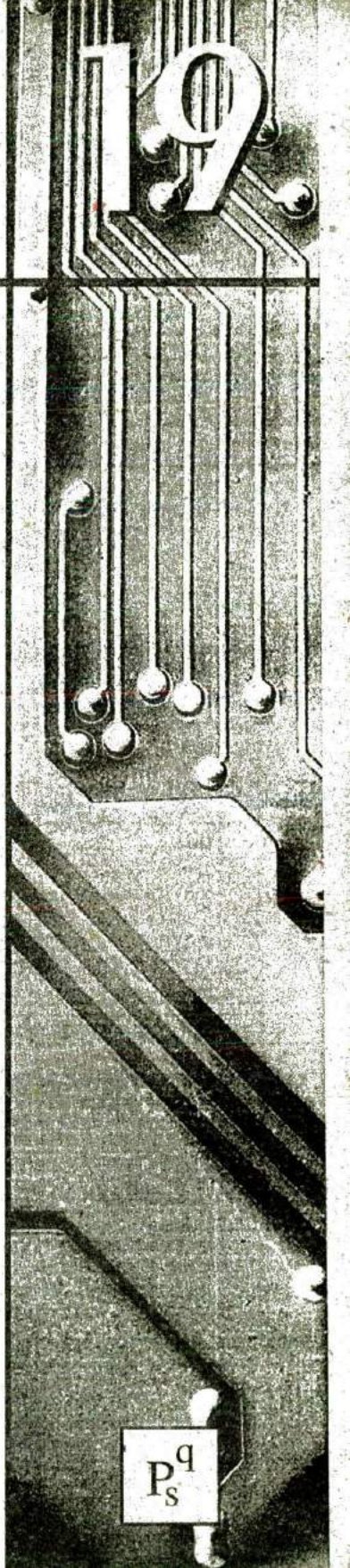
POWER (AC)



Objectives

- *Become familiar with the differences between average, apparent, and reactive power and how to calculate each for any combination of resistive and reactive elements.*
- *Understand that the energy dissipated by a load is the area under the power curve for the period of time of interest.*
- *Become aware of how the real, apparent, and reactive power are related in an ac network and how to find the total value of each for any configuration.*
- *Understand the concept of power-factor correction and how to apply it to improve the terminal characteristics of a load.*
- *Develop some understanding of energy losses in an ac system that are not present under dc conditions.*

19.1 INTRODUCTION

The discussion of power in Chapter 14 included only the average or real power delivered to an ac network. We now examine the total power equation in a slightly different form and introduce two additional types of power: **apparent** and **reactive.**

19.2 GENERAL EQUATION

For any system such as in Fig. 19.1, the power delivered to a load at any instant is defined by the product of the applied voltage and the resulting current; that is,

$$p = vi$$

In this case, since v and i are sinusoidal quantities, let us establish a general case where

$$v = V_m \sin(\omega t + \theta)$$

and

$$i = I_m \sin \omega t$$

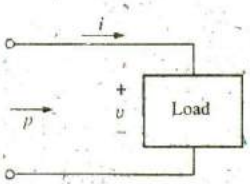


FIG. 19.1
Defining the power delivered to a load.

The chosen v and i include all possibilities because, if the load is purely resistive, $\theta = 0°$. If the load is purely inductive or capacitive, $\theta = 90°$ or $\theta = -90°$, respectively. For a network that is primarily inductive, θ is positive (v leads i). For a network that is primarily capacitive, θ is negative (i leads v).

Substituting the above equations for v and i into the power equation results in

$$p = V_m I_m \sin \omega t \sin(\omega t + \theta)$$

If we now apply a number of trigonometric identities, the following form for the power equation results:

$$\boxed{p = VI \cos \theta(1 - \cos 2\omega t) + VI \sin \theta(\sin 2\omega t)} \qquad \textbf{(19.1)}$$

where V and I are the rms values. The conversion from peak values V_m and I_m to rms values resulted from the operations performed using the trigonometric identities.

It would appear initially that nothing has been gained by putting the equation in this form. However, the usefulness of the form of Eq. (19.1) is demonstrated in the following sections. The derivation of Eq. (19.1) from the initial form appears as an assignment at the end of the chapter.

If Eq. (19.1) is expanded to the form

$$p = \underbrace{VI \cos \theta}_{\text{Average}} - \underbrace{VI \cos \theta \cos 2\omega t}_{\text{Peak}\ \ 2x} + \underbrace{VI \sin \theta \sin 2\omega t}_{\text{Peak}\ \ 2x}$$

there are two obvious points that can be made. First, the average power still appears as an isolated term that is time independent. Second, both terms that follow vary at a frequency twice that of the applied voltage or current, with peak values having a very similar format.

In an effort to ensure completeness and order in presentation, each basic element (R, L, and C) is treated separately.

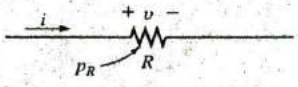


FIG. 19.2
Determining the power delivered to a purely resistive load.

19.3 RESISTIVE CIRCUIT

For a purely resistive circuit (such as that in Fig. 19.2), v and i are in phase, and $\theta = 0°$, as appearing in Fig. 19.3. Substituting $\theta = 0°$ into Eq. (19.1), we obtain

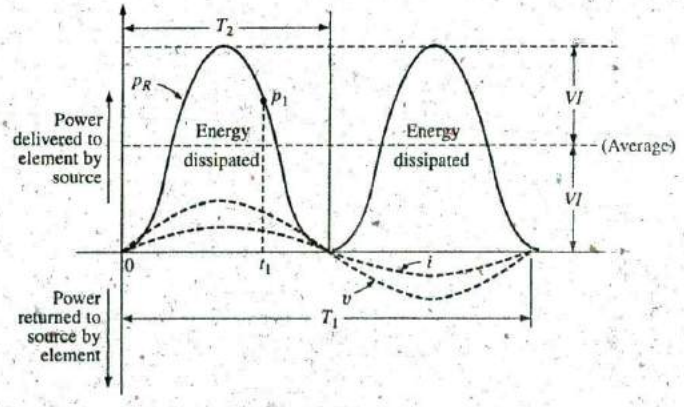


FIG. 19.3
Power versus time for a purely resistive load.

$$p_R = VI\cos(0°)(1 - \cos 2\omega t) + VI\sin(0°)\sin 2\omega t$$
$$= VI(1 - \cos 2\omega t) + 0$$

or,

$$\boxed{p_R = VI - VI\cos 2\omega t} \qquad (19.2)$$

where VI is the average or dc term and $-VI\cos 2\omega t$ is a negative cosine wave with twice the frequency of either input quantity (v or i) and a peak value of VI. The plot in Fig. 19.3 has the same characteristics as obtained in Fig. 14.30.

Note that

T_1 = period of input quantities

T_2 = period of power curve p_R

In addition, the power curve passes through two cycles about its average value of VI for each cycle of either v or i ($T_1 = 2T_2$ or $f_2 = 2f_1$). Consider also that since the peak and average values of the power curve are the same, the curve is always above the horizontal axis. This indicates that

the total power delivered to a resistor will be dissipated in the form of heat.

The power returned to the source is represented by the portion of the curve below the axis, which is zero in this case. The power dissipated by the resistor at any instant of time t_1 can be found by simply substituting the time t_1 into Eq. (19.2) to find p_1, as indicated in Fig. 19.3. The **average (real) power** from Eq. (19.2), or Fig. 19.3, is VI; or, as a summary,

$$\boxed{P = VI = \frac{V_m I_m}{2} = I^2R = \frac{V^2}{R}} \quad \text{(watts, W)} \qquad (19.3)$$

as derived in Chapter 14.

The energy dissipated by the resistor (W_R) over one full cycle of the applied voltage is the area under the power curve in Fig. 19.3. It can be found using the following equation:

$$W = Pt$$

where P is the average value and t is the period of the applied voltage; that is,

$$\boxed{W_R = VIT_1} \quad \text{(joules, J)} \qquad (19.4)$$

or, since $T_1 = 1/f_1$,

$$\boxed{W_R = \frac{VI}{f_1}} \quad \text{(joules, J)} \qquad (19.5)$$

EXAMPLE 19.1 For the resistive circuit in Fig. 19.4,

a. Find the instantaneous power delivered to the resistor at times t_1 through t_6.
b. Plot the results of part (a) for one full period of the applied voltage.
c. Find the average value of the curve of part (b) and compare the level to that determined by Eq. (19.3).

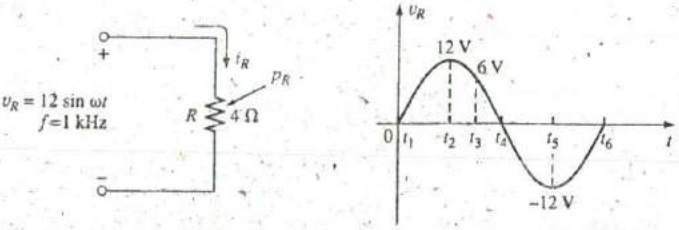


FIG. 19.4
Example 19.1.

d. Find the energy dissipated by the resistor over one full period of the applied voltage.

Solutions:

a. t_1: $v_R = 0$ V and $p_R = v_R i_R = \mathbf{0\ W}$
t_2: $v_R = 12$ V and $i_R = 12\ \text{V}/4\ \Omega = \mathbf{3\ A}$
$p_R = v_R i_R = (12\ \text{V})(3\ \text{A}) = \mathbf{36\ W}$
t_3: $v_R = 6$ V and $i_R = 6\ \text{V}/4\ \Omega = \mathbf{1.5\ A}$
$p_R = v_R i_R = (6\ \text{V})(1.5\ \text{A}) = \mathbf{9\ W}$
t_4: $v_R = 0$ V and $p_R = v_R i_R = \mathbf{0\ W}$
t_5: $v_R = -12$ V and $i_R = -12\ \text{V}/4\ \Omega = \mathbf{-3\ A}$
$p_R = v_R i_R = (-12\ \text{V})(-3\ \text{A}) = \mathbf{36\ W}$
t_6: $v_R = 0$ V and $p_R = v_R i_R = \mathbf{0\ W}$

b. The resulting plot of v_R, i_R, and p_R appears in Fig. 19.5.

c. The average value of the curve in Fig. 19.5 is 18 W, which is an exact match with that obtained using Eq. (19.3). That is,

$$P = \frac{V_m I_m}{2} = \frac{(12\ \text{V})(3\ \text{A})}{2} = 18\ \text{W}$$

d. The area under the curve is determined by Eq. (19.5):

$$W_R = \frac{VI}{f_1} = \frac{V_m I_m}{2 f_1} = \frac{(12\ \text{V})(3\ \text{A})}{2(1\ \text{kHz})} = 18\ \text{mJ}$$

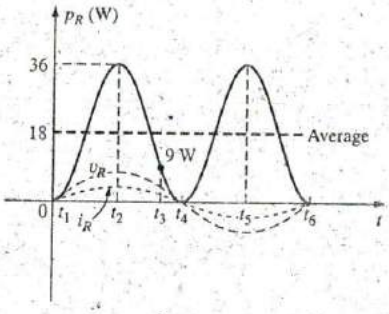


FIG. 19.5
Power curve for Example 19.1.

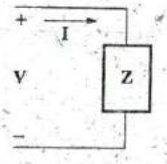


FIG. 19.6
Defining the apparent power to a load.

19.4 APPARENT POWER

From our analysis of dc networks (and resistive elements above), it would seem *apparent* that the power delivered to the load in Fig. 19.6 is determined by the product of the applied voltage and current, with no concern for the components of the load; that is, $P = VI$. However, we found in Chapter 14 that the power factor ($\cos \theta$) of the load has a pronounced effect on the power dissipated, less pronounced for more reactive loads. Although the product of the voltage and current is not always the power delivered, it is a power rating of significant usefulness in the description and analysis of sinusoidal ac networks and in the maximum rating of a number of electrical components and systems. It is called the **apparent power** and is represented symbolically by S.* Since it is simply the product of voltage and current, its units are *volt-amperes* (VA). Its magnitude is determined by

*Prior to 1968, the symbol for apparent power was the more descriptive P_a.

$$\boxed{S = VI} \quad \text{(volt-amperes, VA)} \tag{19.6}$$

or, since $V = IZ$ and $I = \dfrac{V}{Z}$

then $$\boxed{S = I^2Z} \quad \text{(VA)} \tag{19.7}$$

and $$\boxed{S = \frac{V^2}{Z}} \quad \text{(VA)} \tag{19.8}$$

The average power to the load in Fig. 19.4 is

$$P = VI \cos \theta$$

However, $$S = VI$$

Therefore, $$\boxed{P = S \cos \theta} \quad \text{(W)} \tag{19.9}$$

and the power factor of a system F_p is

$$\boxed{F_p = \cos \theta = \frac{P}{S}} \quad \text{(unitless)} \tag{19.10}$$

The power factor of a circuit, therefore, is the ratio of the average power to the apparent power. For a purely resistive circuit, we have

$$P = VI = S$$

and $$F_p = \cos \theta = \frac{P}{S} = 1$$

In general, power equipment is rated in volt-amperes (VA) or in kilovolt-amperes (kVA) and not in watts. By knowing the volt-ampere rating and the rated voltage of a device, we can readily determine the *maximum* current rating. For example, a device rated at 10 kVA at 200 V has a maximum current rating of $I = 10{,}000\ \text{VA}/200\ \text{V} = 50\ \text{A}$ when operated under rated conditions. The volt-ampere rating of a piece of equipment is equal to the wattage rating only when the F_p is 1. It is therefore a maximum power dissipation rating. This condition exists only when the total impedance of a system $Z \angle \theta$ is such that $\theta = 0°$.

The exact current demand of a device, when used under normal operating conditions, can be determined if the wattage rating and power factor are given instead of the volt-ampere rating. However, the power factor is sometimes not available, or it may vary with the load.

The reason for rating some electrical equipment in kilovolt-amperes rather than in kilowatts can be described using the configuration in Fig. 19.7. The load has an apparent power rating of 10 kVA and a current rating of 50 A at the applied voltage, 200 V. As indicated, the current demand of 70 A is above the rated value and could damage the load element, yet the reading on the wattmeter is relatively low since the load is highly reactive. In other words, the wattmeter reading is an indication of the watts dissipated and may not reflect the magnitude of the current drawn. Theoretically, if the load were purely reactive, the wattmeter reading would be zero even if the load was being damaged by a high current level.

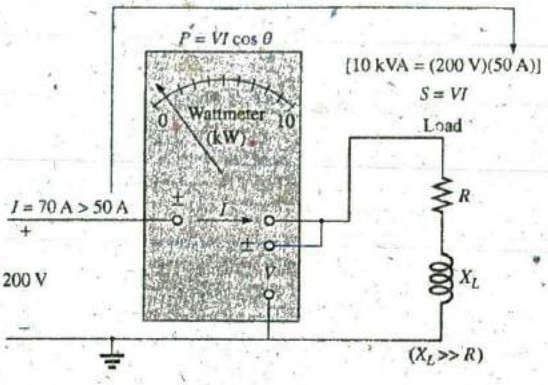


FIG. 19.7
Demonstrating the reason for rating a load in kVA rather than kW.

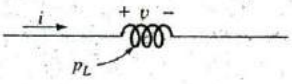


FIG. 19.8
Defining the power level for a purely inductive load.

19.5 INDUCTIVE CIRCUIT AND REACTIVE POWER

For a purely inductive circuit (such as that in Fig. 19.8), v leads i by 90°, as shown in Fig. 19.9. Therefore, in Eq. (19.1), $\theta = 90°$. Substituting $\theta = 90°$ into Eq. (19.1) yields

$$p_L = VI\cos(90°)(1 - \cos 2\omega t) + VI\sin(90°)(\sin 2\omega t)$$
$$= 0 + VI\sin 2\omega t$$

or

$$\boxed{p_L = VI\sin 2\omega t} \tag{19.11}$$

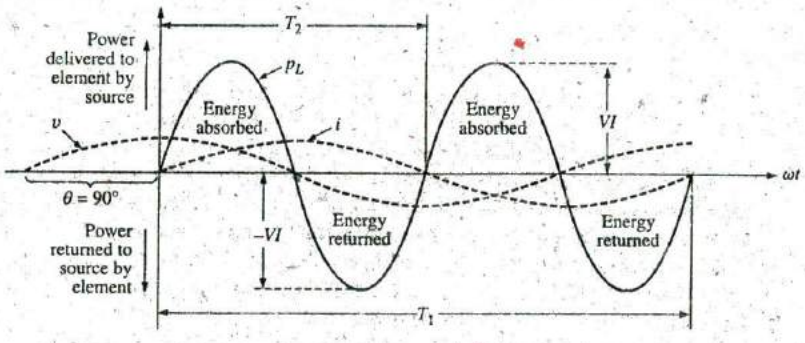


FIG. 19.9
The power curve for a purely inductive load.

where $VI\sin 2\omega t$ is a sine wave with twice the frequency of either input quantity (v or i) and a peak value of VI. Note the absence of an average or constant term in the equation.

Plotting the waveform for p_L (Fig. 19.9), we obtain

T_1 = period of either input quantity

T_2 = period of p_L curve

Note that over one full cycle of p_L (T_2), the area above the horizontal axis in Fig. 19.9 is exactly equal to that below the axis. This indicates that over a full cycle of p_L, the power delivered by the source to the inductor is exactly equal to that returned to the source by the inductor.

The net flow of power to the pure (ideal) inductor is zero over a full cycle, and no energy is lost in the transaction.

The power absorbed or returned by the inductor at any instant of time t_1 can be found simply by substituting t_1 into Eq. (19.11). The peak value of the curve VI is defined as the **reactive power** associated with a pure inductor.

In general, the reactive power associated with any circuit is defined to be $VI \sin \theta$, a factor appearing in the second term of Eq. (19.1). Note that it is the peak value of that term of the total power equation that produces no net transfer of energy. The symbol for reactive power is Q, and its unit of measure is the *volt-ampere reactive* (VAR).* The Q is derived from the quadrature (90°) relationship between the various powers, to be discussed in detail in a later section. Therefore,

$$\boxed{Q_L = VI \sin \theta} \quad \text{(volt-ampere reactive, VAR)} \qquad \textbf{(19.12)}$$

where θ is the phase angle between V and I.

For the inductor,

$$\boxed{Q_L = VI} \quad \text{(VAR)} \qquad \textbf{(19.13)}$$

or, since $V = IX_L$ or $I = V/X_L$,

$$\boxed{Q_L = I^2X_L} \quad \text{(VAR)} \qquad \textbf{(19.14)}$$

or

$$\boxed{Q_L = \frac{V^2}{X_L}} \quad \text{(VAR)} \qquad \textbf{(19.15)}$$

The apparent power associated with an inductor is $S = VI$, and the average power is $P = 0$, as noted in Fig. 19.9. The power factor is therefore

$$F_p = \cos \theta = \frac{P}{S} = \frac{0}{VI} = 0$$

If the average power is zero, and the energy supplied is returned within one cycle, why is reactive power of any significance? The reason is not obvious but can be explained using the curve in Fig. 19.9. At every instant of time along the power curve that the curve is above the axis (positive), energy must be supplied to the inductor, even though it will be returned during the negative portion of the cycle. This power requirement during the positive portion of the cycle requires that the generating plant provide this energy during that interval. Therefore, the effect of reactive elements such as the inductor can be to raise the power requirement of the generating plant, even though the reactive power is not dissipated but simply "borrowed." The increased power demand during these intervals is a cost factor that must be passed on to the industrial consumer. In fact, most larger users of electrical energy pay for the apparent power demand rather than the watts dissipated since the volt-amperes used are sensitive to the reactive power requirement (see Section 19.7). In other words, the closer the power factor of an industrial outfit is to 1, the more efficient is the plant's operation since it is limiting its use of "borrowed" power.

*Prior to 1968, the symbol for reactive power was the more descriptive P_q.

The energy stored by the inductor during the positive portion of the cycle (Fig. 19.9) is equal to that returned during the negative portion and can be determined using the following equation:

$$W = Pt$$

where P is the average value for the interval and t is the associated interval of time.

Recall from Chapter 14 that the average value of the positive portion of a sinusoid equals 2(peak value/π) and $t = T_2/2$. Therefore,

$$W_L = \left(\frac{2VI}{\pi}\right) \times \left(\frac{T_2}{2}\right)$$

and

$$\boxed{W_L = \frac{VIT_2}{\pi}} \quad \text{(J)} \tag{19.16}$$

or, since $T_2 = 1/f_2$, where f_2 is the frequency of the p_L curve, we have

$$\boxed{W_L = \frac{VI}{\pi f_2}} \quad \text{(J)} \tag{19.17}$$

Since the frequency f_2 of the power curve is twice that of the input quantity, if we substitute the frequency f_1 of the input voltage or current, Eq. (19.17) becomes

$$W_L = \frac{VI}{\pi(2f_1)} = \frac{VI}{\omega_1}$$

However,

$$V = IX_L = I\omega_1 L$$

so that

$$W_L = \frac{(I\omega_1 L)I}{\omega_1}$$

and

$$\boxed{W_L = LI^2} \quad \text{(J)} \tag{19.18}$$

providing an equation for the energy stored or released by the inductor in one half-cycle of the applied voltage in terms of the inductance and rms value of the current squared.

EXAMPLE 19.2 For the inductive circuit in Fig. 19.10:

a. Find the instantaneous power level for the inductor at times t_1 through t_5.
b. Plot the results of part (*a*) for one full period of the applied voltage.

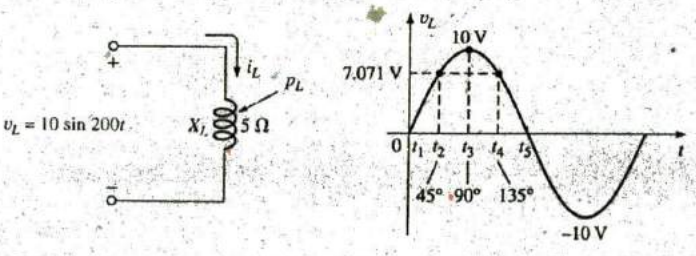


FIG. 19.10
Example 19.2.

c. Find the average value of the curve of part (b) over one full cycle of the applied voltage and compare the peak value of each pulse with the value determined by Eq. (19.13).

d. Find the energy stored or released for any one pulse of the power curve.

