this and other traditional measures of payment behavior (such as DSO) may continue to

Table 7-8. Facts Regarding the Survey Articles on the Management of Accounts Receivable Referenced.

Abbreviation	*Authors*	*Article Title*
BO	S. Besley and J. Osteryoung	"Survey of Current Practices in Establishing Trade-Credit Limits"
HWS	N. Hill, R. Wood, and D. Sorenson	"Factors Influencing Corporate Credit Polic A Survey"
Scherr	F. Scherr	"Managing Accounts Receivable"
Stone	B. Stone	"The Payments-Pattern Approach to the Forecasting and Control of Accounts Receivable"
SS	K. Smith and S. Sell	"Working Capital Management in Practice"
Walker	D. Walker	"Factors Influencing Trade Credit for Small Businesses"

*Estimated from discussion in article.

the present.[20] It is not known how or if managers correct for the bias caused in these measures by variations in sales. If firms use aging fractions or DSO, switching to a more accurate method of monitoring can significantly enhance their ability to detect nonrandom changes in payment behavior.

SUMMARY

The major discussion in this chapter concerned the monitoring of accounts receivable. Monitoring means comparing the actual payment patterns and bad debt losses from this asset to those that are expected because of the firm's terms-of-sale and credit-granting policies. Monitoring is important because the rapid turnover of accounts receivable means that the characteristics

[20]In a reference to an unpublished survey, Srinivasan and Kim ("Credit Management") indicate that aging fractions and DSO continue to be popular as payment monitoring methods.

Reference	Date of Survey	Sample(s) Used	Response Rate
Financial Review (February 1985), pp. 70–81	1982	510 members of the Credit Research Foundation, mostly engaged in manufacturing.	45.9%
Journal of Cash Management, Vol. 1, No. 2 (Fall 1981)	1979	500 members of the National Association of Credit Management	55.4%
Nevada Review of Economics and Business, Vol. 8, No. 2 (Summer 1984), pp. 14–16	1981	Fortune's 500 Industrials	49.0%
Financial Management (Autumn 1976), pp. 65–82	1975*	Executives of 148 firms participating in executive development seminars	100.0%*
Readings in the Management of Working Capital, 2nd. Ed. (N.Y.: West Pub., 1980), pp. 51–84	1978	668 of Fortune's 1000, with selection based on size and profitability	31.4%
Eastern Finance Association, 1985; available from the author at Georgetown University	1983*	27 small businesses near Washington, D.C.	100.0%*

of this asset can change quickly, and because cash inflows from the collections of accounts receivable are important in the seller's working capital cycle. Deviations from expectations can signal changing customer payment characteristics, inaccurate forecasts, or improper implementation of terms-of-sale or credit-granting policies.

The simplest methods of monitoring the payment patterns are aging fractions and traditional days' sales outstanding statistics. Unfortunately, both of these measures are biased when sales fluctuate over time: they may show a deviation from expectations when no change has occurred and may not signal a deviation when one has occurred. Tools that do not contain this inherent bias include payment proportions, ratios of receivables outstanding, and sales-weighted days' sales outstanding. These tools compare outstanding receivables and collections with sales from the period in which these receivables and collections originated. In deciding among these unbiased tools, the choice is between simplicity and descriptive power. Sales-weighted days' sales outstanding is the simplest measure of payment patterns, but will not signal offsetting deviations in payment patterns among collections from different

periods. Payment proportions and ratios of receivables outstanding will signal such deviations, but are more complex to use. One possible compromise is to monitor payment patterns via sales-weighted days' sales outstanding, and then use payment proportions, ratios of receivables outstanding, or the accounting technique of "variance analysis" to pinpoint the sources of deviations once the need for investigation has been signaled.

As in the monitoring of payment patterns, the monitoring of bad debt losses requires the analyst to carefully match the event with original sales. For bad debt losses, this process is complicated by two factors. First, there is a substantial time lag between sales and the eventual recognition of bad debts from sales. Second, due to the "lumpiness" of bad debt losses, effective detection of trends requires that these losses be averaged over a period of time. Consequently, it is often many months before a deviation from expected bad debt levels can be detected.

A review of surveys in the practice of accounts receivable management showed areas where this practice is in accord with the latest developments in the theory of working capital management and areas where there are substantial differences between theory and practice. Firms use the expected criteria in evaluating terms-of-sale policy, but do not change these terms very frequently. Unlike other price-related variables, firms do not use terms-of-sale policy to aggressively pursue the goals of the firm. Firms follow cost-of-information principles in setting their credit-investigation policies; more information is gathered for applicants when expected sales are greater. Contradictions appear in the surveys of procedures that firms use to make credit-granting decisions. Surveys report that firms use judgmental procedures in these decisions. However, these surveys also report that firms make accurate estimates of default probabilities and of opportunity costs; estimation of these parameters is not required in judgmental methods, but is critical to a net present value analysis of credit applicants. No recent published information is available with regard to the measures that firms use in monitoring accounts receivable. If firms use aging fractions or DSO, switching to more accurate techniques will improve their ability to monitor this asset.

This concludes our discussion of the management of accounts receivable. The next chapter starts a new topic: the management of inventory.

Problems

7-1. A firm's sales have been as follows:

Month	June	July	August	September	October
Sales	$5,000	$10,000	$25,000	$35,000	$20,000

And its receivables balances at the end of September and October were:

	September	*October*
From Current Month	$32,000	$17,000
From First Prior Month	$15,000	$21,000
From Second Prior Month	$3,500	$12,000
From Third Prior Month	$500	$1,500
From All Other Months	$0	$0

a. Compute the firm's traditional DSO at the end of September and October using a 90-day sales averaging period.

b. Compute the firm's ratios of receivables outstanding at the end of September and October.

c. Compute the firm's sales-weighted DSO at the end of September and October.

d. Compute the firm's payment proportions for October.

7-2. A firm has observed sales-weighted DSOs of 50.1 days, 62.2 days, 53.1 days, 49.8 days, and 57.6 days over the prior five months. For the current month, the sales-weighted DSO is 59.2 days. Using a Student's t test, assess whether the current month's sales-weighted DSO is significantly different from those of the prior five months at the 10 percent confidence level using a two-tailed test.

7-3. Referring to the data presented in Problem 1, assume that the firm had the following expected sales:

Month	June	July	August	September	October
Sales	$7,500	$20,000	$30,000	$30,000	$25,000

Assume also that the firm expected the following ratios of receivables outstanding:

Month	*Current*	*First Prior*	*Second Prior*	*Third Prior*
RRO	0.90	0.50	0.25	0.05

Using these data, compute the expected SWDSO and the collection, sales, and total dollar variances of accounts receivable for the month of October.

Selected Readings

Besley, S., and J. Osteryoung, "Survey of Credit Practices in Establishing Trade-Credit Limits," *Financial Review* (February 1985), pp. 70–81.

Carpenter, M. D., and J. E. Miller, "A Reliable Framework for Monitoring Accounts Receivable," *Financial Management* (Winter 1979), pp. 37–40.

Gallinger, G. W., and A. J. Ifflander, "Monitoring Accounts Receivable Using Variance Analysis," *Financial Management* (Winter 1986), pp. 69–76.

Gentry, J. A., and J. M. De La Garza, "A Generalized Model for Monitoring Accounts Receivable," *Financial Management* (Winter 1985), pp. 28–38.

Hill, N., R. Wood, and D. Sorensen, "Factors Influencing Corporate Credit Policy: A Survey," *Journal of Cash Management*, Vol. 1, No. 2 (Fall 1980).

Lewellen, W. G., and R. W. Johnson, "Better Way to Monitor Accounts Receivable," *Harvard Business Review* (May–June 1972), pp. 101–109.

Scherr, F., "Managing Accounts Receivable," *Nevada Review of Economics and Business*, Vol. 8, No. 2 (Summer 1984), pp. 14–16.

Srinivasan, V., and Y. Kim, "Credit Management: A Variance Framework for Monitoring Accounts Receivable," available from Srinivasan at Northeastern University.

Stone, B. K., "The Payment-Patterns Approach to Forecasting and Control of Accounts Receivable," *Financial Management* (Autumn 1976), pp. 65–82.

case

Foxtrot Incorporated[1] *Monitoring Accounts Receivable*

Foxtrot Incorporated, a medium-sized manufacturer of chemicals and machinery, had personnel policies that fostered long-term employment. Promotions were made when vacancies arose; there was little hiring from outside the firm. Employees who seemed promising were nurtured slowly and given an opportunity to learn the various areas of their specialty in anticipation of the eventual promotion or retirement of their boss.

However, the accidental death of Foxtrot's treasurer, Louis Newington, had disturbed the firm's plans. Lucy Krenn, the assistant treasurer, had been elevated suddenly to the treasurer's job. Ms. Krenn had started with the firm about five years ago and had previous experience with a larger firm in the areas of capital planning and acquisitions. Since being promoted to assistant treasurer three years ago, her principal assignments had been related to the firm's cash management system. Her experience with credit and receivables management was very limited. Early in her tenure as treasurer, she was reviewing the firm's recent financial statements and noted that receivables balances had been rising rapidly. Bad-debt levels, however, seemed normal. Ms. Krenn had scheduled an appointment with Jay Natalli (the firm's General Credit Manager) to discuss the receivables balances. In Foxtrot's organizational structure, the "Accounts Receivable Management Department" was part of the firm's accounting section and reported to the controller. This department was charged with applying incoming cash receipts to accounts receivable records and keeping these records current. The firm's "Credit Department," by contrast, was part of the firm's finance function and was charged with administering policy with regard to collections, credit granting, and terms of sale. The general credit manager reported directly to the firm's treasurer. Therefore, Mr. Natalli was the appropriate person with whom to discuss terms of sale and credit-granting policies and the effects of the policies on accounts receivable levels.

"I recognize that the monitoring system used in the past was not very sophisticated, Lucy, but it suited Mr. Newington," said a seemingly harried Mr. Natalli. "We gave him data on days' sales outstanding and aging frac-

[1]This case has been adapted from Iqbal Mathur and Frederick C. Scherr, *Cases in Financial Management* (New York: Macmillan, 1979), pp. 45–49.

tions. His critical indicator was the figure representing the overall DSO for all our receivables. We used a ninety-day sales averaging period for DSO calculations. Any tools of these sorts are, of course, just aids to the manager in understanding and evaluating customers' payment patterns relative to the estimates we anticipate because of our credit policies. As you know, our sales are not very seasonal, and until recently haven't been growing substantially. This recent surge of sales has really put my department in a bind. We've been hit by a rash of new credit applicants and requests for increased credit from old customers. My staff is working very hard analyzing customers' financial statements and making credit-granting decisions. This, and the increased collection work from the higher sales volume, doesn't make for short days. By Mr. Newington's standards, we were doing fine."

Ms. Krenn then examined Mr. Newington's file of reports from Mr. Natalli. These reports had been prepared on a quarterly basis; the most recent report was for the last quarter of the year just passed. (This report is presented as Exhibit 1.) She was surprised to see that the overall DSO statistic had risen substantially during 1984, but on closer examination decided that this was probably due to a greater concentration of receivables in export accounts, which were sold on substantially longer terms than domestic accounts. DSO figures for disaggregated domestic and export accounts seemed essentially level for the year. She wondered, however, if patterns of customer payments were, in fact, in accord with the firm's expectations based on prior years' experience. She also had on hand a breakdown, generated from the firm's aging schedules of accounts receivable, of dollar re-

Exhibit 1

Foxtrot Incorporated
DSO and Aging Fractions for 1984

Quarter Ending	*March 31*	*June 30*	*Sept. 30*	*Dec. 31*
Domestic DSO	44.98	46.46	45.23	44.92
Due from Current Month	0.697	0.657	0.680	0.657
Due from First Prior Month	0.241	0.252	0.262	0.280
Due from Second Prior Month	0.040	0.054	0.045	0.051
Due from Third Prior Month	0.020	0.032	0.012	0.011
Due from Fourth Prior Month	0.002	0.005	0.001	0.002
Export DSO	153.20	160.10	155.11	153.27
Export Receivables Balances:				
Due from Current Month	0.212	0.191	0.209	0.206
Due from First Prior Month	0.194	0.185	0.174	0.200
Due from Second Prior Month	0.180	0.185	0.196	0.181
Due from Third Prior Month	0.203	0.177	0.172	0.187
Due from Fourth Prior Month	0.132	0.140	0.136	0.125
Due from Fifth Prior Month	0.069	0.089	0.086	0.084
Due from Sixth Prior Month	0.010	0.033	0.027	0.018
Overall DSO	61.71	64.50	63.25	63.62

Exhibit 2

Foxtrot Incorporated
Quarterly Receivables Aging for 1984

Quarter Ending	*March 31*	*June 30*	*Sept. 30*	*Dec. 31*
Domestic Receivables Balances:				
Due from Current Month	$17,500	$18,400	$19,800	$19,500
Due from First Prior Month	6,050	7,060	7,620	8,320
Due from Second Prior Month	1,000	1,500	1,300	1,500
Due from Third Prior Month	500	900	360	320
Due from Fourth Prior Month	50	140	20	60
Total Domestic Receivables	$25,100	$28,000	$29,100	$29,700
Export Receivables Balances:				
Due from Current Month	$ 3,305	$ 3,480	$ 4,087	$ 4,343
Due from First Prior Month	3,029	3,377	3,406	4,219
Due from Second Prior Month	2,817	3,360	3,837	3,816
Due from Third Prior Month	3,176	3,222	3,358	3,948
Due from Fourth Prior Month	2,062	2,548	2,668	2,646
Due from Fifth Prior Month	1,082	1,622	1,689	1,776
Due from Sixth Prior Month	155	596	521	383
Total Export Receivables	$15,626	$16,205	$19,566	$21,131

Exhibit 3

Foxtrot Incorporated
Domestic and Export Monthly Sales
(thousands of dollars)

Year	*Month*	*Domestic Sales*	*Export Sales*
1983	September		$2,575
1983	October	$18,000	$2,775
1983	November	$15,400	$2,864
1983	December	$15,780	$3,312
1984	January	$16,000	$2,846
1984	February	$16,720	$3,029
1984	March	$17,500	$3,305
1984	April	$17,850	$3,377
1984	May	$17,990	$3,377
1984	June	$18,400	$3,480
1984	July	$19,000	$3,860
1984	August	$19,100	$3,406
1984	September	$19,800	$4,087
1984	October	$19,700	$3,846
1984	November	$20,300	$4,219
1984	December	$19,500	$4,343

ceivables outstanding in various age categories as of the end of each quarter of 1984 (see Exhibit 2) and a schedule of monthly sales for 1984 and late 1983 (see Exhibit 3). As a first step toward additional evaluation of the firm's collection patterns during 1984, she calculated Foxtrot's sales-weighted days' sales outstanding statistics as of the end of each month for the prior three years (1981 through 1983). This 36-month period was a time when the firm's sales volumes and customer payment patterns were considered to be more or less as anticipated. She found the average sales-weighted days' sales outstanding figure to be 43.3 days with a standard deviation of 1.0 days for domestic sales, and the average sales-weighted days' sales outstanding figure to be 152.6 days with a standard deviation of 7.8 days for export sales.

Chapter -8

Inventory Management Certainty Approaches

This is the first of two chapters on the management of *inventory*. The management of this asset is unlike the management of any of the other assets and liabilities discussed in this text, in that the assets are *physical* rather than purely *financial* in nature. A dollar of accounts receivable, for example, represents a debt owed by one firm to another. Such a debt may be evidenced by an invoice, but it is primarily a financial transaction. However, inventory represents items with a physical existence: a bushel of corn, an automobile headlamp, or another good. In the management of inventory, our models relate to physical rather than financial circumstances.

The management of inventory is one of the oldest concerns of management science. As such, there is a broad and deep volume of literature on the management of this asset in various situations, the interaction of this asset with other assets, and similar issues. A complete and comprehensive discussion of the issues in inventory management is beyond the scope of this text. We limit our coverage to only the most basic inventory problem: the determination of ordering strategies for individual items of inventory. These ordering strategies are formulated by the application of mathematical models. A limited set of these models is developed in the following chapters.[1] Readers who are interested in other inventory issues, such as material requirements planning and the use of inventory to improve the quality of goods produced, are referred to the many available texts in the area.[2]

[1]For a more extensive description of the various models available, see S. Love, *Inventory Control* (New York: McGraw-Hill, 1979).

[2]Love, *Inventory Control*, provides a good discussion of many of these issues. See J. Orlicky, *Material Requirements Planning* (New York: McGraw-Hill, 1975), for a discussion of material requirements planning. A good text relating inventory decisions to other strategies in operations management is J. Evans and others, *Applied Production and Operations Management*, Second Edition (St. Paul: West Publishing.

In the inventory situations addressed in this text, the challenge is to formulate strategies for the ordering and holding of inventory that will be the most advantageous to the firm. However, because of uncertainties regarding the various parameters that are estimated in making these decisions, there is risk associated with these strategies. Two approaches are used in addressing this risk. One method, called the "certainty approach," is discussed in this chapter. In this approach, a strategy is first formulated to deal with the *expected values* of these inventory parameters, under the unrealistic assumption that these parameters are certain. Then, a *hedging strategy* is formulated to address the uncertainties in these parameters. This is called the *certainty approach* not because the parameters are certain, but *because they are treated as certain in the first phase of policy making*. An alternate methodology, in which the risk of these parameters is considered simultaneously with their expected values, is called the "uncertainty approach" to inventory management. The uncertainty approach to inventory management is discussed in Chapter 9.

WHY CARRY INVENTORY?

Like all other assets, inventory represents a costly investment to the firm. In order for this investment to be worthwhile, there must be some advantage in making it. In this section, we discuss some reasons why the firm may want to carry inventory. These reasons vary with the *type of inventory* carried. For purposes of discussion, we will use the accountant's convention of dividing inventory into three types: raw materials inventory, work-in-process inventory, and finished goods inventory.

Raw Materials Inventory There are several reasons why firms might want to carry inventories of raw materials. First, having an available stock of raw materials inventory *makes production scheduling easier*. Since the raw materials for the production process are on hand, scheduling of the firm's production equipment can proceed without concern over when these goods will arrive. Without a stock of raw materials on hand, this scheduling would be tentative, and subject to the arrival of such materials.

A second reason to keep raw materials inventory is to *avoid price changes* for these goods. If the firm keeps a stock of raw materials, the firm can purchase these materials when it believes prices are low and can decline to purchase when it believes prices are high. This reduces the firm's costs and also the variability of these costs. By keeping a stock of raw materials for this purpose, the firm is pursuing a hedging strategy against price fluctuations of these materials.

Third, the firm may keep extra raw materials inventory to *hedge against supply shortages*. When prices of raw materials are controlled (such as by

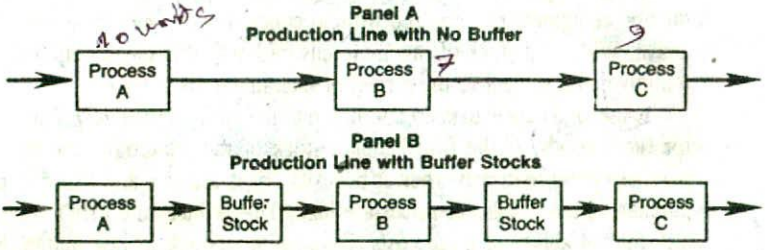
when goods are unavailable at the controlled price. During these times, the firm may draw down its existing inventory to continue production.

Finally, the firm may order and keep additional inventories of raw materials to *take advantage of quantity discounts*. Quantity discounts are reductions in the price per unit purchased that occur when the firm orders larger amounts. For example, the firm may pay $10 per unit if orders are less than a truckload, and $9.75 if a truckload or more is ordered. By ordering more, the purchasing firm may reduce the price paid per unit, but it must hold more inventory on average.[3]

Work-In-Process Inventory In manufacturing firms, a certain amount of work-in-process inventory occurs as products move from one production process to another. A major reason that firms keep work-in-process inventory beyond this minimum level is to *buffer production*. Buffering is part of the planning process and allows flexibility and economies that would not otherwise occur. Buffering is diagrammed in Figure 8-1. In Panel A, a production line with three processes produces goods. Without buffer stocks, production is not very flexible. For example, the production line can produce continuously only at the pace of the slowest process. If, at maximum, Process A can produce 10 units per hour, Process B can produce 7 units per hour, and Process C can produce 9 units per hour, the production line must be set up at a rate of 7 units per hour. Otherwise, because A's rate of production is greater than B's, inventory will accumulate between A and B. Also, if one process breaks down, the entire line must be shut down, resulting in the unplanned idling of the machinery and workers involved with all the processes.

If buffer stocks of inventory are employed between the production processes, these problems are avoided. (The production process with buffer stocks is portrayed in Panel B of Figure 8-1.) Process A can be run at its maximum output, accumulating inventory for Process B. Once sufficient in-

Figure 8-1. *Work-In-Process Inventory as a Production Buffer (arrows indicate flow of materials).*



[3]Later in this chapter, we will formulate a model to assess whether ordering higher volumes to take advantage of quantity discounts is of advantage to the firm.

ventory is accumulated to employ Process B for a time, the firm can plan to take Process A out of the production line temporarily and redeploy its workers and machinery to other useful endeavors. Also, if one of the processes breaks down, the other processes can continue to run. For example, if Process B is shut down for maintenance, Process A can continue to produce goods. These goods are accumulated as inventory in the first buffer stock. Further, Process C can continue to operate using materials from the second buffer stock. Once Process B is reactivated, the excess inventory in the first buffer stock can be depleted and the reduced inventory in the second buffer stock can be replenished.

Finished Goods Inventory. One reason to keep finished goods inventory is to *provide immediate service*. When customers want to purchase goods and services, ready availability of these is a substantial advantage in making a sale. Sellers who do not have goods available will find buyers ordering elsewhere.

Intertwined with this notion of providing immediate service is the presumption that demand for goods or services is uncertain. If demand and the future arrivals of goods could be known for certain, there would be no need to keep inventories of finished goods (beyond those inventories resulting from the minimization of production and ordering costs). Plans could be made for the goods to be produced or to arrive from suppliers just as the buyer wished to purchase them. We will see later in the chapter that dealing with uncertainty of demand is a major consideration in the formulation of inventory strategies.

A second reason for keeping finished goods inventory is to *stabilize production*. When firms produce several types of products using the same equipment, there are costs and delays in changing from the production of one product to another. The longer are the firm's production runs, the lower are these transition ("setup") costs. However, longer production runs result in higher finished goods and work-in-process inventories.

ALTERNATIVES TO HOLDING INVENTORY

In the prior section, rationales for holding various types of inventory were listed. The holding of inventory for these various reasons are *solutions* to different *problems* that the firm faces: fluctuations in the prices of raw materials, the formulation of economical production strategy, the need for goods availability, and so forth. *However, the holding of inventory is not the only possible approach to these problems.* Like investment in any other asset, *investment in inventory should be carefully considered in light of the alternative approaches to the problem at hand.* A little thought will show that each of these problems can be addressed via alternative strategies to holding inven-

tory, and that in some cases, these alternative strategies may result in lower costs. Let us discuss a few examples.

One reason for holding raw materials inventory is to avoid fluctuations in prices of these materials. However, holding inventory is not the only way to avoid these fluctuations. The firm could enter into long-term private contracts with the producers of these raw materials. These contracts would specify the prices to be paid. Alternatively, if the raw material involved is one on which a futures contract is traded, the firm can purchase such a contract for future delivery. This futures strategy is pursued by many firms—producers of chocolate do not keep inventories of cocoa; they purchase futures contracts on this commodity to ensure delivery and price.

One reason for using buffer stocks of work-in-process inventory is to hedge against the breakdown of one of the processes within the production line. The same effect can be achieved by replacing processes prone to breakdown with others that are more reliable. In this circumstance, the capital expenditure made in improving the processes substitutes for the buffer stock.

Similar strategies can be formulated to reduce finished goods inventories. For example, one common reason to keep finished goods is to provide immediate availability of goods to customers. By satisfying customer needs promptly, the seller hopes to increase sales. However, there are other techniques that will obtain the same effect. For example, the firm could offer longer terms of sale or lower prices as an alternative to immediate availability. Many customers will be willing to wait for goods if they can obtain them on more advantageous terms or at a lower price.

Just-In-Time Systems One of the newest and most interesting alternatives to holding inventory is to use *just-in-time* (JIT) systems. JIT systems were developed by Japanese firms. In these systems, holding inventory is considered to be an uneconomical solution to the various problems facing the firm. Inventories are minimized. Instead, JIT substitutes demand-based systems based on flexible production, small lot sizes, and high-quality output.[4] To reduce inventory, goods are produced and delivered only as they are needed. JIT systems can be used within the firm to reduce work-in-process inventories and, in conjunction with suppliers, to reduce raw materials inventories. Let us discuss each of these applications in turn.

Within the firm, the JIT system is, to some extent, an alternative to traditional production planning systems. Production planning is done only in the aggregate, and is based on total output. After this aggregate plan is made,

[4]The just-in-time inventory system is discussed in most texts on production and operations management; see, for example, Evans and others, *Applied Production*, pp. 530–543. For a report of its application in the United States, see E. Hoeffer, "GM Tries Just-In-Time American Style," *Purchasing*, August 19, 1982.

a signaling system is used among the production processes, starting with the last process. The last process signals the next-to-last that goods are needed, the next-to-last signals the process before that, and so forth. This signaling is done by means of *Kanbans* (Japanese for cards). Kanbans travel through the processes, indicating the need for goods. In response to these Kanbans, the employees set up production processes and produce the needed goods. Because goods are produced only as needed, work-in-process buffer stocks are minimized.

The JIT system requires quite different procedures than does the typical American production system. The biggest differences concern the lot sizes and quality of production runs. In order to achieve a reduction in inventory, set-up costs for processes must be small so that small lot sizes can be economically produced in response to demand. Small lot sizes also require that workers be flexible; they must be able to change their work stations from one production process to another in response to demand (as signaled by the Kanbans). Workers cannot be specialists performing only one task. Also, because goods move directly from one production process to another, there are few en route quality control inspections. Workers must perform their own quality control as they execute the production processes. Finally, since there are no buffer stocks, output must be of very high quality (or later processes will not be able to function, since they have no inventory from which to draw if the delivered goods are defective).

In JIT, suppliers are treated as another step in the production process. They are required to produce small lots of high-quality goods for frequent delivery. This minimizes the producer's raw materials inventory. To obtain the necessary quality, buyers frequently must work with their suppliers to assure that the output from supplier's manufacturing processes fits as perfectly as possible with the producer's manufacturing processes.

Japanese firms have proved that JIT is a viable alternative to the costly holding of inventory. However, JIT systems have their own costs. JIT essentially substitutes worker education costs and capital costs for inventory costs. For JIT to work properly, workers must be highly skilled and flexible, and the machinery they use must be capable of high-quality output while being cheap and quick to set up.

The point of this section is that *there are alternative potential strategies in response to each of the problems that holding inventory addresses*. JIT is one of these strategies. Because of the existence of these alternatives, the firm should think *very broadly* about the problem to be addressed and about other possible solutions before adopting an inventory approach. The costs and benefits of these other solutions should be compared to the costs and benefits of holding inventory, given an optimal inventory management strategy. The costs to the firm in inventory-based solutions to such problems are discussed in the next section of this chapter.

COSTS IN INVENTORY SYSTEMS

Any changes in the firm's cash flows that are caused by inventory management strategies are relevant in the evaluation of these strategies. Inventory strategies can have several different effects on cash flows. As in accounts receivable management, the different types of cash flow effects that occur in inventory management decisions are usually discussed in terms of the different costs faced by the firm in making these decisions. These different types of costs are discussed in the following paragraphs.

Costs Directly Proportional to Amount of Inventory Held Certain costs are directly proportional to the level of inventory carried by the firm. These are usually called "carrying costs of inventory" or "holding costs of inventory." Included here are any costs that fluctuate directly with the level of inventory, such as the opportunity cost of inventory investment, insurance on the inventory, storage costs of inventory, taxes on inventory investments, and so forth. The formula for this type of cost is:

$$\text{Cost} = (a)\ (\text{amount of inventory}) \tag{8-1}$$

where a is a coefficient representing the sum of all costs that are directly proportional to the level of inventory.

