Chapter -1

Introduction to the Management of Working Capital

This chapter sets forth the themes and concepts which form the basis for the discussion and analysis in this text. We first define working capital, then show why it exists and why it is important in the financial management of firms. We then present three sets of ideas that we will use extensively later: the principles of modern finance (shareholder wealth maximization and cash flow decision making), the importance of risk, and the role of the computer in its relationship to financial analysis. The organization of the rest of the book is also outlined, followed by a summary that concludes this chapter. As with the introductory chapters of most advanced texts, it is assumed that students are familiar with many of the concepts and ideas presented. However, the principles outlined in this chapter are so important that a thorough review and grounding is worthwhile.

WHAT IS WORKING CAPITAL MANAGEMENT?

The financial management of business firms involves three functions: the management of long-term assets, the management of long-term capital, and the management of short-term assets and liabilities. The first of these is *capital budgeting*, the second is the management of *capital structure*, and the last is *working capital management*. The management of working capital, which is the topic of this book, is concerned with the managment of the assets and liabilities in the *top half* of the balance sheet (see Figure 1-1). Included here are assets such as cash, marketable securities, accounts receivable, inventory, prepaid expenses, and other current assets; also, liabilities such as accounts payable, wages payable, and accruals. We will analyze decisions such as:

- ☐ How should the firm manage its cash?
- ☐ To whom should the firm grant credit?

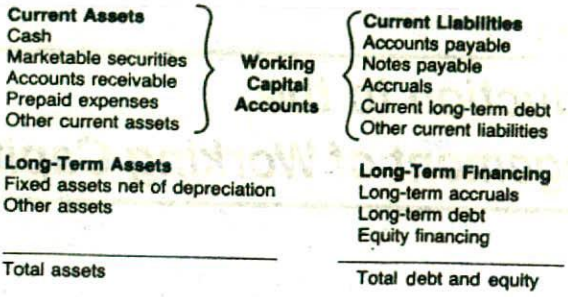


Figure 1-1. A Typical Firm's Balance Sheet.

- ☐ How much inventory should the firm keep?
- ☐ What should be the composition of the firm's current debt?

In any finance text, the authors must make a decision regarding what should be presented. Specifically, should the text concentrate on the current financial practices and procedures of existing firms or should it present the best that finance has to offer (the state of the art)? In this text, while some recognition of current procedure is included, we will concentrate on the *best* methods currently available. In this sense, the text is *prescriptive*: it recommends the most advantageous ways of addressing the problems of working capital management.

WHY IS WORKING CAPITAL NECESSARY?

To produce the best possible returns, firms should keep no unproductive assets and should finance with the cheapest available sources of funds. In this section, we discuss why, in general, it is often advantageous for the firm to invest in short-term assets and to finance with short-term liabilities. More detailed explanations for the existence of specific working capital accounts are provided later in the text.

In a "perfect" world, there would be no necessity for working capital assets and liabilities. In such a world, there would be no uncertainty, no transaction costs, information search costs, scheduling costs, or production and technology constraints. The unit cost of producing goods would not vary with the amount produced. Firms would borrow and lend at the same interest rate. Capital, labor, and product markets would reflect all available information and would be perfectly competitive. In such a world, it can be shown that there would be no advantage for firms to invest or finance in the short term.[1]

[1]See W. Beranek, "Towards a Positive Theory of Working Capital," available from W. Beranek at the University of Georgia.

But the world in which real firms function is not perfect. It is characterized by the firm's considerable uncertainty regarding the demand, market price, quality, and availability of its own products and those of suppliers. There are transactions costs for purchasing or selling goods or securities. Information is costly to obtain, and the firm is faced with limits on the production capacity and technology that it can employ. There are fixed as well as variable costs associated with producing goods for sale, and there are spreads between the borrowing and lending rates for investments and financings of equal risk. Information is not equally distributed and may not be fully reflected in the prices in product and labor markets, and these markets may not be perfectly competitive.

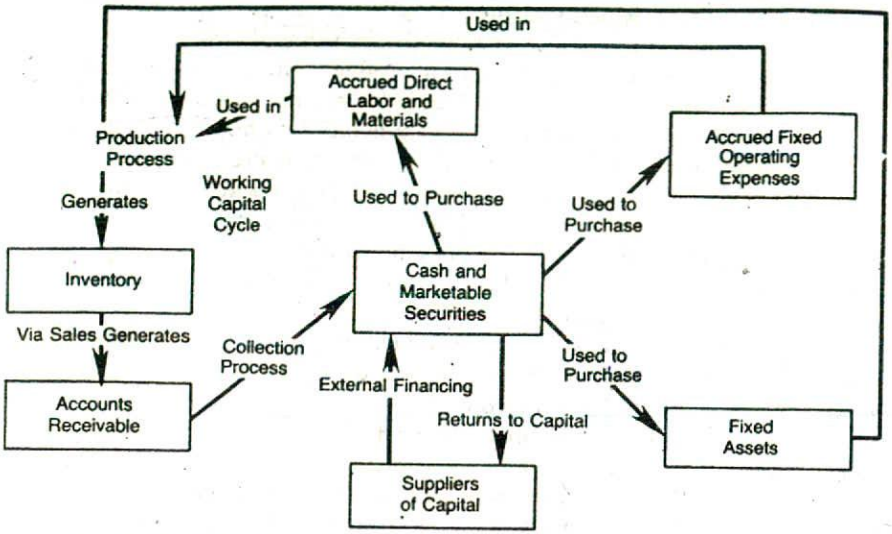
These real-world circumstances introduce problems with which the firm must deal. While the firm has many strategies available to address these circumstances, *strategies that utilize investment or financing with working capital accounts often offer a substantial advantage* over other techniques. For example, assume that the firm is faced with uncertainty regarding the level of its future cash flows and will incur substantial costs if it has insufficient cash on hand to meet expenses. Several strategies may be formulated to address this uncertainty and the costs that it may engender. Among these strategies are some that involve working capital investment or financing such as holding additional cash balances beyond expected needs, holding a reserve of short-term marketable securities, or arranging for the availability of additional short-term borrowing capacity.[2] One of these strategies (or a combination of them) may well be the least costly approach to the problem. Similarly, the existence of fixed set-up costs in the production of goods or of uncertainty regarding the future price or availability of goods may be addressed in several ways, but one possible alternative is to hold inventory.

By these examples, we see that strategies using working capital accounts are some of the possible ways firms can respond to many of the problems engendered by the imperfect and constrained world in which they deal. One of the major features of this world is uncertainty (risk), and it is this feature that gives rise to many of the strategies involving working capital accounts.

THE IMPORTANCE OF WORKING CAPITAL MANAGEMENT

In addition to its use as a means of handling uncertainty, the management of working capital plays an important role in maintaining the financial health of the firm during the normal course of business. This critical role can be seen by examining Figure 1-2, which loosely portrays the flow of resources through the firm. By far the major flow, in terms of its yearly magnitude, is

[2]These and other strategies for dealing with the uncertainty in the firm's cash flows will be discussed in Chapter 3.



A square is a balance sheet account. An arrow indicates a flow. Note that, for a service firm, there is little or no inventory, so the inventory generation step is deleted. All the other parts of the resource flow cycle are unaffected.

Figure 1-2. Resource Flows for a Manufacturing Firm.

the *working capital cycle*. This is the loop which starts at the cash and marketable securities account, goes through the current accruals accounts as direct labor and materials are purchased and used to produce inventory, which is in turn sold and generates accounts receivable, which are finally collected to replenish cash. The major point to notice about this cycle is that *the turnover (or velocity) of resources through this loop is very high* relative to the other inflows and outflows of the cash account.

To see why this cycle is critically important to the firm's survival, visualize the cash account as a bathtub with both the drain and the faucet open. While there are other inflows and outflows, the *major* flow is the working capital cycle. As long as the firm has cash or marketable securities on hand, it can pay bills and thus survive. But if for some reason the resources stop flowing in, as when one of the working capital accounts slows in turnover and starts draining off resources, the level of the tub starts to fall. Unless the firm takes relatively costly action, such as raising new external funds, reducing dividend outflows, or postponing capital expenditures, the resources will all drain out, the firm will be unable to pay bills, and financial embarrassment will occur. It is clear that *the working capital cycle is the lifeblood of the firm.*

It would seem a practical solution to this problem to keep a very high balance in the cash and marketable securities account (analogous to a very high level of water in the bathtub). In this way, when flow problems occur,

hasty and painful remedial actions will not be necessary since the large reserve provides some safety; the leaks can be plugged before the water is exhausted. However, as we will see in later chapters, such a strategy can depress the level of a firm's profitability, since cash and marketable securities are two of the firm's least profitable assets. Too little water (liquidity), and the firm is subject to bankruptcy risk if the inflow slows even slightly. Too much liquidity, and profits are lower than necessary. A major function of decision making for working capital is the management of the various working capital accounts with regard to the firm's level of liquidity: not too much liquidity and not too little liquidity.

Besides this basic and continuing reason for the importance of working capital management to the firm, several developments in the late 1970s and early 1980s led to increasing concern for the management of these accounts. One of these developments was a level of interest rates that climbed extremely high by historic standards. Working capital accounts, particularly accounts receivable and inventory, are major investments of the firm; the rise in interest rates caused the financing of investments to become more costly. A second development was the rapid evolution of deregulated money markets. This evolution provided financial managers with access to many new instruments for short-term financing and investment. Together, these developments kept corporate treasurers seeking new and better ways of managing individual working capital accounts to balance the firm's overall liquidity position.

PRINCIPLES OF MODERN FINANCE

To be most effective, management should be guided by a set of principles that can be applied to decision making in diverse situations. Over the last 30 years, financial researchers, including theoreticians and empiricists, have developed very significant insights about how financial markets work and how financial decisions should be made. The principles for financial decision making that were developed during this period form the basis for the management of working capital as discussed in this text. Two of the most important of these principles are *shareholder wealth maximization* and *cash flow decision making*.

Shareholder Wealth Maximization Let us first address the question of the appropriate goal of management in a publicly held firm. Shareholders, who own the firm, hire managers to act in such a way as to maximize their wealth. The only part of the shareholder's wealth that the manager can influence is the *market value of the firm's shares* held by the shareholder. We therefore say that *to maximize the wealth of shareholders, the firm should maximize the price of its shares*. In the finance literature, this is called the

principle of *shareholder wealth maximization*.[3] The methodologies for the management of working capital presented in this text follow this principle.

Cash Flow Decision Making The principle of shareholder wealth maximization must be put into operation. Just what actions should managers take to maximize the trading price of the firm's shares? Since shares are valued in the capital market, specifying these actions requires an understanding of how such markets value shares. Until a few years ago, it was thought that the market valued (looked at in determining price) the accounting earnings of the firm. However, there has been considerable recent evidence that the market reflects, instead, the public information regarding the firm's *cash flows*. Accounting earnings appear to be valuable information only when they reflect cash flow.[4] Therefore, in order to maximize shareholder wealth, the firm should *maximize the value of the firm's cash flows*.

The principle of maximizing the value of the firm's cash flows raises two additional questions. First, what types of cash flows are we talking about? Second, how is the value of a cash flow to the firm determined? Referring to the first question, if markets allow for *existing* cash flows, then only *changes in cash flows* result in changes in the market price of shares. Thus changes in cash flows are relevant considerations in making financial decisions. We call these changes in cash flows the *marginal cash flow* associated with a financial decision.[5] For example, if the firm's cash flow is expected to be $100,000 in a particular period, and a given decision will increase the expected flow to $110,000, the marginal cash flow associated with this decision in that period is $10,000.

Turning now to the question of valuation, shareholders are assumed to be risk averse and (because of the time value of money) are assumed to value cash flows further in the future less than flows closer to the present. Thus,

[3]Note that it is assumed here that shareholders prefer more wealth to less. Note also that it is not implied that all real-world managers follow this principle; in fact, the structuring of compensation for managers so that they follow the principle of shareholder wealth maximation is a major concern in the business literature.

[4]Important articles in the area include S. Sunder, "Relationship between Accounting Changes and Stock Prices: Problems of Measurement and Some Empirical Evidence," in *Empirical Research in Accounting: Selected Studies*, 1973, pp. 1–45, and R. S. Kaplan and R. Roll, "Investor Evaluation of Accounting Information: Some Empirical Evidence," *Journal of Business* (April 1972), pp. 225–257. A review of the major literature is presented in T. E. Copeland and J. F. Weston, *Financial Theory and Corporate Policy*, Second Edition (Reading, Mass.: Addison-Wesley, 1983), pp. 319–327.

[5]For a relevant marginal cash flow to occur, the cash flows for the firm taken as a whole must change. Note that some financial decisions will change the cost allocation pattern of cash flows among the firm's units without changing the overall cash flows of the firm; such variations in allocation among units are irrelevant from a financial standpoint.

the riskier the cash flow and the further in the future it occurs, the less its contribution to the value of the firm's stock and thus to shareholder wealth. This risk and time pattern can be addressed via the use of *net present value with risk-adjusted discount rates*. In the risk-adjusted discount rate, net present-value procedure, the value of the cash flow is computed as:

$$\text{Net Present Value} = E(NCF_t)/(1+k)^t \qquad (1\text{-}1)$$

where $E(NCF_t)$ is the expected net cash flow, k is the per-period risk-adjusted discount rate, and t is the number of periods in the future at which the cash flow is expected to occur. The risk-adjusted discount rate is determined as:

$$k = R_f + \text{risk premium} \qquad (1\text{-}2)$$

where R_f is the risk-free rate and accounts for the time value of money, and the risk premium accounts for the risk of the cash flow.

For example, suppose that a marginal cash flow of $10,000 is expected to occur two periods in the future, the risk-free rate is 6 percent per period, and the risk premium for a flow of this level of riskiness is 9 percent per year. The risk-adjusted discount rate would be 15 percent and the net present value of the flow would be:

$$\text{Net Present Value} = \$10{,}000/(1.15)^2 = \$7{,}561 \qquad (1\text{-}3)$$

The risk-free rate is usually estimated as the per-period interest rate on short-term federal government debt obligations. Methods of estimating the risk premium are discussed in the next section of this chapter.

The principles of modern finance may be summarized as follows: In making their decisions, the firm's managers should act so as to maximize the wealth of shareholders, which means to maximize the market price of their shares. This market price is maximized by making decisions based on the net present value of the marginal cash flows associated with the firm's decisions; higher net present values are preferred. While this sounds like a relatively simple procedure, a few caveats are in order. Many complications arise in the execution of this rule, and management consequently must be very careful in its application. Decisions regarding the firm's policies do not affect only shareholders; there are other stakeholders in the firm besides shareholders, including bondholders, employees, customers, and so forth. The reactions of these stakeholders to management decisions can affect shareholder wealth via the firm's cost of funds, labor expense, and sales revenue.[6] Also, because the market prices of shares reflect only public information, differences in

[6]See B. Cornell and A. Shapiro, "Corporate Stakeholders and Corporate Finance," *Financial Management* (Spring 1987), pp. 5–14.

information on the expected future cash flows of projects and their risk must be communicated to capital markets. This communication is a costly and difficult process.[7] While such difficulties pose a considerable challenge to management, they do not negate the basic principles of shareholder wealth maximization.

RISK AND FINANCIAL DECISION-MAKING

Because risk management is a major part of many working capital strategies, a substantial portion of this text will deal with risk in relation to working capital management. In finance, risk is dealt with in two steps. First, the amount and types of risk are *assessed* (measured or estimated in some way). Second, the assessed risk is *addressed* (accounted for in the decision-making process). Addressing risk takes two forms: either the risk may be *priced* within the financial decision process and/or a *hedging strategy* may be formulated to remove all or part of the risk. In this section, we review methods of assessing and addressing risk.

Assessing Risk For the purposes of this text, we use two concepts of risk: *total risk* and *systematic risk*. "Total risk" is the total variability of returns from a decision, while "systematic risk" is the degree of comovement between the decision's returns and the returns on a large group of assets. The amounts of these risks are assessed in quite different ways. Total risk is assessed by *sensitivity analysis* and *simulation analysis*, and systematic risk is assessed by estimating the *beta* of the decision or project.

Sensitivity analysis is a method of estimating the effect of variation in *individual output variables* on important outcome variables.[8] Sensitivity anal-

[7]Providing accounting information (annual reports, and so forth) to the market does not solve this problem, since such accounting data deals with past cash flows, and is not primarily concerned with the future cash flows of the firm (which are used by the market to value shares). Research has shown that markets react to publicly available information, and not any information known to the firm's managers but not to the public (for a literature review, see Copeland and Weston, *Financial Theory*, pp. 285–307 and 327–346). Consequently, management must signal its estimates of future cash flows to the market in order to affect shareholder wealth. It may do this through public announcements, dividend policy, capital structure policy, management's holdings of the firm's shares, or in various other ways, though the resulting signals are subject to misinterpretation. See, for example, S. Ross, "The Determination of Financial Structure: The Incentive-Signalling Approach," *Bell Journal of Economics* (Spring 1977), pp. 23–40.

[8]An excellent discussion of sensitivity analysis is presented in K. L. Hastie, "One Businessman's View of Capital Budgeting," *Financial Management* (Winter 1974), pp. 36–44. Simulation analysis is discussed in B. D. Hertz, "Risk Analysis in Capital Investments," *Harvard Business Review* (January–February 1964), pp. 95–106 and in most major corporate finance texts.

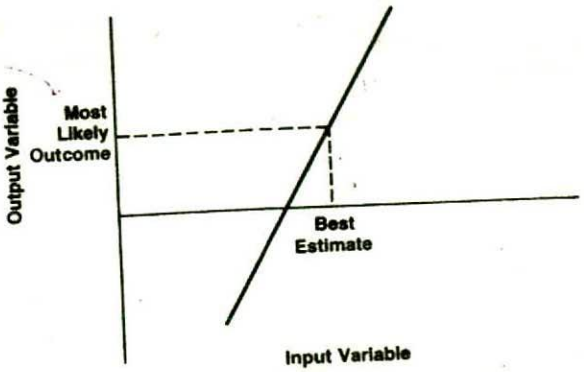


Figure 1-3. A Possible Sensitivity Analysis Plot.

ysis involves asking "what if" questions regarding the effect of one variable in a financial situation on another. For example, in a cash forecasting situation, the analyst might make estimates of the likely future levels of sales, fixed expenses, material and labor expenses, and so forth. The result of the analysis procedure, as we will see later in this text, is a forecast of future cash needs and surpluses that is very useful for planning purposes. But this forecast is subject to various uncertainties. What if sales are more or less than estimated? What if fixed expenses are higher than estimated? Sensitivity analysis addresses questions such as these, which occur frequently in the management of working capital and in other financial decisions.

To use sensitivity analysis to assess the relationship between an input variable and an important output variable of a financial analysis algorithm, the estimate of the input variable is changed, the algorithm is recomputed, and the effect on the outcome variable observed. After several such changes and recalculations, the results are graphed, with a result similar to that shown in Figure 1-3. A steeply sloped function indicates that the outcome variable is very sensitive to changes in the input variable; a relatively flat function indicates that the output variable is not very sensitive to changes in the input variable. Hence the name, "sensitivity analysis."

Sensitivity analysis is a very useful tool for the assessment of the risk associated with input variables. Using it, the analyst can decide which input variables are critical to a particular decision and which are not. Sensitivity analysis does not require the specification of probability distributions of the input variables. It can provide important information in many decision situations. Sensitivity analysis is, however, restricted to the assessment of *individual variable effects*. Firms, on the other hand, are often more concerned with the *total variability* of critical outcome variables. One method of estimating this total variability is simulation analysis.

Simulation analysis is an attempt to assess the total risk (variability) of outcomes based on variation in *all* the uncertain input variables taken si-

Table 1-1. Some Possible Probability Distributions Used as Part of the Simulation Analysis of a Cash Forecast.

Direct Labor as a Percent of Sales		*Sales for January*	
Percent	*Probability*	*Dollars*	*Probability*
61	.20	10,000	.10
62	.60	12,000	.25
63	.20	14,000	.30
		16,000	.25
		18,000	.10

multaneously. To do simulation analysis, the analyst starts by making estimates of the probability distributions of the variables which serve as inputs to the analysis process. For a cash forecast, the analyst would have to obtain estimates not only of the *most likely levels* of each uncertain variable (sales, fixed expenses, direct labor as a percent of sales, and so forth) but also of the *probabilities of other levels*. The estimated probability distributions may be continuous or discrete, but as a practical matter, businessmen are generally more comfortable estimating discrete distributions. Some probability distributions that might result are shown in Table 1-1.

Once every input variable that has any reasonable degree of uncertainty has been quantified in this way, analysis can begin. First, using a random number table or random number generator, an outcome for each uncertain variable for the first trial is determined. For example, consider the probability distribution of direct labor as a percent of sales in Table 1-1. If a two-digit random number was being used, there are 100 possible random numbers from 00 to 99. The analyst might assign the numbers 00–19 to 61 percent, 20–79 to 62 percent, and 80–99 to 63 percent. This way, the probabilities of occurrence of the three outcomes in the simulation procedure for this variable match those estimated by the analyst (for example, there is a 20 percent chance that direct labor will be 61 percent of sales, since this outcome is represented by the 20 numbers—00 through 19—of the possible 100). A random number would then be drawn; assume it is 82. This falls in the range for direct labor to be 63 percent of sales, which would then be the estimate of direct labor for the first trial. All the other uncertain variables would be determined for the trial in the same fashion. The analysis algorithm (in this example, the cash forecasting procedure) would then be executed using these trial variable values and the critical outcome variables (cash needs and surpluses by period in the case of a cash forecast) would be calculated. Several hundred trials would be run and a frequency histogram of the results generated. A typical histogram for a cash forecasting example is presented in Figure 1-4.

It should be obvious that a histogram of this sort gives a lot of useful information about the ranges of critical outcome variables and the probabilities of these ranges. The simulation analysis method is, however, not without

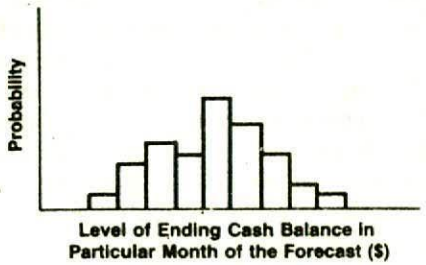


Figure 1-4. A Possible Frequency Histogram Generated from the Results of a Simulation Analysis of a Cash Forecast.

its difficulties. It requires estimates of the probability distributions of many variables. It is difficult to execute when there are significant covariances among the uncertain variables (that is, when drawings for input variables are not independent). Finally, it is computationally cumbersome.

In addition to total risk, which is assessed by sensitivity and simulation analysis, our other concept of the relevant risk for a decision is *systematic risk*, which is measured by beta. In this concept, total risk is divided into two components: *diversifiable risk* and *nondiversifiable risk* (also called systematic risk). Diversifiable risk is the risk particular to an individual asset, that is, the fluctuation in the asset's returns that is due to peculiarities of that asset. Nondiversifiable risk is the inherent economic risk common to all assets to varying degrees. Nondiversifiable risk can also be thought of as *market* or *general economic risk*. The total risk of any asset or security is the sum of its diversifiable and nondiversifiable risks. Diversifiable risk can, however, be avoided if the investor holds a sufficiently large number of assets (10 or 15 will do). Since investors do not like risk, they diversify away the diversifiable risk of their individual security holdings. Thus, in this concept only the nondiversifiable risk of the projects undertaken by the firms is said to matter to investors; only nondiversifiable risk then affects security prices.

Beta is a measure of nondiversifiable risk; it is the relative comovement of the project's returns with the returns on all assets and securities. Mathematically, beta is measured as:

$$b_i = COV(R_i,R_m)/VAR(R_m) \qquad (1\text{-}4)$$

where b_i is the beta of project i, $COV(R_i,R_m)$ is the covariance of the return on the project with the market portfolio (a completely diversified portfolio of assets), and $VAR(R_m)$ is the variance of the return on the market portfolio. Betas of a firm's common shares are measured by regressing the returns on these shares with those of the market as a whole. Firms whose projects' returns do not vary as much as market returns (such as utility firms) tend to have common share betas less than one; firms whose projects' returns vary much more with market returns (such as manufacturers of recreational vehicles)

tend to have common share betas greater than one. The beta of a firm's common shares can be used in evaluating new projects if these projects are expected to have the same beta as the firm's current average project.

However, when new projects are expected to differ in their response to nondiversifiable risk relative to existing projects, estimation of the appropriate beta is a bit more difficult. The finance literature suggests two methods of estimating a beta of such an individual project. The first, called the *pure-play technique*, involves finding a firm engaged solely in the line of business represented by the project and then using that firm's beta as an estimate of the nondiversifiable risk of the project. The other method involves finding several firms with divisions engaged in the line of business of the project and using *mathematical programming* to assess the betas of the relevant divisions. This divisional beta would then be used as the project's beta.[9]

Addressing Risk Once the amount of risk in a working capital or other financial decision has been assessed, the risk must be *priced* to reflect shareholders' risk aversion and/or a *hedging strategy* must be formulated to remove the risk. The firm may consider using either or both of these techniques relative to any financial decision. When risk is priced but not hedged, the cash flows are less valuable (because of shareholder risk aversion), but the firm does not bear the cost of a hedging strategy. Conversely, when a hedging strategy is employed, the expected cash flows are lower (because of the cost of the hedge), but are less risky.

Pricing risk when using net present value procedures involves the determination of the appropriate risk premium to use in discounting risky cash flows [see equation (1-2)]. There are two commonly used approaches to determining this risk premium: benchmark risk-adjusted discount rates and the Capital Asset Pricing Model (CAPM).[10]

Benchmark risk-adjusted discount rates can be used to reflect either total risk or systematic risk. In the benchmark risk-adjusted discount rate method, the analyst determines the appropriate risk premium by comparing the risk of the marginal cash flow with that of traded assets and employing

[9]For more extensive discussion of these methodologies, see R. J. Fuller and H. S. Kerr, "Estimating the Divisional Cost of Capital: An Analysis of the Pure-Play Technique," *Journal of Finance* (December 1981), pp. 997–1009, and J. A. Boquist and W. T. Moore, "Estimating the Systematic Risk of an Industry Segment: A Mathematical Programming Approach," *Financial Management* (Winter 1983), pp. 11–18.

[10]A third approach to the pricing of risk is the Arbitrage Pricing Theory (APT), originally presented in S Ross, "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory* (1976), pp. 341–360. In the APT, other factors besides the risk-free rate of interest and nondiversifiable risk (which are priced in the CAPM) may affect required return. However, more development is required before the APT can be widely used as a risk-pricing mechanism; see D. Bower, R. Bower, and D. Logue, "A Primer on Arbitrage Pricing Theory," *Midland Corporate Finance Journal*, Vol. 2, No. 3 (Fall 1984), pp. 31–40, particularly page 40.

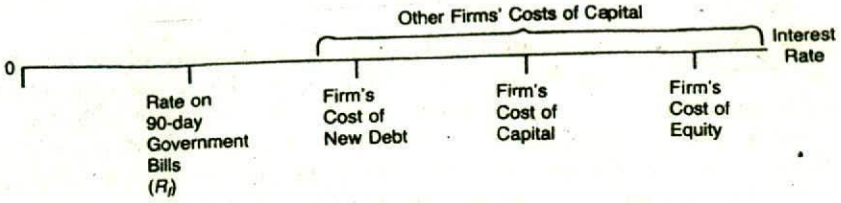


Figure 1-5. Some Benchmark Market Interest Rates.

the market's required returns for traded assets of comparable risk. Sometimes, a traded asset of similar risk to the cash flow being evaluated cannot be found, and the analyst must start with a market-required return and adjust upward or downward based on the relative risk of the cash flow being evaluated. This is called the "benchmark" procedure because the analyst starts with the market's required return on assets of known risk—the "benchmarks." These benchmarks may include the risk-free rate of interest, the firm's cost of debt, the firm's cost of capital, the firm's cost of equity, and other firms' costs of capital. Figure 1-5 presents some of these rates.

Let us replicate the logic of an analyst in deciding on a risk-adjusted discount rate using the benchmark method. He or she knows the current risk-free rate; this represents the time value of money with no risk. No risky cash flow can be discounted at a rate less than this. The analyst also knows the firm's marginal cost of capital, which represents the appropriate market-determined discount rate on projects that are as risky as the average project within the firm's current portfolio of projects. Assume that the cash flow being considered is perceived by the analyst to be slightly more risky than the firm's current cash flows from projects. (This risk estimate might be based on any of the risk assessment methodologies discussed previously, or it might be made in a more informal manner.) At this point, the analyst might judgmentally adjust the firm's cost of capital upward to a discount rate that is appropriate for this higher risk. Alternatively, he or she might compute the costs of capital of firms with similar financial structures (financial risk) where these firms are of business risk similar to that of the cash flow to be analyzed. In this way, a discount rate reflecting the time value of money and the appropriate level of business and financial risk would be estimated. It is obvious that this is in part a subjective process since adjustments are often made judgmentally from the benchmarks.

The capital asset pricing model (CAPM), like the risk-adjusted discount rate method, is used to determine the appropriate discount rate to apply to the expected value of a risky future cash flow.[11] The CAPM implies that the risk premium used in obtaining the appropriate discount rate should be based only on the nondiversifiable risk, not on the total risk, of a particular project.

[11]An extensive treatment of the capital asset pricing model can be found in Copeland and Weston, *Financial Theory*, pp. 185–211.

In the CAPM, only systematic risk is the relevant measure of the project's business risk. Unlike the benchmark method, the CAPM enables the analyst to directly compute the risk-adjusted discount rate associated with the risk of such a cash flow. Once the project's beta has been estimated, this statistic is used in the CAPM formula to obtain the required rate of return of shareholders for the equity investment in the project. The CAPM formula is:

$$R_i = R_f + b_i(R_m - R_f) \tag{1-5}$$

where R_i is the required return on the equity investment in project i, R_m is the expected return on the market portfolio, and the other symbols are as before. This required return on equity investment is used in conjunction with the costs of the other sources of the firm's long-term capital and its target capital structure weights to estimate a required return for that particular project. For example, assume that a firm is trying to decide on an appropriate required rate of return for a particular project via the CAPM method. Based on the business line of the project, its beta is estimated as 1.4. The risk-free rate of interest is 6 percent per year and the expected return on a fully diversified portfolio of assets is 13 percent. The firm's capital structure includes 40 percent interest-bearing debt; new issues of debt are expected to yield 9 percent. The remaining 60 percent of the firm's capital structure is equity. The firm is in the 33 percent marginal tax bracket. To find the required return, we first use equation (1-5) to obtain the required return on equity for projects with a beta of 1.4 and the assumed market conditions:

$$R_i = 0.06 + 1.4(0.13 - 0.06) = 0.158 \tag{1-6}$$

so the appropriate required return on equity is 15.8 percent per year. We then compute the required return for the project:

$$\text{Required Return} = 0.6(0.158) + 0.4(1 - 0.33)(0.09) = 11.9\% \tag{1-7}$$

A comment on the benchmark risk-adjusted discount rate and CAPM methods is appropriate. Both are methods for determining the market's required returns for projects of a particular risk. These returns are then used in the net present value methodology to evaluate projects and other decisions. This project/decision evaluation procedure for physical assets is essentially a search for *excess returns*—projects/decisions whose net present value is greater than their market required returns.[12] A great deal is sometimes made about the fact that these two approaches do not necessarily produce exactly the same discount rate. There are several reasons for this: the judgmental nature

[12]Note that the search for excess returns is restricted to *physical assets*. In an arbitrage-free capital market, all traded securities are priced such that their net present values are zero; there are no excess returns to be made without insider information. See R. Roll, "A Critique of the Asset Pricing Theory's Tests," *Journal of Financial Economics* (March 1977), pp. 129–176.

of the benchmark process, statistical difficulties in the estimation of a project's beta, and so forth. However, there are about equal difficulties associated with the application of *either* method; neither method completely dominates. Thus, sophisticated firms should be familiar with each method and should make an informed choice based on their view of the situation. We will use both methods in determining risk-adjusted market rates of return at various points in this text.

Hedging Hedging means to adopt a strategy that negates, in whole or in part, the risk associated with a decision. Hedging strategies are used every day by financial managers and by the man on the street. "Saving for a rainy day" is a hedging strategy. The "Principle of Conservancy" in accounting is a hedging strategy. *The very act of carrying cash* is partly a hedging strategy. Hedging is quite different from risk pricing; risk pricing is a way of *allowing for risk that is not hedged*. Hedging as a method of controlling risk is very important in working capital management. In no other area of finance are there so many means for dealing with risk by hedging, though each of these means is costly. It is useful to think of hedging strategies as being of two types: *hedging strategies that do not use financial instruments* and *hedging strategies that use financial instruments*.

Hedging strategies that do not involve financial instruments may take several forms, but the most common involves the arrangement of additional available financing or the investment in additional assets beyond what is expected to be needed. Keeping a reserve of cash or of marketable securities to hedge cash flow risk is an example of this type of hedging. The best level for this type of hedge depends on several factors: the riskiness of the variable being hedged, the potential costs of going unhedged, and the costs of the hedge. One method of weighing these factors to produce an optimum strategy is *separate risk modeling*.

In separate risk modeling, two parameters of an uncertain variable—its expected value and its variability—are handled separately, using different modeling procedures. One strategy deals with the expected value of the variable under the assumption of certainty, and a second strategy hedges the uncertainty inherent in the variable. For example, in a popular approach to inventory management, optimal ordering strategies are generated based on expected sales levels, assuming that these levels are certain. Then, in a separate procedure, the uncertainty with regard to the sales level, the potential costs of events relative to this uncertainty, and the cost of hedges for this uncertainty are recognized (after the fact, so to speak) and a second methodology (determination of the safety stock level) is used to deal with these factors.

Another example of separate risk modeling and hedging can occur in cash forecasting. Here, a cash forecast can be generated using the expected values of all variables and an investment and financing plan formulated as if the cash flows are certain. As a hedge against uncertainty, the firm can keep a mix of

cash, marketable securities, and a reserve line of credit. The amount of these reserves can be determined by the cost of carrying them, the degree of uncertainty in the cash forecast (determined by simulation), and the cost of running out of funds. We will use these separate risk modeling strategies to formulate hedges in inventory and cash forecasting situations later in the text.

Another hedging strategy is to use *financial instruments* to hedge risk. When these are available, they can provide effective and low-cost hedges to the uncertainties facing the firm. Financial hedging instruments include *futures contracts*, *forward contracts*, *options*, and *options on futures contracts*. Currently available instruments can be used to hedge changes in interest rates, changes in exchange rates among currencies, changes in the price of purchased inventory, and other risks. Options can be valued via the Option Pricing Model (OPM). We will discuss these instruments at length later in the text.

Risk analysis and risk hedging are relatively new aspects of financial analysis and financial management. Sophisticated strategies of this sort are, to a great extent, the outgrowth of the improved calculation and data analysis capacity available to financial managers. We discuss this new calculation capacity in the next section.

THE COMPUTER IN FINANCE

By far the most important technological innovation affecting financial analysis in recent years has been the advent of inexpensive and readily available computing capacity. A microcomputer is on the desk of virtually every financial analyst at progressive and innovative firms. The primary impact of this cheap computing capacity and user-friendly software has been the ease with which previously laborious financial analysis can now be performed and, in particular, the heightened ability of firms to assess risk. Previously, if the firm was going to perform a particularly tedious procedure, such as a forecast for the next year's cash flows for each month, it was quite costly in time and effort to go through the procedure even once. Thus, analysis was often limited to one pass through the algorithm, utilizing the best estimates of the input variables. Sensitivity and simulation analysis procedures could be justified only for the most critical decisions. But the microcomputer revolution has changed all that. Now, the changing of input variables and the observation of the effects on output variables takes seconds, not hours. Procedures to assess risk, which in the past were too expensive, can now be performed routinely.

Three types of microcomputer software are currently available and are useful for the analysis of working capital decisions: the *spreadsheet*, the *mathematical programming package*, and the *simulation package*. Spreadsheets are now a commodity item; dozens are on the market. Spreadsheets consist of rows and columns, and are useful in the analysis of many types of finance problems. The power of the spreadsheet stems from its *copying feature* and its *quick execution of recalculations*. The former is useful if the financial

analysis procedure requires more than a few similar calculations; it makes model building in such circumstances much easier and faster than manual methods. If the analyst wants to see the effect of a deviation from the original estimate on important outcome variables, the quick recalculation feature of the spreadsheet makes substantial analysis feasible; the analyst merely changes the data or formula, and the spreadsheet automatically recalculates all the other cells. However, one note of warning is appropriate. Spreadsheets are a financial modeling tool, not an end in themselves. The data outputs they provide are only as good as the estimated input data and the financial algorithms used.[13]

While spreadsheets are very useful in a number of respects, they are tools of *analysis*, not *optimization*. Some of the problems we study in the management of working capital are such that an *optimum* solution can be found by the proper application of *mathematical programming techniques*. These techniques should be familiar to all students of finance; among them are *linear programming* (where all the variables may assume noninteger values) and *integer programming* (where only one or several variables may assume integer values). Mathematical programming packages, like spreadsheets, are now available from numerous software houses. In this text, we will be using an integer programming approach to several working capital decisions. However, because of the complexity of the theory and calculation procedures necessary to solve this mathematical algorithm, it is not our purpose in this text to discuss solution procedures. Instead, our emphasis will be on describing the formulation of the objective function and constraint equations necessary to execute this algorithm. Students interested in the mathematics of solving integer programming problems may consult any good operations research text.

Simulation programs are less common and less well developed than spreadsheets and mathematical programming software. Consequently, simulation is often a very time-consuming analysis methodology for the student to execute, though its results are quite useful. While several opportunities for simulation analysis occur in this text, our discussion of it will focus mainly on the use of simulation results rather than on the actual programming of simulation models.

ORGANIZATION OF THIS TEXT

The reader should now have some feeling for what working capital is, why its management is critical to the firm's survival and prosperity, and the concepts and methodologies used in its analysis. It is appropriate at this point

[13]For a good discussion of these problems, see R. M. Freeman, "A Slip of the Chip on Computer Spread Sheets Can Cost Millions," *The Wall Street Journal*, August 20, 1984, p. 11.

to discuss the organization of the remainder of this text. Chapters 2 through 4 discuss the management of cash and marketable securities, addressing decisions regarding how much cash the firm should keep and how investments in marketable securities can best be made. The next part of the text, consisting of Chapters 5 through 7, concerns the management of accounts receivable. Here, we discuss decisions regarding the monitoring of accounts receivable balances, the firm's terms of sale, and the granting of credit to the firm's customers. Inventory management is discussed in Chapters 8 and 9. These chapters address the firm's inventory strategy: how much of various types of goods to keep, when and how much to order, and so forth. Chapter 10 discusses the firm's aggregate liquidity decision, which involves the firm's overall structure of current assets and liabilities. The measurement of total liquidity is investigated, and the risk-return tradeoffs involved are discussed. Chapters 11 and 12 involve the structuring of the firm's current liabilities accounts such that the costs of these liabilities are minimized. The final chapter, Chapter 13, discusses the management of international working capital.

SUMMARY

In this chapter, several important concepts that form the basis for the remainder of the text were introduced. Working capital was defined as the firm's short-term assets and liabilities. Working capital is made necessary by the imperfect nature of the real market and production situations faced by firms. Working capital management is a critical concern of management, since the working capital cycle is the major cash flow through the firm. Basic principles of modern finance were reviewed, including the idea that the appropriate goal of managers is maximizing the market price of their firm's common shares. This requires that managers act so as to maximize the net present value of marginal cash flows. Also reviewed were the methods by which risk will be assessed and addressed in this text. Sensitivity analysis, simulation, and beta estimation will be used for assessing risk. The risk estimated via these techniques may be addresed via pricing or hedging. We will use benchmark risk-adjusted discount rates and the CAPM for pricing; hedging procedures will include separate risk modeling (in which nonfinancial instruments provide the hedge) and the use of futures, forwards, options, and options on futures as hedging vehicles. The chapter included a discussion of the importance of the microcomputer to modern financial management, recounting the types of programs of primary use in the management of working capital and their advantages over manual procedures.

Working capital management, and finance as a whole, is a rapidly changing specialization. We trust that serious students will be challenged and enlightened by the remainder of this text as we put the principles outlined in this chapter to work.

Problems

1-1. Lane Toys, a small toy manufacturer, has experienced modest growth and reasonable profitability over the last three years. Purchases of property, plant, and equipment have been carefully controlled, as have dividend payouts and debts. Despite this, the firm has run out of cash:

Lane Toys
Financial Statements
(rounded thousands)

Date	*12/31/85*	*12/31/86*	*12/31/87*
Statements of Income and Retained Earnings			
Sales	$13,000	$13,650	$14,060
Cost of Goods Sold	9,100	9,555	9,842
Selling and Admin. Expenses	1,950	2,048	2,109
Earnings Before Taxes	$ 1,950	$ 2,048	$ 2,109
Taxes	683	717	738
Earnings After Taxes	$ 1,268	$ 1,331	$ 1,371
Common Dividends	$ 507	$ 532	$ 548
Changes in Retained Earnings	$ 761	$ 799	$ 822
Balance Sheets			
Cash	$ 150	$ 118	$ −6
Accounts Receivable	1,561	1,852	2,228
Inventories	2,090	2,551	3,076
Other Current Assets	100	105	108
Total Current Assets	$ 3,901	$ 4,627	$ 5,406
Net Fixed Assets	$ 7,000	$ 7,350	$ 7,570
Total Assets	$10,901	$11,977	$12,977
Accounts Payable	$ 379	$ 398	$ 410
Accrued Wages	455	478	492
Other Current Liabilities	100	110	120
Total Current Liabilities	$ 934	$ 986	$ 1,022
Long-Term Debt	4,500	4,725	4,867
Common Equity	5,467	6,266	7,088
Total Liabilities and Equity	$10,901	$11,977	$12,977

Compute the following ratios as of the end of each of the last three fiscal years (12/31/8X) for Lane Toys:

a. Earnings After Taxes/Total Assets
b. Sales/Net Fixed Assets
c. Year-to-Year Sales Growth
d. Total Debt/Total Assets
e. Current Ratio
f. Sales/Accounts Receivable
g. Cost of Goods Sold/Inventory

Discuss these results relative to the working capital cycle. Indicate why the firm has run out of cash despite its continuing profitability.

Selected Readings

Beranek, W., "Towards a Positive Theory of Working Capital," available from the author at the University of Georgia.

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case

Citizens' Band Antenna Company The Working Capital Cycle

The Citizens' Band (CB) is a set of radio frequencies near 27 megahertz of the radio spectrum that has been set aside in the United States for personal, low-power communication. This type of communication underwent an upsurge in popularity in the mid-1970s. Communication on any radio frequency requires certain equipment comprised principally of a transmitter and receiver (or, more commonly, a combination of the two, called a transceiver) and an antenna. The main beneficiaries of the CB boom were manufacturers of transceivers (mostly Japanese electronics firms) and manufacturers of antennas. The Citizens' Band Antenna Company was one of the latter.

The Citizens' Band Antenna Company was a family-owned firm. Prior to the upsurge in the popularity of CB radio, the firm had sold a mix of antennas for industrial, military, and amateur ("ham") radio applications as well as antennas for CB frequencies. Antennas are generally frequency-specific. For example, those made for the frequencies used in amateur radio do not work well in other applications. When CB radio became popular, executives of the Citizens' Band Antenna Company decided to devote the firm's full manufacturing capability to this marketplace, and had changed the firm's name to the current one to reflect this new emphasis.

The various types of CB antennas (called "beam antennas," "vertical antennas," and so forth) were all manufactured from tubing, wire, plastic parts, and other components. The firm did some fabrication in-house, including all final assembly work, while some of the manufacturing steps (including the molding of all plastic parts) were subcontracted. Labor and materials accounted for approximately equal portions of the firm's cost of goods sold.

As of the end of fiscal 1974, the firm was financed by long-term loans from a local bank and by equity. The bank loans were secured by the firm's fixed assets, and new term loans were taken out as needed when such assets were purchased. The original contribution of equity capital by family members had grown substantially over the years, and as of December 31, 1974, the book net worth of the firm stood at $1,178,000. Many of the family members were retired and living in part on the dividends paid by the firm. Several younger family members were employed there.

While some of the firm's sales were directly to the public, the majority were made

Exhibit 1

Financial Statements
Citizens' Band Antenna Company
Fiscal Years 1974 to 1976
(thousands of dollars)

Statements of Income and Retained Earnings

Fiscal Year Ending	*12/31/74*	*12/31/75*	*12/31/76*
Sales	$2,896	$3,695	$4,878
Cost of Goods Sold	2,062	2,712	3,464
Depr., Int., and Admin. Exp.	356	484	590
Earnings Before Taxes	$ 478	$ 499	$ 824
Taxes	220	230	379
Earnings After Taxes	$ 258	$ 269	$ 445
Common Dividends	129	134	156
Changes in Retained Earnings	$ 129	$ 135	$ 289

Balance Sheets

	12/31/74	*12/31/75*	*12/31/76*
Cash	$ 145	$ 185	$ 73
Marketable Securities	174	0	0
Accounts Receivable	278	409	588
Inventory	255	401	572
Other Current Assets	28	36	49
Total Current Assets	$ 880	$1,031	$1,282
Gross Fixed Assets	$2,172	$2,571	$3,258
Less: Accumulated Depreciation	1,086	1,125	1,194
Net Fixed Assets	$1,086	$1,446	$2,064
Total Assets	$1,966	$2,477	$3,346
Accounts Payable	$ 82	$ 108	$ 337
Accrued Wages	82	108	139
Other Accruals	74	98	118
Short-Term Notes Due	0	150	300
Current Portion—L.T.D.	50	60	70
Total Current Liabilities	$ 288	$ 524	$ 964
Long-Term Debt	$ 500	$ 640	$ 780
Common Stock	$ 100	$ 100	$ 100
Retained Earnings	1,078	1,213	1,502
Total Common Equity	$1,178	$1,313	$1,602
Total Liabilities and Equity	$1,966	$2,477	$3,346

to jobbers and electronics supply houses. Fiscal 1974 had been a banner year for the firm in terms of sales and profits, but things were actually just starting to move. Demand for the firm's products was even higher in 1975. In response to this, the firm added extra workers, scheduled a second production shift, purchased some new equipment, and increased inventories.

Despite this seemingly rosy picture, the firm started to have cash flow problems in fiscal 1975. Executives of the firm were forced to liquidate the firm's portfolio of marketable securities, which had been used as temporary investments during periods of the year when the firm had surplus funds, and instead funded seasonal variations in asset needs by borrowing on a new short-term credit line from the firm's bank. Because of funding shortages, the firm was forced to curtail some fixed asset acquisition plans.

The firm's financial executives initially blamed the firm's problems on lower-than-expected profitability; although sales had increased by 27.6 percent from fiscal 1974 to fiscal 1975, earnings after taxes had increased by only 4.3 percent. (Exhibit 1 shows the firm's balance sheets and income statements for the fiscal years 1974 through 1976.) Their initial effort in 1976 was therefore directed at cost cutting. Overtime work and administrative expenses were controlled closely, even as sales kept expanding. Still, the firm seemed continually short of cash. More long-term debt was borrowed. Fixed asset purchases were postponed again. More short-term borrowings were utilized. The firm's cash account was allowed to decline to what the firm's treasurer believed was a dangerously low level.

As the fiscal year 1976 progressed, the situation grew desperate. The firm's management took two further steps which they had been loathe to do previously: they cut the firm's dividend payout rate and then implemented a policy of delaying payments to trade suppliers beyond the suppliers' stated terms of sale.

In summary, the firm's two-year record involved an increase in sales of 68.4 percent and an increase in profits of 72.5 percent. Despite these seemingly magnificent results, the executives of the firm had the strong suspicion that if the firm had one more year of this sort of success, it would end up in bankruptcy court. They wondered what had happened and what they could do about it (although their exact sentiments were almost always expressed in much stronger language).

Chapter -2

Managing Cash Inflows And Outflows

This is the first of three chapters dealing with *cash management*. The term "cash management" refers to the management of cash from the time it starts its transit to the firm until it leaves the firm in payments. Cash management encompasses the design of collection and disbursement systems for cash and the temporary investment of cash while it resides with the firm.[1] In this chapter, systems for efficiently gathering, investing, and disbursing cash are discussed. But first, it is useful to consider why it is advantageous for firms to hold cash and similar assets at all.

WHY HOLD CASH AND MARKETABLE SECURITIES?

Cash and short-term, interest-bearing investments (marketable securities) are the firm's least productive assets. They are not required in producing goods or services, unlike the firm's fixed assets. They are not part of the process of selling as are inventory and accounts receivable. When firms hold cash in currency or in noninterest-bearing checking accounts, they obtain no direct return on their investment. Even if the cash is temporarily invested in marketable securities, its return is much less than the return on other assets held by the firm. So why hold cash or marketable securities at all? Couldn't the firm's resources be better deployed elsewhere?