Solutions:

a. t_1: $v_L = 0$ V, $p_L = v_L i_L = \mathbf{0\ W}$

t_2: $v_L = 7.071$ V, $i_L = \dfrac{V_m}{X_L}\sin(\alpha - 90°)$

$= \dfrac{10\text{ V}}{5\ \Omega}\sin(\alpha - 90°) = 2\sin(\alpha - 90°)$

At $\alpha = 45°$, $i_L = 2\sin(45° - 90°) = 2\sin(-45°) = -1.414$ A

$p_L = v_L i_L = (7.071\text{ V})(-1.414\text{ A}) = \mathbf{-10\ W}$

t_3: $i_L = 0$ A, $p_L = v_L i_L = \mathbf{0\ W}$

t_4: $v_L = 7.071$ V, $i_L = 2\sin(\alpha - 90°) = 2\sin(135° - 90°)$

$= 2\sin 45° = 1.414$ A

$p_L = v_L i_L = (7.071\text{ V})(1.414\text{ A}) = \mathbf{+10\ W}$

t_5: $v_L = 0$ V, $p_L = v_L i_L = \mathbf{0\ W}$

b. The resulting plot of v_L, i_L, and p_L appears in Fig. 19.11.

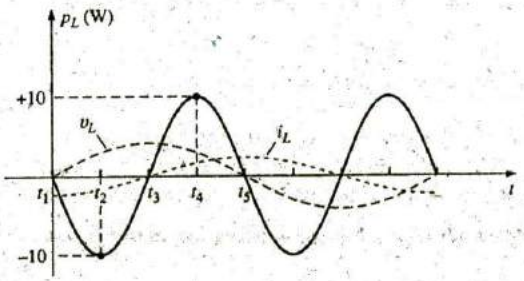


FIG. 19.11
Power curve for Example 19.2.

c. The average value for the curve in Fig. 19.11 is 0 W over one full cycle of the applied voltage. The peak value of the curve is 10 W, which compares directly with that obtained from the product

$$VI = \frac{V_m I_m}{2} = \frac{(10\text{ V})(2\text{ A})}{2} = \mathbf{10\ W}$$

d. The energy stored or released during each pulse of the power curve is

$$W_L = \frac{VI}{\omega_1} = \frac{V_m I_m}{2\omega_1} = \frac{(10\text{ V})(2\text{ A})}{2(200\text{ rad/s})} = \mathbf{50\ mJ}$$

19.6 CAPACITIVE CIRCUIT

For a purely capacitive circuit (such as that in Fig. 19.12), i leads v by 90°, as shown in Fig. 19.13. Therefore, in Eq. (19.1), $\theta = -90°$. Substituting $\theta = -90°$ into Eq. (19.1), we obtain

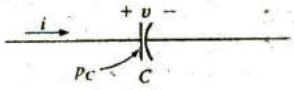


FIG. 19.12
Defining the power level for a purely capacitive load.

$$p_C = VI\cos(-90°)(1-\cos 2\omega t) + VI\sin(-90°)(\sin 2\omega t)$$
$$= 0 - VI\sin 2\omega t$$

or
$$\boxed{p_C = -VI\sin 2\omega t} \tag{19.19}$$

where $-VI\sin 2\omega t$ is a negative sine wave with twice the frequency of either input (v or i) and a peak value of VI. Again, note the absence of an average or constant term.

Plotting the waveform for p_C (Fig. 19.13) gives us

T_1 = period of either input quantity

T_2 = period of p_C curve

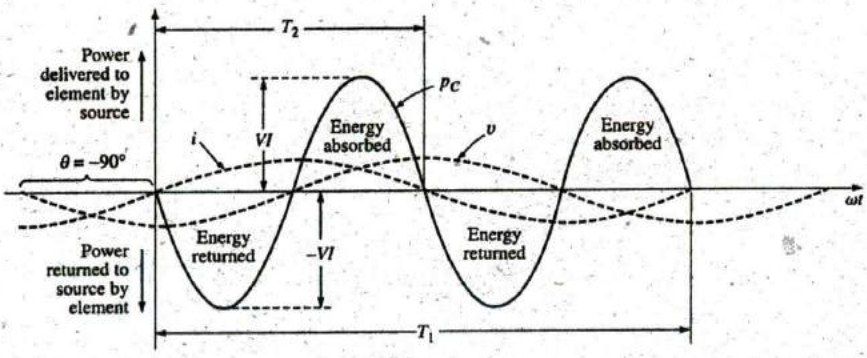


FIG. 19.13
The power curve for a purely capacitive load.

Note that the same situation exists here for the p_C curve as existed for the p_L curve. The power delivered by the source to the capacitor is exactly equal to that returned to the source by the capacitor over one full cycle.

The net flow of power to the pure (ideal) capacitor is zero over a full cycle,

and no energy is lost in the transaction. The power absorbed or returned by the capacitor at any instant of time t_1 can be found by substituting t_1 into Eq. (19.19).

The reactive power associated with the capacitor is equal to the peak value of the p_C curve, as follows:

$$\boxed{Q_C = VI} \quad \text{(VAR)} \tag{19.20}$$

However, since $V = IX_C$ and $I = V/X_C$, the reactive power to the capacitor can also be written

$$\boxed{Q_C = I^2X_C} \quad \text{(VAR)} \tag{19.21}$$

and
$$\boxed{Q_C = \frac{V^2}{X_C}} \quad \text{(VAR)} \tag{19.22}$$

The apparent power associated with the capacitor is

$$\boxed{S = VI} \quad \text{(VA)} \tag{19.23}$$

and the average power is $P = 0$, as noted from Eq. (19.19) or Fig. 19.13. The power factor is, therefore,

$$F_p = \cos\theta = \frac{P}{S} = \frac{0}{VI} = 0$$

The energy stored by the capacitor during the positive portion of the cycle (Fig. 19.13) is equal to that returned during the negative portion and can be determined using the equation $W = Pt$.

Proceeding in a manner similar to that used for the inductor, we can show that

$$\boxed{W_C = \frac{VIT_2}{\pi}} \quad \text{(J)} \qquad (19.24)$$

or, since $T_2 = 1/f_2$, where f_2 is the frequency of the p_C curve,

$$\boxed{W_C = \frac{VI}{\pi f_2}} \quad \text{(J)} \qquad (19.25)$$

In terms of the frequency f_1 of the input quantities v and i,

$$W_C = \frac{VI}{\pi(2f_1)} = \frac{VI}{\omega_1} = \frac{V(V\omega_1 C)}{\omega_1}$$

and

$$\boxed{W_C = CV^2} \quad \text{(J)} \qquad (19.26)$$

providing an equation for the energy stored or released by the capacitor in one half-cycle of the applied voltage in terms of the capacitance and rms value of the voltage squared.

19.7 THE POWER TRIANGLE

The three quantities **average power**, **apparent power**, and **reactive power** can be related in the vector domain by

$$\boxed{\mathbf{S} = \mathbf{P} + \mathbf{Q}} \qquad (19.27)$$

with

$$\mathbf{P} = P\angle 0° \qquad \mathbf{Q}_L = Q_L\angle 90° \qquad \mathbf{Q}_C = Q_C\angle -90°$$

For an inductive load, the *phasor power* **S**, as it is often called, is defined by

$$\mathbf{S} = P + jQ_L$$

as shown in Fig. 19.14.

The 90° shift in Q_L from P is the source of another term for reactive power: *quadrature power*.

For a capacitive load, the phasor power **S** is defined by

$$\mathbf{S} = P - jQ_C$$

as shown in Fig. 19.15.

If a network has both capacitive and inductive elements, the reactive component of the power triangle will be determined by the *difference* between the reactive power delivered to each. If $Q_L > Q_C$, the resultant

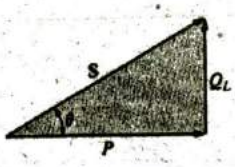


FIG. 19.14
Power diagram for inductive loads.

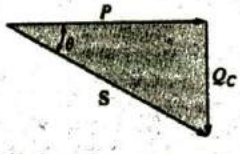


FIG. 19.15
Power diagram for capacitive loads.

power triangle will be similar to Fig. 19.14. If $Q_C > Q_L$, the resultant power triangle will be similar to Fig. 19.15.

That the total reactive power is the difference between the reactive powers of the inductive and capacitive elements can be demonstrated by considering Eqs. (19.11) and (19.19). These equations have been used to plot the reactive power delivered to each reactive element for a series L-C circuit on the same set of axes in Fig. 19.16. The reactive elements were chosen such that $X_L > X_C$. Note that the power curve for each is exactly 180° out of phase. The curve for the resultant reactive power is therefore determined by the algebraic resultant of the two at each instant of time. Since the reactive power is defined as the peak value, the reactive component of the power triangle is as indicated in Fig. 19.16: $I^2 (X_L - X_C)$.

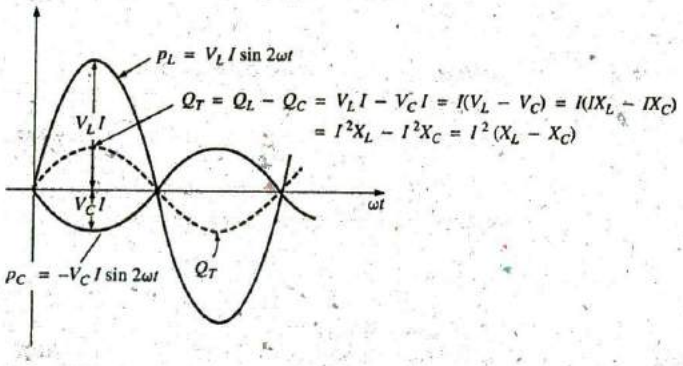


FIG. 19.16

Demonstrating why the net reactive power is the difference between that delivered to inductive and capacitive elements.

An additional verification can be derived by first considering the impedance diagram of a series R-L-C circuit (Fig. 19.17). If we multiply each radius vector by the current squared (I^2), we obtain the results shown in Fig. 19.18, which is the power triangle for a predominantly inductive circuit.

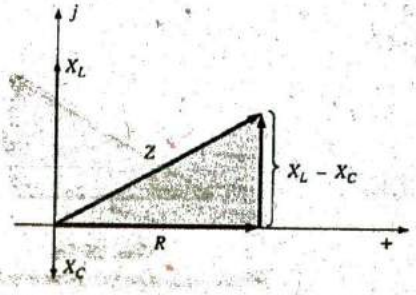


FIG. 19.17

Impedance diagram for a series R-L-C circuit.

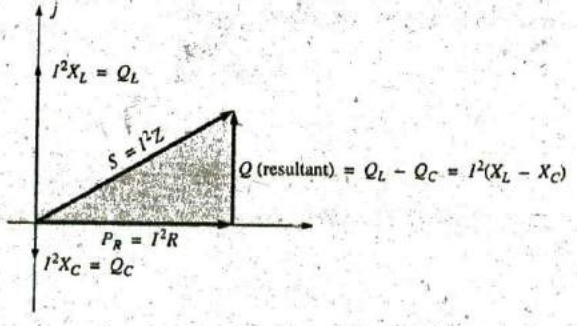


FIG. 19.18

The result of multiplying each vector in Fig. 19.17 by I^2 for a series R-L-C circuit.

Since the reactive power and average power are always angled 90° to each other, the three powers are related by the Pythagorean theorem; that is,

$$S^2 = P^2 + Q^2 \tag{19.28}$$

Therefore, the third power can always be found if the other two are known.

It is particularly interesting that the equation

$$\boxed{\mathbf{S} = \mathbf{V}\mathbf{I}^*} \tag{19.29}$$

will provide the vector form of the apparent power of a system. Here, $\mathbf{V}$ is the voltage across the system, and $\mathbf{I}^*$ is the complex conjugate of the current.

Consider, for example, the simple *R-L* circuit in Fig. 19.19, where

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{10\text{ V}\angle 0^\circ}{3\,\Omega + j4\,\Omega} = \frac{10\text{ V}\angle 0^\circ}{5\,\Omega\angle 53.13^\circ} = 2\text{ A}\angle -53.13^\circ$$

The real power (the term *real* being derived from the positive real axis of the complex plane) is

$$P = I^2R = (2\text{ A})^2(3\,\Omega) = 12\text{ W}$$

and the reactive power is

$$Q_L = I^2X_L = (2\text{ A})^2(4\,\Omega) = 16\text{ VAR}(L)$$

with $$\mathbf{S} = P + jQ_L = 12\text{ W} + j\,16\text{ VAR}\,(L) = 20\text{ VA}\angle 53.13^\circ$$

as shown in Fig. 19.20. Applying Eq. (19.29) yields

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* = (10\text{ V}\angle 0^\circ)(2\text{ A}\angle +53.13^\circ) = 20\text{ VA}\angle 53.13^\circ$$

as obtained above.

The angle θ associated with S and appearing in Figs. 19.14, 19.15, and 19.20 is the power-factor angle of the network. Since

$$P = VI\cos\theta$$

or

$$P = S\cos\theta$$

then

$$\boxed{F_p = \cos\theta = \frac{P}{S}} \tag{19.30}$$

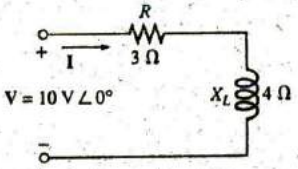


FIG. 19.19
Demonstrating the validity of Eq. (19.29).

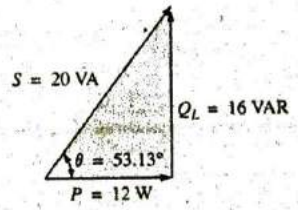


FIG. 19.20
The power triangle for the circuit in Fig. 19.19.

19.8 THE TOTAL *P*, *Q*, AND *S*

The total number of watts, volt-amperes reactive, and volt-amperes, and the power factor of any system can be found using the following procedure:

1. ***Find the real power and reactive power for each branch of the circuit.***
2. ***The total real power of the system (P_T) is then the sum of the average power delivered to each branch.***
3. ***The total reactive power (Q_T) is the difference between the reactive power of the inductive loads and that of the capacitive loads.***
4. ***The total apparent power is $S_T = \sqrt{P_T^2 + Q_T^2}$.***
5. ***The total power factor is P_T/S_T.***

There are two important points in the above procedure. First, the total apparent power must be determined from the total average and reactive powers and cannot be determined from the apparent powers of each branch. Second, and more important, it is *not necessary* to consider the series-parallel arrangement of branches. In other words, the total real, reactive, or apparent power is independent of whether the

loads are in series, parallel, or series-parallel. The following examples demonstrate the relative ease with which all of the quantities of interest can be found.

EXAMPLE 19.3 Find the total number of watts, volt-amperes reactive, volt-amperes, and power factor F_p of the network in Fig. 19.21. Draw the power triangle and find the current in phasor form.

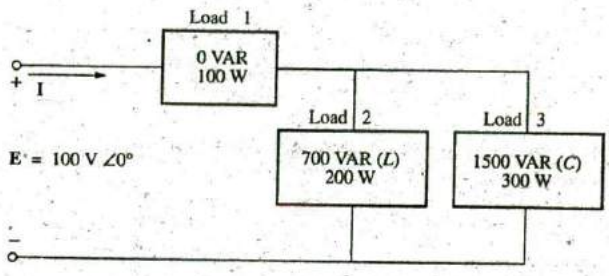


FIG. 19.21
Example 19.3.

Solution: Construct a table such as shown in Table 19.1.

TABLE 9.1

Load	W	VAR	VA
1	100	0	100
2	200	700 (*L*)	$\sqrt{(200)^2 + (700)^2} = 728.0$
3	300	1500 (*C*)	$\sqrt{(300)^2 + (1500)^2} = 1529.71$
	$P_T = 600$	$Q_T = 800(C)$	$S_T = \sqrt{(600)^2 + (800)^2} = 1000$
	Total power dissipated	Resultant reactive power of network	(Note that $S_T \neq$ sum of each branch: $1000 \neq 100 + 728 + 1529.71$)

Thus,

$$F_p = \frac{P_T}{S_T} = \frac{600 \text{ W}}{1000 \text{ VA}} = \mathbf{0.6 \text{ leading } (C)}$$

The power triangle is shown in Fig. 19.22.

Since $S_T = VI = 1000$ VA, $I = 1000$ VA/100 V $= 10$ A; and since θ of $\cos\theta = F_p$ is the angle between the input voltage and current,

$$\mathbf{I = 10 \text{ A} \angle +53.13^\circ}$$

The plus sign is associated with the phase angle since the circuit is predominantly capacitive.

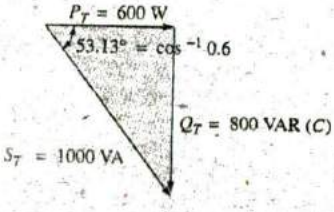


FIG. 19.22
Power triangle for Example 19.3.

EXAMPLE 19.4

a. Find the total number of watts, volt-amperes reactive, volt-amperes, and power factor F_p for the network in Fig. 19.23.
b. Sketch the power triangle.
c. Find the energy dissipated by the resistor over one full cycle of the input voltage if the frequency of the input quantities is 60 Hz.

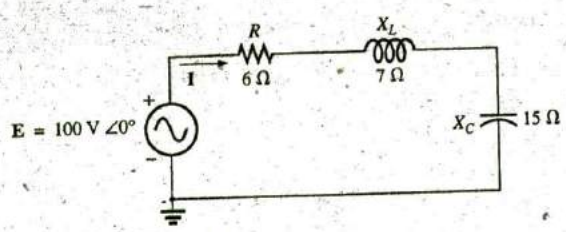


FIG. 19.23
Example 19.4.

d. Find the energy stored in, or returned by, the capacitor or inductor over one half-cycle of the power curve for each if the frequency of the input quantities is 60 Hz.

Solutions:

a. $$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\text{ V}\angle 0^\circ}{6\,\Omega + j\,7\,\Omega - j\,15\,\Omega} = \frac{100\text{ V}\angle 0^\circ}{10\,\Omega\angle -53.13^\circ}$$
$$= 10\text{ A}\angle 53.13^\circ$$

$$\mathbf{V}_R = (10\text{ A}\angle 53.13^\circ)(6\,\Omega\angle 0^\circ) = 60\text{ V}\angle 53.13^\circ$$

$$\mathbf{V}_L = (10\text{ A}\angle 53.13^\circ)(7\,\Omega\angle 90^\circ) = 70\text{ V}\angle 143.13^\circ$$

$$\mathbf{V}_C = (10\text{ A}\angle 53.13^\circ)(15\,\Omega\angle -90^\circ) = 150\text{ V}\angle -36.87^\circ$$

$$P_T = EI\cos\theta = (100\text{ V})(10\text{ A})\cos 53.13^\circ = \mathbf{600\text{ W}}$$
$$= I^2R = (10\text{ A})^2(6\,\Omega) = \mathbf{600\text{ W}}$$
$$= \frac{V_R^2}{R} = \frac{(60\text{ V})^2}{6} = \mathbf{600\text{ W}}$$

$$S_T = EI = (100\text{ V})(10\text{ A}) = \mathbf{1000\text{ VA}}$$
$$= I^2Z_T = (10\text{ A})^2(10\,\Omega) = \mathbf{1000\text{ VA}}$$
$$= \frac{E^2}{Z_T} = \frac{(100\text{ V})^2}{10\,\Omega} = \mathbf{1000\text{ VA}}$$

$$Q_T = EI\sin\theta = (100\text{ V})(10\text{ A})\sin 53.13^\circ = \mathbf{800\text{ VAR}}$$
$$= Q_C - Q_L$$
$$= I^2(X_C - X_L) = (10\text{ A})^2(15\,\Omega - 7\,\Omega) = \mathbf{800\text{ VAR}}$$

$$Q_T = \frac{V_C^2}{X_C} - \frac{V_L^2}{X_L} = \frac{(150\text{ V})^2}{15\,\Omega} - \frac{(70\text{ V})^2}{7\,\Omega}$$
$$= 1500\text{ VAR} - 700\text{ VAR} = \mathbf{800\text{ VAR}}$$

$$F_p = \frac{P_T}{S_T} = \frac{600\text{ W}}{1000\text{ VA}} = \mathbf{0.6\text{ leading}\,(C)}$$

b. The power triangle is as shown in Fig. 19.24.

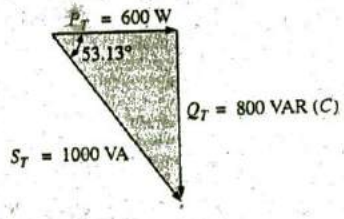


FIG. 19.24
Power triangle for Example 19.4.

c. $$W_R = \frac{V_R I}{f_1} = \frac{(60\text{ V})(10\text{ A})}{60\text{ Hz}} = \mathbf{10\text{ J}}$$

d. $$W_L = \frac{V_L I}{\omega_1} = \frac{(70\text{ V})(10\text{ A})}{(2\pi)(60\text{ Hz})} = \frac{700\text{ J}}{377} = \mathbf{1.86\text{ J}}$$

$$W_C = \frac{V_C I}{\omega_1} = \frac{(150\text{ V})(10\text{ A})}{377\text{ rad/s}} = \frac{1500\text{ J}}{377} = \mathbf{3.98\text{ J}}$$

EXAMPLE 19.5 For the system in Fig. 19.25,

a. Find the average power, apparent power, reactive power, and F_p for each branch.

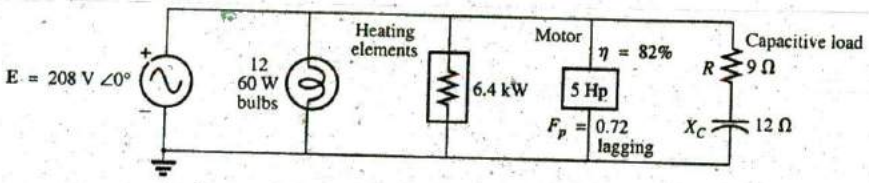


FIG. 19.25
Example 19.5.

b. Find the total number of watts, volt-amperes reactive, volt-amperes, and power factor of the system. Sketch the power triangle.
c. Find the source current I.