Costs Not Directly Proportional to the Amount of Inventory Held In addition to those costs that are directly proportional to the amount of inventory held, there is also a group of costs that vary with inventory size but not in direct proportionality. Instead, they assume some other mathematical relationship with the amount of inventory held. Consider costs such as spoilage and obsolescence. These costs vary with the length of time that an item is in inventory. The more inventory ordered at one time, the higher the firm's average inventory, and the longer, on average, that items are in inventory. For several reasons, the cost of spoilage and obsolescence is nonlinear in this average time in inventory. For example, some types of spoilage are caused by bacterial action, and bacteria multiply at a geometric rather than arithmetic rate. For technological obsolescence, the inventory often retains a substantial part of its value until a new technological advance is made. Initial time periods involve little loss of value, then value declines in an accelerating fashion. While some processes of this sort can be approximated by linear relationships, there are many cases in which such an approximation distorts results. The general formula for these costs is:

$$\text{Cost} = f(\text{inventory level}) \tag{8-2}$$

where f(inventory level) means that the cost is a function of inventory level,

with the particular mathematical relationship depending upon the type of cost being considered.

Costs Directly Proportional to the Number of Orders Whether the firm produces its own materials for inventory or orders goods from another firm, there are costs to the ordering, delivery, and payment processes. These costs depend directly on the number of times that orders are placed and received. Costs of this sort include set-up costs on machines to produce inventory, costs of generating a purchase order for the inventory, costs of writing a check and mailing it in payment for the order, fixed costs of unloading the order, and so forth. The formula for this type of cost is:

$$\text{Cost} = (c)(\text{number of orders}) \tag{8-3}$$

where the c is a coefficient representing the sum of all the costs of this type.

The Price per Unit of Inventory Obtained Due to quantity discounts and economies of scale in production, the price per unit of goods purchased or produced for inventory may vary with the amount ordered. When this occurs, the change in the total cost of the inventory that results can be a major determinant of the most advantageous order quantity. The formula for the total cost of the inventory is:

$$\text{Cost} = P_q S \tag{8-4}$$

where P_q is the unit price for the quantity ordered by the firm and S is the yearly usage of the good.

Stockout Cost Another cost that is dependent on the firm's inventory strategy is stockout cost. Stockout cost occurs when immediate service is required but inventory is unavailable. If the inventory is raw materials or work-in-process, the cost of stocking out will include the cost of changing the firm's production plans, costs of idling or rescheduling equipment, and so forth. If the inventory is a finished good, stockout costs may include the lost cash flow from the sale if the customer does not make the purchase because immediate service cannot be provided. Also, when stockouts of finished goods occur, customers are less likely to purchase from the firm in the future, so the opportunity costs of lost *future* cash flows (in addition to lost flows from missing the current sale) can also occur.

All these types of inventory costs will vary with the type of inventory under consideration. Some types of bulky or high-value inventory may have high storage costs, while other inventory may be stored for years at relatively little expense. The size of these costs, plus the other characteristics of the

inventory situation, help determine the best approach to inventory strategy. These other characteristics of inventory situations are discussed in the next section.

OTHER CHARACTERISTICS OF INVENTORY SITUATIONS

Besides the various types of costs involved, there are other characteristics of the situation that vary among types of inventory, and must be captured if the decision model is to be an accurate representation of the physical circumstances. Several of these characteristics are listed here.

Lead Times Obtaining inventory usually requires a time lag from the initiation of the process until the inventory starts to arrive. This lead time may be a few minutes or it may be many months, and depends in part on whether the firm is producing goods for its inventory or is ordering these goods from another firm. To produce goods for its own use, the firm must schedule, set up, and adjust manufacturing equipment. When purchasing from a vendor, the firm must order the goods and await delivery. Deliveries from a vendor may be almost immediate, or may be months away from imported or specially built items.

Sources and Levels of Risk Uncertainties play a significant role in inventory situations. Uncertainties usually involve lead times and demand levels, but situations where other variables are uncertain also occur. Where there are substantial uncertainties and where the costs of stockout are important, strategies for addressing risk must be formulated.

Static versus Dynamic Problems Inventory problems are usually divided into two types based on the characteristics of the goods involved. In *static inventory problems*, the goods have a one-period life; there can be no carryover of goods from one period to the next. Inventory situations where decisions involve the number of newspapers to print, the number of Christmas trees to purchase, or the number of calendars to produce are static inventory problems. In *dynamic inventory problems*, the goods have value beyond the initial period; they do not lose their value completely over time. Inventory situations such as that faced by a service station in determining how much gasoline to purchase are dynamic inventory problems.

Replenishment Rate Once goods start to be received from a vendor or from the firm's own production processes, there are differences among goods in the *rate* at which they are received. Small orders from vendors are likely to be received all at once. For example, assume that a firm has placed an order for 10 cases of paper towels. For such a small order, the rate of replenish-

ment is essentially infinite; the firm's inventory will go up by 10 cases in a very short time as the goods are quickly unloaded. For larger orders from vendors, or for inventory produced within the firm, the replenishment rate may be slower. For example, suppose that the firm orders 10 *truckloads* of paper towels. Given the size of such an order, it is likely to take some time to unload. During this time, inventory will increase as cases of paper towels are removed from the trucks and added to the firm's inventory. Because inventory increases slowly, this is called a "finite replenishment rate" situation.

DECIDING ON AN APPROACH IN INVENTORY STRATEGY

To formulate an inventory strategy, the firm first considers the costs and characteristics of the particular inventory item involved. What costs are important? What are the characteristics of the inventory situation? Once this is assessed, the analyst then chooses a basic approach: either the *certainty approach*, in which the expected values of uncertain variables and their risk are addressed separately, or the *uncertainty approach*, in which these facets are addressed simultaneously. The certainty approach is tractable, well-tested, and easy to implement. However, because of the way it handles risk, the certainty approach is not applicable to *static* inventory problems. In the certainty approach, risk is usually addressed via the use of a *safety stock*. A safety stock is an additional amount of inventory kept permanently on hand to absorb fluctuations in inventory usage. However, a safety stock strategy is inappropriate in *static* inventory situations because such a safety stock would have no value after the initial period. The uncertainty approach, on the other hand, is applicable in these static inventory problems, because this approach does not rely on the safety stock methodology to handle risk. The uncertainty approach can also address dynamic inventory problems, but simulation is required and the resulting methodology does not lead to a definitive strategy. Thus, in our two chapters on inventory management we will concentrate on illustrating the *certainty* approach for *dynamic* inventory problems and the *uncertainty* approach for *static* inventory problems.

Once the costs and characteristics of the problem are identified and a modeling approach is chosen, a model must be developed for the formulation of policy. This model must capture the costs and characteristics of the inventory situation. This modeling process is critical; it cannot be stressed sufficiently that the *domain of a mathematical model is limited to situations covered by the model's assumptions*. The application of a model to other situations can lead to nonoptimal and sometimes *disastrously incorrect* strategies. For example, assume that an analyst applies an inventory model that does not allow for spoilage to a grocery chain's ordering policy for lettuce and formulates the strategy of ordering lettuce in large amounts every 14 days. A

little thought will show that *this is obviously nonsense*. This strategy implies that lettuce will be on the shelves for an average of seven days, and in seven days most lettuce will be spoiled. However, *this is not a failure of the inventory model; it is a failure to apply the correct model*. With this firmly in mind, let us proceed to discuss inventory models of dynamic inventory situations using the certainty approach.

THE BASIC EOQ MODEL

The economic order quantity (EOQ) model is presented in most introductory textbooks in finance and in management science. In this section, we present the basic version of this model under the assumption that all variables are certain. In later sections, we present other versions of this model for other inventory situations and present methods for developing safety stock strategies to address risk. The basic EOQ model is simple, but it is applicable only to those inventory situations described by its assumptions, which are:

1. There are only two types of costs: costs that are directly proportional to the amount of inventory held and costs that are directly proportional to the number of orders received. Call the former *carrying or holding cost* and the latter *ordering cost*. There are no costs that are indirectly proportional to the amount of inventory held, and the price per unit of inventory obtained does not vary with the amount of inventory received (there are no quantity discounts). Stockout costs are modeled separately and are not considered in the EOQ model.
2. There may be lead times of any length.
3. There is no risk (risk is modeled separately in determining safety stock strategy).
4. The replenishment rate is infinite.

The time-pattern of inventory for these assumptions is portrayed in Panel A of Figure 8-2. When an order arrives, the level of inventory held by the firm increases instantaneously from zero to Q_A, the order quantity. It is a right triangle because of the instantaneous increase in inventory. (Remember, the replenishment rate is infinite, so replenishment occurs in zero time.) Over time, inventory is used up, and the level of inventory declines; let the usage rate be S, which will be the slope of hypotenuse of the triangle. During the inventory cycle, Q_A units are used up, and a new order is received just as inventory falls to zero.

The Optimal Order Quantity The question in this inventory situation is *how much to order*; that is, what should be the order quantity, Q? A relatively

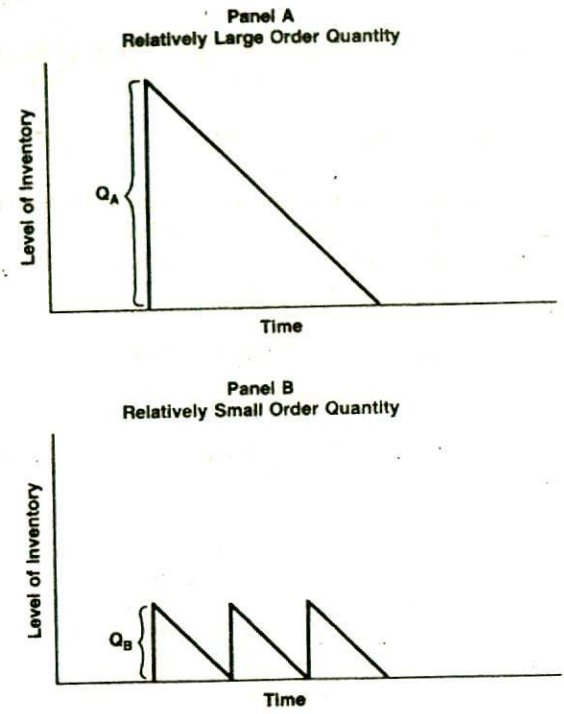


Figure 8-2. Time Pattern of Inventory Level for Basic EOQ Model.

large Q (as in Panel A) results in a high average inventory but infrequent orders. A smaller order quantity (Q_B in Panel B) results in lower average inventory but more frequent orders.[5] The first strategy will result in higher holding cost but lower ordering cost; the latter strategy will have the reverse effect. To decide on the optimal Q, we need a mathematical model of the trade-offs between the two costs. This requires the development of expressions for the holding cost and carrying cost of inventory; these are operationalizations of equations (8-1) and (8-3) for this inventory situation. Equations (8-1) and (8-3) are:

$$\text{Cost} = (a)\ (\text{amount of inventory}) \tag{8-1}$$

$$\text{Cost} = (c)\ (\text{number of orders}) \tag{8-3}$$

Taking equation (8-1) first, inspecting the time-pattern of inventory in

[5]Note that the usage rate, as represented by the slope of the hypotenuse, is the same for the smaller triangles in Panel B as for the larger triangle in Panel A.

Wor- Cap-20

Figure 8-2 shows that the inventory level goes from Q to zero in a linear fashion. The average amount of inventory, in units, is then:

$$\text{Average inventory} = (Q + 0)/2 = Q/2 \tag{8-5}$$

The logical way to value this inventory investment is in dollars. If the dollar investment per unit is P, and the yearly cost of holding a dollar of inventory (equal to the sum of all the costs that are directly proportional to inventory level) is C, then equation (8-1) becomes:[6]

$$\text{Cost} = (a)\ (\text{amount of inventory}) \tag{8-1}$$

$$= CP(Q/2) \tag{8-6}$$

since $a = CP$. This is our expression for the holding cost of the inventory for this inventory situation.[7]

Turning now to equation (8-3), for any level of order quantity, over an entire year the amount ordered must be sufficient to cover the yearly usage (S). The number of orders required to do this is:

$$\text{Number of orders} = S/Q \tag{8-7}$$

For example, if the firm uses 10,000 units of a product per year and obtains 1,000 units each time it orders, 10 orders must be placed per year. Letting F be the cost of ordering, equation (8-3) becomes:

$$\text{Cost} = (c)\ (\text{number of orders}) \tag{8-3}$$

$$= F(S/Q) \tag{8-8}$$

[6]In this equation, P represents the price (out of pocket investment) per unit; this was represented by P_q in equation (8-4). However, since P is invariant to Q (there are no quantity discounts), the use of the P_q notation from equation (8-4) is unnecessary for this inventory situation. The simpler P notation is therefore used.

[7]In this method of calculating the holding cost, a required rate of return is applied to the inventory investment to account for the time value of money. This required return is part of C. This approach to the time value of money was also used in Chapter 2, where lockbox location problems were discussed. As discussed in that chapter, this approach is not quite equivalent to discounting cash flows, which is the proper technique for making decisions that result in shareholder wealth maximization. However, the opportunity cost approach is much simpler and has been shown empirically to produce almost exactly the same strategies for inventory situations; see W. R. McDaniel, "The Economic Ordering Quantity and Wealth Maximization," *Financial Review*, Vol. 21, No. 2 (November 1986), pp. 527–36. For more on the relationship between inventory models and net present value, see Y. H. Kim, G. C. Philippatos, and K. H. Chung, "Evaluating Investment in Inventory: A Net Present Value Framework," *Engineering Economist*, Vol. 31, No. 2 (Winter 1986), pp. 119–25.

since $c = F$. This is our expression for the cost of ordering in this inventory situation. Looking at equations (8-6) and (8-8), we see that the holding cost rises linearly with the order quantity, while the ordering cost declines inversely with the order quantity. The total cost in this situation is the sum of these two cost types:

$$\text{Total cost} = CP(Q/2) + F(S/Q) \tag{8-9}$$

To see how the two cost types interact, let us examine an example inventory problem. We will use several variations of this problem at different times in this chapter.

A firm purchases 10,000 units of a particular product per year. The product costs \$8 per unit. The sum of insurance, storage, and the opportunity cost of invested funds is 20 percent per year of the average dollar investment in inventory. Each time the firm places an order, it costs \$50 in out-of-pocket expenses to generate the purchase order, receive the goods, and so forth. How much should the firm order each time an order is placed?

One possible approach to the problem is to compute the costs for several possible order quantities [from equations (8-6), (8-8), and (8-9)] and graph the costs versus order quantity. Cost data for order quantities between 100 and 1,200 units are presented in Table 8-1. To see how these data were computed, let us examine the cost figures of an order quantity of 500. For this order quantity, the average inventory will be 250 units (equal to $Q/2$). At \$8 per unit, this results in an average investment of \$2,000, which generates a yearly holding cost of \$400 since the sum of all the costs that are directly related to inventory level is 20 percent. Since yearly demand is 10,000 units, an order quantity of 500 necessitates that 20 orders be placed per year. At a cost of \$50 per order, this is an ordering cost of \$1,000. The total cost of the inventory strategy for an order quantity of 500 is then \$1,400 per year.

The cost data from Table 8-1 are graphed in Figure 8-3. Inspection of this graph shows that the minimum total cost occurs for an order quantity of about 800 units. We also note from this graph that the total cost function attains a minimum.[8] This suggests a more direct way to obtain the optimal ordering strategy: we could take the derivative of the total cost function, set this derivative equal to zero, and solve for the optimum Q:

$$\text{Total cost} = TC = CP\,(Q/2) + F(S/Q) \tag{8-9}$$

$$dTC/dQ = CP/2 - F(S/Q^2) \tag{8-10}$$

[8]For some products this function is not quite continuous, since the order quantity can take only integer values. A fractional unit of such products would be quite useless to the firm. An integer restriction does not, however, affect the major conclusions of the analysis in this chapter.

Table 8-1. Holding, Ordering, and Total Costs for Basic EOQ Example Problem.

Initial Data:

Yearly Usage (S)	10,000 units
Yearly Cost per Dollar of Inventory (C)	\$.20/yr.
Investment per Unit of Inventory (P)	\$8
Fixed Cost of Placing an Order (F)	\$50/order

Order Quantity	*Avg. Inv. in Units*	*Holding Cost*	*Orders per Year*	*Ordering Cost*	*Total Cost*
Q	$Q/2$	$CP(Q/2)$	S/Q	$F(S/Q)$	$CP(Q/2)$ $+F(S/Q)$
100	50	\$ 80	100	\$5,000	\$5,080
200	100	\$160	50	\$2,500	\$2,660
300	150	\$240	33	\$1,667	\$1,907
400	200	\$320	25	\$1,250	\$1,570
500	250	\$400	20	\$1,000	\$1,400
600	300	\$480	17	\$ 833	\$1,313
700	350	\$560	14	\$ 714	\$1,274
800	400	\$640	13	\$ 625	\$1,265
900	450	\$720	11	\$ 556	\$1,276
1,000	500	\$800	10	\$ 500	\$1,300
1,100	550	\$880	9	\$ 455	\$1,335
1,200	600	\$960	8	\$ 417	\$1,377

$$0 = CP/2 - F(S/Q^2) \tag{8-11}$$

$$Q^* = (2FS/CP)^{0.5} \tag{8-12}$$

where Q^* is the optimum (lowest total cost) ordering strategy. Substituting the figures from the example problem into this formula, the optimum order quantity is calculated:

$$Q^* = (2FS/CP)^{0.5} \tag{8-12}$$

$$= [2(50)\ (10{,}000)/(0.20(8))]^{0.5}$$

$$= 790.57 \tag{8-13}$$

The optimal order quantity is therefore 791 units per order. This will result in 12.64 orders per year (equal to 10,000/791). For a 360-day year, orders will be placed about every 28 days (equal to 360/12.64). The total cost of holding and ordering the inventory will be:

$$\text{Total cost} = CP(Q/2) + F(S/Q) \tag{8-9}$$

$$= 0.20(8)\ (791/2) + 50(10{,}000/791) = \$1{,}264 \tag{8-14}$$

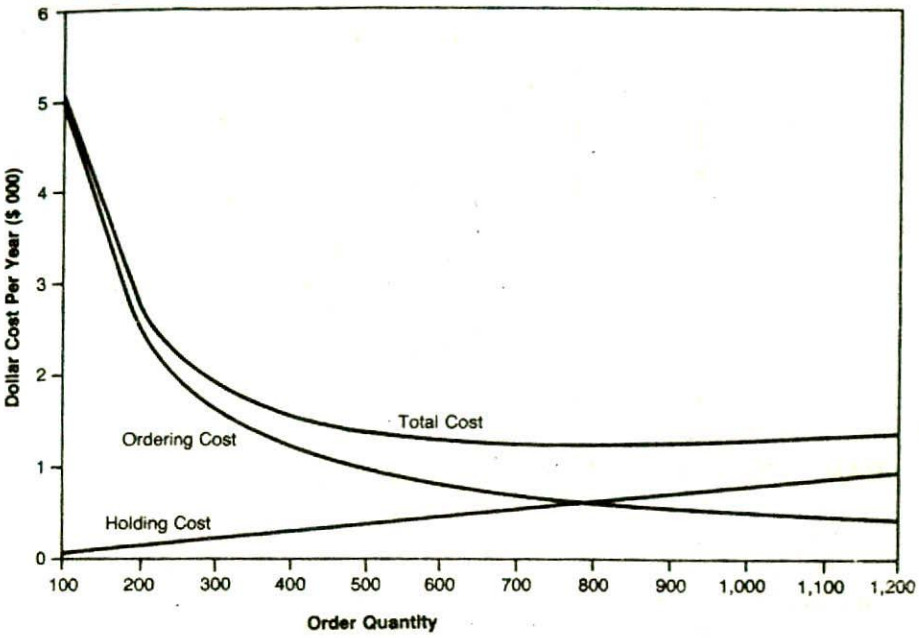


Figure 8-3. Example Problem Data From Table 8-1.

The Order Point The order quantity strategy outlined above is very simple to implement: order 791 units every 28 days. However, rather than using a *timed* ordering approach, it is often advantageous to place orders based on *inventory levels*. In this way, if the usage rate is greater than expected, an order will be placed earlier and the goods received sooner. The triggering of orders based on inventory levels is called the *order point system*, and it works well with modern methods of inventory record keeping. If the firm's data base of inventory levels is adjusted each time a unit of inventory is used (as occurs, for example, when a sale is recorded on an electronic cash register), then the firm's computer can compare inventory levels to order points and generate a list of items to be ordered.

The order point is calculated as the expected usage during the lead time. For the prior example problem, assume that the lead time is seven days. The usage over this time will be (7/360) (10,000) = 194 units. When the level of inventory reaches 194 units, the firm should place an order for 791 units. Just as this order arrives (seven days later), the firm will have run out of inventory. The optimal strategy and the order point for this example problem are portrayed in Figure 8-4.

This basic EOQ model with order points is easy to understand and to put into practice, but its range is limited to those situations described by its assumptions. Other situations require different models to portray their different circumstances.

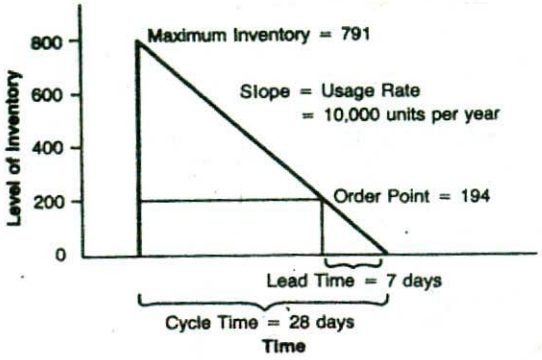


Figure 8-4. Optimal Strategy for Example EOQ Problem.

VARIATIONS ON THE BASIC EOQ MODEL

In this section, we present three variations of the basic EOQ model: the production order quantity model, the EOQ model with quantity discounts, and the EOQ model with decay. These variations are intended to illustrate how models are developed to capture the relevant aspects of the various situations faced by firms in managing inventory.

The Production Order Quantity Model There are many inventory situations where the increase in the inventory level during the replenishment portion of the inventory cycle is not instantaneous. While such situations may occur because of other circumstances (such as limitations in the ordering firm's ability to unload materials quickly), one common circumstance happens when the firm *produces its own goods for inventory*. For example, as a manufacturing firm turns work-in-process inventory into finished goods, the finished goods inventory increases *gradually rather than all at once*. This *finite replenishment rate* situation results in a different time-pattern of inventory than does the basic EOQ model. This time-pattern is portrayed in Panel A of Figure 8-5. Here, the triangle representing inventory level is not a right triangle; there is a gradual buildup of inventory over time. However, the problem of how much to order remains the same. Either infrequent large orders (Panel A) or frequent small orders (Panel B) will get the job done, although different ordering strategies have divergent effects on ordering and holding costs.

The primary difference between the basic EOQ model and the model for the finite-replenishment case (which is called the *production order quantity* (POQ) model) concerns the maximum inventory that occurs during the inventory cycle, and therefore the level of holding costs of inventory. Since the replenishment portion of the inventory cycle takes place over time, and there is a continual usage of inventory, *the maximum inventory that occurs in the POQ model is less than the order quantity*. For example, assume a firm makes

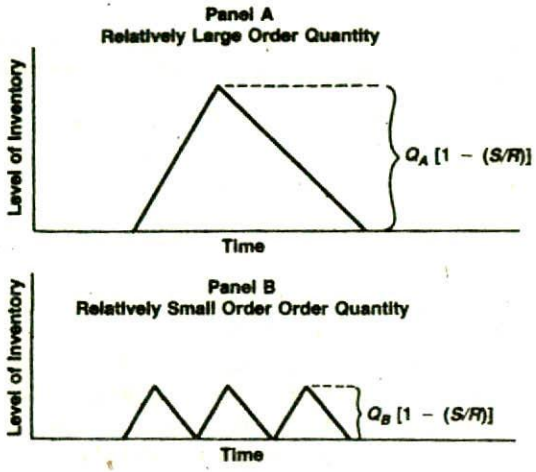


Figure 8-5. Time Pattern of Inventory Level for Finite Replenishment Rate.

500 units of a product for inventory and takes a week to produce these units. Also assume that usage is 20 units per day. Over the seven-day week during which the firm is producing these goods, 140 units of product will be used, so the *net increase* in inventory will be 360 units: the 500 units that were *added* to inventory, less the 140 units that were *withdrawn* from inventory during this week. If inventory started the week at zero, the final inventory after replenishment will be 360 units, not 500.

Since the physical situation is different from that of the basic EOQ model, a different mathematical model is needed. Let R be the *replenishment rate*, the rate at which items are put into inventory. During the replenishment portion of the inventory cycle, Q items will be received. The length of the replenishment portion of the inventory cycle must then be equal to Q/R (for example, if the firm wants to receive 1,000 items, and the items arrive at the rate of 250 per day, the replenishment will take 4 days). During the replenishment portion of the cycle, inventory will be used up at rate S; since this part of the cycle is Q/R in length, the usage over this portion of the cycle will be $S(Q/R)$. For example, if usage is 50 units per day, the order quantity is 1,000, and replenishment is 250 units per day, 200 units will be used during the replenishment part of the inventory cycle. The *maximum inventory* will then be the *increase* in inventory due to replenishment minus the *usage* during replenishment, or:

$$\text{Maximum inventory} = Q - S(Q/R) = Q[1 - (S/R)] \qquad (8\text{-}15)$$

Since the inventory level varies from zero to this maximum in a linear fashion, the average inventory will be one-half of the maximum, or:

$$\text{Average inventory} = Q[1 - (S/R)]/2 \qquad (8\text{-}16)$$

Applying the pricing and holding cost parameters to this average inventory gives the holding cost expression for the finite replenishment case:

$$\text{Holding cost} = CPQ[1 - (S/R)]/2 \tag{8-17}$$

The number of orders is S/Q as before, so the ordering cost is $F(S/Q)$. The total cost equation for the finite replenishment case is then:

$$\text{Total cost} = CPQ[1 - (S/R)]/2 + F(S/Q) \tag{8-18}$$

To see how the finite replenishment rate affects strategy, let us modify our prior example problem by assuming that the firm produces the product (rather than purchases it) and that the production process results in replenishment at a rate of 40,000 units per year during the portion of the inventory cycle when replenishment occurs. As before, yearly usage is 10,000 units, the yearly cost per dollar of inventory is \$0.20, investment per unit is \$8, and it costs \$50 to place each order. The cost terms are calculated in Table 8-2 and plotted in Figure 8-6. As can be seen from these presentations, the optimum for the finite replenishment case is about 900 units per order.

To obtain an exact optimum, we can follow the same procedure as for the infinite replenishment rate situation by taking the derivative of the total

Table 8-2. Costs for Finite Replenishment Rate Example Problem.

Initial Data:

Yearly Usage (S)	10,000 units/yr.
Yearly Cost per Dollar of Inventory (C)	\$.20/yr.
Investment per Unit of Inventory (P)	\$8
Fixed Cost of Placing an Order (F)	\$50/order
Replacement Rate (R)	40,000 units/yr.