Despite the seemingly low returns, there are several good reasons why firms hold cash and marketable securities. It is useful to think of the firm's portfolio of cash and marketable securities as comprised of three parts with each part addressing a particular reason for holding these assets.

[1]For a critical review of the techniques available to address these tasks, see V. Srinivasan and Y. Kim, "Deterministic Cash Flow Management: State of the Art and Research Directions," *Omega*, Vol. 14 No. 2 (1986), pp. 145–166.

Cash for Transactions One very important reason for holding cash in the form of noninterest-bearing currency and checking deposits is *transactions demand*. Since debts are settled via the exchange of cash, the firm must hold some cash in the bank to pay suppliers and some currency to make change if it makes sales for cash.

Cash and Near-Cash Assets as Hedges Unfortunately, the firm's future cash needs for transactions purposes are often quite uncertain; emergencies may arise for which the firm needs immediate cash. The firm must hedge against the possibility of these unexpected needs. Several types of hedges are possible. For example, the firm can arrange to be able to borrow from its bank on short notice should funds suddenly be needed. Another approach is to hold extra cash and near-cash assets beyond what would be needed for transactions purposes. By "near-cash assets," we mean interest-earning marketable assets that have *very* short maturities (a few days or less), and thus can be liquidated to provide funds on short notice with very little risk of loss.

Clearly, the more of this total hedging reserve held in near-cash assets and the less held in cash, the greater the interest earned. However, there is a trade-off between this interest revenue and the transactions costs involved in purchasing and selling such near-cash assets. These transactions costs have a fixed cost component; the firm bears these fixed costs when it buys or sells these assets regardless of the size of investment. Thus, whether it is economical to invest part or all of the hedging reserve in near-cash assets depends on the dollar amount of the reserve. Firms that keep smaller reserves (because their transactions needs are either smaller or more certain) are more likely to hold these reserves in cash, while firms with larger reserves keep them in near-cash assets.

Temporary Investments Many firms experience some seasonality in sales. Often, there will be times during the year when such firms have excess cash that will be needed later in the year. Firms in this situation have several choices. One alternative is to pay out the excess cash to its security holders when this cash is available, and then issue new securities later in the year when funding is needed. However, the costs of issuing new securities usually make this a disadvantageous strategy. More commonly, firms will temporarily invest the cash in interest-earning marketable securities from the time the cash is available until the time it is needed. Proper planning and investment selection for this strategy can yield a reasonable return on such temporary investments; we will discuss this planning and strategy in Chapter 3.

All of these are valid reasons for holding cash and marketable securities in response to the needs and uncertainties faced by the firm. In fact, firms generally hold a surprisingly large portion of their assets in these forms, despite the disadvantage of low returns. Between 1978 and 1982,

large firms held, on average, 7.7 percent of their assets in cash and marketable securities.[2]

Those readers who are familiar with the requirements of some firm's relations with banks will want to know why *compensating balances* have not been listed as a reason for holding cash. "Compensating balances" are non-interest-bearing deposits made by firms with their banks as part of their banking arrangements. Historically, banks have required firms to make such deposits (which the banks then loaned out at interest) as repayment for the availability of reserve credit lines and other functions that banks perform for their clients. However, recent developments in the banking industry have rendered this approach increasingly less common relative to fee-based systems of bank compensation. Since the orientation of this text is toward *modern* working capital management, we will concentrate on these fee-based systems.[3] If the firm uses fee-based banking, it does not need to hold cash for the purposes of compensating balances.

THE MONEY MARKET

In the above discussion of the reasons for holding cash and near-cash assets, it was suggested that near-cash, interest-earning assets could be substituted for cash when the motivation for holding these assets is the hedging of cash flow uncertainties or is the temporary investment of surplus funds. In either case, a primary concern of the cash manager is the *safety and liquidity* of the investment instrument used, since the money will eventually be needed for the operations of the firm. Near-cash assets of high safety and liquidity are traded in the *money market*. For larger corporations with strong financial positions, the money market also provides a source of short-term borrowings. It is important for the student of working capital management to be familiar with the basic instruments of the money market that are used by many firms as investment alternatives to cash and by larger firms as financing vehicles. While all the instruments traded in this market are quite safe relative to other investments (such as common stock), they differ somewhat in risk and return. One difference in per-period investment return is the *yield curve effect*, also known as the *term structure of interest rates*.

[2]See James Vander Weide and Steven Maier, *Managing Corporate Liquidity: An Introduction to Working Capital Management* (New York: John Wiley, 1985), p. 4.

[3]See Jarl Kallberg and Kenneth Parkinson, *Current Asset Management: Case, Credit, and Inventory* (New York: Wiley, 1984), pp. 76–8. Further, if firms choose to use one or more banks that require compensating balances rather than fees, it is possible to convert required compensating balances into equivalent fees; see Kallberg and Parkinson, *Current Asset Management*, pp. 67–75.

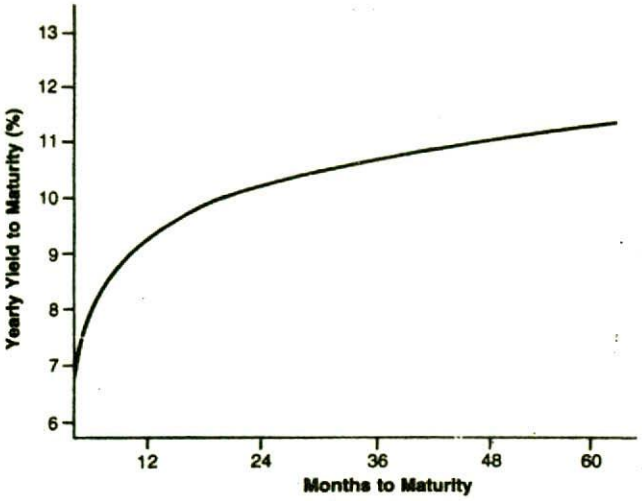


Figure 2-1. Yield Curve as of January 7, 1985 Treasury Bonds, Notes, and Bills.

Term Structure of Interest Rates The term structure of interest rates relates the market's required return on an asset to the time to maturity of that asset. The market relationship between time to maturity and required return can be observed in the plot of the yield to maturity of securities versus their time to maturity; this plot is called the *yield curve*. An example is shown in Figure 2-1.

This curve represents the required returns on securities of a given default risk for various times to maturity. That is, it plots the supply/demand intersections for securities of various maturities. These required returns may be influenced by various factors, such as the level of inflationary expectations, the relative levels of supply and demand for securities of different maturities, and differences in investors' perceptions of the interest rate risk of securities of different maturities. The three most common theories of the determination of term structure of interest rates are based on these differences in inflation, relative demand, and risk among securities of various maturities.[4] *Expectations theory* holds that the differences in per-period required returns among securities of various maturity dates reflect expectations that inflation will change over time. If the near-term inflation is expected to be high, for example, that expectation will raise interest rates on the short-maturity end of

[4]More complete discussions of theories of interest rate determination and yield curve level and shape can be found in T. E. Copeland and J. F. Weston, *Financial Theory and Corporate Policy*, Second Edition (Reading, Mass.: Addison-Wesley, 1983) pp. 65–70 and in E. F. Brigham and L. C. Gapenski, *Intermediate Financial Management* (Chicago: Dryden, 1985) pp. 584–92.

the yield curve more than on the long-maturity end. *Market segmentation theory* holds that investors have preferences for securities of different maturities, and the yield curve's level and shape reflect the availability of various maturities relative to these preferences. For example, if there is a large supply of securities with a particular maturity relative to demand for that maturity, the price of securities with that maturity declines and the yield to maturity rises. Upsloping yield curves in this theory are present when there are relatively larger supplies of longer maturities, and downsloping yield curves are present when there are relatively larger supplies of shorter maturities. *Maturity preference theory* holds that the shape of the yield curve is merely a reflection of the differences in interest rate risk among maturities. In particular, longer maturities are said to contain more interest rate risk, and therefore have higher required rates of return than do shorter maturity issues. Interest rate risk is the relationship between the interest rate prevailing at the time and the trading price of the security. Because the repayment of principal is further in the future, the prices of longer-term securities fluctuate much more with changes in interest rate levels than do the prices of shorter-term securities. They are, therefore, riskier, and investors require a higher return on them.

Maturity preference theory has substantial appeal because it explains one of the important *empirical* observations about the yield curve: *it is usually positively sloped*. Yield curves are upsloping most of the time, although they can sometimes be flat or even negatively sloped. One implication of this empirical phenomena is that, on average, the longer the maturity of the security, the higher the per-period rate of return.

One important aspect of the yield curve with respect to the firm's investments in cash and near-cash assets is the risk-return trade-off that is caused by its (usually) positive slope. The longer the term to maturity of the investment, the higher the per-period yield. Therefore, there is an incentive for firms to invest in longer-term securities. Offsetting this is the greater interest rate risk for longer maturities; if the investing firm chooses a security with a longer maturity and is forced to sell that security before maturity and if interest rates have risen, the return achieved may be less than had the firm invested in shorter-term securities. This interest rate risk can be reduced by hedging with interest rate futures, a technique that will be discussed in Chapter 3.

In addition to yield curve effects, risks and returns among near-cash assets differ because of the *default risk* characteristics of their issuers. Some securities are extremely safe while others entail a nonzero probability of default, and carry higher interest rates to compensate buyers for this risk. At this point it is useful to present a list of typical money market instruments that are used by firms as investment alternatives to cash with discussion of their relative returns and default risks. We call this a list of "typical" money market instruments because of the fluid and consumer-oriented nature of this market. The money market is relatively unregulated and is very competitive

Consequently, there is quite a large amount of continuing product innovation; issuers and dealers continually seek instruments that draw investment funds, generating commissions for dealers and lowering financing costs for the firms issuing the securities. With this diversity of products, many strategies are used, and firms will rapidly change strategy in response to new instruments and conditions. Thus, any list of currently popular money market instruments is subject to rapid obsolescence.[5]

Treasury Bills These are securities sold by the U.S. government, generally with maturities of 91, 182, or 365 days. They are considered to be free of default risk, although interest rate risk occurs, particularly in the longer maturities. They are sold at a discount and redeemed at their principal value at maturity. They can be bought through banks, dealers in government securities, or directly from the government.

Treasury bills (called T-bills) are quite popular as investment alternatives to cash. There are several reasons for this. First, they can be purchased in very low denominations (as little as $10,000), although when investments are made in small amounts the costs of purchase and sale are relatively higher and will lower the net yield. Second, they are extremely liquid. There is a very active secondary market for T-bills, and the firm's position can be liquidated at any time. This is a very valuable feature if the firm holds securities as a hedge against cash flow uncertainties, since the money may be needed quickly. This active secondary market also means that the firm's investments in T-bills need not be limited to the maturities of new issues. For example, if the firm needs a three-day investment, it need only purchase, in the secondary market, a T-bill with three days to maturity.

Treasury bills are issued directly by the U.S. government. The federal government also issues debt instruments through its federal agencies, such as the Federal National Mortgage Association and the Federal Home Loan Bank Board. These debt instruments are issued in various maturities, many under one year, and in various denominations. While there is a slight chance that the federal government might let these quasi-public agencies default, the probability is so small that these investments are generally considered to be almost free of default risk. They are traded through securities dealers. The traded volume of these securities is less than that of T-bills and other securities issued directly by the federal government.

The default-free nature of T-bills is both a positive and a negative feature in cash management. While it means that the principal is safe (except for

[5]For a more complete discussion of short-term investment vehicles, see Timothy Cook and Bruce Summers, eds., *Instruments of the Money Market*, Fifth Edition (Richmond: Federal Reserve Bank of Richmond, 1981) or James Vander Weide and Steven Maier, *Managing Corporate Liquidity: An Introduction to Working Capital Management* (New York: John Wiley, 1985), pp. 169–90.

interest rate risk), it also means that T-bills earn a lower return than do instruments with a bit more default risk. To earn extra return, many firms are willing to invest in slightly riskier instruments, such as commercial paper.

Commercial Paper Commercial paper is large-denomination, unsecured debt, generally in maturities of about 30 days. It is issued by firms, finance companies, and bank holding companies and provides financing for such companies as well as an investment vehicle for the surplus funds of firms. Some larger issuers sell a portion of their commercial paper directly to investors, but the majority is sold through commercial paper dealers.

Since there is some default risk involved, the yields on commercial paper are often quite a bit higher than the yields on T-bills of the same maturity. This risk is somewhat lessened by the general requirement that issuers of commercial paper obtain backup lines of credit from commercial banks in the amount of their commercial paper borrowings to refund the paper if the firm is unable to sell new debt when the paper comes due. Default risk on commercial paper can be estimated by the ratings of this paper; these ratings are issued by ratings services. Fitch Investor Service, Moody's Investor Service, and Standard & Poors Corporation all issue ratings on commercial paper.

Certificates of Deposit These are very large-denomination debt instruments issued by banks. They are of various maturities, sometimes longer than one year. Shorter maturities carry a prespecified interest rate, but some variable-rate certificates are sold among the longer maturities. Because the Federal Deposit Insurance Corporation insures bank debts only up to $100,000, and principal on the certificates of deposit used as investments for large firms' surplus cash are commonly over $1 million, these securities are subject to the risk of failure of the issuing bank, and yields are consequently above those of U.S. government issues. Certificates of deposit are placed directly with investors; some are also traded in the secondary market by dealers. In addition to the standard certificates, firms with a substantial amount of funds to lend may also negotiate private purchases of certificates or time deposits with major banks; these private deals can be tailored to the firm's specific needs.

Banker's Acceptances Like certificates of deposit, these are issued by banks, but are slightly safer because of the security arrangements entailed. Banker's acceptances are generated in the course of international trade, and the holder of a banker's acceptance has recourse to both the issuing bank and the purchaser of the goods. Given the very small likelihood of *both* the purchaser and the bank defaulting, banker's acceptances are relatively safe investments. Banker's acceptances are traded through dealers and are quite liquid.

Repurchase Agreements Repurchase agreements (repos) are used for

very short-term investments and financing. In a repo, the seller agrees to sell a security (usually a T-bill) to the buyer, and the seller also agrees to repurchase the security a few days later at a higher price. The return on the investment is the difference between the sale price and the agreed-upon buyback price, and represents the appreciation, over the period, of the discounted security that serves as collateral. Since the loan of the funds to the seller of the repo is collateralized, the transaction is substantially safer than an unsecured loan. Aside from the security provided by the collateral, the repo transaction is a loan and can also be used for financing purchases (by selling the contract rather than purchasing it). Government agencies (both federal and state), banks, and firms make extensive use of repos for short-maturity (often overnight) borrowing and lending.

The danger in repos is fraud. In purchasing a repo from a dealer, the investor trusts the dealer to, in fact, possess the security used as collateral; the security itself is not exchanged. If, however, the dealer does not possess the collateral, investors can sustain substantial losses.[6]

Eurodollars These are dollar-denominated loans and certificates of deposit in non-U.S. banks. (This is called the "Eurodollar Market," although the funds may be located anywhere in the world.) Because of their international character, we will defer detailed discussion of Eurodollar investments and financing until Chapter 13. Suffice to say that these must be made in large denominations, are commonly of one-week or six-month maturities, entail modest default risk, and carry slightly higher interest rates than domestic securities.

Hedged Dividend Capture Strategies The securities previously described constitute only a partial list of the more popular short-term investment vehicles used by firms. Dozens of other investments are used less frequently. Cash managers are constantly searching for short-term, low-risk investment strategies that yield higher returns, and this search has led to very innovative strategies, some involving instruments outside of the type usually considered for temporary investment purposes. One set of strategies revolves around the relative tax rates on interest income and on intercorporate dividends. While interest income from investments is taxed at the firm's corporate tax rate (which is 33 percent for large firms, based on tax legislation passed in 1986), the effective rate for dividend income is only 20 percent of this (or 6.6 percent) for dividends on stock held sufficiently long to qualify for the 80 percent dividend exclusion (46 days under the same law). Thus, there is a substantial

[6]For discussion, see Daniel Hertzberg and Martha Brannigan, "Dealer Losses in Collapse of ESM Unit Are Manageable, New York Fed Says," *Wall Street Journal*, March 7, 1985, p. 5, and "Former ESM Chief Nicholas Wallace Gets 30-Year Prison Term," *Wall Street Journal*, October 20, 1986, p. 14.

incentive to structure short-term investments to obtain dividend income. However, the use of dividend-paying securities as temporary investments requires special strategies—holding the stock alone would be far too risky, given the short-term nature of the investment and the eventual need of the firm for the invested cash. In particular, when some of these instruments are purchased for dividend income, a hedging strategy must also be employed to lessen the risk from price fluctuations. Some possible strategies are:

1. Holding adjustable rate preferred stock. Since the rate on this stock floats with general interest rates, the interest rate risk inherent in the typical perpetual preferred stock is lessened considerably.[7]
2. Holding common shares of firms and hedging with call options. In this strategy, the firm buys the shares of a firm that is expected to pay a dividend and at the same time sells an option to sell the stock at a specific price (a call option). In this way, the risk of stock price movement is lessened, since any swings in the price of the stock are partially offset by changes in the price of the call. When the stock is sold, the call is rebought; since the gains and losses on the call and the stock tend to offset, the firm is left with the dividend income.[8]
3. Holding shares of an index fund and hedging with index fund call options. This is the same as holding shares in individual firms, except that the investing firm benefits from the wide diversification of the index fund.[9]

All these strategies yield higher after-tax returns than does the strategy of holding money market instruments. Unfortunately, none of these three strategies reduces risk to the levels inherent in holding T-bills, repos, banker's acceptances, or commercial paper. Regarding the first strategy, the prices of adjustable rate preferred stock are somewhat volatile; although the adjustable nature of the preferred returns lessens interest rate risk, it does not remove it entirely. The second and third strategies are also risky, since it is very difficult to construct the perfect hedge required to completely eliminate the price risk in shares of common stock or in index funds, although the hedging strategy does reduce this price risk significantly. Consequently, these strat-

[7]See Bernard Winger and others, "Adjustable Rate Preferred Stock," *Financial Management* (Spring 1986), pp. 48–57.

[8]See Keith Brown and Scott Lummer, "The Cash Management Implications of a Hedged Dividend Capture Strategy," *Financial Management* (Winter 1984), pp. 7–17, and Keith Brown and Scott Lummer, "A Reexamination of the Covered All-Option Strategy for Corporate Cash Management," *Financial Management* (Summer 1986), pp. 13–17.

[9]See Terry Zivney and Michael J. Alderson, "Hedged Dividend Capture Strategy with Stock Index Options," *Financial Management* (Summer 1986), pp. 5–12.

egies remain less popular than those based on standard money market investments.

Summary To summarize this discussion of the money market, the normally positive slope of the yield curve gives incentives for the firm to invest cash in instruments with longer maturities. This adds interest rate risk to investments unless the firm hedges this risk with interest rate futures. In the rapidly evolving money market, several vehicles are popular: Treasury bills, commercial paper, certificates of deposit, banker's acceptances, repurchase agreements, and Eurodollar investments. New and innovative strategies, such as hedged dividend capture, are being developed continuously. In order to select the most advantageous investment strategies for their circumstances, firms must consider the amount involved, the purpose for the investment of cash (hedging of cash flow uncertainty or the investment of a temporary surplus of funds), the size of the firm, and the amount of risk that the firm is willing to take. Very short-term investments (as alternatives to holding cash in hedging the firm's cash flow risk) are commonly made by purchasing nearly matured Treasury bills, commercial paper, or repurchase agreements. All but the very smallest firms can purchase Treasury bills; larger investments are required for the other vehicles. Treasury bills are the safest of these in terms of default risk. Longer-term investments (as necessary for the higher-return investment of temporary surplus funds when the yield curve is upsloping) can be made in longer-maturity Treasury bills, in banker's acceptances, in Eurodollar securities, and in certificates of deposit. Because of the required minimum investments, smaller firms will tend to use Treasury bills here also. With regard to default risk, Treasury bills are the safest, followed by banker's acceptances, commercial paper, and Eurodollar investments. Interest rate risk depends on maturity. Some of these instruments can also be used for financing; larger firms sell commercial paper, Eurodollar securities, and repurchase agreements ("reverse repos"). Firms may also use banker's acceptances to finance foreign trade.

FLOTATION AND CHECK CLEARING

To this point, this chapter has recounted the reasons and mechanisms for holding cash and near-cash assets. The management of these holdings by the firm is an important part of the cash management function. But another important function in this area of finance is the management of cash *when it is not in the firm's hands*, that is, when it is in transit to and from the firm. To understand this transit process, let us track the flow of cash and documents in a typical passage from one firm to another in payment of a debt. Since the most common method of remittance is by check, we will use a check remittance as an example. We will also assume that the firm receiving the cash has

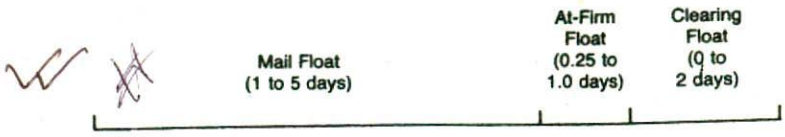


Figure 2-2. Transit Times for a Typical Check.

not taken any steps to accelerate this transit. Later in this chapter, we will discuss possible steps to achieve this acceleration.

The check starts its passage to the receiving firm when the firm making the remittance mails the check.[10] The check then winds its way through the mail service to the receiving firm. This usually takes between one and five days, depending on the distance the check must travel and the efficiency of the parts of the postal system through which the check passes. When the check is received by the firm, the documents enclosed with the check (that indicate what the check is intended to pay) are removed; the check is then forwarded to the firm's bank. Depending on the receiving firm's procedures, the check may spend a few hours or even a day at the firm before it is forwarded to the bank. The bank then sends the check to the Federal Reserve Bank (the "Fed") or the Fed's Regional Check Processing Center, which undertakes to present the check for payment to the bank on which it was written.[11] The Fed guarantees that this clearing process will take at most two days; it if takes longer, the Fed credits the deposit in two days anyway. *Only after this entire process is completed* does the receiving firm receive credit for the cash so that it is able to use the money. This transit process, with estimated delays, is depicted in Figure 2-2.

Why should the firm care about this process? The answer is that every delay in the receipt of money by the firm lowers the firm's returns, and therefore its shareholders' wealth. To see this, suppose that a firm receives $2 million per day via this transit system. Assume that checks were in the mail an average of 4 days, that the firm takes 0.5 days to process checks and get them to the bank, and that the firm's bank takes 1.3 days to obtain funds for the checks. The total delay is then 5.8 days. Since the firm receives $2 million per day, at any time there is $11.6 million in incoming funds that the firm cannot access, and thus cannot use to produce increases in shareholder

[10]We assume here that the check is *truly* in the mail, not that the remitting firm merely says that it is (truly or less truly). In most firms, it is the task of the firm's credit department to get the check put in the mail; the task of cash management starts where that of credit management ends.

[11]There are some special cases here in which the receiving firm is credited with the funds at the time of deposit or almost immediately thereafter. Any cash deposits are immediately credited to the receiving firm's account. Also, checks received by the firm that are drawn on the bank at which the firm makes the deposit (that is, cases where the paying and receiving firms use the same bank) are cleared simply by the bank's reducing one firm's account and increasing the other's.

Table 2-1. *Opportunity Cost of Float for Text Example.*

Float Type	*Float Time (days)*	*Receipts (per day)*	*Float in Dollars*
Mail Float	4.0	$2 mil.	$8.0 mil.
At-Firm Float	0.5	$2 mil.	$1.0 mil.
Clearing Float	1.3	$2 mil.	$2.6 mil.
Total Transit Time	5.8	Total Float	$11.6 mil.
		Required Return	0.10 per year
		Opportunity Cost of Float	$1.16 mil. per year

wealth. If the firm could accelerate the transit process such that it had these funds in hand, and could invest them at the firm's cost of capital (10 percent, for example), it could earn an additional $1.16 million, and shareholders' wealth would increase because of these increased earnings. This lost opportunity for investment is called the *opportunity cost of float*. Calculation of the opportunity cost for this example is presented in Table 2-1.

There are several strategies that firms can use to reduce the delay in receiving funds. Each of these strategies addresses one or more of the three float times (mail float time, at-firm float time, and clearing float time) that make up the total transit time of funds from one firm to another. In the following paragraphs, several such strategies are discussed.

Selection of Banks with Accelerated Clearing Capabilities There is no requirement that the bank must clear all checks through the Fed. Banks have several alternative routes for check clearing, some of which can significantly reduce clearing float. One route is the *clearinghouse*. Clearinghouses are places where banks meet to exchange checks drawn on each other. Clearinghouses may involve only local banks or may involve both local banks and banks outside the local area, as many clearinghouses do. Banks that are members of clearinghouses can clear checks against other clearinghouse banks more rapidly than through the Fed; often, checks processed through clearinghouses can be cleared in one-half day or less. Another method by which a bank may clear checks is through the use of its *correspondent banks*. Correspondent banks in other parts of the country have agreements with the clearing bank. Often, they are participants in local clearinghouses in their own areas. In this system, the clearing bank sends the check to the correspondent bank nearest the bank on which the check was drawn. The correspondent bank then proceeds with the check clearing process. Well-connected banks can often accelerate check clearing in this way.

Acceleration of Check Processing at the Firm A surprising number of firms are not very careful about the delay between when the checks are

received at their various locations and the eventual deposit of these items. While good cash management requires that the firm adopt processing procedures that speed checks into the clearing process, they are sometimes left to sit at the firm for several days (or longer!). Government agencies are particularly prone to this practice, as several studies have shown.

Use of Electronic Collection Procedures It should be clear from the prior discussion that the time necessary for transmittal of cash from one firm to another revolves largely around the passing from one hand to another of a piece of paper—the remitting firm's check. One alternative to this process is to substitute an electronic message of payment for the check. This eliminates the need for the paper and thus the various floats associated with taking the paper from one place to another. Efforts to convert to electronic remittance systems began in the 1970s, but they have met with little success and are currently used only for an insignificant proportion of transactions. Further, the outlook for more widespread adoption is not very bright. Given the appeal (in terms of reduced flotation) of electronic remittances, it is useful to briefly review the history of the proposed electronic payment systems and the problems in adopting these systems.[12]

The major experiment in the development of a system for electronic remittances was the corporate trade payments (CTP) system developed by the National Automated Clearing House Association. This system was tested in 1983 and introduced in 1984, but has not attracted a significant volume of transactions. Several causes of this failure are commonly cited, such as the costs of converting to electronic payments, the absence of adequate marketing efforts. and the difficulty in sending proper advice as to what the remittance is supposed to pay.[13] The *major* problem with electronic payments, however, does not relate to these factors, but to the relative benefits and costs of the system to the paying and receiving firms. To see this, it is first necessary to recognize that the elimination of float on checks benefits the receiving firm *but extracts the same costs from the paying firm*, and that paying firms *must agree to remit electronically*. While outgoing checks in nonelectronic systems remain uncollected, the paying firm retains the cash which it can then use. But in electronic systems, this float is eliminated, to the advantage of the receiving firm but to the disadvantage of the paying firm. This disparity in

[12]Required reading for those interested in electronic trade payments systems are the March and April 1986 issues of *Economic Review*, published by the Federal Reserve Bank of Atlanta. These two issues present nine articles summarizing the history and problems of electronic trade payments systems.

[13]In response to the latter problem, another service has been developed (the corporate trade exchange, or CTX) to ease the transmittal of remittance advice information.

benefits has led to considerable reluctance by paying firms to participate in electronic payment systems. The remaining advantages of electronic payments, such as the reduction in printing costs for checks and the reduction in charges to the receiving firm by its bank for processing these checks, are not large enough to offset the initial costs of instituting an electronic payments system. Unless these economics change, electronic trade payments will continue to be unattractive to remitting firms.

Use of Lockboxes A "lockbox" is a post office box number to which some or all of the firm's customers are instructed to send their checks. The firm grants permission to its bank to take these checks and immediately start them in the clearing process. In fact, the mail addressed to this "post office box" is actually delivered directly to the firm's lockbox bank. In this way, at-firm float is eliminated. Judicious placement of lockboxes and instructions to customers on where to send their checks can also serve to reduce mail and clearing float substantially.

While lockboxes are very useful, not all firms will find them of advantage. Their use entails giving customers two mailing addresses for the firm: the lockbox (for checks) and the firm's usual business address (for all other documents). This inevitably leads to the misrouting of some documents to the lockbox that the firm would prefer to go directly to its business. A consequent delay in the delivery of these documents to the firm occurs, since banks often do not forward items received at the lockbox until the next day. This entails costs to the firm. For example, misrouted purchase orders from customers, which the firm would like to process as soon as possible, are delayed. Further, lockbox check processing systems at banks are oriented toward the rapid processing of routine checks, not extraordinary items (such as postdated checks, unsigned checks, and promissory notes). While lockbox personnel are instructed to look for these items and not to send them on through the banking system, errors occur rather frequently; with consequent inconvenience for the firm.

While lockboxes are not a problem-free panacea for flotation problems, *their proper use can reduce all the types of flotation* on incoming checks. The important decisions in the formulation of a lockbox strategy are:

- Where should the firm locate its lockboxes?
- To which lockboxes should each of the firm's customers send their checks?

The best solutions to the lockbox problem require that these questions be answered simultaneously. The next section, in which the general solution to this lockbox strategy problem is presented, provides the major analytic thrust of this chapter.

IE LOCKBOX LOCATION PROBLEM

The *lockbox location problem* involves answering the two questions just posed. A note on the mathematical and programming procedures presented here for solving this problem is appropriate. While we present the mathematics of a general solution to the lockbox problem, firms designing lockbox systems do not directly use these mathematics. Computer programs to determine lockbox location are sufficiently complex and the lockbox location decisions to be made by various firms are sufficiently similar that the development of this software by individual firms is not cost effective. Instead, the firm usually undertakes the study of this problem in conjunction with a commercial bank that has developed or purchased lockbox location software and that sells the use of this software to corporate clients. The intent of this section is to make the reader able to understand in a specific way *how the software works*: the inputs it requires, the mathematical techniques it uses, and how its outputs are to be interpreted. With this knowledge, the reader should be able to use an appropriate microcomputer package to solve a small example lockbox location problem and interpret the results (full-size lockbox location problems require a mainframe because of the number of computations necessary to solve them). More importantly, the reader should *understand the lockbox solution procedure* sufficiently to be able to work intelligently with a commercial bank in the formulation of a lockbox strategy for a firm.

The lockbox location problem is a cost minimization problem. The firm seeks to minimize the sum of (1) the opportunity cost of the float on incoming cash, and (2) the costs of the lockbox system. The opportunity cost of the incoming cash is the total float in dollars times the firm's required return (as in Table 2-1). The costs of the lockbox system include the fixed costs that banks charge to provide lockbox service (regardless of the number of checks processed), the per-check processing charges, and any charges necessary to gather up the cash from the lockbox banks after it is collected. There is a trade-off between these two sets of costs. As the firm increases the number of lockboxes (properly placed), the opportunity cost of float will be reduced.[14] However, each additional lockbox adds another fixed charge to the costs of the lockbox system. Eventually, the addition of another lockbox increases rather than decreases total costs. This relationship is portrayed in Figure 2-3.

To solve the lockbox location problem, the firm must collect four sets of data:

1. The mail and clearing times for sending checks from each part of the

[14]Note that, for a multiple-lockbox system to be considered by the firm, there must be gains from reduced opportunity costs in adding lockboxes in several locations. This is true only for firms whose receipts come from several different areas. Unless receipts are widely dispersed, multiple lockboxes will not reduce mail and clearing float; one lockbox, near the origin of the receipts, is sufficient.

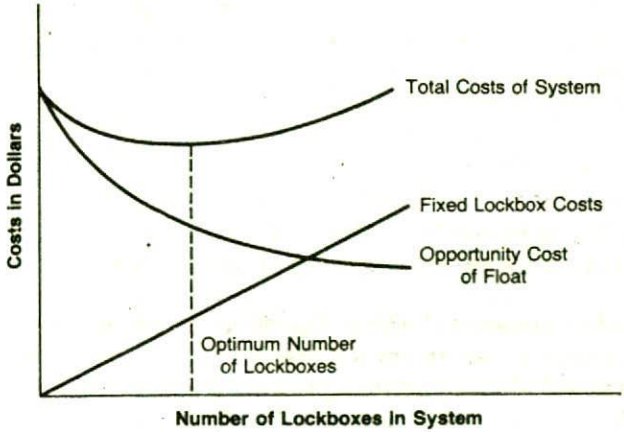


Figure 2-3. Costs in Lockbox Systems Versus Number of Lockboxes.

firm's geographic sales area to each possible lockbox. Mail times are usually obtained from Phoenix-Hecht, Inc., a firm that supplies mail data on a commercial basis from one three-digit zip code location to another. Clearing times (called "availability schedules") are generally obtained from the banks which are potential lockbox sites.

2. The total amount of daily funds and number of checks received by the firm from each part of the sales area. This is usually collected by an examination of the postmarks on a sample of the firm's incoming checks. A stratified sample of high-value checks may be used to reduce collection costs for these data.
3. The required rate of return (for computing opportunity costs).
4. The variable and fixed costs of each proposed lockbox site. To speed clearing times, lockboxes are usually located in cities where the Fed has a major district or branch bank, plus a few other major locations; there are about 50 possible lockbox cities. There are often several banks which offer lockbox services in each of these cities, so the total potential lockbox sites is over 100.[15]

With these data in hand, it is relatively simple to calculate the *one best lockbox* if the firm is constrained to have only one. This optimal solution to the *one-*

[15]See Steven Maier and James Vander Weide, "What Lockbox and Disbursement Models Really Do," *Journal of Finance* (May 1983), p. 364. To keep the following discussion as simple as possible, we will hereafter assume that the firm has examined the banks providing lockbox services in each location and has selected the most advantageous of these; there will then be only one candidate lockbox bank per lockbox city. The mathematical procedures described do not, however, require this assumption.

lockbox problem is simply the lockbox which has the lowest total cost, defined again as the sum of the opportunity cost of float and the costs of the lockbox. Beyond this, it is rarely clear whether a *group* of lockboxes would do a better job (in the sense of lower total costs) than this single best lockbox. For example, suppose that the firm sells over the continental United States, with about equal receipts from the various parts of the country. In this case, a centrally located lockbox (perhaps in Kansas City) might be the best one-box solution. But would two lockboxes, perhaps one in the East and one in the West, be better? If so, to which lockbox should customers from each part of the country be instructed to send their checks? To answer these questions, some sort of optimization technique (an optimization algorithm) is necessary.

A Lockbox Location Problem Algorithm There are several lockbox location and customer assignment routines in the cash management literature. Some of these will always find the optimum (least cost) combination of lockboxes and assignments while others will not. In general, the routines that always lead to optimum solutions are more difficult to calculate (particularly by hand) than those which may not lead to optimums. The trade-off has been between ease of computation versus the guarantee of an optimum solution. However, one of the mathematical approaches that will always lead to an optimum solution—*integer programming*—is now commonly available as a user-friendly microcomputer software package. Thus, there no longer seems any need to compromise on technique in illustrating lockbox location decisions because of computational difficulty considerations. The solution to the lockbox location problem presented here is based on the integer programming approach.

Integer programming can be conceived as linear programming in which some of the variables are constrained to take on only integer values. In the formulation presented here, the variables that must be integers are those that represent whether a lockbox is in use ("open") or whether it is not ("closed") and whether the customers from a particular region are assigned to a particular lockbox or not.[16] Let j designate the prospective lockbox bank location and i designate the zone (area) from which the customer's check originates. Let the number of possible lockboxes be n and number of zones be m. To evaluate a lockbox at location j we need to know the charges that the firm will bear from opening and using that lockbox. These will be composed of the fixed charges from the bank for opening the lockbox, the check processing charges for checks received there, and the opportunity cost of float on checks from customers who send their remittances to that lockbox. Let d_j be the fixed charges for lockbox j. The check processing charges will be the product of

[16]The notation and procedure here follows Robert M. Nauss and Robert E. Markland, "Solving Lock Box Location Problems," *Financial Management* (Spring 1979), pp. 21–31.

the bank's processing charge per check and the number of incoming checks. Let s_j be the processing charge per check at lockbox j and let h_i be the expected number of checks originating from customers in zone i.

The opportunity cost of float from the checks of customers originating in zone i and going to lockbox j is designated as c_{ij}. This is the product of the total float in days on these checks times the dollar amount of checks originating daily in zone i times the firm's required return (a risk-adjusted discount rate). For example, suppose that $5 million in customers' checks originate in a particular zone, that these customers are assigned to a lockbox where the total float time on these checks will be 4 days, and that the required return on checks is 10 percent. The c_{ij} term for this assignment would be ($5 million) (4) (0.10) = $200,000 per year. The total float time, in turn, is the sum of the mail and clearing times for checks originating in zone i and directed to lockbox j.

To complete the integer programming formulation, a few more variables must first be defined. Let K be the maximum number of lockboxes allowed by the analyst in the solution. For the solution involving the lowest possible cost, K would be set equal to n, the total number of possible lockbox sites. Let y_j be a zero-one integer variable indicating whether the lockbox at location j is open (1.0 if it is, zero if it is not) and x_{ij} be a zero-one integer variable indicating whether customers originating checks in zone i are assigned to lockbox j (1.0 if they are, zero if they are not). Of course, since the intent of the lockbox location problem is to determine which lockboxes should be open and which customers should be assigned to which lockboxes, the solution algorithm determines the optimum y_j and x_{ij} terms based on the minimization of total cost. The objective function is:

$$\text{Minimize} \sum_{\text{all } j} d_j y_j + \sum_{\text{all } i} \sum_{\text{all } j} (c_{ij} + h_i s_j)\, x_{ij} \qquad (2\text{-}1)$$

which says that the objective is to minimize the sum of the total fixed costs of opening lockboxes (the sum of $d_j y_j$), the opportunity costs of flotation ($c_{ij} x_{ij}$), and the check processing costs ($h_i s_j x_{ij}$).

The solution to this problem is subject to some very logical constraints. The checks from each customer zone must be received at a lockbox somewhere. Also, a lockbox must be open for receipts from a customer zone to be received there. There should be one and only one lockbox open at location j and customers from zone i may send their checks to one and only one lockbox. Finally, it is often useful to constrain the solution to the problem to the best solution for a maximum of K lockboxes (in part, to limit the computer's required computational time in the initial stages of examining the problem). The following equations model these constraints:

$$\sum_{\text{all } j} x_{ij} = 1 \qquad (2\text{-}2)$$

$$\sum_{\text{all } i} x_{ij} \leq m y_j \tag{2-3}$$

$$\text{All } x_{ij} \text{ and } y_j \geq 0, \leq 1 \tag{2-4}$$

$$\text{All } x_{ij} \text{ and } y_j \text{ integer} \tag{2-5}$$

$$\sum_{\text{all } j} y_j \leq K \tag{2-6}$$

The meaning of these constraints is fairly straightforward. Equation (2-2) says that one of the x_{ij} variables for each origin zone (each i) must be 1.0 and the others zero, so that the checks from zone i are assigned to one and only one lockbox. Equation (2-3) bounds the solution to assign customers only to open lockboxes. In this equation, if y_j is zero, all the x_{ij} terms representing checks to be received at location j must also be zero; if y_j is 1.0, any of the x_{ij} terms may be 1.0 (up to a maximum of m, the number of zones and therefore the number of x_{ij} terms for a given j). Equation (2-4) bounds the x_{ij} and y_j terms to zero or 1.0 and equation (2-5) bounds these variables to be integers, since customers from a zone can send checks to only one lockbox and lockboxes may be either open or closed. Equation (2-6) limits the maximum number of lockboxes in the solution; the integer programming algorithm will compute the least-cost lockbox strategy for this maximum number.

AN EXAMPLE LOCKBOX LOCATION PROBLEM

The application of lockbox location analysis to a small problem is illustrative. Assume that the firm receives checks from four customer zones (zones A, B, C, and D), and has solicited lockbox proposals from banks in three of the zones (zones B, C, and D). It has also collected the other data necessary to analyze costs from various lockbox strategies. Data from lockbox proposals, the mail times from zone to zone, the dollar amount of checks originating in each zone, and the clearing times for the proposed lockboxes are presented in Table 2-2. The firm's required rate of return is 9 percent and the average size of an incoming check is $2,500. This average size does not vary among the origin zones.

It is instructive to first address this example by solving the one-lockbox problem: if the firm is constrained to having only one lockbox, and all checks are sent to that lockbox, where should that lockbox be located? To answer this question, a spreadsheet to calculate the opportunity cost of float, the check processing cost, and the total cost for each of the three potential lockboxes can be generated. One such spreadsheet is presented in Table 2-3.

Table 2-2. *Data for the Example Lockbox Location Problem.*

Data on Lockbox Proposals from Banks:

Proposal Number:	1	2	3
Lockbox Location	B	C	D
Fixed Cost per Year	$3,000	$10,000	$4,000
Processing Charge per Check	$0.10	$0.15	$0.13

Mail Time Data (days):

	Destination		
Origin	*B*	*C*	*D*
A	2.6	2.8	3.0
B	2.5	3.1	3.3
C	2.7	2.8	3.5
D	3.3	3.5	1.8

Clearing Time Data (days):

	Destination		
Origin	*B*	*C*	*D*
A	1.2	1.8	1.7
B	0.8	1.3	1.5
C	1.3	1.1	1.3
D	1.5	1.3	1.0

Check Origination Data (per day):

Origin	*Dollars*
A	$3,500,000
B	$4,500,000
C	$3,000,000
D	$2,500,000
Total	$13,500,000

After setting up the necessary headings, the data on mail float times and clearing float times from Table 2-2 were entered into columns B and C for the first lockbox. The entries in column D, cells 8 through 11, are the sum of the corresponding cells in B and C. The dollar origins appear in column E; column F, cells F8 through F11, is E times D (for example, cell F8 contains "D8*E8"). Summing these gives the total dollar float for this one-lockbox strategy.

The remainder of the calculations takes place in column F. The total dollar float (cell F13) times the required return (F14) gives the opportunity cost of float (F16). The fixed cost of this lockbox is contained as data in cell F18. The number of checks processed per year is calculated in cell F20. Here, the total dollars received per day ($13.5 million) is divided by the average check size ($2,500) and multiplied by 360 to get the number of checks processed yearly. This times the cost per check for this lockbox (cell F21) gives the yearly cost of check processing (F23). The total cost of this lockbox

Table 2-3. Spreadsheet Analysis of One-Lockbox Solution to Example Problem.

	A	B	C	D	E	F
1	*Origination*	*Mail*	*Clearing*	*Total*	*Dollar*	*Total*
2	*Zone*	*Float*	*Float*	*Float*	*Originations*	*Float*
3		*Time*	*Time*	*Time*	*per Day*	*Dollars*
4					*(thousands)*	*(thousands)*
5						
6	Proposal 1 (Lockbox Located in Zone B)					
7						
8	A	2.6	1.2	3.8	$3,500	$13,300
9	B	2.5	0.8	3.3	$4,500	$14,850
10	C	2.7	1.3	4	$3,000	$12,000
11	D	3.3	1.5	4.8	$2,500	$12,000
12						
13	Total Dollar Float for This Lockbox					$52,150
14	Required Return					0.09
15						
16	Opportunity Cost of Float—One Lockbox Solution					$4,694
17						
18	Fixed Cost of Lockbox for This Location					$4
19						
20	Number of Checks Processed per Year					1,944
21	Processing Cost per Check					$0.10
22						
23	Cost of Check Processing—One Lockbox Solution					$194
24						
25	Total Cost (Op. Cost of Float + Fixed + Proc.)					$4,892
26						
27						
28	Proposal 2 (Lockbox Located in Zone C)					
29						
30	A	2.8	1.8	4.6	$3,500	$16,100
31	B	3.1	1.3	4.4	$4,500	$19,800
32	C	2.8	1.1	3.9	$3,000	$11,700
33	D	3.5	1.3	4.8	$2,500	$12,000
34						
35	Total Dollar Float for This Lockbox					$59,600
36	Required Return					0.09
37						
38	Opportunity Cost of Float—One Lockbox Solution					$5,364
39						
40	Fixed Cost of Lockbox for This Location					$10
41						
42	Number of Checks Processed per Year					1,944
43	Processing Cost per Check					$0.15
44						
45	Cost of Check Processing—One Lockbox Solution					$292

46						
47	Total Cost (Op. Cost of Float + Fixed + Proc.)					$5,666
48						
49						
50	Proposal 3 (Lockbox Located in Zone D)					
51						
52	A	3.0	1.7	4.7	$3,500	$16,450
53	B	3.3	1.5	4.8	$4,500	$21,600
54	C	3.5	1.3	4.8	$3,000	$14,400
55	D	1.8	1.0	2.8	$2,500	$7,000
56						
57	Total Dollar Float for This Lockbox					$59,450
58	Required Return					0.09
59						
60	Opportunity Cost of Float—One Lockbox Solution					$5,351
61						
62	Fixed Cost of Lockbox for This Location					$4
63						
64	Number of Checks Processed per Year					1,944
65	Processing Cost per Check					$0.13
66						
67	Cost of Check Processing—One Lockbox Solution					$253
68						
69	Total Cost (Op. Cost of Float + Fixed + Proc.)					$5,607

strategy is presented in cell F25 (the contents of this cell are "F16+F18+F23"). This part of the spreadsheet was then copied (replicated) into rows 28 through 47 and 50 through 69, and modifications made to reflect the characteristics of the other two possible lockbox locations.

Comparison of the three total costs shows that proposal 1, entailing a lockbox in zone B, is the least costly, with a yearly total cost of $4.892 million. But would some combination of lockboxes do a better job? Can costs be reduced further? These questions can be answered by executing the integer programming approach previously described. To execute this approach, the coefficients of the x_{ij} and y_j variables in the objective function [the d_j, c_{ij}, h_i, and s_j terms in equation (2-1)] must first be calculated. The resulting objective function and the problem's constraint equations are then input into an integer programming package and the optimum solution is found via the software.

For the example lockbox problem, there are three possible lockboxes and thus three y_j variables (y_B, y_C, and y_D). There are four origin zones and thus twelve x_{ij} variables (since customers in each zone could be assigned to send checks to any of the lockboxes, the number of possible assignments of customers to lockboxes is the number of lockboxes times the number of zones). There are thus 15 terms in the objective function. The coefficients

Table 2-4: Spreadsheet Computation of Coefficients of x_{ij} for Example Problem

	A	B	C	D	E	F	G	H	I	J	K	L
1	Origination	Mail	Clearing	Total	Dollar	Total	Required	c_{ij}	h_i	s_i	$h_i s_i$	$c_{ij} + h_i s_i$
2	Zone	Float	Float	Float	Originations	Float	Return					
3		Time	Time	Time	per Day	Dollars						
4					(thousands)	(thousands)						
5												
6	Proposal 1 (Lockbox Located in Zone B)											
7												
8	A	2.6	1.2	3.8	$3,500	$13,300	0.09	1197.0	504	$0.10	50.4	1247.4
9	B	2.5	0.8	3.3	$4,500	$14,850	0.09	1336.5	648	$0.10	64.8	1401.3
10	C	2.7	1.3	4	$3,000	$12,000	0.09	1080.0	432	$0.10	43.2	1123.2
11	D	3.3	1.5	4.8	$2,500	$12,000	0.09	1080.0	360	$0.10	36.0	1116.0
12												
13												
14												
15	Proposal 2 (Lockbox Located in Zone C)											
16												
17	A	2.8	1.8	4.6	$3,500	$16,100	0.09	1449.0	504	$0.15	75.6	1524.6
18	B	3.1	1.3	4.4	$4,500	$19,800	0.09	1782.0	648	$0.15	97.2	1879.2
19	C	2.8	1.1	3.9	$3,000	$11,700	0.09	1053.0	432	$0.15	64.8	1117.8
20	D	3.5	1.3	4.8	$2,500	$12,000	0.09	1080.0	360	$0.15	54.0	1134.0
21												
22	Proposal 3 (Lockbox Located in Zone D)											
23												
24	A	3.0	1.7	4.7	$3,500	$16,450	0.09	1480.5	504	$0.13	65.5	1546.0
25	B	3.3	1.5	4.8	$4,500	$21,600	0.09	1944.0	648	$0.13	84.2	2028.2
26	C	3.5	1.3	4.8	$3,000	$14,400	0.09	1296.0	432	$0.13	56.2	1352.2
27	D	1.8	1.0	2.8	$2,500	$7,000	0.09	630.0	360	$0.13	46.8	676.8

of the y_j variables in this function (the d_j terms) are simply the fixed costs of the lockboxes in these locations. The major analytic problem in specification of the objective function is therefore the calculation of the coefficients of the x_{ij} terms (equal to $c_{ij} + h_i s_j$ for each x_{ij}). A spreadsheet approach to calculating these coefficients is presented in Table 2-4.