Solutions:

a. *Bulbs:*

Total dissipation of applied power

$$P_1 = 12(60\text{ W}) = \mathbf{720\text{ W}}$$
$$Q_1 = \mathbf{0\text{ VAR}}$$
$$S_1 = P_1 = \mathbf{720\text{ VA}}$$
$$F_{p_1} = \mathbf{1}$$

Heating elements:

Total dissipation of applied power

$$P_2 = \mathbf{6.4\text{ kW}}$$
$$Q_2 = \mathbf{0\text{ VAR}}$$
$$S_2 = P_2 = \mathbf{6.4\text{ kVA}}$$
$$F_{p_2} = \mathbf{1}$$

Motor:

$$\eta = \frac{P_o}{P_i} \rightarrow P_i = \frac{P_o}{\eta} = \frac{5(746\text{ W})}{0.82} = \mathbf{4548.78\text{ W}} = P_3$$

$F_p = \mathbf{0.72\text{ lagging}}$

$$P_3 = S_3 \cos\theta \rightarrow S_3 = \frac{P_3}{\cos\theta} = \frac{4548.78\text{ W}}{0.72} = \mathbf{6317.75\text{ VA}}$$

Also, $\theta = \cos^{-1} 0.72 = 43.95°$, so that

$$Q_3 = S_3 \sin\theta = (6317.75\text{ VA})(\sin 43.95°)$$
$$= (6317.75\text{ VA})(0.694) = \mathbf{4384.71\text{ VAR}}\ (L)$$

Capacitive load:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{208\text{ V}\angle 0°}{9\,\Omega - j\,12\,\Omega} = \frac{208\text{ V}\angle 0°}{15\,\Omega\angle -53.13°} = 13.87\text{ A}\angle 53.13°$$

$$P_4 = I^2R = (13.87\text{ A})^2 \cdot 9\,\Omega = \mathbf{1731.39\text{ W}}$$

$$Q_4 = I^2X_C = (13.87\text{ A})^2 \cdot 12\,\Omega = \mathbf{2308.52\text{ VAR}}\ (C)$$

$$S_4 = \sqrt{P_4^2 + Q_4^2} = \sqrt{(1731.39\text{ W})^2 + (2308.52\text{ VAR})^2}$$
$$= \mathbf{2885.65\text{ VA}}$$

$$F_p = \frac{P_4}{S_4} = \frac{1731.39\text{ W}}{2885.65\text{ VA}} = \mathbf{0.6\text{ leading}}$$

b. $P_T = P_1 + P_2 + P_3 + P_4$
$= 720\text{ W} + 6400\text{ W} + 4548.78\text{ W} + 1731.39\text{ W}$
$= \mathbf{13{,}400.17\text{ W}}$

$Q_T = \pm Q_1 \pm Q_2 \pm Q_3 \pm Q_4$
$= 0 + 0 + 4384.71\text{ VAR}(L) - 2308.52\text{ VAR}(C)$
$= \mathbf{2076.19\text{ VAR}(L)}$

$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(13{,}400.17\text{ W})^2 + (2076.19\text{ VAR})^2}$
$= 13{,}560.06\text{ VA}$

$F_p = \dfrac{P_T}{S_T} = \dfrac{13.4\text{ kW}}{13{,}560.06\text{ VA}} = \mathbf{0.988\text{ lagging}}$

$\theta = \cos^{-1} 0.988 = 8.89°$

Note Fig. 19.26.

c. $S_T = EI \rightarrow I = \dfrac{S_T}{E} = \dfrac{13{,}559.89\text{ VA}}{208\text{ V}} = 65.19\text{ A}$

Lagging power factor: **E** leads **I** by 8.89°, and

$$\mathbf{I = 65.19\text{ A} \angle -8.89°}$$

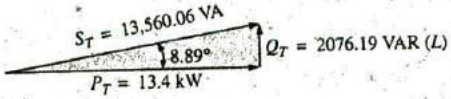


FIG. 19.26
Power triangle for Example 19.5.

EXAMPLE 19.6 An electrical device is rated 5 kVA, 100 V at a 0.6 power-factor lag. What is the impedance of the device in rectangular coordinates?

Solution:

$$S = EI = 5000\text{ VA}$$

Therefore,
$$I = \frac{5000\text{ VA}}{100\text{ V}} = 50\text{ A}$$

For $F_p = 0.6$, we have

$$\theta = \cos^{-1} 0.6 = 53.13°$$

Since the power factor is lagging, the circuit is predominantly inductive, and **I** lags **E**. Or, for $\mathbf{E} = 100\text{ V} \angle 0°$,

$$\mathbf{I} = 50\text{ A} \angle -53.13°$$

However,

$$\mathbf{Z}_T = \frac{\mathbf{E}}{\mathbf{I}} = \frac{100\text{ V} \angle 0°}{50\text{ A} \angle -53.13°} = 2\,\Omega \angle 53.13° = \mathbf{1.2\,\Omega + j\,1.6\,\Omega}$$

which is the impedance of the circuit in Fig. 19.27.

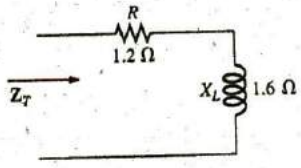


FIG. 19.27
Example 19.6.

19.9 POWER-FACTOR CORRECTION

The design of any power transmission system is very sensitive to the magnitude of the current in the lines as determined by the applied loads. Increased currents result in increased power losses (by a squared factor since $P = I^2R$) in the transmission lines due to the resistance of the lines. Heavier currents also require larger conductors, increasing the amount of copper needed for the system, and, quite obviously, they require increased generating capacities by the utility company.

Every effort must therefore be made to keep current levels at a minimum. Since the line voltage of a transmission system is fixed, the apparent power is directly related to the current level. In turn, the smaller is the net

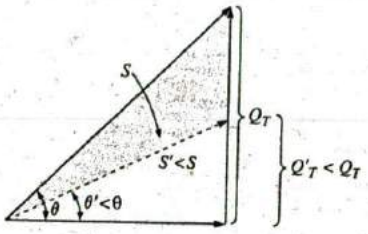


FIG. 19.28
Demonstrating the impact of power-factor correction on the power triangle of a network.

apparent power, the smaller is the current drawn from the supply. Minimum current is therefore drawn from a supply when $S = P$ and $Q_T = 0$. Note the effect of decreasing levels of Q_T on the length (and magnitude) of S in Fig. 19.28 for the same real power. Note also that the power-factor angle approaches zero degrees and F_p approaches 1, revealing that the network is appearing more and more resistive at the input terminals.

The process of introducing reactive elements to bring the power factor closer to unity is called **power-factor correction.** Since most loads are inductive, the process normally involves introducing elements with capacitive terminal characteristics having the sole purpose of improving the power factor.

In Fig. 19.29(a), for instance, an inductive load is drawing a current I_L that has a real and an imaginary component. In Fig. 19.29(b), a capacitive load was added in parallel with the original load to raise the power factor of the total system to the unity power-factor level. Note that by placing all the elements in parallel, the load still receives the same terminal voltage and draws the same current I_L. In other words, the load is unaware of and unconcerned about whether it is hooked up as shown in Fig. 19.29(a) or Fig. 19.29(b).

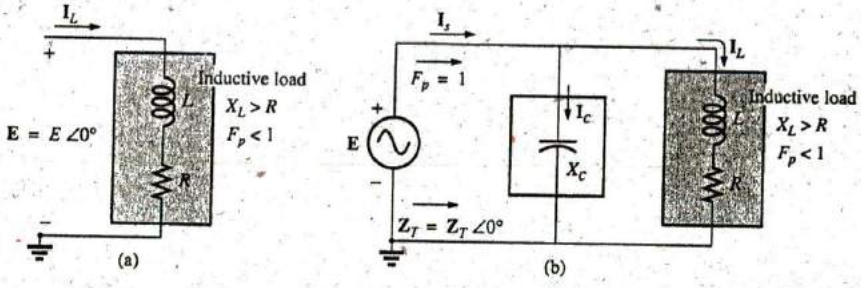


FIG. 19.29
Demonstrating the impact of a capacitive element on the power factor of a network.

Solving for the source current in Fig. 19.29(b) gives

$$\begin{aligned}\mathbf{I}_s &= \mathbf{I}_C + \mathbf{I}_L \\ &= jI_C(I_{mag}) + I_L(R_e) + jI_L(I_{mag}) = jI_C + I_L - jI_L \\ &= I_L(R_e) + j[I_L(I_{mag}) + I_C(I_{mag})] = I_L + j[I_C + I_L]\end{aligned}$$

If X_C is chosen such that $I_C = I_L$, then

$$\mathbf{I}_s = I_L + j(0) = I_L \angle 0°$$

The result is a source current whose magnitude is simply equal to the real part of the inductive load current, which can be considerably less than the magnitude of the load current in Fig. 19.29(a). In addition, since the phase angle associated with both the applied voltage and the source current is the same, the system appears "resistive" at the input terminals, and all of the power supplied is absorbed, creating maximum efficiency for a generating utility.

EXAMPLE 19.7 A 5 hp motor with a 0.6 lagging power factor and an efficiency of 92% is connected to a 208 V, 60 Hz supply.

a. Establish the power triangle for the load.
b. Determine the power-factor capacitor that must be placed in parallel with the load to raise the power factor to unity.

c. Determine the change in supply current from the uncompensated to the compensated system.
d. Find the network equivalent of the above, and verify the conclusions.

Solutions:

a. Since 1 hp = 746 W,

$$P_o = 5 \text{ hp} = 5(746 \text{ W}) = 3730 \text{ W}$$

and $P_i(\text{drawn from the line}) = \dfrac{P_o}{\eta} = \dfrac{3730 \text{ W}}{0.92} = 4054.35 \text{ W}$

Also, $F_p = \cos\theta = 0.6$

and $\theta = \cos^{-1} 0.6 = 53.13°$

Applying $\tan\theta = \dfrac{Q_L}{P_i}$

we obtain $Q_L = P_i \tan\theta = (4054.35 \text{ W}) \tan 53.13°$
$= 5405.8 \text{ VAR}(L)$

and

$$S = \sqrt{P_i^2 + Q_L^2} = \sqrt{(4054.35 \text{ W})^2 + (5405.8 \text{ VAR})^2}$$
$$= 6757.25 \text{ VA}$$

The power triangle appears in Fig. 19.30.

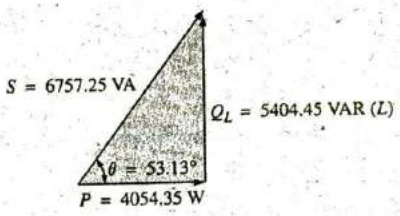


FIG. 19.30
Initial power triangle for the load in Example 19.7.

b. A net unity power-factor level is established by introducing a capacitive reactive power level of 5405.8 VAR to balance Q_L. Since

$$Q_C = \frac{V^2}{X_C}$$

then $X_C = \dfrac{V^2}{Q_C} = \dfrac{(208 \text{ V})^2}{5405.8 \text{ VAR}(C)} = 8\ \Omega$

and $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{(2\pi)(60 \text{ Hz})(8\ \Omega)} = \mathbf{331.6\ \mu F}$

c. **At $0.6F_p$,**

$$S = VI = 6757.25 \text{ VA}$$

and $I = \dfrac{S}{V} = \dfrac{6757.25 \text{ VA}}{208 \text{ V}} = \mathbf{32.49 \text{ A}}$

At unity F_p,

$$S = VI = 4054.35 \text{ VA}$$

and $I = \dfrac{S}{V} = \dfrac{4054.35 \text{ VA}}{208 \text{ V}} = \mathbf{19.49 \text{ A}}$

producing a 40% reduction in supply current.

d. For the motor, the angle by which the applied voltage leads the current is

$$\theta = \cos^{-1} 0.6 = 53.13°$$

and $P = EI\cos\theta = 4054.35$ W, from above, so that

$$I = \frac{P}{E\cos\theta} = \frac{4054.35 \text{ W}}{(208 \text{ V})(0.6)} = \mathbf{32.49 \text{ A}} \quad \text{(as above)}$$

resulting in

$$\mathbf{I} = 32.49\ \text{A} \angle -53.13^\circ$$

Therefore,

$$\mathbf{Z} = \frac{\mathbf{E}}{\mathbf{I}} = \frac{208\ \text{V} \angle 0^\circ}{32.49\ \text{A} \angle -53.13^\circ} = 6.4\ \Omega \angle 53.13^\circ$$
$$= 3.84\ \Omega + j\,5.12\ \Omega$$

as shown in Fig. 19.31(a).

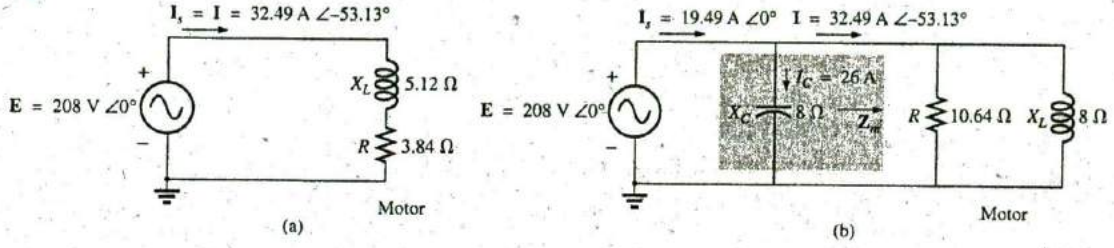


FIG. 19.31

Demonstrating the impact of power-factor corrections on the source current.

The equivalent parallel load is determined from

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{Z}} = \frac{1}{6.4\ \Omega \angle 53.13^\circ}$$
$$= 0.156\ \text{S} \angle -53.13^\circ = 0.0936\ \text{S} - j\,0.125\ \text{S}$$
$$= \frac{1}{10.68\ \Omega} + \frac{1}{j\,8\ \Omega}$$

as shown in Fig. 19.31(b).

It is now clear that the effect of the 8 Ω inductive reactance can be compensated for by a parallel capacitive reactance of 8 Ω using a power-factor correction capacitor of 332 μF.

Since

$$\mathbf{Y}_T = \frac{1}{-jX_C} + \frac{1}{R} + \frac{1}{+jX_L} = \frac{1}{R}$$

$$I_s = EY_T = E\left(\frac{1}{R}\right) = (208\ \text{V})\left(\frac{1}{10.68\ \Omega}\right) = \mathbf{19.49\ A} \quad \text{as above}$$

In addition, the magnitude of the capacitive current can be determined as follows:

$$I_C = \frac{E}{X_C} = \frac{208\ \text{V}}{8\ \Omega} = \mathbf{26\ A}$$

EXAMPLE 19.8

a. A small industrial plant has a 10 kW heating load and a 20 kVA inductive load due to a bank of induction motors. The heating elements are considered purely resistive ($F_p = 1$), and the induction motors have a lagging power factor of 0.7. If the supply is 1000 V at 60 Hz, determine the capacitive element required to raise the power factor to 0.95.

b. Compare the levels of current drawn from the supply.

Solutions:

a. For the induction motors,

$$S = VI = 20\text{ kVA}$$
$$P = S\cos\theta = (20 \times 10^3\text{ VA})(0.7) = 14\text{ kW}$$
$$\theta = \cos^{-1} 0.7 \cong 45.6°$$

and

$$Q_L = VI\sin\theta = (20\text{ kVA})(0.714) = 14.28\text{ kVAR}(L)$$

The power triangle for the total system appears in Fig. 19.32. Note the addition of real powers and the resulting S_T:

$$S_T = \sqrt{(24\text{ kW})^2 + (14.28\text{ kVAR})^2} = 27\ 93\text{ kVA}$$

with $$I_T = \frac{S_T}{E} = \frac{27.93\text{ kVA}}{1000\text{ V}} = \mathbf{27.93\text{ A}}$$

The desired power factor of 0.95 results in an angle between S and P of

$$\theta = \cos^{-1} 0.95 = 18.91°$$

changing the power triangle to that in Fig. 19.33:

$$\text{with } \tan\theta = \frac{Q'_L}{P_T} \rightarrow Q'_L = P_T\tan\theta = (24\text{ kW})(\tan 18.19°)$$
$$= (24\text{ kW})(0.329) = 7.9\text{ kVAR}(L)$$

The inductive reactive power must therefore be reduced by

$$Q_L - Q'_L = 14.28\text{ kVAR}(L) - 7.9\text{ kVAR}(L) = 6.38\text{ kVAR}(L)$$

Therefore, $Q_C = 6.38$ kVAR, and using

$$Q_C = \frac{E^2}{X_C}$$

we obtain

$$X_C = \frac{E^2}{Q_C} = \frac{(10^3\text{ V})^2}{6.38\text{ kVAR}} = 156.74\ \Omega$$

and $$C = \frac{1}{2\pi f X_C} = \frac{1}{(2\pi)(60\text{ Hz})(156.74\ \Omega)} = \mathbf{16.93\ \mu F}$$

b. $$S_T = \sqrt{(24\text{ kW})^2 + [7.9\text{ kVAR}(L)]^2}$$
$$= 25.27\text{ kVA}$$
$$I_T = \frac{S_T}{E} = \frac{25.27\text{ kVA}}{1000\text{ V}} = \mathbf{25.27\text{ A}}$$

The new I_T is

$$I_T = \mathbf{25.27\text{ A}\ \angle 27.93\text{ A}} \quad \text{(original)}$$

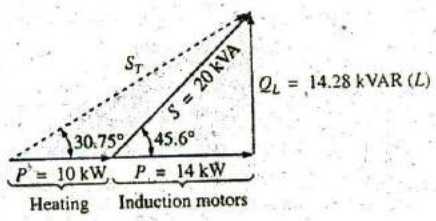


FIG. 19.32
Initial power triangle for the load in Example 19.8.

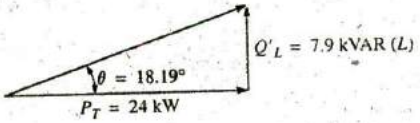


FIG. 19.33
Power triangle for the load in Example 19.8 after raising the power factor to 0.95.

19.10 POWER METERS

The power meter in Fig. 19.34 uses a sophisticated electronic package to sense the voltage and current levels and has an analog-to-digital conversion unit that displays the levels in digital form. It is capable of providing



FIG. 19.34
Digital single-phase and three-phase power meter.
(Courtesy of AEMC® Instruments, Foxborough, MA.)



FIG. 19.35
Power quality analyzer capable of displaying the power in watts, the current in amperes, and the voltage in volts.
(Courtesy of Fluke Corporation. Reproduced with Permission.)

a digital readout for distorted nonsinusoidal waveforms, and it can provide the phase power, total power, apparent power, reactive power, and power factor. It can also measure currents up to 500 A, voltages up to 600 V, and frequencies from 30 Hz to 1000 Hz.

The power quality analyzer in Fig. 19.35 can also display the real, reactive, and apparent power levels along with the power factor. However, it has a broad range of other options, including providing the harmonic content of up to 51 terms for the voltage, current, and power. The power range extends from 250 W to 2.5 MW, and the current can be read up to 1000 A. The meter can also be used to measure resistance levels from 500 Ω to 30 MΩ, capacitance levels from 50 nF to 500 μF, and temperature in both °C and °F.

19.11 EFFECTIVE RESISTANCE

The resistance of a conductor as determined by the equation $R = \rho(l/A)$ is often called the *dc, ohmic,* or *geometric* resistance. It is a constant quantity determined only by the material used and its physical dimensions. In ac circuits, the actual resistance of a conductor (called the **effective resistance**) differs from the dc resistance because of the varying currents and voltages that introduce effects not present in dc circuits.

These effects include radiation losses, skin effect, eddy currents, and hysteresis losses. The first two effects apply to any network, while the latter two are concerned with the additional losses introduced by the presence of ferromagnetic materials in a changing magnetic field.

Experimental Procedure

The effective resistance of an ac circuit cannot be measured by the ratio V/I since this ratio is now the impedance of a circuit that may have both resistance and reactance. The effective resistance can be found, however, by using the power equation $P = I^2R$, where

$$R_{\text{eff}} = \frac{P}{I^2} \tag{19.31}$$

A wattmeter and an ammeter are therefore necessary for measuring the effective resistance of an ac circuit.

Radiation Losses

Let us now examine the various losses in greater detail. The **radiation loss** is the loss of energy in the form of electromagnetic waves during the transfer of energy from one element to another. This loss in energy requires that the input power be larger to establish the same current I, causing R to increase as determined by Eq. (19.31). At a frequency of 60 Hz, the effects of radiation losses can be completely ignored. However, at radio frequencies, this is an important effect and may in fact become the main effect in an electromagnetic device such as an antenna.

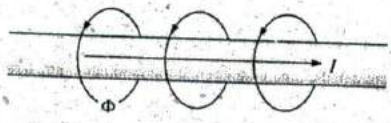


FIG. 19.36
Demonstrating the skin effect on the effective resistance of a conductor.

Skin Effect

The explanation of **skin effect** requires the use of some basic concepts previously described. Remember from Chapter 12 that a magnetic field exists around every current-carrying conductor (Fig. 19.36). Since the amount of charge flowing in ac circuits changes with time, the magnetic

field surrounding the moving charge (current) also changes. Recall also that a wire placed in a changing magnetic field will have an induced voltage across its terminals as determined by Faraday's law, $e = N \times (d\phi/dt)$. The higher the frequency of the changing flux as determined by an alternating current, the greater is the induced voltage.

For a conductor carrying alternating current, the changing magnetic field surrounding the wire links the wire itself, thus developing within the wire an induced voltage that opposes the original flow of charge or current. These effects are more pronounced at the center of the conductor than at the surface because the center is linked by the changing flux inside the wire as well as that outside the wire. As the frequency of the applied signal increases, the flux linking the wire changes at a greater rate. An increase in frequency therefore increases the counter-induced voltage at the center of the wire to the point where the current, for all practical purposes, flows on the surface of the conductor. At 60 Hz, the skin effect is almost noticeable. However, at radio frequencies, the skin effect is so pronounced that conductors are frequently made hollow because the center part is relatively ineffective. The skin effect, therefore, reduces the effective area through which the current can flow, and it causes the resistance of the conductor, given by the equation $R\uparrow = \rho(l/A\downarrow)$, to increase.

Hysteresis and Eddy Current Losses

As mentioned earlier, hysteresis and eddy current losses appear when a ferromagnetic material is placed in the region of a changing magnetic field. To describe eddy current losses in greater detail, we consider the effects of an alternating current passing through a coil wrapped around a ferromagnetic core. As the alternating current passes through the coil, it develops a changing magnetic flux ϕ linking both the coil and the core that develops an induced voltage within the core as determined by Faraday's law. This induced voltage and the geometric resistance of the core $R_C = \rho(l/A)$ cause currents to be developed within the core, $i_{core} = (e_{ind}/R_C)$, called **eddy currents.** The currents flow in circular paths, as shown in Fig. 19.37, changing direction with the applied ac potential.

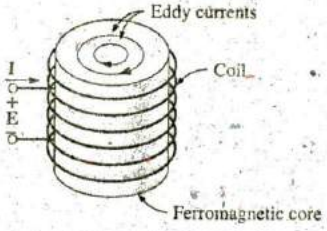


FIG. 19.37
Defining the eddy current losses of a ferromagnetic core.