Order Quantity Q	*Avg. Inv. in Units* $Q[1 - (S/R)]/2$	*Holding Cost* $CPQ[1 - (S/R)]/2$	*Orders per Year* S/Q	*Ordering Cost* $F(S/Q)$	*Total Cost* $CPQ[1 - (S/R)]/2 + F(S/Q)$
100	38	\$ 60	100	\$5,000	\$5,060
200	75	\$120	50	\$2,500	\$2,620
300	113	\$180	33	\$1,667	\$1,847
400	150	\$240	25	\$1,250	\$1,490
500	188	\$300	20	\$1,000	\$1,300
600	225	\$360	17	\$ 833	\$1,193
700	263	\$420	14	\$ 714	\$1,134
800	300	\$480	13	\$ 625	\$1,105
900	338	\$540	11	\$ 556	\$1,096
1,000	375	\$600	10	\$ 500	\$1,100
1,100	413	\$660	9	\$ 455	\$1,115
1,200	450	\$720	8	\$ 417	\$1,137

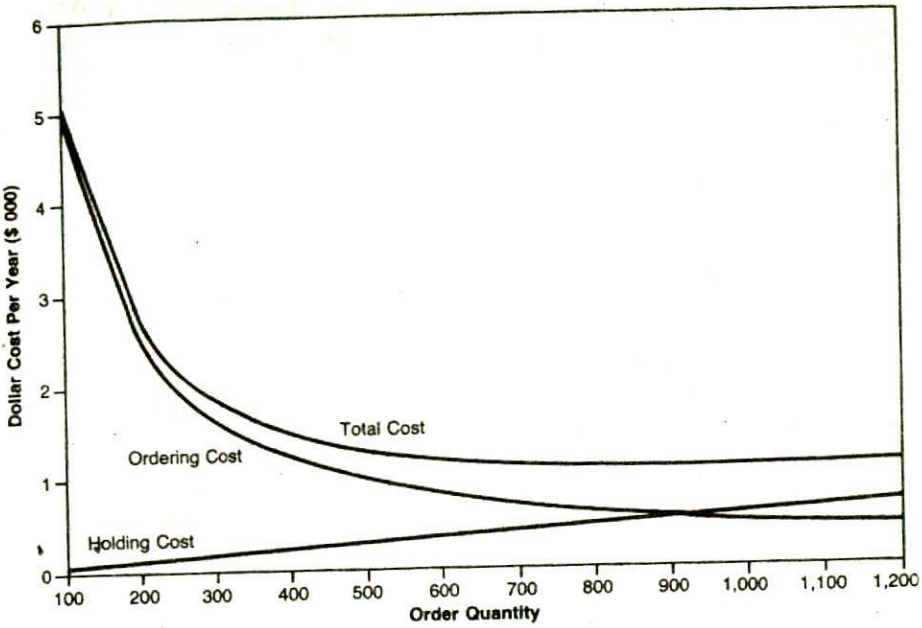


Figure 8-6. Finite R *Problem Data from Table 8-2.*

cost function with respect to Q, setting this derivative equal to zero, and solving for the optimal Q:

$$\text{Total cost} = TC = CPQ[1 - (S/R)]/2 + F(S/Q) \tag{8-18}$$

$$dTC/dQ = CP[1 - (S/R)]/2 - F(S/Q^2) \tag{8-19}$$

$$0 = CP[1 - (S/R)]/2 - F(S/Q^2) \tag{8-20}$$

$$Q^* = (2FS/CP[1 - (S/R)])^{0.5} \tag{8-21}$$

where Q^* is again the optimum order quantity.[9] Substituting the figures for the problem into equation (8-21):

$$Q^* = (2FS/CP[1 - (S/R)])^{0.5} \tag{8-21}$$

$$= [2(50)\ (10{,}000)/(0.20)(8)(1 - (10{,}000/40{,}000))]^{0.5}$$

$$= 912.87 \tag{8-22}$$

[9]Note that formula (8-21) reduces to formula (8-12) as the replenishment rate tends toward infinity:

$$\lim_{R \to \text{Inf.}} (2FS/CP[1-(S/R)])^{0.5} = (2FS/CP)^{0.5}$$

since, as $R \to$ Infinity, $1 - (S/R) \to 1$.

so the optimal order quantity is 913 units per order. This will result in 10.95 order per year: for a 360-day year, orders will be placed about every 33 days. The order point, for a seven-day lead time from the time when the firm decides to produce the material until production is started, is 194 units. The total cost of holding and ordering the inventory will be:

$$\text{Total cost} = CPQ[1 - (S/R)]/2 + F(S/Q) \tag{8-18}$$

$$= 0.20(8)913(1 - (10{,}000/40{,}000))/2 + 50(10{,}000/913)$$

$$= \$1{,}095 \tag{8-23}$$

Quantity Discounts The prior model deals with cases where the replenishment rate is finite. Let us return to the basic infinite replenishment rate situation but now model the circumstance where the cost of acquiring inventory varies with the amount acquired. The most common circumstance that would cause this is the existence of quantity discounts. In this case, there is an additional cost that varies with the firm's ordering strategy and that must be included in the model: the cost of materials. Equation (8-4) gives this cost as:

$$\text{Cost} = P_q S \tag{8-4}$$

Here, the P variable (which represents the investment in each unit of inventory) has a q subscript since it varies with order quantity. In situations where there are no quantity discounts, the cost of materials does not vary with order quantity, and this term need not appear in the total cost equation. But with quantity discounts, the firm's ordering strategy affects the total cost of materials, and this effect must be incorporated in the decision model. Adding the cost of materials term to equation (8-9), the total cost function for the infinite replenishment case becomes:

$$\text{Total Cost} = CP_q(Q/2) + F(S/Q) + P_q S \tag{8-24}$$

To find the optimum order quantity, it would be useful if we could take the derivative of equation (8-24) with respect to Q, set this equal to zero, and solve for Q^* as before. Unfortunately, the quantity discount policies of most selling firms (the manner in which P_q varies with Q) are not such that P_q is a continuous function of Q. One alternative approach to the problem is to evaluate the total cost function of various levels of Q via a spreadsheet. Let us develop an inventory policy for an example problem in this way.

Most of the parameters of the problem are as before. The replenishment rate is infinite, the fixed cost of ordering (F) is \$50, the carrying cost parameter (C) is 20 percent per year, and yearly usage (S) is 10,000 units. However,

the product is to be purchased from a vendor who has established a pricing schedule that includes quantity discounts. If the buyer places an order for less than 500 units of the product, the price is $8.25 per unit. For order quantities between 500 and 1,499, the price is $8 per unit. If the firm's order is for more than 1,500 units, the price is $7.75 per unit. We know that if the price is $8 per unit, the optimal order quantity is 791 units. However, this analysis did not allow for the quantity discounts. The minimum order quantity to obtain the $7.75 price is about twice the EOQ with no quantity discounts. What is the optimal ordering strategy?

A spreadsheet evaluating equation (8-24) for various levels of Q is presented as Table 8-3. The data on yearly usage, the holding cost parameter, and the fixed cost of ordering is entered in cells B6 through B8. Cells A15 through B17 contain a table detailing the relationship between the order quantity and the price per unit. Column A contains the *minimum* order quantity necessary to receive this price. For example, the buyer must order at least 500 units (cell A16) in order to receive the price of $8 per unit (cell B16).

The parts of equation (8-24) are calculated in rows 25 through 44. Order quantities from 100 through 2,000 are listed in column A. Column B contains the unit prices for these order quantities. To obtain these prices, the "LOOKUP" function contained in most spreadsheets is utilized. The LOOKUP function reports the value of a cell based on a table contained elsewhere in the spreadsheet. In this spreadsheet, the values in cells B25 through B44 depend on the order quantities and the quantity/price relationship in cells A15 through B17.[10]

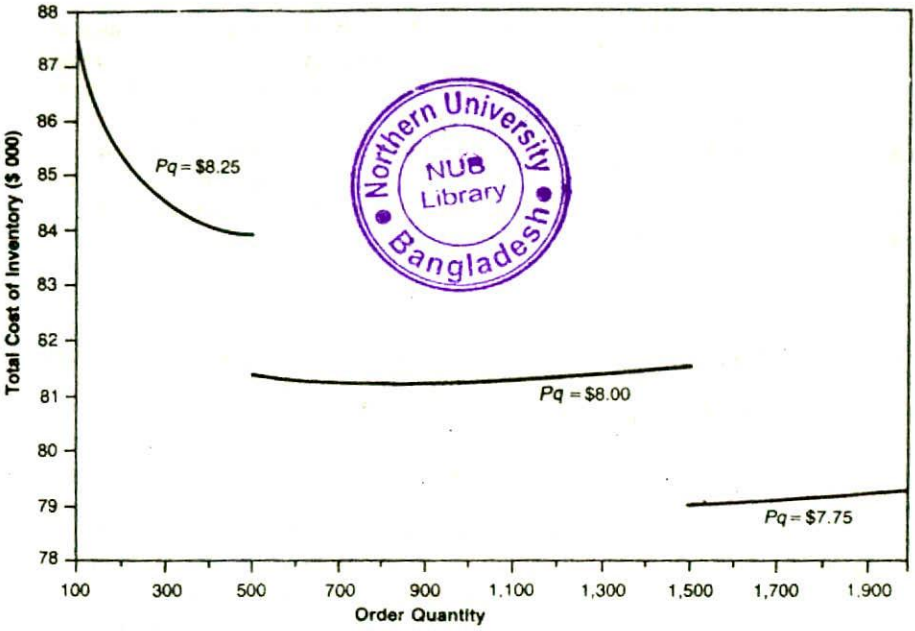
With the price data now available, the holding, ordering, and material costs can be calculated for each order quantity. Holding cost appears in column C; cell C25, for example, contains "B7*B25*(A25/2)". Ordering cost is calculated in column D; cell D25 contains "B8*(B6/A25)". The total cost of materials for the orders appears in column E, with cell E25 containing "A25*B6". These three costs are summed to obtain the total cost figures in column F.

These total cost figures are plotted in Figure 8-7. The total cost function is discontinuous because of the quantity discount pricing schedule. While, for the $8 price, total cost obtains a minimum for an order quantity of 791, placing an order sufficient to obtain the $7.75 price will reduce total cost still further. However, beyond the minimum necessary to get the price break, total cost again rises (because of the higher holding costs for these larger order quantities). Thus, if the seller offers the assumed quantity discount schedule, the

[10]LOOKUP functions vary substantially from program to program; some allow both vertical and horizontal references within tables while others do not. This particular spreadsheet was generated using a program which allows only horizontal lookups. Cell B25, for example, contains "@LOOKUP(A25,A15 . . . A18)".

Table 8-3. *Spreadsheet for Evaluation of EOQ Problem with Quantity Discounts.*

! A !!	B !!	C !!	D !!	E !!	F !
Initial Data:					
S	10000				
C	.20				
F	50				
Lookup Table for Price:					
Q	P_q				
0	8.25				
500	8.00				
1500	7.75				
O. Q.	Price	Hold. Cost	Ord. Cost	Mtl. Cost	Total Cost
Q	P_q	$CP_q(Q/2)$	$F(S/Q)$	P_qS	$CP_q(Q/2) + F(S/Q) + P_qS$
100	8.25	83	5000	82500	87583
200	8.25	165	2500	82500	85165
300	8.25	248	1667	82500	84414
400	8.25	330	1250	82500	84080
500	8.00	400	1000	80000	81400
600	8.00	480	833	80000	81313
700	8.00	560	714	80000	81274
800	8.00	640	625	80000	81265
900	8.00	720	556	80000	81276
1000	8.00	800	500	80000	81300
1100	8.00	880	455	80000	81335
1200	8.00	960	417	80000	81377
1300	8.00	1040	385	80000	81425
1400	8.00	1120	357	80000	81477
1500	7.75	1163	333	77500	78996
1600	7.75	1240	313	77500	79053
1700	7.75	1318	294	77500	79112
1800	7.75	1395	278	77500	79173
1900	7.75	1473	263	77500	79236
2000	7.75	1550	250	77500	79300



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Figure 8-7. Total Cost Versus Q with Quantity Discount.

optimum ordering strategy is to order 1,500 units, incurring higher holding costs but lower material and ordering costs.[11]

Nonlinear Decay Inventory situations will occur where one or more costs are a continuous, but nonlinear, function of order quantity. Since costs of this type are not directly proportional to Q, they cannot be accurately represented as part of the holding cost of inventory. As in the modeling of quantity discounts, the appropriate method is to add a term to the total cost equation representing this cost. Equation (8-2) gives the general form of this additional term:

$$\text{Cost} = f(\text{inventory level}) \tag{8-2}$$

Here, the functional notation indicates that there is a mathematical relationship between the cost and the level of inventory. The specific mathematical

[11]In this particular example, the minimum-cost order quantity occurs at an endpoint of a price range. Note that, if an optimum does not occur at a price range endpoint, but instead occurs for some value of Q between the price range endpoints, equation (8-12) could be used to compute the exact optimum order quantity for that price.

relationship will depend on the type of cost involved. For processes such as decay and obsolescence, the cost may be nonlinearly related to the *average time that a unit is in inventory*. In the EOQ model, the length of an inventory cycle is Q/S. For example, if the firm orders 250 items and yearly demand is 1,000, the inventory cycle will last one-fourth of a year. Since usage is linear, any particular unit will be in inventory an average of one-half this time, or $(1/2)\ (Q/S) = Q/2S$. The relationship between this average time in inventory and loss of value of inventory due to decay might be exponential:

$$\text{Cost} = Ge^{HQ/2S} \tag{8-25}$$

where G and H are parameters that depend on the characteristics of the decay or obsolescence process. Alternately, higher powers of the average time in inventory might be involved:

$$\text{Cost} = G_1(Q/2S)^2 + G_2(Q/2S)^3\ . \tag{8-26}$$

where the G_i $(i = 1, 2, \ldots)$ are the coefficients of the various terms of the polynomial. The ease of finding an optimum for a total cost function including such terms depends on the functional form of the relationship. The derivative of some functional forms is not easily taken. Further, for some functional forms the derivative of the total cost function is not easily solved for the optimum order quantity. When it is reasonable to take the necessary derivative and solve for the optimal order quantity, this approach will directly yield an optimization formula. However, for other functions, as long as the function relating decay costs to order quantity is well-behaved (always increases with increasing order quantity), there will be one global optimum. This optimum can be found using a spreadsheet approach.

Let us try an example problem. The replenishment rate is infinite and the fixed cost of placing an order, the holding cost of inventory, the price per unit, and the usage rate are \$50, 20 percent, \$8, and 10,000 respectively as before. However, inventory loses value over time because of decay. This decay is proportional to the square of the average time that a unit is in inventory. The formula for total inventory cost is then:

$$\text{Total Cost} = CP(Q/2) + F(S/Q) + G_1(Q/2S)^2 \tag{8-27}$$

The firm has researched this particular decay process and found that the appropriate coefficient of the decay term (the value of G_1) is 400,000 when the average time in inventory is expressed in years.[12] Let us find the optimum order quantity via the spreadsheet method.[13]

[12]The units of this coefficient are dollars per year squared. Such a coefficient would be estimated based on historical or theoretical data regarding the decay process.

[13]The derivative of equation (8-27) is easily taken but the derivative is not readily solved for Q^*, since it is a polynomial.

Table 8-4. *Spreadsheet for Evaluation of EOQ Problem with Decay.*

! A !!	B	!! C !!	D	!! E !!	F !
Initial Data:					
S	10000				
C	.20				
F	50				
P	8.00				
G_1	400000				
O. Q.	Hold. Cost	Ord. Cost	Avg. Time in Inv. (yrs)	Decay Cost	Total Cost
Q	CP(Q/2)	F(S/Q)	Q/2S	$G_1(Q/2S)^2$	CP(Q/2) + F(S/Q) + $G_1(Q/2S)^2$
By 100s					
100	80	5000	.005	10	5090
200	160	2500	.01	40	2700
300	240	1667	.015	90	1997
400	320	1250	.02	160	1730
500	400	1000	.025	250	1650
600	480	833	.03	360	1673
700	560	714	.035	490	1764
800	640	625	.04	640	1905
900	720	556	.045	810	2086
1000	800	500	.05	1000	2300
1100	880	455	.055	1210	2545
1200	960	417	.06	1440	2817
By 10s around 500					
450	360	1111	.0225	203	1674
460	368	1087	.023	212	1667
470	376	1064	.0235	221	1661
480	384	1042	.024	230	1656
490	392	1020	.0245	240	1653
500	400	1000	.025	250	1650
510	408	980	.0255	260	1648
520	416	962	.026	270	1648
530	424	943	.0265	281	1648
540	432	926	.027	292	1650
550	440	909	.0275	302	1652

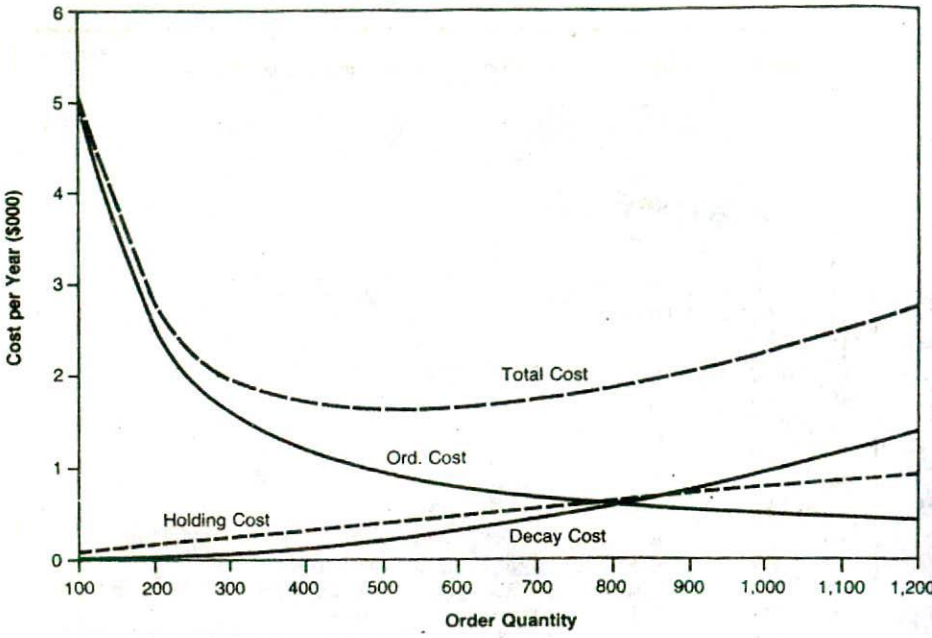


Figure 8-8. Costs in EOQ Model with Decay.

The necessary analysis is presented in Table 8-4 and graphed in Figure 8-8. The formulas for the holding cost and ordering cost in columns B and C are similar to those in Table 8-3 except that price does not vary among order quantities. The average time that a unit is in inventory is calculated in column D. Cell D19, for example, contains "A19/(2*B6)" since the order quantity for this row is in cell A19 and the usage rate is in cell B6. The decay cost is calculated in column E based on the G_1 coefficient in cell B10 and this average time; cell E19 contains "B10*(D19^2)". The total cost in column F is the sum of the holding, ordering, and decay costs.

In rows 19 through 30, the order quantity is iterated by 100 units to search for the approximate optimum. Among these order quantities, the lowest-cost outcome is found to be 500 units, with a yearly total cost of $1,650. In rows 35 through 45, the order quantity is increased by 10 per row starting at 450 units in order to find a more precise optimum. The order quantity of 520 units per order produces the lowest yearly cost (of $1,648 per year). Ordering 520 units is the firm's optimum strategy.

In this section, we have developed three additional models that extend the basic EOQ model to other inventory situations. However, given the diversity of inventory situations faced by firms, no complete listing of such models is possible. Instead, the intent has been to introduce the student, by example, to the principles of *how these models are developed*. In this way,

students may understand the models presented in other texts addressing the particular situations they face in their business experiences.

Thus far, the models presented have not addressed the risk aspects of inventory modeling. Instead, it was assumed that all variables were certain. Since many situations do not fit this assumption, some mechanism for addressing uncertainty is needed. The formulation of safety stock strategies to address this uncertainty and the possible stockout costs caused by it is discussed in the next section.

SAFETY STOCK STRATEGIES FOR ADDRESSING UNCERTAINTY

There are two major uncertain variables in inventory situations: the *demand for the goods* and the *lead time from the order to the arrival of the goods*. If neither of these variables is uncertain, the firm can plan perfectly: inventory will arrive just as it is needed, and the firm will never face an unplanned stockout of goods. However, when either or both of these variables is uncertain, there will be times when the firm will not have sufficient goods available, and will then incur stockout costs. To avoid these stockout costs in dynamic inventory situations, the firm may keep a permanent safety stock of inventory, trading off the holding costs of this safety stock against the stockout costs that would result from not maintaining it. The important question, of course, is *how much safety stock to hold*, given the firm's order quantity strategy. In the next few pages, models for the optimal level of safety stock are developed based on the trade-off between stockout costs and the holding costs of the safety stock. Models will be developed for three circumstances regarding uncertainty: (1) only demand is uncertain, (2) only lead time is uncertain, (3) both lead time and demand are uncertain. In these models, the following assumptions will be made:

1. The firm uses the EOQ/order-point system to generate order quantity strategies.
2. If the firm stocks out of the good, it incurs a one-time cash cost which is independent of the amount of the shortage.[14]
3. Holding costs of the safety stock are a linear function of the amount of safety stock held.
4. The probability distributions of the uncertain variables are normal.[15]

[14]This is the simplest possible assumption regarding stockout cost. A more realistic assumption is that the firm incurs a stockout cost that is proportional to the amount of the shortage. Unfortunately, the latter assumption considerably complicates model building; interested readers are referred to more comprehensive texts in inventory management.

[15]Some safety stock strategies for Poisson and discrete distributions can be found in J. Kallberg and K. Parkinson, *Current Asset Management* (New York: Wiley, 1984), pp. 217–26.

Uncertain Demand Levels Since, for most firms, sales are uncertain, so is their usage of finished goods, work-in-process, and raw materials. To assess the probability of stockout for a given level of safety stock, we need an estimate of the uncertainty in demand. If the firm is using the EOQ/order-point system, the relevant uncertainty of demand is the uncertainty from the time the order is placed until the order is received, that is, *uncertainty of demand during the lead time.* To see this, consider a firm that has highly uncertain demand but can obtain goods immediately as needed. Such a firm need never stock out; if demand is greater than expected, the firm can immediately obtain goods to satisfy this demand. However, when there is a *significant lead time* between order and acquisition of the goods, there may be stockouts. When demand during this lead time is larger than expected, a stockout will occur. For example, assume that the firm has calculated an optimum order quantity of 100 units and an order point of 15 units, based on a lead time of 10 days. During a particular inventory cycle, an order will be placed when inventory reaches 15 units. If, over the next 10 days (until the order is received) demand is greater than 15 units, then the firm will stock out. Thus, *it is the uncertainty of demand between order and delivery that is relevant in formulating safety stock policy.*

Let us present an example problem, and then look at its implications for the optimum safety stock. Recall that, in our basic EOQ example, the optimal ordering strategy was to order 791 units when the level of inventory reached an order point of 194 units, given a lead time of 7 days and a yearly demand of 10,000 units. There were 12.64 inventory cycles per year. The price was $8 per unit and the carrying cost of inventory was 20 percent per year. Assume that the yearly demand is uncertain, with a coefficient of variation of 0.10. Assume also that the lead time is certain and that the cost of a stockout is $100. How much safety stock should the firm keep?

Let A be the level of the safety stock in units. The yearly holding cost of this safety stock will then be CPA. Let the probability of stockout during an inventory cycle based on this level of safety stock be X_a. The expected cost per cycle of stocking out will then be X_aK, where K is the cost of the stockout. Since there are S/Q^* cycles per year, the expected stockout cost will be $X_aK(S/Q^*)$, and the cost associated with the safety stock will be:

$$\text{Total Safety Stock Cost} = CPA + X_aK(S/Q^*) \qquad (8\text{-}28)$$

Equation (8-28) is evaluated for the example problem in Table 8-5; these data are plotted in Figure 8-9. Note here that since the coefficient of variation is 0.10, the standard deviation of demand during the lead time is (0.10) (194) = 19.4. In this table, the calculations are straightforward with the exception of the calculation of the probability of stockout. Let us review this calculation for a safety stock of 10 units. The expected usage during the lead time is 194 units. If the firm carries 10 units of safety stock, actual demand must be greater than 204 units (194 plus 10) in order for a stockout to occur. The Z-

Table 8-5. *Evaluation of Safety Stock Strategies for Example with Demand Uncertainty Only.*

Initial Data:

Carrying Cost per Year (C)	\$.2
Stockout Cost (K)	\$100
Price per Unit (P)	\$8
Standard Deviation of Demand	19.4
Yearly Usage (S)	10,000 units/yr.
Economic Order Quantity (Q^*)	791 units

Level of Safety Stock A	*Holding Cost* CPA	*Z-Score for Stockout*	*Probability of Stockout* X_a	*Expected Yearly Stockout Cost* $X_a K(S/Q^*)$	*Total Cost* $CPA + X_a K(S/Q^*)$
0	\$ 0	0.00	0.5	\$632	\$632
10	\$ 16	0.52	0.3015	\$381	\$397
20	\$ 32	1.03	0.1515	\$192	\$224
30	\$ 48	1.55	0.06057	\$ 77	\$125
40	\$ 64	2.06	0.01970	\$ 25	\$ 89
50	\$ 80	2.58	0.00494	\$ 6	\$ 86
60	\$ 96	3.09	0.001001	\$ 1	\$ 97
70	\$112	3.61	0.0006119	\$ 1	\$113
80	\$128	4.12	0.00008222	\$ 0	\$128
90	\$144	4.64	0.00000843	\$ 0	\$144

score for this outcome is $(204 - 194)/19.4 = 0.52$. From a table of the cumulative normal distribution, the probability that an outcome over 0.52 standard deviation above the mean will occur is 0.3015. Thus, the probability of stockout during each cycle for a safety stock of 10 units is 0.3015.

From this table and figure, the minimum-cost strategy is to keep about 50 units of safety stock, which results in a yearly total cost of \$86. The optimum can be found more directly by taking the derivative of equation (8-28), setting this equal to zero, and solving for the optimum A. The derivative of equation (8-28) is:

$$dTC/dA = CP + (dX_a/dA)K(S/Q^*) \tag{8-29}$$

A closed-form solution to this equation requires an expression for dX_a/dA. For the normal distribution, this expression is:[16]

$$dX_a/dA = -\frac{1}{SD_d\sqrt{2\pi}}\, e^{-(1/2SD_d^2)A^2} \tag{8-30}$$

where SD_d is the standard deviation of demand during the lead time. Sub-

[16]See W. Peters and G. Summers, *Statistical Analysis for Business Decisions* (Englewood Cliffs, N.J.: Prentice Hall, 1968), p. 83, or any good statistics text.

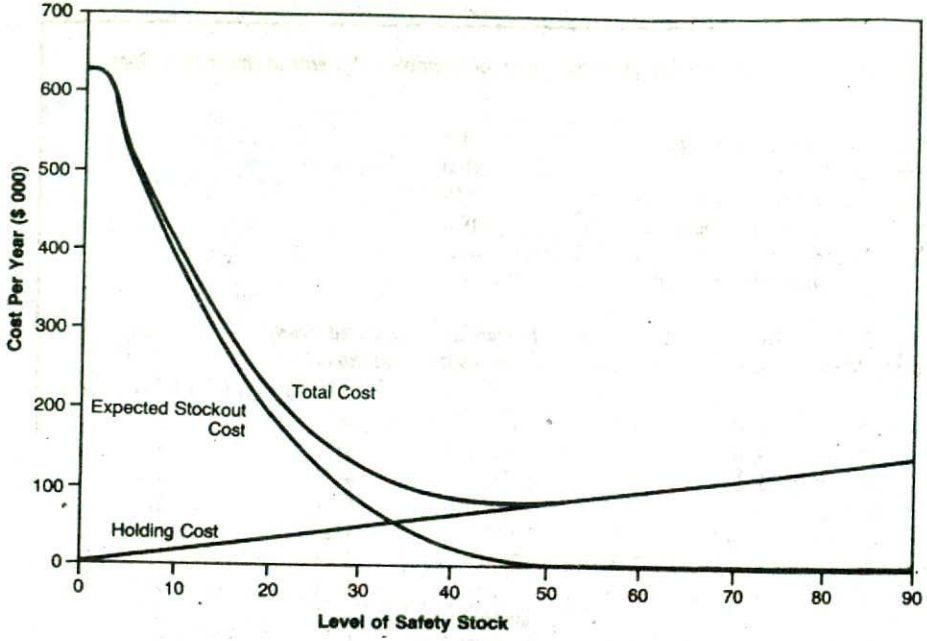


Figure 8-9. Data for Safety Stock Example Problem.

stituting this into equation (8-29), setting this equal to zero, and simplifying, we obtain:

$$A^* = [-2SD_d^2 ln(CPQ^*SD_d\sqrt{2\pi}/SK)]^{0.5} \tag{8-31}$$

Inserting the data from the example problem into this equation, the optimum safety stock can be calculated:

$$\begin{aligned} A^* &= [-2(19.4)^2 \ln\{0.2(8)791(19.4)[2(3.14159)]^{.5}/10{,}000(100)\}]^{.5} \\ &= [-752.72 \ln\{61{,}544.32/1{,}000{,}000\}]^{.5} = 45.81 \end{aligned} \tag{8-32}$$

So the optimal strategy is to carry a safety stock of 46 units. Since the expected usage during the lead time is 194 units, the firm should place an order for 791 units when the inventory level reaches 240 units. This order point reflects an expected usage of 194 units and a safety stock of 46 units. This level of safety stock results in a Z-score of 46/19.41 = 2.37 and a stockout probability of 0.008894 for each inventory cycle. Substituting these figures into equation (8-28) gives the yearly cost of the optimal safety stock strategy:

$$\text{Total Safety Stock Cost} = CPA + X_a K(S/Q^*) \tag{8-28}$$

$$= 0.20(8)46 + 0.008894(100)(10{,}000/791)$$

$$= \$74 + \$11 = \$85 \tag{8-33}$$

Uncertain Lead Times Firms may also face situations where demand is certain but the *lead time between order and the arrival of the inventory is uncertain*. Uncertainty lead times often occur when the firm orders materials from a supplier. In such a case, if the firm kept no safety stock, 50 percent of the time the order would be received after the firm had stocked out, and stockout costs would have been incurred.