In this table, cells A1 through F11 are the same as those in Table 2-3. Cells F8 through F11 are multiplied by the required return in column G to obtain the c_{ij} terms in column H. The number of checks originating yearly from each zone is calculated in column I by dividing the dollar origins in column E by the average check size (to get daily check numbers) and multiplying by 360 (thus, cell I8 contains "(E8/2500*360"). Column K (which contains $h_i s_j$) is column j (which contains the processing charge per check for this lockbox, s_j) times column I. Column L, the coefficient of the respective x_{ij} term, is column H plus column K. For example, the number 1247.4 in cell L8 is $c_{AB} + h_A s_B$, since for this cell checks originate in zone A and the lockbox is in zone B. This will be the coefficient of x_{AB} in the objective function.

Once the coefficients of the x_{ij} and y_j are calculated, objective function and the constraint equations are input into an integer programming package. The appropriate formulation for the example problem is presented in Table 2-5. We have used the popular LINDO mathematical programming package here (from LINDO Systems, Inc.). Other popular integer programming packages use similar input and output formats. The objective function is equation (2-1) with the appropriate coefficients of the three y_j and the 12 x_{ij} variables. Lines 2 through 5 represent equation (2-2), which says that the checks from each origin zone must be sent to one of the lockbox locations. There is one such equation for each origin zone. Here, since there are three possible lockboxes, there are three x_{ij} variables in each equation, and since there are four origin zones, there are four equations. Line 6 represents equation (2-6), which constrains the maximum number of lockboxes in the solution. We have not constrained the solution to any number of lockboxes less than the maximum; there are three possible lockboxes, any of which we allow to be open. If we wished to constrain the solution to a lesser number of lockboxes, this would be reflected on the right-hand side of this line. Lines 7 through 30 and 34 through 36, along with the integer constraints (which do not print in the LINDO package), constrain values of the y_j and x_{ij} variables to be zero-one integers, representing equations (2-4) and (2-5). Lines 31, 32, and 33 represent equation (2-3), and allow the assignment of the customer's checks from a zone to a lockbox location only if that lockbox is open. Since there are three lockboxes, there are three such constraints.[17]

The results of the integer programming solution to this lockbox problem

[17]Some of these constraints are redundant. We include them only to follow the problem format as given in equations (2-1) through (2-6).

Table 2-5. LINDO Formulation for Example Problem (any number of lockboxes up to n allowed).

```
MIN      3 YB + 10 YC + 4 YD + 1401.3 XBB + 1247.4 XAB
       +1524.6 XAC + 1546 XAD + 1879.2 XBC + 2028.2 XBD + 1123.2 XCB
       +1117.3 XCC + 1352.2 XCD + 1116 XDB + 1134 XDC + 676.8 XDD
SUBJECT TO
        2)     XAB + XAC + XAD = 1
        3)     XBB + XBC + XBD = 1
        4)     XCB + XCC + XCD = 1
        5)     XDB + XDC + XDD = 1
        6)     YB + YC + YD <= 3
        7)     XAB <=   1
        8)     XAC <=   1
        9)     XAD <=   1
       10)     XBB <=   1
       11)     XBC <=   1
       12)     XBD <=   1
       13)     XCB <=   1
       14)     XCC <=   1
       15)     XCD <=   1
       16)     XDB <=   1
       17)     XDC <=   1
       18)     XDD <=   1
       19)     XAB >=   0
       20)     XAC >=   0
       21)     XAD >=   0
       22)     XBB >=   0
       23)     XBC >=   0
       24)     XBD >=   0
       25)     XCB >=   0
       26)     XCC >=   0
       27)     XCD >=   0
       28)     XDB >=   0
       29)     XDC >=   0
       30)     XDD >=   0
       31)  - 4 YB + XBB + XAB + XCB + XDB <=   0
       32)  - 4 YC + XAC + XBC + XCC + XDC <=   0
       33)  - 4 YD + XAD + XBD + XCD + XDD <=   0
       34)     YB <=   1
       35)     YC <=   1
       36)     YD <=   1
       END
       INTEGER-VARIABLES=        15
```

Table 2-6. LINDO Solution for Example Problem (any number of lockboxes up to n *allowed).*

OBJECTIVE FUNCTION VALUE

1) 4455.70000

VARIABLE	VALUE	REDUCED COST
YB	1.000000	3.000000
YC	.000000	10.000000
YD	1.000000	4.000000
XBB	1.000000	1401.300000
XAB	1.000000	1247.400000
XAC	.000000	1524.600000
XAD	.000000	1546.000000
XBC	.000000	1879.200000
XBD	.000000	2028.200000
XCB	1.000000	1123.200000
XCC	.000000	1117.800000
XCD	.000000	1352.200000
XDB	.000000	439.200000
XDC	.000000	457.200000
XDD	1.000000	.000000

are presented in Table 2-6.[18] Only two of the possible lockboxes appear in the optimal formulation: proposal number 1 (with a lockbox in zone B) and proposal number 3 (with a lockbox in zone D). Proposal number 2, with a lockbox in zone C, does not appear since it does not reduce the total cost of the system. The cost with these two lockboxes, given an optimal assignment of customers to these lockboxes, is $4.456 million per year, which is $336,000 a year less expensive than the best one-box solution. In the optimal strategy, customers' checks originating in zones A, B, and C are sent to the lockbox in zone B (since x_{AB}, x_{BB}, and x_{CB} are all 1.0), while checks originating in zone D are sent to the lockbox in zone D (since x_{DD} is 1.0). All the remaining assignment variables are, of course, zero.

One of the substantial advantages of approaching the lockbox location problem in this way is *flexibility*. While Table 2-5 presents the formulation for the *optimal* (lowest total cost) solution, judicious changes in the constraint equations can provide very interesting information to the firm performing the analysis. For example, changing the constraints on the y_j terms in lines 34 through 36 to force one of these terms to 1.0 and the others to zero will produce the one-lockbox solution for that lockbox. The formulation and

[18]In the interest of space, and because of the difficulty in interpreting slack values and dual prices in the solution to an integer programming problem, the slacks and duals are not presented here or for any of the other integer programming solutions in this text.

Table 2-7. LINDO Formulation for Example Problem (one-lockbox problem; lockbox at location B).

```
MIN      3 YB + 10 YC + 4 YD + 1401.3 XBB + 1247.4 XAB + 1524.6 XAC
       + 1546 XAD + 1879.2 XBC + 2028.2 XBD + 1123.2 XCB + 1117.8 XCC
       + 1352.2 XCD + 1116 XDB + 1134 XDC + 676.8 XDD
SUBJECT TO
         2)   XAB + XAC + XAD =   1
         3)   XBB + XBC + XBD =   1
         4)   XCB + XCC + XCD =   1
         5)   XDB + XDC + XDD =   1
         6)   YB + YC + YD <=   3
         7)   XAB <=   1
         8)   XAC <=   1
         9)   XAD <=   1
        10)   XBB <=   1
        11)   XBC <=   1
        12)   XBD <=   1
        13)   XCB <=   1
        14)   XCC <=   1
        15)   XCD <=   1
        16)   XDB <=   1
        17)   XDC <=   1
        18)   XDD <=   1
        19)   XAB >=   0
        20)   XAC >=   0
        21)   XAD >=   0
        22)   XBB >=   0
        23)   XBC >=   0
        24)   XBD >=   0
        25)   XCB >=   0
        26)   XCC >=   0
        27)   XCD >=   0
        28)   XDB >=   0
        29)   XDC >=   0
        30)   XDD >=   0
        31) - 4 YB + XBB + XAB + XCB + XDB <=   0
        32) - 4 YC + XAC + XBC + XCC + XDC <=   0
        33) - 4 YD + XAD + XBD + XCD + XDD <=   0
        34)   YB =   1
        35)   YC =   0
        36)   YD =   0
END
INTEGER-VARIABLES=          15
```

A-76229

Table 2-8. LINDO Solution for Example Problem (one-lockbox problem; lockbox at location B).

OBJECTIVE FUNCTION VALUE

1) 4890.900000

VARIABLE	VALUE	REDUCED COST
YB	1.000000	3.000000
YC	.000000	10.000000
YD	.000000	1752.800000
XBB	1.000000	1401.300000
XAB	1.000000	1247.400000
XAC	.000000	1524.600000
XAD	.000000	1985.200000
XBC	.000000	1879.200000
XBD	.000000	2467.400000
XCB	1.000000	1123.200000
XCC	.000000	1117.300000
XCD	.000000	1791.400000
XDB	1.000000	.000000
XDC	.000000	18.000000
XDD	.000000	.000000

output for this alteration is presented in Tables 2-7 and 2-8 for lockbox y_B. Note that the value of total cost in the objective function is the same as that calculated via the spreadsheet approach in Table 2-3, allowing for rounding.

The integer programming approach can also be used to optimally schedule other configurations of lockbox locations so that the firm might contrast the cost of these strategies with the lowest-cost solution. For example, suppose that the firm wants to assess the cost of including the lockbox bank represented by proposal 2 (lockbox location C) in the firm's lockbox system. Perhaps the firm desires to have borrowing capacity available from this bank and believes that having a lockbox location at the bank would help in negotiations, or perhaps there is an existing lockbox at location C which the firm does not wish to close. In the lowest-cost solution, there is no lockbox at this location; y_C, as determined by the algorithm, is zero. To be sure that this lockbox is included in the system, we can change a constraint on y_C in line 35 from "YC $\leq$ 1.0" to "YC = 1.0". This requires the algorithm to give the minimum-cost solution which includes an open lockbox at location C. This formulation and solution is presented in Tables 2-9 and 2-10. With this constraint on y_C, the optimal solution is to open all the lockboxes. Checks for zones A and B are assigned to the lockbox at B, those from C to C, and those from D to D. The total cost is $4.460 million per year, which is $4,000 more expensive than the lowest-cost solution. With this information, the firm could decide

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A-76229

Table 2-9. LINDO Formulation for Example Problem (solution constrained to include lockbox location C).

```
MIN       3 YB + 10 YC + 4 YD + 1401.3 XBB + 1247.4 XAB + 1524.6 XAC
        + 1546 XAD + 1879.2 XBC + 2028.2 XBD + 1123.2 XCB + 1117.8 XCC
        + 1352.2 XCD + 1116 XDB + 1134 XDC + 676.8 XDD
SUBJECT TO
          2)    XAB + XAC + XAD =   1
          3)    XBB + XBC + XBD =   1
          4)    XCB + XCC + XCD =   1
          5)    XDB + XDC + XDD =   1
          6)    YB + YC + YD <=   3
          7)    XAB <=   1
          8)    XAC <=   1
          9)    XAD <=   1
         10)    XBB <=   1
         11)    XBC <=   1
         12)    XBD <=   1
         13)    XCB <=   1
         14)    XCC <=   1
         15)    XCD <=   1
         16)    XDB <=   1
         17)    XDC <=   1
         18)    XDD <=   1
         19)    XAB >=   0
         20)    XAC >=   0
         21)    XAD >=   0
         22)    XBB >=   0
         23)    XBC >=   0
         24)    XBD >=   0
         25)    XCB >=   0
         26)    XCC >=   0
         27)    XCD >=   0
         28)    XDB >=   0
         29)    XDC >=   0
         30)    XDD >=   0
         31)  - 4 YB + XBB + XAB + XCB + XDB <=   0
         32)  - 4 YC + XAC + XBC + XCC + XDC <=   0
         33)  - 4 YD + XAD + XBD + XCD + XDD <=   0
         34)    YB <=   1
         35)    YC =   1
         36)    YD <=   1
END
INTEGER-VARIABLES=      15
```

Table 2-10. LINDO Solution for Example Problem (solution constrained to include lockbox at location C).

OBJECTIVE FUNCTION VALUE

1) 4860.30000

VARIABLE	VALUE	REDUCED COST
YB	1.000000	3.000000
YC	1.000000	.000000
YD	1.000000	4.000000
XBB	1.000000	.000000
XAB	1.000000	.000000
XAC	.000000	277.200000
XAD	.000000	298.600000
XBC	.000000	477.899900
XBD	.000000	626.899900
XCB	.000000	.000000
XCC	1.000000	−5.399902
XCD	.000000	229.000000
XDB	.000000	439.200000
XDC	.000000	457.200000
XDD	1.000000	.000000

whether having a lockbox relationship with the bank at location C is worth this additional yearly cost.

SOME WARNINGS ABOUT LOCKBOX LOCATION DECISIONS

This integer programming approach to lockbox location analysis should seem a relatively straightforward approach to a rather complex problem. However, there are difficulties associated with both the collection of the necessary data and with the lockbox location algorithm. These difficulties do not invalidate this approach to the lockbox location problem, but they do limit its accuracy. They should be kept in mind when the firm is considering, analyzing, and implementing a lockbox strategy.[19]

Determining Customer Zones Because the opportunity cost of float is computed as the sum of the mail and clearing floats, the customer zones must be defined such that these should be as homogeneous as possible within the zone. Otherwise, nonoptimal strategies may result. For example, suppose we define one zone as "checks originating in Chicago" and one of the possible

[19]Many of these problems are discussed in Maier and Vander Weide, "What Lockbox and Disbursement Models Really Do."

lockboxes is located in Chicago. If this lockbox is opened, all the customers from Chicago might be assigned to this lockbox. But suppose that there are, within this customer group, some customers who pay with checks written on Chicago banks and some who pay with checks written on West Coast banks. Depending on the disadvantage in mail time and the advantage in clearing time, it might be better to have the customers issuing checks drawn on West Coast banks send their checks to a West Coast lockbox. This possibility would be allowed if the customers with checks originating in Chicago were broken up into two zones: "customers with checks originating in Chicago drawn on Chicago banks" and "customers with checks originating in Chicago drawn on West Coast banks." Greater accuracy in defining zones leads to more nearly optimal solutions, but increases the cost of data collection and the cost of computation.

Obtaining Bank Cost Data Estimating the least-cost placement of lockboxes requires data on the fixed costs of opening a lockbox at a particular location and the variable cost of processing checks at that location. The logical approach to collecting this data is to conduct a telephone or mail survey of potential bank lockbox sites, inquiring as to these costs. However, such surveys do not necessarily produce accurate results. Banks seem prone to offer their services in packages, with the costs of individual services tied to the size of the package. For example, a firm requesting only a lockbox service might pay more for that service than would a firm requesting a package of a lockbox service and a borrowing arrangement. Thus, the cost of a lockbox service might depend on the amount of other services that the firm is willing to purchase from the lockbox bank.

The criticality of this bank cost data depends on the average size of the checks received by the firm. If the firm receives many checks for relatively small amounts, data on bank charges (particularly processing costs) are very important, as these will be an important fraction of the total costs associated with the lockbox. If the average size of the checks received by the firm is relatively large, bank cost data will be a good deal less important, as the dominant cost will be the opportunity cost of float and not check processing cost.[20]

Obtaining a Representative Sample of Check Volume and Origination Since a one-month sample of the receiving firm's incoming checks is usually used for this purpose, the firm must be very careful that this sample is representative of the entire year's check patterns. If the firm has seasonality in its sales, or if receipts are concentrated from a few remitting firms that pay infrequently, a small sample may not adequately represent the total pattern of receipts.

[20]See Kallberg and Parkinson, *Current Asset Management*, pp. 21–23.

Further, *the very redesign* of the firm's cash receipt system may cause changes in this pattern of receipts if some of the firm's customers practice "disbursement management." Disbursement management, we will see later in this chapter, can entail an attempt by the disbursing firm to stretch the clearing float on outgoing checks. This strategy may negate the receiving firm's attempts to speed float via lockbox strategy. For example, suppose that a major customer of a firm currently pays from Cleveland with checks drawn on a bank in Cleveland. Based on this data, the firm locates a lockbox in Cleveland to reduce clearing float and instructs the major customer to send remittances to this box. The disbursing firm may counter this by paying to the Cleveland lockbox with checks drawn on a bank in Los Angeles, increasing clearing float to counter the receiving firm's acceleration strategy.

The Costing of Float In most lockbox location analysis systems, the opportunity cost of float is determined by multiplying the total float in dollars times the required rate of return. This approach is intended to account for the time value of money, but it is exactly equivalent to net present value only when the future cash flow stream is a perpetuity.[21] When there is growth in the future cash flows, the opportunity cost approach will misstate the time cost of funds relative to their proper discounted value. In the analysis of lockbox strategies, an understatement occurs; the amount of this bias is a nonlinear function of the rate of growth, the float time, and the required return.[22] Because of problems such as this, in this text we try to avoid using the opportunity cost approach to the time value of money, calculating instead the net present values whenever possible. However, it does not appear that a reasonably simple solution to the lockbox location problem can be achieved unless the opportunity cost system is adopted.

Interaction with the Availability of Borrowing Capacity As noted previously, banks are prone to provide services in packages. Thus, the location of a lockbox at a bank may make the bank more likely to lend to the firm, possibly at favorable rates.[23] This borrowing availability and lower cost have a value to the firm. In such a case, the cash cost of the lockbox overstates the disadvantage to the firm of having a lockbox at a particular bank.

To restate, these difficulties do not negate the analysis methodologies presented in this text as a valid approach to the lockbox location problem. However, managers should be aware of these difficulties and their implications for lockbox analysis.

[21]We will prove this in Chapter 5.

[22]See Howard Lanser and John Halloran, "Evaluating Cash Flow Systems Under Growth," *Financial Review* (May 1986), pp. 309–18.

[23]See C. A. Batlin and Susan Hinko, "Lockbox Management and Value Maximization," *Financial Management* (Winter 1981), pp. 39–44.

CASH CONCENTRATION STRATEGIES

Once the remittances from the firm's customers have been received and cleared, the resulting cash balances are available in the firm's lockbox (depository) banks. It is useful for the firm to gather these balances from the lockbox banks into a central bank account. The process of collecting funds is called *cash concentration*. There are two reasons why cash concentration is advantageous. First, the collection process results in a larger pool of funds. The larger pool makes any temporary interest-earning investments more economical because it reduces the transaction costs per dollar invested. For example, if the firm gathers $10,000 from each of 10 accounts, the firm will have $100,000 to invest, resulting in one purchase of securities rather than 10. Also, the larger amount of funds may enable the firm to take advantage of higher-interest investments that require a larger minimum purchase. Second, with all the cash in a central location, keeping track of the cash (controlling the cash) is considerably simplified.[24]

In concentrating cash, firms channel collected funds from the depository banks to the central concentration bank, sometimes through regional concentration banks (see Figure 2-4). The main questions in the generation of a cash concentration system revolve around the design of the system and the scheduling of transfers. Typical questions include:

- What banks should be the firm's concentration banks?
- What mechanism should the firm use to concentrate funds?
- How frequently should transfers to the concentration banks be made?[25]

Concentration Banks. The choice of concentration banks is not a simple one and is not easily quantified. For best clearing availabilities, these banks are usually located in major Fed cities, and firms often have a regional concentration bank located in each of the Federal Reserve districts in which it does a major amount of business. Cost is one consideration in choosing con-

[24]Another function of the cash concentration system, according to some authors, is to generate *dual balances*. Dual balances are caused by the Fed's policy of crediting cash to the firm on a fixed schedule even when the cash has not actually been collected, with the Fed funding the difference. Moving funds from one account to another sometimes creates dual balances. However, since 1979 the Fed has greatly reduced the amount of these dual balances, and is striving to reduce them further. Also, the Fed is certainly not in support of cash management strategies aimed at creating dual balances. For these reasons, the author does not consider the creation of dual balances to be a valid consideration in the design of a cash concentration system.

[25]In the interests of tractability, it will be assumed that the firm formulates separate strategies to address each of these questions. Simultaneous approaches to more than one of these questions tend to be extremely complex. For a critique of this approach, see Srinivasan and Kim, "Deterministic Cash Flow Management," pp. 154–156.

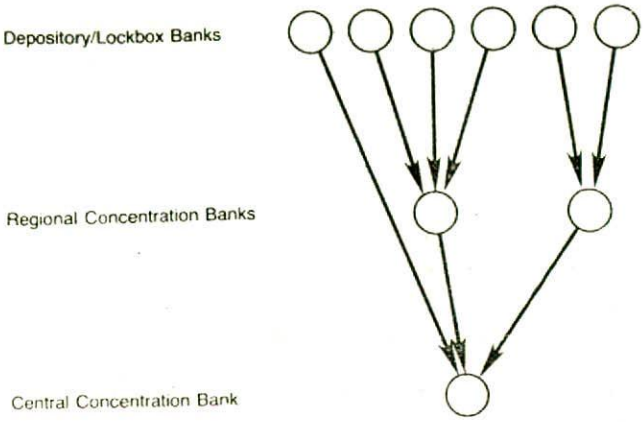


Figure 2-4. A Typical Cash Concentration System (arrows indicate transfers of collected funds).

centration banks, but more important is the array of other services that concentration banks can provide, particularly by assisting with the firm's credit needs and arrangements.

Concentration Mechanisms Several mechanisms are available for the firm to use in transferring funds from its collection banks to regional concentration banks, and from there to the central concentration bank. These mechanisms differ in cost and in the availability of funds that they provide. The cheapest transfer mechanism is the *depository transfer check*. This document instructs one bank to send funds to another, and is treated the same as any other check. A depository transfer check typically costs less than $1.00 and provides availability of the funds at the depositing bank in one or two days. A second alternative is the *automated clearinghouse (ACH) electronic transfer*. This vehicle is essentially an electronic version of the depository transfer check, and can be used between banks that participate in the automated clearinghouse system. Depending on the expense of preparation, these typically cost less than $5.00 and provide one-day availability. The most expensive vehicle is the *wire transfer*. These are electronic messages between banks; they cost about $10.00 and provide same-day availability.

Decisions regarding concentration mechanisms *usually hinge on the size and spread of the firm's deposits*. Firms with small deposits spread over a substantial number of banks will tend to have more extensive concentration systems and will transfer among accounts using low-cost transfer vehicles that offer only delayed availability (such as depository transfer checks). These characteristics are made necessary by the small individual amounts involved in the firm's deposits; concentration systems for supermarket chains and fast-food restaurants have these characteristics. Firms with larger deposits will

have fewer accounts (since a local deposit for transfer to an upstream bank is not needed) and will use more expensive and more rapid transfer mechanisms (such as wire transfers). The concentration systems of major chemical companies, for example, tend to be structured in this way.

The choice among alternative mechanisms for the transfer of cash in concentration systems can be analyzed based on their relative cost. Comparison of the opportunity cost of float during transfer and the transfer costs gives the optimum. For example, assume that a particular lockbox of a firm receives an average of $75,000 per day in receipts. The firm follows a policy of transferring the collected cash from this lockbox bank to a regional or central concentration bank at the end of each business day. For this particular situation, a depository transfer check would cost $1.50 and provide two-day availability; an ACH electronic transfer would cost $3.00 and provide one-day availability; and a wire transfer would cost $16.00 and provide one-half-day availability. The required return is 8 percent per year.

The alternatives are compared in Table 2-11. In this table, the transfer cost per year is the number of transfers times the cost per transfer. For example, for the alternative of depository transfer checks, there are 250 transfers per year (the assumed number of business days) and each costs $1.50, so the transfer cost per year is $375. The float resulting from the transfer policy is the daily deposit amount times the availability from the transfer mechanism (for depository transfer checks, $75,000 per day times the two-day availability gives a $150,000 float). Costing this float at the required return produces its opportunity cost, and the total transfer cost of each alternative is that alternative's transfer cost plus its opportunity cost of float. For this

Table 2-11. Costing of Transfer Mechanisms for Example Problem.

Daily Deposit	$75,000		
Transfers/Year	250		
Transfer Mechanism	*Depository Transfer Check*	*ACH Electronic Transfer*	*Wire Transfer*
Cost per Transfer	$ 1.50	$ 3.00	$ 16.00
Availability (Days)	2.00	1.00	0.50
Transfer Cost per Year	$ 375	$ 750	$ 4,000
Float	$150,000	$75,000	$37,500
Required Return	0.08%	0.08%	0.08%
Opportunity Cost of Float	$ 12,000	$ 6,000	$ 3,000
Total Transfer Cost	$ 12,375	$ 6,750	$ 7,000

problem, the lowest cost solution is to use the ACH electronic transfer mechanism.[26]

Frequency of Transfer Similar to the decision of what transfer vehicle to use is the decision on how frequently to make transfers. Frequent transfers to the concentration banks are costly but result in funds being available more quickly for use. Less frequent transfers reduce the out-of-pocket cost of these transfers but slow the availability of funds. While most large firms make daily transfers, other policies may be more economical.[27] Three of these policies are *scheduled transfers*, *trigger points*, and *anticipation transfers*.

Daily transfers are one variant of the *scheduled transfers* system. In this system, the firm decides on a predetermined time pattern of transfers from the depository bank to the concentration bank. These transfers are made regardless of the actual amounts of money received at the depository banks. For example, the firm might transfer every third business day or might set up a more complex schedule to reflect its historic pattern of cash receipts. Thus, a firm with heavy receipts early in each week and lighter receipts later in the week might choose to make transfers each Tuesday and Friday. The advantage of the scheduled transfers system is simplicity. The disadvantage is that transfers of uneconomically small amounts of funds may be made if receipts for a particular period happen to be less than expected.

The *trigger point* system avoids the transfer of small amounts by basing the transfer decision on account balance rather than on timing. In this system, the firm transfers all or part of the balance once the balance of collected funds in the depository bank reaches some predetermined level. This eliminates the transfer of small amounts, but results in a more complex decision process; balances at depository accounts must be monitored by the firm, or the depository bank must be given instructions to execute the trigger point transfers.

Anticipation transfers are a method of reducing the float within the transfer process. Here, the transfer is initiated based on anticipated (not actual) balances in the depository bank. For example, a firm with $100,000 in a depository account and expecting receipts of $900,000 over the next two days might initiate a depository transfer check for $1 million with a two-day availability. If receipts are as planned, the depository transfer check will reach the depository bank just as the funds are available for transfer. If the firm's daily cash forecasts are very accurate, this is an effective mechanism. How-

[26]The student may verify that for an average daily deposit of $100,000 the lowest cost solution is to use the wire transfer mechanism.

[27]The following discussion draws heavily from Berell Stone and Ned Hill, "Cash Transfer Scheduling for Efficient Cash Concentration," *Financial Management* (Autumn 1980), pp. 35–43.

ever, if there is uncertainty in these forecasts, the firm must hedge against the possibility that the depository transfer check will arrive but that there will not be sufficient funds to cover it (because collections over the two-day period were lower than expected). One such hedge is to keep a reserve of funds at the depository bank; the cost of this hedge offsets against the gains from the reduction of float in the transfer system.

Alternative frequency-of-transfer policies may be evaluated based on the cost of the transfers and the float resulting from various policies in a fashion similar to the evaluation of transfer mechanisms. However, advances in information technology may soon make all preplanned frequency-of-transfer systems a good deal less useful, particularly for larger firms. These preplanned systems are partly a result of the costliness of monitoring the balances in the various depository accounts maintained by the firm. The usual procedure for monitoring these balances used to require that a clerk (or a team of clerks) telephone all the firm's lockbox banks early in the morning to assess collected balances. This was an expensive procedure, and was prohibitively costly and complex if the firm had more than a few depository accounts; prespecified frequency-of-transfer systems avoid this data collection cost. However, software is now available to transfer balance information to the cash manager's computer before the opening of the work day. Investments and transfers based on actual account balances can then be initiated from the manager's computer work station. The Treasury Management System from BYAD Inc. (Arlington Heights, Illinois) is one software product with these capabilities. Systems such as this reduce the need for predetermined, inflexible transfer policies.

DISBURSEMENT MANAGEMENT

Lockbox systems address the collection of cash from the point at which the customer mails the check until the money is collected. Concentration systems address the gathering of this collected cash so that it may be used. *Disbursement management* addresses the efficient paying out of this cash once it is concentrated.

The firm's objective in disbursement management is to retain the cash for as long as possible. In this way, the firm will have the maximum amount of funds available for investment and transactional purposes. Obviously, this management entails making disbursements only when they are due and not before. Beyond this, the firm has several available sets of techniques for disbursement management. Included among these are the management of *disbursement float*, the use of *zero-balance accounts*, and the use of *controlled disbursing*.

Management of Disbursement Float In this set of techniques, the disbursing firm tries to defeat the receiving firm's attempts to reduce the float

on incoming checks. It does this by attempting to increase the length of time between the mailing of its checks and the eventual withdrawal of funds from its banks. This involves strategies for *increasing* mail float, at-firm float, and clearing float on its *outgoing* checks. For example, the disbursing firm may intentionally address checks to the firm's office address rather than its lockbox, creating at-firm float. Mathematical methodologies are available to maximize the expected mail and clearing float on outgoing checks.[28] These methods are mirror images of lockbox location routines, which *minimize* float on *incoming* checks.

This text does not present analysis techniques for the maximization of disbursement float nor does it recommend their use. There are four reasons for this position. First, in maximizing mail float, these techniques assume that a receiving firm calculates the receipt date for checks based on the check's postmark. Many firms, particularly larger ones, do not use this practice, but instead use the date of receipt at the firm's lockbox. When the receiving firm uses the lockbox receipt date, attempts to increase mail float by disbursing firms are irrelevant. Second, in maximizing clearing float, these algorithms often recommend disbursement locations in remote cities, cities where the disbursing firm does relatively little business. Having bank accounts with the small banks in these cities is of little advantage to the firm in negotiating other banking requirements (such as credit lines; remember, banks like to offer services in packages). Third, the maximization of disbursement float will not go unnoticed by sophisticated creditors for very long. When it is noticed, it will negatively affect relations with these creditors, reducing the firm's bargaining power with them. This may eventually cost the disbursing firm dearly when future prices and delivery schedules for goods and services purchased from trade creditors are negotiated. Finally, our experience with businesspersons indicates that the practice of lengthening disbursement float is regarded as being somewhat unethical, rather than as a legitimate business strategy. Firms that use this strategy will find their reputations with potential suppliers and others damaged.

Zero-Balance Accounts The remaining two methods of managing disbursements, zero-balance accounts and controlled disbursements, do not entail significant extensions of disbursement float to trade suppliers and other creditors. Instead, they are methods of synchronizing the arrival of funds at the disbursement bank accounts with the presentation of checks for disbursement at these accounts. That is, the firm holds the cash until the checks arrive (or are expected to arrive) at the disbursing bank. This coordination of funds inflows to disbursement banks with the presentation of checks does not work

[28]See Lawrence Gitman, D. Keith Forrester, and John R. Forrester, "Maximizing Cash Disbursement Float," *Financial Management* (Summer 1976), pp. 15–24.

to the detriment of creditors since the disbursing firm's checks to them are honored as presented, and these strategies do not affect float in any major way.

Zero-balance accounts are a very common strategy for funding disbursements as the checks are presented. In this strategy, an account for disbursement is first established at a bank. For the zero-balance system to be effective, the participating bank *must* be one on which most disbursements are made via the Fed's clearance system (which presents disbursements to banks early in the morning), and not a bank where disbursements occur throughout the day (as with a major money-center bank). Consequently, the banks used in zero-balance strategies are usually branches of major banks and not their main locations.

The once-a-morning disbursement requirement is *critical* to the zero-balance system. As implied by the name, the disbursing firm does not keep any permanent stock of cash in the disbursing account. Instead, the participating bank agrees that when the morning disbursements for the firm are presented to it, the bank will advise the firm of the amount of the cash required to cover these disbursements. The money will then be wire-transferred into the zero-balance account and the checks honored. In this way, the disbursing firms' checks are honored as presented, but the firm does not tie up cash while the checks are in the mail and while they are clearing.

Controlled Disbursing. If the zero-balance system is not feasible, another (though less attractive) is the use of *controlled disbursing*, which is often used when the firm's disbursing bank receives checks throughout the day. In this system, the firm projects the dollar amount of checks to arrive each day at the disbursement bank (based on the checks written in previous days and historic statistics on disbursement float) and transfers the amount of the expected checks to the account on that day or just before.[29] Of course, the firm does not know exactly what outstanding checks will be presented on any particular day; the forecasting procedure is subject to error because the disbursement float figures are uncertain. To hedge this uncertainty, firms keep a *safety stock of cash* in the disbursement account. The amount of this safety stock may be calculated if the probability distribution of disbursements is known.[30] For example, suppose that the probability distribution of disbursements is normal, that the firm wishes to have only a 1.0 percent chance that the initial cash will be insufficient to cover the presented checks, and that for a particular day the expected presentments are $500,000 with a standard deviation of $50,000. Since the Z-score for 1.0 percent of the normal distribution is 2.72, the firm will need to start with an additional $136,000 (2.72

[29]See Vander Weide and Maier, *Managing Corporate Liquidity*, pp. 142–65, for methods of projecting disbursement float times.

[30]We will extensively discuss this approach to the hedging of the risk of cash flows in the next chapter.

times $50,000) in its disbursement account, above the expected requirement of $500,000, to have only a 1.0 percent chance of having insufficient funds. Thus, the firm should arrange to have $636,000 in its account to cover disbursements for that day. The additional $136,000 is a hedge, and its carrying cost is the result of uncertainty regarding the firm's disbursement figure for that day.

SUMMARY

This chapter concerns the management of cash inflows and outflows. The management of these flows is a major task of the cash management group within the firm. This task starts when checks are in the mail to the firm and includes the management of these checks while they are in the postal system and while they are in the process of being cleared against the banks on which they are drawn. The major technique for the management of cash during this time is *lockbox location strategy*. Once the incoming checks have been cleared, the firm must gather the cash together so it can be efficiently used. The firm's system for the management of this gathering process is *cash concentration strategy*. When the firm is required to disburse the cash, it must also manage this process so the firm may retain the cash for the longest possible time. The management of outgoing cash is done via the firm's *disbursement management strategy*. All three strategies are discussed in this chapter.

This chapter also gives three reasons for holding cash and near-cash assets: for transactions needs, as a hedge against uncertainties in the firm's cash flow, and as a temporary investment of surplus funds. The popular near-cash assets used as alternatives to cash in hedging and as temporary investments for surpluses are discussed. Included in this discussion is an assessment of the relative risks (based on default and interest rate risks) and returns of these assets. What is not discussed (except in a very general way) is how firms develop strategies for the timing and amount of their investments in these assets. The following chapter, Chapter 3, discusses cash forecasting. The formulation of strategy for the temporary investment of surplus funds is a major outcome of cash forecasting procedures. Strategy for investment in the near-cash assets that are used as alternatives to cash in the hedging of the firm's cash flow uncertainty will be discussed in Chapter 4.

Looking to the future, one factor which may substantially impact lockbox location analysis and cash concentration strategies is the trend toward the deregulation of banks. A primary response by the financial sector to deregulation has been the geographic expansion of well-managed banks. Should true interstate banking come to pass, banks with interstate branches may be able to offer additional lockboxes to their customers at costs substantially less than the current costs of opening lockboxes at remote locations. Firms would then have additional incentives to locate lockboxes so as to reduce flotation

on incoming checks. Also, the costs of transferring funds to and from depository, concentration, and disbursement accounts at branch locations of multistate banks is likely to be much lower than the current costs of transfer mechanisms among banks, and more frequent transfers of funds will result. It is clear that interstate banking would be likely to increase the amount of firms' activity in the collection, concentration, and disbursement of cash flows while decreasing the cost of these systems. Firms' relationships with their banks would also become less complex since larger interstate banks would be able to provide a wider range of services, and firms would then be able to reduce the number of banks with which they deal.[31]

Problems

2-1. A firm is trying to decide where to locate its lockbox(es). Checks come to the firm from three different locations, and the firm has received proposals for lockboxes in two of these locations. Data on the lockbox proposals are:

Lockbox Location	*A*	*B*
Yearly Fixed Lockbox Charge	$10,000	$15,000
Processing Charge per Check	$0.10	$0.15
Clearing and Processing Float	1.5 days	1.0 day

The mail float matrix (in days) is:

		Origin Location		
		A	*B*	*C*
Destination Location	A	2.3	3.0	3.5
	B	3.0	2.2	3.2
	C	3.5	3.2	2.0

The check origination data is:

Origin Location	*Daily Receipts from This Origin Location*	*Yearly Number of Checks from This Origin Location*
A	$10,000,000	2,500,000
B	$15,000,000	3,500,000
C	$12,000,000	4,500,000

The firm's required return is 8.5 percent per year. Use a 360-day year. Based on the above data, calculate the yearly cost of: (a)

[31]For additional discussion, see Srinivasan and Kim, "Deterministic Cash Flow Management," p. 159.

using only one lockbox, with that lockbox located at A, and (b) using only one lockbox, with that lockbox located at B.

2-2. Develop the objective function and the necessary constraint equations for the lockbox location problem given in Problem 1. Use the following variable definitions:

YA = a 0–1 integer variable indicating whether there is a lockbox open at location A (1 if there is, 0 if there is not).

YB = a 0–1 integer variable indicating whether there is a lockbox open at location B (1 if there is, 0 if there is not).

XAA = a 0–1 integer variable indicating whether checks originating at location A are sent to the lockbox at location A (1 if they are, 0 if they are not).

XAB = a 0–1 integer variable indicating whether checks originating at location A are sent to the lockbox at location B (1 if they are, 0 if they are not).

XBA = a 0–1 integer variable indicating whether checks originating at location B are sent to the lockbox at location A (1 if they are, 0 if they are not).

XBB = a 0–1 integer variable indicating whether checks originating at location B are sent to the lockbox at location B (1 if they are, 0 if they are not).

XCA = a 0–1 integer variable indicating whether checks originating at location C are sent to the lockbox at location A (1 if they are, 0 if they are not).

XCB = a 0–1 integer variable indicating whether checks originating at location C are sent to the lockbox at location B (1 if they are, 0 if they are not).

2-3. Assume that the firm described in Problem 1 has solved its lockbox location problem via integer programming, with the following result:

YA = 1.0
YB = 1.0
XAA = 1.0
XAB = 0.0
XBA = 0.0
XBB = 1.0
XCA = 0.0
XCB = 1.0

Calculate the yearly cost of this solution to the lockbox location problem.

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case

Butler Manufacturing[1] Lockbox Location

The Butler Manufacturing Company was started in the mid-1950s, and was a subsidiary of a large German firm. By 1983, annual sales had reached the half-billion dollar level. The company was organized into five divisions of unequal size. This organization resulted, in part, from the piecemeal fashion in which the company had grown through acquisitions and, in part, from attempts to parallel the parent organization's structure with respect to product lines. Each division had, until recently, been operated quite autonomously; in fact, the divisions had been separately incorporated subsidiary operations until 1970.

In 1982, the firm had decided to centralize certain operations. Operating separately, each division had developed (or had inherited from its predecessor) different methods for handling various operations. Top management felt that some of these operations could be better addressed on a centralized basis. The corporate offices in New York City were closed and the company's top management was moved to recently constructed corporate headquarters in Philadelphia. The site was adjacent to the headquarters of Butler's largest division in terms of sales volume.

An example of how differently the five divisions were operated was provided by their respective accounts receivable management and cash application systems. The largest division of the firm had annual sales of $325 million. Its cash application system was a manual-computer hybrid, wherein the cash application clerk identified the items being paid from alphabetical listings and manually removed a computer card from a deck of such cards signifying the outstanding items. The computer then compared the cards pulled against the tape record of the division's accounts receivable to determine which items were to be removed. The firm's second-largest division, headquartered in St. Louis (with sales of $100 million per year), used an entirely computer-based system. As checks came in, data were encoded; a computer program then scanned the tape of outstanding accounts receivable items to determine which had been paid. The cash application clerk audited this cash application program by examining the check and invoice information presented on a CRT. A slightly smaller division, head-

[1]This case has been adapted from Iqbal Mathur and Frederick C. Scherr. *Cases in Financial Management* (New York: Macmillan, 1979), pp. 37–44.

Exhibit 1

Butler Manufacturing
Some Characteristics of Five Divisions

Headquarters Location	*Plant Locations*	*Annual Sales (Millions)*	*Number of Checks Received per Year*
Philadelphia	Virginia Texas	$325	67,000
St. Louis	Missouri	$100	25,000
Charlottesville	Virginia New Jersey	$80	53,000
Los Angeles	California	$40	81,000
New York	None—Products imported from Germany	$10	20,000

quartered in Charlottesville, Virginia (annual sales of $80 million), used a totally manual system; copies of invoices generated by this division were physically filed in alphabetical order as they were generated. When paid, they were removed from the file and stored by date of payment. The firm's smallest division, headquartered in New York (with annual sales of $10 million), used a similar system. The remaining division was headquartered in Los Angeles and had annual sales of $40 million. Its cash application and accounts receivable system was similar to that of the Philadelphia division except that no manual cards were used. The cash application clerk worked from computer printouts; after he had determined what item had been paid, this data was encoded and the paid item removed from the accounts receivable data tape. A chart of the divisions and some of their characteristics is presented in Exhibit 1.

Like the divisions' cash application and accounts receivable systems, their cash management policies also differed substantially, though top management had recently made some attempts at coordination and rationalization. Each division kept a central receiving and disbursing bank account for cash receipts and trade payments. Each division also had a payroll account at a local bank for each plant and/or headquarters location. Money was wire transferred into these payroll accounts on the day that paychecks were written, and drawn out by employees shortly thereafter. Sales offices (which were scattered about the United States) were not allowed to have local bank accounts; emergency needs were met from petty cash. Other disbursements were made from the divisional receipts and disbursements accounts. Paychecks for field salesmen were mailed to their homes, and were drawn on payroll accounts at their division's headquarters locations. The corporate headquarters used the accounts of the Philadelphia division. It was the company's policy to pay for all bank services on a straight fee (rather than compensating balance) basis.

In addition to the receipts and disbursements and the payroll accounts, the Philadelphia division had several small accounts with international banks. This division did a sizable export business, and it was felt that having these accounts was necessary to facilitate receipts from export sales. A chart of

Exhibit 2

Butler Manufacturing
Current Location and Number of Bank Accounts

Headquarters Location	*Rec.-Disb. Account Locations*	*Payroll Account Locations*	*International Account Locations*	*Total Account Locations*
Philadelphia	Philadelphia	Virginia Texas Pennsylvania	New York (5)	9
St. Louis	St. Louis	Missouri	None	2
Charlottesville	Char.	Virginia New Jersey	None	3
Los Angeles	Los Angeles	California	None	2
New York	New York	None—Products Imported from Germany	None	1
				17

the bank accounts of the firm is presented in Exhibit 2.

To facilitate the deposit of checks, most of the divisions kept lockboxes at the bank handling their receipts and disbursements account. The exception was the Los Angeles division. The credit manager of this division was opposed to using a lockbox because he felt that too large a portion of the checks for his division were nonroutine items: they were restrictively endorsed, postdated, and so on. Lockboxes do not handle these items well. It had not been determined if this contention was valid, but the Los Angeles division did deal (on average) with smaller customers than the other divisions.

Centralization of Butler's cash management functions had already resulted in several changes. The division had been required to submit weekly and monthly cash forecasts showing expected future bank balances. The cash management staff at the corporate headquarters then combined these forecasts and planned the firm's borrowing and investment strategies. However, with five divisions and thus five receipts and disbursements accounts, coordination was a problem. After the cash managers at headquarters had stripped off excess cash, unexpectedly low receipts at any division could cause a cash stockout at that division's disbursements account. This was handled by transferring cash from the accounts of one or more other divisions that had experienced greater-than-expected receipts. This transfer procedure led to substantial additional transfer costs and had necessitated significant time and effort on the part of the corporate cash management group to make and record such transactions.

Also, it was clear that the rather arbitrary location of the firm's lockboxes needed to be reconsidered. It was obvious that the divisionalized character of the lockbox locations did not lead to the quickest receipt of

funds. For example, the Philadelphia division did substantial business with customers in Virginia and in the Carolinas. However, under the current system all the checks from these customers were sent to the division's lockbox in Philadelphia even though the firm had a much closer lockbox in Charlottesville serving the division based there.

To address these problems, the firm's treasurer decided to rationalize the firm's cash collection system. The divisions would no longer operate their own collections systems. Instead, a series of lockboxes would be set up across the country; a customer of any division would be instructed to send remittances to the lockbox that would result in the shortest flotation. Concentration bank accounts would be set up at the firm's Philadelphia bank, and the receipts to these lockboxes would be transferred to the concentration account at the end of each day. Needed funds would be transferred to the disbursement accounts at the divisions as needed, similar to the procedure currently used for payroll disbursements at plant locations. It was felt that this concentration system would eliminate the transfers between divisions and result in the minimum economical flotations on incoming funds. The main question was where to put the lockboxes.

To address this problem, the firm divided the originating zip codes of incoming checks into six regions and solicited bids from one bank in each of these regions that could provide lockbox services.[2] The details of these bids are presented in Exhibit 3.

Prior to soliciting these bids, Butler had to decide what level of lockbox service was acceptable for the firm. These levels related to how much time and attention the bank providing the lockbox service would give to each check, how much information would be forwarded to Butler regarding each check, and how this information would be forwarded. The most expensive service involved the manual handling of each check, and the forwarding of the envelope in which the check was received, any remittance advice on how the check was to be applied, and a photocopy of the check itself. This careful handling and complete forwarding gave Butler the most information about each received check but was relatively costly (about $0.25 per check). The least expensive service provided only minimal data about each check (usually a computer printout listing only the payer and the check amount), and generally cost about $0.11 per check. Butler decided that an intermediate level of information (and thus processing charges) would be sufficient to identify the items paid by each check received, and had specified this level of service in soliciting lockbox bids from each of the banks.

A few more details on these bids are useful. The bid from the bank in the Northeast region was from the concentration bank in Philadelphia. Thus, no daily wire transfers

Exhibit 3

Butler Manufacturing
Details of Lockbox Bids

Lockbox Location	*Processing Cost per Check*	*Yearly Fixed Fee*
Northeast	$0.145	$2,500
Southeast	$0.130	$2,000
North Central	$0.160	$1,500
South Central	$0.140	$2,500
Northwest	$0.175	$4,000
Southwest	$0.180	$3,500

[2]The situation here has been considerably simplified to aid in calculation. Typically, two-digit zip code designations are used to designate regions (resulting in 100 regions) and many more offers are solicited, including some from several banks in the same region.

Exhibit 4

Butler Manufacturing Company
Average Mail Times between Parts of the Country in Days

	Destination					
Origin	*Northeast*	*Southeast*	*N. Central*	*S. Central*	*Northwest*	*Southwest*
Northeast	2.2	2.3	2.8	3.0	3.5	4.2
Southeast	2.3	2.1	3.2	2.9	4.2	3.6
North Central	2.8	3.2	2.5	3.0	2.9	3.0
South Central	3.0	2.9	3.0	2.3	3.1	3.2
Northwest	3.5	4.2	2.9	3.1	2.5	2.6
Southwest	4.2	3.6	3.0	3.2	2.6	2.5

Exhibit 5

Butler Manufacturing Company
Analysis of Customer Pay—from Location by Division
(percent of total receipts)

	Division				
	Philadelphia	*St. Louis*	*Charlottesville*	*Los Angeles*	*New York*
Northeast	31	12	30	0	76
Southeast	21	32	61	0	20
North Central	20	2	0	0	0
South Central	9	27	9	0	0
Northwest	2	10	0	26	0
Southwest	17	17	0	74	4

would be required with respect to this bid. For the other location proposals, daily wire transfers of collections would add $6,000 per year to the lockbox costs. Average mail times between regions are given in Exhibit 4. Butler's management decided to add one day to these mail times to account for clearing and processing float for all checks received from within a region and 1.5 days for all checks received from outside a region.[3] Butler's required rate of return on any funds freed due to decreased float was 8 percent. Remittance origin locations (pay-from locations) for Butler's customers are given in Exhibit 5.

[3]This structure of clearing times is assumed for simplicity. Clearing times usually vary from bank to bank, and thus from lockbox location to lockbox location.

Chapter -3

Cash Forecasting

In this chapter, methods for generating the firm's *cash forecast* (sometimes known as the firm's *cash budget*) are discussed. The cash forecast is an estimation of the flows in and out of the firm's cash account over a particular period of time, usually a quarter, month, week, or day. The cash forecast is primarily intended to produce a very useful piece of information: an *estimation of the firm's borrowing and lending needs* and the uncertainties regarding these needs during various future periods.

Cash forecasting is extremely important to most firms. It enables them to *anticipate* periods of surplus cash and periods where financing will be necessary. This anticipation is the reason that cash forecasts are generated. Anticipation enables the firm to *plan* much more effectively for investment and financing, and via this planning, produce superior returns.

WHY FORECAST CASH FLOWS?