The eddy current losses are determined by

$$P_{eddy} = i_{eddy}^2 R_{core}$$

The magnitude of these losses is determined primarily by the type of core used. If the core is nonferromagnetic—and has a high resistivity like wood or air—the eddy current losses can be neglected. In terms of the frequency of the applied signal and the magnetic field strength produced, the eddy current loss is proportional to the square of the frequency times the square of the magnetic field strength:

$$P_{eddy} \propto f^2 B^2$$

Eddy current losses can be reduced if the core is constructed of thin, laminated sheets of ferromagnetic material insulated from one another and aligned parallel to the magnetic flux. Such construction reduces the magnitude of the eddy currents by placing more resistance in their path.

Hysteresis losses were described in Section 12.6. You will recall that in terms of the frequency of the applied signal and the magnetic field strength produced, the hysteresis loss is proportional to the frequency to the 1st power times the magnetic field strength to the nth power:

$$P_h \propto f^1 B^n$$

where n can vary from 1.4 to 2.6, depending on the material under consideration.

Hysteresis losses can be effectively reduced by the injection of small amounts of silicon into the magnetic core, constituting some 2% or 3% of the total composition of the core. This must be done carefully, however, because too much silicon makes the core brittle and difficult to machine into the shape desired.

EXAMPLE 19.9

a. An air-core coil is connected to a 120 V, 60 Hz source as shown in Fig. 19.38. The current is found to be 5 A, and a wattmeter reading of 75 W is observed. Find the effective resistance and the inductance of the coil.

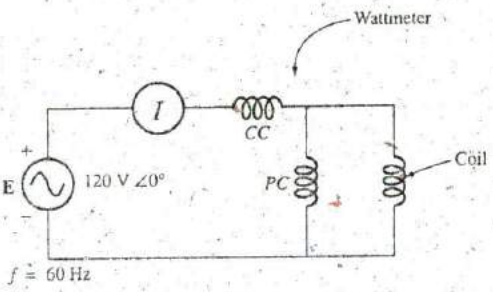


FIG. 19.38
The basic components required to determine the effective resistance and inductance of the coil.

b. A brass core is then inserted in the coil. The ammeter reads 4 A, and the wattmeter 80 W. Calculate the effective resistance of the core. To what do you attribute the increase in value over that in part (a)?

c. If a solid iron core is inserted in the coil, the current is found to be 2 A, and the wattmeter reads 52 W. Calculate the resistance and the inductance of the coil. Compare these values to those in part (a), and account for the changes.

Solutions:

a. $$R = \frac{P}{I^2} = \frac{75\text{ W}}{(5\text{ A})^2} = 3\ \Omega$$

$$Z_T = \frac{E}{I} = \frac{120\text{ V}}{5\text{ A}} = 24\ \Omega$$

$$X_L = \sqrt{Z_T^2 - R^2} = \sqrt{(24\ \Omega)^2 - (3\ \Omega)^2} = 23.81\ \Omega$$

and $$X_L = 2\pi fL$$

or $$L = \frac{X_L}{2\pi f} = \frac{23.81\ \Omega}{377\text{ rad/s}} = \mathbf{63.16\text{ mH}}$$

b. $$R = \frac{P}{I^2} = \frac{80\text{ W}}{(4\text{ A})^2} = \frac{80\ \Omega}{16} = 5\ \Omega$$

The brass core has less reluctance than the air core. Therefore, a greater magnetic flux density B will be created in it. Since $P_{\text{eddy}} \propto f^2B^2$, and

$P_{hys} \propto f^1 B^n$, as the flux density increases, the core losses and the effective resistance increase.

c. $$R = \frac{P}{I^2} = \frac{52\ \text{W}}{(2\ \text{A})^2} = \frac{52\ \Omega}{4} = 13\ \Omega$$

$$Z_T = \frac{E}{I} = \frac{120\ \text{V}}{2\ \text{A}} = 60\ \Omega$$

$$X_L = \sqrt{Z_T^2 - R^2} = \sqrt{(60\ \Omega)^2 - (13\ \Omega)^2} = 58.57\ \Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{58.57\ \Omega}{377\ \text{rad/s}} = \mathbf{155.36\ mH}$$

The iron core has less reluctance than the air or brass cores. Therefore, a greater magnetic flux density B will be developed in the core. Again, since $P_{eddy} \propto f^2 B^2$, and $P_{hys} \propto f^1 B^n$, the increased flux density will cause the core losses and the effective resistance to increase.

Since the inductance L is related to the change in flux by the equation $L = N(d\phi/di)$, the inductance will be greater for the iron core because the changing flux linking the core will increase.

19.12 APPLICATIONS

Portable Power Generators

Even though it may appear that 120 V ac are just an extension cord away, there are times—such as in a remote cabin, on a job site, or while camping—that we are reminded that not every corner of the globe is connected to an electric power source. As you travel farther away from large urban communities, gasoline generators such as shown in Fig. 19.39 appear in increasing numbers in hardware stores, lumber yards, and other retail establishments to meet the needs of the local community. Since ac generators are driven by a gasoline motor, they must be properly ventilated and cannot be run indoors. Usually, because of the noise and fumes that result, they are placed as far away as possible and are connected by a long, heavy-duty, weather-resistant extension cord. Any connection points must be properly protected and placed to ensure that the connections will not sit in a puddle of water or be sensitive to heavy rain or snow. Although there is some effort involved in setting up generators and constantly ensuring that they have enough gas, most users think that they are priceless.

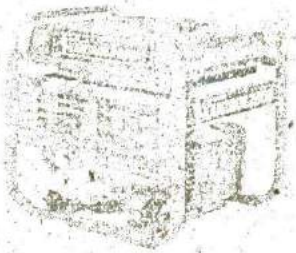


FIG. 19.39
Single-phase portable generator.
(Courtesy of Coleman Powermate, Inc.)

The vast majority of generators are built to provide between 1750 W and 5000 W of power, although larger units can provide up to 20,000 W. At first encounter, you may assume that 5000 W are more than adequate. However, keep in mind that the unit purchased should be rated at least 20% above your expected load because of surge currents that result when appliances, motors, tools, and so on, are turned on. Remember that even a light bulb develops a large turn-on current due to the cold, low-resistance state of the filament. If you work too closely to the rated capacity, experiences such as a severe drop in lighting can result when an electric saw is turned on—almost to the point where it appears that the lights go out altogether. Generators are like any other piece of equipment: If you apply a load that is too heavy, they will shut down. Most have protective fuses or circuit breakers to ensure that the excursions above rated conditions are monitored and not exceeded beyond reason. The 20% protective barrier drops the output power from a 5000 W unit to 4000 W, and already we

begin to wonder about the load we can apply. Although 4000 W are sufficient to run a number of 60 W bulbs, a TV, an oil burner, and so on, troubles develop whenever a unit is hooked up for direct heating (such as heaters, hair dryers, and clothes dryers). Even microwaves at 1200 W command quite a power drain. Add a small electric heater at 1500 W with six 60 W bulbs (360 W), a 250 W TV, and a 250 W oil burner, and then turn on an electric hair dryer at 1500 W—suddenly you are very close to your maximum of 4000 W. It doesn't take long to push the limits when it comes to energy-consuming appliances.

Table 19.2 provides a list of specifications for the broad range of portable gasoline generators. Since the heaviest part of a generator is the gasoline motor, anything over 5 hp gets pretty heavy, especially when you add the weight of the gasoline. Most good units providing over 2400 W will have receptacles for 120 V and 220 V at various current levels, with an outlet for 12 V dc. They are also built so that they tolerate outdoor conditions of a reasonable nature and can run continuously for long periods of time. At 120 V, a 5000 W unit can provide a maximum current of about 42 A.

TABLE 19.2

Specifications for portable gasoline-driven ac generators.

Continuous output power	1750–3000 W	2000–5000 W	2250–7500 W
Horsepower of gas motor	4–11 hp	5–14 hp	5–16 hp
Continuous output current	At 120 V: 15–25 A At 220 V(3ϕ): 8–14 A	At 120 V: 17–42 A At 220 V(3ϕ): 9–23 A	At 120 V: 19–63 A At 220 V(3ϕ): 10–34 A
Output voltage	120 V or 3ϕ: 120 V/220 V	120 V or 3ϕ: 120 V/220 V	120 V or 3ϕ: 120 V/220 V
Receptacles	2	2–4	2–4
Fuel tank	½ to 2 gallons gasoline	½ to 3 gallons gasoline	1 to 5 gallons gasoline

Business Sense

Because of the costs involved, every large industrial plant must continuously review its electric utility bill to ensure its accuracy and to consider ways to conserve energy. As described in this chapter, the power factor associated with the plant as a whole can have a measurable effect on the drain current and therefore the kVA drain on the power line. Power companies are aware of this problem and actually add a surcharge if the power factor fades below about 0.9. In other words, to ensure that the load appears as resistive in nature as possible, the power company asks users to try to ensure that their power factor is between 0.9 and 1 so that the kW demand is very close to the kVA demand.

Consider the following monthly bill for a fairly large industrial plant:

kWh consumption	146.5 MWh
peak kW demand	241 kW
kW demand	233 kW
kVA demand	250 kVA

The rate schedule provided by the local power authority is the following:

Energy First 450 kWh @ 22.3¢/kWh Next 12 MWh @ 17.1¢/kWh
Additional kWh @ 8.9¢/kWh

Power First 240 kW @ free
Additional kW @ $12.05/kW

Note that this rate schedule has an energy cost breakdown and a power breakdown. This second fee is the one sensitive to the overall power factor of the plant.

The electric bill for the month is then calculated as follows:

$$\begin{aligned}\text{Cost} &= (450\text{ kWh})(22.3¢/\text{kWh}) + (12\text{ MWh})(17.1¢/\text{kWh}) \\ &\quad + [146.2\text{ MWh} - (12\text{ MWh} + 450\text{ kWh})](8.9¢/\text{kWh}) \\ &= \$100.35 + \$2052.00 + \$11{,}903.75 \\ &= \mathbf{\$14{,}056.10}\end{aligned}$$

Before examining the effect of the power fee structure, we can find the overall power factor of the load for the month with the following ratio taken from the monthly statement:

$$F_p = \frac{P}{P_a} = \frac{233\text{ kW}}{250\text{ kVA}} = \mathbf{0.932}$$

Since the power factor is larger than 0.9, the chances are that there will not be a surcharge or that the surcharge will be minimal.

When the power component of the bill is determined, the kVA demand is multiplied by the magic number of 0.9 to determine a kW level at this power factor. This kW level is compared to the metered level, and the consumer pays for the higher level.

In this case, if we multiply the 250 kVA by 0.9, we obtain 225 kW, which is slightly less than the metered level of 233 kW. However, both levels are less than the free level of 240 kW, so there is no additional charge for the power component. The total bill remains at $14,056.10.

If the kVA demand of the bill were 388 kVA with the kW demand staying at 233 kW, the situation would change because 0.9 times 388 kVA would result in 349.2 kW, which is much greater than the metered 233 kW. The 349.2 kW would then be used to determine the bill as follows:

$$\begin{aligned}349.2\text{ kW} - 240\text{ kW} &= 109.2\text{ kW} \\ (109.2\text{ kW})(\$12.05/\text{kW}) &= \mathbf{\$1315.86}\end{aligned}$$

which is significant.

The total bill can then be determined as follows:

$$\begin{aligned}\text{Cost} &= \$14.056.10 + \$1{,}315.86 \\ &= \mathbf{\$15{,}371.96}\end{aligned}$$

Thus, the power factor of the load dropped to 233 kW/388 kVA = 0.6, which would put an unnecessary additional load on the power plant. It is certainly time to consider the power-factor-correction option as described in this text. It is not uncommon to see large capacitors sitting at the point where power enters a large industrial plant to perform a needed level of power-factor correction.

All in all, therefore, it is important to fully understand the impact of a poor power factor on a power plant—whether you someday work for the supplier or for the consumer.

19.13 COMPUTER ANALYSIS

PSpice

Power Curve: Resistor The computer analysis begins with a verification of the curves in Fig. 19.3, which show the in-phase relationship between the voltage and current of a resistor. The figure shows that the power curve is totally above the horizontal axis and that the curve has a frequency

twice the applied frequency and a peak value equal to twice the average value. First, set up the simple schematic of Fig. 19.40. Then, use the **Time Domain(Transient)** option to get a plot versus time, and set the **Run to time** to 1 ms and the **Maximum step size** to 1 ms/1000 = 1 μs. Select **OK** and then the **Run PSpice** icon to perform the simulation. Then **Trace-Add Trace-V1(R)** results in the curve appearing in Fig. 19.41. Next, **Trace-Add Trace-I(R)** results in the curve for the current as appearing in Fig. 19.41. Finally, plot the power curve using **Trace-Add Trace-V1(R)*I(R)** from the basic power equation, and the curve of Fig. 19.41 results.

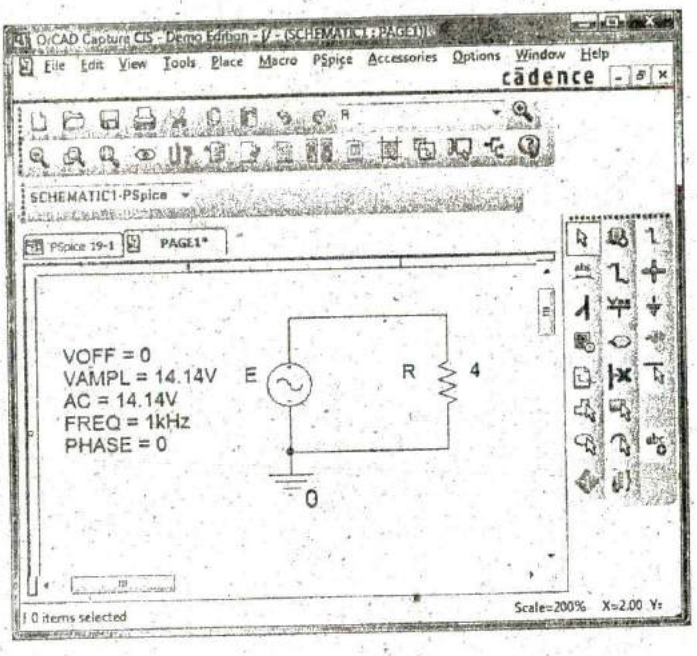


FIG. 19.40

Using Pspice to review the power curve for a resistive element in an ac circuit.

You can distinguish between the curves by looking at the symbol next to each quantity at the bottom left of the plot. In this case, however, to make it even clearer, a different color was selected for each trace by right-clicking on each trace, selecting **Properties,** and choosing the color and width of each curve. However, you can also add text to the screen by selecting the **ABC** icon to obtain the **Text Label** dialog box, entering the label such as **P(R),** and clicking **OK.** The label can then be placed anywhere on the screen. By selecting the **Toggle cursor** key and then clicking on **I(R)** at the bottom of the screen, you can use the cursor to find the maximum value of the current. At **A1** = 250 μs or ¼ of the total period of the input voltage, the current is a peak at 3.54 A. The peak value of the power curve can then be found by right-clicking on **V1(R)*I(R),** clicking on the graph, and then finding the peak value (also available by clicking on the **Cursor Peak** icon to the right of the **Toggle cursor** key). It occurs at the same point as the maximum current at a level of 50 W. In particular, note that the power curve shows two cycles, while both v_R and i_R show only one cycle. Clearly, the power curve has twice the frequency of the applied signal. Also note that the power curve is totally above the zero line, indicating that power is being absorbed by

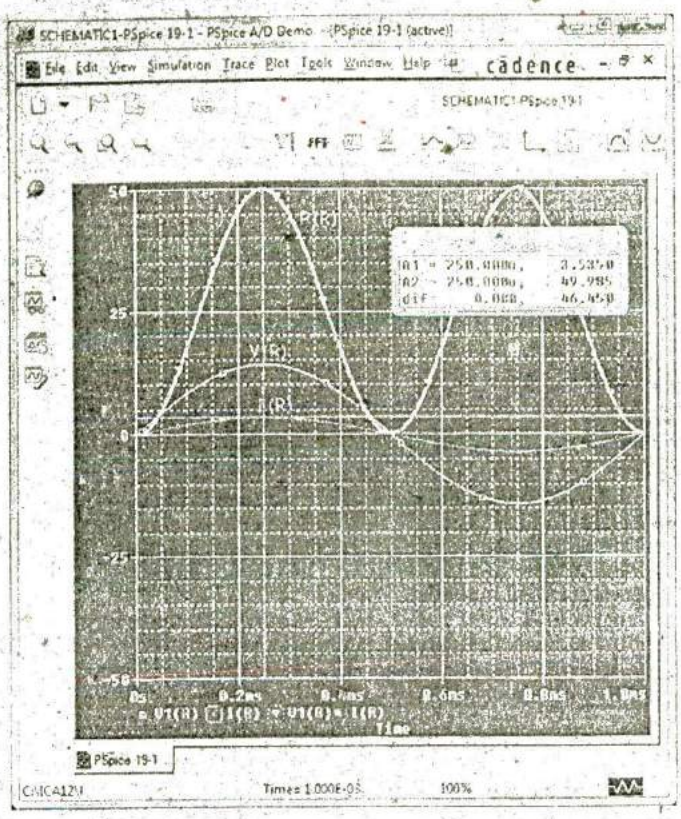


FIG. 19.41

The resulting plots for the power, voltage, and current for the resistor in Fig. 19.40.

the resistor through the entire displayed cycle. Further, the peak value of the power curve is twice the average value of the curve; that is, the peak value of 50 W is twice the average value of 25 W.

The results of the above simulation can be verified by performing the longhand calculation using the rms value of the applied voltage. That is,

$$P = \frac{V_R^2}{R} = \frac{(10\text{ V})^2}{4\ \Omega} = \mathbf{25\ W}$$

Power Curves: Series *R-L-C* Circuit The network in Fig. 19.42, with its combination of elements, is now used to demonstrate that, no matter what the physical makeup of the network, the average value of the power curve established by the product of the applied voltage and resulting source current is equal to that dissipated by the network. At a frequency of 1 kHz, the reactance of the 1.273 mH inductor will be 8, and the reactance of the capacitor will be 4 Ω, resulting in a lagging network. An analysis of the network results in

$$Z_T = 4\ \Omega + j\,4\ \Omega = 5.657\ \Omega\ \angle 45°$$

with

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{10\text{ V}\angle 0°}{5.657\ \Omega\ \angle 45°} = 1.768\text{ A}\ \angle -45°$$

and

$$P = I^2R = (1.768\text{ A})^2 4\ \Omega = \mathbf{12.5\ W}$$

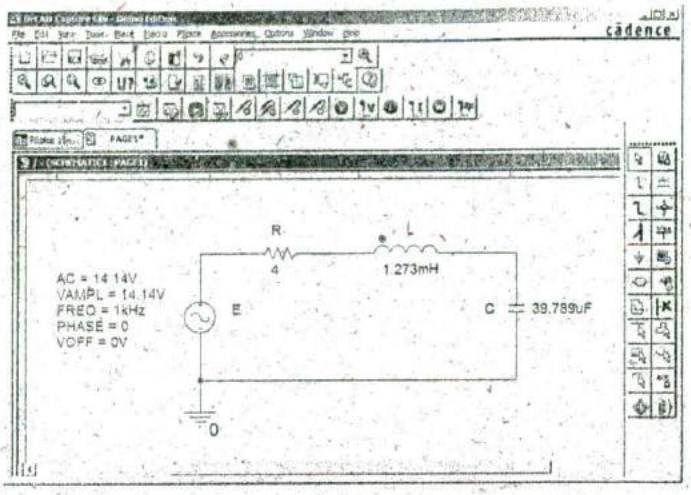


FIG. 19.42
Using PSpice to examine the power distribution in a series R-L-C circuit.

In the **Time Domain (Transient)** listing, insert **Run to Time** as 20 ms and **Maximum step size** as 1 μs. The three curves in Fig. 19.43 are obtained using the **Simulation Output Variables V(E:+), I(R)**, and **V(E:+)*I(R)**. The **Run to time** under the **Simulation Profile** listing was 20 ms, although 1 μs was chosen as the **Maximum step size** to ensure a good plot. In particular, note that the horizontal axis does not start until $t = .18$ ms to ensure that you are in a steady-state mode and not in a transient stage (where the peak values of the waveforms could

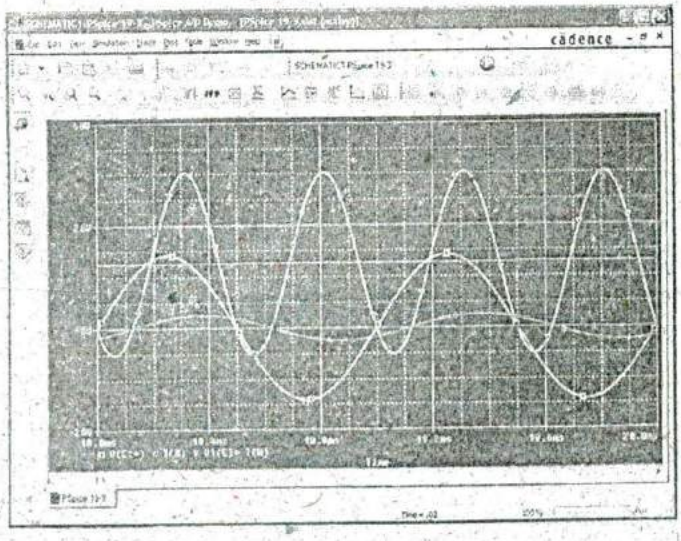


FIG. 19.43
Plots of the applied voltage e, current $i_R = i_s$, and power delivered $p_s = e \cdot i_s$ for the circuit in Fig. 19.42.

change with time). Set the horizontal axis to extend from 18 ms to 20 ms by selecting **Plot-Axis Settings-X Axis-User Defined-18ms to 20ms-OK.** First note that the current lags the applied voltage as expected for the lagging network. The phase angle between the two is 45° as determined above. Second, be aware that the elements are chosen so that the same scale can be used for the current and voltage. The vertical axis does not have a unit of measurement, so the proper units must be mentally added for each plot. Using **Plot-Label-Line,** draw a line across the screen at the average power level of 12.5 W. A pencil appears that can be clicked in place at the left edge at the 12.5 W level. Drag the pencil across the page to draw the desired line. Once you are at the right edge, release the mouse, and the line is drawn. Obtain the different colors for the traces by right-clicking on a trace and selecting from the choices under **Properties.** Note that the 12.5 W level is indeed the average value of the power curve. It is interesting to note that the power curve dips below the axis for only a short period of time. In other words, during the two visible cycles, power is being absorbed by the circuit most of the time. The small region below the axis is the return of energy to the network by the reactive elements. In general, therefore, the source must supply power to the circuit most of the time, even though a good percentage of the power may be delivering energy to the reactive elements, not being dissipated.