The appropriate model for determining the optimum safety stock with uncertain lead time is essentially the same as that where only demand is uncertain. To see this, recall that in this model the measure of uncertainty was the *standard deviation of demand from the order point to the receipt of the goods*. Variation in lead time produces uncertainty in demand *over this period* in the same way that a simple variation in demand would produce this uncertainty. For example, suppose that a firm's demand for a particular good was 5 units per day for certain, but the lead time from order to delivery was 10 days with a standard deviation of 2 days. This is the same as saying that, *between the placing of an order and its receipt*, the expected demand is 50 units with a standard deviation of 10 units (2 days times 5 units per day). If SD_1 is the standard distribution of demand during the time from order to receipt due to uncertainty regarding the lead time, formula (8-31) still gives the optimum order quantity, but SD_1 is substituted for SD_d.

Uncertain Demand and Uncertain Lead Times There are many circumstances where both demand and lead time are uncertain. In such cases, *both uncertainties* must be taken into account in formulating safety stock strategy. Here, the probability distributions of demand and lead time uncertainty must be combined to obtain an estimate of the total uncertainty during the time between the placing of the order and its receipt. The formula for the standard deviation of two combined probability distributions is:

$$SD_c = (SD_a^2 + SD_b^2 + 2SD_aSD_b\text{COR}_{ab})^{0.5} \qquad (8\text{-}34)$$

where SD_c is the standard deviation of the combined distributions, SD_a is the standard deviation of the first distribution, SD_b is the standard deviation of the second distribution, and COR_{ab} is the correlation coefficient between the two distributions. If both of the probability distributions are normal, the resulting probability distribution will be normal. Further, if the correlation coefficient between the distributions of demand and lead time is zero, then the standard deviation of the combined distributions is simply:

$$SD_c = (SD_a^2 + SD_b^2)^{0.5} \qquad (8\text{-}35)$$

To illustrate the appropriate safety stock levels with uncertainty in both demand and lead time, return to our example problem where demand was uncertain over the seven-day lead time, with a standard deviation of 19.4 units. Let us also assume that lead time is uncertain, with a standard deviation

of 1.2 days, and that the two uncertainties are uncorrelated. Since demand is 10,000/360 = 27.78 units per day, the standard deviation of the lead time translates into a standard deviation in usage of 33.33 units (1.2 times 27.78). The standard deviation of the combined probability distributions of lead time and demand uncertainty is:

$$SD_c = (SD_d^2 + SD_l^2)^{0.5} \tag{8-34}$$

$$= (19.4^2 + 33.33^2)^{0.5} = 38.56 \tag{8-36}$$

This figure can then be used in equation (8-31) to obtain the optimal safety stock:

$$A^* = [-2SD_c^2 \ln(CPQ^* SD_c \sqrt{2\pi}/SK)]^{0.5} \tag{8-31}$$

$$= [-2(38.56)^2 \ln\{0.2(8)791(38.56)\ [2(3.14159)]^{.5}/10{,}000(100)\}]^{.5}$$

$$= [-2{,}973.75 \ln(122{,}327.26/1{,}000{,}000)]^{0.5} = 79.04 \tag{8-37}$$

so the optimum strategy is to carry 79 units of safety stock. This strategy will minimize the total costs to the firm, where these costs are composed of the expected stockout costs (due to uncertainty regarding demand and lead time) and the carrying costs of the safety stock.

SUMMARY

Firms make costly investments in inventory to address problems they face. These problems include variability in the price of goods over time, the need for flexibility in production processes, and the need for ready availability of goods to sell. Holding raw materials, work-in-process, and finished goods inventories will address these problems, but so will other strategies. For example, the problem hedging variability in the price of future purchases of raw materials can be addressed via futures contracts (for materials on which these contracts are available). Similarly, the just-in-time system can provide materials to the firm's production processes without the necessity of extensive investments in inventories.

In this chapter and the following one, we discuss optimal strategies for holding and ordering inventory for firms that choose to use the inventory approach to address some of their problems. We discuss strategies only for single-item inventory problems; those interested in more advanced inventory management topics are referred to the extensive literature in the area. The development of mathematical models for these single-item inventory situations requires that these models describe the physical circumstances regarding the purchasing and holding of these items. The great diversity of these physical

situations leads to the great diversity of inventory models that appear in the literature.

In this chapter, the basic EOQ model was first developed. This model describes situations where the replenishment rate is infinite, and there are only two types of relevant costs in the inventory decision: ordering costs and holding costs. Other inventory models were then developed to address situations where the replenishment rate was finite, where there were quantity discounts on materials purchased, and where there were decay costs. These models were developed to illustrate the process of model building in inventory situations. Like all inventory models, these optimization routines are limited to situations described by the assumptions under which they are derived.

In the "certainty" approach to inventory management, an ordering and holding strategy is developed assuming that the levels estimated for each of the parameters in the inventory situation are certain. A separate safety stock strategy is then developed to address the uncertainty inherent in these estimates. This strategy trades off the holding costs of the permanent safety stock with the expected stockout costs inherent in that level of safety stock. Under restrictive assumptions, models were developed to generate optimal safety stock levels where demand and/or lead time were uncertain.

While this two-phase approach to the generation of inventory strategies is tractable, it is primarily oriented toward dynamic inventory situations, where a permanent safety stock can be used as a hedge against uncertainty since this stock does not become worthless after the initial period. "Uncertainty" approaches can also address such dynamic inventory situations, but are of most use in static inventory problems. Uncertainty approaches to inventory problems are the primary topic of the next chapter.

Problems

8-1. A firm is trying to decide on an ordering strategy for a particular item. The item can be delivered in unlimited quantities; unloading times are inconsequential. The firm uses 3,000 units of the item per year. Each item has a purchase price of $3. The firm has calculated that storage and insurance costs of inventory are 12 percent per year. The firm's cost of capital is 10 percent per year. Out-of-pocket paperwork costs associated with generating a purchase order total $10 per order. Costs of writing a check for an order are $5. Unloading costs are $12, regardless of the amount of the order. In general, the items ordered are expected to arrive 7 days after the order is placed. Using a 360-day year, calculate the optimum order quantity, the order point, and the total cost of the ordering strategy assuming that the firm does not keep a safety stock.

8-2. A firm is trying to decide how much of a particular liquid should be ordered. The liquid is kept in a storage tank; there are separate inflow and outflow pipes for the tank. The firm uses 500 gallons of liquid per day. When an order is placed with the supplier of the liquid, the truck arrives 7 days later to pump liquid into the tank. Because of the small size of the input pipe, only 2,500 gallons per day can be pumped into the tank. The fixed costs of placing an order are $500 per order. Carrying costs are 20 percent per year, and each gallon of liquid costs $10. The firm keeps a safety stock of 1,000 gallons of liquid on hand at all times. Use a 360-day year.

a. Calculate the economic order quantity and the order point.

b. Assume that the maximum capacity of the tank is 10,000 gallons. For your ordering strategy, will the tank overflow? Explain why or why not.

8-3. You are manager of the Manero Paint Store, a large paint outlet. One big-demand item is Color Number 107 (Passion Fire) latex paint in one-gallon containers. Manero Paint sells 10,000 containers of this paint per year. Demand is normally distributed with a standard deviation of 2,000 units. Passion Fire costs $7.50 per container. The lead time from order to delivery is 10 days with a standard deviation of 3 days. Holding costs are 16 percent per year and the fixed costs of placing an order are $80. If Manero Paint stocks out of Passion Fire, it incurs $500 in stockout costs. The replenishment rate is infinite. Use a 360-day year.

a. Calculate the optimum order quantity.

b. Calculate the optimum safety stock.

c. Calculate the cost of this inventory strategy per year.

8-4. A firm uses 7,500 units per year of a particular item. Holding costs are 15 percent per year and the fixed costs of ordering are $15 per order. The replenishment rate is infinite. If the firm orders in quantities of 1,000 units or less, the item costs $6 per unit. If the firm orders in quantities of greater than 1,000 units, the item costs $5.95 per unit. Find the optimum ordering strategy by evaluating the total costs (including material costs) of ordering strategies between 250 and 1200 units per order.

Selected Readings

Austin, L., "Project EOQ: A Success Story in Implementing Academic Research," *Interfaces*, Vol. 7, No. 4 (August 1977).

Kallberg, J., and K. Parkinson, *Current Asset Management* (New York: John Wiley, 1984), Chapter 6.

Kim, Y. H., G. C. Philippatos, and K. H. Chung, "Evaluating Investment in Inventory: A Net Present Value Framework," *Engineering Economist*, Vol. 31, No. 2 (Winter 1986), pp. 119–35.

Love, S., *Inventory Control* (New York: McGraw-Hill, 1979), Chapters 1, 2, and 4.

McDaniel, W. R., "The Economic Ordering Quantity and Wealth Maximization," *Financial Review*, Vol. 21, No. 2 (November 1986), pp. 527–36.

Snyder, A., "Principles of Inventory Management," *Financial Executive*, Vol. 32, No. 4 (April 1964), pp. 13–21.

case

Koss TBA[1]
Inventory Management

In April of 1985, Dan Olson (the manager of retail sales for Koss TBA, Inc.) had asked the operations research staff of the firm to develop an inventory management system for the company's chain of TBA stores. TBA, in the automotive business, stands for "tires, batteries, and accessories." TBA stores cater to the do-it-yourself trade in auto maintenance. In the early 1970s, a trend had developed in automobile maintenance. Due to the high cost of labor, automobile owners had started to do their own routine maintenance chores rather than pay a service station to do the work. Included among the routine chores were changing oil and oil filters, replacing windshield wipers, and changing spark plugs. (Another manifestation of this do-it-yourself attitude was the self-service gas station). A growing portion of the market for oil, oil filters, and so forth was therefore the car owner rather than the service station operator. To service this demand, enterprising firms had opened chains of retail stores selling auto parts to the general public. Koss TBA was a chain of such stores. Koss purchased TBA items from suppliers, warehoused them in central locations, and distributed them from the warehouse network to their retail stores.

Mr. Olson did not consider the existing inventory system at the retail stores to be very effective. In this system, managers were given the authority to place orders from the warehouses at their discretion. This worked fairly well when the store manager had considerable experience, since their "feel" for demand levels was sufficient for effective inventory control. However, less experienced managers frequently had considerable trouble and often kept either too much inventory or too little. Too much inventory resulted in excessive carrying costs, but too little resulted in overly frequent orders and stockouts.

Once an ordering system was developed, implementation at the store level would be fairly simple. When customers paid for their purchases, checkout personnel at the firm's electronic cash registers input the item numbers. The current price for each item was then automatically drawn from the store's computer system; checkout personnel did not input prices. The computer system also kept

[1]This case has been adapted from Iqbal Mathur and Frederick C. Scherr, *Cases in Financial Management* (New York: Macmillan, 1979), pp. 56–61.

running records of the amount of inventory for all parts, updating this based on sales (via the register checkouts) and parts received. The accuracy of these computer records was maintained via periodic physical checks of inventory level (these checks are, of course, known as "taking inventory"). It would be a fairly easy programming task to have the computer system print daily lists of items on which inventory levels had fallen below critical cutoffs. These lists of merchandise would then be ordered from the warehouses and sent to the stores via one of the firm's trucks. The problem for the operations research staff was to set the cutoffs.

Any inventory system involves trade-offs among different types of costs. For TBA items, at the retail level, these costs were of three types: ordering costs, carrying costs, and stockout costs. Ordering costs have to do with the costs of placing and receiving the ordered quantity of the item. In Koss's system, these costs consisted of the incremental costs of adding a quantity of another item to the daily delivery which was made to each store. Compared to ordering from an outside source, these costs were relatively small. Since there was a daily delivery to each store regardless of what items were ordered, there were no additional shipping costs. Since there was no check to be written to an outside vendor, there were no check generation costs. The only additional costs, the operations research staff reasoned, would relate to any additional computer and paperwork expenses necessary to process the order, plus the additional labor necessary to unload the items from the delivery truck. They estimated the computer and paperwork expenses at $5 per order. While the personnel necessary to unload such items were already employed at the store, the unloading of trucks diverted their time from more profitable endeavors, such as servicing customers. Thus, the cost of the unloading crew was not a sunk cost. The bulkier the item, the longer it took to unload each order of the item, and the higher were unloading costs.

To test the feasibility of the new inventory system, the operations research staff decided to estimate ordering and other costs and to formulate an inventory strategy for six sample items for a sample store: a particular brand of GR-78/14 tires, F22 batteries, 323 filters, 16-inch windshield wiper blades, 5-inch lamps, and 6C tuneup kits. These items varied in size, yearly demand, uncertainty of demand, bulk, unloading cost, and out-of-pocket investment. Estimates of some of these parameters for the six sample items were developed by the operations research department based on sampling and on some calculations. A summary is presented in Exhibit 1.

The costs of carrying inventory for the retail stores consisted of taxes, insurance, and cost of tied-up capital. Taxes occurred because some states in which the stores were located levied inventory taxes on the value of the inventory stored. For the sample store, taxes were 4 percent per year of inventory investment. Insurance was 3 percent per year of investment. The firm's cost of capital was 10 percent.

Stockout costs occurred when the store did not have inventory on hand to service a customer. In such a case, the profit on the sale was lost. However, the firm's true loss was more than this. The TBA business was very competitive; several other firms in areas where Koss stores operated could provide essentially equivalent auto parts. A prime selling feature of TBA stores was the ability to service demand with a minimum of inconvenience. When a customer tried to purchase an item and did not find it in stock, the customer was far less likely to return for future purchases, and most TBA customers needed auto parts and accessories several times per year. The operations research staff decided to model

Exhibit 1

Koss TBA
Statistics on Six Items Sold

Part	Expected Yearly Demand (Units)	Investment Per Item	Standard Deviation of Yearly Demand	Unloading Cost Per Order
GR-78/14 Tires	757	$30	298	$30
F22 Batteries	252	$30	78	$30
323 Filters	720	$ 5	150	$10
16-Inch Wiper Blades	360	$ 2	100	$10
5-Inch Lamps	108	$ 2	20	$10
6C Tuneup Kits	720	$15	168	$10

the stockout process by assigning a $50 stockout cost if the store was unable to service demand.

The operations research staff decided to use a two-step strategy for addressing the costs of inventory. To address the ordering and carrying costs associated with the normal inflow and outflow of material from the stores, they would assume that the demand for the item was certain, and they would develop an ordering strategy using standard economic order quantity methods. The uncertainty of this demand would be addressed via the use of a safety stock, the level of which would depend on the holding costs of this stock and the penalties for stockout. The store's computer would be programmed to place an order for the order quantity in such a way as to have the order arrive just as the store's inventory for the item was expected to reach the safety stock level. This order would then be scheduled for a truck making deliveries to the store. Given the firm's delivery system, lead times from order to delivery were five days for all items. Orders could be received later or earlier in the day, de-

Exhibit 2

Koss TBA
Monthly Sales Data for GR-78/14 Tires

Month Number	Sales in Units	Month Number	Sales in Units
1	84	21	55
2	36	22	101
3	62	23	90
4	30	24	20
5	5	25	72
6	70	26	55
7	54	27	65
8	24	28	87
9	65	29	91
10	77	30	36
11	39	31	74
12	81	32	20
13	66	33	90
14	83	34	86
15	44	35	87
16	86	36	57
17	42	37	61
18	40	38	99
19	70	39	40
20	91	40	88

pending on the amounts delivered to the other stores on the truck's delivery route. For this reason, the lead time was not certain, but had a standard deviation of 0.20 days.

The determination of the safety stock for each item posed one additional problem. To formulate a strategy based on the trade-off between the holding costs of a level of safety stock and the expected stockout costs that would result from that level, it was necessary to assess the probability distribution of demand. The operations research staff suspected, for several reasons, that demand and lead time were normally distributed, but sound and careful business practice required that this suspicion be checked. In collecting the data on demand presented in Exhibit 1, the staff had gathered monthly sales data for the last 40 months for each of the six products. These data for the GR-78/14 tires are presented in Exhibit 2.[2]

[2]The operations research staff also checked this data for seasonality. Note that, while this is monthly data, if these data are normally distributed, the probability distribution of sales is also normal for any longer or shorter period.

Chapter -9

Inventory Management Uncertainty Approaches

In Chapter 8, we discussed the "certainty" approach to inventory management. In that approach, a strategy for inventory management was developed in two parts. First, a minimum-cost ordering strategy was formulated using the expected parameters of the particular inventory situation. This involved the computation of an economic order quantity based on expected demand, carrying costs, ordering costs, and so forth. Second, a separate hedging strategy was developed to handle deviations from these expectations. This involved the computation of an optimum safety stock, the level of which minimized the sum of the holding costs and the stockout costs associated with this safety stock.

The "uncertainty" methods discussed in this chapter are an alternate approach to inventory management problems. We label as "uncertainty methods" those methods that incorporate the riskiness of the situation, in addition to the firm's expectations, as a primary part of the analysis procedures. In this chapter, we consider uncertainty methods for both static and dynamic inventory problems. However, "static" problems (in which no carryover of inventory is allowed from one inventory cycle to the next) receive the major portion of our attention. We address static problems using three methods: (1) the Black-Scholes option pricing model; (2) risk-return analysis using the expected net present value of the cash flows and the coefficient of variation of this net present value; and (3) net present value using risk-adjusted discount rates. For simplicity, in our analysis of static problems, risk is assumed to occur only because of uncertainty regarding demand levels.

The second type of inventory problem allows carryover of inventory from one cycle to the next. This is called the "dynamic" inventory problem. We address this problem via simulation analysis, with the relative merits of various strategies assessed via their expected net present values and the risk of these net present values. Since simulation can incorporate the variability

in any number of parameters without undue difficulty, our analysis of dynamic inventory problems allows for uncertainty in both lead times and demand levels. The chapter concludes with sections on the monitoring of inventory and on the management of inventory in practice.

THE STATIC INVENTORY PROBLEM

The static inventory problem is the simplest of all inventory problems. One and only one order is placed; there is no variation in ordering costs from one strategy to another. The question is how much of the good in question to order. Cash flows may include those related to initial purchase, overstocking, stockout costs, liquidation of excess inventory, and so forth. This type of problem occurs when the product is perishable in a physical or stylistic sense. Fashion or fad items, seasonal items, and other items with one and only one selling period (such as calendars) are typical items of this sort.

To enable the student to get a feel for such inventory problems and the methods by which they may be addressed, it is useful to introduce an example problem.[1] A nontaxable firm is deciding how many Christmas trees to purchase. The order must be placed in early October and prepaid. These trees are to be resold to the public during late December (90 days after ordering). Each tree costs $12. If there is demand for a particular tree, it will be sold for $20. Overstocked trees will sell for $2. The firm has estimated a discrete distribution of potential demand levels; it is expected that there will be a 20 percent chance that demand will be 100, 125, 150, 175, or 200 trees.[2] There are no repeat customers and thus no stockout costs of unfilled demand. How many trees should the firm order?

The simplest way, though not the best way, to address this problem is to calculate the expected profits from ordering strategies of 100 trees, 125 trees, and so forth. The profit from each strategy-outcome pair is presented in Table 9-1. For example, the entry of $950 for an ordering strategy of 175 trees and an outcome of 150 trees (cell D8) is computed as $-175(12)$ plus

[1]This problem is abstracted from J. D. Stowe and A. K. Gehr, Jr., "An Arbitrage Solution to the Christmas Tree Problem," a paper presented at the Financial Management Association Meeting in October, 1984, and available from authors at the University of Missouri at Columbia.

[2]In this problem and throughout this chapter, we have used discrete distributions of uncertain variables because we have found that managers are more comfortable estimating ranges and probabilities of these variables than estimating means and standard deviations. However, when historical data is used to estimate probability distributions, continuous distributions may be more appropriate. For a treatment of the static inventory problem where demand is distributed as a truncated normal distribution, see Y. H. Kim and K. H. Chung, "Inventory Management: A Financial Theory of the Firm Under Uncertainty," available from Kim at the University of Cincinnati.

Table 9-1. Expected Profits for Example Problem

	A	B	C	D	E	F	G
1	Demand	100	125	150	175	200	
2	Prob.	.2	.2	.2	.2	.2	
3							
4	O. Q.		PROFIT				E(Prof)
5	100	800	800	800	800	800	800
6	125	550	1000	1000	1000	1000	910
7	150	300	750	1200	1200	1200	930
8	175	50	500	950	1400	1400	860
9	200	-200	250	700	1150	1600	700

20(150) plus 2(25) equals \$950. All the entries above the diagonal running from the upper left to the lower right of the profits matrix are situations where quantity demanded exceeds that supplied (that is, the firm stocks out). Entries on this diagonal represent situations where just enough is ordered to satisfy demand. For entries below the diagonal, the firm has ordered more than is sold. Cell entries in the profits matrix take the form of "if statements." In this example problem, if quantity demanded is less than quantity supplied for a particular cell, the spreadsheet returns \$8 (the margin per unit sold) times quantity demanded minus \$10 per unit on overages (the loss per extra unit). If quantity demanded is greater than quantity supplied, the spreadsheet returns \$8 times quantity supplied. Cell D8 thus contains "@IF (D1 <= A8, D1*8, (D1*8) - ((A8-D1)*10))". The expected profits are calculated by multiplying the probabilities times the outcomes for each strategy; for example, the expected profit of \$930 for the strategy of ordering 150 trees is 0.2(300) + 0.2(750) + (0.2 + 0.2 + 0.2) (1,200). To perform this operation on the spreadsheet, the contents of the probabilities row is multiplied by the contents of the outcomes row and these contributions summed. Thus, cell G7 contains "(B2*B7) + (C2*C7) + (D2*D7) + (E2*E7) + (F2*F7)". Since the strategy of ordering 150 trees produces the highest expected profit, this strategy would be preferred under this simple methodology.

This methodology is unsatisfactory from a financial analysis standpoint for two reasons. First, it does not take into account the time value of money; all the cash flows are lumped together without discounting. Since many static inventory problems involve cash flows that are spread over substantial periods, ignoring time value is a serious problem with this methodology. Second, this method does not address risk. For example, it is clear in our sample problem that the 150-trees order strategy is considerably more risky than the 125-trees order strategy in the sense that the variation in outcomes is more substantial. A method is needed to account for time value and risk. Three such methods are described in the following sections.

USING THE OPTION PRICING MODEL

There are interesting parallels between the put options that are available on some common stocks and the purchase of inventory in static inventory situations. A put is a contract to sell the stock at a predetermined price, which can be exercised or not at the holder's option.[3] The value of this contract depends on the future market price of the stock, which is uncertain. European put options (which can be exercised only on their maturity date) derive their value from the chance that the market price of the stock will be higher than the exercise price of the option on the maturity date. Similarly, in static inventory problems the value of a unit of inventory depends on the uncertain level of demand at the time of sale. The acquisition of a unit of inventory can be thought of as *an option to sell that unit, the value of which will depend on the eventual demand level.*

Because of these parallel circumstances, an option pricing approach has appeal as an analysis method in static inventory situations. One possible approach involves the application of the Black-Scholes option pricing model (OPM).[4] The OPM is a method of determining the current value of an option where the future payoffs from the option depend on the level of an exogenous variable at the option's maturity. In the OPM, the current value of the option depends on (1) the relationship between the current level of the exogenous variable and the exercise level of the option, (2) the variability of the exogenous variable, (3) the time remaining until the option's maturity, and (4) the risk-free rate of interest.

The relationships between these variables and the value of the put option are almost all obvious. The value of the put option increases directly with the probability that the level of the exogenous variable will be above the exercise price of the option at the option's maturity (and thus the probability that the holder of the option will be able to exercise it at a gain). This probability (and thus the value of the put option) is larger when:

1. The current price of the exogenous variable is higher relative to the exercise price. The higher the current price of the exogenous variable

[3]The basics of options contracts were described in Chapter 3.

[4]A complete presentation of the OPM is beyond the scope of this text. See F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* (May–June 1973), pp. 637–54. In this article, the option pricing model is derived using the advanced mathematical technique of stochastic calculus. For a derivation of the OPM without the use of this technique, see J. R. Garven, "A Pedagogic Note on the Derivation of the Black-Scholes Option Pricing Formula," *Financial Review*, Vol. 2, No. 2 (May 1986), pp. 337–44. A good overview of the OPM is presented in T. Copeland and J. F. Weston, *Financial Theory and Corporate Policy*, Second Edition (Reading, Mass.: Addison-Wesley, 1983), pp. 230–84.

is, the more likely that this price will be above the exercise price at maturity.

2. The variability of the price of the exogenous variable is greater. The greater this variability is, the greater is the spread of the probability distribution of the price of the exogenous variable at maturity. The greater the spread of this probability distribution, the greater is the probability that the price of the exogenous variable at maturity will be higher than the exercise price.
3. The time to maturity is longer. The longer is the time to maturity, the greater the chance that the value of the exogenous variable will rise above the exercise price.

By more obtuse reasoning, the value of the option in the OPM will increase with the risk-free rate of interest.

The primary figure to be calculated in the option pricing model is the "d_2 statistic" and its cumulative normal value. The cumulative normal value of the d_2 statistic can be thought of as the probability that the level of the exogenous variable will be less than the exercise level of the option. The formula for the d_2 statistic is:

$$d_2 = -\{[ln(P_0/Y) + (r + (1/2)SD^2)t]/SD(t^{.5}) - SD(t^{.5})\} \qquad (9\text{-}1)$$

where P_0 = the current level of the exogenous variable
Y = the exercise level for the option on the exogenous variable
r = the yearly risk-free rate of interest
SD = the instantaneous standard deviation of the exogenous variable
t = the number of years to maturity of the option

Once the d_2 statistic is calculated, its cumulative normal value is obtained, and this cumulative normal value is discounted to the present to obtain the value today of an option that pays $1 at the maturity date if and only if the level of the exogenous variable is less than the exercise level for the option. The formula (using continuous discounting) is:

$$P(Y) = e^{-rt}[N(d_2(Y))] \qquad (9\text{-}2)$$

where $P(Y)$ is the value of the option with an exercise price of Y and $N(d_2(Y))$ is the cumulative normal value of the d_2 statistic for this Y.

In applying the option-pricing model to static inventory problems, it is useful to value an option that pays off if the value of the exogenous variable falls in a particular *range* (rather than merely falling below a particular level). In such a case, the $P(Y)$ statistics are first computed for the upper and lower cutoffs of the range and their normal values obtained. The following formula

is then applied to obtain the value of an option which can be exercised only if the level of the exogenous variable falls within the range:

$$P(Y_1,Y_2) = e^{-rt}[N(d_2(Y_2)) - N(d_2(Y_1))] \tag{9-3}$$

where $P(Y_1,Y_2)$ is the value of the option, $N(d_2(Y_2))$ is the normal value of the d_2 statistic for the *upper limit* of the range, and $N(d_2(Y_1))$ is the normal value of the d_2 statistic for the *lower limit* of the range.

In option pricing, the exogenous variable can be almost any uncertain variable, but it is usually the market price of a traded asset; this asset is then referred to as the "comparison asset." The comparison asset is often an issue of common stock. The firm's own common stock or the stock of any other firm not expected to pay a dividend over the period can be used.

The OPM and the formulas associated with it may seem daunting, but their application to the static inventory problem is quite straightforward. Estimates or actual data on four parameters are needed:

1. The risk-free rate of interest.
2. The current price of the comparison asset.
3. The instantaneous variance of the price of the comparison asset.[5]
4. The future price ranges of the comparison asset for the various levels of demand.

Let us illustrate the use of the OPM per our example Christmas tree problem. Assume that the firm's common stock is to be used as the comparison asset and that the current price of this stock is \$42. The risk-free rate is 8 percent per year, and the instantaneous standard deviation (square root of the variance) of the price of the firm's stock is 20 percent. The firm expects that 100 trees will be demanded only if economic conditions near Christmas are very poor. In that case, the firm's stock price might be expected to decline from its present level to less than \$36. Similarly, it has estimated that if its stock price falls in the range of \$36 to \$40, demand will be 125 trees; if the price falls in the range of \$40 to \$46, demand will be 150 trees; if the price falls in the range of \$46 to \$52, demand will be 175 trees; and if price is greater than \$52, demand will be 200 trees. With this information in hand, we can now execute the option pricing model's formulas and derive an optimum strategy.