To see why the cash forecasting process produces additional returns, let us consider the situation where the firm *does not* use cash forecasting.[1] Unanticipated cash shortages could occur. During these periods, the firm would have to slow down its cash outflows, dip into reserves, or get emergency financing. A typical method of slowing outflows in such a situation would be to delay payments to trade suppliers.[2] Because of these late payments, sup-

[1]For an excellent discussion of the need for cash forecasting, see E. M. Lerner, "Simulating a Cash Budget," *California Management Review* (Winter 1968), pp. 79–86, particularly pages 79–80.

[2]We refer here to an *unanticipated* delay in paying suppliers *beyond* their stated terms of sale. The *planned* use of delaying suppliers as a financing tool is discussed in Chapters 11 and 12, where we cover short-term financing.

pliers might delay delivery of critical materials, causing expensive production delays. The firm's slowness in payment might become known to customers and shareholders, who would question the firm's financial soundness. Alternatively, if the firm tried to get an emergency bank loan to provide the needed funds under these circumstances, the firm's bank might be reluctant to grant such a loan on reasonable terms, since their confidence in the ability of the firm's management would be lessened. Higher interest rates or onerous constraints on management actions would likely be the result.

Now let us consider the situation where the firm has an unanticipated, unplanned *surplus* of cash. This happy surprise causes almost as many problems as an unplanned deficit. Without a cash forecast, the firm has no way of knowing how long the surplus will persist, that is, if and when the cash will be needed for expenses. Without such knowledge, the firm cannot make an investment plan that will maximize interest income on the surplus funds. To see this, assume that the yield curve (discussed in Chapter 2) is upsloping. The longer the maturity of the investment, the higher the interest yield per period. But without a cash forecast, the firm cannot take advantage of this effect *because it does not know how long it will have the surplus funds*. If the money is going to be available for only a few days, it would be a mistake to buy a 90-day instrument; the firm would have to sell it prior to maturity, and subject itself to unnecessary interest rate risk. On the other hand, if the money is going to be available for six months, putting it in a relatively low-interest but highly liquid near-cash instrument would not produce the highest interest income. Thus we see that the cash forecast is a critical tool for effective financing of temporary deficits and investment of temporary surpluses.

While the firm's cash forecast is an important document, it is not generated nor does it function in isolation. The cash forecast is an important part of the firm's *cash control system* and is one of the forecasts that is part of the firm's *financial plan*. The firm's cash control system includes its lockboxes, its marketable securities portfolio, its short-term borrowing structure, and its cash management and transfer system. (All of these topics are discussed in other chapters.)[3]

Besides the cash forecast, the firm also makes numerous other forecasts as part of its financial planning process. In fact, these other documents are *critical inputs* to the cash forecast.[4] The firm's *capital budget* details planned outflows for capital expenditures. The firm's *production plan* provides significant information on expected manufacturing expenses. The firm's *sales forecast* is important in determining many other expected cash inflows and outflows. In fact, the firm's other financial plans and estimates are usually

[3]For more discussion of the relationship between the firm's cash budget and the other aspects of its cash management system, see F. W. Searby, "Use Your Hidden Cash Resources," *Harvard Business Review* (March-April 1968), pp. 71–80.

[4]For a discussion of the relationship between the firm's cash forecast and its other financial planning documents, see K. V. Smith, *Guide to Working Capital Management* (New York: McGraw-Hill, 1979), pp. 39–61.

generated *before* the cash forecast is made because so many of the estimates used in the cash forecast are dependent on these other plans.

TYPES OF CASH FORECASTS

The types of cash forecasts generated by firms can be differentiated along two dimensions: the *length of the periods* included within the cash forecast and the *approach to cash flows* used in the cash forecast.

The *length of the periods* refers to the units of time into which the cash forecast is divided. Firms may make cash flow forecasts over periods of various lengths: yearly flows, quarterly flows, monthly flows, weekly flows, or even daily flows. The most popular forecast involves monthly flows, but most firms do not confine themselves to a single forecast. Instead they use several forecasts with periods of various lengths.[5] The important question is what the length of the *shortest* period to be forecast should be. This depends critically on the volume of the firm's cash inflows and outflows because of the transaction costs involved in short-term investments. If the firm is sufficiently large that the investment of a day's surplus is profitable, it will probably pay the firm to forecast on a daily basis (as well as on periods of longer lengths). Small firms with lesser amounts to lend are probably better off using a week or even a month as the length of their shortest period (assuming that any temporary shortages of funds within these periods can be covered without undue cost).

Another question that arises when the firm makes forecasts involving multiple and overlapping period lengths is how one forecast relates to another. To see how this question arises, assume that a firm is practicing multiple period-length forecasting and is generating forecasts for the next quarter, months within this quarter, and weeks within these months. Does the forecaster start with quarterly data and break this down into months, then break the months down into weeks, or does the forecaster start with weekly data and aggregate this into months and quarters? Starting with data on relatively long periods and breaking it down into smaller periods is called *distribution* starting with data on relatively short periods and aggregating into longer periods is called *scheduling*.[6] Both methodologies have advantages and disadvantages. Scheduling requires more data manipulation, but distribution requires more sophisticated statistical techniques. The cash forecasting systems of most sophisticated firms involve a combination of the two methods.[7]

[5]See D. I. Fisher, *Cash Management* (New York: The Conference Board, 1973), page 5.

[6]See B. K. Stone and R. W. Wood, "Daily Cash Forecasting: A Simple Method for Implementing the Distribution Approach," *Financial Management* (Fall 1977), pp 40–50.

[7]See Stone and Wood, "Daily Cash Forecasting," page 41.

Table 3-1. A Typical Spreadsheet Cash Forecast by the Receipts and Disbursements Method.

	A	B	C	D
1	Cash Forecast			
2				
3	Month:	Jan.	Feb.	
4				
5	Receipts:			
6	Collections on Receivables	2,500	3,000	
7				
8	Disbursements:			
9	Labor Expenses	1,000	1,200	
10	Materials Expenses	900	1,300	
11	Other Expenses	300	600	
12				
13	Net Cash Flow (Rec − Disb)	300	−100	
14				
15	Beginning Cash	100	400	
16	Net Cash Position	400	300	
17	Required Cash Level	300	450	
18	Surplus Cash	100		
19	Deficit Cash		−150	

Firms use two common *approaches to cash flows* in generating the cash forecast: the *receipts and disbursements approach* and the *adjusted net income* approach. The receipts and disbursements approach uses the amounts of cash expected to be received and disbursed by the firm over the periods chosen for the forecast. This method minutely traces the movement of cash and is preferred by firms that exercise very close cash control. It is usually used for cash forecasts with relatively short period lengths (daily, weekly, or monthly) and short time horizons (up to a year or so in the future). The format of a typical spreadsheet cash forecast using the receipts and disbursements method is illustrated in Table 3-1; we will discuss the possible types of entries in such a spreadsheet forecast later in this chapter.

The adjusted income statement approach is sometimes called the sources and uses approach. Here, the forecaster starts with projected net income on an accrual basis and adjusts to a cash basis. The format of a typical spreadsheet cash forecast using the adjusted net income method is provided in Table 3-2. This method provides a representation of changes in asset and liability accounts; since the levels of these accounts are of interest to the firm, this aspect of the adjusted income statement approach is an advantage over the receipts and disbursements method. However, the adjusted net income method does not permit the tracing of the individual types of cash inflows and outflows for any given period, which is often useful information for the firm. The adjusted net income method is usually used by firms for longer-

Table 3-2. A Typical Spreadsheet Cash Forecast by the Adjusted Net Income Method.

	! A	!! B	!! C	!! D !
1	Cash Forecast			
2				
3	Month:	Jan.	Feb.	. . .
4				
5	Sources:			
6	Earnings After Taxes	100	150	. . .
7	Depreciation Addback	150	150	. . .
8	Increases in Liabilities	200	50	. . .
9	Decreases in Assets	50	300	. . .
10	Total Sources	500	650	. . .
11				
12	Uses:			
13	Dividends	25	25	. . .
14	Decreases in Liabilities	125	225	. . .
15	Increases in Assets	50	500	. . .
16	Total Uses	200	750	
17				
18	Net Cash Flow (Sour − Use)	300	−100	. . .
19	Beginning Cash	100	400	. . .
20	Net Cash Position	400	300	. . .
21	Required Cash Level	300	450	. . .
22	Surplus Cash	100		. . .
23	Deficit Cash		−150	. . .

term forecasts of surpluses and deficits (a year or more in the future). Since our concern in this text is with the management of short-term assets and liabilities, we will concentrate on the receipts and disbursements approach to forecasting. In our examples, we will center on monthly forecasts because of the popularity of this period length.

ITEMS TO BE FORECAST

In the receipts and disbursements cash forecasting method, estimates need to be made of the numerous major and minor items that the firm collects (receipts) and that it pays (disbursements). The more individual categories of items the firm includes in its forecast procedure, the more accurate the forecast may be, but the more costly in terms of time and effort it will be to generate.

Most firms find that the receipts portion of the cash forecast contains far fewer potential individual categories of items than does the disbursements forecast. Most firms' primary source of receipts is the collection of monies

Table 3-3. Some Possible Types of Cash Receipts.

Accounts Receivable Collections—Domestic Customers
Accounts Receivable Collections—Foreign Customers
Notes Receivable
Rental Income
Interest Income
Principal on Maturing Investments
Dividends from Subsidiaries
Miscellaneous Receipts

from sales: either via direct inflows from cash sales or via inflows resulting from the collection of accounts receivable, which are the results of prior sales. However, inflows from other sources not related to sales are sometimes non-trivial. In deciding what categories of inflows to include in the receipts portion of its cash forecast, management should first list all the firm's major types of inflows. A list of some possible types of inflows is provided in Table 3-3. Whether relatively minor categories of inflows should be examined in detail or merely aggregated and approximated depends on their size relative to other inflows *and their seasonality*. A type of inflow that contributes relatively little to total flow during one month may be quite important in another.

Table 3-4. Some Possible Types of Cash Disbursements.

Cash Purchases of Materials
Payroll—Executive
Payroll—Nonexecutive
Taxes—Local
Taxes—State
Taxes—Federal Income
Taxes—FICA Withheld
Maturing Accounts Payable
Mortgage Payments
Maturing Notes Payable—Interest and Principal
Capital Expenditures
Utility Payments
Dividends—Preferred Stock
Dividends—Common Stock
Sinking Fund Payments—Outstanding Bonds
Interest Payments—Outstanding Bonds
Maturing Bonds—Principal Repayments
Payments to Pension Fund
Market Repurchase of Outstanding Stock or Bonds
Miscellaneous Disbursements

The types of disbursements that a firm makes are usually more diverse than the types of receipts. The firm's major outflows are often for payroll, payments to suppliers, capital expenditures, and interest and principal payments. However, as with receipts, types and patterns of disbursements vary considerably with the firm's line of business, business style, and business strategy. Rent and lease payments, for example, are often a substantial portion of outflows for firms that do not own their facilities, but are less important to firms following other ownership policies. A list of potential outflow categories in the disbursements portion of the firm's cash forecast is provided in Table 3-4.

A primary question, after the firm has determined what types of receipts and disbursements are important in its overall cash flow, is how to forecast the future levels of these types of inflows and outflows. A brief review of the procedures most commonly used for financial forecasting is in order.

METHODS OF FINANCIAL FORECASTING

Financial forecasting is the estimation of the future level of a financial variable, often a cash flow, asset level, or liability level. It is usually assumed that the relationship between the financial variable and other variables is linear. The *general linear model* can then be used:

$$Y_t = a_o + a_1X_1 + a_2X_2 \ldots a_nX_n \qquad (3\text{-}1)$$

Here, Y_t is the financial variable (Y) to be forecast in period t. The X's are the *explanatory variables*; they are assumed to cause the level of Y in period t. The a_o term represents a constant unaffected by the X's. The other a terms are the estimated coefficients of the explanatory X variables. There are n terms with X's in them. This general methodology will be clearer as examples are presented. It is understood that any forecast made in this way is subject to some prediction error because of uncertainty about both the future levels of the X's and uncertainty about the exact relationship between the explanatory variables (the X's) and the outcome variable (the Y; that is, uncertainty about the a coefficients). There are four common approaches to forecasting financial variables, but they are all special cases of the general linear model. These four methods are discussed below.

Spot Method Here it is assumed that the variable to be forecast is independent of all other variables, or alternatively, is predetermined. The variable is forecast by using its expected or predetermined level. All other explanatory variables are presumed to be irrelevant and the formula used is

$$Y_t = a_o \qquad (3\text{-}2)$$

were a_o is the expected or predetermined level of Y_t. For example, if we are doing a cash forecast and we know that the level of a particular type of disbursement (such as rental payments) will be $12,000 in every month because of the firm's lease agreement, it would be reasonable to use the spot method to estimate rental payments as $12,000 per month.

Proportion of Another Account This technique is used to project financial variables that are expected to vary directly with the level of another variable. The formula used is:

$$Y_t = a_1 X_1 \tag{3-3}$$

where X_1 is the other variable to which Y is related and a_1 is the constant of proportionality between the two. The "percent of sales" method is a variation of this technique, wherein X_1 is sales for a particular period and a_1 is the percent. The "proportion of another account" method is widely used when there is a causal link from the explanatory variable to the variable to be forecast. For example, if sales volume (units sold) increases, it is natural that more units will have to be produced to replenish inventory. It is then reasonable to project certain direct costs of production, such as direct materials, as a percent of sales. In this circumstance, if costs of direct materials have historically been 50 percent of sales, and sales for a particular period have been forecast as $100,000, the firm would normally project direct material purchases at $50,000 for that period.

Compounded Growth This method is used when a particular financial variable is expected to grow at a steady growth rate over time. The formula is the same as equation (3-3), but the explanatory variable X_1 is the *prior period's* level of Y, and a_1 is one plus the expected growth rate. That is:

$$Y_t = (1 + g)\, Y_{t-1} \tag{3-4}$$

where g is the period's growth rate. For example, if it is expected that a firm's level of selling expenses will grow at 10 percent per year, and this year's selling expenses are $1 million, we would project next year's selling expenses as $1.1 million.

Multiple Dependencies Here the variable is thought to depend on more than one factor; not just sales or some other variable but a combination of several variables. The general linear model as expressed in equation (3-1) is used, and the statistical technique of *linear regression* is often employed with historic data to estimate which explanatory variables are significant in determining Y_t and to estimate the coefficients of these variables. A classic example of multiple dependency is inventory level. Firms often keep a "base level"

or "safety stock" of inventory to hedge uncertainty and vary the remaining portion of inventory in response to demand. In such a system, there are two appropriate variables associated with inventory level:

$$Y_t = a_o + a_1 X_1 \tag{3-5}$$

The a_o term represents the base inventory level, the X_1 the square root of the sales level, and a_1 the proportionality constant.[8] If the firm's base level of inventory is \$50,000, the proportionality constant is 15 and sales are expected to be \$500,000, we could estimate inventory as:

$$Y_t = 50{,}000 + (15)\ ((500{,}000)^{1/2}) = \$60{,}607 \tag{3-6}$$

All these forecasting methods will be used at times in the course of this text.

In deciding which of these methods to use to forecast a particular variable, a primary consideration is the *term of the forecast*: how far into the future are we projecting? To see this, the concepts of *the short run* and *the long run* from economics can be employed. In the short run, most things are predetermined or preplanned; very little can be changed. In the long run, almost everything is variable. In terms of financial forecasting, this means that in short-term forecasts, many things will result from plans and events that are already in place (contracts, capital budgets, long-range financing plans, and so forth). But in the long run, most things can vary and are dependent on outside influences such as the firm's long-term growth rate. Since cash forecasts deal mostly with the near future, *many of the items on the cash forecast are estimated by some variation of the spot method*. The bases for these spot estimates are usually the firm's other financial plans. Remaining estimates are mostly on a "proportion of another account" basis, with this "other account" often being a particular period's sales; the other two methods are employed less frequently. This is quite unlike longer-term forecasting, where compounded growth and multiple dependency methodologies play a more important role.

FORECASTING COLLECTIONS FROM ACCOUNTS RECEIVABLE

The firm's *cash disbursements* are generally forecast in a straightforward fashion using the firm's other plans and a combination of the forecasting methods described previously. However, a major challenge for the analyst comes in estimating the *receipts from the collection of the firm's receivables*. There are two major decisions to be made: the *forecasting methodology* to

[8]The rationale for this approach to inventory management strategy will be discussed in Chapters 8 and 9.

be used and the *degree of aggregation* of the forecast across the firm's lines of business.[9]

One forecasting methodology used by firms is the *turnover method*. This is a variation of the percent of sales method in which it is assumed that receivables will all be collected based on the average turnover of the receivables of the firm. For example, if the firm's average receivables' turnover is 30 days, it is projected that all sales for one month will be collected in the following month. The problem with this system is that it does not do a very good job of capturing the actual process of receivables collection, and therefore it does not produce accurate forecasts except in special cases. This problem stems from the turnover statistic as a *weighted average* of the payment patterns of various customers paying at different times. A better receipt forecasting method is the *payment patterns approach* (also known as payment fractions or payment proportions approach). In this method, the analyst determines (using historical data) the proportion of customers that pay at various times after the date of sale and from this information projects future receipts.

The payment patterns method gives forecasts quite different from the turnover method. To see this, let us develop an example. Assume that last month's sales were $20,000, that this month's sales are expected to be $30,000, and that next month's sales are expected to be $20,000. Assume that 30 percent of the customers purchased products for cash, 40 percent paid in 30 days, and the remaining 30 percent paid in 60 days. These payment patterns would produce a 30-day turnover in periods of constant sales. Forecasts of cash receipts for next month by the turnover and payment patterns methods are presented in Table 3-5. The forecast of collections made by the turnover method of $30,000 is quite different from the forecast of $24,000 made by the payment proportions method. Since the payment patterns method is a better representation of the underlying customer remittance habits, the cash receipt forecast associated with this method is likely to be more accurate.[10]

The second issue that must be addressed in forecasting receivables collections is the *level of aggregation* of these forecasts across lines of business for multibusiness firms. Firms selling various types of products often employ different terms of sale for their product lines and often sell these different products to customers with quite different payment habits. For example, it would not be expected that a unit of a firm selling on terms of 3 percent 10 days, net 30 days, to a group of very well-financed customers would have the same receipt patterns as a unit selling to less well-financed firms on net 60-

[9]The following discussion draws from B. K. Stone, "The Payment-Patterns Approach to the Forecasting and Control of Accounts Receivable," *Financial Management* (Autumn 1976), pp. 65–82.

[10]Note that, when the firm has bad debts, the payment pattern coefficients will not sum to 1.0. Note also that it is possible to use statistical techniques to adjust the payment pattern coefficients for seasonality and other effects. See Stone, "The Payment-Patterns Approach," for discussion of both issues.

Table 3-5. Collections Forecasts by the Turnover and Payment Patterns Methods.

Month:	*Last Month*	*This Month*	*Next Month*
Sales:	$20,000	$30,000	$20,000
Receipts Forecast Based on 30 Days Turnover (Last Month's Sales Figure)			30,000
Receipts Forecast Based on Payment Patterns:			
Collections from Cash Sales (30 Percent of the Month's Sales)			6,000
Collections from Last Month's Sales (40 Percent of Last Month's Sales)			12,000
Collections from Sales Two Months Ago (30 Percent of That Month's Sales)			6,000
Total Collections Forecast by Payment Patterns			$24,000

day terms. In a situation where the firm is in multiple business lines, the use of *overall* payment patterns to forecast receipts will be accurate *only when the proportions of total sales made in each business line are constant*—an unlikely situation, particularly since the different lines usually have different seasonal variations. In such a multiline situation, the most accurate forecasting result is achieved by forecasting receipts for the *different units of the firm individually* based on their own receipt patterns, then summing these receipt forecasts to obtain total cash receipts for the firm.[11]

AN EXAMPLE SPREADSHEET CASH FORECAST

To see how cash forecasting works, let us generate a hypothetical cash forecast. An analyst for a firm is charged with creating a monthly cash forecast for the next six months. Consultation with the firm's credit manager indicates that, historically, 10 percent of the firm's sales have been for cash, 60 percent

[11]See Stone, "The Payment Patterns Approach," page 79.

of sales have been collected in the month following sale, and 29 percent of sales have been collected in the second month following sale; the remaining 1 percent of sales are bad debts and are never collected. A check of accounting records shows that sales for the prior two months (May and June) were $500,000 and $550,000, respectively. The analyst then meets with the firm's sales manager and is advised that the current sales forecast calls for sales of $600,000 in July, $700,000 in August, $800,000 in September, $750,000 in October, $650,000 in November, and $500,000 in December. The firm's production manager has made a production plan based on this sales forecast; the analyst is informed that direct materials will be 42 percent of sales. Direct labor will be 34 percent of sales for July, August, and September based on the production plan, but will rise to 35 percent for the remainder of the year because of a new wage scale to take effect in October. Consultation with the payables manager reveals that direct labor is generally paid for in the month incurred, but that trade suppliers grant terms of sale of net 60 days, which are fully utilized. Also, the payables manager advises that monthly fixed expenses such as rents, executive payroll, and so forth will total $40,000 per month, and that tax payments of $220,000 per month are due in July and October. No repayments on the firm's outstanding debt are due during the next six months. A conference with the firm's capital planners yields the information that the progress payments on the firm's new construction in the amount of $90,000 will be due in August, October, and December. The firm has a starting cash balance of $25,000, no temporary investment of surplus cash in marketable securities or temporary short-term borrowings to cover deficits, and wishes to keep cash on hand at a level of 5 percent of monthly sales.

A few notes on the structure of this particular cash forecast are appropriate. In it, we will ignore the effect of interest income from the investment of surplus funds and interest expense from the financing of deficits on cash flow. This is the standard procedure; the amounts involved are usually small relative to other flows. Where these inflows and outflows are a significant part of total flow, the appropriate procedure is to first calculate the net cash flow for a period; from this, calculate the amount of interest income or expense; and finally, include this income or expense in the cash flow of the appropriate future periods. If this income or expense is sufficient to affect the firm's tax payments, pro forma income statements for tax purposes are also required. We will also ignore the disbursement flotation between the time when checks are written for outflows and the time when these checks are actually cashed. Because it is a few days, this disbursement flotation effect is not important in monthly forecasts, though it must be accounted for in cash forecasts of shorter period lengths.[12] Note that the payment proportions method

[12]See J. H. Vander Weide and S. F. Maier, *Managing Corporate Liquidity: An Introduction to Working Capital Management* (New York: Wiley, 1984), pp. 142–56, for discussion.

Table 3-6. An Example of Spreadsheet Cash Forecast.

	A	B	C	D	E	F	G	H	I
1	Cash Forecast								
2									
3	Month	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
4		(act.)	(act.)	(est.)	(est.)	(est.)	(est.)	(est.)	(est.)
5	Sales	550	550	600	700	800	750	650	500
6									
7	Cash Receipts Forecast								
8	Cash Sales			60	70	80	75	65	50
9	Collections: 1st Prior Mo.			330	360	420	480	450	390
10	Collections: 2nd Prior Mo.			145	159.5	174	203	232	217.5
11									
12	Total Receipts			535	589.5	674	758	747	657.5
13									
14	Cash Disbursements Forc.								
15	Direct Labor Payments			204	238	272	262.5	227.5	175
16	Direct Materials Payments			210	231	252	294	336	315
17	Monthly Fixed Expenses			40	40	40	40	40	40
18	Tax Payments					220			220
19	New Construction				90		90		90
20									
21	Total Disbursements			454	599	784	686.5	603.5	840
22									
23	Net Cash Flow			81	−9.5	−110	71.5	143.5	−182.5
24	Beginning Cash			25	106	96.5	−13.5	58	201.5
25									
26	Total Cash			106	96.5	−13.5	58	201.5	19
27	Desired Cash			30	35	40	37.5	32.5	25
28									
29	Surplus or (Borrowings)			76	61.5	−53.5	20.5	169	−6

used in our forecast of receipts by its very nature accounts for the flotation on incoming checks, assuming that the payment proportions are calculated using cleared items. Later in this chapter, we will discuss the implications of the firm's strategy regarding the level of desired cash, which is in part a hedge against the uncertainties inherent in the cash forecast.

The resulting forecast, in rounded thousands of dollars, is presented in Table 3-6.[13] This spreadsheet was started by expanding column A to allow for longer titles. The necessary titles were then entered in column A and the month abbreviations in line 4. The "act." and "est." notations under the month abbreviations indicate which sales data are actuals and which are projections. The actual and projected sales data are entered in line 5. The formula "D5*.1" was entered in cell D8 and replicated (copied) into E8 through I8 to get receipts from cash sales. The formula "C5*.60" was inserted into cell D9 and replicated, and the formula "B5*.29" placed into cell D10 and replicated, giving projections of collections from last month's sales and from the sales of two months ago, respectively. Cells D8 through D10 were summed in cell D12 to obtain total receipts.

The disbursements section contains both formulas and data. The cells in columns D, E, and F of line 15 (Direct Labor Payments) contain 34 percent of the corresponding month's sales (sales appear on line 5) while the cells in columns G, H, and I contain 35 percent of the corresponding sales figures. The direct materials payments are made with a two-month lag, so ".42*B5" was entered into cell D16 and replicated for the other months. The remaining cells in this section are data.

Net cash flow (line 23) was computed as the contents of the cell in lines 12 (Total Receipts) minus the contents of the cell in line 21 (Total Disbursements) for each column. The initial cash balance was entered as data; after the first month, formulas were inserted in this line such that the contents would be the same as the cell one column to the left and two rows down (for example, cell E24 contains "D26"). Total cash is net cash flow plus beginning cash (cell D26 contains "D23+D24"). In cell D27, "0.05*D5" was entered to make the desired level of cash equal to 5 percent of the month's sales and replicated this across the line. Total cash less desired cash determines the firm's cumulative surpluses or required borrowings.

USING THE CASH FORECAST

Line 29, the cumulative surpluses and required borrowings for the firm, is the important result of a cash forecast. This estimate of available funds for investment and needed financing enables the firm to plan so as to obtain the

[13]In this example, we have generated a spreadsheet of a level of sophistication similar to that which a small firm might employ.

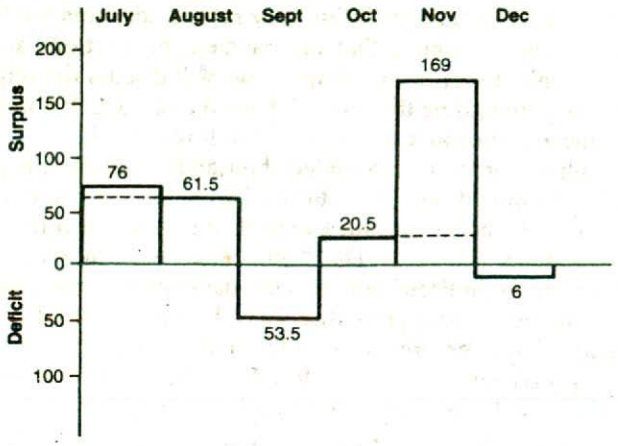


Figure 3-1. Bar Chart of Cash Surpluses and Deficits from Table 3-6 (rounded thousands of dollars).

most advantageous borrowing terms for deficits and achieve the greatest interest income on surplus. A useful chart for these planning purposes is a bar chart of this line. A bar chart of the surpluses and deficits from Table 3-6 is presented in Figure 3-1.

Let us assume that the yield curve is upsloping. The firm starts the forecast with no surplus or deficit. During July, a surplus of $76,000 is accumulated. For maximum interest income, these funds should be invested in temporary instruments with the longest possible maturity. In examining the forecast for August we see that the surplus for *this* month is only $61,500; of July's $76,000 surplus, $14,500 will be needed to cover cash outflows and increases in the desired level of cash. Further, there is a deficit in September. Therefore, of the $76,000 surplus in July, $61,500 can be invested until these funds are needed in September; the remaining $14,500 will be needed in August and must be invested in shorter maturity instruments. The firm will need to borrow $53,500 in September to cover the deficit in this month. They should make arrangements to have these funds available at that time. The entire $20,500 surplus in October can be invested until December, since the surplus of $169,000 in November is larger than the surplus in October (November being a month of large anticipated cash inflows). The additional $148,500 surplus in November can be invested at that time until it is needed in December. The firm's maximum borrowing needs over this six-month period are $53,500, reached in September.[14]

[14]For many firms this simple strategy regarding borrowing and lending is insufficiently descriptive. One possible approach to planning more complex strategies is to add a section to the cash forecast detailing the intended remedies to the projected shortfalls and surpluses. This methodology is suggested in M. A. Nunes, *Operational Cash Management and Control* (Englewood Cliffs, N.J.: Prentice-Hall, 1982), pp. 57–61.

Some firms that borrow on short-term credit lines are required to be out of debt for a specific period during the year (often 60 to 90 consecutive days) under these credit agreements. For such firms, the cash forecast has an additional significant benefit: it can help determine whether the firm will fulfill this out-of-debt requirement, given the firm's current plans. For example, examination of Table 3-6 reveals that the firm in the example will not be out of debt for 90 consecutive days during the July through December period.

DISTRIBUTION TO A DAILY CASH FORECAST

The monthly cash forecast thus can be used to enhance the firm's planning process. In addition, a monthly forecast can be *scheduled* to a quarterly or yearly forecast or *distributed* to a daily forecast. Scheduling to longer periods is simply a matter of summing the forecast monthly cash inflows and outflows over the longer period. Distribution requires a bit more analysis, but the results can be useful for planning over the very short term.[15]

Distributing the monthly cash forecast to a daily basis requires that the pattern of cash flows within the month be estimated for each type of cash inflow and outflow. The estimated pattern is then applied to the month's forecast for each cash flow type to generate the daily forecast. One formula which can be used is:

$$Forecast_{nw} = (P_{nw})\ (\text{forecast of monthly cash flow}) \qquad (3\text{-}7)$$

where $Forecast_{nw}$ is the daily forecast for a particular type of cash flow on day n of week w and P_{nw} is the proportion of the month's total that is expected to occur on day n of week w for this type of cash flow. The P_{nw} terms reflect two important cash flow effects within months: the *week-of-the-month effect* and the *day-of-the-week effect*. Due to numerous circumstances, a firm's various types of cash inflows and outflows do not tend to be the same over the weeks within the month of the days within these weeks. They tend to exhibit time patterns that must be reflected in daily cash forecasts. One way to reflect these patterns is to estimate separate coefficients representing the week-of-the-month and day-of-the-week effects and then use these coefficients in a multiplicative fashion to estimate the proportion of the month's total flow that will occur as a result:

$$P_{nw} = A_n B_w \qquad (3\text{-}8)$$

[15]The discussion in this section draws from T. W. Miller and B. K. Stone, "Daily Cash Forecasting and Seasonal Resolution: Alternative Models and Techniques for Implementing the Distribution Approach," *Journal of Financial and Quantitative Analysis*, Vol. 20, No. 3 (September 1985), pp. 335–51.

Table 3-7. Distribution of July's Monthly Forecast to Daily Cash Flows.

	A	B	C	D	E	F	G	H	I	J
1	Week	1	1	1	1	1	2	2	2	2
2	Week Coefficient (An)	.30	.30	.30	.30	.30	.20	.20	.20	.20
3	Day of Week	1	2	3	4	5	1	2	3	4
4	Day Coefficient (Bw)	.30	.25	.20	.10	.15	.30	.25	.20	.10
5	Proportion of Month (Pnw)	.09	.075	.06	.03	.045	.06	.05	.04	.02
6										
7	Total Receipts	48	40	32	16	24	32	27	21	11
8										
9	Direct Labor Payments					51				
10	Direct Materials Payments				53				53	
11	Monthly Fixed Expenses				10				10	
12	Tax Payments									
13	New Construction									
14										
15	Total Disbursements	0	0	0	63	51	0	0	0	63
16										
17	Net Cash Flow	48	40	32	−46	−27	32	27	21	−52
18	Beginning Cash	25	73	113	145	99	72	104	131	152
19										
20	Total Cash	73	113	145	99	72	104	131	152	100
21	Desired Cash	30	30	30	30	30	30	30	30	30
22										
23	Surplus or (Borrowings)	43	83	115	69	42	74	101	122	70

where A_n and B_w represent coefficients for the week-of-the-month and day-of-the-week effects.[16]

Let us apply this technique to the prior example cash forecasting problem. Suppose that the firm generating our example monthly cash forecast wants to distribute the monthly forecast for July to a daily basis. Ignore any holidays and assume that there are 21 working days in this month: five in each of the first four weeks (designate these weeks as numbered 1 through 4) and one in the final week (week number 5). Number Monday as 1, Tuesday as 2, and so forth. The firm must estimate patterns within this month (the A_n and B_w) for all the types of cash flows on its cash forecast.

For the six-month period of the original month-by-month forecast, re-

[16]This particular approach to the estimation of P_{nw} assumes that the day-of-the-week and week-of-the-month effects are independent. See Miller and Stone, "Daily Cash Forecasting and Seasonal Resolution," for statistical estimation techniques, alternate approaches, and methods for handling seasonality, month-endings, and holidays.

Table 3-7. Continued.

K	L	M	N	O	P	Q	R	S	T	U	V	W
2	3	3	3	3	3	4	4	4	4	4	5	
.20	.25	.25	.25	.25	.25	.2	.2	.2	.2	.2	.05	
5	1	2	3	4	5	1	2	3	4	5	1	
.15	.30	.25	.20	.10	.15	.30	.25	.20	.10	.15	1	
.03	.075	.063	.05	.025	.038	.06	.05	.04	.02	.03	.05	Monthly Totals
16	40	33	27	13	20	32	27	21	11	16	27	535
51					51					51		204
				53					53			210
				10					10			40
												0
												0
51	0	0	0	63	51	0	0	0	63	51	0	454
−35	40	33	27	−49	−31	32	27	21	−52	−35	27	81
100	66	106	139	166	117	86	118	145	166	114	79	25
66	106	139	166	117	86	118	145	166	114	79	106	106
30	30	30	30	30	30	30	30	30	30	30	30	30
36	76	109	136	87	56	88	115	136	84	49	76	76

ceipts are from sales, and disbursements in various months are for direct labor payments, direct materials payments, monthly fixed expenses, tax payments, and payments for new construction. However, since there are no forecast payments for taxes or for new construction to be made in July, these categories can be ignored for purposes of this distribution example. Turning first to the cash receipts, assume that the firm has found, from research on prior patterns, that for months with the structure of July, 30 percent of cash receipts occur in the first week of the month (week number 1), 20 percent occur in week number 2, 25 percent occur in week 3, 20 percent occur in week 4, and 5 percent occur in week 5. A_1 would then be 0.30, A_2 would be 0.20, and so forth. Assume also that, within each week, 30 percent of receipts occur on Monday (day number 1), 25 percent on Tuesday, 20 percent on Wednesday, 10 percent on Thursday, and 15 percent on Friday (except for week 5, which has only one day—all that week's receipts will occur on day number 1). The B_w coefficients would reflect this pattern; B_1 would be 0.30, B_2 would be 0.25, and so forth.

Assume that payments for direct labor clear the firm's bank on Friday, and that each payment is to be for one-fourth of the month's wage bill.[17] Finally, assume that checks representing one-fourth of the monthly direct material payments and fixed monthly expenses are expected to clear the firm's bank on Thursday of weeks 1 through 4.[18]

The resulting distributed daily cash forecast is presented in Table 3-7 in rounded thousands of dollars. As the distribution of the cash disbursements in this example is obvious, the main point of this table is to illustrate the distribution method as applied to the daily forecast of the cash receipts. The week numbers and their associated A_n coefficients appear in rows 1 and 2. The days of the week and their associated B_w coefficients appear in rows 3 and 4. In this distribution method, the product of A_n and B_w is P_{nw}; this proportion appears in row 5. For example, cell B5 contains "B2*B4"; the result of 0.09 indicates that 9 percent of the month's expected cash receipts (of $535 thousand; see Table 3-6) are expected to occur on the first day of week number 1. The remaining cells in row 5 are similarly interpreted. Multiplying these cells times $535 thousand gives the expected daily receipts in row 7.

Once the various receipts and disbursement types are distributed among the days of the month, the remainder of the cash forecast proceeds as with any other forecast of this sort. For illustrative purposes, monthly totals (across columns B through V) have been included in column W; note that these totals are exactly the same as July's monthly forecast in column D of Table 3-6. However, the distributed daily forecast enables the firm to plan not only on a month-to-month but also on a day-to-day basis. This is a useful addition if the firm's cash flows are sufficiently large to make day-to-day planning of investment and borrowing strategy worthwhile. In the case of the example for July, while there is an overall surplus of $76,000 for the month, there are several times during the month when the surplus is considerably larger than this, though the funds are needed shortly thereafter. These temporarily higher surpluses offer opportunities during the month for short-term, temporary investments having maturities of a few days.

Up until now, we have discussed cash forecasting as if the future cash flows were known with certainty. In practice, the estimated inflows and outflows (and thus the computed surpluses and deficits) are often subject to substantial *uncertainty*. Because of the costs that this uncertainty introduces, *it must be addressed within the firm's planning process*. Uncertainty assessment techniques and strategies for the management of this uncertainty in the cash

[17]While the disbursements in this example will not be dealt with in as much detail as the receipts, this is equivalent to assuming that for direct labor A_1 through A_4 are 0.25 while A_5 is zero and that B_5 is 1.00 while all the other B_w are zero.

[18]Therefore, for these types of expenses, the A_n terms are the same as for direct labor, B_4 is 1.0, and the other B_w are zero.

forecast are addressed in the remainder of this chapter.[19] In discussing these uncertainties and their assessment, we will use, as a primary illustration, the example monthly cash forecast previously developed.

SOURCES OF UNCERTAINTY IN CASH FORECASTING

Given the short-run nature of the cash forecast, with most things occurring in the near future and the forecast period typically being a year, one would tend to think that most financial transactions over this period could be forecast very accurately. Unfortunately, this is far from true. Even with this short period, there are numerous sources of risk. Among the sources are *sales uncertainty, collection rate uncertainty, production cost uncertainty*, and *capital outflow uncertainty*. Let us look at each one of these in turn.

Sales uncertainty refers to the risk regarding the firm's future levels of sales. Most firms try to forecast accurately enough to hold errors in short-run sales forecasts to less than 10 percent, but are often unsuccessful in these efforts.[20] Sales-dollar projections are a product of two other projections, units to be sold and price per unit. Both are often quite uncertain and depend on economic and competitive conditions. Note that *any errors in sales forecasts have multiple impacts* on the firm's cash flows; they impact on receivable levels (and therefore collections) and also on production expenses (and therefore disbursements).

Collection rate uncertainty is the uncertainty regarding the firm's actual future collection patterns of receivables. The firm may historically have collected an average of a certain percent of its outstanding receivables from a particular period in another particular period, but this average contains considerable variability. Further, changing market and economic conditions may make for chancy extrapolation of past historic data into future periods. Because of this and the uncertainties in forecasting sales, forecasts of the collection of future receivables contain at least three sources of uncertainty: uncertainty regarding the number of units that will be sold, uncertainty regarding the price at which these units will be sold, and uncertainty regarding the patterns with which the receivables generated by these sales will be collected.

Production cost uncertainty has to do with the risk of the actual labor and material costs that go into the making of a product or service.[21] Labor productivity may be more or less than expected, making labor costs uncertain.

[19]It is assumed in the forthcoming discussion that sources of uncertainty are uncorrelated.

[20]See J. Pan, D. R. Nichols, and O. M. Joy, "Sales Forecasting Practices of Large U.S. Industrial Firms." *Financial Management* (Fall 1977), pp. 72–77.

[21]See Lerner, "Simulating a Cash Budget," page 79, for discussion.

The cost of materials used may vary due to unexpected changes in price or in the amount of materials necessary to produce products and services.

Capital outflow uncertainty is one of the biggest sources of surprises in cash flow forecasting. This is the uncertainty regarding the timing of cash disbursements related to the firm's major capital expenditure and construction programs.[22] The uncertainty arises from the nature of payments made for new construction. When the firm undertakes to build a new plant or other project of this sort, the total price (subject to specified revisions) is generally agreed upon in advance. The construction firm then starts the project. After a certain percent of the project is done, the construction firm submits a "progress report" to the firm and is paid for what has been completed, less a retainage. For example, assume that the firm has contracted to have built a $10 million building with a 10 percent retainage. Once the construction firm completes the first 20 percent of the project, a payment of $1.8 million will be due ($10 million times 20 percent times 90 percent). Such payments are subject to at least two uncertainties. First, the weather, a very risky variable, plays a significant part in the rate of construction completion. Second, construction firms are notorious for filing late progress reports and then expecting immediate payment. While only a small percent of the firm's total bills are from capital construction programs, the amounts involved are usually very large. One unexpected item of this sort can destroy a carefully planned cash flow management strategy.

PROBLEMS WITH THE CERTAINTY APPROACH

These sources of uncertainty cause risk in the individual items of the cash forecast, and therefore risk in the net cash flow and the resultant surpluses and required borrowings. To see the difficulties this uncertainty causes, think about a manager looking at a cash forecast based on the assumption of certainty and who is attempting to develop a cash management plan. He or she knows the expected flows and knows that these are subject to considerable risk, but he or she has no estimate of this risk. Going back to the cash forecasting example previously done in this chapter, the maximum financing needed occurred in September in the expected amount of $53,500 (see Table 3-6). This is the *expected value of a probability distribution of borrowings.* Assume this probability distribution is symmetric; if the manager arranges to borrow *exactly* $53,500, then *this amount will be insufficient 50 percent of the time*, since for a symmetric distribution the observed value will be greater than the mean 50 percent of the time. Yet arranging to borrow more funds than needed is costly; for example, banks often charge a commitment fee

[22]See Fisher, *Cash Management*, p. 8, for discussion.

based on the *maximum* amount of the firm's line of credit.[23] What maximum amount of financing should the firm arrange to have available in order to be safe? Answering this question *requires an estimate of the uncertainty present* in the estimation procedure. If it is low, a maximum amount of $60,000 may be adequate; if there is very substantial uncertainty, a maximum of $125,000 may be necessary.

Uncertainty as to the amount and length of cash surpluses causes other, equivalent problems. Returning again to the example forecast, the firm is expected to have a $76,000 surplus in July and a $61,500 surplus in August. We previously argued that in a period of upsloping yield curves, the highest-yield strategy involved investing $61,500 of the initial $76,000 for 60 days and the balance for 30 days. However, the cash flows are uncertain. The surplus in August may be less than $61,500; in this case, the firm might have to liquidate part of the 60-day investment before maturity. If interest rates have risen since the investment was purchased, a capital loss may result. To avoid this, the manager may elect to put part of the $61,500 in a more liquid but lower-yielding instrument, sacrificing interest yield for some hedging against interest rate risks. As with the borrowing case, uncertainty regarding the expected cash flows is costly.[24] Therefore, an estimation of the amount of the uncertainty would greatly assist the manager in generating an investment policy as well as a financing policy.

ESTIMATING UNCERTAINTY IN CASH FORECASTS

There are two basic approaches to the assessment of risk in cash forecasting. First, we could assess the effects of *individual sources* of uncertainty on important *individual outcome variables*. Second, we could assess the effects of *all the uncertainties* in all the risky estimated variables on *all the important outcome variables* with all the uncertainties allowed to vary simultaneously. Both of these methodologies are very useful. The first requires sensitivity anal-

[23]When the firm finances deficits of this sort in other ways, the cost arguments are more complicated, but the conclusion that the uncertainty is costly remains valid. For example, if the firm finances deficits with commercial paper and must issue more of this paper than they may eventually need due to lack of an estimation of the uncertainty, additional interest expenses will be incurred.

[24]As with borrowing, other strategies to deal with uncertainty in investments are also costly. Note that at this point in the chapter we are discussing uncertainties in the cash forecast that may cause a cash shortage and the costs that these uncertainties may engender. We will later discuss another source of uncertainty related to the cash forecast: the interest rates on the borrowing and lending necessary to finance temporary deficits. The uncertainty in this case arises not from the uncertain cash flows, but entirely from the possibility of changing interest rates.

ysis; the second requires simulation. To illustrate these, let us return to our cash forecasting example problem, whose solution is contained in Table 3-6.

Sensitivity Analysis of the Cash Forecast We know that there are uncertainties in the estimation of sales, collection rates, production and other cost amounts, and the timing and amount of capital disbursements, at the least. Using sensitivity analysis, we can assess some of the effects of these individual sources of uncertainty. For example, recall that the direct labor payments were estimated as 34 percent of sales for the first three months of the forecast, but that this was expected to rise to 35 percent of sales for the last three months because of a new wage scale. If this new wage scale was the result of a new labor contract that is yet to be negotiated when the forecast is made, the actual wage rate would be subject to uncertainty. Let us assess the effects of variation in the direct labor payments as a percent of sales on the projected surpluses and deficits in October, November, and December. As discussed in Chapter 1, this is done by changing the input variable and observing changes in the output variable. The first step might be to estimate the surpluses and deficits with direct labor at 36 percent of sales. Cell G15 presently contains "G5*.35". We change this to "G5*.36" and replicate this into H15 and I15. The results are presented in Table 3-8. The projected surplus in October drops from $20,500 to $13,000, the surplus in November drops from $169,000 to $155,000, and the projected deficit in December increases from $6,000 to $25,000. Trying various values of direct labor as a percent of sales—from 32 to 38 percent—produces the graphs of deficits and surpluses for the three months that are presented in Figure 3-2. The slopes of the functions in this table indicate the relative sensitivities of the three months' balances to changes in the wage cost.

Another uncertain variable is the timing of payments for new construction. As was discussed previously, the weather plays a significant part in the rate of progress of construction. It is possible that the weather may be good and that the firm's construction company may progress ahead of schedule; instead of $90,000 being due in August, October, and December, this could result in $135,000 being due in August and October and nothing due in December. To model this, the "90" in cells E19 and G19 is replaced with "135," and cell I19 is erased. The results are presented in Table 3-9. Other variations in the timing of construction payments could also be investigated.

This kind of analysis provides very useful information about the amounts of possible surpluses and deficits in various future periods. With regard to the construction payments example, the expected amounts of surpluses and borrowings in the beginning and ending months are unaffected, but the pattern from September to November is significantly altered. The maximum amount of the firm's necessary borrowings are now $98,500, not $53,500 as in the original calculation. If there is a significant chance that this speedup of con-

Table 3-8. Sensitivity Analysis of Example Cash Forecast, Direct Labor for October through December Changed to 36 Percent of Sales.

	A	B	C	D	E	F	G	H	I
1	Cash Forecast								
2									
3	Month	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
4		(act.)	(act.)	(est.)	(est.)	(est.)	(est.)	(est.)	(est.)
5	Sales	500	550	600	700	800	750	650	500
6									
7	Cash Receipts Forecast								
8	Cash Sales			60	70	80	75	65	50
9	Collections: 1st Prior Mo.			330	360	420	480	450	390
10	Collections: 2nd Prior Mo.			145	159.5	174	203	232	217.5
11									
12	Total Receipts			535	589.5	674	758	747	657.5
13									
14	Cash Disbursements Forc.								
15	Direct Labor Payments			204	238	272	270	234	180
16	Direct Materials Payments			210	231	252	294	336	315
17	Monthly Fixed Expenses			40	40	40	40	40	40
18	Tax Payments					220			220
19	New Construction				90		90		90
20									
21	Total Disbursements			454	599	784	694	610	845
22									
23	Net Cash Flow			81	−9.5	−110	64	137	−187.5
24	Beginning Cash			25	106	96.5	−13.5	50.5	187.5
25									
26	Total Cash			106	96.5	−13.5	50.5	187.5	0
27	Desired Cash			30	35	40	37.5	32.5	25
28									
29	Surplus or (Borrowings)			76	61.5	−53.5	13	155	−25

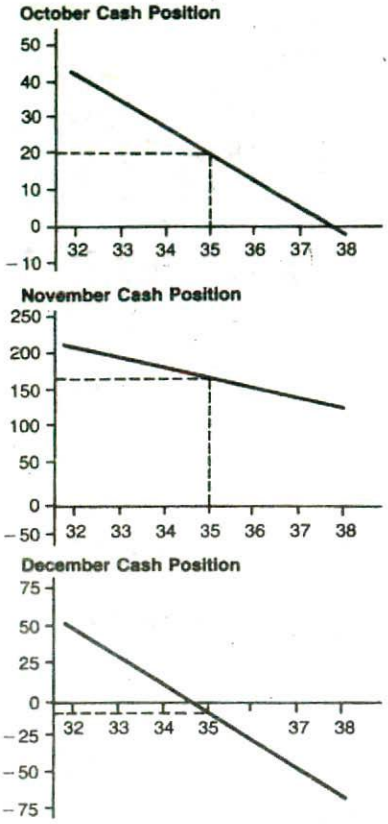


Figure 3-2. Sensitivity Analysis Graphs of October, November, and December Cash Position in Example Cash Forecast for Variations in Direct Labor as a Percent of Sales for These Months.

struction may occur, the firm should make far different financing arrangements than were originally anticipated.