PROBLEMS

SECTIONS 19.1 THROUGH 19.8

1. For the battery of bulbs (purely resistive) appearing in Fig. 19.44:
   a. Determine the total power dissipation.
   b. Calculate the total reactive and apparent power.
   c. Find the source current I_s.
   d. Calculate the resistance of each bulb for the specified operating conditions.
   e. Determine the currents I_1 and I_2.

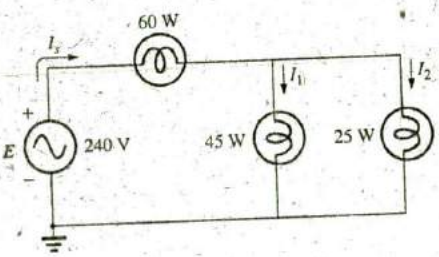


FIG. 19.44
Problem 1.

2. For the network of Fig. 19.45:
   a. Find the average power delivered to each element.
   b. Find the reactive power for each element.
   c. Find the apparent power for each element.
   d. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor F_p of the circuit.
   e. Sketch the power triangle.
   f. Find the energy dissipated by the resistor over one full cycle of the input voltage.
   g. Find the energy stored or returned by the capacitor and the inductor over one half-cycle of the power curve for each.

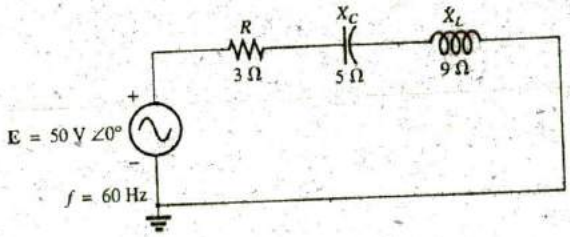


FIG. 19.45
Problem 2.

3. For the system of Fig. 19.46:
   a. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor F_p.
   b. Draw the power triangle.
   c. Find the current $\mathbf{I}_s$.

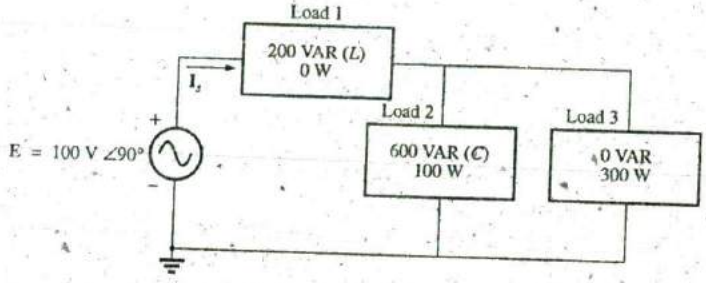


FIG. 19.46
Problem 3.

4. For the system of Fig. 19.47:
   a. Find P_T, Q_T, and S_T.
   b. Determine the power factor F_p.
   c. Draw the power triangle.
   d. Find $\mathbf{I}_s$.

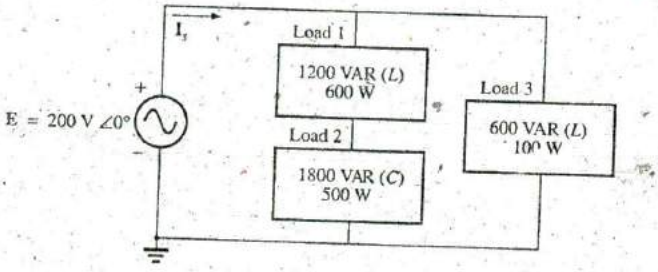


FIG. 19.47
Problem 4.

5. For the system of Fig. 19.48:
   a. Find P_T, Q_T, and S_T.
   b. Find the power factor F_p.
   c. Draw the power triangle.
   d. Find $\mathbf{I}_s$.

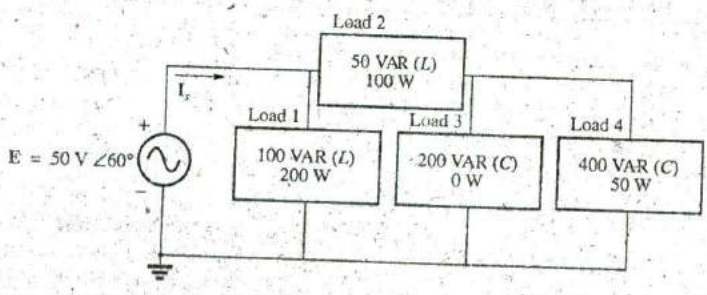


FIG. 19.48
Problem 5.

6. For the circuit of Fig. 19.49:
   a. Find the average, reactive, and apparent power for the 20 Ω resistor.
   b. Repeat part (a) for the 10 Ω inductive reactance.
   c. Find the total number of watts, volt-amperes reactive, volt-amperes, and power factor F_p.
   d. Find the current $\mathbf{I}_s$.

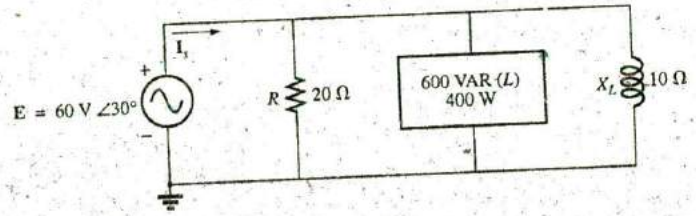


FIG. 19.49
Problem 6.

7. For the network of Fig. 19.50:
   a. Find the average power delivered to each element.
   b. Find the reactive power for each element.
   c. Find the apparent power for each element.
   d. Find P_T, Q_T, S_T, and F_p for the system.
   e. Sketch the power triangle.
   f. Find $\mathbf{I}_s$.

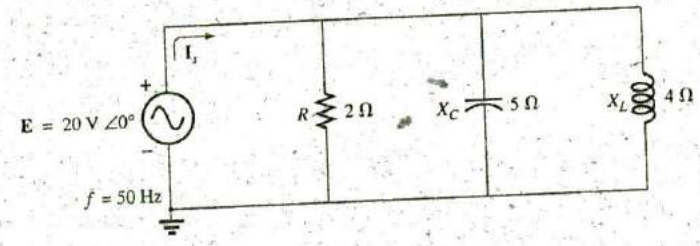


FIG. 19.50
Problem 7.

8. Repeat Problem 7 for the circuit of Fig. 19.51.

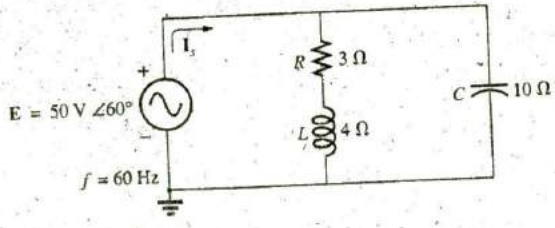


FIG. 19.51
Problem 8.

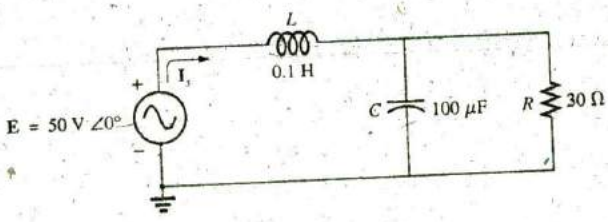


FIG. 19.52
Problem 9.

*9. For the network of Fig. 19.52:
   a. Find the average power delivered to each element.
   b. Find the reactive power for each element.
   c. Find the apparent power for each element.
   d. Find the total number of watts, volt-amperes reactive, volt-amperes, and power factor F_p of the circuit.
   e. Sketch the power triangle.
   f. Find the energy dissipated by the resistor over one full cycle of the input voltage.
   g. Find the energy stored or returned by the capacitor and the inductor over one half-cycle of the power curve for each.

10. An electrical system is rated 10 kVA, 200 V at a 0.5 leading power factor.
    a. Determine the impedance of the system in rectangular coordinates.
    b. Find the average power delivered to the system.
11. An electrical system is rated 5 kVA, 120 V, at a 0.8 lagging power factor.
    a. Determine the impedance of the system in rectangular coordinates.
    b. Find the average power delivered to the system.

*12. For the system of Fig. 19.53:
    a. Find the total number of watts, volt-amperes reactive, volt-amperes, and F_p.
    b. Find the current $\mathbf{I}_s$.
    c. Draw the power triangle.
    d. Find the type of elements and their impedance in ohms within each electrical box. (Assume that all elements of a load are in series.)

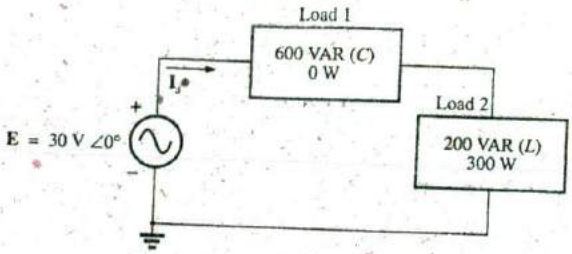


FIG. 19.53
Problem 12.

    e. Verify that the result of part (b) is correct by finding the current $\mathbf{I}_s$ using only the input voltage $\mathbf{E}$ and the results of part (d). Compare the value of $\mathbf{I}_s$ with that obtained for part (b).

*13. Repeat Problem 12 for the system of Fig. 19.54.

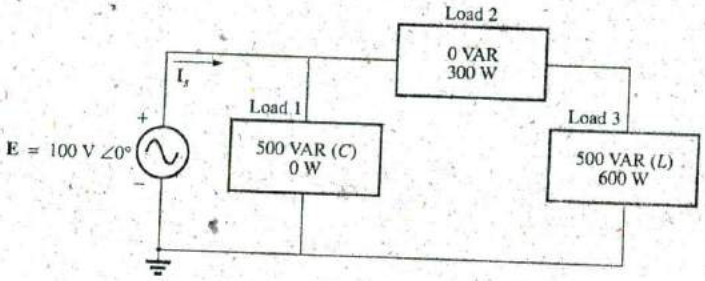


FIG. 19.54
Problem 13.

*14. For the circuit of Fig. 19.55:
    a. Find the total number of watts, volt-amperes reactive, volt-amperes, and F_p.
    b. Find the current $\mathbf{I}_s$.
    c. Find the type of elements and their impedance in each box. (Assume that the elements within each box are in series.)

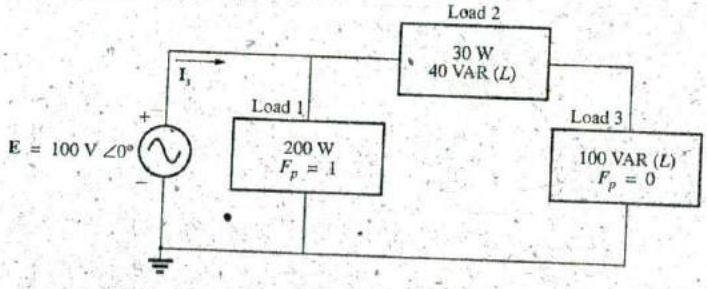


FIG. 19.55
Problem 14.

15. For the circuit of Fig. 19.56:
    a. Find the total number of watts, volt-amperes reactive, volt-amperes, and F_p.
    b. Find the voltage **E**.
    c. Find the type of elements and their impedance in each box. (Assume that the elements within each box are in series.)

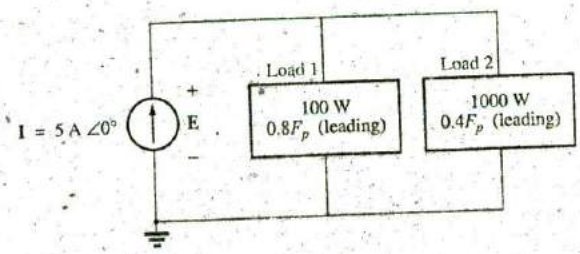


FIG. 19.56
Problem 15.

SECTION 19.9 Power-Factor Correction

*16. The lighting and motor loads of a small factory establish a 10 kVA power demand at a 0.7 lagging power factor on a 208 V, 60 Hz supply.
    a. Establish the power triangle for the load.
    b. Determine the power-factor capacitor that must be placed in parallel with the load to raise the power factor to unity.
    c. Determine the change in supply current from the uncompensated to the compensated system.
    d. Repeat parts (b) and (c) if the power factor is increased to 0.9.

17. The load on a 120 V, 60 Hz supply is 5 kW (resistive), 8 kVAR (inductive), and 2 kVAR (capacitive).
    a. Find the total kilovolt-amperes.
    b. Determine the F_p of the combined loads.
    c. Find the current drawn from the supply.
    d. Calculate the capacitance necessary to establish a unity power factor.
    e. Find the current drawn from the supply at unity power factor, and compare it to the uncompensated level.

18. The loading of a factory on a 1000 V, 60 Hz system includes:
    20 kW heating (unity power factor)
    10 kW (P_i) induction motors (0.7 lagging power factor)
    5 kW lighting (0.85 lagging power factor)
    a. Establish the power triangle for the total loading on the supply.
    b. Determine the power-factor capacitor required to raise the power factor to unity.
    c. Determine the change in supply current from the uncompensated to the compensated system.

SECTION 19.10 Power Meters

19. a. A wattmeter is connected with its current coil as shown in Fig. 19.57 and with the potential coil across points *f*-*g*. What does the wattmeter read?
    b. Repeat part (a) with the potential coil (*PC*) across *a*-*b*, *b*-*c*, *a*-*c*, *a*-*d*, *c*-*d*, *d*-*e*, and *f*-*e*.

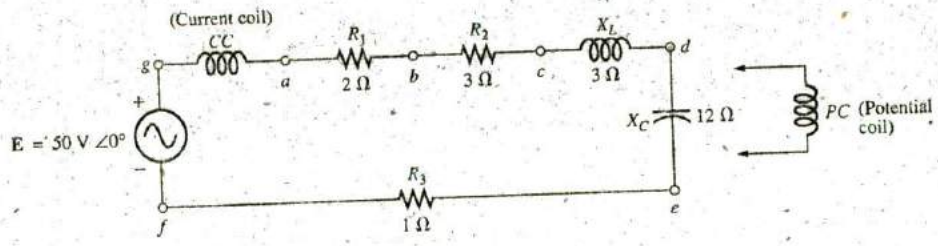


FIG. 19.57
Problem 19.

20. The voltage source in Fig. 19.58 delivers 660 VA at 120 V, with a supply current that lags the voltage by a power factor of 0.6.
    a. Determine the voltmeter, ammeter, and wattmeter readings.
    b. Find the load impedance in rectangular form.

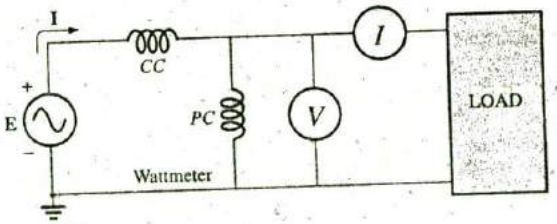


FIG. 19.58
Problem 20.

SECTION 19.11 Effective Resistance

21. a. An air-core coil is connected to a 200 V, 60 Hz source. The current is found to be 4 A, and a wattmeter reading of 80 W is observed. Find the effective resistance and the inductance of the coil.
    b. A brass core is inserted in the coil. The ammeter reads 3 A, and the wattmeter reads 90 W. Calculate the effective resistance of the core. Explain the increase over the value in part (a).
    c. If a solid iron core is inserted in the coil, the current is found to be 2 A, and the wattmeter reads 60 W. Calculate the resistance and inductance of the coil. Compare these values to the values in part (a), and account for the changes.
22. a. The inductance of an air-core coil is 0.08 H, and the effective resistance is 4 Ω when a 60 V, 50 Hz source is connected across the coil. Find the current passing through the coil and the reading of a wattmeter across the coil.
    b. If a brass core is inserted in the coil, the effective resistance increases to 7 Ω, and the wattmeter reads 30 W. Find the current passing through the coil and the inductance of the coil.
    c. If a solid iron core is inserted in the coil, the effective resistance of the coil increases to 10 Ω, and the current decreases to 1.7 A. Find the wattmeter reading and the inductance of the coil.

SECTION 19.13 Computer Analysis

PSpice or Multisim

23. Using PSpice or Multisim, obtain a plot of reactive power for a pure capacitor of 636.62 μF at a frequency of 1 kHz for one cycle of the input voltage using an applied voltage $\mathbf{E} = 10\ \text{V} \angle 0°$. On the same graph, plot both the applied voltage and the resulting current. Apply appropriate labels to the resulting curves to generate results similar to those in Fig. 19.41.
24. Repeat the analysis in Fig. 19.42 for a parallel *R-L-C* network of the same values and frequency.
25. Plot both the applied voltage and the source current on the same set of axes for the network in Fig. 19.31(b), and show that they are both in phase due to the resulting unity power factor.

GLOSSARY

Apparent power The power delivered to a load without consideration of the effects of a power-factor angle of the load. It is determined solely by the product of the terminal voltage and current of the load.

Average (real) power The delivered power dissipated in the form of heat by a network or system.

Eddy currents Small, circular currents in a paramagnetic core causing an increase in the power losses and the effective resistance of the material.

Effective resistance The resistance value that includes the effects of radiation losses, skin effect, eddy currents, and hysteresis losses.

Hysteresis losses Losses in a magnetic material introduced by changes in the direction of the magnetic flux within the material.

Power-factor correction The addition of reactive components (typically capacitive) to establish a system power factor closer to unity.

Radiation losses The losses of energy in the form of electromagnetic waves during the transfer of energy from one element to another.

Reactive power The power associated with reactive elements that provides a measure of the energy associated with setting up the magnetic and electric fields of inductive and capacitive elements, respectively.

Skin effect At high frequencies, a counter-induced voltage builds up at the center of a conductor, resulting in an increased flow near the surface (skin) of the conductor and a sharp reduction near the center. As a result, the effective area of conduction decreases and the resistance increases as defined by the basic equation for the geometric resistance of a conductor.

20 RESONANCE

Objectives

- *Become familiar with the frequency response of a series resonant circuit and how to calculate the resonant and cutoff frequencies.*
- *Be able to calculate a tuned network's quality factor, bandwidth, and power levels at important frequency levels.*
- *Become familiar with the frequency response of a parallel resonant circuit and how to calculate the resonant and cutoff frequencies.*
- *Understand the impact of the quality factor on the frequency response of a series or parallel resonant network.*
- *Begin to appreciate the difference between defining parallel resonance at the frequency either where the input impedance is a maximum or where the network has a unity power factor.*

20.1 INTRODUCTION

This chapter introduces the very important *resonant* (or *tuned*) *circuit*, which is fundamental to the operation of a wide variety of electrical and electronic systems in use today. The resonant circuit is a combination of R, L, and C elements having a frequency response characteristic similar to the one appearing in Fig. 20.1. Note in the figure that the response is a maximum for the frequency f_r, decreasing to the right and left of this frequency. In other words, for a particular range of frequencies, the response will be near or equal to the maximum. The frequencies to the far left or right have very low voltage or current levels and, for all practical purposes, have little effect on the system's response. The radio or television receiver has a response curve for each broadcast station of the type indicated in Fig. 20.1. When the receiver is set (or tuned) to a particular station, it is set on or near the frequency f_r in Fig. 20.1. Stations transmitting at frequencies to the far right or left of this resonant frequency are not carried through with

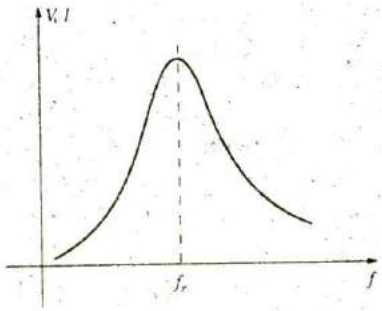


FIG. 20.1
Resonance curve.

significant power to affect the program of interest. The tuning process (setting the dial to f_r) as described above is the reason for the terminology *tuned circuit*. When the response is at or near the maximum, the circuit is said to be in a state of **resonance.**

The concept of resonance is not limited to electrical or electronic systems. If mechanical impulses are applied to a mechanical system at the proper frequency, the system will enter a state of resonance in which sustained vibrations of very large amplitude will develop. The frequency at which this occurs is called the *natural frequency* of the system. The classic example of this effect was the Tacoma Narrows Bridge built in 1940 over Puget Sound in Washington State. Four months after the bridge, with its suspended span of 2800 ft, was completed, a 42 mi/h pulsating gale set the bridge into oscillations at its natural frequency. The amplitude of the oscillations increased to the point where the main span broke up and fell into the water below. It was replaced by the new Tacoma Narrows Bridge, completed in 1950.

The resonant electrical circuit *must* have both inductance and capacitance. In addition, resistance will always be present due either to the lack of ideal elements or to the control offered on the shape of the resonance curve. When resonance occurs due to the application of the proper frequency (f_r), the energy absorbed by one reactive element is the same as that released by another reactive element within the system. In other words, energy pulsates from one reactive element to the other. Therefore, once an ideal (pure C, L) system has reached a state of resonance, it requires no further reactive power since it is self-sustaining. In a practical circuit, there is some resistance associated with the reactive elements that will result in the eventual "damping" of the oscillations between reactive elements.

There are two types of resonant circuits; *series* and *parallel*. Each will be considered in some detail in this chapter.

SERIES RESONANCE

20.2 SERIES RESONANT CIRCUIT

A resonant circuit (series or parallel) must have an inductive and a capacitive element. A resistive element is always present due to the internal resistance of the source (R_s), the internal resistance of the inductor (R_l), and any added resistance to control the shape of the response curve (R_{design}). The basic configuration for the series resonant circuit appears in Fig. 20.2(a) with the resistive elements listed above. The

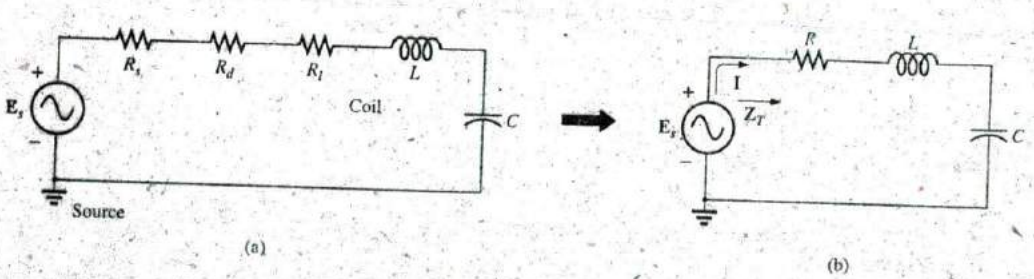


FIG. 20.2
Series resonant circuit.