In the example problem, P_0 is \$42, the first comparison stock range is

[5]See H. Latane and R. J. Rendleman, Jr., "Standard Deviations of Stock Price Ratios Implied in Option Prices," *Journal of Finance* (May 1976), pp. 369–82, and M. Parkinson, "Option Pricing: The American Put," *The Journal of Business* (January 1977), pp. 21–36, for methods of obtaining estimates of the instantaneous variance of the price of common stock.

that below \$36, and the inventory cycle is 1/4 year. The d_2 statistic for the price of the comparison stock being less than \$36 is:

$$d_2 = -\{[ln(42/36) + (0.08 + (1/2)0.2^2)0.25]/0.2(0.25^{.5}) - 0.2(0.25^{.5})\}$$
$$= -1.69 \qquad (9\text{-}4)$$

The normal value of this statistic, from a cumulative normal distribution table, is 0.04551. This is the probability that the stock price for the comparison stock will be less than \$36. The second calculation needed in the option pricing model is to discount this probability back to the present. For the "below \$36" range, the lower limit of the range (Y_1) is minus infinity (with a probability of zero), and the upper limit is \$36 (with a probability of 0.04551). The $P(Y_1,Y_2)$ statistic for this range is:

$$P(Y_1,Y_2) = e^{-0.08(0.25)}\,(0.04551 - 0) = 0.04461 \qquad (9\text{-}5)$$

This calculated result can be regarded as the value of an option which pays \$1 if the price of the comparison stock falls below \$36 and nothing otherwise. Calculation of this value for the other ranges of the comparison stock's future price is parallel. The d_2 statistics are first computed for the two values of Y, and the difference between the normal values of these statistics ($N(d_2(Y_2))$ minus $N(d_2(Y_1))$) is then discounted via equation (9-3).[6] Let us examine the next stock price range, which extends from \$36 to \$40. We already know the value of d_2 and the normal value for a Y of \$36 (the d_2 value is -1.69, and the cumulative normal value for this d_2 statistic is 0.04551). The d_2 statistic for a Y of \$40 is -0.64, and its normal value is 0.2611. The difference in cumulative normal values is 0.2611 minus 0.04551 or 0.21559. Using equation (9-3), the value of an option that pays \$1 if the price of the comparison stock is between \$36 and \$40 is \$0.21132.

Since these calculations are quite burdensome, it makes sense to set up the entire procedure on a spreadsheet, particularly if more than a very few ranges of the comparison stock are to be used. The results of the analysis for the sample problem are presented in Table 9-2. The possible demand levels and comparison stock upper limits are presented in lines 1 and 2. Additional needed data are in cells C3 through C6. These cells are referenced in the computation of the d_2 statistics and the option values in lines 8 through 13; the formulas for the d_2 statistics, option values, and so forth, are contained in the appropriate cells in these rows.

Once the option values (line 13) are calculated, these are applied to the future cash flows to assess the present values of these flows. For example, the value of an option that pays \$1 if the comparison stock price is between

[6]See D. T. Breeden and R. H. Litzenberger, "Prices of State-Contingent Claims Implicit in Option Prices," *Journal of Business* (October 1978), pp. 621–51.

Table 9-2. Net Present Values for Example Problem From Option Pricing Model.

	A	B	C	D	E	F	G	H
1	Demand		100	125	150	175	200	
2	Comp. Stock Upper		36	40	46	52		
3	Comparison Stk. Var.		.04					
4	Yearly Risk-Free Rt.		.08					
5	Period Length		.25					
6	Init. Comp. Stk.		42					
7								
8	d2(Y1)		− Inf.	−1.69	− .64	.76	1.99	
9	N(d2(Y1))		0.00	.04551	.2611	.7764	.9767	
10	d2(Y2)		−1.69	− .64	.76	1.99	+ Inf.	
11	N(d2(Y2))		.04551	.2611	.7764	.9767	1.00	
12	N(d2(Y1))−N(d2(Y2))		.04551	.21559	.5153	.2003	.0233	
13	P(Y1,Y2)		.04461	.21132	.50510	.19633	.02284	
14								
15	O. Q.	Int. Flow		Future Cash Flows				NPV
16	100	−1200	2000	2000	2000	2000	2000	760.40
17	125	−1500	2050	2500	2500	2500	2500	930.42
18	150	−1800	2100	2550	3000	3000	3000	1003.4
19	175	−2100	2150	2600	3050	3500	3500	852.99
20	200	−2400	2200	2650	3100	3550	4000	612.28

$36 and $40 is $0.21132. If we order 100 trees and this state occurs, the futur net cash flow will be $2,000. The present value of this potential future cash flo is thus 0.21132 times $2,000 or $422.64. This is the contribution to the net presen value of this ordering strategy from this potential future cash flow. Summin these contributions for all the potential cash flows and including the initia expense gives the total net present value of the strategy as computed via th option pricing approach. We see from the net present value column (column H that the maximum net present value occurs for the 150-trees ordering strategy based on the option pricing model, this strategy should be selected.

While the option pricing model provides an interesting and well-base approach to the one-period inventory problem, its application is not witho difficulty. First, it requires a particular input that is fairly difficult to estimate the set of price ranges of the comparison stock that are associated with eac level of demand. Second, while this approach accounts for the time value c money, the option pricing model is risk-neutral; it does not price risk. In th option pricing model, risk in the option is not relevant because this risk ca be hedged completely via an appropriate investment position in the compa ison asset. While it is possible for the firm to make the necessary transactio so as to hedge an investment in an inventory position with an investment i the comparison asset, such strategies may be costly. Other uncertainty mode of static inventory decisions provide more direct methods of dealing wit risk.

USING THE EXPECTED NPV AND ITS COEFFICIENT OF VARIATION

Another approach to time value and risk in the static inventory problem is to employ a two-phase strategy in which time value is addressed first and then risk is assessed. The time value of money is properly addressed if the net present value technique is used to calculate the worth of cash flows. Since risk is to be addressed separately, discounting need only reflect time value, and the risk-free rate is the appropriate discount rate. In this technique, the expected net present value of each strategy is calculated first. Once these expected net present values are computed, one possible measure of their relative risk is the coefficient of variation of each net present value. A risk-return decision can then be made among strategies, based on their expected net present values and their dispersions of net present value. Let us execute this technique for the example problem.

In this problem, there is a three-month time lag between the initial cash outflow and the final cash inflow. Assuming an 8 percent yearly risk-free rate gives a discount factor of $1/1.02 = 0.9804$ for this interval. Multiplying this factor times the future cash flows for each strategy-outcome pair and incorporating the initial expense gives the result presented in Table 9-3. For example, cell D8 has a value of \$890.20, which is a net present value. For this cell, the initial cash outflow is $175(12) = \$2{,}100$. The future cash inflow is $20(150) + 2(175 - 150) = \$3{,}050$. The value of this future flow discounted at 2 percent is \$2,990.20, so the net present value is \$890.20. The cell contains "@IF (D1 ⇐ A8, −(D1*12) + (D1*20)/1.02, −(D1*12) + ((D1*20) − ((A8−D1)*2)))/1.02". Columns H and I contain the standard deviation and coefficient of variation of these net present values for each order quantity strategy. Examining these, it is clear that strategies involving order quantities above 150 trees are ruled out on dominance grounds; they have *lower* expected net present values and *higher* risk (in a coefficient of variation sense) than do other alternatives. It is *not* clear, however, whether the strategy of ordering

Table 9-3. Expected Net Present Value and Coefficient of Variation of Net Present Value for Example Problem.

	A	B	C	D	E	F	G	H	I
1	Demand	100	125	150	175	200			
2	Prob.	.2	.2	.2	.2	.2			
3									
4	O. Q.			NPV			E(NPV)	SD(NPV)	COV(NPV)
5	100	760.78	760.78	760.78	760.78	760.78	760.78	0	0
6	125	509.80	950.98	950.98	950.98	950.98	862.75	176.4706	.20454545
7	150	258.82	700	1141.2	1141.2	1141.2	876.47	352.9412	.40268456
8	175	7.8431	449.02	890.20	1331.4	1331.4	801.96	514.4958	.64154728
9	200	−243.1	198.04	639.22	1080.4	1521.6	639.22	623.9177	.97606764

150 trees is superior to that of ordering 125 trees when risk is considered. The expected net present value of the 150-tree order strategy is $13.72 higher ($876.47 − $862.75), *but this strategy is also much riskier*; the coefficient of variation for this order strategy is about 0.40 versus 0.20 for the 125-tree order quantity strategy.

It should be obvious that while this approach is easy to understand and allows us to rule out some strategies and to assess the risk of others, the procedure could still be improved upon in at least one respect: it does not lead to a definitive recommendation in terms of strategy. Once risk and expected net present value are assessed and dominated strategies eliminated, the risk-return trade-off decision must still be made among the remaining alternatives. In the example problem, the manager still has to decide among the 100-, 125-, and 150-tree ordering strategies, although his or her decision is now a more informed one.

NPV WITH RISK-ADJUSTED DISCOUNT RATES

One method that leads to a definitive recommendation on strategy is to use risk-adjusted discount rates to incorporate the differential risk among the expected future cash flows from the various strategies. In this technique, the risk of future cash flows is assessed first and then addressed in the discounting process simultaneously with the time value of money. The discount rate does double duty: it reflects the time value of money and also the risk of the cash flows.

In using risk-adjusted discount rates (RAD), an important decision involves the choice of an RAD that properly reflects the amount of risk in the uncertain cash flows. If the firm uses the market risk concept and can estimate the beta of the cash flows from the decision, this estimate could form the basis for using the CAPM to obtain a risk-adjusted rate.[7] However, let us assume for example purposes that the firm uses the benchmark method, comparing the risk of the cash flows from the various inventory strategies with the risk of other cash flows and selecting a RAD accordingly. To make such an assignment of rates, the firm might assess the risk of the uncertain future cash flows by calculating their standard deviation and then use this measure of variability to assign a RAD.[8] Let us illustrate this procedure for the example problem.

The standard deviations of the future cash flows for the various ordering

[7]Kim and Chung, "Inventory Management," use this technique.

[8]Note the difference from the technique presented in the prior section. In that procedure, the NPV was computed first (to account for time value) and the coefficient of variation of this NPV formed the basis for second-phase risk assessment. However, if RADs are used, the risk adjustment is concurrent with discounting, and risk must be assessed before discounting is done.

Table 9-4. Risk-Adjusted Net Present Values for Example Problem.

	A	B	C	D	E	F	G	H	I	J	K
1		Demand	100	125	150	175	200				
2		Prob.	.2	.2	.2	.2	.2				
3											
4	O. Q.	Int. Flow	Future Cash Flows					E(FCF)	SD(FCF)	RAD	NPV(RAD)
5	100	-1200	2000	2000	2000	2000	2000	2000	0	.02	760.78
6	125	-1500	2050	2500	2500	2500	2500	2410	175.442	.03	839.81
7	150	-1800	2100	2550	3000	3000	3000	2730	339.146	.04	823.00
8	175	-2100	2150	2600	3050	3500	3500	2960	465.918	.05	719.05
9	200	-2400	2200	2650	3100	3550	4000	3100	492.950	.055	538.89

strategies are presented in column I of Table 9-4. Risk-adjusted discount rates are then assigned. (Note that the rates shown in the table are for the 90-day discounting period of the example problem, not yearly rates.) These appear in column J. The order quantity of 100 trees has a certain return, so its future cash flow is discounted at the risk-free rate. Various premiums have been added to this rate to reflect the relative riskiness of the other strategies. Risk-adjusted net present values are then computed using these rates (for example, cell K6 contains "B6 + (H6/(1+J6))"). From this analysis, it would seem that the 125-tree ordering strategy (not the 150-tree strategy) is superior, since it has the highest risk-adjusted net present value (column K).

This risk-adjusted discount approach is similar to standard capital budgeting procedures. It allows for the time value of money and prices risk. The problem, as in many situations where risk-adjusted discount rates are used, is the selection of the appropriate rates in line with the risk of each alternative strategy. The betas of demand for individual products may be difficult to estimate. If the benchmark method is used, the assignment of RADs may require substantial judgment, even though the analyst has reference discount rates from which to make adjustments.

THE DYNAMIC INVENTORY PROBLEM

In dynamic inventory problems, just as in static inventory problems, different ordering strategies produce divergent patterns of future cash flows. The major difference between the static and dynamic problems is that for dynamic problems, the cash flows occur over longer periods of time (since inventory can be carried from one period to another) and are, therefore, more complex. One uncertainty technique that can be applied to dynamic inventory problems is to discount future cash flows to obtain their NPVs, then assess risk via the coefficients of variation of these net present values.[9]

[9]Recall that we applied exactly this technique to the static inventory problem in the second prior section.

However, because the static inventory problem involves uncertainty in several future periods, simulation analysis is required to assess the risk of the various ordering strategies.[10]

While simulation is a more complex analysis technique, there are some advantages to its application. Recall that simulation analysis enables the management of the firm to assess the level of risk of a particular strategy by generating a mathematical model of the system under consideration and the random processes that affect it. These random factors are then allowed to vary, and the effects on important outcome variables are observed. In the inventory situation, a major random factor is sales volume, and the previous discussion of static problems centered on addressing the effects of sales fluctuations. However, within a simulation, the effects of other uncertain variables (such as delivery time, interest rate, and so forth) can be easily introduced.

Rather than continue the discussion in the abstract, let us introduce a sample dynamic inventory problem and address it via simulation analysis.[11] An appliance retailer is trying to formulate a strategy for managing the inventory of a particular major appliance (a dishwasher, clothes dryer, or similar item). His strategy variables are the number of appliances ordered (the order quantity) and the level of inventory at which an order is placed (the order point). Each appliance has an out-of-pocket cost of $300 and sells for $400. It costs $10 to place an order (no matter how many units are ordered). Orders can be placed only once a month. If a customer goes to the store to make a purchase and the appliance is out of stock, the retailer bears a $20 stockout cost per unit stocked out, representing the present value at the time of the stockout of lost future sales to the customer. All sales are for cash. Terms of sale from the appliance supplier to the retailer are net 30 days from the date of delivery, and the retailer pays these bills on the net date. Sales are not seasonal, and the retailer wants to formulate a policy to be in effect for the next 12 months.[12] The yearly risk-free rate of interest (used as the discount rate in simulation analysis) is 8 percent. Five appliances are currently in stock.

In this problem, assume that both the monthly demand and the delivery time from the month the order is placed are uncertain. The firm has collected demand and delivery time data from the last 50 months; these are presented in Table 9-5. Assuming that the future is like the past, the observed frequencies can be interpreted as probabilities, and random numbers can be assigned to the frequencies to stimulate the probability distributions of demand and delivery time.

[10]Simulation analysis was discussed in general in Chapter 1. This analysis technique was used in Chapter 3 to assess the risk of future cash balances in cash forecasting.

[11]This example problem is adapted from Ali Mansour, "Monte Carlo Simulation," available from the author at West Virginia University.

[12]For simplicity we assign a zero value to any remaining inventory at the end of this time.

Table 9-5. *Historical Data on Demand and Delivery Times for Example Dynamic Inventory Problem.*

Demand/Month	*Frequency*	*Relative Frequency*	*Cumulative Frequency*
0	2	0.04	0.04
1	4	0.08	0.12
2	14	0.28	0.40
3	20	0.40	0.80
4	8	0.16	0.96
5	1	0.02	0.98
6	1	0.02	1.00
	50	1.00	

Delivery Time (Months)	*Frequency*	*Relative Frequency*	*Cumulative Frequency*
1	6	0.60	0.60
2	3	0.30	0.90
3	1	0.10	1.00
	10	1.00	

To obtain expected net present values and coefficients of variation for these net present values, the usual procedure would be to utilize a simulation package. However, simulation analysis of small problems can also be conducted on a spreadsheet. A spreadsheet simulation of this inventory situation, with a set of random uniform numbers to illustrate the procedure, is presented in Table 9-6. The initial data and test values for the strategy variables are presented in columns A and B. The rest of the spreadsheet draws on these for the values in the remaining columns. The demand data in cells A7 through B15 and the delivery time data (time from order to delivery) in cells A18 through B22 are used in conjunction with the spreadsheet's lookup feature to turn random uniform numbers into demand and lead time statistics with the appropriate historically based probability distributions. In this speadsheet, we use two-digit random uniform numbers between 0.00 to 0.99. Cutoff points in the lookup tables must be specified such that the proper probabilities occur for each possible outcome. To do this, the cutoff points have been generated by deducting 0.005 from the cumulative frequencies given in Table 9-5. Thus, for example, if the random number for demand is 0.00, 0.01, 0.02, or 0.03, the lookup feature will return a demand of 0, since all these numbers are less than the cutoff for the first range of 0.0395. These four possible outcomes, which produce this demand level, are 4 percent of the 100 possible random numbers between 0.00 and 0.99, so the proper historic probabilities are represented.

To see how this simulation works, let us examine the results in columns E through P in detail. The random uniform numbers to generate demand and delivery time are in cells E3 through P4. These could be input manually from a uniform random number table or generated internally if the spreadsheet has this facility. Reading down column E, the beginning inventory is 5. The demand uniform random number for the first month is 0.19 (cell E3), so demand for this month is 2 appliances (because 0.19 is greater than 0.115 but less than 0.395 on the demand lookup table). We assume that there are no previous orders yet to be delivered, so the cells representing deliveries for this month (E8 through E10) are zero. The ending inventory is thus three units. Since this is equal to or less than the reorder point (cell B27), an order for the order quantity (cell B26) is placed. That is, cell E13 contains an "if statement"—that if ending inventory (cell E11) is equal to or less than the reorder point, an order for the order quantity is placed. The delivery time random number is 0.20 (cell E4), so the delivery time assigned to this order is one month (via the lookup table for delivery time). This order then appears as an addition to inventory in the next month (cell F8). For the second month, the beginning inventory is three units, the demand is three units, and a delivery of five units is received. The ending inventory is thus five units, and no order is placed. The simulation continues on this basis for the remaining months. Note that in month three, a stockout occurs; demand is six units, but available inventory is only five units.

Sales in units are recorded in cells E16 through P16. These sales figures are the lesser of (1) demand, or (2) available inventory (beginning inventory plus deliveries) for each month. Using these sales and inventory transactions, cash flows are calculated in cells E18 through P23. Since all sales are for cash, cash receipts from this particular appliance are sales from the month (row 16) times the selling price per unit (cell B3). Terms are net 30 days, and so are calculated as the prior month's deliveries (rows 8 through 10) times the cost per item (cell B2). A stockout cost (cell B4) is included for each unit by which demand exceeds available inventory (row 20 thus contains a series of if statements referring to rows 6 through 10), and row 21 contains an ordering cost (cell B4) in each month during which an order is placed (the if statements in row 21 refer to the contents of the "orders placed" row, which is row 13). Looking at the cash flow for month three, for example, cash receipts are $2,000, representing the sale of five units at $400 per unit. Payments for purchases are $1,500, representing $300 per unit paid for the five units received in month two (which were ordered in month one). While sales in month three were five units, demand was six units, so a stockout charge of $20 is incurred ($20 per unit times the one unit stockout). The net cash flow for the month is therefore $470, equal to the $2,000 inflow minus the $1,530 in outflows. The net present value of $4,265.60 for this simulation run is presented in cell P25, using 1/12 of the yearly risk-free rate (cell B29) as a monthly discount factor.

Table 9-6. Spreadsheet Simulation of Example Dynamic Inventory Problem.

	A	B	C	D	E
1	Data:			Month:	1
2	Cost per Item		300		
3	Selling Price per Item		400	Demand Random Number	.19
4	Ordering Cost		10	Delivery Time Random Number	.2
5	Stockout Cost		20		
6				Beginning Inventory	5
7	Demand Data:			Demand	2
8		R. N.	Demand	Rec. From Fst. Prior Mo.	0
9		0	0	Rec. From Snd. Prior Mo.	0
10		.0395	1	Rec. From Trd. Prior Mo.	0
11		.115	2	Ending Inventory	3
12		.395	3		
13		.795	4	Orders Placed	5
14		.955	5	Delivery Time	1
15		.975	6		
16				Sales (units)	2
17					
18	Delivery Time Data:			Cash Receipts	800
19		R. N.	Lead Time	Payments for Pur.	0
20		0	1	Stockout Costs	0
21		.595	2	Ordering Cost	10
22		.895	3		
23				Cash Flows	790
24					
25	Strategy Variables:				
26	Order Quantity		5		
27	Reorder Point		3		
28					
29	Yearly Risk-Free Rate:		8		

The value of simulation analysis lies not in the results of any particular simulation run but in the results of numerous simulation runs, which together give estimates of both the expected values of outcome variables and their riskiness. In the context of this example dynamic inventory problem, the strategy variables are order quantity and order point, and the outcome variable is net present value. The retailer wants to know what combinations of order point and order quantity strategy will yield his preferred combination of risk and return. To assess the risk and expected return from various order point and order quantity strategies, the analyst generates a number of simulation runs for each order-point/order-quantity strategy. The resulting net present values are recorded and the mean and standard deviation of the net present value calculated for each strategy combination. The mean net present value and the estimated coefficients of variation of these net present values

F	G	H	I	J	K	L	M	N	O	P
2	3	4	5	6	7	8	9	10	11	12
.45	.99	.8	.15	.34	.1	.74	.02	.13	.73	.43
.95	.13	.14	.45	.61	.54	.67	.87	.39	.23	.36
3	5	0	1	4	2	1	8	8	6	3
3	6	4	2	2	1	3	0	2	3	3
5	0	5	5	0	0	5	0	0	0	5
0	0	0	0	0	0	5	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
5	0	1	4	2	1	8	8	6	3	5
0	5	5	0	5	5	0	0	0	5	0
3	1	1	1	2	1	2	2	1	1	1
3	5	4	2	2	1	3	0	2	3	3
1200	2000	1600	800	800	400	1200	0	800	1200	1200
0	1500	0	1500	1500	0	0	3000	0	0	0
0	20	0	0	0	0	0	0	0	0	0
0	10	10	0	10	10	0	0	0	10	0
1200	470	1590	−700	−710	390	1200	−3000	800	1190	1200

Net Present Value 4265.60

for combinations of order points from 1 to 7 and order quantities from 1 to 7 using the simulation model previously presented are given in Table 9-7. These sample statistics are estimates of the true population parameters; the more simulation runs are made for each possible strategy, the closer will these estimates converge on the actual parameters.

Assessing the risk-return trade-offs encompassed in a table of this sort seems foreboding at first, but fortunately a substantial number of the possible strategies can be eliminated on dominance grounds. However, in looking for dominated strategies, we must keep in mind that we are examining estimates of population parameters and not true population parameters. Thus, small differences among results can be sampling variation and not actual differences in expected risk and return.

A detailed examination of Table 9-7 gives some very interesting infor-

Table 9-7. *Results of Simulation Analysis of Example Dynamic Inventory Problem.*

		Order Quantity						
		1	*2*	*3*	*4*	*5*	*6*	*7*
Order Point	*1*	1371 (0.06)	1368 (0.05)	1671 (0.24)	3017 (0.23)	3570 (0.22)	3107 (0.30)	3171 (0.18)
	2	1333 (0.05)	1348 (0.06)	1737 (0.30)	3128 (0.17)	3948 (0.20)	3676 (0.26)	3039 (0.25)
	3	1363 (0.05)	1341 (0.04)	1853 (0.30)	3214 (0.22)	3826 (0.24)	3604 (0.32)	2978 (0.36)
	4	1342 (0.05)	1363 (0.05)	1730 (0.21)	3158 (0.28)	3697 (0.23)	4028 (0.22)	3208 (0.36)
	5	1342 (0.05)	1365 (0.04)	1800 (0.26)	3397 (0.23)	3351 (0.37)	3735 (0.27)	2731 (0.61)
	6	1367 (0.04)	1345 (0.05)	1781 (0.25)	2979 (0.27)	3373 (0.31)	3724 (0.23)	2685 (0.43)
	7	1348 (0.04)	1300 (0.05)	1684 (0.18)	2867 (0.25)	3085 (0.40)	3272 (0.30)	2017 (0.85)

Estimated means, standard deviations, and coefficients of variation are based on 25 simulation runs per order-point/order-quantity combination (a total of 960 runs). Numbers without parentheses are mean net present values for each set of 25 runs; numbers in parentheses below these are the associated estimated coefficients of variation.

mation. It can be calculated from Table 9-5 that the expected demand is 2.70 units per month. The problem specifies that only one order can be placed per month, and that the beginning stock is five units. As a result, for strategies involving order quantities of less than 3 the firm sells out in almost every month and incurs frequent stockout costs. This produces relatively low but safe profits, as can be observed in the entries in the columns representing order quantities of 1 and 2. The order point has little effect on these results; the retailer orders in virtually every period in most of the simulation runs for these order quantities.

Reading across the rows of the table, it would seem that the strategies using order quantities of 6 and 7 for order points of 1, 2, and 3 are less advantageous than lower order quantity strategies for these same order points. For example, looking at the row for a two-unit order point, the maximum mean profit occurs for an order quantity of 5 (at $3,948); higher order quantities for this order point produce lower and riskier profits. Similar logic allows us to discard order quantities of 7 for order points of 5, 6, and 7.

The patterns of expected return and risk are less clear when looking down the columns. It appears that order points of 5, 6, and 7 for a seven-

unit order quantity produce lower and riskier profits than do lower order points for this same order quantity. Similarly, order points of 7 seem to produce inferior expected results to lower order point strategies for order quantities of 4, 5, and 6.

Finally, a sizable increase in expected profitability comes from increasing an order quantity of 3 to an order quantity of 4 for all order points. This occurs with no estimated increase in risk. The average coefficient of variation across all order points for the three-unit order quantity is 0.25, while the average for the four-unit order quantity is 0.24; yet the respective average net present values are $1,751 and $3,109. It would thus seem that the three-appliance order strategy is less advantageous than the four-appliance order strategy.

Figure 9-1 presents the results of this examination of dominance among the simulated strategies. Managers representing very risk-averse shareholders would choose order quantity strategies of one or two units; for these strategies, order point is irrelevant. Less risk-averse shareholders would choose to undertake more risk in exchange for more expected present value. Such strategies are represented in the rightmost unshaded portion of the chart. Within this portion of the chart, the estimated coefficients of variation indicate that risk is fairly uniform among strategies. It would seem that the strategies involving either: (1) an order quantity of 5 and an order point of 2; (2) an order quantity of 5 and an order point of 3; and (3) an order quantity of 6

Figure 9-1. Dominated Strategies in Example Dynamic Inventory Problem.

Order Point	Order Quantity 1	2	3	4	5	6	7
1	1,371 (0.06)	1,368 (0.05)	1,671 (0.24)	3,017 (0.23)	3,570 (0.22)	3,107 (0.30)	2,171 (0.19)
2	1,333 (0.05)	1,348 (0.06)	1,737 (0.30)	3,128 (0.17)	3,948 (0.20)	3,878 (0.26)	3,036 (0.26)
3	1,363 (0.05)	1,341 (0.04)	1,853 (0.30)	3,214 (0.22)	3,826 (0.24)	3,604 (0.32)	2,979 (0.36)
4	1,342 (0.05)	1,363 (0.05)	1,730 (0.21)	3,158 (0.28)	3,697 (0.23)	4,028 (0.22)	3,818 (0.36)
5	1,342 (0.05)	1,365 (0.04)	1,600 (0.20)	3,397 (0.23)	3,351 (.037)	3,735 (0.27)	2,741 (0.41)
6	1,367 (0.04)	1,345 (0.05)	1,781 (0.25)	2,979 (0.27)	3,373 (0.31)	3,724 (0.23)	2,685 (0.43)
7	1,348 (0.04)	1,300 (0.05)	1,644 (0.18)	2,867 (0.25)	3,085 (0.40)	3,272 (0.30)	2,017 (0.36)

Data are from Table 9–7. The dominated strategies are indicated by shading. The remaining strategies are not dominated by others and will be preferred by some decisionmakers.

and an order point of 4 would be prime candidates for implementation, since these produce the highest (and nearly equivalent) mean net present values.

Even a small simulation analysis of this sort is complicated and expensive relative to the certainty approaches to the dynamic inventory problem. Also, as seen by our example, simulation analysis does not produce a definitive recommendation regarding strategy; the choice among nondominated risk-return combinations must be made by the firm's management. However, simulation analysis is very flexible in the probability distributions that it can address. In addressing risk in the certainty method, we generated optimal safety stock strategies by taking the derivative of the total cost expression for the safety stock. Included in this expression was the probability density function for uncertainty in usage. Obviously, there will be probability density functions that are discontinuous (as in our example) and others where obtaining or solving a mathematical expression for the derivative is very difficult. However, the simulation method can address these situations. This is a prime advantage of the simulation approach to dynamic inventory problems.