Simulation Analysis of the Cash Forecast While sensitivity analysis methodologies give useful information, it is generally the *overall variation* from the means of the monthly cash deficits and surpluses that concerns management for planning purposes. This information on the probability distributions of cash surpluses and deficits is necessary to plan advantageous strategies. To estimate these probability distributions, a simulation of the overall uncertainty in the ending cash balances for each of the periods within the forecast is needed. To get these, the methods of simulation analysis described in Chapter 1 are used. First, probability distributions for each of the major uncertain variables are developed. For a cash forecast, the variables

Table 3-9. Sensitivity Analysis of Example Cash Forecast, New Construction Payments Accelerated.

	A	B	C	D	E	F	G	H	I
1	Cash Forecast								
2									
3	Month	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
4		(act.)	(act.)	(est.)	(est.)	(est.)	(est.)	(est.)	(est.)
5	Sales	550	550	600	700	800	750	650	500
6									
7	Cash Receipts Forecast								
8	Cash Sales			60	70	80	75	65	50
9	Collections: 1st Prior Mo.			330	360	420	480	450	390
10	Collections: 2nd Prior Mo.			145	159.5	174	203	232	217.5
11									
12	Total Receipts			535	589.5	674	758	747	657.5
13									
14	Cash Disbursements Forc.								
15	Direct Labor Payments			204	238	272	262.5	227.5	175
16	Direct Materials Payments			210	231	252	294	336	315
17	Monthly Fixed Expenses			40	40	40	40	40	40
18	Tax Payments					220			220
19	New Construction				135		135		
20									
21	Total Disbursements			454	644	784	731.5	603.5	750
22									
23	Net Cash Flow			81	−54.5	−110	26.5	143.5	−92.5
24	Beginning Cash			25	106	51.5	−58.5	−32	111.5
25									
26	Total Cash			106	51.5	−58.5	−32	111.5	19
27	Desired Cash			30	35	40	37.5	32.5	25
28									
29	Surplus or (Borrowings)			76	16.5	−98.5	−69.5	79	−6

involved would include sales, collection rates, production costs, and capital expenditures. Statistical estimation procedures or management estimates could be used; discrete or continuous distributions are possible. Then, a large number of trials are run. From these trial results, frequency histograms of the important outcome variables would be developed and these compared to known probability distributions via goodness-of-fit methods.[25]

To apply simulation analysis in estimating the total uncertainty in a cash forecast, one of the uncertainties that must be quantified is that of the *collection rates on accounts receivable*. Uncertainty in the collection rates of receivables is an important component in the overall uncertainty of the cash forecast. The usual method of estimating these rates is to compute individual collection rates on various period's sales using historic data. Another approach to the problem aids in quantifying the multivariate uncertainty in these rates. This approach estimates all the collection rates simultaneously by regressing past sales figures against past collections.[26] The estimated coefficients of the sales figures in the regression can be interpreted as the collection proportions, and the standard errors of the estimated regression coefficients as the uncertainty inherent in the estimation of these collection proportions.[27] For example, assume that the firm has regressed its monthly collections for past months against the appropriate past monthly sales figures and has obtained the following results:

$$\underset{}{C_t} = \underset{(0.250)}{0.754}\ S_{t-1} + \underset{(0.087)}{0.241}\ S_{t-2} \tag{3-9}$$

In this equation, C_t is the collection from receivables in period t, S_{t-1} is the sales in period $t-1$, and S_{t-2} is the sales in period $t-2$. Assume also that these were the only statistically significant explanatory variables, and that the overall estimated equation was highly significant. The estimated collection rates are 75.4 percent of the previous month's sales and 24.1 percent of the sales from two months previously. The implied bad debt rate is 0.5 percent, equal to one minus the sum of the collection rates. The figures in

[25]The primary concern of this text is not with the methodologies of simulation analysis; rather, simulation is one of the methodologies used herein to assess risk. A good reference on simulation analysis is J. C. T. Mao, *Quantitative Analysis for Financial Decisions* (New York: Macmillan, 1969), pp. 553–77.

[26]See J. K. Shim, "Estimating Cash Collection Rates from Credit Sales: A Lagged Regression Approach," *Financial Management* (Winter 1981), pp. 28–30. Additional discussion of such time series approaches to cash forecasting can be found in Vander Weide and Maier, *Managing Corporate Liquidity*, pp. 125–38.

[27]See Shim, "Estimating Cash Collection Rates," pp. 29–30. Shim uses a correction factor applied to the standard error in obtaining confidence intervals of collections. We shall omit this adjustment, interpreting the standard error of the estimate as the estimated standard deviation.

Table 3-10. Hypothetical Simulation Results for the Example Cash Forecasting Problem.

Month:	July	Aug.	Sept.	Oct.	Nov.Dec.
Mean Ending Cash Position (Rounded Thousands):	76.0	61.5	−53.5	20.5	169.0 −6.0
Standard Deviation of Ending Cash Position (Rounded Thousands):	20.0	25.0	30.0	20.0	60.0 30.0

Goodness-of-fit tests on histograms produced by the simulation showed no significant deviation from the normal distribution.

parentheses below the estimated collection rates are the standard errors of these collection rates. These standard errors of these estimated proportions are interpreted as the standard deviations of these estimates. If the number of observations in the sample used to estimate this regression was over 30, it is assumed that these estimated collection rates are normally distributed.[28]

Assume that the simulation analysis methodology has been applied for the example cash forecasting problem, and that all the uncertain variables have been simulated as part of this procedure. Hypothetical results are presented in Table 3-10.

With data of this type in hand, we can now address the formulation of appropriate investment, financing, and hedging strategies, taking into consideration the uncertainties of the projected cash surpluses and deficits. The usefulness of these data stems from the ability to convert the summary statistics regarding the probability distributions of the cash surpluses and deficits into *probabilities that outcomes will fall into various ranges*. Since the probability distributions and estimates of the means and standard deviations of the borrowings and surpluses are now known, it is relatively simple to compute the probability that the actual outcome in any of the periods will lie in any particular range.[29] For the normal distribution, the probability is computed from the Z statistic, which is calculated as:

$$Z = (X - E(X))/SD(X) \qquad (3\text{-}10)$$

[28]An underlying normal distribution is, of course, implied by the linear regression technique used to estimate the equation.

[29]See Lerner, "Simulating a Cash Budget," pp. 82–92, for discussion of these procedures and for extensions to questions involving more than one period's cash flows.

where X is the test value, $E(X)$ is the mean of X, and $SD(X)$ is the standard deviation of X.

Let us proceed with some example calculations under the assumption that the surpluses and deficits in Table 3-10 are normally distributed. For example, assume that the costs of having insufficient cash and the costs of hedges are such that the firm wishes to incur, at maximum, a 5 percent chance of having insufficient cash to cover expenses. What is the maximum amount the firm should arrange to borrow so that they will have only a 5 percent chance of having insufficient borrowing capacity to cover deficits while keeping cash at the desired level? The maximum expected borrowing is in September, with a mean of $53,500 and a standard deviation of $30,000. Let us assume, for purposes of this particular analysis, that September is the month when the probability of having insufficient cash is greatest. The Z statistic for 5 percent is 1.645, and 1.645 times $30,000 is $49,350. The amount that the firm should arrange to have available is $49,350 plus $53,500, or $102,850. There is a 5 percent chance that the actual requirements in September will be greater than this and a 95 percent chance that the requirements will be less than this. Another example: the firm is trying to decide how much of the estimated July surplus of $76,000 to invest in a 60-day investment. The estimated August surplus is $61,500 and the standard deviation in August is $25,000. How much can the firm invest and have only a 10 percent chance of having to resell the investment in August? The Z statistic for 10 percent is 1.28; 1.28 times $25,000 is $32,000; $61,500 less $32,000 is $29,500. There is a 10 percent chance that the cash surplus in August will be less than $29,500 and a 90 percent chance that it will be greater than $29,500. Thus, the firm can invest $29,500 in the 60-day investment and have a 10 percent chance that they will have to liquidate these bills prior to maturity.

These examples highlight the costs that the firm bears as a result of the cash flow uncertainty and hedging strategies that the firm must pursue to address these uncertainties. In the first example, the firm must arrange for an additional $49,350 in available borrowing to cover the potential shortfall; in the second, the firm can invest only $29,500 of $61,500 in longer-term, higher-yield investments. But these examples are intended to illustrate the mechanics of manipulating means, standard deviations, and probabilities of cash balances rather than to present realistic hedging strategies. In practice, the array of possible hedging strategies is quite a bit more complicated. In the second example, the firm could elect to invest more than $29,500 in the longer-term investment and plan to reduce its cash balance rather than liquidate the securities if the cash surplus in August turned out to be less than expected. That is, the firm has hedging strategies other than the very simple and obvious ones. The costs, alternatives, and risks in hedging strategies are described in the following section.

HEDGING CASH BALANCE UNCERTAINTIES

To understand the mechanics and alternatives in hedging, it is first necessary to understand a bit better why it is of advantage to the firm to hedge. At the end of each period of the cash forecast, the firm expects to be in either a surplus or a deficit position. Let us examine the risks and costs that the firm would face if it did not hedge in each of these cases. That is, assume that the firm keeps no cash, near-cash marketable securities, additional borrowing arrangements, or any other possible hedge.

If the firm is in a period of borrowing, is at its maximum available borrowing limits (as determined by its credit line arrangements), and cash flows turn out to be less than expected (so that borrowing needs would be greater than expected), the firm would be faced with a substantial problem. All the solutions to this problem are costly. For example, the firm could raise cash to cover the deficit by obtaining an emergency loan from its bank. However, bankers are not very receptive to emergency requests of this type, and this solution could endanger the firm's relationship with its bank. Alternatively, the firm could delay one or more types of outflows, such as payments to trade suppliers. This would, of course, endanger relationships with these suppliers. Another strategy the firm might consider would be to sell an asset quickly to generate cash; but the rushed sale might net the firm less for the asset than if it sold the asset in a more considered fashion. In a time of surplus, where the firm has invested the extra funds in longer-maturity securities (to take advantage of yield curve effects), the firm has another alternative: it may sell the investment prior to maturity. But by purchasing longer-term securities, the firm has subjected itself to interest rate risk; if interest rates have increased, the return on the investment will be reduced.

The point is that, without some kind of hedge against the uncertainties of future cash flows, *the firm incurs costs that could be avoided by the use of a hedging strategy*. Of course, there is a trade-off between the cost of the hedge and the expected costs that it avoids. It would not be cost-effective to hedge against all possible future costs if their probability of occurrence was very small. Because of this trade-off, it is necessary to understand the relative costs and other characteristics of the various methods commonly used to hedge the uncertainty of the firm's cash flows. The following pages contain descriptions of some of the possible hedging methods and their costs.

Holding a Stock of Extra Cash We refer here to a stock of cash kept by the firm beyond that needed for transactions.[30] Cash is the most flexible but the most costly hedge available to the firm. It is the most flexible in that

[30]Recall that keeping extra cash or near-cash assets for hedging purposes was one of the reasons for holding cash given in Chapter 2.

it can hedge a shortage in any circumstances, at any time, with no transaction costs. If the firm holds a stock of extra cash as a hedge and experiences a shortage during a period of borrowing or a period of lending, it can cover the deficit simply by drawing funds from its cash account. There are no significant transaction costs involved, since the firm simply writes checks.

The problem with holding a stock of extra cash as a hedge is the high cost of this hedging strategy. When we say that the firm holds a stock of cash, we mean cash held in a noninterest-earning checking account. If the firm, instead, invested this cash in its operations, it would earn a return of at least the firm's cost of capital. If the firm's after-tax cost of capital is 9 percent, this entire return is lost to the firm, since the cash hedge earns no interest.

Holding a Stock of Near-Cash Assets One strategy that is nearly as flexible as holding a stock of cash and that reduces the cost disadvantage of cash is holding a stock of near-cash assets. Near-cash assets are securities such as repurchase agreements or nearly matured Treasury bills. Since maturity is very near, these assets carry almost no interest rate risk and thus are almost as safe as cash. However, their use to cover deficits requires that they be sold, with attendant transactions costs. Like keeping extra cash, holding near-cash assets can be used to cover deficits whether the firm is in a period of temporary borrowing or lending. The cost of near-cash assets as a hedge depends on the return of these investments relative to the firm's cost of capital. If these instruments yield 6 percent per year, and the firm's income is taxed at 33 percent, the after-tax yield is 4 percent. If the firm's cost of capital is 9 percent, the 5 percent difference is the cost of this hedge. By using this method of hedging, the firm cuts the cost of the hedge relative to holding cash, but experiences some transactions costs in investing and reinvesting the near-cash assets as they mature and in liquidating these assets when funds are needed. For large firms, these expected transactions costs are small relative to the interest that accrues from the use of near-cash assets as a hedge. Consequently, larger firms commonly invest a substantial portion of their hedging funds in these assets.

Extra Borrowing Capacity One common strategy for funding the firm's expected financing needs is the establishment of a reserve credit line with a bank or group of banks. In doing this, the firm arranges for the maximum credit line with the bank(s) to exceed the *expected* amount of borrowing. This *extra* credit line, beyond that which the firm is expected to need, is a *hedge against cash flow uncertainty*.[31] For example, suppose that a firm expects to

[31]For simplicity, at this point in the text it is assumed that the only possible source of emergency short-term borrowing is the reserve portion of the firm's bank credit line. However, other strategies may also be used to raise emergency short-term debt-generated funds; we will discuss hedging strategies using other sources of short-term debt in Chapter 12.

borrow a maximum of $1 million, but sets up the credit line for a maximum of $1.2 million. The additional $200,000 can be used to fund the firm if cash flows turn out to be less than expected.

This approach to the hedging problem is fairly convenient and low-cost. It requires that the firm negotiate with its banker(s) well in advance of any borrowing needs, providing the financial statements and projections necessary to apply for the loan. Any additional inconvenience and expense necessary to apply for the extra amount desired for hedging purposes is a cost of this hedging strategy. Also, the firm must advise the bank(s) when it wants to use ("draw down") the credit line, and it must arrange for the funds to be transferred to its disbursement account(s). Finally, banks generally require a "commitment fee" that varies with the maximum amount of the credit line. This fee is often in the 0.25 to 0.5 percent range (tax deductible): the additional commitment fee for the hedging portion of the credit line is another cost of this hedge. Note that since bank credit line commitments often cover substantial periods of time (a year is not uncommon), and the amount of needed hedge varies with the firm's projected cash flows and their uncertainty over such periods, there will be times when the credit line arrangement will provide much more hedge than is needed.

For large firms arranging to borrow large amounts, the fixed costs of application for the credit line are small relative to the variable costs of the commitment fees. Because these commitment fees are inexpensive relative to the costs of most other hedging strategies, the establishment of additional available borrowing capacity is often the cheapest hedge for large firms. This is true even though the use of reserve bank borrowing requires that firms must pay fees on the *maximum* needed hedge, which results in excess hedging capacity during some times of the year.

Investing Temporary Surpluses in Near-Cash Assets If the yield curve is upsloping, the firm has an incentive to invest surplus funds for the longest possible time, matching the maturity of the investment made with the surplus funds to the length of time until the funds will be needed. However, if there are unexpectedly low cash flows before the maturity of the investment, the firm may be forced to sell before the maturity date. This exposes the firm to interest rate risk. One method of addressing this risk is to invest the surplus funds in near-cash investments. When the firm pursues this strategy, it incurs two costs which would not have occurred had the maturity of the instrument been matched to the time of future cash needs. First, the firm will obtain less interest income. Second, the firm will incur more investment transaction costs. To see this, assume that the firm has cash available that will not be needed for the next 90 days. It can invest the cash by purchasing an instrument maturing in 90 days and yielding a yearly interest rate of 6.5 percent. Alternatively, it can purchase a series of 10-day instruments that currently yield 6 percent per year. If the firm chooses to hedge the interest rate risk by using

the second strategy, it will lose 0.5 percent interest. However, the firm will also have to purchase nine different instruments to reinvest the cash (one every 10 days) rather than one instrument. Also, there are transaction costs incurred when the near-cash assets are liquidated to cover shortages. These transaction costs and interest differentials are the costs of this hedging strategy.

Note that, while this strategy removes the interest rate risk from the 90-day investment, another type of risk is generated. Since the reinvestment of the 10-day instruments will be made at market rates at the time of the reinvestment, a decline in these rates during the 90-day period will cause still-lower yields for this strategy.

HEDGING VIA INTEREST RATE FUTURES AND OPTIONS ON FUTURES

Up to this point in the text, we have alluded to the use of interest rate and other futures without any extensive discussion of these vehicles. In this section, we present discussion of interest rate futures, options on these futures, and how these futures and options can be used in hedging two types of uncertainties that arise from the cash forecast.[32] First, like the holding of cash balances and near-cash assets, these futures and options can hedge the *interest rate risk inherent in the temporary investment of short-term funds where unexpected future needs for these funds can cause premature liquidation of the temporary investment*. Second, interest rate futures and options can also be used to hedge interest rate risks that do not arise as the result of uncertainties in the firm's cash flow, but instead occur solely because of *changing interest rates between the time of the forecast and the planned investment or financing*. The intent in this section is to review the basics of futures and options and to provide an introduction to the hedging of risks in the cash forecast with futures and options on financial instruments.

Basics of Interest Rate Futures Contracts The discussion of futures can be obscured by the jargon of the futures market, leaving the reader bewildered and uninformed. If a few basic principles are kept in mind, much of this confusion can be avoided. First, we must always remember that the proper

[32]For more on interest rate futures contracts and options on these futures contracts, see two articles by Anatoli Kuprianov, both of which apeared in *Economic Review* (published by the Federal Reserve Bank of Richmond): "Short Term Interest Rate Futures" (September/October 1986), pp. 12–26, and "Options on Short-Term Interest Rate Futures" (November/December 1986), pp. 3–11. For additional discussion of hedging strategies using futures contracts, see Michael T. Belongia and G. J. Santoni, "Hedging Interest Rate Risk with Financial Futures: Some Basic Principles," *Review*, published by the Federal Reserve Bank of St. Louis (October 1984), pp. 15–25.

name for any future is a *futures contract*, with the emphasis on *contract*. A futures contract is just a contract between one party and another for the future delivery of a commodity. In this contract, the price and delivery date are specified. By specifying the price, both the buyer and seller are hedged against price fluctuations between the date on which the contract is sold and the delivery date. Futures contracts are commonly made on numerous types of goods. In this text, we will discuss the use of three types of futures contracts in the context of working capital management: interest rate futures (which hedge interest rate risk), foreign exchange futures (which hedge exchange risk among currencies), and commodity futures (which hedge the price risk of commodities purchased for inventory). Interest rate futures are discussed in this chapter.

Since the futures contract specifies delivery at a *fixed* price but the *market* price of the commodity fluctuates with supply and demand, *the value of the futures contract will fluctuate with the market price of the commodity*. If the price of the commodity rises, so will the value of the futures contract specifying the purchase of the commodity at a lower price. However, one major difference between run-of-the-mill legal contracts for future delivery of goods and the futures contracts discussed here is that *these futures contracts are traded on exchanges*. This enables the parties to satisfy the contract and to realize gains or losses before the maturity of the contract by selling the contract to a third party.

In these exchanges, the *clearinghouse* (which manages the exchange) guarantees the performance of both parties to the contract and offsets opposing transactions. An example of this offset procedure is useful. Suppose that a firm *purchases* a contract for the future delivery of a commodity. Later, it decides to take its gain or loss on this contract. It may do so by *selling* a contract for the delivery of the same commodity on the same delivery date at the same price. In the language of the futures market, the firm is both *long in the contract* (because it contracted to take delivery of the commodity) and *short in the contract* (because it contracted to deliver the commodity). In futures markets, the clearinghouse will net out these offsetting transactions, and the firms need not be involved thereafter.

When a firm decides to take its gain or loss on a futures contract, a large transfer of cash between the firm and its futures broker is not required. Most futures contracts are purchased *on the margin*; the firm gives the futures broker a relatively small deposit toward the total price of the contract. In the futures market, the amount of the deposit (the *margin*) is adjusted every day, and the firm gains or loses the difference in value. If the value of its contract goes up, the margin account with the futures broker is credited; if it goes down, the margin account is reduced. This process is called *marking to market*. When the futures transaction is closed, the firm reclaims the balance in its margin account.

A very common mistake regarding futures contracts is to think of the

margin requirement as an out-of-pocket transaction cost. Like a deposit on any purchase contract, *the margin is eventually applied to the purchase price* should the buyer take delivery. If the buyer cancels the contract by taking an opposing position, the margin is returned to the trader (less any losses and plus any gains). Thus, the margin requirement is a partial payment for the final goods, not a transaction cost.

While futures contracts on other financial instruments are traded, the major hedging instrument is the contract for the *future delivery of Treasury bills*. The major trading volume occurs in the contracts traded on the International Money Market (IMM). These contracts are for the delivery of 91-day Treasury bills with a maturity value of $1 million maturing in March, June, September, and December of the following two years. Therefore, eight contracts are being traded at any time, and the furthest delivery date for the Treasury bills is 24 months in the future. Transactions cost for the purchase of these futures contracts (represented by the difference between the prices at which the same contracts are bought and sold at the same time on the IMM) is quite small.

Basics of Options on Interest Rate Futures Contracts An *option* is a contract to purchase or sell something at a fixed price *which may be exercised or not at the buyer's discretion*. This is distinct from a futures contract, which *must* be exercised, unless the futures contract is canceled via an opposing transaction. In the case of a futures contract, once the contract is canceled, the cost or benefit to the trader is the difference in price between the original cost of the contract and its cancellation cost, which is the difference in the trading price of the contract between when the trader went long and when he or she canceled via the short (or vice versa). In the case of the option, *the cost of the option does not count toward the purchase price of the items in the contract*, and the option does not have to be canceled; it merely expires.

Thus, for an option, the firm faces an initial out-of-pocket cost. But relative to the futures contract, which locks in the future price of the commodity, the option offers greater flexibility. Since the option need not be exercised, the firm's costs are limited to the initial price of the option. If it is not profitable to exercise the option, the firm need not do so. However, if movement in the price of the commodity on which the option is written turns out to be such that exercising the option is profitable, the firm may make gains by exercising the option or by taking an opposing position to the option and offsetting the positions. An option to *sell* an item for a fixed price over a fixed period is called a *put option*. An option to *buy* an item for a fixed price over a fixed period is called a *call option*.

Options on the purchase or sale of interest-bearing instruments are not widely used. Instead, the popular option is the *option on a contract for the future purchase of a Treasury bill*, that is, an option on a Treasury bill futures contract. Options on futures contracts for Treasury bills are traded on the

IMM for the three-month bills. The options expire about three weeks before the maturity of the futures contract on the Treasury bill. Thus, the holder of the option must decide at the time of the option's maturity whether or not to take delivery on a three-week futures contract to purchase 90-day Treasury bills or to let the option expire (the option can of course be exercised, sold, or canceled via an offsetting transaction at any time before this).

The market prices of futures contracts on financial instruments and options on these contracts depends on the market price of the underlying financial instrument. For futures contracts, while the cash price of the instrument and the price of the instrument implied in the price future on this instrument are always closely related, they may diverge when the futures contract is far from maturity. However, as the futures contract moves closer to maturity, the prices of the cash purchase of the good and the price of the good implied by the price of the futures contract for the good *converge*, until *just before maturity the cash price of the good and the price implied by the futures contract on this good are the same.* This occurs because purchasing a futures contract with a short time to maturity is virtually the same as purchasing the instrument itself.

For options on these futures contracts, the market price of the option depends on the relationship between the market price of the instrument and the *exercise price* of the option. The exercise price is the price at which an option is executed. For a put option, it is the price at which the instrument is *sold* to satisfy the option; for a call option it is the price at which the instrument is *bought.* If the exercise price of a put option on a financial future is greater than the current market price of that future, then the value of the option must be at least the difference between the prices. This is necessary because, if it were less, any investor could buy the future, exercise the option to sell the future, and make a gain of the difference. If the current market price of the future is more than the exercise price of the put option, the only value of the put option is that, over the remaining period, the market price might fall below the exercise price, giving the ability to make money. The reverse case between exercise price, market price, and option value occurs for call options.[33]

Using Futures and Options on Futures to Hedge the Risk in the Cash Forecast As previously discussed, interest rate futures contracts, and options on these contracts, can be used in two ways to hedge risk in a cash forecast: (1) they can be used to hedge the interest rate risk on *future borrowings and*

[33]Additional discussion of the relationship between the price of the futures contract on a Treasury bill and the market price of the bill can be found in Kuprianov, "Short-Term Interest Rate Futures," pp. 24–25. Additional discussion of the pricing of options on futures can be found in Kuprianov, "Options on Short-Term Interest Rate Futures," pp. 4–7.

investments, or (2) they can be used to hedge the interest rate risk inherent in investing in *longer-term instruments where an unexpected cash shortage may lead to selling* these instruments before maturity. Each of these uses will be discussed in turn.

Let us first consider the hedging of interest rates on future expected borrowing and investing. Futures contracts on Treasury bills or options on these contracts can be used in conjunction with the cash forecast to lock in future rates on this expected borrowing and investment, and thus to hedge interest rate risk from the fluctuation in these rates between the time the cash forecast is generated and the time the borrowing or investing is to be executed. Using futures in this way is a classic futures application. Futures contracts were originally developed to hedge the risk of price fluctuations in agricultural commodities; by selling contracts for future delivery at a specified price, both buyer and seller were protected against price fluctuations in the intermediate period.

To hedge the risk of changes *in interest yields on investments*, the firm may *purchase a financial future* in the investment instrument. For example, assume that it is January and the firm has just completed its cash forecast which shows a surplus of $1.96 million beginning in June and lasting through September. To invest these funds from June to September (90 days), it intends to purchase Treasury bills. The current interest rate on a 90-day T-bill is 2 percent. If interest rates do not change and no hedging is undertaken, the firm may purchase $2 million (maturity value) in Treasury bills in June for $1,960,784.31 (since $1,960,784.31 times 1.020 is $2 million). The resulting cash flows are portrayed in Panel A of Table 3-11.

The firm will receive $39,215.69 in interest income using this strategy (ignoring any commissions). This unhedged strategy requires no initial investment or cash flows between January and June. The problem with the unhedged strategy is risk. If interest rates fall to 1.50 percent (per 90 days) between January and June, the firm will have to pay $1,970,443.35 for the Treasury bills, and will receive only $29,556.65 in interest.

However, if the firm purchases a futures contract in January for the acquisition in June of 90-day Treasury bills (September maturity), it is guaranteed that the rate of investment will be that of the futures contract. The disadvantage of this strategy is the requirement of initial margin and the possibility of other cash flows from January through June. If the initial requirement is 1.0 percent, the firm will have to deposit $19,607.84 with its futures broker. Also, if interest rates rise so that the value of the future falls, the firm may be subject to margin calls. Of course, any margin the firm deposits with its broker will be used to offset the cost of the contract when it is executed in June. The expected cash flows from this hedging strategy, ignoring any cash flows from margin calls, are portrayed in Panel B of Table 3-11.

The difference between Panel A and Panel B is that part of the cash flow to purchase the T-bills (the initial margin deposit) occurs at time zero

Table 3-11. Net Cash Flows from Various Hedging Strategies for Example Investment Situation.

Day 0 (January) NCF Transactions			Day 180 (June) NCF Transactions		Day 270 (September) NCF Transactions
Panel A: No Hedge					
0	None	−$1,960,784.31	Purchase of T-Bills	$2,000,000	
					Maturity of T-Bills
Panel B: Futures Hedge, Contract Executed to Obtain T-Bill					
−$19,607.84	Futures Contracts Purchased	−$1,941,176.47	Futures Contracts Executed	$2,000,000	
					Maturity of T-Bills
Panel C: Futures Hedge, Contract Sold, Interest Rates Decline					
−$19,607.84	Futures Contracts Purchased	−$1,970,443.35	Purchase of T-Bills	$2,000,000	Maturity of T-Bills
		9,659.04	Gain on Futures		
		19,607.84	Return of Margin		
		−$1,941,176.47	Net Cash Flow		
Panel D: Futures Hedge, Contract Sold, Interest Rates Rise					
−$19,607.84	Futures Contracts Purchased	−$1,955,034.21	Purchase of T-Bills	$2,000,000	Maturity of T-Bills
		−5,750.10	Loss on Futures		
		19,607.84	Return of Margin		
		−$1,941,176.47	Net Cash Flow		

in Panel B, whereas the entire cost of the T-bills is paid at 180 days in Panel A. Since money has a time value, the firm would prefer the cash flows in Panel A to those in Panel B *if these two sets of flows were equal in risk, but they are not.* The cash outflow at 180 days in Panel A is *risky* because of potential changes in interest rates, while the cash flows in Panel B are *riskless.* Therefore, many firms would prefer the cash flows in Panel B, and will choose the hedging strategy.

For illustrative purposes, Panels C and D portray the cash flows that would occur if the firm chose to liquidate the futures contract and purchase T-bills directly rather than via the futures contract. Panel C presents the cash flows associated with a decline in the 90-day interest rate to 1.50 percent, and Panel D presents the flows associated with a rise in this interest rate to 2.30 percent. The important thing to note in these two panels is that, once the firm has entered into the futures contract, *the cash flows (ignoring margin transactions) are the same regardless of any changes in interest rates*. By purchasing the futures contract, *the firm is truly locked into the rate on that contract*. If interest rates go up before the June surplus is available for investment (Panel D, which would be advantageous if the firm were unhedged), the firm must still either honor the futures contract (and take a lower rate on its investment than the market rate), or sell the contract (and take a loss on this sale). The firm is protected against declines in rates, but cannot profit from rises.

Because the use of a futures contract to hedge the interest rate risk of a future investment precludes the firm from investing should rates fall, some firms use options on futures contracts (rather than the futures contracts themselves) to hedge future investments even though there is an out-of-pocket cost to use the option. If options are used, the exercise of the option is at the discretion of the firm, and it may purchase securities directly in the market rather than through the option. In this way, it may benefit from rises in rates, though it is protected from their decline. To do this in the prior example, the firm would *purchase a call option on the June futures contract*. This option would mature in early May. At that time, the firm could either (1) exercise the option, or (2) let the option expire, go unhedged for the remaining three weeks, and purchase the bills in June. The first strategy would lock in the rate via the futures contract, and would be advantageous if rates had dropped, while the second strategy would be advantageous if rates had risen.

The process of hedging the interest cost on future needed financing is parallel to hedging the interest yield on future investing. However, with regard to financing, the firm wants to hedge a rate of *borrowing* (equivalent to selling debt securities) as opposed to *lending* (equivalent to buying debt securities). To hedge future borrowings, the firm may *sell a futures contract* on an investment instrument for future delivery (go short in the contract). To see how this hedges borrowings, consider the firm that, as of January, expects to borrow $3 million from September until December. If it sells a futures contract for September delivery of Treasury bills, it is hedged against changes in overall interest rate levels. If interest rates increase between January and September, the costs of its borrowing will increase, but the price of the futures contract will decline. When the firm must buy the investments for delivery to fulfill (cover) the short in the futures contract, the price of these instruments will have fallen, and the firm will make a gain on the futures transaction between the original selling price of the instrument (in the short sale) and

the eventual covering price. If interest rates decline, the opposite effect occurs; the firm's borrowings are cheaper, but it takes a loss on the futures transaction (since it will now cost more to buy the securities to cover the short sale than the original short sale netted to the firm).[34]

Like the purchase of a futures contract in anticipation of investment, the short sale of a futures contract in anticipation of borrowing locks in the rate of borrowing. Similar to the investment case, the use of options on futures (rather than futures themselves) enables the firm to profit from fortuitous interest rate movements, but at the cost of the option. To use an option to hedge future borrowing rates, the firm should *purchase a put option on the futures contract.* In the prior financing example, the firm would purchase a put option on a futures contract for Treasury bills to be issued in September. This would mature in early August. If interest rates had increased, the firm's cost of borrowing will have increased, but so will the value of the put. The firm can sell the put (or exercise it and then sell the futures contract), and use these gains to offset the increased cost of borrowing. If interest rates have not increased, the firm would let the put expire, go unhedged over the remaining three weeks, and borrow at the lower rates.

Now let us consider the second application of these contracts in hedging the risks from the cash forecast: *the risk of funds shortage.* We previously discussed how hedges such as keeping a stock of cash and near-cash assets, investing temporarily surplus cash in near-cash assets rather than longer maturities, or arranging for excess borrowing capacity can address this risk. Recall that when the firm has invested temporary cash in investments with maturities matching the expected future times when the cash will be needed (to take advantage of upsloping yield curve effects), the firm is subject to interest rate risk should actual cash flows be such that the firm must liquidate these investments before maturity. Futures contracts themselves will not work in this situation because the firm does not know whether there will be a shortage which will necessitate the selling of the investments. The firm needs an instrument it can exercise *if the shortage occurs*, and it is forced to sell its investments. One possible strategy is to *purchase a put option* on a futures contract for investment securities.

To see how this strategy would work in practice, assume that it is January and the firm has $1 million in surplus which it will need in March. If the yield curve is upsloping, it is more advantageous to invest this by purchasing a Treasury bill with a March maturity than to buy bills of shorter maturity and reinvest the proceeds as these bills mature. But if the cash is needed before

[34]As before, the firm does not actually have to purchase the securities to fulfill the contract for delivery (the short sale). The same losses and gains occur if the firm merely purchases a futures contract to cover the short and has the clearinghouse offset the firm's short and long positions in the contract. The difference between the prices of the short and long contracts is the firm's gain or loss, which offsets the changes in the cost of borrowing.

this, the firm may have to liquidate the investment (depending on the amount of the deficit and the amount of the firm's other hedges). Since interest rates may rise, this possibility exposes the firm to interest rate risk. However, *if the firm also purchases a put option* on a futures contract for the future delivery of Treasury bills, this interest rate risk will have been partially hedged. If the firm is forced to liquidate the investment before maturity *and interest rates have fallen*, it will allow the put option to expire, sell the investment, and realize a gain. (Since interest rates have fallen, the value of the bill will have risen by more than its normal appreciation.) If the firm is forced to liquidate the investment in February *and interest rates have risen*, it will sell its securities *and sell the put*, which will have risen in value. The increase in the value of the put because of increased interest rates will offset the lessened rise in the value of the bill.

Problems in Using Interest Rate Options and Futures for Hedging There are two factors inhibiting the firm in efforts to construct hedges that completely eliminate the risk of interest rate fluctuations via futures contracts or options on these futures contracts. These problems are *thin markets* and *basis risk*.

In order to close out any position in futures or options on futures, the firm must either (1) fulfill the contract by taking delivery of the instrument, (2) take a position opposite to the original contract by selling a contract (for a long) or buying a contract (for a short), take the loss or gain, and have the clearinghouse offset the transactions. The latter strategy is by far the most popular; however, it requires that the firm make at least one additional transaction in the futures marketplace. In making these market transactions, it is very useful for the firm to be able to find a ready buyer or seller without the action of buying or selling affecting the market price of the contract. In *thin markets*, there are so few contracts traded that this may not be possible; the very act of closing out the position may reduce the firm's returns via the act's effect on the market price of the contracts. While volume on near-term IMM futures contracts is quite high, volume in contracts with maturities beyond nine months is significantly lower, and may not be sufficient for large firms that wish to close out positions far in advance of maturity.[35] The same thin market condition exists in many options on financial futures contracts.

[35]To avoid this problem of thin markets and still hedge interest rate risk for longer time horizons, the firm may purchase a future on a relatively near-term instrument, close out the futures position just before maturity, and purchase another future. This procedure hedges most of the interest rate risk and is called "rolling the hedge forward." The same procedure can be used to hedge interest rate risk beyond the longest available future. See George M. McCabe and Charles T. Franckle, "The Effectiveness of Rolling the Hedge Forward in the Treasury Bill Futures Market," *Financial Management* (Summer 1983), pp. 21–29, and Dwight Grant, "Rolling the Hedge Forward: An Extension," *Financial Management* (Winter 1984), pp. 26–28.

The other problem, *basis risk*, also is related to trading volume. While Treasury bills are issued weekly, the futures contracts are traded on only eight different Treasury bill issues. This standardization of trading in a relatively few instruments generates the economies of scale and exchange that make modern futures trading possible. However, this standardization means that firms can rarely find a futures contract or option that *exactly* matches their hedging needs in terms of maturity date, amount, and instrument. For example, suppose it is January and that the firm expects to borrow $1.3 million in May from its bank for 60 days. To try to hedge this, it may purchase a futures contract on $1 million in Treasury bills to be issued in June with a maturity of 90 days (if this were the closest match available). This contract is different in instrument (Treasury bills versus bank lending), amount ($1 million versus $1.3 million), and maturity (the Treasury bills will mature in September, but the loan will mature in July). As long as the *difference in rates* between the future loan and 90-day Treasury bills (the *basis*) stays the same, the changes in the value of the futures contract will offset changes in the cost of borrowing for $1 million of the firm's future loan. But if the basis changes, or if the firm is not able to exactly match the principal value of the instrument with the principal value of the hedge (as in this case), *the changes in the value of the instrument will not be exactly offset with changes in the value of the future or option*.

While basis risk is a significant consideration in using futures and options to hedge risk, its importance should not be overemphasized. When the firm uses these hedges, the prime variable it is attempting to hedge is the overall level of interest rates. Compared to changes in these general levels, changes in the basis are often rather small. Despite problems of thin markets and basis risk, the use of futures, and options on these futures, provides results that are significantly safer than unhedged strategies.

HEDGING STRATEGIES IN PERSPECTIVE

One final note on the hedging of the risk implied in the cash forecast is useful. In the prior pages, we have discussed several different types of hedges: keeping a stock of cash, keeping a stock of near-cash assets, arranging for extra borrowing capacity, investing temporary surpluses of cash in near-cash rather than longer-term instruments, and using interest rate futures and options on these futures. It is important to remember that these differ significantly in their cost, flexibility, and the types of risk that they can hedge. Keeping a stock of cash or near-cash assets will hedge the risk of cash stockout (unexpectedly low cash balance). Cash is expensive but very flexible. Near-cash assets are less expensive but entail transaction costs. Excess borrowing will also hedge cash stockout risk but requires prearrangement and commitment fees. Investing surpluses in near-term instruments rather than in longer-

Table 3-12. Features of Some Hedging Strategies for Risks in the Cash Forecast.

	Risks of Cash Shortage	
Hedge	*When Applicable*	*Costs*
1. Keep stock of extra cash.	Hedges any shortage.	1. Opportunity loss on investment in amount of the firm's cost of capital.
2. Keep stock of near-cash assets.	Hedges any shortage.	1. Opportunity loss on investment in amount of the difference between the after-tax return on the near-cash investments and the firm's cost of capital. 2. Transaction costs of purchase, reinvestment, and sales of assets.
3. Arrange for extra borrowing capacity.	Hedges any shortage.	1. Out-of-pocket expenses of applying for and drawing down extra borrowing. 2. Commitment fees on extra borrowing capacity for maximum needed hedge.
4. Invest temporary surpluses in near-cash assets rather than longer-term assets.	During surpluses only.	1. If yield curve is upsloping, difference in yield between near-cash and longer-term investments. 2. Transaction costs of purchase, reinvestment, and sale of assets.
5. Invest temporary surpluses in longer-term investments and hedge interest rate risk with options on financial futures.	During surpluses only.	1. Cost of the options contracts. 2. Possible losses due to changes in the basis.

Interest Rate Risk on Future Investments and Borrowings

Hedge	*When Applicable*	*Costs*
1. Buy futures contract on T-bills.	Future investment.	1. Small transactions costs of purchase. 2. Opportunity cost of lost income if rates rise before time of expected investment.
2. Buy call option on a futures contract for T-bills.	Future investment.	1. Cost of the options contract. 2. Possible losses during three-week unhedged period if rates decline during this period.
3. Sell futures contract on T-bills.	Future borrowing.	1. Small transactions costs of purchase. 2. Opportunity cost of lost income if rates decline before time of expected investment. 3. Possible losses due to change in basis between T-bill and borrowing rates.
4. Buy put option on a futures contract for T-bills.	Future borrowing.	1. Cost of the options contract. 2. Possible losses during three-week unhedged period if rates rise during this period. 3. Possible losses due to change in basis between T-bills and borrowing rates.

term investments will provide a hedge only in times of surplus. Its costs are the difference in yield between near-term and longer-term investments and the cost of the additional transactions required. The use of put options on financial futures will hedge the interest rate risk inherent in using longer-term temporary investments when cash flows are uncertain. Interest rate futures and options can also hedge the interest rate risk involved in planned future

investments and borrowings. Options have significant transaction costs, but allow the firm to profit from fortuitous changes in interest rates. Futures contracts do not allow such profit, but lock in future interest rates and have low transactions costs. Table 3-12 summarizes the features and risks of the various hedging strategies used as a result of uncertainty in the cash forecast. This mix of applicability and cost means that firms have a rather difficult strategic decision in hedging the cash shortage uncertainties inherent in the cash forecast. To correspond with this mix of advantage and disadvantage, it makes sense to use a mix of hedging strategies, and *many firms use more than one of these strategies* to address the uncertainties they face.[36]

SUMMARY

In this chapter, the methods and uses of cash forecasting have been discussed. Firms make and use cash forecasts in order to be able to plan for expected surpluses and deficits. This planning process produces lower borrowing costs during periods of cash need and higher interest income during periods of temporary cash surplus than would otherwise be the case. The early portions of this chapter discussed methods of generating cash forecasts. The receipts and disbursements approach to cash forecasting was reviewed in detail. This method requires the estimation of individual cash inflows and outflows for each of the periods within the forecast. Many of the inflows and outflows are prescribed by the firm's other financial plans and forecasts, such as its capital budget and its sales forecast. Forecasting methods used for cash flows are usually the spot or percent of sales methods; the use of these methods is a consequence of the near-term nature of the forecasting period. Accounts receivable collections, an important inflow for many firms, can be forecast by the payment patterns approach. In this approach, expected collections on the sales of a particular period are spread over other periods based on historic payment proportions. An example spreadsheet cash forecast was executed, illustrating these forecasting techniques.

To use the cash forecast for more comprehensive planning, it is necessary to estimate the uncertainty in it and to develop strategies to deal with this uncertainty. Even though the cash forecast is a very near-term estimate, there are significant uncertainties involved. These uncertainties can be costly if ignored. Major uncertainties come from sales forecasts, estimates of receivables collection proportions, forecasts of production costs, and schedules of capital expenditures. One approach to assessing uncertainty is sensitivity anal-

[36]Because the cheapest strategies for hedging the risk of cash shortages often involve arranging for the availability of extra borrowing capacity, we defer the remaining discussion of hedging such shortages to Chapter 12, which deals with short-term financing. In that chapter, a methodology for developing the optimal hedging strategy for a limited variety of hedging vehicles will be presented.

ysis. This involves trying alternate estimates, forecasts, and assumptions to assess their effect on the timing and amount of cash surpluses and required borrowings.

The sensitivity analysis approach provides useful insights, but major concerns of the firm regarding its future cash position require the more complex technique of simulation analysis. Simulation analysis yields probability distributions of required borrowings and surpluses. These probabilities can then be used to develop hedging strategies to address the possible variations of the firm's borrowings and surpluses from their expected values. The choice of hedging strategies employed by the firm depends on the costs of the hedges and the costs of shortage. Possible hedges include keeping a stock of extra cash or near-cash assets, arranging for the availability of extra borrowing capacity, keeping temporary surpluses in near-cash investments rather than longer maturities, and using long-term investments but hedging the interest rate risk in these temporary investments with options on interest rate futures. Futures and options can also be used to hedge the interest rate risk in planned borrowings and investments, though the construction of perfect hedges is difficult.

Problems

3-1. It is January and the XYZ Company wishes to prepare a monthly cash forecast over the next four months. Sales for December were $1,100,000. Expected sales for the next four months are:

Month	*January*	*February*	*March*	*April*
Expected Sales	$600,000	$800,000	$1,100,000	$800,000

Thirty percent of the firm's sales are for cash; the remainder are collected in the month following the sale (there are no bad debts). Disbursements consist of payments for raw materials, direct labor, other operating expenses, purchases of fixed assets, and taxes. Costs of sales are 75 percent of sales. Of these costs of sales, 38 percent are raw material costs and 62 percent are direct labor costs. Direct labor costs are paid in the month incurred, while raw materials are purchased on net 30-day terms. Other operating expenses total $150,000 per month. Expenditures for fixed assets of $75,000 are to be made in February and April, and tax payments of $110,000 are to be made in January and March. As of January 1, there are no surpluses or deficits and the firm's cash balance is $83,000. The firm keeps a cash balance equal to 10 percent of the month's cost of sales. Generate a monthly cash forecast for the upcoming four months.

3-2. A firm makes monthly cash forecasts and distributes these forecasts

to a daily basis. For the upcoming month, anticipated cash inflows are $1,000,000. Each of the first two weeks has five working days. Twenty percent of the month's total cash receipts are expected to be received during the first week and 28 percent are expected to be received during the second week. Within each week, receipts are expected to be received as follows:

Day	*Monday*	*Tuesday*	*Wednesday*	*Thursday*	*Friday*
Proportion	0.20	0.23	0.07	0.20	0.30

The firm writes checks on Tuesday for the entire week's expenses, except for taxes. Ten percent of these checks are cashed on Tuesday, 5 percent on Wednesday, 60 percent on Thursday, and the remainder on Friday. Checks totaling $250,000 are to be written on Tuesday of the first week, and checks totaling $200,000 are to be written on Tuesday of the second week. A tax payment in the amount of $100,000 will be made on Wednesday of the second week via wire transfer. Beginning cash is $50,000; this is also the desired cash at all times during the first two weeks of the month. Generate the distributed daily cash forecast for the first two weeks of the upcoming month.

3-3. A firm has generated a cash forecast that shows the following pattern of surpluses over the next four months:

Month	*March*	*April*	*May*	*June*
Surplus	$2,500,000	$1,700,000	$2,000,000	$0

The yield curve is upsloping and has the following rates and maturities:

Time to Maturity	*Uncompounded Yearly Rate*
1 month	9.00%
2 months	9.60%
3 months	10.00%

Generate a bar chart of the surpluses over time. Using this bar chart, formulate an investment strategy for the investment of surplus funds. In formulating this investment strategy, assume that the firm has hedged the cash stockout risk; the surplus amounts can thus be treated as certain. Ignoring transaction costs, calculate the interest income from your investment strategy.

3-4. A firm has performed a simulation of its cash budget, with the following results:

Month	Expected Surplus or (Deficit)	Standard Deviation of Expected Surplus or (Deficit)
1	($80,000)	$100,000
2	($20,000)	$150,000
3	$70,000	$200,000
4	$20,000	$250,000

The firm keeps two hedges against cash flow uncertainty: excess borrowing capacity with its bank and a $50,000 cash reserve. It must commit to its maximum level of borrowing over the next four months, at a commitment fee of 0.2 percent of the maximum amount of borrowing arranged. The opportunity cost of holding the cash hedge is 10 percent per year (3.33 percent for the upcoming four months). The firm wishes to limit its probability of running out of cash (of exceeding the total available hedge) to a maximum of 5 percent in any month. Calculate the amount of borrowing capacity that the firm should arrange for with its bank and the total cost of its hedges of cash flow uncertainty, assuming that the probability distributions of surpluses and deficits are normal and serially uncorrelated. (Hint: The maximum amount of borrowing required will be the maximum needed for hedging and for expected borrowings less that needed for expected borrowings only.)

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case

Calais Belting Company Cash Forecasting

Calais Belting Company was a subsidiary of a French firm with a similar name. The American subsidiary was headquartered in a relatively remote section of the Virginia countryside. When asked why that particular section of the country was selected, Jacques Beaudine, the president of the subsidiary, cited the physical similarity to the terrain near the firm's main plant and headquarters facility in France. "One of the considerations in plant location is that the region should make you comfortable," he said, "and when executives from our parent firm come to visit us here, it seems like home to them."

The firm's primary business line was plastic industrial belting for applications in motion transfer (between pulleys) and material transfer (via conveyor belts). Some belts were fabric-backed while others were made only of resilient plastic materials. The raw material used in making the firm's products was mostly rolled plastic sheet which was manufactured at the firm's headquarters facility and shipped to the United States by the parent firm. Some sales were directly to end users, while other sales were made via jobbers. All sales to customers from the U.S. subsidiary were made on stated terms of net 30 days from date of invoice.

The U.S. firm's production facility was a modest one-story plant. The plant was previously used as a regional warehouse by an auto parts firm. While Calais Belting considered the building to be a good investment because of its excellent condition and location, floor space was only partially utilized; the firm had substantial room to expand their production facilities. This was an advantage since the firm felt that there were excellent market opportunities for high quality belt products in the American market. The firm intended to make substantial capital investment purchases during the 1984 calendar year, including a new roof for part of the plant, shelves, grinders, racks, and similar equipment. Some of this would be purchased from the French parent firm while some would be bought locally.