"cleaner" appearance in Fig. 20.2(b) is a result of combining the series resistive elements into one total value. That is,

$$\boxed{R = R_s + R_l + R_d} \tag{20.1}$$

The total impedance of this network at any frequency is determined by

$$\mathbf{Z}_T = R + jX_L - jX_C = R + j(X_L - X_C)$$

The resonant conditions described in the introduction occurs when

$$\boxed{X_L = X_C} \tag{20.2}$$

removing the reactive component from the total impedance equation. The total impedance at resonance is then

$$\boxed{\mathbf{Z}_{T_s} = R} \tag{20.3}$$

representing the minimum value of $\mathbf{Z}_T$ at any frequency. The subscript s is employed to indicate series resonant conditions.

The resonant frequency can be determined in terms of the inductance and capacitance by examining the defining equation for resonance [Eq. (20.2)]:

$$X_L = X_C$$

Substituting yields

$$\omega L = \frac{1}{\omega C} \quad \text{and} \quad \omega^2 = \frac{1}{LC}$$

and

$$\boxed{\omega_s = \frac{1}{\sqrt{LC}}} \tag{20.4}$$

or

$$\boxed{f_s = \frac{1}{2\pi\sqrt{LC}}} \qquad \begin{matrix} f = \text{hertz (Hz)} \\ L = \text{henries (H)} \\ C = \text{farads (F)} \end{matrix} \tag{20.5}$$

The current through the circuit at resonance is

$$\mathbf{I} = \frac{E\angle 0^\circ}{R\angle 0^\circ} = \frac{E}{R}\angle 0^\circ$$

which is the maximum current for the circuit in Fig. 20.2 for an applied voltage $\mathbf{E}$ since $\mathbf{Z}_T$ is a minimum value. Consider also that *the input voltage and current are in phase at resonance.*

Since the current is the same through the capacitor and inductor, the voltage across each is equal in magnitude but 180° out of phase at resonance:

$$\left.\begin{aligned} \mathbf{V}_L &= (I\angle 0^\circ)(X_L\angle 90^\circ) = IX_L\angle 90^\circ \\ \mathbf{V}_C &= (I\angle 0^\circ)(X_C\angle -90^\circ) = IX_C\angle -90^\circ \end{aligned}\right\} \begin{matrix} 180^\circ \\ \text{out of} \\ \text{phase} \end{matrix}$$

and, since $X_L = X_C$, the magnitude of V_L equals V_C at resonance; that is,

$$\boxed{V_{L_s} = V_{C_s}} \tag{20.6}$$

Fig. 20.3, a phasor diagram of the voltages and current, clearly indicates that the voltage across the resistor at resonance is the input voltage, and $\mathbf{E}$, $\mathbf{I}$, and $\mathbf{V}_R$ are in phase at resonance.

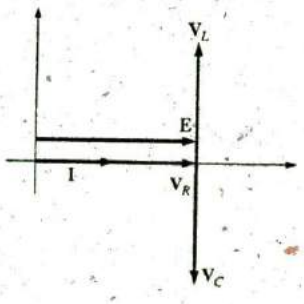


FIG. 20.3
Phasor diagram for the series resonant circuit at resonance.

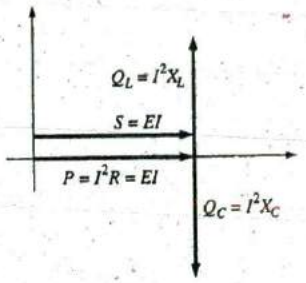


FIG. 20.4
Power triangle for the series resonant circuit at resonance.

The average power to the resistor at resonance is equal to I^2R, and the reactive power to the capacitor and inductor are I^2X_C and I^2X_L, respectively.

The power triangle at resonance (Fig. 20.4) shows that the total apparent power is equal to the average power dissipated by the resistor since $Q_L = Q_C$. The power factor of the circuit at resonance is

$$F_p = \cos\theta = \frac{P}{S}$$

and

$$\boxed{F_{p_s} = 1} \qquad (20.7)$$

Plotting the power curves of each element on the same set of axes (Fig. 20.5), we note that, even though the total reactive power at any instant is equal to zero (note that $t = t'$), energy is still being absorbed and released by the inductor and capacitor at resonance.

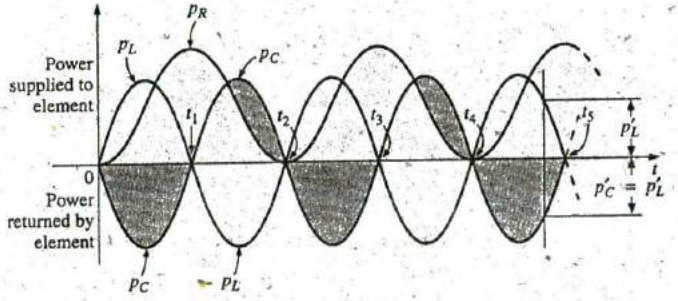


FIG. 20.5
Power curves at resonance for the series resonant circuit.

A closer examination reveals that the energy absorbed by the inductor from time 0 to t_1 is the same as the energy released by the capacitor from 0 to t_1. The reverse occurs from t_1 to t_2, and so on. Therefore, the total apparent power continues to be equal to the average power, even though the inductor and capacitor are absorbing and releasing energy. This condition occurs only at resonance. The slightest change in frequency introduces a reactive component into the power triangle, which increases the apparent power of the system above the average power dissipation, and resonance no longer exists.

20.3 THE QUALITY FACTOR (Q)

The **quality factor** Q of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance; that is,

$$\boxed{Q_s = \frac{\text{reactive power}}{\text{average power}}} \qquad (20.8)$$

The quality factor is also an indication of how much energy is placed in storage (continual transfer from one reactive element to the other) compared to that dissipated. The lower the level of dissipation for the same reactive power, the larger is the Q_s factor and the more concentrated and intense is the region of resonance.

Substituting for an inductive reactance in Eq. (20.8) at resonance gives us

$$Q_s = \frac{I^2 X_L}{I^2 R}$$

and

$$\boxed{Q_s = \frac{X_L}{R} = \frac{\omega_s L}{R}} \qquad (20.9)$$

If the resistance R is just the resistance of the coil (R_l) then

$$\boxed{Q_s = Q_{\text{coil}} = Q_l = \frac{X_L}{R_l}} \qquad R = R_l \qquad (20.10)$$

Since the quality factor of a coil is typically the information provided by manufacturers of inductors, it is often given the symbol Q without an associated subscript. It appears from Eq. (20.10) that Q_l increases linearly with frequency since $X_L = 2\pi fL$. That is, if the frequency doubles, then Q_l also increases by a factor of 2. This is approximately true for the low range to the midrange of frequencies such as shown for the coils in Fig. 20.6. Unfortunately, however, as the frequency increases, the effective resistance of the coil also increases, due primarily to skin effect phenomena, and the resulting Q_l decreases. In addition, the capacitive effects between the windings increases, further reducing the Q_l of the coil. For this reason, Q_l must be specified for a particular frequency or frequency range. For wide frequency applications, a plot of Q_l versus frequency is often provided. The maximum Q_l for most commercially available coils is less than 200, with most having a maximum near 100. Note in Fig. 20.6 that for coils of the same type, Q_l drops off more quickly for higher levels of inductance.

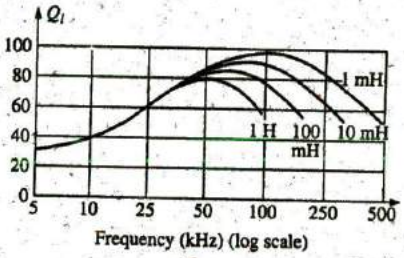


FIG. 20.6
Q_l versus frequency for a series of inductors of similar construction.

If we substitute

$$\omega_s = 2\pi f_s$$

and then

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

into Eq. (20.9), we have

$$Q_s = \frac{\omega_s L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi}{R}\left(\frac{1}{2\pi\sqrt{LC}}\right)L$$
$$= \frac{L}{R}\left(\frac{1}{\sqrt{LC}}\right) = \left(\frac{\sqrt{L}}{\sqrt{L}}\right)\frac{L}{R\sqrt{LC}}$$

and

$$\boxed{Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}} \qquad (20.11)$$

providing Q_s in terms of the circuit parameters.

For series resonant circuits used in communication systems, Q_s is usually greater than 1. By applying the voltage divider rule to the circuit in Fig. 20.2, we obtain

$$V_L = \frac{X_L E}{Z_T} = \frac{X_L E}{R} \qquad \text{(at resonance)}$$

and

$$\boxed{V_{L_s} = Q_s E} \qquad (20.12)$$

or $$V_C = \frac{X_C E}{Z_T} = \frac{X_C E}{R}$$

and $$\boxed{V_{C_s} = Q_s E} \qquad (20.13)$$

Since Q_s is usually greater than 1, the voltage across the capacitor or inductor of a series resonant circuit can be significantly greater than the input voltage. In fact, in many cases the Q_s is so high that careful design and handling (including adequate insulation) are mandatory with respect to the voltage across the capacitor and inductor.

In the circuit in Fig. 20.7, for example, which is in the state of resonance,

$$Q_s = \frac{X_L}{R} = \frac{480\ \Omega}{6\ \Omega} = 80$$

and $$V_L = V_C = Q_s E = (80)(10\ \text{V}) = \mathbf{800\ V}$$

which is certainly a potential of significant magnitude.

The phrase commonly used is there is a Q rise in voltage across the reactive components in a series resonant circuit.

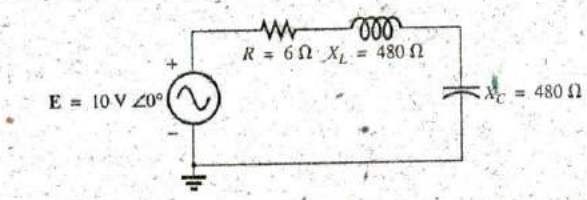


FIG. 20.7
High-Q series resonant circuit.

20.4 Z_T VERSUS FREQUENCY

The total impedance of the series *R-L-C* circuit in Fig. 20.2 at any frequency is determined by

$$\mathbf{Z}_T = R + jX_L - jX_L - jX_C \quad \text{or} \quad \mathbf{Z}_T = R + j(X_L - X_C)$$

The magnitude of the impedance $\mathbf{Z}_T$ versus frequency is determined by

$$Z_T = \sqrt{R^2 + (X_L - X_C)^2}$$

The total-impedance-versus-frequency curve for the series resonant circuit in Fig. 20.2 can be found by applying the impedance-versus-frequency curve for each element of the equation just derived, written in the following form:

$$\boxed{Z_T(f) = \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2}} \qquad (20.14)$$

where $Z_T(f)$ "means" the total impedance as a *function* of frequency. For the frequency range of interest, we assume that the resistance R does not change with frequency, resulting in the plot in Fig. 20.8. The curve for the inductance, as determined by the reactance equation, is a straight line intersecting the origin with a slope equal to the inductance of the coil. The mathematical expression for any straight line in a two-dimensional plane is given by

$$y = mx + b$$

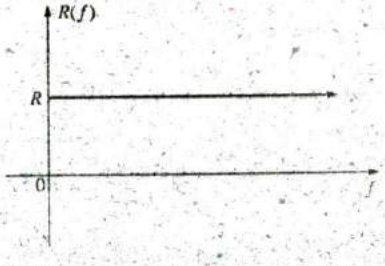


FIG. 20.8
Resistance versus frequency.

Thus, for the coil,

$$X_L = 2\pi fL + 0 = (2\pi L)(f) + 0$$
$$\downarrow \qquad\qquad \downarrow\ \downarrow\ \downarrow$$
$$y = \qquad\qquad m \cdot x + b$$

(where $2\pi L$ is the slope), producing the results shown in Fig. 20.9.

For the capacitor,

$$X_C = \frac{1}{2\pi fC} \quad \text{or} \quad X_C f = \frac{1}{2\pi C}$$

which becomes $yx = k$, the equation for a hyperbola, where

$$y(\text{variable}) = X_C$$
$$x(\text{variable}) = f$$
$$k(\text{constant}) = \frac{1}{2\pi C}$$

The hyperbolic curve for $X_C(f)$ is plotted in Fig. 20.10. In particular, note its very large magnitude at low frequencies and its rapid dropoff as the frequency increases.

If we place Figs. 20.9 and 20.10 on the same set of axes, we obtain the curves in Fig. 20.11. The condition of resonance is now clearly defined by the point of intersection, where $X_L = X_C$. For frequencies less than f_s, it is also quite clear that the network is primarily capacitive ($X_C > X_L$). For frequencies above the resonant condition, $X_L > X_C$, and the network is inductive.

Applying

$$Z_T(f) = \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2}$$
$$= \sqrt{[R(f)]^2 + [X(f)]^2}$$

to the curves in Fig. 20.11, where $X(f) = X_L(f) - X_C(f)$, we obtain the curve for $Z_T(f)$ as shown in Fig. 20.12. The minimum impedance occurs at the resonant frequency and is equal to the resistance R. Note that the curve is not symmetrical about the resonant frequency (especially at higher values of Z_T).

The phase angle associated with the total impedance is

$$\boxed{\theta = \tan^{-1}\frac{(X_L - X_C)}{R}} \tag{20.15}$$

For the $\tan^{-1}x$ function (resulting when $X_L > X_C$), the larger x is, the larger is the angle θ (closer to 90°). However, for regions where $X_C > X_L$, one must also be aware that

$$\boxed{\tan^{-1}(-x) = -\tan^{-1}x} \tag{20.16}$$

At low frequencies, $X_C > X_L$, and θ approaches −90° (capacitive), as shown in Fig. 20.13, whereas at high frequencies, $X_L > X_C$, and θ approaches 90°. In general, therefore, for a series resonant circuit:

$f < f_s$: network capacitive; **I** leads **E**
$f > f_s$: network inductive; **E** leads **I**
$f = f_s$: network resistive; **E** and **I** are in phase

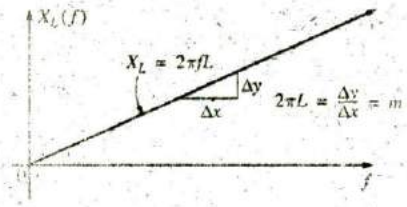


FIG. 20.9
Inductive reactance versus frequency.

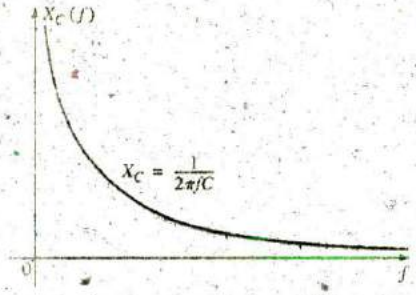


FIG. 20.10
Capacitive reactance versus frequency.

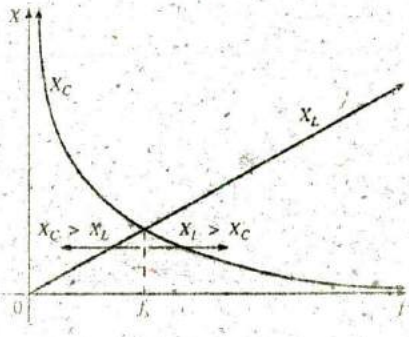


FIG. 20.11
Plotting the frequency response of the inductive and capacitive reactance of a series R-L-C circuit on the same set of axes.

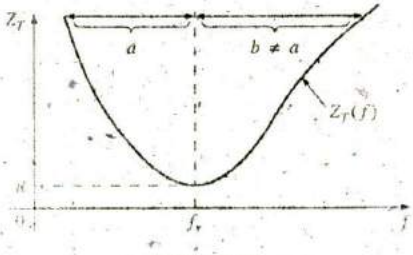


FIG. 20.12
Z_T versus frequency for the series resonant circuit.

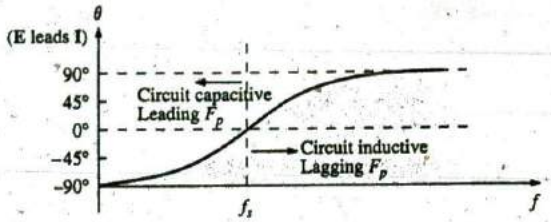


FIG. 20.13
Phase plot for the series resonant circuit.

20.5 SELECTIVITY

If we now plot the magnitude of the current $I = E/Z_T$ versus frequency for a *fixed* applied voltage E, we obtain the curve shown in Fig. 20.14, which rises from zero to a maximum value of E/R (where Z_T is a minimum) and then drops toward zero (as Z_T increases) at a slower rate than it rose to its peak value. The curve is actually the inverse of the impedance-versus-frequency curve. Since the Z_T curve is not absolutely symmetrical about the resonant frequency, the curve of the current versus frequency has the same property.

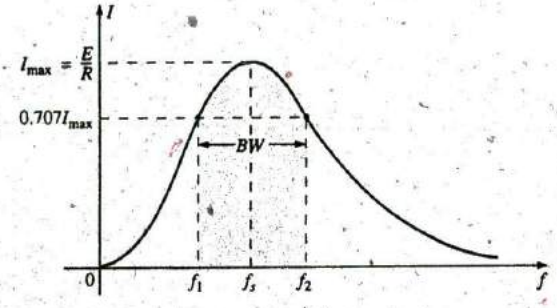


FIG. 20.14
I versus frequency for the series resonant circuit.

There is a definite range of frequencies at which the current is near its maximum value and the impedance is at a minimum. Those frequencies corresponding to 0.707 of the maximum current are called the **band frequencies, cutoff frequencies, half-power frequencies,** or **corner frequencies.** They are indicated by f_1 and f_2 in Fig. 20.14. The range of frequencies between the two is referred to as the **bandwidth** (abbreviated BW) of the resonant circuit.

Half-power frequencies are those frequencies at which the power delivered is one-half that delivered at the resonant frequency; that is,

$$\boxed{P_{HPF} = \frac{1}{2} P_{max}} \tag{20.17}$$

The above condition is derived using the fact that

$$P_{max} = I_{max}^2 R$$

and $$P_{HPF} = I^2 R = (0.707 I_{max})^2 R = (0.5)(I_{max}^2 R) = \frac{1}{2} P_{max}$$

Since the resonant circuit is adjusted to select a band of frequencies, the curve in Fig. 20.14 is called the **selectivity curve.** The term is derived from the fact that one must be *selective* in choosing the frequency to ensure that it is in the bandwidth. The smaller the bandwidth, the higher is the selectivity. The shape of the curve, as shown in Fig. 20.15, depends on each element of the series *R-L-C* circuit. If the resistance is made smaller with a fixed inductance and capacitance, the bandwidth decreases and the selectivity increases. Similarly, if the ratio L/C increases with fixed resistance, the bandwidth again decreases with an increase in selectivity.

In terms of Q_s, if R is larger for the same X_L, then Q_s is less, as determined by the equation $Q_s = \omega_s L/R$.

A small Q_s, therefore, is associated with a resonant curve having a large bandwidth and a small selectivity, while a large Q_s indicates the opposite.

For circuits where $Q_s \geq 10$, a widely accepted approximation is that the resonant frequency bisects the bandwidth and that the resonant curve is symmetrical about the resonant frequency.

These conditions are shown in Fig. 20.16, indicating that the cutoff frequencies are then equidistant from the resonant frequency.

For any Q_s, the preceding is not true. The cutoff frequencies f_1 and f_2 can be found for the general case (any Q_s) by first using the fact that a drop in current to 0.707 of its resonant value corresponds to an increase in impedance equal to $1/0.707 = \sqrt{2}$ times the resonant value, which is R.

Substituting $\sqrt{2}R$ into the equation for the magnitude of Z_T, we find that

$$Z_T = \sqrt{R^2 + (X_L - X_C)^2}$$

becomes

$$\sqrt{2}R = \sqrt{R^2 + (X_L - X_C)^2}$$

or, squaring both sides, that

$$2R^2 = R^2 + (X_L - X_C)^2$$

and

$$R^2 = (X_L - X_C)^2$$

Taking the square root of both sides gives us

$$R = X_L - X_C \quad \text{or} \quad R - X_L + X_C = 0$$

Let us first consider the case where $X_L > X_C$, which relates to f_2 or ω_2. Substituting $\omega_2 L$ for X_L and $1/\omega_2 C$ for X_C and bringing both quantities to the left of the equal sign, we have

$$R - \omega_2 L + \frac{1}{\omega_2 C} = 0 \quad \text{or} \quad R\omega_2 - \omega_2^2 L + \frac{1}{C} = 0$$

which can be written

$$\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$

Solving the quadratic, we have

$$\omega_2 = \frac{-(-R/L) \pm \sqrt{[-(R/L)]^2 - [-(4/LC)]}}{2}$$

and

$$\omega_2 = +\frac{R}{2L} \pm \frac{1}{2}\sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}$$

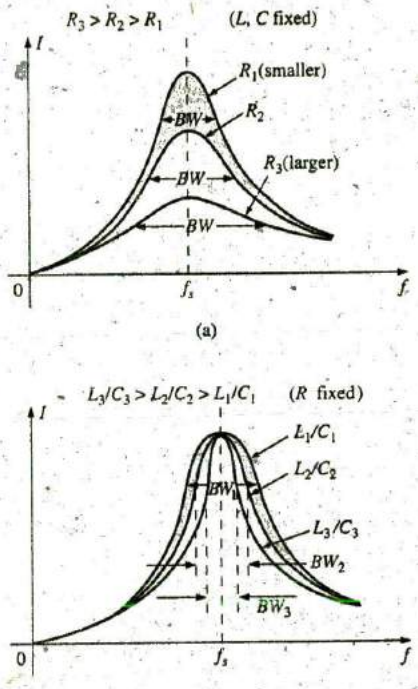


FIG. 20.15
Effect of R, L, and C on the selectivity curve for the series resonant circuit.

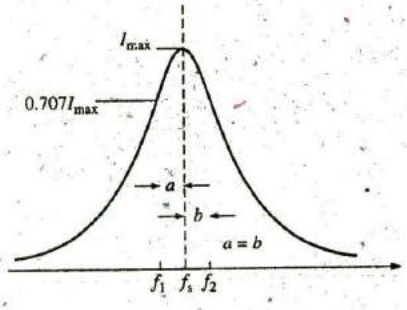


FIG. 20.16
Approximate series resonance curve for $Q_s \geq 10$.

with $$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right] \quad \text{(Hz)} \qquad \textbf{(20.18)}$$

The negative sign in front of the second factor was dropped because $(1/2)\sqrt{(R/L)^2 + 4/LC}$ is always greater than $R/(2L)$. If it were not dropped, there would be a negative solution for the radian frequency ω.