MONITORING INVENTORY BALANCES

As with accounts receivable balances, inventory balances turn over rapidly. Also, inventory is a major investment for many firms, particularly firms engaged in manufacturing, retailing, or wholesaling. For these reasons, the continuous monitoring of inventory investments is a necessary function in the management of working capital.

The monitoring of inventory entails several problems not encountered in the monitoring of accounts receivable. First, there is a *cost-of-information problem* in keeping track of the physical inventories of some goods. Second, because of the number of variables involved, *it is very difficult to develop an accurate measure of inventory turnover.* We will discuss these two problems in the following paragraphs. To add interest to the discussion, we will contrast the monitoring of inventory with that of accounts receivable, which was discussed in detail in Chapter 7.

The cost-of-information problem in the monitoring of inventory levels has to do with the nature of inventory and the many types of inventory that are held by firms. Records of accounts receivable are kept in electronic form; the cost of maintaining such records is not large. But the electronic records of inventory are only *representations of the actual physical items* that are the firm's inventory. Physically counting large numbers of items to update electronic records is extremely expensive because it is a labor-intensive process. Also, because of the great number of items kept by some firms and the costliness of the monitoring process, there is always a question about how much time and money should be spent on the monitoring of the levels of some inventory items. If the carrying costs and stockout costs associated with

holding some types of inventory are small, it makes sense to avoid spending substantial amounts to access the level of these types of inventory; the costs of monitoring outweigh the benefits.

To address these information cost problems, many firms use the *A-B-C System* of inventory management. In this system, types of inventory are given ratings of A, B, or C based on their cost and criticality to the firm. Items rated A are large-dollar investments or are critical to the operations of the firm. Items rated C are small-dollar items or items where immediate availability is not required. For example, in a hospital, oxygen supplies would be type A inventory, while pencils would be type C inventory. Items of middle rank in terms of investment and need are classified as type B inventory items. Depending on the type of firm, inventory rated A and B will encompass a relatively small proportion of the *number of items* in the firm's inventories, but will include the majority of the *dollar inventory investment* and of the items that are absolutely necessary in order for the firm to function. When using this system, the firm will keep very close track on the ordering and inventory levels of type A items, spend somewhat less effort in the management of type B items, and be fairly cavalier about inventories of type C items. In this way, information and management costs are minimized.[13]

The second problem in the monitoring of inventory is developing a procedure to accurately measure the behavior of inventory. Recall that this problem also occurred in the measurement of customer payment patterns of accounts receivable balances. In monitoring accounts receivable, changes in sales caused distortions in turnover-based measures of receivables' quality, such as days' sales outstanding. Measures such as sales-weighted days' sales outstanding were developed to correct for this problem. These measures eliminated the distortions caused by fluctuating sales levels. Unfortunately, the problem of distortion in turnover-based monitoring statistics is even more severe in the management of inventory. For inventory, distortions occur not only because of variations in *sales*, but also variations in *unit investment* in inventory, *usage rates* (unit turnover), and the *mix of different types* of inventory carried by the firm.

To see this, let us address an example problem. Using the A-B-C System of inventory management, a firm has decided to divide its inventory into three types: inventories of Product 1 and Product 2 (which are type-A inventory items) and all other inventories (which are type-C items). Based on expected costs of obtaining inventories, sales rates, inventory management procedures, and so forth, the firm has estimated that year-end inventory will be $40.25 million, and that aggregate dollar inventory turnover (cost of sales divided by inventory) will be 6.32. Sales turn out to be somewhat higher than expected,

[13]The A-B-C System is widely discussed in the literature on inventory management. For more on this system, see S. Love, *Inventory Control* (New York: McGraw Hill, 1979), pp. 244–47.

Table 9-8. Example Inventory Monitoring Problem.

Projected Inventory Balances

	Sales (Units)	*Unit Turnover*	*Investment per Unit*	*Cost of Goods Sold*	*Inventory (Units)*	*Inventory (Dollars)*
Product 1	40,000	8.00	2.50	100,000	5,000	12,500
Product 2	18,000	6.00	7.25	130,500	3,000	21,750
All Other Prdts.	48,000	4.00	.50	24,000	12,000	6,000
Totals				254,500	20,000	40,250
CGS/Inventory						6.32
Actual Inventory Balances						
Product 1	35,000	5.83	2.75	96,250	6,000	16,500
Product 2	25,000	7.14	6.75	168,750	3,500	23,625
All Other Prdts.	50,000	4.55	.60	30,000	11,000	6,600
Totals				295,000	20,500	46,725
CGS/Inventory						6.31

and inventory turnover is actually 6.31 (data are presented in Table 9-8). Is any review of inventory policies and levels needed, or should the firm be satisfied with this result?

While the actual dollar inventory turnover is very near the expected levels, even a cursory glance at the data shows that something may be happening that the firm did not expect. Some unit inventory turnovers (calculated based on units sold and held) and investments in the three types of inventory items were much more or less than expected; *it is only a coincidence that these differences balance out* in the aggregate dollar turnover ratio calculated for this firm.

To find out what has actually occurred, one useful technique is *variance analysis*. We also used this accounting-based procedure in connection with the monitoring of accounts receivable in Chapter 7. However, this procedure is even more useful in connection with the monitoring of inventory. Because of the difficulty in obtaining and interpreting a measure of inventory behavior that corrects for the biases introduced by changes in sales, price, and so forth, variance analysis is our primary means of monitoring inventory patterns. The variance analysis technique addresses changes in the *mix* of inventory by disaggregating the various types of inventory kept by the firm. Then, for each type of inventory, dollar variances due to the differences between actual and expected levels of *sales, unit turnover,* and *price* (where price is the investment in each unit of inventory) are computed to isolate the effects of each of these important variables on inventory investment.

The variance analysis technique for inventory monitoring is best illus-

trated by example. Table 9-9 presents a spreadsheet variance analysis of the data in Table 9-8. Let us examine this spreadsheet in detail.[14]

The computation of the three variances occurs in rows 4 through 40. In these rows, column B contains sales in units, column C contains unit turnover (based again on units sold and held), D contains investment (price) per unit of inventory, E contains inventory in units, F contains inventory in dollars (computed as inventory in units times investment), and G contains the variances for each type of inventory. Rows 4 through 13 contain the projected (budgeted) figures for each of these items; rows 33 through 40 contain the actual outcomes. The total variance is $6.475 million, computed as the actual inventory balance of $46.725 million minus the projected balance of $40.25 million. The intervening rows are used to calculate the portions of this variance resulting from sales, turnover, and price for each type of inventory held.

In rows 15 through 22, the actual sales volumes in units are used and the inventory investment computed assuming that the investment per unit and the turnover were as projected. Because of these variations in sales, inventory balances are lower than expected for Product 1 and higher than expected for the remaining products. When sales are greater or less than expected, changes in inventory investments are required, and this type of variance in inventory balances due to sales is to be anticipated.

The variances due to unit turnover are computed in rows 24 through 31 by using the actual ending inventories in units. These appear in column E. For example, actual sales on Product 1 were 35 million units and actual ending inventory in units was 6 million, so unit inventory turnover was 5.83 (cell C27) for this inventory item. We see in column G that there are large variances caused by the differences between projected and actual turnovers. All these differences should cause substantial concern regarding the inventory management procedures of the firm. If the firm keeps too much inventory (has too low a turnover), there is excessive investment. This has occurred for Product 1. However, if the firm keeps too little inventory, it risks stockouts. This has occurred for Product 2 and for the type-C inventory items.

The variance in inventory investment due to price is calculated in rows 33 through 40. There are substantial variances for each of the three inventory types. Product 1 costs more than expected to purchase or make, as did the type-C items. Product 2 costs less than expected to obtain.

A summary of the variances for the three products is presented in rows 42 through 50. Since the effect of sales is factored out, we can discuss the effects of turnover and price in isolation. The pattern of these two variances differs among the three types of inventory. Product 1 has a large turnover variance; turnover has slowed from expected levels. The firm may be carrying

[14]Additional discussion of variance analysis for the monitoring of inventory balances is provided in G. Gallinger and P. Healey, *Liquidity Analysis and Management* (Reading, Mass.: Addison-Wesley, 1987), pp. 417–42.

Table 9-9. Variance Analysis of Example Monitoring Problem via Spreadsheet

!	A	!!	B	!!	C	!!	D	!!	E	!!	F	!!	G	!
4	Projected Sales, Projected													
5	Turnover, and Projected Price													
6														
7			Sales				Inv./		Inventory		Inventory			
8			(Units)		Turnover		Unit		(Units)		(Dollars)			
9	Product 1		40000		8.00		2.50		5000		12500			
10	Product 2		18000		6.00		7.25		3000		21750			
11	All Other Prdts.		48000		4.00		.50		12000		6000			
12														
13	Totals								20000		40250			
14														
15	Actual Sales, Projected													
16	Turnover, and Projected Price													
17														Sales Variance
18	Product 1		35000		8.00		2.50		4375		10938			−1563
19	Product 2		25000		6.00		7.25		4167		30208			8458
20	All Other Prdts.		50000		4.00		.50		12500		6250			250
21														
22	Totals								21042		47396			7146
23														

24 Actual Sales, Actual						
25 Turnover, and Projected Price						Turnover Variance
26						
27 Product 1	35000	5.83	2.50	6000	15000	4063
28 Product 2	25000	7.14	7.25	3500	25375	−4833
29 All Other Prdts.	50000	4.55	.50	11000	5500	−750
30						
31 Totals				20500	45875	−1521
32						
33 Actual Sales, Actual						
34 Turnover, and Actual Price						Price Variance
35						
36 Product 1	35000	5.83	2.75	6000	16500	1500
37 Product 2	25000	7.14	6.75	3500	23625	−1750
38 All Other Prdts.	50000	4.55	.60	11000	6600	1100
39						
40 Totals				20500	46725	850
41						
42 Summary of Variances						
43						
44	Proj.	Sales	Turn.	Price	Actual	
45	Balance	Var.	Var.	Var.	Balance	
46 Product 1	12500	−1563	4063	1500	16500	
47 Product 2	21750	8458	−4833	−1750	23625	
48 All Other Prdts.	6000	250	−750	1100	6600	
49						
50 Totals	40250	7146	−1521	850	46725	

too much of this type of inventory. The firm has also paid more than expected for inventory of this product.

For Product 2, turnover was much faster than expected, resulting in inventory being $4.833 million lower than expected, given actual sales levels. Investigation is needed to assess whether this has resulted in excessive stock-outs for this type-A inventory item. The firm also paid less than expected for this inventory over the period of the analysis.

The turnover variance for the type-C inventory items is negative, indicating that these items turned over less rapidly than expected. This is of a good deal less concern than the turnovers of the type-A items, since (by their classification as C items) these inventories are not critical to the firm's operations. However, the price variance for these items is substantial ($1.1 million) and requires investigation.

By this example, we see the usefulness of variance analysis in the monitoring of inventory patterns. By applying variance analysis to each type of inventory, we are able to separate the effects of mix, sales, turnover, and price. With these effects isolated, investigation of any disadvantageous deviations from expectations can be undertaken.

INVENTORY MANAGEMENT IN PRACTICE

This is the final chapter concerning the management of inventory. In this chapter and the prior one, techniques have been presented for managing inventory and for monitoring inventory levels. Students are naturally interested in the extent to which these techniques are used in the practice of inventory management.

Unlike the decision methodologies presented in many other parts of this text, the basic technologies of inventory management are not new; many of them date from the 1940s or before. Consequently, there has been relatively little recent interest in assessing whether firms have adopted such basic methodologies as the EOQ approach, safety stock strategies, the A-B-C System, and so forth. However, a review of trade publications, where businessmen often discuss their views on management, indicates that the understanding of these basic methodologies is widespread.

Some supporting evidence on the use of these basic approaches to inventory management is presented in Smith and Sell's questionnaire survey of large firms, which was conducted in 1978.[15] As part of this survey, they asked firms to specify the technique used for establishing inventory restocking points (reorder points). Sixty-eight percent of respondents indicated that they used

[15]K. V. Smith and S. B. Sell, "Working Capital Management in Practice," in K. V. Smith, Editor, *Reading on the Management of Working Capital*, Second Edition (St. Paul: West Publishing, 1980), pp. 51–86.

either cost-balancing models (such as the models presented in Chapter 9) or computerized inventory control. This result depended to some extent on the size of the firm; 82.4 percent of larger firms indicated the use of these two types of reorder point determination, while 53.1 percent of smaller firms made similar indications. If firms' computerized inventory order point systems are based on EOQ-type models, it would seem that certainty methodologies are a widely used approach to the order strategy problem, particularly for larger firms.

Smith and Sell also found that changes in profits and return on investment were important criteria in evaluating changes in inventory policy. To the extent that criteria of these sorts are proxies for shareholder wealth maximization, they are generally in keeping with the approaches used in this text. This survey evidence, together with the discussion of these techniques in trade publications, suggests that the basic models of inventory management are widely used. However, much less is known about the use of more complex or more recently developed approaches (such as uncertainty models) to the management of inventory.

SUMMARY

In this chapter, we discussed several methods that simultaneously address the expected parameters of an inventory situation and their uncertainty. Both static and dynamic inventory problems were discussed. Three uncertainty methods for addressing static inventory problems were outlined: (1) valuation of the uncertain future flow via the option-pricing model; (2) the calculation of expected net present value, with risk assessed via the coefficient of variation of this net present value; and (3) net present value using risk-adjusted discount rates to adjust the value of the uncertain expected future flows for their riskiness. All three of these methodologies addressed both the time value and the riskiness of future cash flows in static inventory problems.

Dynamic inventory problems were addressed via simulation analysis of the expected net present value and the variability of net present value for various inventory strategies. Unlike certainty methods, simulation analysis can address any probability distribution of the uncertain variables in inventory problems. However, simulation analysis of inventory problems is costly to execute and eliminates only dominated risk-return strategy combinations; management must make the choice among the nondominated strategies.

The monitoring of inventory levels was discussed. This monitoring is a difficult process because of the effects of variations in sales, the price of the inventory, the mix of inventory among inventory types, and unit inventory turnover on monitoring statistics. These variations make the aggregate dollar turnover ratio (which compares sales to inventory investment) a misleading indicator of inventory behavior. To control for price, sales, unit turnover,

and mix in assessing behavior, the accounting technique of variance analysis was used.

The available survey evidence, along with observations of businesspersons' contributions to the trade press, indicates that the basic inventory management concepts of economic order quantity, safety stock for hedging uncertainty, and so forth, are widely used and understood, particularly among larger firms. Less is known about the application of more advanced techniques to inventory management situations.

This chapter concludes our discussion of the management of working capital assets. Chapters thus far have concerned the management of the major short-term assets held by firms: cash, marketable securities, accounts receivable, and inventory. Numerous tools and concepts for determining the most advantageous composition of these various assets have been discussed. The next three chapters concern the management of short-term liabilities.

Problems

9-1. A student at Enormous State University (ESU) has decided to scalp tickets to ESU's football games. The prime question is, of course, how many tickets to purchase for a particular game. The student anticipates that the number of tickets that he or she can sell on the day of the game will be dependent on the level of the Dow-Jones Industrial Average (DJIA) at the time of the game:

Level of the DJIA	*Less Than 1,500*	*1,500–1,900*	*Greater Than 1,900*
Number of tickets sold	20	40	60

The DJIA is currently 1850. The first game is 6 months away. The risk-free rate of interest is 6.5 percent per year. Tickets cost $15, payable immediately, and sell for $25 if they are sold. There are no taxes or stockout costs. Assume that the instantaneous variance of the DJIA is 0.70. Calculate the number of tickets that the student should purchase (choices: 20, 40, or 60) using the option-pricing model.

9-2. A firm is trying to develop an inventory policy for an item with a one-period life using uncertainty methods. Items can be acquired now for sale three months hence. Each item costs $6, payable immediately, and will sell for $10 if it is sold at all. If there are not sufficient items to service demand, the firm will incur an after-tax stockout cost of $1 per unit stocked out. Excess items have no salvage value. Tax bills will be paid at the time of sale; the firm is in the 40 percent tax rate, and will have sufficient other earnings

to utilize tax shields in the event of losses. The estimated probability distribution of demand is:

Probability	*0.1*	*0.4*	*0.4*	*0.1*
Demand (units)	100	200	300	400

The feasible ordering strategies are 100, 200, 300, or 400 units.

a. Generate a matrix of initial cash flows and future after-tax cash flows.

b. Calculate the expected net present value and the coefficient of variation of net present value for each ordering strategy. The risk-free rate of interest is 7.5 percent per year. Indicate which strategy should be chosen by this methodology.

c. Calculate the net present value of each ordering strategy using risk-adjusted discount rates. Use the following risk-adjusted rates (in percent per year):

Order Quantity	*100*	*200*	*300*	*400*
RAD	8	9	10	11

Indicate which strategy should be chosen by this methodology.

9-3. A firm carries two types of inventory, Type 1 and Type 2. Expected sales, turnover, and price were:

Inventory	*Sales (in Units)*	*Unit Turnover*	*Price per Unit*
Type 1	1,000	20.00	$150
Type 2	3,000	15.00	$75

Actual sales, turnover, and price were:

Inventory	*Sales (in Units)*	*Unit Turnover*	*Price per Unit*
Type 1	750	10.00	$170
Type 2	4,000	18.00	$65

Perform a variance analysis for the two types of inventory; calculate sales, turnover, and price variances for each inventory type.

Selected Readings

Copeland, T. E., and J. F. Weston, *Financial Theory and Corporate Policy*, Second Edition (Reading, Mass.: Addison-Wesley, 1983), pp. 230–84.

Gallinger, G., and P. Healey, *Liquidity Analysis and Management* (Reading, Mass.: Addison-Wesley, 1987), Chapter 15.

Kim, Y. H., and K. H. Chung, "Inventory Management: A Financial Theory of the Firm Under Uncertainty," available from Kim at the University of Cincinnati.

Love, S., *Inventory Control* (New York: McGraw-Hill, 1979), Chapter 9.

Mansour, Ali, "Monte Carlo Simulation," unpublished manuscript, available from the author at West Virginia University.

Stowe, J. D., and A. K. Gehr, Jr., "An Arbitrage Solution to the Christmas Tree Problem." Paper presented at the Financial Management Association Annual Meeting, October, 1984. Available from the authors at the University of Missouri at Columbia.

case

Simpson Stores Inventory Management: One-Period Products

In the clothing business, there are two general types of goods: fashion-related and nonfashion-related. Fashion-related items are those with a high "fad" component. Demand for items of this sort is heavily related to the strength and length of fashion trends, and thus is highly uncertain. Further, since these trends tend to disappear as quickly as they appear, the recovery value of unsold fashion items is usually minimal. Numerous retailers, left with stocks of unsalable merchandise, will sadly attest to this fact.

Nonfashion-related items become outmoded more slowly. Demand trends in such items are less perceptible from season to season. Basic athletic apparel and some types of blue jeans are examples of such items.

For Ms. Dora Lee, a merchandise buyer for Simpson Stores, this difference required a variation in purchase and inventory management techniques. For all types of goods, demand was seasonal, and overstocked items could not be returned to vendors. For nonfashion-related items, while some merchandise could be sold in end-of-season sales with minor price reductions, it was generally feasible to carry over some merchandise from one year to another. Many fashion-related items, however, had a selling life of one season or less. Drastic markdowns were used to dispose of such merchandise as the end of the season approached. Any unsold merchandise of this sort was written off at cost at the end of the season and given to charity. Thus, nonfashion-related items represented a dynamic (multiperiod) inventory problem, while fashion-related items represented a static (one-period) inventory problem.

One item with a heavy fad component was a particular blouse (style number 111703) purchased from No Bikini Atoll Sportswear. The blouse was of a color and style that led Ms. Lee to believe that, while it might be quite popular this year, the market for the item next year was likely to be very limited. Further, the blouse could be ordered only once for the upcoming selling season (a common situation for fashion items). The order had to be placed six months before the selling season. Merchandise would be received about four months after the order was placed, and the invoice for the merchandise would be due one month after the merchandise was received. Based on her assessment of the demand for such items among the stores in the Simpson chain, Ms. Lee expected that 200 blouses could be sold

Exhibit 1

Simpson Stores
Estimated Main-Season Sales of Style Number 111703 for Each Size Range

Economic Conditions	*Sales Range (units sold)*	*Price Range for Firm's Stock*
Very Poor	25–75	Below $75
Recession	76–125	$75–$90
Weak	126–175	$90–$105
Average	176–225	$105–$120
Good	226–275	$120–$135
Excellent	276–325	$135–$150
Boom	326–375	Above $150

in the upcoming year in each of the common sizes. There was, however, substantial uncertainty about this demand level. Ms. Lee estimated the continuous distribution presented in Exhibit 1 to represent demand and economic conditions. She decided to use the midpoints of the demand ranges to analyze the outcomes. Consultation with the firm's financial staff also gave her estimates of the levels of the firm's common stock for each of these economic conditions. The current price of the firm's common stock was $110 per share.

The blouses could be purchased from the vendor for $20 each. During the relatively short prime selling season, they would be priced at $30 each. At the end of this season, they would be marked down to $15 each; it was expected that 85 percent of the remaining stock would be sold at this price. The remaining 15 percent would be given to charity. The firm was taxable and was in the 46 percent marginal tax bracket. Ms. Lee felt, given the character of Simpson Stores and its customers, that stockout costs would be incurred if a customer wished to purchase the blouse and found the store to be out of stock. She decided to assign a $2 after-tax stockout cost for each unit stocked out. This cost would be incurred at the time of the stockout and would represent the present value of lost future after-tax cash inflows due to decreased customer patronage.

Chapter -10

The Firm's Level of Aggregate Liquidity

Throughout this text, we are concerned with the effects of uncertainty on the management of the firm's current assets and liabilities. At each stage, we provide methods for addressing the risks that occur for a particular current liability or asset. What is not discussed in other chapters is the *gross* hedge that occurs via the relationship between *total* short-term assets and *total* short-term liabilities. The overall relationship between current assets (which produce cash inflows) and current liabilities (which require cash outflows) will determine the size of this gross hedge. If current assets can provide much more cash than is needed for current liabilities, then the chance of a cash stockout is lessened. We call this overall relationship between a firm's potentially available cash and its potential cash needs the firm's *aggregate liquidity position*. The proper measurement and management of aggregate liquidity position is the concern of this chapter.

The chapter begins with a section discussing why the measurement and management of aggregate liquidity is important to the firm. Following this, issues in the measurement of aggregate liquidity are discussed. A reliable measure of aggregate liquidity can serve several purposes, but we will see that the traditional measures of liquidity are not very accurate indicators of a firm's true aggregate liquidity position. After these traditional measures are reviewed, some that are more accurate and comprehensive are suggested. The risk-return trade-off that faces the firm in determining its aggregate liquidity position is then discussed, and some measures of the inherent risks and returns are illustrated. A section on the management of aggregate liquidity in practice concludes the chapter.

WHY MEASURE AND MANAGE AGGREGATE LIQUIDITY?

Firms measure and manage their aggregate liquidity positions because

see later in the chapter that the amount of current debt relative to current assets affects the level of expected cash flows to shareholders and the risk of these cash flows. Since any such variations in the risk and return affect shareholder wealth, assessing and addressing these variations is a necessary function of financial management.

The management of aggregate liquidity starts with the measurement of the size of this hedge for various potential financial strategies. Since this management is facilitated by more precise measurement, the development and application of *accurate* measures of aggregate liquidity is of substantial advantage to the firm. However, the availability of accurate measures of aggregate liquidity has a side benefit. In addition to the measurement of the firm's own liquidity for the purpose of managing this position, measures of liquidity can also be applied to outside firms as an aid in making credit-granting decisions. Aggregate liquidity is an important determinant of the probability of default, and the accurate assessment of this aggregate liquidity position can therefore lend important insights in estimating the default probabilities of credit applicants.

TRADITIONAL MEASURES OF THE AGGREGATE LIQUIDITY OF THE FIRM

In prior chapters, we have discussed aggregate liquidity rather simplistically in terms of the relative amounts of current debt and current assets, but the concept is really more complex than this. Liquidity can be thought of as the *firm's ability to quickly generate cash versus the firm's need for cash on short notice.* Beginning textbooks usually cite several financial ratios as measures of liquidity. Unfortunately, each of the ratios that are traditionally cited has problems as an omnibus measure of the firm's liquidity position. Commonly cited ratios include:

Current Ratio. This is the ratio of current assets to current liabilities. The higher this ratio, the more liquid the firm is said to be. This ratio is widely used by practitioners and has substantial intuitive appeal. It uses the standard accounting convention in designating which assets and debts are "liquid" and which are not. The convention, of course, states that any asset or debt expected to mature in less than a year is "liquid" (that is, "current").

The problem with the current ratio is that it mixes assets and liabilities that are, in reality, quite different in terms of their nearness to cash (that is, in their time to maturity). A debt due in six months is treated the same way as a debt due in six days, since they are both part of current liabilities. Inventory, which requires a sale to become receivables and the collection of the resulting receivables to become cash, is treated in the same way as marketable securities, which can be sold quickly and thus are clearly more liquid.

Therefore, while it is widely used, the current ratio can be improved upon as a measure of aggregate liquidity.

Quick Ratio This is also called the "acid test" ratio. Here, inventories (which require a sale to become collectible) are deducted from the current assets account and the result divided by current liabilities.[1] Interpretation is similar to the current ratio. The idea behind the quick ratio is to generate an index that better compares short-term cash-generating ability with short-term cash needs by excluding the most obviously illiquid current asset (the one furthest from cash in the working capital cycle) from the near-cash assets total. This is a reasonable step since the sales of any firm are always quite uncertain; however, this ratio still entails a substantial mixing of maturities, particularly among liabilities.

Accounts Receivable Turnover This ratio is usually calculated as sales divided by accounts receivable. The inverse of this ratio times the number of days in a year gives the *average collection period*: the weighted-average time that a receivable is outstanding.[2] The higher the turnover (and the lower the average collection period), the quicker is a receivable turned into cash, and the more liquid is the firm said to be.

Inventory Turnover Ratio This is usually computed as cost of sales divided by inventory. Like the accounts receivable turnover ratio, the higher the turnover, the more liquid the asset.[3] The basic difficulty with the accounts receivable turnover ratio and the inventory turnover ratio as liquidity measures is that they focus on only one asset's nearness to cash, and thus by themselves do not tell much about the firm's *overall* liquidity position. That is really what we want to measure.

The four ratios discussed above are easy to understand and are commonly used in assessing the liquidity of firms. But as the discussion indicates, all are flawed to some extent as omnibus liquidity measures. *They may give the wrong signals, contradictory signals, or no signals at all of actual changes in liquidity position.* To see this, let us examine the liquidity position of Hacker Corporation, a designer and manufacturer of computer games. Hacker's year-

[1]Note that this procedure still leaves other, less liquid assets (such as prepaid expenses) included in the numerator of the ratio. This is usually justified because the amounts of these other assets are generally small; cash, marketable securities, accounts receivable, and inventories comprise the vast majority of current assets for most firms. A more careful but less common definition of the quick ratio is: (cash + marketable securities + accounts receivable)/current liabilities.

[2]When used for the monitoring of customer payments, this ratio is known as the firm's *days' sales outstanding*; it was discussed in Chapter 7.

[3]We discussed the use of this aggregate dollar inventory turnover ratio for the monitoring inventory in Chapter 9.

end financial data are presented in Table 10-1. The "Old" columns indicate Hacker's current financial position.

In assessing Hacker's financial position, the four traditional measures of liquidity can be computed:

Current Ratio: \$140,000/\$170,000 = 0.82 (10-1)

Quick Ratio: (\$140,000 − \$75,000)/\$170,000 = 0.38 (10-2)

Accts. Rec. Turnover: \$1,000,000/\$50,000 = 20 times per year (10-3)

Inv. Turnover: (\$200,000 + \$700,000)/\$75,000 = 12 times per year (10-4)

To see the problems with these traditional measures, let us hypothesize some changes in Hacker's financial position. Suppose that buyers start paying more slowly, so that accounts receivable build up by \$10,000. Hacker Corporation finances this by reducing cash by the same amount. This change is portrayed in Panel A of Table 10-1. The firm's overall liquidity has been reduced since it has less cash and more accounts receivable (which are less liquid than cash). However, *the firm's quick, current, and inventory turnover ratios are unchanged.* These measures do not show this change in liquidity. The accounts receivable turnover ratio would, however, decrease to 16.67 (\$1,000,000/\$60,000).