The firm was financed by a package of equity (invested by the parent firm), long-term notes to the firm's bank (principal and interest payments on these notes were made on a monthly basis), and temporary short-term bank borrowings to cover deficits, in addition to the usual accruals and amounts due trade suppliers.

One problem faced by Mr. Beaudine was that of cash forecasting and control. Arrangements for shipments from the parent firm had

Exhibit 1

Calais Belting Company
Forecast of Monthly Sales
(thousands of dollars)

Month 1984	*Pessimistic Estimate*	*Realistic Estimate*	*Optimistic Estimate*
January		468.00 (actual)	
February		455.00 (actual)	
March	426.08	448.50	470.93
April	389.03	409.50	429.98
May	389.03	409.50	429.98
June	401.38	422.50	443.63
July	397.80	442.00	486.20
August	409.50	455.00	500.50
September	418.50	465.00	511.50
October	427.50	475.00	522.50
November	412.25	485.00	557.75
December	425.00	500.00	575.00
1985			
January	437.58	514.80	592.02
February	420.75	495.00	569.25

Note: For any month, the probability of the expected sales estimate is 0.6, and the probabilities of the optimistic and pessimistic estimates are 0.2 each. Each month's outcome is independent of the result in any other month.

Exhibit 2

Calais Belting Company
Projected Future Purchases of Franc Currency
(thousands of dollars)

Month 1984	*Purchase*
March	299.00
April	234.00
May	208.00
June	299.00
July	234.00
August	208.00
September	328.90
October	257.40
November	228.80
December	328.90
1985	
January	257.40
February	228.80

to be made many months in advance, and these purchases were a large portion of the firm's cash outflows. The firm had only limited opportunities to adjust purchases (and thus cash outflows) downward to reflect lower sales (and thus cash inflows). Further, it was the firm's policy not to lay off workers in the event of a temporary sag in demand. Within a short planning horizon, the firm's payroll expenses were also fixed. Without the ability to quickly cut payrolls and material purchases in response to any circumstances involving unexpectedly lower sales, the firm could face significant cash shortages. In light of these facts, Mr. Beaudine was very careful about his cash forecasting procedures. At the beginning of each month, he generated a monthly cash forecast for the following twelve months, showing months where surpluses and deficits could be expected. These deficits were financed by short-term bank borrowings.

Exhibit 3

Calais Belting Company
Estimates of Several Types of Cash Expenses (thousands of dollars)

	1984									1985	
	March	*April*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*	*Jan.*	*Feb.*
Salaries and Taxes	61.10	65.00	67.60	67.60	67.60	67.60	70.00	70.00	70.00	70.00	70.00	70.00
Travel and Entertainment	17.55	22.75	26.65	26.65	26.65	26.65	30.00	30.00	30.00	30.00	30.00	30.00
Interest Payments	4.03	26.39	4.03	4.06	26.39	4.03	4.03	26.39	4.03	4.03	26.39	4.03
Principal Pays.– Notes	6.63	6.63	6.63	6.63	6.63	6.63	6.63	6.63	6.63	6.63	6.63	6.63
Insurance Payments	5.20	5.20	11.96	5.20	5.20	11.96	5.20	5.20	11.96	5.20	5.20	11.96
Overheads	18.85	18.85	18.85	18.85	18.85	18.85	18.85	18.85	18.85	18.85	18.85	18.85
Equipment Purchases	10.56	22.50	18.99	10.10	2.30	7.80	20.50	7.30	5.40	5.70	19.80	24.50

In early March of 1984, Mr. Beaudine was making a cash forecast for the upcoming year. He first made optimistic, realistic, and pessimistic estimates of monthly sales (see Exhibit 1). Naturally, he felt that his estimates of near-term sales figures were more certain than those further in the future. He also knew the schedule of purchases of materials from the parent firm in France. As a matter of policy, when the materials were shipped from France, the U.S. subsidiary entered into a futures contract for the delivery of francs at the time the bills were due. A schedule of projected currency purchases is presented in Exhibit 2. Mr. Beaudine also made schedules of expected payments for several other types of expenses which were estimated over the planning period via the spot method. These are presented in Exhibit 3.

Mr. Beaudine knew that two types of expenses were likely to vary with sales volume. These were sales commissions and shop materials. Sales commissions were paid to the firm's sales force during the month of the sale and were 2.3 percent of sales volume. Shop materials were materials, other than those shipped from the parent firm, that were used in the production process. These could be varied in the short term to reflect sales volume and were projected as 6.5 percent of sales. These materials were purchased on net 30-day terms from local suppliers.

Exhibit 4

Calais Belting Company
Results of Regression Analysis of Collection Patterns

$$C_t = \underset{(0.023)}{0.075}\, S_t + \underset{(0.120)}{0.800}\, S_{t-1} + \underset{(0.055)}{0.120}\, S_{t-2}$$

This is an ordinary least squares suppressed-constant regression. The equation is not serially correlated according to the appropriate statistical test. Dummy variables were tried to assess seasonality but these were not significant. The numbers in parentheses under the estimated coefficients are their respective standard errors.

Mr. Beaudine had previously done a regression analysis to estimate the firm's collection rates. The results are presented in Exhibit 4. He decided to follow the European practice of keeping a fairly substantial cash balance in his firm's account; he intended to keep a cash balance of 12 percent of the month's sales. This cash balance, along with reserve borrowing capacity at the firm's bank, formed the firm's hedge against funds shortages. The beginning cash balance for the forecasting period (that is, the firm's actual cash balance at the end of February 1984) was $58,240. As of this time, the firm had no outstanding temporary borrowings or temporary investments.

Chapter -4

Models for the Management of Cash And Temporary Investment

This is the final chapter concerning the management of cash and marketable securities. In Chapter 2, we discussed the gathering and disbursing of cash by the firm. In Chapter 3, we discussed cash forecasting, the temporary investment of cash, and the hedging methods that the firm can use to address the risk inherent in its forecasts of cash flows. This chapter, Chapter 4, has two purposes. First, we discuss models to decide whether it is worthwhile to make short-term investments of cash and to make strategic decisions regarding such investments. Second, we review survey evidence regarding the actual practices used by financial managers in the management of cash and temporary investments.

IS IT WORTHWHILE TO MAKE A SHORT-TERM INVESTMENT?

In the prior two chapters, we discussed strategies for (1) the investment of temporary surpluses of funds until they are needed, and (2) the use of short-term investments as interest-bearing repositories for funds kept as reserves against cash flow shortages. In this discussion, we deemphasized the role of the transactions costs of investing and disinvesting in these investment strategies. In general, it was assumed that the amounts involved were sufficiently large and the time of investment sufficiently long that *transactions costs were very small in comparison to investment income*. For example, in Chapter 3 we discussed cash forecasting, and concentrated on the monthly cash forecast. For most firms, surplus cash that is available for more than a month can be profitably invested. For investments of this duration, the period of investment is sufficiently long that the interest income from the investment will far outweigh the costs of making and withdrawing that investment.

However, the predominance of investment income over transaction costs

of investment does not hold *if the amounts of funds are small or the periods of investment are very short*. For example, assume that the total costs of investing and disinvesting are $50, and that the monthly interest rate is 0.6 percent (0.02 percent per day). If a firm had $100,000 to invest for a month, it would be profitable to make the investment; the interest income of $600 would far outweigh the transaction costs of investment and disinvestment. But if the firm had the funds for only two days, it would be better not to invest; the interest income of $40 would be less than the $100 total cost of making and withdrawing the investment. Since many firms have small surpluses available for short periods of time, a set of models is needed to address temporary investment decisions for circumstances where transactions costs play an important part.

In this chapter, we discuss four models that provide strategies in these circumstances. These models are named after their authors; the models are the Baumol model, the Beranek model, the Miller-Orr model, and the Stone model. Each of these models provides optimum strategies *for a given time pattern of cash flows*. Each model assumes a particular pattern of future cash flows and develops an optimum strategy for investment and disinvestment for that assumed pattern based on the trade-off between investment income and transaction costs.

While these models provide interesting and useful approaches, their limitations must be stressed. Like all optimization models, their domain is constrained by the assumptions made in their derivation. While all models are robust to deviations from these assumptions to a greater or lesser extent, the use of an optimization model outside its intended domain is dangerous, and may lead to uneconomical and perhaps disastrous results. Thus, it makes good sense to pay close attention to the assumptions and derivation of a model in deciding whether to use that model in addressing a particular management decision.

THE BAUMOL MODEL

In this model, the firm is assumed to receive cash periodically but to pay out cash continuously at a steady rate.[1] That is, the firm's inflows are lumpy but its outflows are not. This time pattern of cash balance is portrayed in Panel A of Figure 4-1. When the cash inflow is received, the firm puts enough cash in its disbursement account to cover outflows until the next inflow is received. There are several types of businesses that experience a time pattern of cash flows similar to this. For example, for firms that manage rental

[1]See William Baumol, "The Transactions Demand for Cash: An Inventory Theoretic Approach," *Quarterly Journal of Economics* (November 1952), pp. 545–56.

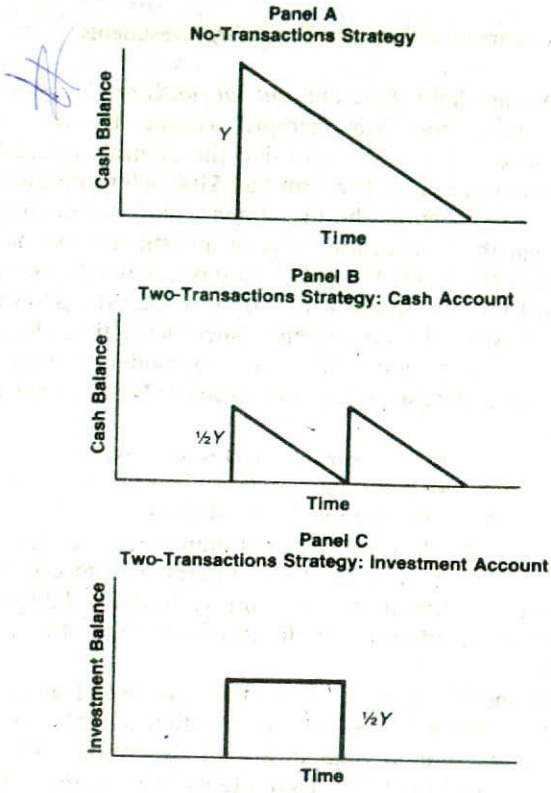


Figure 4-1. Time Pattern of Cash Balance for the Baumol Model.

property, rents are often received over a short period early in the month but other expenses (such as maintenance) continue evenly throughout the month.

The Baumol model also makes several assumptions about the firm's situation. It assumes that investments yield a fixed rate of return per period, regardless of the length of the investment (that is, it assumes that yield curve effects are small over the time period of investment). This is an appropriate assumption if the firm is limited to investments in money market accounts, negotiated order or withdrawal accounts, or similar small-dollar investments. The model assumes, as do all the other models discussed in this chapter, that the transaction cost of investing and disinvesting is a fixed cost that is independent of the amount of the investment.[2] These transaction costs consist of all the out-of-pocket costs of making or selling the investment: commissions, postage, telephone charges, the opportunity cost of diverted managerial effort, and so forth.

[2]Situations where the transaction cost of investing is variable in the amount of the investment are addressed in mathematical programming models of investment, such as Gary Eppen and Eugene Fama, "Cash Balance and Simple Dynamic Portfolio Problems with Proportional Costs," *International Economic Review* (June 1969), and Hans Dallenbach, "A Stochastic Cash Balance Model with Two Sources of Short-Term Funds," *International Economic Review* (June 1969).

Given that the firm has periodic inflows and steady outflows, what is the appropriate strategy for investing the funds until they are needed? This is the question addressed by the Baumol model. To understand the trade-offs in this situation, let us evaluate some possible strategies. One possible strategy is:

1. When the cash inflow is received, invest one-half of the total inflow; put the remaining one-half in the disbursement account.
2. During the first half of the period, pay disbursements from the disbursement account. This account will be drained one-half of the way through the period. At that time, sell the investments and place the resulting funds in the disbursement account.
3. Use these funds to pay disbursements during the remainder of the period.

This is called the *two-transactions strategy* because it involves one investment of funds and one disinvestment of funds. The effects of this strategy on the cash and investment accounts is portrayed in Panels B and C of Figure 4-1. To see the net gain from this strategy, let Y be the amount of the cash inflow and i be the interest rate per period (that is, the interest rate from one periodic cash inflow to the next). Since in this strategy one-half the inflow is invested for one-half the period, the interest income is:

$$\text{Investment Income} = (1/2)(1/2)iY = (1/4)iY \qquad (4\text{-}1)$$

But there are two transactions in this strategy: one investment and one disinvestment. If the cost per transaction is a, the transactions cost of this strategy is $2a$, and the profit is:

$$\text{Profit} = (1/4)iY - 2a \qquad (4\text{-}2)$$

While the simple two-transactions strategy captures one-half of the cash inflow into the investment account for one-half the period, other strategies will capture more of this flow for longer periods and thus earn more interest. Consider the *three-transactions strategy*:

1. When the cash flow is received, initially invest two-thirds of it. Place the remaining one-third in the disbursements account.
2. One-third of the way through the period, the disbursements account will be exhausted. At this time, disinvest one-half of the funds in the investment account [the amount is $(1/2)(2/3)Y = (1/3)Y$] and put this in the disbursement account. Leave the remaining $(1/3)Y$ in the investment account.
3. Two-thirds of the way through the period, the disbursements account will again be exhausted. Disinvest the remaining $(1/3)Y$ in the investment account and move the proceeds to the disbursement ac-

count. This will fund disbursements through the remainder of the period.

This strategy involves one investment transaction and two disinvestment transactions. The balances in the investment and disbursement accounts are portrayed in Figure 4-2. This strategy captures a greater portion of the initial cash inflow in the investment account. Two-thirds of the initial cash inflow resides in the investment account for one-third of the period (until the first withdrawal) and one-third of the cash inflow resides in the investment account for the second one-third of the period (until the second withdrawal). The interest income is then:

$$\text{Interest Income} = (2/3)\,(1/3)iY + (1/3)\,(1/3)iY = (1/3)iY \qquad (4\text{-}3)$$

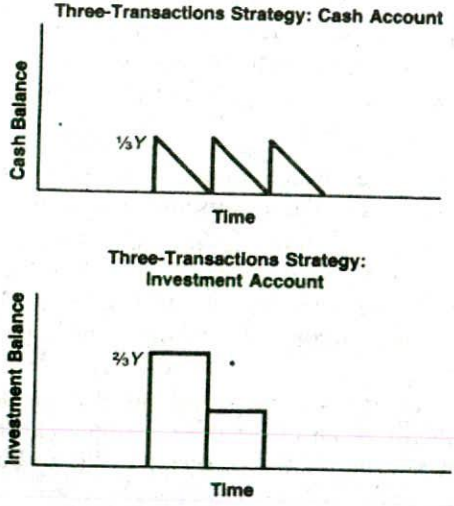
And since there are now three transactions, the profit is:

$$\text{Profit} = (1/3)iY - 3a \qquad (4\text{-}4)$$

Whether the three-transactions strategy is more profitable than the two-transactions strategy depends on the amount of additional interest earned versus the additional transaction cost paid. The Baumol model addresses the question of the optimal strategy by deriving a general expression for the *optimal number of transactions* for a firm with periodic inflow and steady outflows. In general, the interest income from investment strategies in this situation is:

$$\text{Interest Income} = [(n-1)/2n]iY \qquad (4\text{-}5)$$

Figure 4-2. Time Pattern of Cash Balance for the Baumol Model Three-Transactions Strategy.



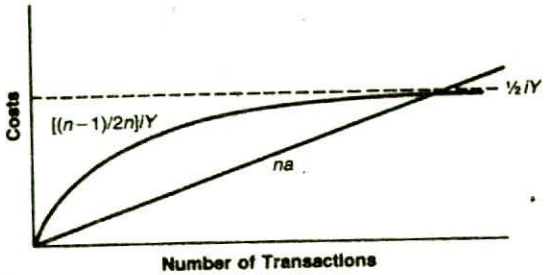


Figure 4-3. *Costs and Revenues versus Number of Transactions for the Baumol Model.*

where n is the number of transactions. The student may verify that this expression gives the proper income for the two- and three-transactions strategies. Since the transactions cost is na, the profit for n transactions is:

$$\text{Profit} = [(n-1)/2n]iY - na \qquad (4\text{-}6)$$

As the number of transactions increases, the interest income increases but so does the total transactions cost. This relationship is plotted in Figure 4-3. Since the total interest income is limited to capturing the entire initial cash inflow in the investment account, which would result in an interest income of $(1/2)iY$, the interest income is asymptotic to this level.[3] For some level of n, the profit (equal to the vertical distance between the cost and interest income functions) is at a maximum. To find this, we take the first derivative of the profit function [equation (4-6)], set this equal to zero, and solve for the optimal n. The result is:

$$n^* = (iY/2a)^{1/2} \qquad (4\text{-}7)$$

where n^* is the optimal number of transactions. In the Baumol model, the firm will always make one deposit and $n^* - 1$ withdrawals from the investment account. The amount of the initial deposit will be $[(n^* - 1)/n^*]Y$; the amount of the withdrawals will be $(1/n^*)Y$. The profit from this strategy can be computed via equation (4-6).

An example in the use of the Baumol model is in order. A firm runs an apartment complex. Cash comes in at the beginning of the month (from rental payments) and is disbursed uniformly throughout the following month. For the upcoming month, the firm expects to have cash expenses of $250,000.

[3]This occurs because:

$$\lim_{n \to \text{Inf.}} [(n-1)/2n]iY = 1/2\,iY$$

By investing in a money market account, it can earn 0.5 percent per month. Transactions costs are estimated at \$50 per transaction. What is the firm's best investment strategy and how much will this earn?

To formulate this strategy, the optimal number of transactions is first computed:

$$n^* = (iY/2a)^{1/2} = [(0.005)\ (250{,}000)/2(50)]^{1/2} = 5.0 \qquad (4\text{-}8)$$

The five-transactions strategy is optimal; there should be one deposit and four withdrawals from the investment account. The amount of the initial deposit will be (4/5) (\$250,000) = \$200,000; each of the four withdrawals will be \$50,000, as will the initial deposit in the disbursement account. This initial deposit will fund the firm for one-fifth of the month (50,000/250,000), or about six days; then the first withdrawal will be made from the investment account. Withdrawals will occur every six days (on the sixth, twelfth, eighteenth, and twenty-fourth days of the month). The profit from this strategy will be:

$$\begin{aligned} \text{Profit} &= [(n-1)/2n]iY - na \\ &= (5-1)/2(5)\ (0.005)\ (250{,}000) - 5(50) \\ &= 500 - 250 = \$250 \end{aligned} \qquad (4\text{-}9)$$

Like all the models in this chapter, the applicability of the Baumol model is limited to situations where the cash flow pattern fits that assumed in the model's derivation. Other patterns are addressed by other models, such as the Beranek model, which is discussed in the next section.

THE BERANEK MODEL

Beranek hypothesized firms where the cash *inflows* were steady, but the *outflows* were periodic.[4] This is the mirror image of the time pattern of cash flows within the Baumol model, where inflows were periodic and outflows were steady. The time pattern of cash balances in the Beranek model is pictured in Panel A of Figure 4-4. Balances build over time, then are disbursed all at once. This time pattern represents the situation faced by many firms. For example, a firm that ships and bills uniformly throughout the month on net 30-day terms but which writes checks only a few times per month would experience this pattern. Cash would be collected continuously at a uniform rate, but would be disbursed over a short time as a group of the firm's checks reached its bank and were paid.

[4]See William Beranek, *Analysis for Financial Decisions* (Homewood, Ill.: Richard D. Irwin, 1963), Chapter 11.

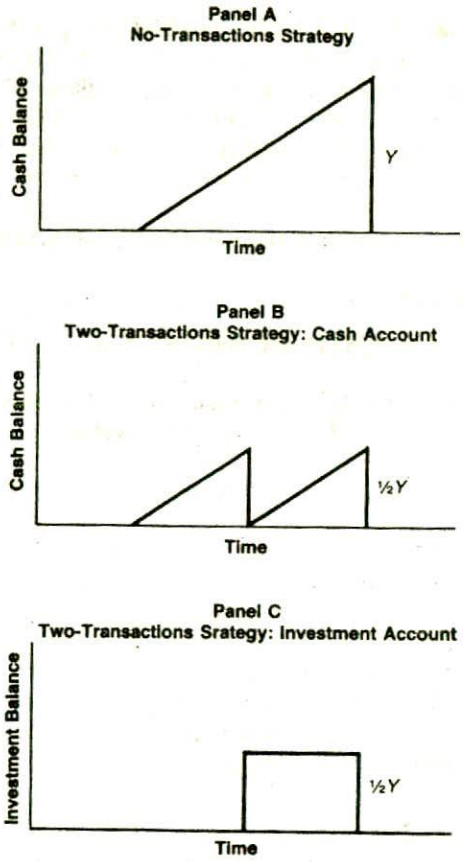


Figure 4-4. Time Pattern of Cash Balance for the Beranek Model.

In this pattern of cash flows, the challenge is to profitably invest the funds between the time of their receipt and the time when a group of checks are presented to the bank for payment. The trade-off between interest income and transactions expense and the strategies employed are parallel to those of the Baumol model. For example, the firm could wait until one-half of the incoming cash has been accumulated, then invest this cash until the end of the period when it is needed. In this two-transaction strategy (see panel B of Figure 4-4), the interest income would be $(1/4)iY$, as in the Baumol model, and the transaction expense would be $2a$. The formula for the optimum number of transactions is the same as that for the Baumol model, and is given in equation (4-7). In the Beranek model, however, *the cash is accumulated gradually* (rather than disbursed gradually), so the transactions pattern would involve *a series of investments followed by one disinvestment* at the end of the period.

Let us try an example. A firm receives $100,000 per day, which it will eventually need to make disbursements, and follows a policy of writing checks every two weeks. Over a two-week period, it accumulates $1,400,000. It can invest at an interest rate of 6.825 percent per year (0.2625 percent for two weeks). The transaction cost of investment or disinvestment is $37.50. What investment strategy should the firm follow and how much will this strategy earn?

From equation (4-7), the optimal number of transactions is:

$$\begin{aligned} n^* &= (iY/2a)^{1/2} \\ &= [(0.002625)\ (1{,}400{,}000)/2(37.50)]^{1/2} = 7.0 \end{aligned} \tag{4-10}$$

In the Beranek model, these seven transactions consist of six deposits and one withdrawal. The amount of the final withdrawal is:

$$[(n^* - 1)/n]Y = 6/7\ (1{,}400{,}000) = \$1{,}200{,}000 \tag{4-11}$$

The amount of the periodic investments will be $(1/n)Y$, or $200,000. The initial investment will require two days to accumulate, and will be made at the end of the second day. The other investments, in the same amount, will be made at the end of the fourth, sixth, eighth, tenth, and twelfth days. The accumulated balance of $1,200,000 will be withdrawn on the fourteenth day, and with the cash accumulated during days 13 and 14, will constitute the $1.4 million needed to cover the checks presented to the bank at that time. From equation (4-6), the profit from this strategy will be:

$$\begin{aligned} \text{Profit} &= [(n-1)/2n]iY - na \\ &= [(7-1)/2(7)]0.002625(1{,}400{,}000) - 7(37.50) \\ &= 1575 - 262.5 = \$1{,}312.50 \end{aligned} \tag{4-12}$$

While they address parallel time patterns of cash flow, neither the Baumol nor the Beranek models deal with risk; they both assume that the amounts of cash inflows and outflows are known with certainty. In using these models, the firm must use a hedging strategy similar to those outlined in the prior chapter to deal with cash flow uncertainty. For example, a firm facing the cash flow pattern in the Beranek model might keep a reserve of cash. If cash inflows then turned out to be less than expected for a particular part of the period, the firm could temporarily draw from the cash reserve to make deposits of economical size.

The remaining two models presented in this chapter (the Miller-Orr model and the Stone model) do not require these additional hedges. Instead, they incorporate uncertainty explicitly within the strategies which they derive.

THE MILLER-ORR MODEL

As in the prior two models, Miller and Orr assume that the yield curve for investments made via their model is flat and that there is a fixed cost of investing and disinvesting that does not vary with the amount of investment or disinvestment.[5] They also assume that investments and disinvestments can take place instantaneously and that there is a lower limit below which the firm's cash balance is not to fall. This lower limit is set by management and is not determined within their model; it may be a zero cash balance (so that the firm does not overdraft) or the firm's minimum compensating balance.

The major difference between the Miller-Orr model and the prior two models concerns the assumed time pattern of cash flows. Baumol and Beranek assume a rise-and-fall pattern of net inflows and outflows. Miller and Orr assume that net cash flows are normally distributed with a mean of zero, that the standard deviation of this distribution does not vary across time, and that there is no correlation of the cash flows across time. Under these assumptions, cash flows must follow a random walk around a zero average net flow. Based on these assumptions, and using the advanced mathematical technique of stochastic calculus, they formulate a profit-maximizing strategy based on *control limits*.

This control-limits approach involves exactly the same decision rules as does the control-limits approach in production management theory. In production management theory, a production process is initially adjusted for optimum performance. However, it is recognized that variations in materials, machine wear, and so forth will cause random fluctuations in the output of the process. Since it is costly to adjust the process, control limits are set up based on the expected variation in the process. Only when these limits are breached is the process adjusted. The Miller-Orr cash management model is basically an application of control-limits theory to the cash/investment decision. Control limits are set up using a formula derived by Miller and Orr. When the firm's total cash goes outside the *upper control limit, investments are made* to bring the cash balance back down to the *return point*; when the firm's cash balance goes below the *lower control limit, disinvestments are made* to bring the balance back up to the return point. The formula developed by Miller and Orr is:

$$R = (3aV/4i)^{1/3} \tag{4-13}$$

[5]See M. H. Miller and D. Orr, "A Model of the Demand for Money by Firms," *Quarterly Journal of Economics* (August 1966), pp. 413–35. For an interesting discussion of the relationship between the Baumol and Miller-Orr models, see B. D. Bagamery, "On the Correspondence Between the Baumol-Tobin and Miller-Orr Optimal Cash Balance Models," *Financial Review* (May 1987), pp. 313–19.

where V is the variance of daily cash flows, i is the daily interest rate on investments, and a is the transactions cost of investing or disinvesting. If L is the lower control limit (set by management), the optimal return point is $R + L$ and the optimal upper control limit is $3R + L$.

To see how this model would work in practice, let us address an example problem. A firm has confirmed that its cash flows satisfy the requirements of the Miller-Orr model (by procedures we will discuss later). It can earn 0.02 percent per day in investments. Based on historic data, the firm has estimated the standard deviation of its daily cash flows at $50,000 and its transaction costs of investment or disinvestment at $45.00. It wishes to maintain a minimum cash balance of $100,000 at all times. Based on this data, the R statistic would be calculated as:

$$R = (3aV/4i)^{1/3} = [3(45)(50{,}000^2)/4(0.0002)]^{1/3}$$
$$= \$75{,}000 \qquad (4\text{-}14)$$

Using this statistic, the firm would set the lower control limit at $100,000, the return point at $175,000, and the upper control limit at $325,000 (3(75,000) + 100,000). Let us track the effects of this strategy for some example cash flows. Assume that the firm's starting balance was $150,000 and the following cash flows occur:

Day	*Net Cash Flow*
1	−$25,000
2	−$75,000
3	$100,000
4	−$25,000
5	$125,000

The control limits, the return point, and the effects of these cash flows are portrayed in Figure 4-5. At the end of day 1, the cash balance would be $125,000; since this is between the control limits, no action would be taken. At the end of day 2, however, the cash balance would be reduced to $50,000 if the firm did nothing. Since this is below the lower control limit, the firm would disinvest sufficient securities to get back the return point. In this case, a disinvestment of $100,000, equal to the return point of $150,000 less the ending cash balance of $50,000, would be made and the firm would start day 3 at the return point. At the end of days 3 and 4, the cash balance would be $250,000 and $225,000 respectively, but no action would be taken since these balances are within the control limits. However, during day 5, $125,000 is collected, and the balance would be $350,000 if no action is taken. However, in the Miller-Orr model, this is a sufficient balance to justify investment, and $200,000 would be invested, bringing the cash balance back to the return point to start day 6.

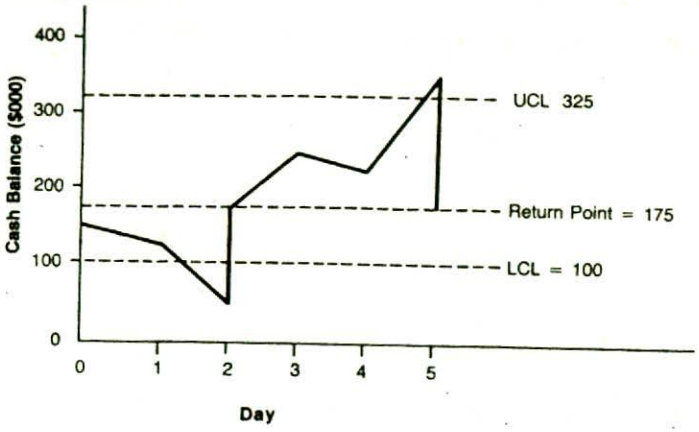


Figure 4-5. Cash Balances for Miller-Orr Example.

Concerns About the Assumptions of the Miller-Orr Model While the Miller-Orr model has intuitive appeal as a control-limits approach, the time pattern of cash flows hypothesized within the model has been subject to substantial criticism. The random walk nature of this time pattern assumes that management has no knowledge at all about the direction of future cash flows; in reality, management may forecast these flows, but with a substantial error component within the forecast.[6] Also, while existence of the time patterns of cash flows in the Baumol and Beranek models may be suspected by the business patterns and policies of the firm, the assumption of normality in the Miller-Orr model relies on the central limits theorem, and the convergence to normality of a sample under this theorem depends on the shape of the underlying population distribution.

Because of doubts about the normality of firms' cash flows, and about the Miller-Orr assumptions of no correlations in cash flows over time and no changes in standard deviation over time, several researchers in finance have conducted empirical studies of the probability distributions of firms' net cash flows. These studies have produced mixed results. In their original study, Miller and Orr found that the distributions of cash flows for the Union Tank Car Corporation were more "fat tailed" (had more outlying observations) than would be expected under normality.[7] Homonoff and Mullins also found the cash flows to be nonnormal for their sample firm.[8] Emery tested cash

[6]See Bernell Stone, "The Use of Forecasts and Smoothing in Control-Limit Models for Cash Management," *Financial Management* (Spring 1972), pp. 72–84, and Hans Daellenbach, "Are Cash Management Optimization Models Worthwhile?" *Journal of Financial and Quantitative Analysis* (September 1974), pp. 607–26.

[7]See Miller and Orr, "A Model of the Demand for Money."

[8]See R. Homonoff and D. W. Mullins, Jr., *Cash Management* (Lexington, Mass.: Lexington Books, 1975).

Table 4-1. *Example Net Cash Flows and Summary Statistics.*

Day	*NCF*
1	−100
2	100
3	−200
4	0
5	−100
6	300
7	0
8	200
9	−200
10	−100
Sample Mean	−10.00
Sample Standard Deviation	157.80
Standard Deviation for Mean = 0	158.11

flow data from three firms in different industries.[9] He found that daily net cash flows from two of these firms were normally distributed, but the distributions changed over time. He also found evidence regarding the existence of correlations in cash flows over time. Further, his simulation of the Miller-Orr model found that the existence of such a correlation affected the behavior of this model.

In summary, these empirical results suggest that a firm should not automatically adopt the Miller-Orr if no discernible time pattern of cash flows is obvious. Instead, the firm needs to perform statistical tests to assess the agreement between the probability distributions of the firm's cash flows and the distributions assumed in the model. Tests for nonnormality and for the correlation of the cash flow over time (autocorrelation) are generally sufficient to confirm or deny this agreement. To illustrate this procedure, let us test the data in Table 4-1 for these two properties.[10]

[9]See Gary Emery, "Some Empirical Evidence on the Properties of Daily Cash Flow," *Financial Management* (Spring 1981), pp. 21–28.

[10]In the following discussion, we treat the test for equality of distribution over time as tested simultaneously with that of normality, since shifting distribution parameters over time would bias the statistical tests against normality. (See Emery, "Some Empirical Evidence," for discussion.) We test only for first-order autocorrelation (correlation between one day's cash flow and the next) because this is the correlation that Emery found to be important. Finally, we use an impractically small sample in the interests of simplicity; the chi-square test we use to test for normality requires that the expected number of outcomes in each range be greater than five, though it is robust to deviations from this condition.

Summary statistics regarding this distribution of cash flows are presented at the bottom of the table. The Miller-Orr model requires that the cash flows be normally distributed with a mean of zero,[11] so for testing purposes an additional standard deviation of 158.11 has been calculated using a zero mean, and the other test statistics will be calculated using a mean of zero.

To test for normality, the frequency distribution of the sample is compared with the frequency distribution that would be expected of a normally distributed sample with the same mean and standard deviation. We will use a chi-square test for this purpose.[12] A test of this sort is illustrated in Table 4-2. To use this test, the data are first divided up into ranges. This division is rather arbitrary; we have used four ranges with endpoints of −150, 50, and 250 (four ranges is the minimum number for using this test). The upper limits of these ranges are in column A. To compare the expected frequency if the distribution is normal with a mean of zero to the observed frequencies, we need to calculate the probability that an outcome will fall in each range. The first range extends from minus infinity to −150. If the mean is zero and the standard deviation is 158.11, −150 is 0.95 standard deviations to the left of the mean [(−150−0)/158.11 = −0.95]; cell B5 contains "(A5−0)/158.11)." This formula is then copied into cells B6 and B7 to obtain the Z scores for the upper cutoffs for each range (the

Table 4-2. Test for Nonnormality for Example Net Cash Flows

	A	B	C	D	E	F	G
1	Upper	Z for	Prob.	Prob.	Expected	Actual	Cont.
2	Cutoff	Upper	for Upper	for	Number	Number	to Chi-
3	of Range	Cutoff	Cutoff	Range	for Range	in Range	Square
4							
5	+150	−.95	.1711	.1711	1.71	2	.0260
6	50	.32	.6255	.4544	4.54	5	.0004
7	250	1.58	.9430	.3175	3.18	2	.1435
8	+ Inf.	+ Inf.	1.0000	.0570	.57	1	.0086
9							
10	Totals			1.0000	10.00	10.00	.1786

[11]Alternately, if the firm has a positive or negative level of expected cash flows (for example, if it has expected cash inflows of a certain amount per day), it might deduct or add this expected amount from the actual net cash flow during each day and use the Miller-Orr model to manage the remainder, which would then have a zero mean. The net cash flows in Table 4-1 have been adjusted in this way.

[12]A more powerful test, particularly for small samples, is the Kolmogorov-Smirnov goodness-of-fit test [see J. V. Bradley, *Distribution-Free Statistical Tests* (Englewood Cliffs, N.J.: Prentice-Hall, 1968)]. We use the chi-square test here because it is more familiar to students and because it is simpler.

upper cutoff for the topmost range is infinity). A cumulative probability up to these Z scores (from a cumulative normal distribution table) is then entered as data in column C. The probability between these cutoffs is the probability for the range, which is calculated in column D. For example, cell D6 contains "C6-C5." The total in cell D10 verifies that the probabilities of the ranges sum to 1.0. The probabilities in column D times the sample size of 10 give the expected frequencies in column E.

These frequencies in column E are those that would be expected if the distribution is normal with a mean of zero and a standard deviation of 158.11. The actual frequency counts in the sample appear in column F. The larger the differences between the expected frequencies and the actual frequencies, the less likely that the distribution is normal. The chi-square statistic measures the differences between the expected and actual frequencies. The overall chi-square statistic is the sum of the contributions to that statistic based on the differences between the expected and actual frequencies for each range. The contribution to the chi-square statistic for each range is calculated as:

$$\frac{(|\text{actual frequency} - \text{expected frequency}| - 0.5)^2}{\text{expected frequency}} \qquad (4\text{-}15)$$

The particular spreadsheet used here has an absolute value function denoted by @ABS. Therefore, cell G5 contains "((@ABS(F5-E5) − 0.5)²)/E5" and parallel entries are in cells G5 through G8. The contributions to the chi-square statistic are summed in cell G10; this total chi-squared statistic measures the difference between the actual and the expected frequencies, and the distribution of this statistic is known. The number of degrees of freedom is the number of ranges minus three, so in this case the number of degrees of freedom is 1. From a table of critical chi-square values, at the 90 percent confidence level with one degree of freedom, a chi-square statistic of greater than 2.71 is required to accept the hypothesis that the two frequency distributions are not the same. Since the calculated chi-square statistic is 0.1786, *the hypothesis of nonnormality is not supported* at the 90 percent confidence level.

The second test is for autocorrelation. Autocorrelation has to do with the existence of positive or negative trends. Does knowing one value tell us anything about the next value? If it does, there is autocorrelation. If a higher-than-average value on an outcome is associated with a higher-than-average value in the next outcome, there is a positive correlation coefficient among the cash flows and positive autocorrelation. If a higher-than-average value on an outcome is associated with a lower-than-average value in the next outcome, there is a negative correlation coefficient among the cash flows and negative autocorrelation. Any significant autocorrelation, positive or negative, is a violation of the assumptions of the Miller-Orr model.

In a testing for autocorrelation, one possible approach is based on the statistical significance of the correlation coefficient between one day's cash flow and the next day's cash flow. To perform such a test, the correlation coefficient is first calculated, then tested to see if it is significantly different from zero. A significantly nonzero correlation coefficient, either positive or negative, is evidence of autocorrelation. The formula for the Pearson product-moment correlation coefficient between these two cash flows is:

$$r = \frac{\sum_{\text{all } t} (NCF_t - E(NCF_t))\,(NCF_{t-1} - E(NCF_{t-1}))}{(SS_{NCFt} SS_{NCFt-1})^{1/2}} \tag{4-16}$$

where r is the correlation coefficient, NCF_t is the net cash flow on day t, $E(NCF_t)$ is the expected net cash flow on day t, NCF_{t-1} is the net cash flow on day $t-1$, and $E(NCF_{t-1})$ is the expected net cash flow on day $t-1$. Also, SS_{NCFt} and SS_{NCFt-1} are the *sums of squared deviations* for NCF_t and NCF_{t-1} respectively. The sum of squared deviations for NCF_t is:

$$SS_{NCFt} = \sum_{\text{all } t} (NCF_t - E(NCF_t))^2 \tag{4-17}$$

and SS_{NCFt-1} is a corresponding formula. In this test, we want to constrain the expected net cash flows ($E(NCF_t)$ and $E(NCF_{t-1})$) to zero (if they are not actually zero) since a zero mean is required by the Miller-Orr model. In this special case, equation (4-16) reduces to:

$$= \frac{\sum_{\text{all } t} (NCF_t)\,(NCF_{t-1})}{(SS_{NCFt} SS_{NCFt-1})^{1/2}} \tag{4-18}$$

and equation (4-17) reduces to:

$$SS_{NCFt} = \sum_{\text{all } t} (NCF_t)^2 \tag{4-19}$$

A spreadsheet to perform this test for the data from Table 4-1 is presented in Table 4-3. The original data are in columns A and B. Column C contains the prior day's net cash flow (for example, cell C6 contains "B5"). Because no prior data is available for day 1, that data point is lost, and analysis proceeds using data from days 2 through 10. We use these data to calculate the numerator and denominator of equation (4-18). The sums of squared deviations from zero required for the denominator are calculated in columns D and E.

Table 4-3. Calculation for First-Order Correlation Coefficient for Example Net Cash Flows

!	A	!!	B	!!	C	!!	D	!!	E	!!	F	!
1					Prior		Day's		Prior Day's		Product of	
2			Day's		Day's		Squared		Squared		Deviations	
3		Day	NCF		NCF		Deviation		Deviation		from Zero	
4							from Zero		from Zero			
5		1	−100									
6		2	100		−100		10000		10000		−10000	
7		3	−200		100		40000		10000		−20000	
8		4	0		−200		0		40000		0	
9		5	−100		0		10000		0		0	
10		6	300		−100		90000		10000		−30000	
11		7	0		300		0		90000		0	
12		8	200		0		40000		0		0	
13		9	−200		200		40000		40000		−40000	
14		10	−100		−200		10000		40000		20000	
15												
16	Sums of Squared Deviations						240000		240000			
17	Sum of Product of Deviations										−80000	
18	Correlation Coefficient										−0.333	

In column D, each of the day's cash flows from column B is squared, and the sum of these is presented in cell D16; column E performs a parallel calculation for the prior day's cash flows. For this particular example, the sums of square deviations from zero for the day's cash flows and the prior day's cash flows are both 240,000.

The numerator of equation (4-18) is calculated in column F. The numerator is the sum of the products of the day's cash flow and the prior day's cash flow, so each row in column F contains the day's cash flow (from column B) times the prior day's cash flow (from column C). Thus, cell F6 contains "B6*C6". Cell F17 gives the sum of cells F6 through F14, which is the numerator of equation (4-18). Cell F18 contains the formula for the correlation coefficient, which in this spreadsheet is "F17/((D16*E16)^1/2)."

The calculated value for the correlation coefficient is −0.333. To test whether this is significantly different from zero, we use a Student's t-test. The formula for converting a correlation coefficient into a t-score is:

$$t = r[(N-2)/(1-r^2)]^{1/2} \qquad (4\text{-}20)$$

where t is the t statistic and N is the number of data points used (nine in this example). This statistic has $N-2$ degrees of freedom. The t statistic for the example is then:

$$t = r[N-2)/(1-r^2)]^{1/2}$$
$$= -0.333[(9-2)/(1-(-0.333)^2]^{1/2} = -0.935 \quad (4\text{-}21)$$

The assumptions of the Miller-Orr model are violated if the correlation coefficient is significantly different from zero in a positive or negative direction, so a two-tailed test is required. With seven degrees of freedom, 90 percent of the Student's t distribution is contained between t statistics of -1.895 and $+1.895$. Since the calculated statistic is within this range, *the hypothesis of significant autocorrelation is not accepted* at the 90 percent confidence level. Since the data are neither significantly nonnormal nor significantly autocorrelated, the firm might consider applying the Miller-Orr model as part of its management strategy for cash and temporary investments.

The Miller-Orr model is intended to apply when the firm has no valid future information about day-to-day cash flows; that is, when these cash flows follow a random walk. However, firms may know parts of their cash flows with considerable certainty (such as disbursements), though other parts may be quite uncertain. In the case where the firm has some (though not error-free) knowledge of future cash flows, the Miller-Orr model may produce nonoptimal strategies. For example, if the upper control limit in the Miller-Orr model is pierced, the model suggests that the firm invest. But if the firm knows that a major cash outflow is imminent, this investment may not be the best strategy. It may be better for the firm to ignore the signaled purchase, keep the funds in cash to cover the outflow, and save the transactions costs on the investment and disinvestment. The Stone look-ahead model described in the next section allows the firm to modify investment and disinvestment decisions based on expected future cash flows.

THE STONE MODEL

Like the Miller-Orr model, the Stone model takes a control-limits approach; when cash balances fall outside the control limits, the firm is signaled to do something.[13] But in the Stone model, the signal does not automatically result in an investment or disinvestment: the recommended action depends on management's estimates of future cash flows. That is, the model signals an *evaluation* by management rather than an *action*. To do this, the Stone model uses *two sets* of control limits (see Figure 4-6); the *inner control limits* (UCL_1 and LCL_1) and the *outer control limits* (UCL_2 and LCL_2).

Under the Stone model, strategy proceeds as follows. The firm performs no evaluation until its cash balance falls outside the *outer* control limits. When this occurs, the firm *looks ahead* by adding the expected cash flows for the

[13]This model is presented in Stone, "The Use of Forecasts."

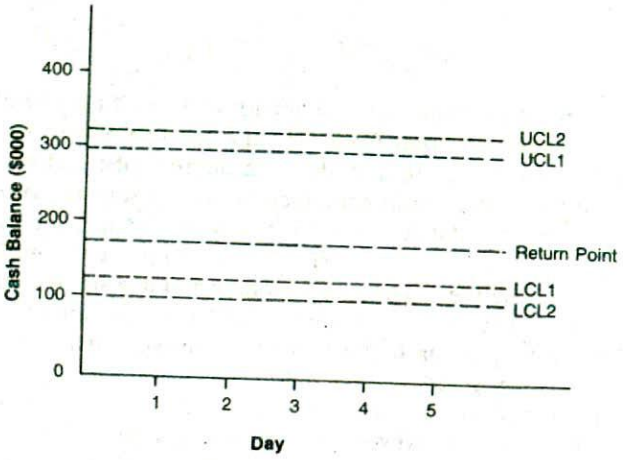


Figure 4-6. Control Limits for the Stone Model.

next few days to the current balance. If the sum of the current balance and these expected future cash flows (which is the expected cash balance a few days hence) falls outside the *inner* control limits, a transaction is made; otherwise, the transaction is foregone. The transactions are the same as those in the Miller-Orr model. Investments are made sufficient to bring the cash balance back to the return point if the upper control limit is exceeded; corresponding disinvestments are made if the lower control limit is exceeded. Optimal procedures for setting the return point, the two sets of control limits, and the number of days a firm looks ahead are not specified; the outer control limits could be set by the Miller-Orr model or could be based on the cash manager's feeling for the best limits.

To see how the Stone model would function in practice, let us consider the first example used in the discussion of the Miller-Orr model. In this discussion, the beginning balance was $150,000, the upper control limit was $325,000, the return point was $175,000, the lower control limit was $100,000. The cash flows for the first five days were:

Day	*Net Cash Flow*
1	−$25,000
2	−$75,000
3	$100,000
4	−$25,000
5	$125,000

Let us also assume that the inner control limits are set $20,000 inside the outer control limits (at $305,000 and $120,000), and that the firm looks ahead at the next two days' cash flows. At the end of day 1, the cash balance is

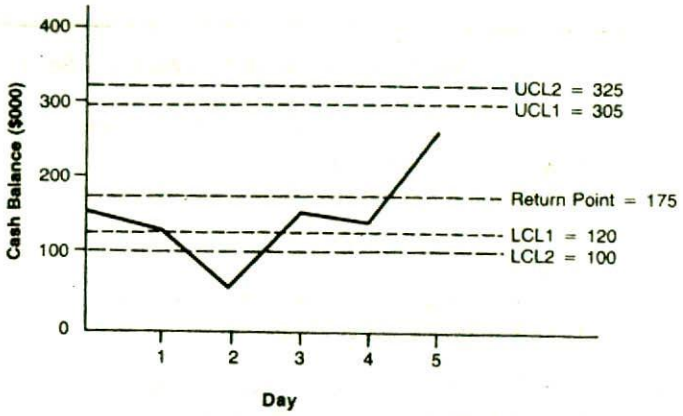


Figure 4-7. Cash Balances for the Stone Example.

$125,000, but since the outer control limits have not been breached, no evaluation is made. At the end of day 2, however, the cash balance has been reduced to $50,000. At this point, the firm totals the next two days' cash flows. Let us assume that this forecast is correct; the total obtained is $75,000 (100,000 − 25,000) as the expected future cash flow. Adding this to the current balance of $50,000 gives an expected balance of $125,000. *Since this expected cash balance is within the inner control limits, no transaction is made.* The pattern of cash balances that would result is illustrated in Figure 4-7; there are no investments or disinvestments over the five-day period of the example (recall the Miller-Orr model required one investment and one disinvestment).

When the firm has some knowledge of future daily cash flows, the Stone model offers an advantage over the Miller-Orr model in reducing the number of investment and disinvestment transactions, though investment income may also be reduced. Also, the Stone model is very flexible in that it does not assume a flat yield curve, and in that the parameters of the model may be changed over time to accommodate the firm's needs. For example, the control limits may be tightened during periods when cash control is critical, or the look-ahead period may be lengthened when the firm has better information about future cash flows. Set against these advantages is a substantial disadvantage: *the model does not specify the optimal levels of the parameters* (the control limits, the return point, and the look-ahead period). Stone states that these parameters should be set by the manager by trying various strategies on past data and observing the results; this is a sensitivity analysis approach. There is no guarantee that this process will lead to optimal (or even near-optimal) strategies. Indeed, though it is flexible and has substantial intuitive appeal, the Stone method might be more properly regarded as an *approach* to the daily management of cash and temporary investments rather than an *optimization model* for this management.