If we repeat the same procedure for $X_C > X_L$, which relates to ω_1 or f_1 such that $Z_T = \sqrt{R^2 + (X_C - X_L)^2}$, the solution f_1 becomes

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right] \quad \text{(Hz)} \qquad \textbf{(20.19)}$$

The bandwidth (BW) is

$$BW = f_2 - f_1 = \text{Eq. (20.18)} - \text{Eq. (20.19)}$$

and $$BW = f_2 - f_1 = \frac{R}{2\pi L} \qquad \textbf{(20.20)}$$

Substituting $R/L = \omega_s/Q_s$ from $Q_s = \omega_s L/R$ and $1/2\pi = f_s/\omega_s$ from $\omega_s = 2\pi f_s$ gives us

$$BW = \frac{R}{2\pi L} = \left(\frac{1}{2\pi}\right)\left(\frac{R}{L}\right) = \left(\frac{f_s}{\omega_s}\right)\left(\frac{\omega_s}{Q_s}\right)$$

or $$BW = \frac{f_s}{Q_s} \qquad \textbf{(20.21)}$$

which is a very convenient form since it relates the bandwidth to the Q_s of the circuit. As mentioned earlier, Eq. (20.21) verifies that the larger the Q_s, the smaller is the bandwidth, and vice versa.

Written in a slightly different form, Eq. (20.21) becomes

$$\frac{f_2 - f_1}{f_s} = \frac{1}{Q_s} \qquad \textbf{(20.22)}$$

The ratio $(f_2 - f_1)/f_s$ is sometimes called the *fractional bandwidth*, providing an indication of the width of the bandwidth compared to the resonant frequency.

It can also be shown through mathematical manipulations of the pertinent equations that the resonant frequeney is related to the geometric mean of the band frequencies; that is,

$$f_s = \sqrt{f_1 f_2} \qquad \textbf{(20.23)}$$

20.6 V_R, V_L, AND V_C

Plotting the magnitude (effective value) of the voltages $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ and the current $\mathbf{I}$ versus frequency for the series resonant circuit on the same set of axes, we obtain the curves shown in Fig. 20.17. Note that the V_R curve has the same shape as the I curve and a peak value equal to the magnitude of the input voltage E. The V_C curve builds up slowly at

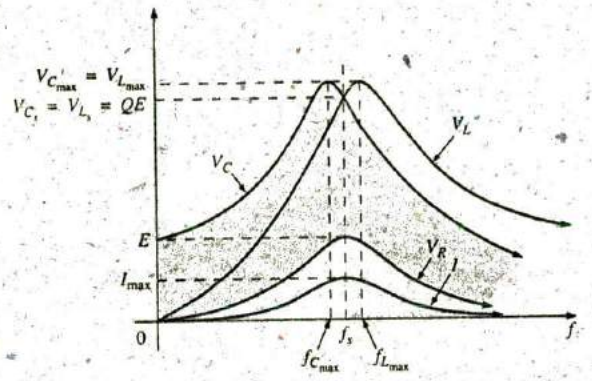


FIG. 20.17

V_R, V_L, V_C, and I versus frequency for a series resonant circuit.

first from a value equal to the input voltage since the reactance of the capacitor is infinite (open circuit) at zero frequency and the reactance of the inductor is zero (short circuit) at this frequency. As the frequency increases, $1/\omega C$ of the equation

$$V_C = IX_C = (I)\left(\frac{1}{\omega C}\right)$$

becomes smaller, but I increases at a rate faster than that at which $1/\omega C$ drops. Therefore, V_C rises and will continue to rise due to the quickly rising current until the frequency nears resonance. As it approaches the resonant condition, the rate of change of I decreases. When this occurs, the factor $1/\omega C$, which decreased as the frequency rose, overcomes the rate of change of I, and V_C starts to drop. The peak value occurs at a frequency just before resonance. After resonance, both V_C and I drop in magnitude, and V_C approaches zero.

The higher the Q_s of the circuit, the closer $f_{C_{max}}$ will be to f_s, and the closer $V_{C_{max}}$ will be to Q_sE. For circuits with $Q_s \geq 10$, $f_{C_{max}} \cong f_s$, and $V_{C_{max}} \cong Q_sE$.

The curve for V_L increases steadily from zero to the resonant frequency since both quantities ωL and I of the equation $V_L = IX_L = (I)(\omega L)$ increase over this frequency range. At resonance, I has reached its maximum value, but ωL is still rising. Therefore, V_L reaches its maximum value after resonance. After reaching its peak value, the voltage V_L drops toward E since the drop in I overcomes the rise in ωL. It approaches E because X_L will eventually be infinite, and X_C will be zero.

As Q_s of the circuit increases, the frequency $f_{L_{max}}$ drops toward f_s, and $V_{L_{max}}$ approaches Q_sE. For circuits with $Q_s \geq 10$, $f_{L_{max}} \cong f_s$, and $V_{L_{max}} \cong Q_sE$.

The V_L curve has a greater magnitude than the V_C curve for any frequency above resonance, and the V_C curve has a greater magnitude than the V_L curve for any frequency below resonance. This again verifies that the series R-L-C circuit is predominantly capacitive from zero to the resonant frequency and predominantly inductive for any frequency above resonance.

For the condition $Q_s \geq 10$, the curves in Fig. 20.17 appear as shown in Fig. 20.18. Note that they each peak (on an approximate basis) at the resonant frequency and have a similar shape.

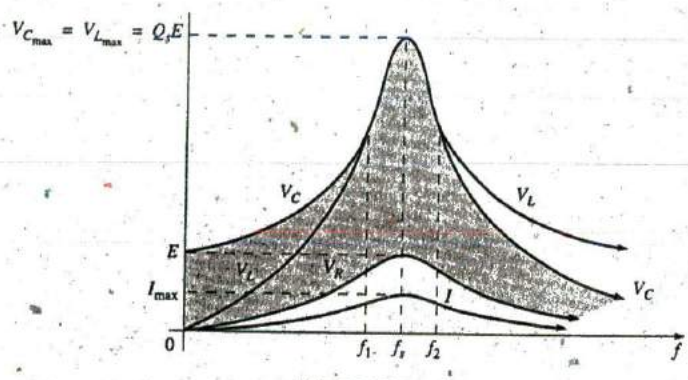


FIG. 20.18
V_R, V_L, V_C, and I for a series resonant circuit where $Q_s \geq 10$.

In review,

1. V_C and V_L are at their maximum values at or near resonance (depending on Q_s).
2. At very low frequencies, V_C is very close to the source voltage and V_L is very close to zero volts, whereas at very high frequencies, V_L approaches the source voltage and V_C approaches zero volts.
3. Both V_R and I peak at the resonant frequency and have the same shape.

20.7 EXAMPLES (SERIES RESONANCE)

EXAMPLE 20.1

a. For the series resonant circuit in Fig. 20.19, find $\mathbf{I}$, $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ at resonance.
b. What is the Q_s of the circuit?
c. If the resonant frequency is 5000 Hz, find the bandwidth.
d. What is the power dissipated in the circuit at the half-power frequencies?

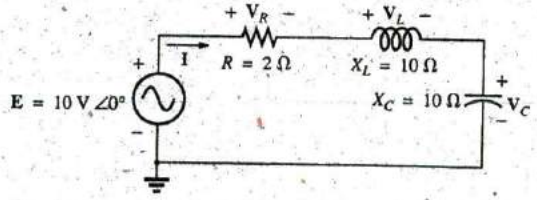


FIG. 20.19
Example 20.1.

Solutions:

a. $\mathbf{Z}_{T_s} = R = 2\,\Omega$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_{T_s}} = \frac{10\,\text{V}\angle 0°}{2\,\Omega\angle 0°} = 5\,\text{A}\angle 0°$$

$$\mathbf{V}_R = \mathbf{E} = 10\,\text{V}\angle 0°$$

$$\mathbf{V}_L = (I\angle 0°)(X_L\angle 90°) = (5\text{ A}\angle 0°)(10\,\Omega\angle 90°)$$
$$= \mathbf{50\text{ V}\angle 90°}$$
$$\mathbf{V}_C = (I\angle 0°)(X_C\angle -90°) = (5\text{ A}\angle 0°)(10\,\Omega\angle -90°)$$
$$= \mathbf{50\text{ V}\angle -90°}$$

b. $Q_s = \dfrac{X_L}{R} = \dfrac{10\,\Omega}{2\,\Omega} = \mathbf{5}$

c. $BW = f_2 - f_1 = \dfrac{f_s}{Q_s} = \dfrac{5000\text{ Hz}}{5} = \mathbf{1000\text{ Hz}}$

d. $P_{HPF} = \dfrac{1}{2}P_{max} = \dfrac{1}{2}I_{max}^2R = \left(\dfrac{1}{2}\right)(5\text{ A})^2(2\,\Omega) = \mathbf{25\text{ W}}$

EXAMPLE 20.2 The bandwidth of a series resonant circuit is 400 Hz.

a. If the resonant frequency is 4000 Hz, what is the value of Q_s?
b. If $R = 10\,\Omega$, what is the value of X_L at resonance?
c. Find the inductance L and capacitance C of the circuit.
d. What are the probable commercial values of L and C?

Solutions:

a. $BW = \dfrac{f_s}{Q_s}$ or $Q_s = \dfrac{f_s}{BW} = \dfrac{4000\text{ Hz}}{400\text{ Hz}} = \mathbf{10}$

b. $Q_s = \dfrac{X_L}{R}$ or $X_L = Q_sR = (10)(10\,\Omega) = \mathbf{100\,\Omega}$

c. $X_L = 2\pi f_sL$ or $L = \dfrac{X_L}{2\pi f_s} = \dfrac{100\,\Omega}{2\pi(4000\text{ Hz})} = \mathbf{3.98\text{ mH}}$

$$X_C = \frac{1}{2\pi f_sC} \quad \text{or} \quad C = \frac{1}{2\pi f_sX_C} = \frac{1}{2\pi(4000\text{ Hz})(100\,\Omega)}$$
$$= \mathbf{397.89\text{ nF}}$$

d. $L = 3.98\text{ mH} \cong \mathbf{3.9\text{ mH}}$
$C = 397.89\text{ nF} \cong \mathbf{390\text{ nF}} = \mathbf{0.39\,\mu F}$

EXAMPLE 20.3 A series *R-L-C* circuit has a series resonant frequency of 12,000 Hz.

a. If $R = 5\,\Omega$, and if X_L at resonance is $300\,\Omega$, find the bandwidth.
b. Find the cutoff frequencies.

Solutions:

a. $Q_s = \dfrac{X_L}{R} = \dfrac{300\,\Omega}{5\,\Omega} = 60$

$BW = \dfrac{f_s}{Q_s} = \dfrac{12{,}000\text{ Hz}}{60} = \mathbf{200\text{ Hz}}$

b. Since $Q_s \geq 10$, the bandwidth is bisected by f_s. Therefore,

$$f_2 = f_s + \frac{BW}{2} = 12{,}000\text{ Hz} + 100\text{ Hz} = \mathbf{12{,}100\text{ Hz}}$$

and $f_1 = 12{,}000\text{ Hz} - 100\text{ Hz} = \mathbf{11{,}900\text{ Hz}}$

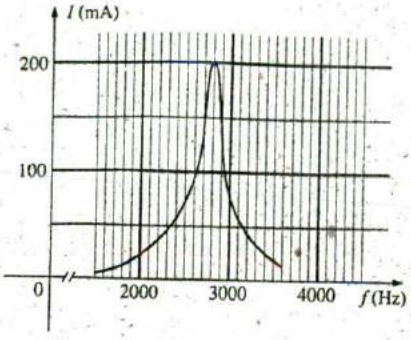


FIG. 20.20
Example 20.4

EXAMPLE 20.4

a. Determine the Q_s and bandwidth for the response curve in Fig. 20.20.
b. For $C = 100$ nF, determine L and R for the series resonant circuit.
c. Determine the applied voltage.

Solutions:

a. The resonant frequency is 2800 Hz. At 0.707 times the peak value,

$$BW = \mathbf{200\ Hz}$$

and
$$Q_s = \frac{f_s}{BW} = \frac{2800\ \text{Hz}}{200\ \text{Hz}} = \mathbf{14}$$

b. $f_s = \dfrac{1}{2\pi\sqrt{LC}}$ or $L = \dfrac{1}{4\pi^2 f_s^2 C}$

$$= \frac{1}{4\pi^2(2.8\ \text{kHz})^2(100\ \text{nF})}$$
$$= \mathbf{32.3\ mH}$$

$$Q_s = \frac{X_L}{R} \quad \text{or} \quad R = \frac{X_L}{Q_s} = \frac{2\pi(2800\ \text{Hz})(32.3\ \text{mH})}{14}$$
$$= \mathbf{40.6\ \Omega}$$

c. $I_{max} = \dfrac{E}{R}$ or $E = I_{max}R$

$$= (200\ \text{mA})(40.6\ \Omega) = \mathbf{8.12\ V}$$

EXAMPLE 20.5

A series *R-L-C* circuit is designed to resonate at $\omega_s = 10^5$ rad/s, have a bandwidth of $0.15\omega_s$, and draw 16 W from a 120 V source at resonance.

a. Determine the value of R.
b. Find the bandwidth in hertz.
c. Find the nameplate values of L and C.
d. Determine the Q_s of the circuit.
e. Determine the fractional bandwidth.

Solutions:

a. $P = \dfrac{E^2}{R}$ and $R = \dfrac{E^2}{P} = \dfrac{(120\ \text{V})^2}{16\ \text{W}} = \mathbf{900\ \Omega}$

b. $f_s = \dfrac{\omega_s}{2\pi} = \dfrac{10^5\ \text{rad/s}}{2\pi} = 15{,}915.49\ \text{Hz}$

$$BW = 0.15f_s = 0.15(15{,}915.49\ \text{Hz}) = \mathbf{2387.32\ Hz}$$

c. Eq. (20.20):

$$BW = \frac{R}{2\pi L} \quad \text{and} \quad L = \frac{R}{2\pi BW} = \frac{900\ \Omega}{2\pi(2387.32\ \text{Hz})} = \mathbf{60\ mH}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}} \quad \text{and} \quad C = \frac{1}{4\pi^2 f_s^2 L}$$
$$= \frac{1}{4\pi^2(15{,}915.49\ \text{Hz})^2(60 \times 10^{-3})}$$
$$= \mathbf{1.67\ nF}$$

d. $Q_s = \dfrac{X_L}{R} = \dfrac{2\pi f_s L}{R} = \dfrac{2\pi(15{,}915.49\text{ Hz})(60\text{ mH})}{900\,\Omega} = \mathbf{6.67}$

e. $\dfrac{f_2 - f_1}{f_s} = \dfrac{BW}{f_s} = \dfrac{1}{Q_s} = \dfrac{1}{6.67} = \mathbf{0.15}$

PARALLEL RESONANCE

20.8 PARALLEL RESONANT CIRCUIT

The basic format of the series resonant circuit is a series *R-L-C* combination in series with an applied voltage source. The parallel resonant circuit has the basic configuration in Fig. 20.21, a parallel *R-L-C* combination in parallel with an applied current source.

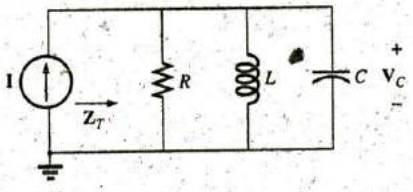


FIG. 20.21
Ideal parallel resonant network.

For the series circuit, the impedance was a minimum at resonance, producing a significant current that resulted in a high output voltage for V_C and V_L. For the parallel resonant circuit, the impedance is relatively high at resonance, producing a significant voltage for V_C and V_L through the Ohm's law relationship ($V_C = IZ_T$). For the network in Fig. 20.21, resonance occurs when $X_L = X_C$, and the resonant frequency has the same format obtained for series resonance.

If the practical equivalent in Fig. 20.22 had the format in Fig. 20.21, the analysis would be as direct and lucid as that experienced for series resonance. However, in the practical world, the internal resistance of the coil must be placed in series with the inductor, as shown in Fig. 20.22. The resistance R_l can no longer be included in a simple series or parallel combination with the source resistance and any other resistance added for design purposes. Even though R_l is usually relatively small in magnitude compared with other resistance and reactance levels of the network, it does have an important impact on the parallel resonant condition, as demonstrated in the sections to follow. In other words, the network in Fig. 20.21 is an ideal situation that can be assumed only for specific network conditions.

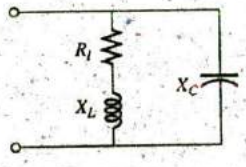


FIG. 20.22
Practical parallel L-C network.

Our first effort is to find a parallel network equivalent (at the terminals) for the series *R-L* branch in Fig. 20.22 using the technique introduced in Section 15.10. That is,

$$Z_{R\text{-}L} = R_l + jX_L$$

and
$$Y_{R\text{-}L} = \frac{1}{Z_{R\text{-}L}} = \frac{1}{R_l + jX_L} = \frac{R_l}{R_l^2 + X_L^2} - j\frac{X_L}{R_l^2 + X_L^2}$$
$$= \frac{1}{\dfrac{R_l^2 + X_L^2}{R_l}} + \frac{1}{j\left(\dfrac{R_l^2 + X_L^2}{X_L}\right)} = \frac{1}{R_p} + \frac{1}{jX_{L_p}}$$

with
$$\boxed{R_p = \frac{R_l^2 + X_L^2}{R_l}} \qquad (20.24)$$

and
$$\boxed{X_{L_p} = \frac{R_l^2 + X_L^2}{X_L}} \qquad (20.25)$$

as shown in Fig. 20.23.

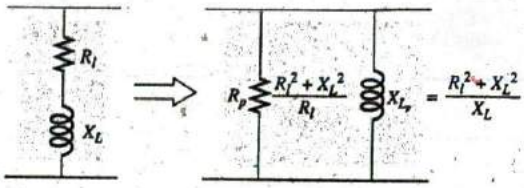


FIG. 20.23
Equivalent parallel network for a series R-L combination.

Redrawing the network in Fig. 20.22 with the equivalent in Fig. 20.23 and a practical current source having an internal resistance R_s results in the network in Fig. 20.24.

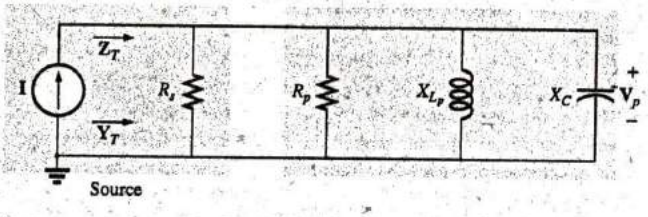


FIG. 20.24
Substituting the equivalent parallel network for the series R-L combination in Fig. 20.22.

If we define the parallel combination of R_s and R_p by the notation

$$\boxed{R = R_s \| R_p} \tag{20.26}$$

the network in Fig. 20.25 results. It has the same format as the ideal configuration in Fig. 20.21.

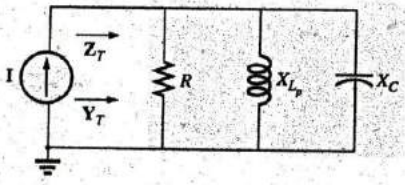


FIG. 20.25
Substituting $R = R_s \| R_p$ for the network in Fig. 20.24.

We are now at a point where we can define the resonance conditions for the practical parallel resonant configuration. Recall that for series resonance, the resonant frequency was the frequency at which the impedance was a minimum, the current a maximum, and the input impedance purely resistive and the network had a unity power factor. For parallel networks, since the resistance R_p in our equivalent model is frequency dependent, the frequency at which maximum V_C is obtained is not the same as required for the unity-power-factor characteristic. Since both conditions are often used to define the resonant state, the frequency at which each occurs is designated by different subscripts.

Unity Power Factor, f_p

For the network in Fig. 20.25,

$$\mathbf{Y}_T = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{R} + \frac{1}{jX_{L_p}} + \frac{1}{-jX_C}$$
$$= \frac{1}{R} - j\left(\frac{1}{X_{L_p}}\right) + j\left(\frac{1}{X_C}\right)$$

and
$$\boxed{\mathbf{Y}_T = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_{L_p}}\right)} \qquad (20.27)$$

For unity power factor, the reactive component must be zero as defined by

$$\frac{1}{X_C} - \frac{1}{X_{L_p}} = 0$$

Therefore,
$$\frac{1}{X_C} = \frac{1}{X_{L_p}}$$

and
$$\boxed{X_{L_p} = X_C} \qquad (20.28)$$

Substituting for X_{L_p} yields

$$\boxed{\frac{R_l^2 + X_L^2}{X_L} = X_C} \qquad (20.29)$$

The resonant frequency, f_p, can now be determined from Eq. (20.29) as follows:

$$R_l^2 + X_L^2 = X_C X_L = \left(\frac{1}{\omega C}\right)\omega L = \frac{L}{C}$$

or
$$X_L^2 = \frac{L}{C} - R_l^2$$

with
$$2\pi f_p L = \sqrt{\frac{L}{C} - R_l^2}$$

and
$$f_p = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R_l^2}$$

Multiplying the top and bottom of the factor within the square root sign by C/L produces

$$f_p = \frac{1}{2\pi L}\sqrt{\frac{1 - R_l^2(C/L)}{C/L}} = \frac{1}{2\pi L\sqrt{C/L}}\sqrt{1 - \frac{R_l^2 C}{L}}$$

and
$$\boxed{f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_l^2 C}{L}}} \qquad (20.30)$$

or
$$\boxed{f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}}} \qquad (20.31)$$

where f_p is the resonant frequency of a parallel resonant circuit (for $F_p = 1$) and f_s is the resonant frequency as determined by $X_L = X_C$ for series resonance. Note that unlike a series resonant circuit, the resonant frequency f_p is a function of resistance (in this case R_l). Note also, however, the absence of the source resistance R_s in Eqs. (20.30) and (20.31). Since the factor $\sqrt{1 - (R_l^2 C/L)}$ is less than 1, f_p is less than f_s. Recognize also that as the magnitude of R_l approaches zero, f_p rapidly approaches f_s.

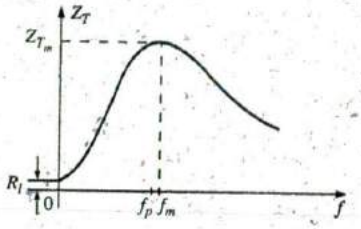


FIG. 20.26
Z_T versus frequency for the parallel resonant circuit.