Even more startling examples are possible. Suppose that, due to better inventory management techniques, the firm is able to reduce inventory investment from \$75,000 to \$50,000 with no increase in total inventory costs. Assume that the funds freed from this reduction in inventory are used to reduce accounts payable. These effects are portrayed in Panel B of Table 10-1. The firm's current ratio would decline from 0.82 to 0.79, its quick ratio would climb from 0.38 to 0.45, its inventory turnover would increase from 12.0 to 18.0, and its accounts receivable turnover would be unchanged. The decreased current ratio indicates a *decline* in liquidity, the increased quick and inventory turnover ratios indicate an *increase* in liquidity, and the accounts receivable turnover ratio indicates that liquidity is *unchanged.* We see by these examples that the traditional measures of liquidity may not signal when changes in liquidity have occurred, or they may give conflicting signals regarding the direction of this change.

Problems and contradictions such as these in the use and interpretation of the traditional ratio measures have led to the development of several improved measures of the overall liquidity of the firm. These measures are discussed in the next section.

IMPROVED INDICES FOR MEASURING AGGREGATE LIQUIDITY

Several improved measures of aggregate liquidity have appeared in the literature. Four of these are: (1) the *Cash Conversion Cycle* of Richards and

Table 10-1. Hacker Corporation: Analysis of Changes in Liquidity.

Last Year's Sales	$1,000,000
Cost of Sales	$ 900,000

Panel A: Change in Accounts Receivable Turnover

Situation Number	*1* *Old*	*2* *New*
Cash	$ 15,000	$ 5,000
Accounts Receivable	50,000	60,000
Inventory	75,000	75,000
Total Current Assets	140,000	140,000
New Fixed Assets	150,000	150,000
Total Assets	$290,000	$290,000
Accounts Payable	$110,000	$110,000
Accrued Wages	60,000	60,000
Total Current Liabilities	170,000	170,000
Equity	120,000	120,000
Total Liabilities and Equity	$290,000	$290,000
Current Ratio	.82	.82
Quick Ratio	.38	.38
Accts. Rec. Turnover	20.00	16.67
Inventory Turnover	12.00	12.00

Panel B: Change in Inventory and Accounts Payable

	1 *Old*	*3* *New*
Cash	$ 15,000	$ 15,000
Accounts Receivable	50,000	50,000
Inventory	75,000	50,000
Total Current Assets	140,000	115,000
Net Fixed Assets	150,000	150,000
Total Assets	$290,000	$265,000
Accounts Payable	$110,000	$ 85,000
Accrued Wages	60,000	60,000
Total Current Liabilities	170,000	145,000
Equity	120,000	120,000
Total Liabilities and Equity	$290,000	$265,000
Current Ratio	.82	.79
Quick Ratio	.38	.45
Accts. Rec. Turnover	20.00	20.00
Inventory Turnover	12.00	18.00

Laughlin,[4] (2) a version of the *Comprehensive Liquidity Index* developed by Melnyk and Birati,[5] (3) the *Net Liquid Balance* measure of Shulman and Cox,[6] and (4) the *Lambda* index developed by Emery.[7] While each of these improved indices measures one or more aspects of liquidity more accurately than do the traditional measures, each is, nonetheless, limited in some respect; none is a perfect liquidity measure. We discuss them in turn in the following paragraphs.

Cash Conversion Cycle This measure is based directly on the concept of the working capital cycle as discussed in Chapter 1 of this text. Recall in this cycle that the firm purchases labor and materials, which it uses to make inventory, which is sold to generate receivables, which are in turn collected to produce cash. Within this cycle, there are two assets, generated at different times, that must be financed (inventory and accounts receivable), and one set of liabilities that provides the financing (accounts payable and wages payable). The Cash Conversion Cycle is the *net time interval* between the expenditure of cash in paying the liabilities and the receipt of cash from the collection of receivables. The lower the Cash Conversion Cycle, the more liquid the firm is said to be. It is calculated as the firm's average collection period plus its "inventory conversion period" (Richards and Laughlin call the sum of these two the "operating cycle") minus the "payment deferral period." The "inventory conversion period" and the "payment deferral period" are turnover statistics similar to the average collection period. They represent, respectively, the weighted-average time that a dollar is tied up in inventory or financed by payables. Inventory conversion period is calculated as the inverse of the inventory turnover ratio times 360 days. The payment deferral period is calculated as the sum of all the sales-related accrual and payable accounts divided by cost of sales, with this calculated figure multiplied by 360 days.

The Cash Conversion Cycle may seem a bit difficult at first, but it is, in fact, quite easily calculated. Let us perform an example calculation using the original data from Hacker Corporation presented in Table 10-1. The firm's accounts receivable turnover is 20 times per year, so its average collection period is 360 divided by 20 or 18 days. The firm's inventory turnover is 12

[4]See V. D. Richards and E. J. Laughlin, "A Cash Conversion Cycle Approach to Liquidity Analysis," *Financial Management* (Spring 1980), pp. 32–38.

[5]See Z. L. Melnyk and A. Birati, "Comprehensive Liquidity Index as a Measure of Corporate Liquidity," *Scientific and Behavioral Foundations of Decision Sciences* (Atlanta, Ga.: Southeastern Region of the American Institute for Decision Sciences, 1974), pp. 162–65.

[6]J. Shulman and R. Cox, "An Integrative Approach to Working Capital Management," *Journal of Cash Management* (November/December 1985), pp. 64–67.

[7]G. Emery, "Measuring Short-Term Liquidity," *Journal of Cash Management* (July/August 1984), pp. 25–32.

times per year, so its inventory conversion period is 360 divided by 12 or 30 days. The sum of these two statistics is 48 days; this is the firm's operating cycle. Hacker's cost of sales is $900,000. Hacker's accounts payable and accrued wages totaled $170,000, so the firm's payment deferral period was $170,000 divided by $900,000 times 360 or 68 days. Hacker's Cash Conversion Cycle is 48 days minus 68 days or minus 20 days.

The Cash Conversion Cycle is an excellent quantification of the working capital cycle. It is often a more accurate measure of overall liquidity than is the current ratio. However, it does not capture the effects of current assets or liabilities other than accounts receivable, inventory, accounts payable, and sales-related accruals. For example, *cash*, the most liquid of assets, is not involved in the calculation. Yet few would argue that a firm with less cash is more liquid than a firm with more cash, all other things being equal.

Comprehensive Liquidity Index This is a liquidity-weighted version of the popular current ratio. Recall that one of the problems with using the current ratio is that it treats all the current assets and liabilities as being of equal liquidity, when in reality their liquidity is quite different. In the prior example of an increase in receivables and a corresponding decrease in cash, the current ratio would not be affected. The Comprehensive Liquidity Index overcomes this by weighting each current asset and liability based on its nearness to cash (its turnover). In computing the Comprehensive Liquidity Index, the dollar amount of each current asset or liability is multiplied by one minus the inverse of the asset or liability's turnover ratio. If there are more than two turnovers required to generate cash from the asset, the inverse of each of these ratios is deducted. The results are summed over all current assets and current liabilities. The summed totals are liquidity-adjusted measures of total current assets and total current liabilities. The current ratio is then computed based on these adjusted figures; in the notation of Melnyk and Birati, X is the adjusted total current assets figure, and Y is the adjusted total current liabilities figure. The Comprehensive Liquidity Index is just X divided by Y.

Again, let's compute this measure for the original data on Hacker in Table 10-1. The cash figure of $15,000 needs no adjustment. Receivables turnover is 20, so the adjusted receivables figure is:

$$\$50{,}000(1 - (1/20)) = \$47{,}500 \qquad (10\text{-}5)$$

The generation of cash from inventory requires both the turnover of inventory to receivables and the turnover of receivables to cash. Inventory turnover is 12, so the adjusted value of inventory is:

$$\$75{,}000(1 - (1/20) - (1/12)) = \$65{,}000 \qquad (10\text{-}6)$$

The adjusted current assets figure (X) is then \$15,000 plus \$47,500 plus \$65,000 or \$127,500 (in thousands). The slower the turnovers, the lower this adjusted figure will be relative to the book figure.

For the adjusted current debt figure, we must compute some new turnover ratios. Accounts payable of \$110,000 are the result of materials purchases; assume that such purchases totaled \$400,000 for the year. The accounts payable turnover ratio is then \$400,000 divided by \$110,000 or 3.64. The adjusted accounts payable figure is:

$$\$110{,}000(1 - (1/3.64)) = \$79{,}750 \qquad (10\text{-}7)$$

Wages payable are \$60,000; assume that the firm's total wage bill is \$500,000, so the turnover of wages payable is 8.33. The adjusted wages payable amount is:

$$\$60{,}000(1 - (1/8.33)) = \$52{,}800 \qquad (10\text{-}8)$$

The adjusted current debt figure (Y) is the sum of these adjusted liability figures, or \$132,550. The Comprehensive Liquidity Index is then \$127,500 divided by \$132,550 or 0.96.

Net Liquid Balance Both this index and the Lambda measure (discussed next) center on the firm's balance of cash and marketable securities. The argument is that this balance represents the firm's true reserve against unanticipated cash needs, since other remedies for cash shortages can be very costly.[8] For example, if the firm runs out of cash, it might try to liquidate inventory via a distress sale, but this would generate costs that would not otherwise occur. The Net Liquid Balance does not view the firm's investments in accounts receivable and inventory as contributions to aggregate liquidity, but, rather, considers them as additional assets to be financed. The accounts payable and accruals that are part of current liabilities are treated not as maturing obligations but as part of the firm's permanent financing package (similar to long-term debt). Only notes payable (short-term, interest-bearing debts) are treated as maturing obligations. The Net Liquid Balance (NLB) is defined as:

$$\text{NLB} = (\text{Cash} + \text{Mar. Sec.} - \text{Notes Payable})/\text{Total Assets} \qquad (10\text{-}9)$$

[8]Recall from the discussion in Chapter 3 that holding cash and marketable securities beyond those amounts necessary for transactions and for the temporary investment of surplus funds is one mechanism for hedging cash flow risk. Note, however, that Net Liquid Balance ignores other possible hedges of this risk, such as the availability of reserve borrowing capacity.

In the original situation, Hacker Corporation has cash of $15,000 and no marketable securities or notes payable balances. Total assets are $290,000, so the NLB is:

$$\text{NLB} = 15{,}000/290{,}000 = 0.052 \qquad (10\text{-}10)$$

Lambda This approach is different from the previous ones in four respects. First, in the Lambda measure, the firm's available credit line (if known) is counted as part of the firm's package of liquid reserves. Second, this index uses a measure of uncertainty to evaluate the firm's potential need for liquidity. Third, this is the only measure that incorporates the firm's expected cash *flows* in addition to its cash and near-cash *stocks* of assets. Finally, the other measures consider only cash flows relating to current assets and liabilities; that is, they consider only flows around the working capital cycle. While working capital flows are extremely important, they are not the only flows of funds within the firm; there are also cash inflows and outflows on the long-term accounts. Lambda considers all the flows through the firm, regardless of whether they originate from short-term or long-term transactions.

The use of a measure of uncertainty in this context makes particularly good sense. By measuring aggregate liquidity, we are attempting to develop an index that indicates the possibility the firm will run out of cash, and thus be forced to take costly actions to remedy this problem. A firm with more certain cash flows will be less likely to default. The Lambda index uses this cash flow uncertainty along with the level of the firm's initial reserve and the expected future cash flows to generate an index akin to a Z-score. The formula is:

$$\text{Lambda} = (\text{Initial Reserve} + E(NCF))/\text{Uncertainty} \qquad (10\text{-}11)$$

In this formula, the initial reserve is defined as cash plus marketable securities plus available credit lines. The expected cash flow ($E(NCF)$) and the uncertainty regarding this expected cash flow depend on the period being forecast. For example, if a firm is using this index to formulate its liquidity strategy over the next year, these expected flows and the uncertainty of these flows would be defined on a yearly basis. However, if the firm is using this index to assess a credit applicant for a trade credit sale on 30-day terms, these flows and their uncertainty are defined on a 30-day basis.

For Hacker Corporation, assume that our analysis is over the next year, and that Hacker's expected cash flow is $50,000 over this year (a net cash inflow). The appropriate measure of uncertainty is the standard deviation of this flow; assume that this is $100,000. Hacker has no reserve line of credit. The corporation's Lambda is then:

$$\text{Lambda} = (15{,}000 + 50{,}000)/100{,}000 = 0.65 \qquad (10\text{-}12)$$

The problem with the net liquid balance and Lambda measures as indices of liquidity is that they ignore all assets other than cash, marketable securities, and available borrowings in determining the firm's liquidity position. While these are certainly major contributors to liquidity, the natural process of the working capital will turn other current assets into cash as time progresses. Also, current liabilities will reduce cash as they mature and are retired. Leaving these other assets and liabilities out of a liquidity measure or dealing with them indirectly can introduce distortion in the measurement of aggregate liquidity. This is the opposite problem from that of the cash conversion cycle, which counts the *other* assets and liabilities but *does not* count cash or marketable securities.

To assess the advantages and disadvantages of these new indices, let us apply them to the two hypothetical changes in Hacker's financial position, as previously postulated. The calculations and results are presented in spreadsheet format in Table 10-2. The Cash Conversion Cycle statistics appear in cells B46 through D46, the Comprehensive Liquidity Index statistics in cells B53 through D53, the Net Liquid Balance in cells B55 through D55, and the Lambda index in cells B57 through D57.

Recall that in going from situation 1 (the firm's original financial position) to situations 2 and 3, the firm's liquidity seemed to have changed, but the common liquidity ratios did not indicate this change or provide contrary indications. In going from situation 1 to situation 2, Hacker's cash is reduced, and accounts receivable are increased. This clearly makes the firm less liquid. *All four of the new measures decline and thus properly detect this change.*

The change in liquidity in going from situation 1 to situation 3 is more complicated. Both current assets and current liabilities decline by the same amount, but the current asset (inventory) that is reduced in this process is further from cash than is the current liability (accounts payable). Inventory requires that a sale and a collection be converted to cash, while cash for accounts payable will be required as these payables mature. Further, initial current assets are less than current liabilities; an equal dollar reduction in both bears more heavily on asset reserves than it does on liability requirements. In comparing situations 1 and 3, then, aggregate liquidity probably decreases. The Net Liquid Balance (cells B55 and D55), however, incorrectly signals an increase in liquidity. The Cash Conversion Cycle (cells B46 and D46) and Lambda (cells B57 and D57) signal no change in liquidity. The Comprehensive Liquidity Index (cells B53 and D53) signals a decrease in liquidity.

The two examples in Table 10-2 of changes in liquidity illustrate the positive and negative features of the improved measures of liquidity relative to traditional measures. The first example illustrates that the new measures are more accurate than traditional ones, and will frequently detect changes in liquidity where traditional methods fail to detect such changes. However, the second example shows that since the four measures do not measure exactly

Table 10-2. Hacker Corporation: Analysis of Liquidity Changes with New Liquidity Indices.

	A	B	C	D
5	Last Year's Sales	1000000		
6				
7	Last Year's Materials	400000		
8	Last Year's Wages	500000		
9				
10	Last Year's Cost of Sales	900000		
11				
12	This Year's E (NCF)	50000		
13	This Year's SD (NCF)	100000		
14				
15	*Situation Number*	*1*	*2*	*3*
16				
17	Cash	15000	50000	15000
18	Accounts Receivable	50000	50000	50000
19	Inventory	75000	75000	50000
20				
21	Total Current Assets	140000	140000	115000
22	Net Fixed Assets	150000	150000	150000
23				
24	Total Assets	290000	290000	265000
25				
26	Accounts Payable	110000	110000	85000
27	Accrued Wages	60000	60000	60000
28				
29	Total Current Debt	170000	170000	145000
30	Equity	120000	120000	120000
31				
32	Total Lia. and Equity	290000	290000	265000
33				
34	Current Ratio	.82	.82	.79
35	Quick Ratio	.38	.38	.45
36	Accts. Rec. Turnover	20.00	16.67	20.00
37	Inventory Turnover	12.00	12.00	18.00
38				
39	Cash Conversion Cycle:			
40	Average Collection Period	18.00	21.60	18.00
41	Inventory Conv. Period	30.00	30.00	20.00
42				
43	Operating Cycle	48.00	51.60	38.00
44	Payment Deferral Period	68.00	68.00	58.00
45				
46	Cash Conversion Cycle	−20.00	−16.40	−20.00
47				
48	Comprehensive Liquidity Index:			
49	Accounts Pay. Turnover	3.64	3.64	4.71
50	Accrued Wages Turnover	8.33	8.33	8.33
51	Calculated X	127500	125650	107222
52	Calculated Y	132550	132550	119737
53	Calculated X/Y	.96	.95	.90
54				
55	Net Liquid Balance:	.052	.017	.057
56				
57	Lambda:	.65	.55	.65

the same aspects of liquidity, they will sometimes produce contradictions in the same way as do the traditional measures.[9]

Some Caveats Regarding Liquidity Measures While all four of these measures are improvements, no index of this sort will *completely* measure the concept we call "aggregate liquidity," particularly when the index is used to assess the liquidity of a firm other than one's own. Two problems in the measurement of liquidity that are not addressed by these new indices concern the effect of *off-balance-sheet relationships* and the *treatment of current long-term debt.*

The existence of off-balance-sheet affectors of liquidity creates special circumstances that may make these indices less than complete representations of a firm's position. These indices measure only "normal course of business" transactions. A firm with seemingly poor liquidity may have hidden reserves (such as some relatively liquid and valuable fixed assets) for use in times of emergency. If such a firm needs cash, it can be raised quickly, and possibly with little cost. Conversely, a firm in a seemingly good liquidity position can be forced into financial difficulties by an unforeseen occurrence, such as the bankruptcy of a major customer or a lawsuit. It is extremely difficult for an outsider to assess the quantity and quality of such special circumstances, and thus to measure the aggregate liquidity of such firms.

A second problem concerns the treatment of the current portion of the firm's long-term debt. Should this be included among current liabilities in assessing liquidity or not? Whether this current long-term debt will actually result in a cash outflow (and a reduction in liquidity) depends on the analyzed firm's long-term financing plans. If the firm considers its long-term debt to be *permanent* financing, it will refund this debt with new debt of the same type as it comes due. In this case, the current portion of its long-term debt will be paid via the sale of new debt, and not from maturing current assets; the existence of current long-term debt does not decrease the firm's liquidity. However, if the firm being analyzed is *paying off* long-term debt as it becomes due (perhaps because it has too much debt in its financial structure), then current long-term debt must be serviced from working capital flows and should be counted among current liabilities.[10] Unfortunately, a firm's plans for managing its long-term debt are not often available to outside analysts.

[9]See J. W. Henderson and T. S. Maness, "An Integration of the Net Liqui l Balance and the Financial Flexibility Approach in Analyzing Liquidity," paper presented at the 1987 Southern Finance Association Meeting and available from Maness at Baylor University, for more discussion of the Net Liquid Balance and the signals it gives relative to other approaches in measuring liquidity. This reference also reviews several improved measures of liquidity in addition to those discussed in this text.

[10]For discussion of the effect of current long-term debt on the firm's liquidity measures, see E. F. Brigham and L. C. Gapenski, *Intermediate Financial Management* (New York: Dryden, 1985), p. 620.

Both of these two problems add uncertainty to the measurement of the liquidity of outside firms, regardless of the measure of liquidity employed. The analyst must be aware of these problems in using measures of aggregate liquidity in the analysis of credit applicants. However, in using any measure in the formulation of the firm's *own liquidity management strategy*, these problems are of less concern, since the analyst will have information on off-balance-sheet relationships and on the firm's plans regarding its long-term debt. The next sections of this chapter concern the formulation of such liquidity management strategies.

IQUIDITY AND THE FIRM'S FINANCIAL MANAGEMENT PROCESS

Now that we have developed measures of the firm's aggregate liquidity position, we can talk about the management of that position within the firm's financial management process. The author of this text sees the firm's financial management process as composed in part of the following steps:[11]

1. The firm decides what assets, current and long-term, it wants to hold using methodologies related to shareholder wealth maximization. Such procedures for various current assets were discussed previously in this text; for fixed assets, the procedures are those of capital budgeting.[12]
2. The firm decides on a target structure of financing based on a desired ratio of total debt to total assets.[13] This ratio determines the amount of debt and equity the firm needs but not the maturity structure of

[11]In reality, no firm operates in the clean, step-by-step fashion described here; the steps are all going on at the same time. The steps represent instead, a *thinking process* about financial management that is consistent with much of financial theory. Other approaches to the problem are also possible. For a classic summary of possible approaches to the liquidity level decision and to several other topics in the management of working capital see K. V. Smith, "State of the Art of Working Capital Management," *Financial Management* (Autumn 1973), pp. 50–55.

[12]The idea that the firm can make these decisions concerning which assets to purchase without regard to how these assets will be financed is called a *separation principal* in finance because the investment and financing decisions are made separately. For discussion, see T. E. Copeland and J. F. Weston, *Financial Theory and Corporate Policy*, Second Edition (Reading, Mass.: Addison-Wesley, 1983), pp. 408–409.

[13]This may be based on the firm's belief about its optimal capital structure: the mix of debt and equity that minimizes the firm's cost of capital. Whether optimal capital structure exists is an issue of significant debate in financial theory. The literature on this debate is extensive; see A. Barnea, R. A. Haugen, and L. W. Senbet, "Market Imperfections, Agency Problems, and Capital Structure: A Review," *Financial Management* (Summer 1981), pp. 7–22 for a partial literature review.

this debt (that is, how total debt is to be split into short- and long-term portions).

3. The firm decides on a target level of aggregate liquidity. This decision may be based on a desired index of aggregate liquidity (such as the current ratio, Cash Conversion Cycle, Comprehensive Liquidity Index, and so forth). In doing this, the firm may evaluate several potential positions to determine the appropriate target. Once the aggregate level of liquidity is chosen, the firm determines a target structure for its current and long-term debt.[14]
4. The firm compares this plan with its present financial structure and makes necessary and cost-effective adjustments to move toward the targeted financial structure. In a particular planning period, the costs of transition (such as the costs of refunding debt) may prevent the firm from achieving these targets; however, the firm moves toward the targets when it is cost-effective to do so.

The part of step 3 in which the firm determines its target level of liquidity and the structure of current debt is the thrust of the remainder of this chapter and of the next two chapters of this text. Procedures and models for choosing among liquidity positions are discussed in this chapter. Procedures for achieving a minimum-cost structure for current debt are discussed in Chapters 11 and 12.

To show how these decisions fit into the financial management process outlined above, let us illustrate this process for an example problem. Assume that the management of GenuSplice, a biological research firm, wants to plan its future financial position. Data on the current financial position of the firm is presented in Table 10-3. In step 1, the firm has assessed its asset position based on shareholder wealth maximization and determined that its desired level of assets is $270 million; fixed assets must be increased by $20 million. An additional investment of $10 million is also required for accounts receivable, as the firm has decided that it would be in the shareholders' interest to lengthen terms of sale. Based on considerations of optimal capital structure

[14]Note that in the methodology presented, the target composition of current debt is determined after its target level is determined. Since the relative costs of long-term and short-term debt are an important determinant of this target level, one alternative approach to the problem would be to use a global optimization procedure that would codetermine the level of short-term debt and its composition. However, such a procedure would be very cumbersome. For purposes of discussion, we assume that the firm can approximate the cost of short-term debt when determining its target level of aggregate liquidity. This estimate can be refined later when the target composition of debt for this target level is determined. If this cost of short-term debt is significantly different from that used in the analysis of aggregate liquidity, the firm may perform this analysis again based on this new data regarding short-term debt cost.

Table 10-3. *GenuSplice*
Actual and Desired Financial Positions (thousands of dollars).

	Actual	*Desired*
Cash	$ 10,000	$ 10,000
Accounts Receivable	100,000	110,000
Inventory	20,000	20,000
Current Assets	130,000	140,000
Fixed Assets	110,000	130,000
Total Assets	$240,000	$270,000
Accounts Payable	$ 5,000	$ 10,000
Accrued Wages	30,000	30,000
Short-Term Bank Borrowings	50,000	30,000
Total Current Debt except Current LTD	85,000	70,000
Current Portion of LTD	10,000	
Long-Term Debt (LTD)	90,000	
Total Long-Term Debt	100,000	92,000
Total Debt	185,000	162,000
Owners' Equity	55,000	108,000
Total Liabilities and Equity	$240,000	$270,000

(step 2), the firm determined that the desired debt-to-assets ratio is 0.60 (a rather highly leveraged position, for one reason or another), so the preferred amount of total debt for this asset structure is 0.60 times $270 million, or $162 million, and equity is the remaining $108 million. Step 3 requires the firm to set a target aggregate liquidity level and to structure current debt. Assume for simplicity that the firm has used the current ratio as its measure of aggregate liquidity and has decided that their preferred aggregate liquidity position involves a current ratio of 2.00 (where current long-term debt is not included in current liabilities in computing this ratio because this debt will be refunded at maturity). Since the desired level of current assets is $140 million, this entails that all current liabilities except the current portion of long-term debt total $70 million. The firm would then make a minimum-cost financing plan for current liabilities to total this amount; assume that this plan requires $10 million in accounts payable, $30 million in accrued wages, and $30 million in short-term bank borrowings. The firm would then compare these desired positions in its assets and liabilities with its planned positions and make cost-effective changes to move toward its desired future financial structure (step 4). In this very simplified example, over the planning period the firm would consider purchasing $20 million in fixed assets, increasing accounts receivable by $10 million, retiring $8 million in long-term debt (since the desired total is $92 million and the present total is $100 million), issuing $53 million in

equity, lengthening payments to trade creditors, and decreasing reliance on short-term bank borrowings.

EVALUATING STRATEGIES FOR AGGREGATE LIQUIDITY

Left unsaid thus far is how the firm decides on the best level of aggregate liquidity. For a given structure of current assets (step 1) and a given amount of total debt (step 2), the level of aggregate liquidity depends mainly on how the total amount of debt is divided into short- and long-term portions. This division of total debt into short- and long-term portions has substantial implications for the risk and return position of the firm. In particular, the use of relatively more short-term debt and less long-term debt is a *higher risk and higher expected return liquidity strategy*. This is due to the way the division of total debt affects the firm's *expected level of interest payments, variability of interest payments,* and *cash shortage risk*. Let us discuss each of these effects in turn.

Expected Level of Interest Payments The use of more permanent, short-term debt (with this debt refinanced as it matures) and less long-term debt will generally reduce the firm's cash outflows for interest expense. There are two reasons for this reduction. First, the interest rates on short-term debt are usually lower than the interest rates on long-term debt (that is, the yield curve is usually upsloping). Second, if the firm is in a business where seasonality affects its need for funds, and it uses a relatively large proportion of long-term debt financing, there may be times during which the firm has excess financing. During these times, the firm will invest these excess funds in short-term instruments, since they will be needed later in the year. However, since the yield curve is usually upsloping, the interest revenue from these investments will be less than the interest cost of the long-term debt financing. During these slack periods, if short-term financing had been used instead, the firm could pay off this financing and incur no interest expense. We see that since the yield curve is usually positively sloped, the strategy of relatively more short-term and less long-term debt financing produces a lower expected interest outflow, and therefore a higher residual cash flow for the shareholders.

Variability of Interest Expense However, the strategy of more short-term debt and less long-term debt involves higher risks. One source of risk in using short-term financing is the changeable nature of short-term interest rates. Minimum-cost, short-term financing strategies for firms often involve the use of interest-bearing obligations, such as bank credit line borrowings or commercial paper. While interest rate variations can be hedged to some

extent with forwards and futures,[15] when the firm uses short-term, interest-bearing obligations, and refunds these with similar instruments at maturity (instead of financing with fixed-rate, long-term obligations) it subjects itself to the interest rate variability, because it must bear market interest rates at the time of refunding. This variability adds an additional component of uncertainty into the firm's cash flow stream. Further, if monetary conditions become such that the yield curve becomes downsloping, the strategy of using more short-term and less long-term financing will result in a *higher* interest outflow (since short-term rates would then be higher than long-term rates).

Cash Shortage Risk Another risk associated with the use of more short-term debt and less long-term debt is the cash shortage risk that accompanies the lower liquidity in this strategy. The higher the short-term borrowing, all other things being equal, the more principal to be refinanced, and consequently, the higher the firm's average inflows and outflows of cash. And the higher the required cash outflows, the greater the chance that a difficulty with inflows will cause a financial crisis and that a financially painful solution (dividend reduction, rushed liquidation of assets, unplanned borrowing, and so forth) will be needed. This cash shortage risk is measured by the indices of aggregate liquidity developed previously.