Table 4-4. *Time Pattern of Cash Flow Assumed in Four Models of Short-Term Investment.*

Model	*Assumptions Regarding Cash Flow Pattern*
Baumol	1. All cash flows are certain. 2. Cash inflows are periodic and instantaneous. 3. Cash outflows occur at a constant rate.
Beranek	1. All cash flows are certain. 2. Cash outflows are periodic and instantaneous. 3. Cash inflows occur at a constant rate.
Miller-Orr	1. Firm has minimum required cash balance. 2. Cash flows are normally distributed. 3. The expected cash flow is zero. 4. There is no autocorrelation in cash flows. 5. The standard deviation of cash flows does not change over time.
Stone	1. Firm has minimum required cash balance. 2. Firm has some knowledge of future cash flows, although this knowledge contains an error component.

OPTIMIZATION MODELS FOR SHORT-TERM INVESTMENTS IN PERSPECTIVE

In the past sections, we have discussed four models that generate strategies for the short-term investment of funds.[14] All of these models are oriented toward small- and medium-sized firms where the amounts of cash available for investment are sufficiently small that transaction costs are an important consideration in formulating strategy. All of these models take a similar approach: they first hypothesize a time-pattern of cash flows, then develop a strategy appropriate for that time pattern. The Miller-Orr and Stone models explicitly address the risk in the net cash flows, while the Baumol and Beranek models assume certainty. In the latter cases, risk must be addressed outside the methods by using the hedging strategies for cash flow uncertainty outlined in Chapter 3.

In deciding whether to use these models, the firm must examine the yield on its investments versus the transactions costs of making such invest-

[14]Numerous models besides those discussed or referenced in this chapter have also been developed to address other cash flow patterns. Many of these models are discussed in G. Gregory, "Cash Flow Models: A Review," *Omega*, Vol. 4, No. 6 (1976), pp. 643–56.

ments. If transaction costs are large relative to investment income and the firm has foregone investments because of such transaction costs, these models should be considered. To choose among these models, the first step is to match the time pattern of the firm's cash flows to the time patterns assumed in the models; Table 4-4 relates the four models to their assumed time patterns. This match should be based on the specifics of the firm's business policies that affect cash flows (how often payments are made to suppliers, the firm's terms of sale, and so forth) and on empirical investigations of the time patterns of cash flow. The latter investigation might include time plots of net cash flow and statistical comparisons of net cash flows with known probability distributions.

THE MANAGEMENT OF CASH AND TEMPORARY INVESTMENTS IN PRACTICE

The first purpose of this chapter was to review several optimization models for the short-term investment decision. The second purpose is to present survey evidence of the practices actually used by the cash managers of firms. Unlike some other areas of working capital management, there is a good deal of available survey evidence describing current practice in the management of cash and marketable securities. Such evidence is helpful because it gives the student useful knowledge in two areas. First, it tells the student which material presented in the past three chapters is likely to be used in the firms that may employ them. Second, it tells which areas of current practice are candidates for improvement by the adoption of new and better analysis methods. For ease of discussion, the survey information will be presented according to the major topics covered in the past three chapters. We will refer to the survey articles in this area by the initials of their authors. For students concerned about the samples used and the dates of the surveys, or who wish to read the original articles, details of the surveys are presented in Table 4-5.

Management of Cash Inflows In general, firms perceive that the acceleration of the receipt of cash is more important than the control of disbursements (GMW, p. 34). To achieve this acceleration, the use of lockboxes is widespread (GMW, pp. 35–36; SS, pp. 54, 65). Virtually all large firms use lockbox systems as do a large percentage of smaller firms (GMW, p. 36). This somewhat lower use by smaller firms is a reflection of the costs versus the gains from lockbox systems; for such systems to be economical, the gain for the acceleration of receipts must be sufficient to offset the costs of the system, and large firms have higher average receipts. To bring collected funds together for use, over one-half of all large firms use concentration banking (GMW, p. 36), with wire transfers and depository transfer checks being the

Table 4-5. *Facts Regarding the Survey Articles on the Management of Cash and Short-Term Investments Used.*

Abbreviation	Authors	Article Title	Reference	Date of Survey	Sample(s) Used	Response Rate
BG	S. Block and T. Gallagher	"The Use of Interest Rate Futures and Options by Corporate Financial Managers"	*Financial Management* (Autumn 1986), pp. 73–78.	1985	Fortune 500	38.6%
GG	L. Gitman and M. Goodwin	"An Assessment of Marketable Securities Management Practices"	*Journal of Financial Research* Vol. II, No. 2 (Fall 1979), pp. 161–69.	1977	300 firms ranging from number 201 to number 631 of Fortune 1,000	50.0%
GMW	L. Gitman, E. Moses, and I. White	"An Assessment of Corporate Cash Management Practices"	*Financial Management* (Spring 1979), pp. 32–41.	1977*	Largest 150 and smallest 150 of Fortune 1,000	32.6%
KKMW	R. Kamath, S. Khaksari, H. Meier, and J. Winklepleck	"Management of Excess Cash: Practices and Developments"	*Financial Management* (Autumn 1985), pp. 70–77.	1984**	Fortune's Second 500	41.5%
RGG	A. Rappaport, P. Goulet, and L. Goulet	"Marketable Securities Portfolio Management: A Survey of Theory and Practice"	Eastern Finance Association, 1985; available from the authors at the University of Northern Iowa.	1984	All publicly traded firms with marketable securities of over $5 million in 1982 (373 firms)	26.5%
SS	K. Smith and S. Sell	"Working Capital Management in Practice"	*Readings in the Management of Working Capital*, 2d. Ed. (N.Y.: West Pub., 1980), pp. 51–84.	1978	668 of Fortune's 1,000, with selection based on size and profitability	31.4%

*Estimated; date not given.

primary means of moving funds from one bank to another (GMW, p. 35; SS, p. 65).

Temporary Investments Almost all large firms (96 percent) invest surplus cash in money market instruments (KKMW, p. 71). To select the instruments for investment, the criteria used (in descending order of importance) are default risk, liquidity, yield, time to maturity, and price risk (KKMW, p. 74; GG, p. 165; SS, p. 66). Firms typically use three of four types of investment instruments over the course of a year (KKMW, pp. 71–2), but at any particular time they have over one-half of their total portfolio in a single type of security (RGG, p. 6). These investments are almost always held to maturity (KKMW, p. 76; GG, p. 164). If differences in the time to maturity of the different instruments are ignored, the most popular investments (in descending frequency of use) are: commercial paper, certificates of deposit, repurchase agreements, treasury securities, and banker's acceptances (KKMW, p. 73, provides a summary of KKMW, GG, SS, and GMW). However, the most popular type of investment varies with the length of time that the funds are available for investment. Repurchase agreements and commercial paper are the most popular investments for maturities under 30 days, while certificates of deposit and commercial paper are most popular investments for maturities over 30 days (KKMW, p. 73). Eurodollar deposits have become more popular over time (KKMW, p. 72). This trend is based on the higher interest rates available on such deposits (RGG, p. 7), a phenomenon which will be discussed in Chapter 13 of this text.

Management of Cash Outflows The primary tools for the management of cash outflows are zero-balance accounts and centrally controlled disbursing. Central control of disbursements is the major tool for about 70 percent of large firms (SS, p. 65; GMW, p. 37). Zero-balance accounts are used by the vast majority of larger firms, although they are used less frequently by smaller firms (GMW, p. 35). There appears to be substantial disagreement among firms as to whether the further delay of payments to suppliers (via maximization of mail float, for example) is useful. Some firms report the use of this technique (GMW, p. 37), while others think it is a bad idea (SS, pp. 54, 65).

Cash Forecasting Almost all large firms prepare cash forecasts (GMW, p. 35; GRR, p. 11). Smaller firms are less likely to prepare such forecasts (GMW, p. 35). These cash forecasts are the primary method used to determine the amount and duration of temporary investments in marketable securities (KKMW, p. 76).

In hedging the risk that cash flows may be less than forecast, many firms use more than one technique. A substantial number of firms keep a stock of short-term investments for precautionary reasons (GMW, p. 37; RGG, pp. 3, 12). Many firms also borrow to address unanticipated cash needs, either

directly from banks or through the commercial paper market (GRR, pp. 12, 26).

Futures and Options for Hedging The number of firms using futures and options on financial instruments to hedge the risk of future borrowings and lendings or to protect against interest rate risk in temporary investments is not yet very large. About 20 percent of large firms use these contracts, and they use them almost exclusively for hedging rather than speculation (BG, pp. 75–6). Bigger firms are much more likely to use the contracts than are smaller firms (BG, p. 75). Treasury bill futures and Eurodollar futures are the most frequently used contracts, with the most important uses being the hedging of expected future borrowings and lendings (BG, p. 76). The reason for the relatively low use of these hedging vehicles appears to be top managements' resistance to these contracts, caused by a lack of knowledge about their characteristics, advantages, and costs (BG, pp. 77–8).

Models for Temporary Investments While, in general, quantitative and statistical models are in wide use in working capital management (GMW, p. 36), the models discussed in this chapter (which balance the transactions costs of investment against interest revenues) have not found wide acceptance. They are in use by less than 10 percent of large firms (SS, p. 68; GRR, p. 26). Further, smaller firms do not use them at all (SS, p. 68). This is perhaps due to an interaction between the need for models of these sorts and the financial sophistication of firms. These models are primarily oriented toward small- and medium-sized firms, where the amounts of investment are such that transaction costs are an important offset to interest income. Yet smaller firms tend to be less financially sophisticated, as much research on the adoption of other advanced financial techniques has shown.[15]

SUMMARY

This chapter addresses two topics. The majority of the chapter is concerned with cost-balancing models for decisions regarding the temporary investment of cash into interest-bearing investments. These models balance the interest revenues from these investments against the fixed cost of making investments and disinvestments. Each of the four models assumes a different time pattern of cash flow, then derives a strategy based on this time pattern. In choosing among these models, the firm must first compare its time pattern of cash flows against those assumed by the various models. The Baumol and

[15]See, for example, J. W. Petty and O. D. Bowlin, "The Financial Manager and Quantitative Decision Models," *Financial Management* (Winter 1976), pp. 32–41.

Beranek models assume certainty, and thus require a hedging strategy outside the context of the model, while the Miller-Orr and Stone models explicitly account for the uncertainty of cash flows within their formulation.

The second part of the chapter reviewed several surveys in the current practice of the management of cash and marketable securities. In general, the management techniques discussed in the text concerning the management of cash inflows, temporary investment of surplus funds, cash budgeting, and hedging cash uncertainties are in use by a substantial portion of larger firms. For the management of cash outflows, some of the techniques we described are frequently used, but some firms also use techniques (such as increasing mail float on outgoing checks) that other firms believe are inappropriate, and which we have not addressed in depth. We have also devoted considerable space in this text to two techniques not in wide use: hedging with interest rate futures and cost-balancing models. The most probable reason for the relatively infrequent use of these techniques is management's lack of familiarity with them. These are areas where employees might make significant contributions to the current practices of their firms, if the circumstances are appropriate for the use of these methodologies.

Problems

4-1. A firm has periodic cash inflows and steady cash outflows and wants to use the Baumol model to formulate a strategy for the temporary investment of some of its cash. The firm receives an inflow of cash every 15 days (use a 360-day year). The yield on invested cash is 12 percent per year. It costs $500 to invest or disinvest, and the portion of the next cash inflow that will be retained to pay bills over the following 15 days is $3.2 million. Calculate:

a. The optimal number of transactions.

b. The amount of the initial investment.

c. The amount of the periodic withdrawals.

d. The net profit from this strategy.

4-2. A firm has steady inflows and periodic outflows and wants to use the Beranek model to manage its cash. Cash outflows occur once per month; the amount of the next outflow is $2.16 million. It costs $200 to make an investment or a disinvestment. The yearly interest rate is 8 percent. Calculate:

a. The optimal number of transactions.

b. The amount of the periodic investments.

c. The amount of the final withdrawal.

d. The net profit from this strategy.

4-3. A firm is trying to decide whether its cash flows fit the assumptions of the Miller-Orr model. A sample of 15 cash flows has been obtained:

Day	*Net Cash Flow*	*Day*	*Net Cash Flow*
1	\$ 0	9	\$500
2	\$100	10	−\$600
3	−\$200	11	\$200
4	−\$300	12	−\$200
5	\$100	13	\$100
6	\$ 0	14	−\$100
7	\$200	15	−\$200
8	\$400		

a. Calculate the standard deviation of these cash flows assuming a zero mean.

b. Using a chi-square test, assess whether the data are normally distributed under that assumption of a zero mean. Utilize five ranges: less than −250, −250 to −75, −75 to +75, +75 to +250, and greater than +250.

c. Using a Student's t-test, assess whether the data are serially correlated.

4-4. A firm has confirmed that its daily cash flows are in accord with the assumptions of the Miller-Orr model. Based on historic data, it has been determined that the standard deviation of daily cash flows is \$400,000 with a mean of zero. Interest rates on short-term investments are 6 percent per year (use a 360-day year). Each investment or disinvestment costs the firm \$100 in paperwork costs, etc. The firm's bank requires a minimum cash balance of \$500,000; this is the firm's lower control limit.

a. Calculate the return point and the upper control limit.

b. Assume that the firm's initial cash balance is \$1,000,000 and that it experiences the following cash flows over the first seven days:

Day	*Net Cash Flow*
1	−\$300,000
2	−\$400,000
3	\$500,000
4	−\$200,000
5	\$900,000
6	\$200,000
7	\$700,000

Using the control limits and return points calculated in part a, indicate the transactions that would occur for this series of cash flows. Give the amounts of any puchases or sales of short-term investments.

4-5. A firm wishes to use the Stone look-ahead model with a one-day look-ahead. It has determined that the outside upper control limit is $1,600,000, the inside upper control limit is $1,300,000, the return point is $1,000,000, the inside lower control limit is $750,000, and the outside lower control limit is $500,000. Assuming that the starting cash balance is $1,000,000 and using the seven days of cash flows from Problem 4, find the transactions that would occur during the first six days.

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case

The Polyurethane Company[1]—Cash and Marketable Securities: The Miller-Orr Model

The Polyurethane Company was the largest division of a middle-sized chemical company and was a primary manufacturer of the chemicals used in the production of polyurethane foam. Polyurethane foam is made by mixing chemicals together under controlled conditions. The chemical reaction is exothermic (that is, it gives off heat). The exact chemicals used determine the consistency of the foam produced: hard or soft, with large or small bubbles. Hard, large-bubbled urethane foams were often used as insulating materials; softer, small-bubbled foams were used in shoe soles. However, the most common use of urethane foam was in cushions and mattresses. To produce mattresses, 80/20 toluene diisocyanate, the largest-selling urethane chemical (80/20 indicates the mix of monomers), was mixed with another chemical and extruded into long, rectangular "buns." These buns were then cut into mattress sizes by means of a hot wire system. The foam produced for this application was large-bubbled and soft. In the early 1980s the Polyurethane Company sold about $200 million in 80/20 toluene diisocyanate and similar chemicals annually.

In middle-sized firms such as the Polyurethane Company, managers often "wear many hats": they do several jobs which in a larger firm would be handled by specialists. Thomas Charles, who held an undergraduate degree and an M.B.A. from a large land-grant university, was the company's assistant credit manager, accounts receivable manager, cash reporter, and cash forecaster. Also (but less frequently), he assisted the accounting department in computing break-even points, performed accounting functions, and helped the firm's planning department with statistical analysis. When the firm's finance department moved to a new office building, Mr. Charles was heard to remark (somewhat jovially), "I hope my new office isn't near the boiler; they'll have me shoveling coal, too!"

Mr. Charles's duties as cash forecaster of the company had become increasingly important (a sample of one of his cash forecasts is presented as Exhibit 1). The firm had recently undertaken a program of active management of its cash and near-cash assets. Because the Polyurethane Company was the firm's largest division, Mr. Charles's forecasts were

[1]This case has been adapted from Frederick C. Scherr, *Cases in Finance* (New York: Macmillan, 1984), pp. 51–55.

Exhibit 1

The Polyurethane Company
Forecast of Daily Cash Flows For the Business Week of 1/11/82 to 1/15/82
(rounded thousands of dollars)

	Monday 1/11/82	*Tuesday 1/12/82*	*Wednesday 1/13/82*	*Thursday 1/14/82*	*Friday 1/15/82*
Starting cash balance	200	827	387	(53)	174
Receipts	1667	600	600	1267	1100
Total funds	1867	1427	987	1214	1274
Disbursements	1040	1040	1040	1040	1040
Cash balance, end of day	827	387	(53)	174	234
Desired cash balance	140	140	140	140	140
Surplus or (deficit)	687	247	(193)	34	94

critical to the process of planning short-term investments and financings. However, Mr. Charles was not happy with the accuracy of his forecasts of daily cash flow. If the forecasts proved to be substantially in error, the cash manager had to alter investment or borrowing plans. This was inconvenient, and the resulting transactions costs were expensive.

Mr. Charles found the inaccuracy of the daily forecasts to be quite perplexing. While the company's sales were somewhat seasonal, by using appropriate estimates of the firm's collection rates, he could make fairly accurate estimates of monthly cash flows. The difficulty was in forecasting daily flows within the monthly periods. Mr. Charles had been distributing the monthly forecasts to daily forecasts based on his feel for the patterns of cash flow within the month. His initial efforts to improve his daily forecasts centered on the statistical estimation of week-of-the-month and day-of-the-week effects. It was his plan to use these estimates in his cash flow distribution system.

He first examined the firm's cash flow patterns near the tenth and twenty-fifth of the month. He knew that, in the past, some customers had tended to concentrate their payments around these dates. However, he found no statistically significant differences between the cash flow patterns around these dates and other times during the month. He then tested cash flow patterns for days early in the week versus those later in the week. He had suspected that there would be heavier cleared cash flows at some point during the week since the firm's bank credited checks received in the mail on Saturday and Monday to the company's account on Monday (on book but not cleared basis). Again, he found no statistically significant differences in cleared balances. Finally, he ran regressions looking for daily and weekly forecasting tools, but the explanatory power of these regressions was minimal.

In desperation, he talked the problem over with his friend and golfing partner, David Feldstein. "Tom," Mr. Feldstein had said, "has it occurred to you that there may be no time

Exhibit 2

The Polyurethane Company Changes in Cleared Bank Balances (Cleared Daily Net Cash Flows) Month of January 1982 (Business Days Only; Rounded Thousands of Dollars)

Date	*Day No.*	*Change in Cleared Balances*
4	1	−445
5	2	−137
6	3	376
7	4	−34
8	5	445
11	6	−479
12	7	274
13	8	−96
14	9	294
15	10	501
18	11	239
19	12	34
20	13	171
21	14	308
22	15	−342
25	16	−205
26	17	−103
27	18	−117
28	19	616
29	20	−137

pattern in these net cash flows at all? If there is not really a time pattern, all this churning of cash and investments on the basis of your daily forecasts is counterproductive. Maybe what we need is a new approach to the daily cash management problem."

Mr. Charles considered this for a time and decided to try a new approach. Instead of looking for a pattern, he assumed that the net cash flows fluctuated randomly around their mean. He knew that if, in such a cash, the cash flows of the firm met certain statistical requirements, the Miller-Orr model of cash and marketable securities management and not the firm's current procedure, was the appropriate strategy. He decided to test the daily cash flow data within the months to assess whether the pattern of cash receipts met these requirements. To perform the necessary tests, he collected daily net cash flow data for several months; an example of the data for the 20 working days in January 1982 is presented in Exhibit 2. For this month, total cash inflows were expected to exceed total cash outflows by $800,000, so Mr. Charles decided to deduct $40,000 per day from these figures under the assumption that this amount would be removed daily from the firm's temporary pool of funds and invested in a more permanent fashion. Once the results of these tests were in hand, he intended to present them to the treasurer of the firm with a recommendation regarding the most advantageous strategy for the management of daily cash flows.

Chapter -5

Terms of Sale Decisions

This is the first of three chapters concerning the *management of accounts receivable*. Accounts receivable management *starts* where the management of inventory *ends* (with the shipment of goods or the performance of services by the seller) and *ends* where the management of cash *begins* (when the remittance in payment for the goods or services is on its way to the seller from the buyer). Accounts receivable management is also known as *credit management*. Credit management is primarily concerned with two questions:

- ☐ What terms of sale should the firm use?
- ☐ To whom should the firm grant credit?[1]

This chapter, Chapter 5, has two purposes. First, it provides an introduction to the management of accounts receivable. Second, it discusses decision methodologies with respect to the first of the above questions: what terms of sale should the firm use? Chapter 6 addresses the second of these questions: to which applicants should credit be granted? Included in Chapter 6 is a substantial discussion of the effects of the costs of information on such credit-granting decisions. Chapter 7, the final chapter concerned with the management of accounts receivable, discusses the monitoring of accounts receivable. Monitoring receivables is a mechanism for assessing the accuracy and stability of the estimates used in making the terms of sale and credit-

[1]Another question sometimes discussed in this area is *collection policy*—how much should the firm spend on collecting from buyers and what mechanisms should be used? While the collection policy problem is not trivial, the amounts of money involved in such policy decisions are usually much smaller than those involved in the other credit policy decisions. Also, the principles used in the analysis of collection policy decisions are the same as those used in the analysis of terms of sale policy decisions. In the interests of space, we do not discuss collection policy in this text.

granting decisions. Chapter 7 also contains a review of survey evidence regarding the methods currently in use by credit managers.

WHY GRANT CREDIT?

If you ask a practicing businessperson *why* his firm grants credit to customers for the purchases of goods and services, the businessperson is likely to be rather perplexed. The use of credit in the purchase of goods or services is so common that it is taken for granted. The granting of credit from one business to another for the purchase of goods or services—*trade credit*—has been part of the American business scene for hundreds of years. Trade credit provided the major means of obtaining debt financing by businesses before the existence of banks.[2] Over 200 years ago, Benjamin Franklin was a founding member of a trade credit organization. Trade credit continues to be a very important source of funds for firms; it provides more financing to businesses than does commercial borrowing or corporate bond financing.[3] Further, the accounts receivable assets that result from granting trade credit are a major investment for the firm. As of 1981, accounts receivable assets averaged 26.5 percent of the total assets of industrial firms.[4]

But business procedures such as the granting of trade credit do not persist and do not provide major financing and investment merely because of historical tradition. They persist because they perform useful economic functions. Trade credit may perform at least three useful functions: it may provide the opportunity for *financial arbitrage*, it may help overcome an *information problem in the sale of goods*, and it may *make the payment for goods less difficult*.[5]

Let us first discuss the possibility of *financial arbitrage*. If firms operated in perfectly competitive capital markets, there would be no financial impetus for trade credit. The extension of time to pay in granting credit is equivalent to the granting of a loan from the seller to the buyer. The seller bears the cost of this loan. If the seller chose not to grant trade credit to buyers, in perfect capital markets, the buyer could obtain the same financing by taking out a loan in these markets and paying cash for the product or service. But

[2]See R. Barzman, *Credit in America* (New York: National Association of Credit Management, 1975).

[3]See M. H. Seiden, *The Quality of Trade Credit* (New York: National Bureau of Economic Research, 1964).

[4]See G. Emery, "A Pure Financial Explanation for Trade Credit," *Journal of Financial And Quantitative Analysis*, Vol. 19, No. 3 (September 1984), pp. 271–85.

[5]If customers are differentially allowed to pay beyond the stated terms of sale, credit policy can also be used to circumvent the equal-price restrictions of the Robinson-Patman Act. We do not discuss this potentially illegal strategy here; see G. J. Alexander and J. M. Gahlon, "Can Credit Terms Cause Stretching?" available from the authors at the University of Minnesota.

this loan would be costly to the buyer, and the buyer would then require a lower price from the seller. In perfect capital and product markets, the lower price that the buyer requires if the seller does not grant trade credit exactly offsets the costs to the seller if trade credit is granted. There is no advantage to the seller of granting or not granting credit, and therefore no financial reason for the existence of trade credit.[6]

However, capital markets may not be perfectly competitive. For one reason or another, buying firms may not be able to borrow at economical interest rates, and thus may not be able to economically replicate the delay in payment that is inherent in trade credit. When such imperfections occur, trade credit can serve as a conduit for funds from the capital markets to buying firms. Sellers can borrow in the capital markets, then lend to buyers via trade credit. This process of borrowing cheaply by sellers in order to relend to buyers substitutes for buyers' access to capital markets and is thus a type of financial arbitrage.

What imperfections exist that might make this financial arbitrage profitable? Several observed imperfections of capital markets might account for it, particularly if the buying firms are smaller than the selling firms. Various restrictions on the amount of borrowing occur for smaller firms. Since they are less well-known than bigger firms, they are less able to borrow in the capital markets. If they do borrow in these markets, their interest costs will be higher than for larger firms because of this information effect and because the fixed costs of issuing securities bear more heavily on their smaller issues. If they instead borrow from banks, usury laws and restrictions on banks' lending to particular customers limit the availability of funds. Finally, as a result of the selling (and possibly the product development) process, the seller may have acquired quite a bit of information about the buyer. For the buyer to borrow, this information would have to be transmitted to the bank (a costly process), while the seller already possesses it. All these factors work to give advantage to an arbitrage transaction where the seller borrows in the capital markets, then relends to buyers.[7]

Another possible function of trade credit has to do with the *buyer's imperfect knowledge regarding the quality of the products purchased*.[8] When payment is delayed until some time after the goods have been received, the

[6]See W. Lewellen, J. McConnell, and J. Scott, "Capital Market Influences on Trade Credit Policies," *Journal of Financial Research*, Vol. III, No. 2 (Fall 1980), pp. 105–13.

[7]For discussion of these effects, see Dana Johnson, "Market Justifications for Trade Credit in Imperfect Capital Markets," available from the author at Virginia Polytechnic Institute and State University. Also see Emery, "A Pure Financial Explanation," and R. Schwartz, "An Economic Model of Trade Credit," *Journal of Financial and Quantitative Analysis* (January 1974), pp. 643–57.

[8]For a review of the literature, see W. Beranek, "Towards a Positive Theory of Working Capital," available from the author at the University of Georgia, pp. 17–19.

buyer has the opportunity to count and inspect the goods. If the goods are not up to the required standards, the buyer may short-remit or not pay at all until the defect is fixed. If the buyer has paid cash for the goods, much of the leverage that the buyer has in obtaining satisfaction on such matters is lost.

Still a third potential function of trade credit concerns the payment process itself. *It is simply less costly, and to some extent, less risky for buyer and seller alike if payment on delivery can be avoided.* The payment of cash on delivery requires that the buyer and seller entrust large sums to employees dealing with the deliveries, opening the firms to the possibility of theft. Alternatively, the seller could obtain a certified check to be presented on delivery, but this is an inconvenient and expensive process, and the check may be lost by delivery personnel. From both the buyer's and seller's viewpoints, there are substantial advantages to the payment for goods or services via the normal check remittance process, even if the time from the receipt of the goods to payment is only a few days.

In addition to these three potential functions of trade credit, more complex explanations for its existence have also been put forward. One of these concerns the use of trade credit as a potentially effective means of dealing with temporary fluctuations in demand for the firm's products or services. In this explanation, terms of sale exist so that firms lengthen them to stimulate demand when it is less than expected and shorten them to curtail demand when it is more than expected.[9] Another explanation centers around the use of the cash discount amount, cash discount date, and net date to obtain information regarding the probability of the buyer's default. Buyers who do not take advantage of generous cash discounts may be more prone to default. Since the financing they obtain by delaying payment is very expensive, they must be foreclosed from cheaper sources because of their poor financial condition.[10]

By this discussion, we see that there are several possible functions of trade credit in the selling of goods from one firm to another. It is not presently known which of these explanations is correct; they may all be true to some extent. But whatever its reasons for existence, trade credit is pervasive. It has been employed for centuries and will likely persist for some time to come.

This section has deliberately discussed credit in the context of *one firm granting credit to another* rather than *a firm granting credit to an individual.* This orientation toward interfirm credit rather than consumer credit will continue throughout this text. This orientation is an outgrowth of this text's direction toward the management of working capital *within business firms* and recent trends in the granting of credit. While the granting of credit from

[9]See G. Emery, "An Optimal Financial Response to Variable Demand," *Journal of Financial and Quantitative Analysis*, Vol. 22, No. 2 (June 1987), pp. 209–25.

[10]See J. K. Smith, "Trade Credit and Information Asymmetry," *Journal of Finance* (September 1987), pp. 863–72.

one firm to another shows no sign of abating, the granting of credit *by firms directly to individuals* is rapidly fading from the economic scene. It is fading because there are substantial economies of scale in credit investigation and administration when granting credit to individuals. This has led to a substantial growth of *financial institutions* such as credit-card companies whose business it is to grant credit to *individuals* for the purchase of goods *from business firms*. By aggregating the sales of many businesses, these institutions achieve the economies of scale in investigation and administration necessary to reduce the costs of consumer credit-granting.[11] These institutions *enable individuals to buy on credit without the seller having to extend credit.* While the study of such institutions is an important part of finance, it is not a topic for this text. Since this text is concerned with the management of business firms rather than financial institutions, we deal with firms granting credit to other firms, rather than with the granting of credit to individuals.

COSTS, REVENUES, AND CREDIT DECISIONS

It should be obvious to most students that *the granting of trade credit is an aspect of price*. The time that the buyer gets before payment is due is one of the dimensions of the product (like quality, service, and so forth) which determine the attractiveness of the product. Like other aspects of price, the firm's terms of sale and credit-granting decisions affect its sales volume. All other things being equal, longer terms of sale and more liberal credit-granting policies increase sales, while shorter terms and more stringent credit-granting policies decrease sales. These policies also affect the level and timing of certain costs. Evaluation of policy changes must *compare these sales and cost effects*. In this section, we list and discuss the various types of costs and revenues that may be affected by the firm's accounts receivable management decisions.

While the discussion of these decisions' effects is usually phrased in terms of costs and revenues, we should recognize that what is really being discussed is the *timing and amount of the firm's cash flows*. Cash flows are critical to firms, and the different types of costs and revenues are just labels for the various cash flows that result from credit management policies. The analysis of terms of sale policy to be presented later in this chapter centers on the timing and amount of these cash flows.

Collections on Sales Whenever the firm institutes a new credit policy, it is anticipated that changes in the firm's sales volume and in the timing of

[11]However, there are a few major firms that still run their own consumer credit operations. Some retailers continue to issue their own credit cards, as do many large oil companies. In such circumstances, sellers must be aware of the morass of constantly changing state and federal legislation dealing with the granting of credit to individuals and the collection of debts from them.

receipts from sales will result. For example, suppose that the firm lengthens its terms of sale. Since this is in effect a lowering of price, sales volume would be expected to increase.[12] However, since the terms of sale are longer, customers would be expected to pay later, and the cash inflows from these sales would occur further into the future.

Investment in Inventory Many credit policy decisions result in an increase or decrease in sales volume. This change in sales volume in turn requires an increase or decrease in inventory.[13] For example, suppose that the firm makes a change in credit policy that reduces sales by $200,000, and that the firm follows a policy of keeping inventory so as to produce an inventory turnover ratio of 5.0 based on sales volume (that is, sales divided by inventory is 5.0). Because of this change in policy, the firm can then liquidate $40,000 in inventory ($200,000/5.0) and retain the same turnover ratio on the remaining inventory balance. This liquidated inventory can be used to make sales that would normally require the firm to spend $40,000 in additional costs of materials. The $40,000 represents a net cash inflow (a saved cash outflow) for the firm.

Cost of Sales When sales change, the firm will have to produce different amounts of goods. This in turn will affect several types of cash outflows, such as direct labor, direct materials, and so forth. The change in the amounts of these cash outflows is relevant to credit policy decisions.

Discount and Bad Debt Expenses These expenses are really deductions from the expected level of cash inflows from future sales. For example, suppose that the firm changes credit policy in such a way that sales increase from $1 million to $1.3 million in a particular period. Assume that the levels of discount and bad debt expense increase from 2 percent and 0.5 percent of sales to 3 percent and 1 percent of sales respectively. Under the old policy, the collections on sales from that period would have been $975,000 ($1 million − (0.02 + 0.005) ($1 million)), but under the new policy collections will be $1,248,000 ($1.3 million − (0.03 + 0.01) ($1.3 million)). The differences in bad debt expense and in discount expense are thus reflected in the changes in future cash flows.

[12]The amount of this increase depends in part on market structure facing the firm and the reactions of competitors to the change in terms. See F. Scherr, "Credit, Oligopoly, and the Prisoners' Dilemma," *Credit and Financial Management*, Vol. 80, No. 1 (January 1978), pp. 14–15, 41, and J. M. Collins, "The Multiperiod Approach to Credit Policy Decision," available from the author at the University of Tulsa, pp. 13–14.

[13]We will discuss inventory policy in later chapters. For now, we assume that the firm's policy relating inventory and sales has already been determined.

Collection Costs The administration of accounts receivable is costly. To collect the monies owed, the credit departments of firms generally employ telephone calls, letters, visits to customers, and other mechanisms. These entail postage, telephone charges, travel expenses, lost management time and effort, and other out-of-pocket and opportunity costs. While these costs usually rise with sales volume, their amounts vary among types of customers; risky customers with financial problems require disproportionate collection costs. However, the out-of-pocket costs of collection are almost always very small relative to the other cash flows that occur in credit policy decision making (such as sales, cost of sales, changes in inventory levels, and so forth). Thus, their effects are frequently ignored in the interests of simplicity when analyzing terms of sale and credit-granting policies.

Timing of Capital Expenditures When a credit policy change affecting sales volume is made, this change may also affect the timing and amount of future capital expenditures. For example, suppose that a firm expects sales of 100,000 units for the upcoming year and that sales are expected to grow at 10 percent per year. Assume also that the life of the product being sold is six years and that the production capacity of the firm's plant is 130,000 units. The firm is trying to decide whether to implement a credit policy change that would increase next year's sales to 115,000 units but result in the same rate of future sales growth and the same product life. The projected future sales volumes for the present and proposed policies are presented in Table 5-1. If no change in credit policy was made, the firm would reach capacity near the end of the fourth year. At that time, if they wanted to continue to service demand through the life of the product, the firm would have to expand the plant to accommodate an eventual demand of 161,051 units (the sales volume for year six under the present policy). But if the change in credit policy was made, the plant would reach capacity between the second and third years, and the final required capacity would be 185,209 units; a larger capacity expansion would be required. Changes such as these in the timing and amount of the capital expenditures for plant expansion may be caused by alterations in the terms of sale, and any such changes need to be considered in conjunction with terms of sale policy.

Income Tax Effects Changes in credit policy may also affect the firm's tax bill. The most obvious effect involves the change in the firm's depreciation

Table 5-1. *Future Sales in Units for Change in Credit Policy.*

Year	*1*	*2*	*3*	*4*	*5*	*6*
Sales–Old Policy	100,000	110,000	121,000	133,100	146,410	161,051
Sales–New Policy	115,000	126,500	139,150	153,065	168,372	185,209

schedule that results from alterations in the timing and amount of the firm's capital expenditures. However, changes in sales and in cost of sales will also have an impact on the firm's required income tax payments by changing other portions of its pretax income.

Salvage and Recovery Values Since changes in credit policy affect the firm's investments in accounts receivable, inventory, and capital assets, they will naturally affect the recoveries on these assets at the end of the life of the product.

All these changes in costs and revenues affect the cash flows of the firm, and therefore the firm's value to shareholders. To properly evaluate credit policy, a methodology is needed to reflect the differences in timing and amount of these cash flows. In the next section, we present the standard methodology for evaluating terms of sale decisions. Variations on this methodology are presented in many textbooks. Unfortunately, this simple methodology does not sufficiently capture many of the changes in cash flows that result from many terms of sale decisions. The presentation of this textbook methodology is followed by a technique, based on spreadsheet analysis, that adequately addresses the cash flows from these terms of sale decisions.[14]

TERMS OF SALE DECISIONS: STANDARD APPROACH

Before getting into the details of technique, a general discussion of terms of sale decisions is appropriate. First, under U.S. antitrust statutes, *terms of sale must, in almost all cases, be the same for all the firm's customers*, although the selling firm may require some specific buyers to pay cash because of the high costs of granting credit to these buyers.[15] Second, terms of sale decisions involve the setting of the three parameters: the cash discount (the amount of discount allowed for payment within a specified period of time), the period of time this discount is to be allowed, and the net date (the due date of the invoice if the cash discount is not taken). For example, the setting of terms of 2 percent 10 days, net 30 days, requires three decisions (the three numbers in these terms). Finally, for any firm selling any product in any competitive situation, there is almost certainly a set of *optimum terms of sale*, that is, a set of terms of sale for a product or service that results in the highest possible

[14]The author is grateful to Tirlochan S. Walia, who is coauthor of the remainder of this chapter.

[15]There are two exceptions to this rule. The selling firm may charge different terms of sale to different customers if the costs of selling to these customers are different (the "cost justification defense") or if other sellers are selling to the customers on terms that are of more advantage to buyers (the "meeting competition" defense).

net present value to the selling firm from that product or service. However, the firm often has little insight into what that set of optimum parameters may be. Consequently, it is usually recommended that the firm analyze the effects of *changes* in its terms; if the proposed change increases the value of the firm, then the change is a move toward the optimum and should be implemented.

To illustrate the traditional methodology for evaluation of the effects of terms of sale changes, let us address a simple example problem. A firm is considering changing its terms of sale from net 30 days to net 60 days (the firm does not offer a cash discount). The change in terms will not affect collection expense or bad debt expense. All the firm's customers pay promptly. The firm's current sales are $50 million per year, and the marketing department estimates that such a change would cause sales to immediately increase to $53 million. The out-of-pocket costs of materials are 80 percent of sales and the required rate of return on accounts receivable is 12 percent. Should the firm make the change in its terms?

To address this question, the traditional terms of sale analysis method compares the one-period, before-tax effects on "profit" of the present and proposed terms of sale. Changes in sales, costs of sales, discount expense, bad debt expense, collection expense, and the opportunity cost of changes in the investment in accounts receivable are netted to determine the advantage of the change in terms. In the example problem, there are no changes in discount expense, bad debt expense, or collection expense. The traditional analysis is presented in Table 5-2. Projected accounts receivable are calculated as the expected time for payment divided by 360 times sales, and opportunity cost for this asset is calculated as the receivables balance times a required return.[16] Since there is an increase in "profit" for this example problem, the change in terms of sale is recommended.

While this analysis methodology is very simple, it turns out to have substantial problems in representing the situations faced by firms and in evaluating the net cash flow effects from terms of sale decisions. Several of these problems stem from a particular assumption made in this methodology regarding the time pattern of future cash flows. To see this assumption, let us look a little more closely at the algebra of this analysis. Let ΔS be the change in sales resulting from the change in policy and V/P to be the cost ratio. For purposes of simplicity, let this cost ratio capture all the costs of the decision (production costs, bad debt expense, and so forth) except for the opportunity cost of investment in receivables. The change in margin can be expressed as $\Delta S(1 - V/P)$; in our example, this is 3,000,000 (1 − 0.8), or

[16]Other methods can also be used to compute this opportunity cost. We do not discuss these methods here because we will later present a better approach to the problem that does not require the use of this opportunity cost methodology to compute the cost of investment in accounts receivable.

Table 5-2. *Traditional One-Period Analysis of a Terms of Sale Change (thousands).*

Analysis of Terms of Sale Change			
Item	*Old*	*New*	*Change*
Sales	$50,000	$53,000	$3,000
COGS	40,000	42,400	2,400
Margin	$10,000	$10,600	$600
Expected Time for Payment	30	60	
Accounts Receivable	$4,167	$8,833	$4,667
Required Return	0.12	0.12	0.12
Accounts Receivable Carrying Cost	$500	$1,060	$560
Profit	$9,500	$9,540	$40

$600,000. Letting ΔR be the change in accounts receivable, the increased carrying cost is $k\Delta R$, where k is the required return. Algebraically, this decision rule becomes:

$$\text{if } \Delta S(1 - V/P) - k\Delta R > 0, \text{ accept} \tag{5-1}$$

Or, rearranging the terms:

$$\text{if } (\Delta S(1 - V/P))/k - \Delta R > 0, \text{ accept} \tag{5-2}$$

Finance theory states that cash flows are the things of importance to firms, and that the net present value of future cash flows is the proper evaluation statistic for decisions. Under what conditions does the rule expressed in equation (5-2) agree with the net present-value rule as applied to the terms of sale decision? Except by luck, *the one-period rule will only give the net present value of the decision when the future cash flows from the decision are a perpetual* in the amount of $\Delta S(1 - V/P)$. If the future cash flows are a perpetual in this amount, then their net present value is $(\Delta S(1 - V/P))/k$, since $1/k$ is the discount rate for a perpetual starting at period 1. Netting this present value of the perpetual future cash flows against the initial cash outflow of ΔR yields the traditional decision rule.

Unfortunately, the sales of most firms are not perpetual nor do the managers of firms expect them to be. The sales of actual firms, both in terms of dollar sales and unit sales, *change over time*. Sometimes these changes are due to the market penetration of new products or the maturity of old ones;

sometimes they are due to external factors such as economic conditions or competition. With these changes in sales come changes in bad debt expense, costs of production, and so forth. Since the traditional decision methodology assumes a perpetual, *the effect of changes in sales and costs over time cannot be captured in the net present value implied within this rule*. For firms whose sales are expected to change over time, the rule therefore misrepresents the effects of the present and proposed terms of sale policies.

In addition to this, there are *other problems* within the traditional decision methodology. There is no provision made within this methodology for assessing the effects of changes in capital expenditures, salvage values, or taxes that occur because of changes in terms of sale. Also, if the rule is portrayed in the form of equation (5-1), there is a substantial question as to the proper specification of both the investment in accounts receivable and the opportunity cost of this investment.[17]

To be fair, the traditional one-period approach to the terms of sale decision can be viewed as an attempt to introduce beginning students to trade-offs required in analyzing terms of sales decisions. The effects of changes in all the different types of costs and revenues relevant to terms of sale decisions are not easy to address. By ignoring some of these costs and revenues and assuming a perpetual, it is possible to reduce the problem in size. But the cost of this simplicity is accuracy and realism. In the next section, we introduce a spreadsheet approach to the problem which, by sequentially considering the various types of cash flows inherent in the terms of sale decision, can give a reasonably accurate representation of the net present values in such decisions.

TERMS OF SALE DECISIONS: MULTIPERIOD APPROACH

The discounted net cash flow approach presented here is based on the net present-value procedures for evaluating terms of sale decisions outlined by Collins and by Sartoris and Hill.[18] To illustrate this spreadsheet application, we model the various types of cash flows for an example terms of sale decision problem. This problem is described in the following paragraph.

[17]The debate about the proper specification of investment in accounts receivable and the opportunity cost of these investments spawned a series of articles on the topic during the late 1970s and early 1980s. These articles are too numerous to reference here. See, for example, E. Dyl, "Another Look at the Evaluation of Investment in Accounts Receivable," and T. S. Walia, "Explicit and Implicit Cost of Changes in the Level of Accounts Receivable and the Credit Policy Decision of the Firm," both in *Financial Management* (Winter 1977), pp. 67–70 and 75–78 respectively.

[18]Collins, "The Multiperiod Approach" and W. Sartoris and N. Hill, "A Generalized Cash Flow Approach to Short-Term Financial Decisions," *Journal of Finance*, Vol. 38, No. 2 (May 1983), pp. 349–60.

An Example Terms of Sale Decision Problem Sweets, Incorporated, is the manufacturer of Joybeans, an "executive" jellybean. This year's sales of Joybeans were $1,904,762. The firm expects to sell $2,000,000 worth of Joybeans during the upcoming year with sales expected to grow at 5 percent per year. It is considering lengthening its terms of sale. This change in policy would result in a decreased accounts receivable turnover from eight times per year (45 days) to six times per year (60 days). Under the proposed terms, next year's sales are expected to increase to $2,200,000, although the growth rate of sales would be the same as that under the present policy for all years after this. Variable costs of production are 65 percent of gross sales, and the appropriate risk-adjusted discount rate for cash flows is 15 percent. The life of the product is five years. The production equipment currently in place will produce enough Joybeans for a maximum of $2,300,000 in sales. Under the present policy, the firm will have to purchase additional production capacity at the end of the fourth year at a cost of $150,000; this capacity will be scrapped at the end of the life of the product at a salvage value of $80,000. If the firm implements the proposed policy, the capital expenditure for additional capacity will be made at the end of the second year in the amount of $175,000 and will have a salvage value of $75,000 at the end of the product's life. The firm's policies are such that inventory turnover is 20 times per year, with this ratio based on gross sales. Under the present policy, bad debts total 2 percent of sales; under the proposed policy, this would increase to 2.5 percent of sales. Collection costs for the firm will not change if the new policy is adopted, and the firm does not offer a cash discount. The firm is in the 34 percent tax bracket. For simplicity, assume that the firm depreciates assets from their initial value to their expected salvage value by the straight-line method. At the end of the product's life, inventory will be liquidated and accounts receivable will be collected. It is anticipated that these will be recovered at book value. Should the firm make the proposed change in its terms of sale?

A Spreadsheet Solution This is a relatively complicated problem, but by modeling the various costs and revenues one at a time, the problem can be solved in a tractable and reasonably accurate fashion. We want to know the net present value of the cash flows from the product under two conditions—with the proposed change in terms of sale and without it. To do this, we start by modeling the cash flows that would occur if the policy is not changed. The required analysis for the present policy is presented in Table 5-3. It will pay the student to study this table in detail, as it will be reviewed on a line-by-line basis.

Much of the basic data is contained in cells B8 through B13. This data has been separated, rather than embedded in the contents of later cells, for two reasons: first, so that the analysis of the proposed policy can be carried out using this same spreadsheet by changing this basic data; and second, to facilitate sensitivity analysis for the assessment of risk in terms of sale decisions

(a procedure we will discuss later). Using this basic data, gross sales and net sales figures are computed in rows 15 and 16. Since there is no discount expense, net sales are gross sales minus expected bad debts. The cell entries here are straightforward; for example, cell C15 contains "B9", cell D15 contains "C15*(1+(B10/100))", and cell D16 contains "D15*(1−(B12/100))". These gross and net sales entries will be used later in the spreadsheet in calculating cash inflows and outflows.

Because sales in this problem are growing, so also will the accounts receivable balances; the amount of uncollected sales will increase over time. At present, ⅛ of last year's net sales are outstanding (assuming that the firm keeps a reserve for bad debt). The amount of $233,333 appears in cell B19, which contains "B16/8" since the turnover for last year was eight times per year. Over the next year, the firm will make $1,960,000 in net sales; $245,000 will remain outstanding at the end of the year (cell C19 contains "C16/B11" since next year's turnover rate is in cell B11). Thus, of the $1,960,000 in net sales, an additional $11,667 will be uncollected at year-end; *this is the amount of the increase in the firm's accounts receivable* ($245,000 − $233,333). The *net cash flow from the collection of sales* will then be only $1,948,333 because of the increase in receivables over the year. This amount is calculated in cell C20, which contains "C16−(C19−B19)"; the remaining cells in this row contain parallel entries. This row contains the timing and amount of the firm's cash inflow from sales, allowing for the growth of the firm and for increasing investment in accounts receivable.[19]

The sale of Joybeans requires that Sweets, Inc., keep inventory on hand. Like the accounts receivable balance, the required inventory balance will increase over time with increases in sales, as will the cost of materials and labor (variable cost of production). Cash outflows associated with the production of goods thus have two parts: (1) cash outflows to increase inventory levels, and (2) cash outflows to produce goods that are actually sold (the costs of sales actually made). These two parts are calculated in rows 22 through 26 of the spreadsheet. The required inventory balances are calculated in row 23 as 1/20 of the gross sales figures in line 15 (cell B23 contains "B15/20"). The required increase in these balances, equal to last year's balance minus this year's balance, appears in line 24 (cell C24 contains "C23−B23"). Costs of sales, at 65 percent of gross sales, are in line 25. Line 26 adds the cash outflows for additions to inventory to the cash outflows for goods actually sold to calculate the total cash outflows for goods produced in line 26.

[19]This is an approximate presentation of this timing and amount because the cash flows are grouped at the end of each year. This is the same grouping generally used in the evaluation of other multiperiod decisions, such as those related to the management of capital assets. A more accurate presentation could be achieved by using more columns (for example, one column for each month rather than for each year).

Table 5-3. Sweets, Inc. Example Problem—Analysis of Present Policy.