The trade-offs involved in setting the level of aggregate liquidity by partitioning total debt are pictured in Figure 10-1.[16] As the proportion of long-term, fixed-rate financing increases, the expected total interest cost increases, but the expected shortage cost and the risk cost of varying interest payments declines. As a result, there is an optimum proportion of long-term versus short-term debt at the minimum of the total cost function.[17] The task

[15]The main reason that the interest rate risk from the strategy of financing with short-term debt and refunding cannot be completely hedged revolves around the maturities of long-term debt, short-term debt, and futures contracts. To obtain the minimum interest expense, short-term financing should be of short maturity, say 90 or 180 days. The interest rate risk from future refundings of such debt can be hedged by the standard futures contracts for a maximum of only 24 months ahead (see Chapter 3). However, long-term instruments (such as corporate bonds) usually have maturities of 10 years or longer. This gap in hedging between the maximum available futures contract and the maturity of corporate bonds results in interest rate risk. Further, strategies where the futures position is closed out and a new futures position is undertaken ("rolling the hedge") do not eliminate all interest rate risk (see footnote 35, Chapter 3).

[16]For additional discussion, see R. Gilmer, "The Optimal Level of Liquid Assets: An Empirical Test," *Financial Management* (Winter 1985), pp. 39–43.

[17]The conclusion that there is an optimal level of aggregate liquidity (that is, for a given structure of current assets, an optimal debt maturity structure) is not without challenge. Under some assumptions about the nature of markets and costs, this structure is irrelevant. See, for example, C. M. Lewis, "A Multiperiod Theory of Corporate Financial Policy under Taxation," available from the author at Vanderbilt University.

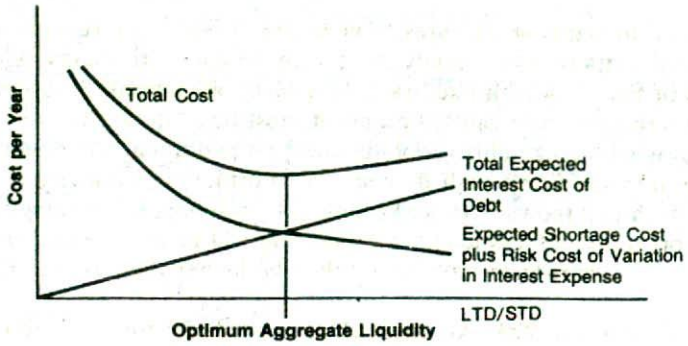


Figure 10-1. Optimum Level of Aggregate Liquidity.

in the management of aggregate liquidity is to find this optimum. Several partial models of the risk-return trade-offs inherent in liquidity decisions have been developed,[18] but a complete model of these trade-offs and their effect on firm value has not yet emerged. In the following section, the methods of financial analysis are used to quantify some of the risk and return aspects of this decision.

AN EXAMPLE SPREADSHEET ANALYSIS OF AGGREGATE LIQUIDITY

The Gridlock Corporation is trying to decide on an aggregate liquidity plan for the upcoming year. The firm wishes to maintain a total debt to total assets ratio of 0.50. Since total assets are expected to be $174.5 million, both total debt and total equity will then be $87.25 million. The firm is evaluating two liquidity strategies: these have been designated Plan 1 and Plan 2 (most firms would, of course, evaluate more than two alternative plans). Both plans would utilize minimum accounts payable and accrued wages of $11.4 and $13.2 million respectively.[19] Plan 1 calls for the use of $30 million in interest-bearing, short-term financing, while Plan 2 calls for the use of $50 million in such financing. The firm expects that, with proper structuring, this interest-

[18]Included among these models are those in J. C. Van Horne, "Corporate Liquidity and Bankruptcy Costs," Research Paper Number 205, Graduate School of Business, Stanford University; H. Bierman, K. Chopra, and L. J. Thomas, "Ruin Considerations: Optimal Working Capital and Capital Structure," *Journal of Financial and Quantitative Analysis* (March 1975), pp. 119–28; and K. Borch, "The Capital Structure of the Firm," *Swedish Journal of Economics* (1969), pp. 1–13.

[19]There is a minimum level of these liabilities below which it is uneconomical for the firm to reduce these items. Beyond this minimum, the firm may utilize these liabilities for financing, or may utilize other financing vehicles. These effects will be discussed in Chapters 11 and 12.

bearing, short-term financing will result in a yearly interest cost at approximately the prime rate. Neither plan will result in excess financing at any time during the current year. The question, of course, is which plan should be undertaken. To address this question, we need to develop measures of the cash stockout risk, expected interest cost, and potential variation in interest cost for the two strategies.[20]

To assess cash stockout risk, the firm could use any or all of the improved indices of aggregate liquidity that were presented previously. The Comprehensive Liquidity Indices for the two strategies are calculated in Table 10-4. The structures of current assets do not depend on the financing plan utilized. We have assumed that the firm's marketable securities mature every seven days, that the firm's "Other Current Assets" contribute nothing to liquidity, and that the interest-bearing, short-term liabilities turn over eight times per year (assuming that 45 days is the average maturity). We have also assumed the long-term debt is permanent; the current portion of this debt is then irrelevant in liquidity calculations. All the remaining calculations are straightforward: the Comprehensive Liquidity Index is 1.91 for Plan 1 and 1.41 for Plan 2.

To calculate the expected interest expense and fluctuations in this expense, we need to know the current rates of interest and the probability distribution of interest expense for interest-bearing, short-term debt (which is assumed to be floating-rate). Gridlock's long-term debt carries a 13.5 percent fixed coupon rate. The current prime rate is 10.5 percent. The firm believes there is a 50 percent chance that this rate will stay the same over the planning period, a 10 percent chance that this rate will fall to 9.5 percent, a 30 percent chance that it will rise to 11.5 percent, and a 10 percent chance that it will rise to 12.5 percent.

A spreadsheet analysis of the firm's after-tax interest expense is presented in Table 10-5. We have assumed a 34 percent tax rate in this example. The basic data for the problem appears in cells B5 through E7. Each column represents an interest rate outcome. Thus, column B, cells B10 through B37, models the interest expenses that will occur to Gridlock under Plans 1 and 2 if the prime rate averages 9.5 percent. The interest-bearing, short-term financing for Plan 1 is in cells B10 through E10; the data were put in column B and replicated into columns C, D, and E. The interest expense calculations in cells B11 through E11 are the interest rates from the "Basic Data" section times the short-term balance (for example, cell B11 contains "B6 * B10"). Multiplying this times 0.66 (equal to 1.0 minus the tax rate of 34 percent) gives the after-tax interest outflow figures in row 12. Similar calculations are performed in rows 14 through 16 for the long-term debt in Plan 1. These two

[20]We assume in this example that the fluctuations in short-term interest rates are undiversifiable or unhedged. In such a case, interest rate fluctuations represent relevant risk even in a CAPM sense.

Table 10-4. Gridlock Corporation: Data on Two Liquidity Plans (rounded thousands for dollar figures).

Sales	$698,000	
Cost of Materials	209,400	
Cost of Labor	349,000	
Total Cost of Sales	558,400	
Cash	5,400	
Marketable Securities	12,000	
Accounts Receivable	54,300	
Inventory	32,000	
Other Current Assets	5,400	
Total Current Assets	109,100	
Fixed Assets	65,400	
Total Assets	$174,500	
	Plan 1	*Plan 2*
Minimum Accounts Payable	$ 11,400	$ 11,400
Minimum Accrued Wages	13,200	13,200
Other Short-Term Financing	30,000	50,000
Total Current Liabilities	54,600	74,600
Long-Term Borrowings	32,650	12,650
Common Equity	87,250	87,250
Total Liabilities and Equity	$174,500	$174,500
Comprehensive Liquidity Index:		
Marketable Securities Turnover	52.00	
Accounts Receivable Turnover	12.85	
Inventory Turnover	17.45	
Accounts Payable Turnover	18.37	
Accrued Wages Turnover	26.44	
Turnover for Other Short-Term Financing	8.00	
Contributions to CLI:		
Cash	$ 5,400	
Marketable Securities	11,769	
Accounts Receivable	50,076	
Inventory	27,677	
Calculated *X*	$ 94,922	$ 94,922
Minimum Accounts Payable	$ 10,779	$ 10,779
Minimum Accrued Wages	12,701	12,701
Other Required Financing	26,250	43,750
Calculated *Y*	$ 49,730	$ 67,230
CLI (*X*/*Y*)	1.91	1.41

Table 10-5. Gridlock Corporation: Spreadsheet Analysis of Alternative Liquidity Strategies.

	A	B	C	D	E
1	Gridlock Corporation				
2	Analysis of Debt Structure Plans				
3					
4	Basic Data:				
5	Interest Rate on Long-Term Debt	.135	.135	.135	.135
6	Prime Rate	.095	.105	.115	.125
7	Prime Rate Probabilities	.1	.5	.3	.1
8					
9	Plan 1:				
10	Short Term Financing	30000	30000	30000	30000
11	Before-Tax Interest Expense	2850	3150	3450	3750
12	After-Tax Interest Expense	1881	2079	2277	2475
13					
14	Long-Term Borrowings	32650	32650	32650	32650
15	Before-Tax Interest Expense	4408	4408	4408	4408
16	After-Tax Interest Expense	2909	2909	2909	2909
17					
18	Total After-Tax Interest Expense	4790	4988	5186	5384
19					
20	Expected Interest Expense				5067
21	Standard Deviation of Interest Expense				158
22					
23					
24	Plan 2:				
25	Short Term Financing	50000	50000	50000	50000
26	Before-Tax Interest Expense	4750	5250	5750	6250
27	After-Tax Interest Expense	3135	3465	3795	4125
28					
29	Long-Term Borrowings	12650	12650	12650	12650
30	Before-Tax Interest Expense	1708	1708	1708	1708
31	After-Tax Interest Expense	1127	1127	1127	1127
32					
33	Total After-Tax Interest Expense	4262	4592	4922	5252
34					
35	Expected Interest Expense				4724
36	Standard Deviation of Interest Expense				264
37					
38					
39				*Ratio*	
40	*Summary of Results*	*Plan 1*	*Plan 2*	*P1/P2*	
41					
42	Comprehensive Liquidity Index	1.91	1.41	1.35	
43	Expected Interest Expense	5067	4724	1.07	
44	Standard Deviation of Interest Expense	158	264	.60	

after-tax interest figures are totaled in row 18 to obtain total interest expense for each level of the prime interest rate for Plan 1. The expected after-tax interest expense appears in cell E20, and is calculated by multiplying the total interest expense figures in row 18 by the probabilities in row 7. For Plan 1, the expected interest expense is $5.067 million. The standard deviation of this interest expense is calculated in cell E21, given the probabilities in row 7. The analysis for Plan 1 was replicated into cells B25 through E37 and the short-term balance data changed to produce the parallel analysis for Plan 2.

A summary of the risk and return measures for the two plans appears in rows 39 through 44. For comparison purposes, we have divided the statistics from Plan 1 by those from Plan 2 in cells D42 through D44. The Comprehensive Liquidity Index for Plan 1 is 35 percent higher than that for Plan 2 indicating that Plan 1 produces a considerably lower stockout risk. Because of the use of less short-term debt, the variability of interest expense due to changes in the prime rate (in a standard deviation sense) for Plan 1 is only 60 percent of that of Plan 2. However, the after-tax interest expense for Plan 1 is 7 percent higher than for Plan 2; Plan 1 will cost an additional $343 thousand in after-tax expense.

In this decision methodology, it is now up to the firm to choose among the plans based on management's assessment of current shareholders' risk preferences. If the firm was one that in the past had pursued high-risk, growth-oriented financial and operating strategies, then it probably would have attracted investors with a preference for strategies such as this, and management should adopt a liquidity strategy in line with these preferences. That is, in this example, it should probably adopt Plan 2. If the firm had in the past been very conservatively managed, then its shareholders would probably prefer a less risky strategy, and Plan 1 should be adopted.[21] Of course, since most firms' operating and financial policies are neither extremely aggressive nor conservative, and since they are likely to consider several alternative strategies with diverse risk-return characteristics, their decisions will rarely be so obvious.

LIQUIDITY MANAGEMENT IN PRACTICE

It is useful to compare the approaches to the management of working capital outlined in this text with those used by practicing financial managers. When this comparison was made in past chapters, survey evidence was used

[21]In this discussion, we assume the existence of shareholder clienteles. These may be induced by the different tax consequences of high-risk, high-growth vs. conservative management policies. The existence of shareholder clienteles is currently under investigation by finance researchers, and the issue is still in doubt; see Copeland and Weston, *Financial Theory*, pp. 506–509.

to assess management practice. We will also cite evidence from surveys in assessing the practice of liquidity management. However, unlike the results of many other decisions in working capital management, the results of decisions regarding the management of aggregate liquidity are *observable*: from published financial statements, measures of aggregate liquidity can be calculated for samples of firms. Using this data, researchers in finance have been able to ascertain patterns in firms' aggregate liquidity decisions. This empirical research provides additional evidence on practice in the management of aggregate liquidity.

Let us first review the available survey evidence. Two surveys that addressed current practices regarding the management of aggregate liquidity were conducted in 1978. Both of these surveys utilized all or part of the Fortune listing of large firms. Smith and Sell found that about 30 percent of responding firms had a formal overall working capital policy, 60 percent had an informal policy, and 10 percent had no policy.[22] Larger firms were more prone to have a formal policy than were smaller firms. With regard to liquidity level, 22 percent of the firms indicated that they had a cautious (high liquidity) working capital policy, 28 percent indicated that they had an aggressive (low liquidity) policy, and the remainder indicated that they changed policy over time. Ratios used to assess liquidity position (in order of popularity) were the current ratio, the working capital turnover ratio (sales divided by net working capital), and working capital as a percentage of total assets. The use of the improved measures of liquidity management described in this chapter was not investigated in this survey.

Where Smith and Sell investigated the types of tools and policies used by firms, Johnson, Campbell, and Wittenbach were primarily concerned with the *importance* of several tools in the management of liquidity.[23] They found that the most popular objective of liquidity management was a very general one: to meet temporary financial problems as they arise. This overall objective was more popular than specific ones related to individual parts of liquidity management, such as the ability to meet creditor obligations. However, they found that measures of aggregate liquidity were important to firms relative to this objective mostly during periods when the firm was having liquidity difficulties. They did not investigate the techniques by which firms manage aggregate liquidity.

Let us now turn from survey evidence to empirical investigations of liquidity choice based on firms' financial statements. If the previously presented logic regarding an optimal level of aggregate liquidity is true and the

[22]K. V. Smith and S. B. Sell, "Working Capital Management in Practice," in K. V. Smith, Editor, *Readings on the Management of Working Capital* (St. Paul: West Publishing, 1980), pp. 51–84.

[23]J. Johnson, D. Campbell, and J. Wittenbach, "Problems in Corporate Liquidity," *Financial Executive* (March 1980), pp. 44–53.

liquidity policies of firms are based on this optimum, then there should be industry-to-industry variations in the aggregate liquidity structures of firms, because there is variation among firms in shortage costs should the firm run out of funds. For example, firms in some industries hold more illiquid fixed assets than others, and consequently, will take greater losses if the firm runs out of cash and is forced to sell these assets. These shortage costs, along with the risk cost of interest rate fluctuations, act to offset the lower expected interest expense on short-term debt and cause an optimum in aggregate liquidity. If there are variations in shortage costs, then the optimal level of liquidity should vary among industries. That is, the minimum point on the total cost curve of aggregate liquidity portrayed in Figure 10-1 should vary among firms in different business lines.

Two empirical studies have confirmed this variation. Gilmer found that the optimal level of liquid assets varied among industries and over time.[24] Gardner and Mills found the level of current liabilities to be associated with the level of assets and the firm's industry.[25] These findings suggest that there is an optimal liquidity position that firms seek, and that this position is dependent on the characteristics of a firm's business.

Another interesting result of these empirical investigations is that firms' aggregate liquidity positions have been declining over time. Gardner and Mills cite such evidence using the quick and current ratios as liquidity measures.[26] Belt finds the same trend when using the Cash Conversion Cycle as the measure of aggregate liquidity.[27] This suggests that either expected costs of cash shortages have declined over time or firms have become less averse to the risk of cash shortage.

These surveys and empirical investigations lead to important conclusions regarding the practice of liquidity management. Most firms have either a formal or informal aggregate liquidity policy. These policies may be aggressive, conservative, or may change over time, and are most important when firms are having problems within the working capital cycle. While not all firms have a formal policy and while many regard the measurement of liquidity as important only at certain times, empirical evidence suggests that *there is an optimal level of liquidity that depends on the characteristics of the firm.* The average value of this optimal level seems to be declining over time.

[24]Gilmer, "The Optimal Level of Liquid Assets."

[25]M. Gardner and D. Mills, "Determinants of Working Capital Policy in Non-Financial Industries, 1969–1979: Some Empirical Evidence," presented at the Financial Management Association Annual Meeting, 1982, and available from the authors at Illinois State University. Gardner and Mills also found that the size of the firm influenced the proportion of current assets held.

[26]Gardner and Mills, "Determinant of Working Capital Policy," p. 2.

[27]B. Belt, "The Trend in the Cash Conversion Cycle and Its Components," *Akron Business and Economic Review* (Fall 1985), pp. 48–54.

SUMMARY

This chapter addressed one of the firm's major decisions: the level of aggregate liquidity. Aggregate liquidity is the relationship between the firm's ability to generate cash and the firm's potential needs for cash on short notice. The level of the firm's aggregate liquidity constitutes a multiasset hedge against multiliability cash demands.

We first discussed issues in the measurement of the firm's aggregate liquidity position. Some traditional measures of the aggregate liquidity are the current ratio, quick ratio, accounts receivable turnover, and inventory turnover. Unfortunately, all of these are seriously flawed in some way as measures of aggregate liquidity. For example, the most popular of these measures (the current ratio) treats the liquidity of several types of current assets and liabilities as equivalent when in reality they are not. These ratios do not always properly signal changes in liquidity when such changes occur.

To accurately measure liquidity, improved indices are required. Four of these were illustrated: the Cash Conversion Cycle, the Comprehensive Liquidity Index, the Net Liquid Balance, and the Lambda index. While the signals regarding liquidity that these measures provide may sometimes conflict, all of these are improvements over the traditional measures in the assessment of liquidity position.

In prior chapters of this text, the structuring of current assets was discussed. For a given structure of current assets and a given amount of total debt, the firm's aggregate liquidity position is determined by how the total amount of debt is divided into floating-rate, short-term borrowings and fixed-rate, long-term borrowings. Even if financial futures are used to hedge against interest rate swings, the use of relatively more floating-rate, short-term debt increases the variability of interest expense. The use of more short-term debt also results in a higher risk of cash shortage. However, expected cost of short-term debt is usually lower, since the yield curve is usually upsloping. In setting the level of aggregate liquidity, the firm is therefore faced with a risk/return trade-off: more short-term debt means more risk but lower expected interest expense. Procedures were outlined to estimate the risk and return effects of various liquidity strategies.

Once the firm has decided on its level of aggregate liquidity, it must generate an advantageous structure for current debt that yields this level of liquidity. The next two chapters deal with the generation of a plan for current liabilities that results in the targeted level of liquidity while minimizing the cost of this financing.

Problems

10-1. A firm is trying to formulate its aggregate liquidity policy. One alternative involves financing with $600,000 in short-term notes

payable, and will result in the following projected financial position at the end of the upcoming year:

Projected Income Statement
Year Ending 12/31/8X

Sales	$3,000
Cost of Sales:	
Materials	$1,000
Labor	$1,500
Depreciation	$100
Interest Expense	$90
Earnings Before Taxes	$310
Taxes	$109
Earnings After Taxes	$201

Projected Balance Sheet
Year Ending 12/31/8X

Cash	$100	Accounts Payable (materials)	$200
Marketable Securities	$300	Accrued Labor	$400
Accounts Receivable	$500	Notes Payable	$600
Inventory	$700	Total Current Debt	$1,200
Total Current Assets	$1,600		
		Long-Term Debt	$200
Fixed Assets	$500	Equity	$700
Total Assets	$2,100	Total Lia. and Equity	$2,100

The average maturity of marketable securities is 90 days. The maturity of all notes payable is 120 days. Ignore the effects of any maturing of long-term debt. With this structure, the firm will have $100 in available borrowing remaining on its credit line. Use a 360-day year.

a. Compute the firm's projected Cash Conversion Cycle.

b. Compute the firm's projected Comprehensive Liquidity Index.

c. Compute the projected Net Liquid Balance.

d. Compute the firm's projected Lambda. In this calculation, assume the firm's cash flows are defined as its projected cash flow from operations (earnings after taxes plus depreciation) and that the standard deviation of these flows for the upcoming year is $250.

10-2. A firm is trying to decide on the structure of its debt financing. Based on capital structure considerations, the firm has decided to utilize a total of $10 million in total debt. It expects to have $1 million in unavoidable short-term funding and $5 million in ex-

isting long-term debt which it does not wish to refund. The total amount of debt to be structured is thus $4 million. Two alternatives are being considered. In alternative A, the firm would issue $1.5 million in long-term, fixed-rate debt and 2.5 million in short-term, floating-rate debt. In alternative B, the firm would issue $3 million in long-term, fixed-rate debt and $1 million in short-term, floating-rate debt. In either case, the interest rate on long-term debt will be 10 percent. The estimated probability distribution of interest rates on short-term debt for the upcoming year is:

Probability	.2	.5	.3
Interest Rate	7%	8%	9%

The firm is in the 35 percent tax bracket. For the upcoming year, compute the standard deviation of after-tax interest expense on the $4 million in new interest-bearing debt for each of the alternative structures.

Selected Readings

Bierman, H., K. Chopra, and L. J. Thomas, "Ruin Considerations: Optimal Working Capital and Capital Structure," *Journal of Financial and Quantitative Analysis* (March 1975), pp. 119–28.

Cohn, R. A., and J. J. Pringle, "Steps Toward an Integration of Corporate Financial Theory," in K. V. Smith, ed., *Readings on the Management of Working Capital*, Second Edition (St. Paul: West Publishing: 1980), pp. 35–42.

Emery, G., "Measuring Short-Term Liquidity," *Journal of Cash Management* (July/August 1984), pp. 25–32.

Gilmer, R., "The Optimal Level of Liquid Assets: An Empirical Test," *Financial Management* (Winter 1985), pp. 39–43.

Melnyk, Z. L., and A. Birati, "Comprehensive Liquidity Index as a Measure of Corporate Liquidity," *Scientific and Behavioral Foundations of Decision Sciences* (Atlanta, Ga.: Southeastern Region of the American Institute for Decision Sciences, 1974), pp. 162–65.

Richards, V. D., and E. J. Laughlin, "A Cash Conversion Cycle Approach to Liquidity Analysis," *Financial Management* (Spring 1980), pp. 32–38.

Shulman, J., and R. Cox, "An Integrative Approach to Working Capital Management," *Journal of Cash Management* (November/December 1985), pp. 64–67.

Van Horne, J. C., "A Risk-Return Analysis of a Firm's Working Capital Position," *The Engineering Economist* (Winter 1969), pp. 71–89.

Van Horne, J. C., "Corporate Liquidity and Bankruptcy Costs," Research Paper Number 205, Graduate School of Business, Stanford University.

case

The Jones Company
Deciding on the Level of Liquidity

The Jones Company was the product of a financial transaction called a "leveraged buyout." In such a transaction, the managers of a unit of a large company purchase the unit from the parent, usually financing the purchase with a relatively small equity contribution. The bulk of the financing is provided by loans secured by the purchased unit's assets and by loans from the selling firm. This method of divestiture is sometimes used by large firms to rid themselves of unwanted divisions.

Four years ago, the Jones Company had been the Jones Division of FCS Incorporated, a large conglomerate. The division manufactured and sold office furniture, a business that the top management of FCS had come to regard as low-growth, marginally profitable, cyclical, and generally not compatible with FCS's long-term corporate strategy. The three upper-level managers of the division were not of the same opinion. Consequently, when the division became available for purchase, they put in a bid, which was accepted. The purchase of the unit by the managers was advantageous in that it preserved their jobs, which would have been at risk under other ownership. The purchase was financed via a $1.5 million equity contribution, a $50 million unsecured note from FCS, and short- and long-term secured loans from a financial intermediary.

Things went well for the first few years. Though sales did not grow much, the new owners were able to use their knowledge of the industry to lessen the effects of the business cycle on the firm's earnings. Equity grew, and the long-term portion of the loan from the financial intermediary was reduced on schedule. (The firm's financial statements as of December 31, 1983, are presented in Exhibit 1.) The beginning of the firm's fifth year brought a crisis, however. The note from FCS allowed the Jones Company to make interest payments only for the first four years to allow the firm to "get on its feet." Very substantial principal payments ($10 million per year) were scheduled to be due starting in 1984, however, and it was clear that these payments could not be made from the firm's cash flow, even if needed fixed asset purchases were delayed (see Exhibit 2). It seemed that a restructuring of the FCS note would be necessary. At the same time, the owners decided to give serious thought to the firm's capital structure and its mix of short- and long-term borrowings.

Exhibit 1

Financial Statements
The Jones Company
Fiscal Year 1983
Rounded Thousands of Dollars

Balance Sheet
December 31, 1983

Cash	$ 4,200
Marketable Securities	410
Accounts Receivable	24,996
Inventories	34,421
Other Current Assets	1,127
Total Current Assets	$ 65,154
Gross Fixed Assets	$ 55,832
Less: Accum. Depreciation	22,333
Net Fixed Assets	33,499
Other Assets	3,790
Total Assets	$102,443
Accounts Payable	14,547
Notes Payable (FCS)	10,000
Accruals, etc.	10,859
Short-Term Borrowings	1,365
Current Portion–LTD	1,000
Total Current Liabilities	$ 37,771
Long-Term Debt	$ 15,000
Long-Term Note to FCS	$ 40,000
Common Stock	$ 1,500
Retained Earnings	8,172
Total Common Equity	9,672
Total Lia. and Equity	$102,443

Statement of Income and Retained Earnings
Fiscal Year Ending December 31, 1983

Sales	$174,972
Cost of Goods Sold	121,605
Selling and Administrative Expenses	42,173
Interest Expense — FCS Note	5,000
Other Interest Expense	2,084
Earnings Before Taxes	$ 4,110
Taxes	1,891
Earnings After Taxes	$ 2,219
Common Dividends	0
Changes in Retained Earnings	$ 2,219

Exhibit 2

The Jones Company
Proforma Statement of 1984 Net Operating Cash Flow
if FCS Note is Not Refinanced

Sales[1]	$180,221
Cost of Goods Sold[1]	125,254
Selling and Admin. Exp.[2]	43,270
Interest Expense — FCS Note[3]	4,500
Other Interest Expense[4]	2,090
Earnings before Tax	$ 5,107
Taxes[5]	2,120
Earnings after Taxes	$ 2,987
Depreciation Addback	5,583
Cash Dividends	0
Operating Net Cash Flow	$ 8,570
Principal Payment — FCS Note	10,000
Principal Payment — Other LTD	1,000
Operating Net Cash Flow after Principal Payments	−$2,430

[1]All entries with this mark are forecast strictly on a 3 percent growth over 1983.

[2]This entry consists of fixed operating expenses plus depreciation. Depreciation is expected to be the same as 1983.

[3]It is assumed that the $10 million payment is made about one-half of the way through the year, and thus average balance on the note is $45 million.

[4]Due to financial intermediary. Includes interest on new required short-term financing.

[5]Based on the firm's expected tax rate.

The owners believed that there was an optimal overall mix between debt and equity that minimized a firm's cost of capital. They also recognized that the capital structure of the Jones Company as of early 1984 was probably far from this optimum. Specifically, with 90 percent debt and 10 percent equity, they knew that interest rates on debt and the risk of returns to equity were far higher than need be. However, they did not want to make additional equity contributions or dilute their ownership interest by selling equity to outside investors. Therefore, they decided not to issue any new equity financing, but instead decided to slowly reduce the firm's level of debt by paying off long-term debt as it became due, at least for the next few years.

The refinancing problem then resolved into how the required total debt financing should be divided into short- and long-term portions. The owners contacted the financial intermediary and were advised that the intermediary would be willing to lend the $50 million needed to refinance the FCS note. The firm's other borrowings from the intermediary would be renegotiated at the same time. Because of the Jones Company's very highly leveraged position, rates on all short-term borrowings would be 13 percent per year. Further, it was standard that such borrowings would

have to be reduced to a zero balance for at least 60 consecutive days per year to evidence that such borrowings were truly short term and intended to cover seasonal needs only. The owners calculated that this would limit short-term borrowings to (at most) an average short-term balance of $12 million. All long-term borrowings would then carry an interest rate of 16 percent per year, with principal payments made in equal installments over the next 20 years (a repayment schedule similar to that for a sinking fund debenture).