	A	B	C	D	E	F	G
3							
4	Year	0	1	2	3	4	5
5							
6	Present Policy:						
7							
8	Last Year's Gross Sales	1904762					
9	First Year's Gross Sales	2000000					
10	Growth Rate	5	percent				
11	Accounts Receivable Turnover	8	Times/Yr.				
12	Bad Debt Expense	2	percent of sales				
13	Discount Rate	15	percent				
14							
15	Gross Sales	1904762	2000000	2100000	2205000	2315250	2431013
16	Sales Net of Bad Debts	1866667	1960000	2058000	2160900	2268945	2382392
17							
18	Collections:						
19	Accounts Receivable Balance	233333	245000	257250	270113	283618	297799
20	Cash Receipts from Sales		1948333	2045750	2148038	2255439	2368211
21							
22	Inventory Investment:						
23	Inventory Balance	95238	100000	105000	110250	115763	121551
24	Increase in Inventory		4762	5000	5250	5513	5788
25	Cost of Goods Produced for Sale		1300000	1365000	1433250	1504913	1580158
26	Total Cost of Goods Produced		1304762	1370000	1438500	1510425	1585946
27							

28 **Capital Expenditures**				150000	
29 **Salvage Value—New Equipment**					80000
30					
31 **Income Tax Calculations:**					
32 Sales Net of Bad Debts	1960000	2058000	2160900	2268945	2382392
33 Cost of Goods Produced for Sale	1300000	1365000	1433250	1504913	1580158
34 Depreciation					70000
35					
36 EBT	660000	693000	727650	764033	732234
37 Taxes at 34 percent	224400	235620	247401	259771	248960
38					
39					
40 Net Cash Flow Table:					
41 Cash Receipts from Sales	1948333	2045750	2148038	2255439	2368211
42 Total Cost of Goods Produced	−1304762	−1370000	−1438500	−1510425	−1585946
43 Capital Expenditures				−150000	
44 Salvage Value—New Equipment					80000
45 Recovery on Receivables					297799
46 Recovery on Inventory					121551
47 Taxes Paid	−224400	−235620	−247401	−259771	−248960
48					
49 Net Cash Flow from Joybeans	419171	440130	462137	335243	1032655
50					
51 Present Value Interest Factors	.8695652	.7561437	.6575162	.5717532	.4971768
52 Contribution to NPV	364497	332802	303862	191676	513412
53 Total Net Present Value					1706249

Table 5-4. Sweets, Inc. Example Problem—Analysis of Proposed Policy.

	A	B	C	D	E	F	G
4	Year	0	1	2	3	4	5
5							
6	Proposed Policy:						
7							
8	Last Year's Gross Sales	1904762					
9	First Year's Gross Sales	2200000					
10	Growth Rate	5 percent					
11	Accounts Receivable Turnover	6 Times/Yr.					
12	Bad Debt Expense	2.5 percent of sales					
13	Discount Rate	15 percent					
14							
15	Gross Sales	1904762	2200000	2310000	2425500	2546775	2674114
16	Sales Net of Bad Debts	1866667	2145000	2252250	2364863	2483106	2607261
17							
18	Collections:						
19	Accounts Receivable Balance	233333	357500	375375	394144	413851	434543
20	Cash Receipts from Sales		2020833	2234375	2346094	2463398	2586568
21							
22	Inventory Investment:						
23	Inventory Balance	95238	110000	115500	121275	127339	133706
24	Increase in Inventory		14762	5500	5775	6064	6367
25	Cost of Goods Produced for Sale		1430000	1501500	1576575	1655404	1738174
26	Total Cost of Goods Produced		1444762	1507000	1582350	1661468	1744541
27							

28	**Capital Expenditures**		−175000			
29	**Salvage Value—New Equipment**					75000
30						
31	Income Tax Calculations:					
32	Sales Net of Bad Debts	2145000	2252250	2364863	2483106	2607261
33	Cost of Goods Produced for Sale	1430000	1501500	1576575	1655404	1738174
34	Depreciation			33333	33333	33333
35						
36	EBT	715000	750750	754954	794369	835754
37	Taxes at 34 percent	243100	255255	256684	270085	284156
38						
39						
40	Net Cash Flow Table:					
41	Cash Receipts from Sales	2020833	2234375	2346094	2463398	2586568
42	Total Cost of Goods Produced	−1444762	−1507000	−1582350	−1661468	−1744541
43	Capital Expenditures		−175000			
44	Salvage Value—New Equipment					75000
45	Recovery on Receivables					434543
46	Recovery on Inventory					133706
47	Taxes Paid	−243100	−255255	−256684	−270085	−284156
48						
49	Net Cash Flow from Joybeans	332971	297120	507059	531846	1201120
50						
51	Present Value Interest Factors	.8695652	.7561437	.6575162	.5717532	.4971768
52	Contribution to NPV	289540	224665	333400	304084	597169
53	Total Net Present Value					1748859

Table 5-5. Sweets, Inc. Example Problem—Analysis of Proposed Policy, No Increase in Sales.

3	! A	!! B	!! C	!! D	!! E	!! F	!! G	!
4	Year		0	1	2	3	4	5
5								
6	Proposed Policy:							
7								
8	Last Year's Gross Sales	1904762						
9	First Year's Gross Sales	2000000						
10	Growth Rate	5	percent					
11	Accounts Receivable Turnover	6	Times/Yr.					
12	Bad Debt Expense	2.5	percent of sales					
13	Discount Rate	15	percent					
14								
15	Gross Sales	1904762	2000000	2100000	2205000	2315250	2431013	
16	Sales Net of Bad Debts	1866667	1950000	2047500	2149875	2257369	2370237	
17								
18	Collections:							
19	Accounts Receivable Balance	233333	325000	341250	358313	376228	395040	
20	Cash Receipts from Sales		1858333	2031250	2132813	2239453	2351426	
21								
22	Inventory Investment:							
23	Inventory Balance	95238	100000	105000	110250	115763	121551	
24	Increase in Inventory		4762	5000	5250	5513	5788	
25	Cost of Goods Produced for Sale		1300000	1365000	1433250	1504913	1580158	
26	Total Cost of Goods Produced		1304762	1370000	1438500	1510425	1585946	
27								

28	Capital Expenditures				150000	
29	Salvage Value—New Equipment					80000
30						
31	Income Tax Calculations:					
32	Sales Net of Bad Debts	1950000	2047500	2149875	2257369	2370237
33	Cost of Goods Produced for Sale	1300000	1365000	1433250	1504913	1580158
34	Depreciation					70000
35						
36	EBT	650000	682500	716625	752456	720079
37	Taxes at 34 percent	221000	232050	243653	255835	244827
38						
39						
40	Net Cash Flow Table:					
41	Cash Receipts from Sales	1858333	2031250	2132813	2239453	2351426
42	Total Cost of Goods Produced	−1304762	−1370000	−1438500	−1510425	−1585946
43	Capital Expenditures				−150000	
44	Salvage Value—New Equipment					80000
45	Recovery on Receivables					395040
46	Recovery on Inventory					121551
47	Taxes Paid	−221000	−232050	−243653	−255835	−244827
48						
49	Net Cash Flow from Joybeans	332571	429200	450660	323193	1117243
50						
51	Present Value Interest Factors	.8695652	.7561437	.6575162	.5717532	.4971768
52	Contribution to NPV	289193	324537	296316	184787	555467
53	Total Net Present Value					1650299

The capital expenditures required of Sweets, Inc., under this credit policy are input as data in line 28 as is the salvage value of these new purchases in line 29. With these and the prior data, the firm's tax bills can now be computed. This is done in lines 31 through 37. In this calculation, *accrual accounting must be used*, since this is the method required under the federal tax statutes. Thus, in line 32 of these pro forma income statements, sales appears rather than collections. Under these statutes, only the costs of goods actually sold (and not those produced to increase inventory) are deductible, so these are the costs reproduced in row 33. Therefore, cell C32 contains "C16" and cell C33 contains "C25". The depreciation deductions appear in row 34. Since we are assuming for simplicity that the firm depreciates assets from their acquisition cost to their expected salvage value by the straight-line method, and since the asset will be held for only one year, cell G34 contains "(F28 – G29)/1". Row 36 contains the contribution to the firm's taxable earnings; cell C36 contains "C32 – (C33 + C34)". The tax bill associated with the Joybeans product is 34 percent of these earnings, and appears in row 37.

At this point, data for all the various types of cash flows associated with the present terms of sale for Joybeans have been calculated; it is just a matter of summing these cash flows and discounting them. For clarity, a separate summary section of this spreadsheet in rows 40 through 49 presents the cash flow data; the discounting is carried out in rows 51 through 53. The contents of the cells in rows 41 through 49, where the different types of net cash flows are detailed, are simply cell addresses referring to other parts of the spreadsheet. For example, cell C41 contains "C20", cell C42 contains "– C26", and cell G45 contains "G19". Rows 41 through 47 are summed in row 49 (cell C49 contains "@SUM(C41 . . . C49)"). Present value interest factors are computed in row 51 by using the discount rate in cell B13 and the year number data in row 4. For example, cell D51 contains "1/(((1 + (B15/100))^D4)". These are multiplied by the net cash flow estimates in line 49 to obtain the contributions to net present value in line 52. The formula in cell G53 is "@SUM(C52 . . . G52)", and gives the total net present value of the cash flow stream from Joybeans under the present terms of sale. This NPV is $1,706,249.

To decide whether the new terms of sale should be adopted, this net present value of the present policy must be compared to the net present value of the cash flows under the proposed policy. To obtain this analysis, the turnover, sales, and bad-debt data in cells B9, B10, and B11 and the data and formulas elsewhere in the spreadsheet on capital expenditures and their depreciation are changed to reflect the results of the new policy. The results are presented in Table 5-4. The net present value is $1,748,859 for the cash flows under the proposed policy, so by this analysis *the change should be made*; it increases the value of the cash flows to Sweets, Inc., by $42,610.

ASSESSING RISK IN TERMS OF SALE DECISIONS

This approach to the analysis of terms of sale decisions is comprehensive and reasonably accurate. It avoids many of the problems inherent in the traditional approach to these decisions and is very flexible. Numerous time patterns of sales, costs, and so forth can be modeled quite easily. This methodology takes the basic capital budgeting view of discounting expected cash flows to obtain their value. Since the changes in net cash flows inherent in terms of sale decisions are generally of similar risk to other cash flows of the firm (since they result from greater or lesser sales and costs for an existing product or service), it makes sense to use a risk-adjusted rate in discounting, with this rate based on the firm's cost of capital. Adjustments to this cost of capital (via benchmarks or via the CAPM) can be made to reflect differences in risk among the firm's products. In this way, the value of the flows to the firm is adjusted for risk as part of the discounting process.

While this risk-adjusted discounting procedure is a reasonable approach to the problem of addressing risk in terms of sale decisions, many firms will want to do additional analysis of risk with respect to these decisions, particularly if the net present values of the cash flow streams are large and the differences in net present value between alternate credit policies are small. The evaluation of various terms of sale policies involves the estimation of future cash flows, and there are numerous sources of potential error that may make actual results different from estimates. Sartoris and Hill cite four sources of possible error in estimates of future outcomes in terms of sale decisions: estimates of the fraction of sales paid with a discount, estimates of the accounts receivable turnover, estimates of sales volume, and estimates of bad debt expense.[20] To this list could be added sources of error in numerous other estimates, including estimates of future production expenses, capital expenses, salvage values, product life, and even future tax rates. Sartoris and Hill suggest several methods for addressing this estimation risk, including the use of risk-adjusted discount rates (as we used previously), simulation, and sensitivity analysis.

Simulation, as was used in Chapter 3 to assess the risk of cash stockout in the cash forecast, is a potentially fruitful avenue for assessing risk in many situations. However, the additional estimation cost inherent in simulation is of most advantage when there are a number of important parameters of the decision which exhibit large variations and which are likely to substantially change the outcome of the analysis. In many terms of sale problems, there is one parameter that plays an extremely important part in the analysis and about which there is considerable uncertainty. That parameter is *the change in future sales resulting from a change in credit policy*. It is our experience

[20]See Sartoris and Hill, "A Generalized Cash Flow Approach," p. 357.

that other parameters of the analysis (future accounts receivable turnover, bad debt expense, and so forth) may be estimated relatively accurately based on the firm's past experience, data from other product lines where the firm offers different terms of sale, publicly available data on turnover ratios, and so forth. However, *the estimation of the change in future sales requires substantial insight into demand curve effects and the reactions of competitors*; accurate estimation in these areas is a good deal harder to achieve.[21] Consequently, we believe that the most useful tool for risk assessment in terms of sale decisions is *sensitivity analysis of the anticipated sales change resulting from a terms of sale decision*.

To understand the criticality of the sales effect, let us return to the example problem used previously. While a change in terms of sale policy could well affect other aspects of the time pattern of future sales, in this problem the firm's first-year sales were expected to undergo a substantial increase, with sales thereafter growing at the same rate regardless of whether the proposed policy is adopted or the present policy is continued. Assume that Sweets, Inc., changes its terms of sale, and that the firm's accounts receivable turnover slows and bad debt increases as a result, but that the first year's sales do not increase. Perhaps the firm is in an oligopolistic industry and other manufacturers of executive jellybeans elect to match the change in terms initiated by Sweets. In such a case, the firm will experience decreased cash inflows due to (1) decreased accounts receivable turnover, and (2) increased bad debt losses, but will experience no offsetting increases in cash inflows due to additional sales volumes (although the firm's time pattern of capital expenditures will be the same as under the present terms of sale policy).[22] The analysis of this possible result is presented in Table 5-5. The net present value of the cash flows from Joybeans is $1,650,299, which is $55,950 less than the net present value for the current terms of sale, so the change reduces the value of the firm. Using this example, we see that the relationship between the firm's terms of sale and the resulting sales volume is a critical element affecting the desirability of any proposed changes in these terms.

Since the changes in sales that result from a change in terms are so critical and so uncertain, sensitivity analysis of the effects of these changes versus the advantage of the proposed new policy is a reasonable approach to addressing the major risk factor in this analysis. A sensitivity analysis of the first-year sales level for the proposed terms of the Sweets, Inc., example problem is presented in Table 5-6 and graphed in Figure 5-1. In this example

[21]The substantial difficulty experienced by firms in estimating future sales levels, even without the added complication of changes in terms of sale, is discussed in J. Pan, D. R. Nichols, and O. M. Joy, "Sales Forecasting Practices of Large U.S. Industrial Firms," *Financial Management* (Fall 1977), pp. 72–77.

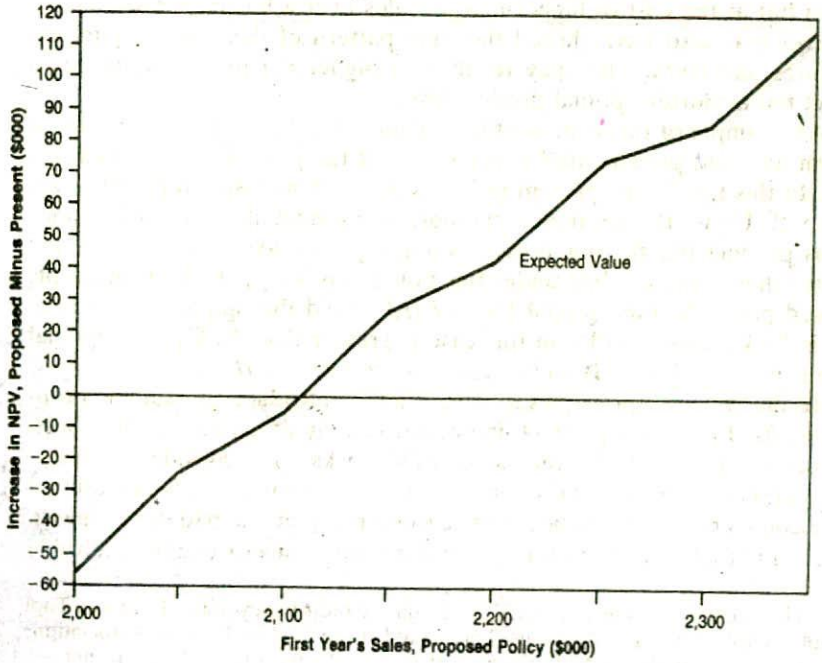
[22]This discussion ignores any increases in industry sales from industry demand curve effects in the interests of simplicity.

Table 5-6. *Sensitivity Analysis of Increase in NPV for Various First Year's Sales.*

First Year's Gross Sales	% Above Present	Year of Cap. Exp.	Amount of Cap. Exp.	Amt. of Salvage	NPV	Increase in NPV
$2,000,000	0.00	4	$150,000	$80,000	$1,650,299	−$55,950
$2,050,000	2.50	4	$156,250	$78,750	$1,682,340	−$23,909
$2,100,000	5.00	3	$162,500	$77,500	$1,701,523	−$4,726
$2,150,000	7.50	3	$168,750	$76,250	$1,733,123	$26,874
$2,200,000	10.00	2	$175,000	$75,000	$1,748,859	$42,610
$2,250,000	12.50	2	$181,250	$73,750	$1,779,947	$73,698
$2,300,000	15.00	1	$187,500	$72,500	$1,791,535	$85,286
$2,350,000	17.50	1	$193,750	$71,250	$1,822,030	$115,781

problem, the initial sales figure for the new policy affects the future schedule of capital expenditures because it affects the levels of future sales; estimates of the required capital expenditures for various sales levels must be generated. Such estimates are also provided in Table 5-6. As can be seen from this table and figure, unless the change in terms increases sales to approximately $2,110,000 (about a 5.5 percent increase), the difference in the old and new net present

Figure 5-1. Increase in NPV Versus First-Year Sales.



values is negative, and the change will not be of advantage to the firm. The expected increase is 10 percent, but Sweets, Inc., needs to take a *very close look at this estimate* and the way in which it was made. What assumptions and procedures were used to make it? How certain is it? What is the chance that the actual increase will be less than 5.5 percent? If the management of Sweets is confident that this estimate was made in a reasonable fashion and that the probability is small that the increase in sales will be less than 5.5 percent, the change in the terms of sale should be made. If not, the results of the analysis need to be considered carefully.

STRATEGIC ASPECTS OF TERMS OF SALE POLICY[23]

In the prior discussion and example of the analysis of terms of sale decisions, it was assumed that the decision to change the firm's terms of sale did not affect the life of the product. However, it is the policy of many firms to exit the market for a particular product as the product moves toward the maturity stage of its product life cycle. In such a case, the firm may use credit policy and other price-related tools to change the *time pattern* of future cash flows as well as the *level* of future cash flows. This strategy may, in turn, affect the optimal life of the product. Assume, for example, that the firm has used credit and other pricing policies to accelerate the acceptance of a new product but at the cost of higher-margin sales over a longer period. Such a maneuver may also have shifted the time pattern of the firm's capital expenditures and costs. This may result in a higher net present value for a product but a shorter optimal product life.

An example of this is presented in Figure 5-2. Here, it is assumed that the firm has used price-related policies to shift the pattern of costs and revenues. In this figure, the present policy is policy A and the proposed policy is policy B. Under the old policy, the optimal product life was $LIFE_A$ (since for this product life the net present value is at a maximum), and the net present value of the product under this policy was NPV_A. If the firm adopts proposed policy B, the optimal life is $LIFE_B$, and the optimal net present value is NPV_B. Since NPV_B in this case is greater than NPV_A, the optimal choice is to adopt Policy B and to exit the product at $LIFE_B$.

To illustrate the principles involved, let us introduce an example problem. Sweets, Inc., has a product line of hard candy which has recently been introduced and is sold under the name "Moonrocks." It is considering changing the terms of sale for this product line. Present terms of sale have resulted in an accounts receivable turnover of 12 times per year and bad debt expense of 2 percent of sales. The new terms of sale will result in an accounts receivable

[23]This section relies heavily on T. S. Walia, "Credit Policy of the Firm as a Tool for Implementing Strategic Decisions," presented at the Eastern Finance Association, April 1984, and available from the author at Indiana University of Pennsylvania.

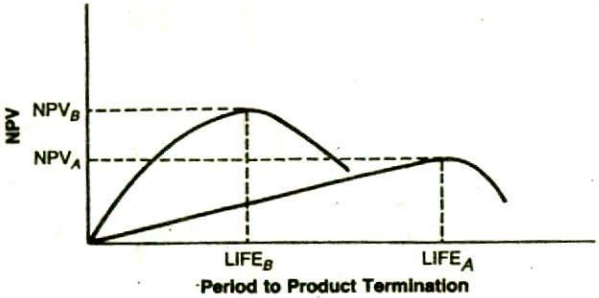


Figure 5-2. Net Present Value Versus Project Life for Two Terms of Sale Policies.

turnover of 4 times per year and bad debt expense of 5 percent of sales. The appropriate risk-adjusted discount rate for this product is estimated at 10 percent per year (the cash flows from Moonrocks are considered to be less risky than those of Joybeans), and inventory turnover will be 15 times per year. Last year's gross sales were $10 million. No capital expenditures will be required under either policy, although the rate of production and length of production will affect the after-tax salvage value of existing fixed assets.

The expected effect of this policy change on sales for Moonrocks is portrayed in Table 5-7. After seven years, sales are expected to be zero for either policy, and the product will be abandoned then in any case. The policy change is expected to increase sales from $123 million to $139 million over the life of the product. However, the new terms will increase sales only in the first two years of the product (due to market penetration effects). After that, the present policy will produce higher sales volumes in the remaining years. In such a case, the changes in the time pattern and level of sales may change the optimum product life.

Analysis of the present and proposed policies is presented in Tables 5-8 and 5-9. Rows 1 through 34 are virtually the same as the spreadsheets

Table 5-7. Sales for Moonrocks Example Problem.

Year	Sales—Present Policy	Sales—Proposed Policy	Difference in Sales
1	$ 20,000	$ 35,000	−$15,000
2	$ 30,000	$ 60,000	−$30,000
3	$ 30,000	$ 25,000	$ 5,000
4	$ 15,000	$ 10,000	$ 5,000
5	$ 12,000	$ 5,000	$ 7,000
6	$ 10,000	$ 3,000	$ 7,000
7	$ 6,000	$ 1,000	$ 5,000
Total	$123,000	$139,000	

Table 5-8. Moonrocks Example Problem—Analysis of Present Policy (rounded thousands of dollars)

4	! A	!! B	!! C	!! D	!! E	!! F	!! G	!! H	!! I	!
5	Year	0	1	2	3	4	5	6	7	
6										
7	Present Policy:									
8										
9	Last Year's Gross Sales	10000								
10	First Year's Gross Sales	20000								
11	Accounts Receivable Turnover	12	Times/Yr.							
12	Bad Debt Expense	2	percent of sales							
13	Discount Rate	10	percent							
14	Inventory Turnover (GS/Inv.)	15								
15										
16	Gross Sales	10000	20000	30000	30000	15000	12000	10000	6000	
17	Sales Net of Bad Debts	9800	19600	29400	29400	14700	11760	9800	5880	
18										
19	Collections:									
20	Accounts Receivable Balance	817	1633	2450	2450	1225	980	817	490	
21	Cash Receipts from Sales		18783	28583	29400	15925	12005	9963	6207	
22										
23	Inventory Investment:									
24	Inventory Balance	667	1333	2000	2000	1000	800	667	400	
25	Increase in Inventory		667	667	0	−1000	−200	−133	−267	
26	Cost of Goods Produced for Sale		13000	19500	19500	9750	7800	6500	3900	
27	Total Cost of Goods Produced		13667	20167	19500	8750	7600	6367	3633	

29	Income Tax Calculations:						
30	Sales Net of Bad Debts	19600	29400	29400	14700	11760	9800	5880
31	Cost of Goods Produced for Sale	13000	19500	19500	9750	7800	6500	3900
32								
33	EBT	6600	9900	9900	4950	3960	3300	1980
34	Taxes at 34 percent	2244	3366	3366	1683	1346	1122	673
35								
36	Net Cash Flow Table:							
37	Cash Receipts from Sales	18783	28583	29400	15925	12005	9963	6207
38	Total Cost of Goods Produced	−13667	−20167	−19500	−8750	−7600	−6367	−3633
39	Taxes Paid	−2244	−3366	−3366	−1683	−1346	−1122	−673
40								
41	Net Cash Flow Selling Moonrocks	2873	5051	6534	5492	3059	2475	1900
42								
43	Recovery on AR and Inv.	2967	4450	4450	2225	1780	1483	890
44	Recoveries on Equipment	6000	4800	3840	3072	2458	1966	1573
45								
46	Present Value Interest Factors	.9090909	.8264463	.7513148	.6830135	.6209213	.5644739	.5131581
47	Contribution to NPV from Oper. NCF	2612	4174	4909	3751	1899	1397	975
48								
49	NPV of Recoveries	8152	7645	6228	3618	2631	1947	1264
50								
51	Net Present Value of Termination	10763	14430	17923	19064	19976	20689	20981

Table 5-9. Moonrocks Example Problem—Analysis of Proposed Policy (rounded thousands of dollars)

4	A	B	C	D	E	F	G	H	I
5	Year	0	1	2	3	4	5	6	7
6									
7	Proposed Policy:								
8									
9	Last Year's Gross Sales	10000							
10	First Year's Gross Sales	35000							
11	Accounts Receivable Turnover	4	Times/Yr.						
12	Bad Debt Expense	5	percent of sales						
13	Discount Rate	10	percent						
14	Inventory Turnover (GS/Inv.)	15							
15									
16	Gross Sales	10000	35000	60000	25000	10000	5000	3000	1000
17	Sales Net of Bad Debts	9800	33250	57000	23750	9500	4750	2850	950
18									
19	Collections:								
20	Accounts Receivable Balance	817	8313	14250	5938	2375	1188	713	238
21	Cash Receipts from Sales		25754	51063	32063	13063	5938	3325	1425
22									
23	Inventory Investment:								
24	Inventory Balance	667	2333	4000	1667	667	333	200	67
25	Increase in Inventory		1667	1667	−2333	−1000	−333	−133	−133
26	Cost of Goods Produced for Sale		22750	39000	16250	6500	3250	1950	650
27	Total Cost of Goods Produced		24417	40667	13917	5500	2917	1817	517

28							
29	**Income Tax Calculations:**						
30	**Sales Net of Bad Debts**	33250	57000	23750	9500	4750	2850	950
31	Cost of Goods Produced for Sale	22750	39000	16250	6500	3250	1950	650
32								
33	EBT	10500	18000	7500	3000	1500	900	300
34	Taxes at 34 percent	3570	6120	2550	1020	510	306	102
35								
36	Net Cash Flow Table:							
37	Cash Receipts from Sales	25754	51063	32063	13063	5938	3325	1425
38	Total Cost of Goods Produced	−24417	−40667	−13917	−5500	−2917	−1817	−517
39	Taxes Paid	−3570	−6120	−2550	−1020	−510	−306	−102
40								
41	Net Cash Flow Selling Moonrocks	−2233	4276	15596	6543	2511	1202	806
42								
43	Recovery on AR and Inv.	10646	18250	7604	3042	1521	913	304
44	Recoveries on Equipment	6000	4200	2940	2058	1441	1008	706
45								
46	Present Value Interest Factors	.9090909	.8264463	.7513148	.6830135	.6209213	.5644739	.5131581
47	Contribution to NPV from Oper. NCF	−2030	3534	11717	4469	1559	679	414
48								
49	NPV of Recoveries	15133	18554	7922	3483	1839	1084	518
50								
51	Net Present Value of Termination	13103	20058	21144	21173	21088	21012	20860

presented in the prior analysis of terms of sale policy, except that (1) the capital expenditure effects have been deleted (since there are no differential effects between the policies on capital expenditures), and (2) the future sales figures are now input as data, since they are being forecast by the spot method. For purposes of this analysis, the cash flows associated with the Moonrocks product are split into two types: the cash flows from sales and the recoveries in the termination of the product line. The cash flows from sales are calculated in rows 37 through 41. In this particular example problem, there are three types of cash flows resulting from sales: the cash receipts from sales (row 37), the cost of goods produced (row 38), and the taxes paid (row 39). As in prior spreadsheet analyses, these figures are taken from calculations performed in prior rows (cell C37 contains "C21", and so forth). The sum of these three, which is the cash flow from sales of Moonrocks, appears in row 41 (cell C41 contains "@ SUM(C37 . . . C39), for example).

Rows 43 and 44 contain the recovery amounts to the firm *if the product is terminated at the end of the year.* The recovery amount for accounts receivable and inventory (row 43) is the sum of the balances of these two accounts at the end of the year from rows 20 and 24 (a 100 percent recovery is assumed in this problem). The recoveries on equipment in line 44 are spot estimates and are input as data. The sum of these two is the cash recovery that the firm would receive if it terminates the product at the end of that year. For example, if the product was terminated at the end of year 2 (column D), the firm would receive a total of $4.45 million on the recovery of receivables and inventory ($2.45 million from receivables and $2 million from inventory) and $4.8 million on the salvage of equipment. The contributions to the net present value from the operating net cash flow (from selling the product) and from liquidation are presented in rows 47 and 49 respectively. These are the future cash flows times the present value interest factors in row 46.

The net present value of the flows from the Moonrocks product line for termination in various years is presented in row 51. To understand how these figures were derived, recognize that if the firm terminates the product line at any time, it receives the cash flows from sales *up through that time* plus the cash flow from recovery that is applicable *at that time.* For example, if the firm terminates the product at the end of the third year, it will receive the cash flows from sales for years 1 through 3 and the recovery for year 3. Therefore, the net present value of termination of the product at the end of the third year is $17.923 million, which is composed of the net present value of the operating cash inflows from years 1, 2, and 3 in the amounts of $2.162 million, $4.174 million, and $4.909 million plus the net present value of the recoveries at the end of year 3 in the amount of $6.228 million. That is, cell E51 contains "@SUM(C47 . . . E47) + E49". The other cells in row 51 contain similar entries, with the summation function always beginning at cell C47

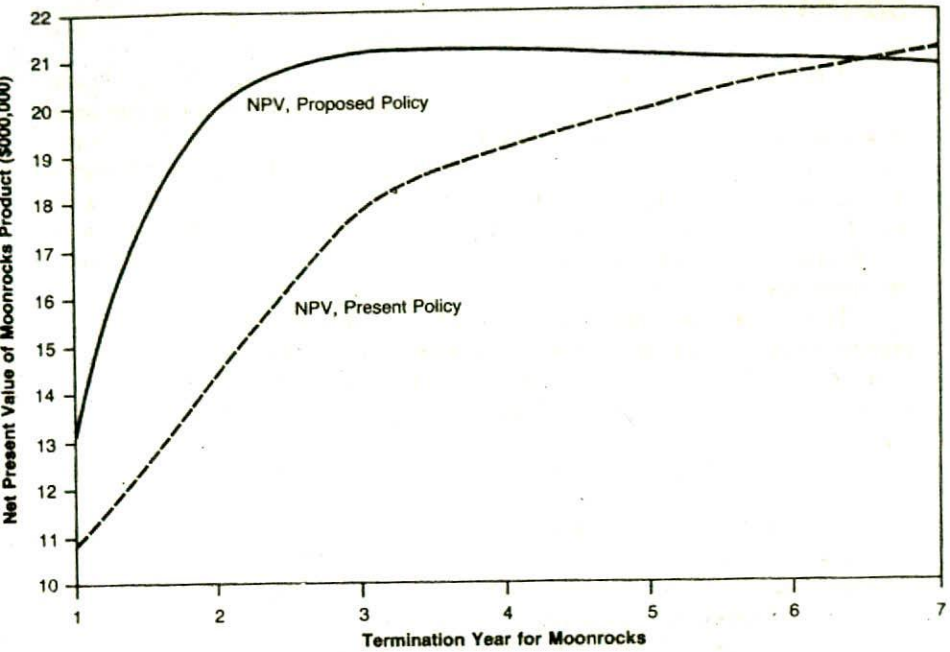


Figure 5-3. NPV Versus Termination Year—Two Policies.

and continuing to the column involved. From row 51, the optimal strategy for the present terms of sale is to continue to sell the product for the entire seven-year life, with a net present value of $20.981 million.

The parallel analysis for the proposed terms of sale in Table 5-9 shows a different picture. Here, the optimum product life is only four years, with a net present value of $21.173 million. Since this net present value is higher than the net present value for the optimum product life under the present policy, *the proposed terms of sale should be adopted, and the product should be terminated after four years.* A graph of the net present values for the two policies is presented in Figure 5-3. This graph highlights the necessity that the analyst consider the effects of optimal product life in assessing any multi-period decision (including terms of sale policy decisions). In this particular problem, if Sweets, Inc., assumes that the change in policy has not affected the optimal life and incorrectly evaluates both the present and proposed policies using a seven-year life, it will continue with the present terms of sale since the net present value of the present policy is higher than that of the proposed policy for a seven-year product life. Stockholders would then not gain the additional value that would accrue to the firm from adopting the proposed terms of sale and terminating the product in the fourth year.

SUMMARY

This chapter introduced the management of accounts receivable. Trade credit, which is the granting of credit from one firm to another for the purchase of goods or services, has historically been an important source of financing for firms. Trade credit continues to exist because it performs several useful functions: it allows firms that are best able to borrow to access capital markets and then relend funds to other firms; it allows firms to check the quality of goods and services before paying for them; and it avoids the conveyance problems associated with the payment of cash on delivery.

This chapter also discussed one of the two major decision areas in the management of accounts receivable: terms of sale policy. The analysis of terms of sale decisions requires that the selling firm estimate the effects of these decisions on the timing and amount of its cash flows. There are several types of cash flows that may be affected by terms of sale decisions. Unfortunately, the standard approach to terms of sale policy deals with only a few of these types, and assumes an unrealistic time pattern for those that it does consider. An alternate spreadsheet analysis system is presented in which the analyst estimates all the major types of cash flows effects from the terms of sale decision, then sums and discounts these to obtain the net present values of alternative terms of sale policies. This approach to terms of sale analysis is very flexible and is reasonably accurate in capturing the timing and value of cash flows. Further, this approach facilitates the assessment (via sensitivity analysis) of the major risk factor in terms of sale decisions: the expected change in future sales from a change in terms of sale policy.

Finally, we present analysis of the strategic aspects of terms of sale policy. In this analysis, the firm may use terms of sale policy to change the time pattern (as well as the level) of future cash flows. By the technique presented, firms may analyze the alternative terms of sale policies which may involve the termination of products before the maturity stage of the product life cycle.

In addition to the analysis of terms of sale policy, the other major decision to be made in the management of accounts receivable is the *credit-granting decision*: to whom should the firm grant credit? We take up the analysis of the credit-granting decision in the next chapter.

Problems

5-1. A firm is considering changing its terms of sale for a particular product. The product has a three-year life. Last year's sales were $1 million. The current terms are net 30 days; 99 percent of customers pay promptly and the remainder default. Under the current terms, expected sales for the upcoming year are $1.1 million. If

these terms are continued, the firm will make a fixed asset purchase in the amount of $30,000 in the second year. The salvage value for this asset is $10,000. The proposed terms are net 60 days. If these terms are instituted, next year's sales are expected to be $1.15 million, and it is expected that 97 percent of all customers will pay promptly and the remaining 3 percent will default. Under the proposed terms, a fixed asset purchase in the amount of $40,000 will be required in the first year; the salvage value of this asset will be $15,000. Whether or not the terms change is instituted, sales are expected to grow at 10 percent per year after the first year. Costs of goods are 80 percent of sales. Inventory levels are kept at 10 percent of sales. The firm's cost of capital is 12 percent, and federal and state income taxes total 40 percent of earnings before taxes. The firm depreciates assets using the straight-line method from the initial purchase price to the expected salvage value.

a. Compute the net present value of the product under the present terms of sale.

b. Compute the net present value of the product under the proposed terms of sale.

c. Indicate whether the proposed terms change should be instituted. Give a rationale for your answer.

5-2. The increment to sales volume resulting from a change in terms of sale is always uncertain. Investigate the sensitivity of your answer to Problem 1 by recalculating the net present value of the proposed policy for initial sales volumes of $1.1 million, $1.2 million, $1.25 million, $1.3 million, $1.35 million, and $1.4 million. Assume that these sales amounts do not require any other capital expenditures beyond the $40,000 amount to be spent in the first year, and do not affect the salvage value of this expenditure. How does this analysis influence your conclusions with respect to the adoption of the proposed terms of sale change?

Selected Readings

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case

Alloyed Companies, Inc.[1] *Terms of Sale Analysis*

Alloyed Companies, Inc., was one of the more successful miniconglomerates, perhaps because of two policies: (1) no firm was acquired that Alloyed did not think it could manage properly, and (2) after acquisition, certain functions of the acquired firms were centralized to take advantage of economies of scale. One of these centralized functions was receivables management. Mr. Juliett, assistant treasurer of Alloyed, acted as credit and receivables manager for all of Alloyed's five divisions.

Sales had been expanding over the past few years, as had Alloyed's customer base, in each of the five divisions. The average receivables balance for each of the credit department's three accounts managers had reached $9 million, which Mr. Juliett thought was uncomfortably high. Though there had been no complaints from the accounts managers (who reported to Mr. Juliett directly) he felt that they could do a better job with a decreased work load and had decided to add another accounts manager to the force. Mr. Juliett also felt that the time was right to review the firm's credit and receivables management policies in general.

Despite the fact that Alloyed's five divisions were similar in terms of the processes used and the physical products sold, they each sold to different markets. There were no common customers. As each firm had been acquired, its receivables had been centralized and sorted into Alloyed's receivables system. This system was a fairly complex one, using much computer assistance and capable of producing the numerous reports requested by management. All sales were within the continental United States.

Credit and receivables management was one of the areas where Alloyed felt that future financial executives should have experience, and so the company rotated promising people into the accounts manager positions on a regular basis. Average tenure within these positions was about two and one-half years, after which time the employee would go on to another position within the firm. The accounts of all five divisions were sorted alphabetically and assigned to the accounts managers in this fashion. Each manager had about the same total dollar exposure and total number of accounts. The management strategy for handling accounts had been to treat each account

[1]This case has been adapted from Iqbal Mathur and Frederick C. Scherr, *Cases in Financial Management* (New York: Macmillan, 1979), pp. 50–55.

Exhibit 1

Alloyed Companies
Divisional Statements of Income for Fiscal 1977
(rounded thousands of dollars)

	Imported Products Division	*Paint Supplies Division*	*Plastic Resin Division*	*Consumer Products Division*	*Chemical Division*
Gross Sales	$41,800	$24,200	$61,600	$26,400	$66,000
Bad Debt Allowance	100	121	462	610	660
Other Allowances	109	121	154	182	0
Net Sales	$41,591	$23,958	$60,984	$25,608	$65,340
Cost of Goods Sold:					
Direct Labor	$ 0	$ 1,978	$ 3,297	$ 1,512	$ 5,213
Direct Materials	36,421	14,916	32,450	15,026	29,657
Direct Overhead	1,011	1,412	1,932	1,020	1,764
Depreciation[1]	0	1,100	4,400	880	4,620
Gross Margin on Sales	$ 4,159	$ 4,552	$18,905	$ 7,170	$24,086
Selling and Administrative Expenses:					
Sales Salaries[2]	$ 1,501	$ 1,121	$ 8,703	$ 3,121	$13,101
Marketing Department[3]	539	1,962	5,232	1,127	3,020
Headquarters Staff[4]	190	110	280	120	300
Computer Department[4]	152	88	224	96	240
Nonplant Depreciation[5]	836	484	1,232	528	1,320
Other Indirect Overhead[4]	105	61	154	66	165
Operating Margin	$ 836	$ 726	$ 3,080	$ 2,112	$ 5,940
Interest Expense[6]	167	97	246	106	264
Other Income[7]	50	0	0	0	111
Profit Before Tax	$ 719	$ 629	$ 2,834	$ 2,006	$ 5,787
Gross Margin on Sales (%)	10	19	31	27	36
Operating Margin (%)	2.0	3.0	5.0	8.0	9.0
Profit Before Tax (% Sales)	1.7	2.6	4.6	7.6	8.8

Notes to Divisional Statements of Income

[1]This depreciation refers to that on the actual facilities producing the materials for sale. It is computed on an accelerated basis, and transferred to the divisional income statements from the corporate tax reports.

[2]Sales salaries are only for the salesmen who are actually selling the division's products. A salesman's time is highly utilized and each salesman has about as much volume as he can handle.

[3]Marketing department expenses include all salaries for this department for the related division. In general, this department's time is not fully utilized.

[4]For all costs with this footnote number, costs were allocated based on sales volume. Charges of this type continue, regardless of changes in the sales volume of the individual divisions.

similarly, and to try to keep the time spent on each account proportional to the dollar exposure. Accounts managers were instructed to be courteous to all, but not to make concessions from the stated terms of sales, which had been set at net 30 days for all divisions. These standardized terms were a policy decision and were an outgrowth of the centralization of the firm's receivables management. To assure that none of the accounts managers grew bored with his or her group of customers, assignments within the department were rotated on a regular basis. Mr. Juliett wondered if these policies were optimal, particularly in view of the differences between the divisions in customer characteristics, competitive situation, and profitability (see Exhibit 1 for divisional statements of income for 1977, the year just completed).

The Imported Products Division brought products into the United States from other countries. Sales were by contract to large, very creditworthy firms. Though the division sold a substantial portion of Alloyed's sales volume in terms of dollars, it handled relatively few orders, and consequently had relatively few employees in the sales and marketing areas. Because bargaining power was on the buyer's side, profit margins tended to be slim. Despite the low risk of default, the collection effort of the credit and receivables management department tended to be fairly high for this division. This was because of paperwork problems. The large firms sold by this division did not respond to computer-generated form letters, which were effective collection tools in dealing with some of the smaller firms sold by other divisions. Also, the sales management of this division tended to make special deals with customers that, because they were not reflected in paperwork, caused some nonpayment of invoices. When this occurred, the division was slow to correct the matter. Consequently, the accounts managers spent a significant amount of time following up unpaid items on a personalized basis and attempting to get the sales management of the division to correct errors so that payments could be collected. The average time to receive payment for an invoice issued by this division was 42 days. Customers of this division almost never requested extended time for payments.

The Paint Supplies Division dealt with paint manufacturers of various sizes. Risk of default for these customers was in the medium range, but payments were often received far beyond terms. Accounts could be collected in a routine fashion with computer assistance. There had been numerous requests in the past for lengthened terms of sale to meet competitive situations, but in keeping with the firm's policy of standard terms, these had been denied. The average time taken to pay an invoice was 60 days.

The Plastic Resin Division represented the highest degree of default risk of any of the divisions. The customers were small job shops with highly variable cash flows, little management expertise, and almost no profitability. Though market power should have been on the seller's side, with few sellers of resin and many buyers, a "huckster" attitude prevailed among the industry sellers, and there were constant requests for extended terms to meet competition. Because of poor financial circumstances, customers of this division were

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[5]Nonplant depreciation includes tax-statement depreciation on administrative buildings, recreational facilities, and so on. It is allocated to the divisions based on sales volume.

[6]This interest-expense entry is an allocation based on the sales volume of interest expense on corporate debt to support assets. Accounts receivable are a minor part of these assets.

[7]Other income is basically royalties on the patents which divisions have licensed out to other firms.

continuously harangued by creditors, and did not generally respond to computer-assisted collection methods; personal collection effort was necessary. Average time to pay an invoice by the customers of this division was 55 days.

The Consumer Products Division sold items to wholesalers and jobbers. Though customers tended to be highly leveraged, and therefore very risky from a default standpoint, their asset turnover was very high and their cash flows substantial; they tended to pay promptly. Routine follow-up was sufficient to collect any past-due items. Though analysis of the financial condition of these customers was necessary on a regular basis, less than an average amount of time of the accounts receivable managers was taken up with accounts from this division. Because demand for the Consumer Products Division's products at the retail level was high, bargaining power was on Alloyed's side, and there were few requests for concessions in terms of sale. The average time taken to pay an invoice was 35 days.

The Chemical Division had the largest share of company sales. The average time to pay an invoice by the customer of this division was 36 days, and customers were average in terms of default risk. The Chemical Division sold to a moderately large group of medium-sized, well-established firms, and it held a major share of the market. However, because the number of buyers of the division's individual product lines was not large, bargaining power was about equally distributed between buyer and seller. There had been frequent requests for extended terms of sale, all of which had been refused. This had resulted in some lost sales. Most of this division's customer base had dealt with Alloyed for several years, and had developed working relationships with the accounts managers. When there was some payment problem (which occurred in a small minority of cases, as indicated by this division's 36-day payment average) some personal contact by accounts managers was required.

Mr. Juliett had already decided that an additional accounts manager would be added to the staff. He was also considering changing the assignment system of these managers from alphabetical to divisional. In a divisional system, the firm's accounts receivable would be sorted by division, and each manager would handle accounts from only one or at most two divisions. His tentative plan was to have one manager handle each of the Imported Products, Plastic Resins, and Chemical Divisions and have the other manager handle the Paint Supplies and Consumer Products Divisions. He felt that this would enable the department to enforce different terms of sale policies for different divisions without confusion. The question then was which divisions should receive terms of sale other than the firm's regular net 30 terms.

To address this question, Mr. Juliett first eliminated from consideration the Imported Products Division and the Consumer Products Division. He knew that firms incur costs when terms of sale are lengthened; to make changes in terms of sale advantageous, these costs must be offset with sales increases. For both these divisions, Mr. Juliett felt that any increases in sales were likely to be very small, and this made terms changes unattractive.

Mr. Juliett approached the divisional sales managers for the remaining three divisions and asked them to prepare proposals for terms of sale adjustments. These proposals were to include the estimated effects of changes in terms on sales, receivables turnover, and bad debt expense (it was assumed that collection expense would not change). These proposals are summarized in Exhibit 2. He then used the standard single-period analysis technique to evaluate the proposed changes in terms. This analysis was presented at the next meeting of the firm's Policy Committee (a committee of

Exhibit 2

Alloyed Companies
Proposed Changes in Terms of Sale

	Paint Supplies	*Plastic Resin*	*Chemical*
Present Terms	*Net 30 Days*	*Net 30 Days*	*Net 30 Days*
Proposed Terms	*3% 10, N 90*	*Net 45 Days*	*Net 60 Days*
Percent of Customers Taking Discount: Present Terms	0		
Expected Percent of Customers Taking Discount: Proposed Terms	70		
Expected 1978 Gross Sales (Thousands, Present Policy)	$25,410	$64,648	$70,620
Expected 1978 Gross Sales (Thousands, Proposed Policy)	$26,681	$71,148	$75,917
Current Accounts Receivable Turnover (Based on Net Sales)	60 Days	55 Days	36 Days
Expected Accounts Receivable Turnover (Based on Net Sales)	40 Days	70 Days	66 Days
Current Bad Debt Expense (Percent of Gross Sales)	0.5%	0.75%	1%
Expected Bad Debt Expense (Percent of Gross Sales)	0.4%	1.2%	1.5%
Inventory Turnover (Based on Gross Sales)	8.0	9.0	9.0

executives that considered strategic proposals which might affect several departments). His recommendation was that changes in the terms of sale proposed by the Plastic Resin and Chemical Divisions should be accepted, but that the Paint Supplies Division should continue with the current terms. Following the lively discussion (to put it mildly) of this recommendation, Fred Turner (Manager of Long-Term Planning for the firm) cautioned against a hasty decision on credit policy. He raised the following specific objections to the analysis presented:

1. The analysis showed only the first-year impact of the proposed changes in credit

Exhibit 3

Alloyed Companies, Inc.
Fred Turner's Memo to Frank Juliett

Subject: Credit Policy for Plastic Resin Division
To: Frank Juliett
From: Fred Turner

John and I have put together the information requested in our last Policy Committee meeting. The necessary data are summarized in Table 1 of this memo. You will notice that John projects the market for the products produced by the Division to last for ten years.

Table 1

	Present	Proposed
Estimated Gross Sales for 1978	$64,680	$71,148
Growth Rate of Sales	6%	8%
Capital Expansion Cost	$50,000	$60,000
Required Year for Capital Expansion	Seventh	Fourth
Salvage Net of Restoration, Tenth Year	$8,000	$15,000

Notes: The product life is ten years. Recovery on accounts receivable on product termination is expected to be 100 percent of the book value of receivables at that time; recovery on inventory is expected to be 70 percent of book value of these assets at that time. Depreciation will be via the straight-line method from the initial book value to the estimated salvage value. The firm is in the 46 percent tax bracket. The firm's weighted-average, after-tax marginal cost of capital is 12 percent, and the cash flows from the products sold by the Plastic Resin Division are about as risky as the firm's average project.

policy, and as such it was difficult to gauge its effect on the value of the firm.

2. The analysis was based on the present idle capacity for manufacturing these products. However, he felt that the new terms of sale would have an impact on the timing and amount of capital investments required for future plant expansion. This would be especially true for the Plastic Resin Division, since the proposed terms were expected to increase first-year sales by about 10 percent.

Fred suggested that the proposed credit policy be evaluated in a net present-value framework. The ensuing discussion indicated that the other members were sympathetic to Fred's concerns. Linda Harper, from the treasurer's staff, indicated special interest in such an analysis since it would indicate future cash flows from the product line, and thus would aid in her financial forecasting tasks. Mr. Juliett agreed to make the analysis, provided he was supplied with future sales figures and plant expansion data. John Lambert, who was responsible for marketing research, and Fred agreed to supply the necessary figures within a week's time (Fred's response is contained in Exhibit 3). Furthermore, because of the time and effort involved, the committee agreed to limit the initial analysis to the Plastic Resin Division.