Maximum Impedance, f_m

At $f = f_p$ the input impedance of a parallel resonant circuit will be near its maximum value but not quite its maximum value due to the frequency dependence of R_p. The frequency at which maximum impedance occurs is defined by f_m and is slightly more than f_p, as demonstrated in Fig. 20.26. The frequency f_m is determined by differentiating (calculus) the general equation for Z_T with respect to frequency and then determining the frequency at which the resulting equation is equal to zero. The algebra is quite extensive and cumbersome and is not included here. The resulting equation, however, is the following:

$$\boxed{f_m = f_s\sqrt{1 - \frac{1}{4}\left(\frac{R_l^2 C}{L}\right)}} \tag{20.32}$$

Note the similarities with Eq. (20.31). Since the square root factor of Eq. (20.32) is always more than the similar factor of Eq. (20.31), f_m is always closer to f_s and more than f_p. In general,

$$\boxed{f_s > f_m > f_p} \tag{20.33}$$

Once f_m is determined, the network in Fig. 20.25 can be used to determine the magnitude and phase angle of the total impedance at the resonance condition simply by substituting $f = f_m$ and performing the required calculations. That is,

$$\boxed{Z_{T_m} = R \parallel X_{L_p} \parallel X_C}_{f=f_m} \tag{20.34}$$

20.9 SELECTIVITY CURVE FOR PARALLEL RESONANT CIRCUITS

The Z_T-versus-frequency curve in Fig. 20.26 clearly reveals that a parallel resonant circuit exhibits maximum impedance at resonance (f_m), unlike the series resonant circuit, which experiences minimum resistance levels at resonance. Note also that Z_T is approximately R_l at $f = 0$ Hz since $Z_T = R_s \parallel R_l \cong R_l$.

Since the current I of the current source is constant for any value of Z_T or frequency, the voltage across the parallel circuit will have the same shape as the total impedance Z_T, as shown in Fig. 20.27.

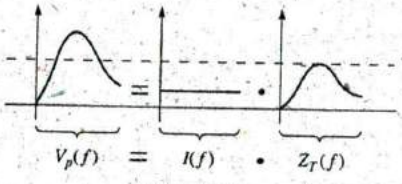


FIG. 20.27
Defining the shape of the $V_p(f)$ curve.

For the parallel circuit, the resonance curve of interest is that of the voltage V_C across the capacitor. The reason for this interest in V_C derives from electronic considerations that often place the capacitor at the input to another stage of a network.

Since the voltage across parallel elements is the same,

$$\boxed{V_C = V_p = IZ_T} \tag{20.35}$$

The resonant value of V_C is therefore determined by the value of Z_{T_m} and the magnitude of the current source I.

The quality factor of the parallel resonant circuit continues to be determined by the ratio of the reactive power to the real power. That is,

$$Q_p = \frac{V_p^2/X_{L_p}}{V_p^2/R}$$

where $R = R_s \| R_p$, and V_p is the voltage across the parallel branches. The result is

$$\boxed{Q_p = \frac{R}{X_{L_p}} = \frac{R_s \| R_p}{X_{L_p}}} \tag{20.36a}$$

or, since $X_{L_p} = X_C$ at resonance,

$$\boxed{Q_p = \frac{R_s \| R_p}{X_C}} \tag{20.36b}$$

For the ideal current source ($R_s = \infty\ \Omega$) or when R_s is sufficiently large compared to R_p, we can make the following approximation:

$$R = R_s \| R_p \cong R_p$$

and

$$Q_p = \frac{R_s \| R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = \frac{(R_l^2 + X_L^2)/R_l}{(R_l^2 + X_L^2)/X_L}$$

so that

$$\boxed{Q_p = \frac{X_L}{R_l} = Q_l}_{R_S >> R_p} \tag{20.37}$$

which is simply the quality factor Q_l of the coil.

In general, the bandwidth is still related to the resonant frequency and the quality factor by

$$\boxed{BW = f_2 - f_1 = \frac{f_r}{Q_p}} \tag{20.38}$$

The cutoff frequencies f_1 and f_2 can be determined using the equivalent network in Fig. 20.25 and the unity power condition for resonance. The half-power frequencies are defined by the condition that the output voltage is 0.707 times the maximum value. However, for parallel resonance with a current source driving the network, the frequency response for the driving point impedance is the same as that for the output voltage. This similarity permits defining each cutoff frequency as the frequency at which the input impedance is 0.707 times its maximum value. Since the maximum value is the equivalent resistance R in Fig. 20.25, the cutoff frequencies are associated with an impedance equal to $0.707R$ or $(1/\sqrt{2})R$.

Setting the magnitude of the input impedance for the network in Fig. 20.25 equal to this value results in the following:

$$|\mathbf{Z}| = Z = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}} = 0.707R = \frac{R}{\sqrt{2}}$$

or

$$\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} = \frac{\sqrt{2}}{R}$$

Squaring both sides gives

$$\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2 = \frac{2}{R^2}$$

which results in

$$\left(\omega C - \frac{1}{\omega L}\right)^2 = \frac{1}{R^2}$$

The resulting fourth-degree equation for ω results in two second-degree equations for ω:

$$\omega C - \frac{1}{\omega L} = -\frac{1}{R}$$

and

$$\omega C - \frac{1}{\omega L} = \frac{1}{R}$$

Solving each for the positive result for ω gives

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

and

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

so that

$$\boxed{f_1 = \frac{1}{4\pi C}\left[-\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]} \qquad \textbf{(20.39a)}$$

$$\boxed{f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]} \qquad \textbf{(20.39b)}$$

The effect of R_l, L, and C on the shape of the parallel resonance curve, as shown in Fig. 20.28 for the input impedance, is quite similar to their effect on the series resonance curve. Whether or not R_l is zero, the parallel resonant circuit frequently appears in a network schematic as shown in Fig. 20.28.

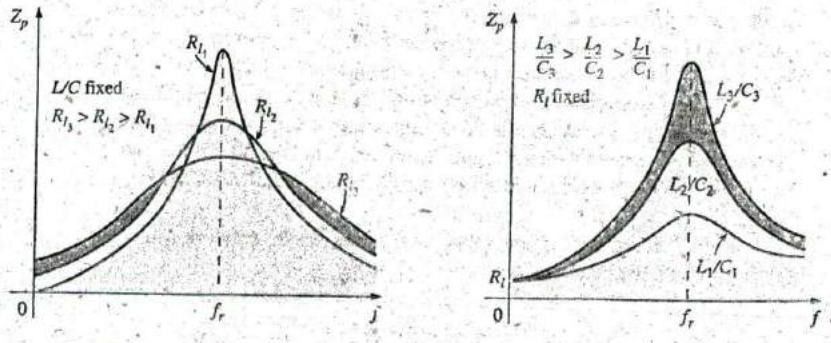


FIG. 20.28
Effect of R_l, L, and C on the parallel resonance curve.

At resonance, an increase in R_l or a decrease in the ratio L/C results in a decrease in the resonant impedance, with a corresponding increase in the current. The bandwidth of the resonance curves is given by Eq. (20.38). For increasing R_l or decreasing L (or L/C for constant C), the bandwidth increases as shown in Fig. 20.28.

At low frequencies, the capacitive reactance is quite high, and the inductive reactance is low. Since the elements are in parallel, the total impedance at low frequencies is therefore inductive. At high frequencies, the reverse is true, and the network is capacitive. At resonance (f_p), the network appears resistive. These facts lead to the phase plot in Fig. 20.29. Note that it is the inverse of that appearing for the series resonant circuit because at low frequencies the series resonant circuit was capacitive and at high frequencies it was inductive.

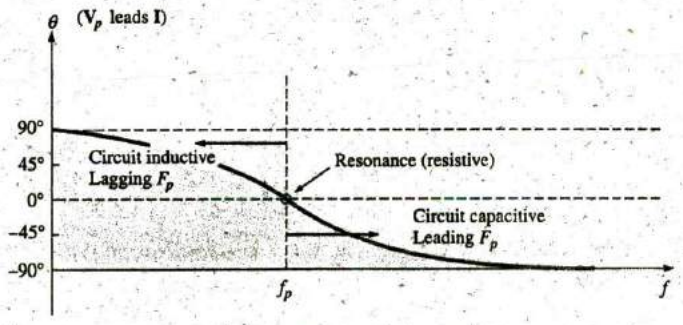


FIG. 20.29
Phase plot for the parallel resonant circuit.

20.10 EFFECT OF $Q_l \geq 10$

The content of the previous section may suggest that the analysis of parallel resonant circuits is significantly more complex than that encountered for series resonant circuits. Fortunately, however, this is not the case since, for the majority of parallel resonant circuits, the quality factor of the coil Q_l is sufficiently large to permit a number of approximations that simplify the required analysis.

Inductive Reactance, X_{L_P}

If we expand X_{L_p} as

$$X_{L_p} = \frac{R_l^2 + X_L^2}{X_L} = \frac{R_l^2(X_L)}{X_L(X_L)} + X_L = \frac{X_L}{Q_l^2} + X_L$$

then, for $Q_l \geq 10$, $1/Q_l^2 \ll 1$, and therefore

$$\frac{X_L}{Q_l^2} + X_L = X_L\left(\frac{1}{Q_l^2} + 1\right) \cong X_L$$

which results in

$$\boxed{X_{L_p} \cong X_L}_{Q_l \geq 10} \qquad (20.40)$$

and since resonance is defined by $X_{L_p} = X_C$, the resulting condition for resonance is reduced to

$$\boxed{X_L \cong X_C}_{Q_l \geq 10} \qquad (20.41)$$

Resonant Frequency, f_p (Unity Power Factor)

We can rewrite the factor R_l^2C/L of Eq. (20.31) as

$$\frac{R_l^2C}{L} = \frac{1}{\frac{L}{R_l^2C}} = \frac{1}{\frac{(\omega)}{(\omega)}\frac{L}{R_l^2C}} = \frac{1}{\frac{\omega L}{R_l^2\omega C}} = \frac{1}{\frac{X_LX_C}{R_l^2}}$$

and substitute Eq. (20.41) ($X_L \cong X_C$):

$$\frac{1}{\frac{X_LX_C}{R_l^2}} = \frac{1}{\frac{X_L^2}{R_l^2}} = \frac{1}{Q_l^2}$$

Eq. (20.31) then becomes

$$\boxed{f_p = f_s\sqrt{1 - \frac{1}{Q_l^2}}}_{Q_l \geq 10} \tag{20.42}$$

clearly revealing that as Q_l increases, f_p becomes closer and closer to f_s.

For $Q_l \geq 10$,

$$1 - \frac{1}{Q_l^2} \cong 1$$

and

$$\boxed{f_p \cong f_s = \frac{1}{2\pi\sqrt{LC}}}_{Q_l \geq 10} \tag{20.43}$$

Resonant Frequency, f_m (Maximum V_C)

Using the equivalency $R_l^2C/L = 1/Q_l^2$ derived for Eq. (20.42), we find that Eq. (20.32) takes on the form

$$\boxed{f_m \cong f_s\sqrt{1 - \frac{1}{4}\left(\frac{1}{Q_l^2}\right)}}_{Q_l \geq 10} \tag{20.44}$$

The fact that the negative term under the square root will always be less than that appearing in the equation for f_p reveals that f_m will always be closer to f_s than f_p.

For $Q_l \geq 10$, the negative term becomes very small and can be dropped from consideration, leaving

$$\boxed{f_m \cong f_s = \frac{1}{2\pi\sqrt{LC}}}_{Q_l \geq 10} \tag{20.45}$$

In total, therefore, for $Q_l \geq 10$,

$$\boxed{f_p \cong f_m \cong f_s}_{Q_l \geq 10} \tag{20.46}$$

R_p

$$R_p = \frac{R_l^2 + X_L^2}{R_l} = R_l + \frac{X_L^2}{R_l}\left(\frac{R_l}{R_l}\right) = R_l + \frac{X_L^2}{R_l^2}R_l$$
$$= R_l + Q_l^2R_l = (1 + Q_l^2)R_l$$

For $Q_l \geq 10$, $1 + Q_l^2 \cong Q_l^2$, and

$$\boxed{R_p \cong Q_l^2 R_l}_{Q_l \geq 10} \tag{20.47}$$

Applying the approximations just derived to the network in Fig. 20.24 results in the approximate equivalent network for $Q_l \geq 10$ in Fig. 20.30, which is certainly a lot "cleaner" in general appearance.

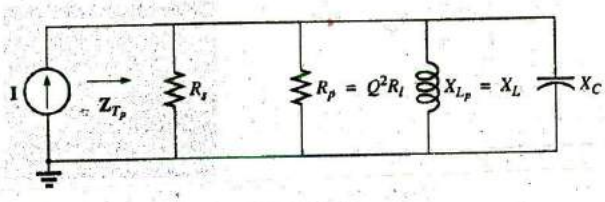


FIG. 20.30
Approximate equivalent circuit for $Q_l \geq 10$.

Substituting $Q_l = \dfrac{X_L}{R_l}$ into Eq. (20.47) gives

$$R_p \cong Q_l^2 R_l = \left(\frac{X_L}{R_l}\right)^2 R_l = \frac{X_L^2}{R_l} = \frac{X_L X_C}{R_l} = \frac{2\pi f L}{R_l(2\pi f C)}$$

and

$$\boxed{R_p \cong \frac{L}{R_l C}}_{Q_l \geq 10} \tag{20.48}$$

Z_{T_p}

The total impedance at resonance is now defined by

$$\boxed{Z_{T_p} \cong R_s \parallel R_p = R_s \parallel Q_l^2 R_l}_{Q_l \geq 10} \tag{20.49}$$

For an ideal current source ($R_s = \infty\,\Omega$), or if $R_s >> R_p$, the equation reduces to

$$\boxed{Z_{T_p} \cong Q_l^2 R_l}_{Q_l \geq 10,\ R_s >> R_p} \tag{20.50}$$

Q_p

The quality factor is now defined by

$$\boxed{Q_p = \frac{R}{X_{L_p}} \cong \frac{R_s \parallel Q_l^2 R_l}{X_L}} \tag{20.51}$$

Quite obviously, therefore, R_s does have an impact on the quality factor of the network and the shape of the resonant curves.

If an ideal current source ($R_s = \infty\,\Omega$) is used, or if $R_s >> R_p$,

$$Q_p \cong \frac{R_s \parallel Q_l^2 R_l}{X_L} = \frac{Q_l^2 R_l}{X_L} = \frac{Q_l^2}{X_L/R_l} = \frac{Q_l^2}{Q_l}$$

and

$$\boxed{Q_p \cong Q_l}_{Q_l \geq 10,\ R_s >> R_p} \tag{20.52}$$

BW

The bandwidth defined by f_p is

$$\boxed{BW = f_2 - f_1 = \frac{f_p}{Q_p}} \tag{20.53}$$

By substituting Q_p from above and performing a few algebraic manipulations, we can show that

$$\boxed{BW = f_2 - f_1 \cong \frac{1}{2\pi}\left[\frac{R_l}{L} + \frac{1}{R_s C}\right]} \tag{20.54}$$

clearly revealing the impact of R_s on the resulting bandwidth. Of course, if $R_s = \infty\,\Omega$(ideal current source), then

$$\boxed{BW = f_2 - f_1 \cong \frac{R_l}{2\pi L}}_{R_s = \infty\,\Omega} \tag{20.55}$$

I_L and I_C

A portion of Fig. 20.30 is reproduced in Fig. 20.31, with I_T defined as shown.

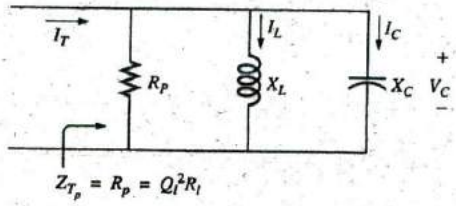


FIG. 20.31
Establishing the relationship between I_C and I_L and the current I_T.

As indicated, Z_{T_p} at resonance is $Q_l^2 R_l$. The voltage across the parallel network is, therefore,

$$V_C = V_L = V_R = I_T Z_{T_p} = I_T Q_l^2 R_l$$

The magnitude of the current I_C can then be determined using Ohm's law as follows:

$$I_C = \frac{V_C}{X_C} = \frac{I_T Q_l^2 R_l}{X_C}$$

Substituting $X_C = X_L$ when $Q_l \geq 10$ gives

$$I_C = \frac{I_T Q_l^2 R_l}{X_L} = I_T \frac{Q_l^2}{\frac{X_L}{R_l}} = I_T \frac{Q_l^2}{Q_l}$$

and

$$\boxed{I_C \cong Q_l I_T}_{Q_l \geq 10} \tag{20.56}$$

revealing that the capacitive current is Q_l times the magnitude of the current entering the parallel resonant circuit. For large Q_l, the current I_C can be significant.

A similar derivation results in

$$\boxed{I_L \cong Q_l I_T}_{Q_l \geq 10} \tag{20.57}$$

Conclusions

The equations resulting from the application of the condition $Q_l \geq 10$ are obviously a great deal easier to apply than those obtained earlier. It is, therefore, a condition that should be checked early in an analysis to

determine which approach must be applied. Although the condition $Q_l \geq 10$ was applied throughout, many of the equations are still good approximations for $Q_l < 10$. For instance, if $Q_l = 5$, $X_{L_p} = X_L/Q_l^2 + X_l = (X_L/25) + X_L = 1.04X_L$, which is very close to X_L. In fact, for $Q_l = 2$, $X_{L_p} = (X_L/4) + X_L = 1.25X_L$, which is not X_L, but it is only 25% off. In general, be aware that the approximate equations can be applied with good accuracy with $Q_l < 10$. The smaller the level of Q_l, however, the less valid is the approximation. The approximate equations are certainly valid for a range of values of $Q_l < 10$ if a rough approximation to the actual response is desired rather than one accurate to the hundredths place.

20.11 SUMMARY TABLE

In an effort to limit any confusion resulting from the introduction of f_p and f_m and an approximate approach dependent on Q_l, the summary in Table 20.1 was developed. You can always use the equations for any Q_l, but a proficiency in applying the approximate equations defined by Q_l will pay dividends in the long run.

For the future, the analysis of a parallel resonant network may proceed as follows:

1. Determine f_s to obtain some idea of the resonant frequency. Recall that for most situations, f_s, f_m, and f_p will be relatively close to each other.
2. Calculate an approximate Q_l using f_s from below, and compare it to the condition $Q_l \geq 10$. If the condition is satisfied, the approximate approach should be the chosen path unless a high degree of accuracy is required.
3. If Q_l is less than 10, the approximate approach can be applied, but it must be understood that the smaller the level of Q_l, the less accurate is the solution. However, considering the typical variations from nameplate values for many of our components and

TABLE 20.1
Parallel resonant circuit ($f_s = 1/(2\pi\sqrt{LC})$).

	Any Q_l	$Q_l \geq 10$	$Q_l \geq 10, R_s >> Q_l^2 R_l$
f_p	$f_s\sqrt{1 - \frac{R_l^2 C}{L}}$	f_s	f_s
f_m	$f_s\sqrt{1 - \frac{1}{4}\left[\frac{R_l^2 C}{L}\right]}$	f_s	f_s
Z_{T_p}	$R_s \| R_p = R_s \| \left(\frac{R_l^2 + X_L^2}{R_l}\right)$	$R_s \| Q_l^2 R_l$	$Q_l^2 R_l$
Z_{T_m}	$R_s \| \mathbf{Z}_{R\text{-}L} \| Z_C$	$R_s \| Q_l^2 R_l$	$Q_l^2 R_l$
Q_p	$\frac{Z_{T_p}}{X_{L_p}} = \frac{Z_{T_p}}{X_C}$	$\frac{Z_{T_p}}{X_L} = \frac{Z_{T_p}}{X_C}$	Q_l
BW	$\frac{f_p}{Q_p}$ or $\frac{f_m}{Q_p}$	$\frac{f_p}{Q_p} = \frac{f_s}{Q_p}$	$\frac{f_p}{Q_l} = \frac{f_s}{Q_l}$
I_L, I_C	Network analysis	$I_L = I_C = Q_l I_T$	$I_L = I_C = Q_l I_T$

that a resonant frequency to the tenths place is seldom required, the use of the approximate approach for many practical situations is usually quite valid.

20.12 EXAMPLES (PARALLEL RESONANCE)

EXAMPLE 20.6 Given the parallel network in Fig. 20.32 composed of "ideal" elements:

a. Determine the resonant frequency f_p.
b. Find the total impedance at resonance.
c. Calculate the quality factor, bandwidth, and cutoff frequencies f_1 and f_2 of the system.
d. Find the voltage V_C at resonance.
e. Determine the currents I_L and I_C at resonance.

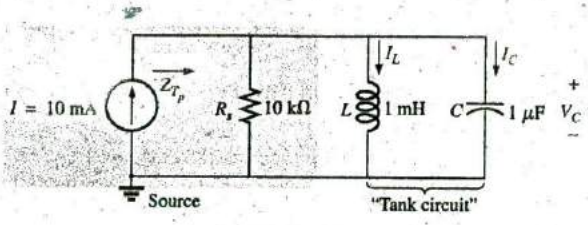


FIG. 20.32
Example 20.6.

Solutions:

a. The fact that R_l is zero ohms results in a very high $Q_l (= X_L/R_l)$, permitting the use of the following equation for f_p:

$$f_p = f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1\text{ mH})\,1\,\mu\text{F}}}$$
$$= \mathbf{5.03\ kHz}$$

b. For the parallel reactive elements,

$$\mathbf{Z}_L \| \mathbf{Z}_C = \frac{(X_L \angle 90°)(X_C \angle -90°)}{+j(X_L - X_C)}$$

but $X_L = X_C$ at resonance, resulting in a zero in the denominator of the equation and a very high impedance that can be approximated by an open circuit. Therefore,

$$Z_{T_p} = R_s \| \mathbf{Z}_L \| \mathbf{Z}_C = R_s = \mathbf{10\ k\Omega}$$

c. $$Q_p = \frac{R_s}{X_{L_p}} = \frac{R_s}{2\pi f_p L} = \frac{10\text{ k}\Omega}{2\pi(5.03\text{ kHz})(1\text{ mH})} = \mathbf{316.41}$$

$$BW = \frac{f_p}{Q_p} = \frac{5.03\text{ kHz}}{316.41} = \mathbf{15.90\ Hz}$$

Eq. (20.39a):

$$f_1 = \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

$$= \frac{1}{4\pi(1\ \mu\text{F})}\left[\frac{1}{10\ \text{k}\Omega} - \sqrt{\frac{1}{(10\ \text{k}\Omega)^2} + \frac{4(1\ \mu\text{F})}{1\ \text{mH}}}\right]$$
$$= \mathbf{5.03\ kHz}$$

Eq. (20.39b):

$$f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$
$$= \mathbf{5.04\ kHz}$$

d. $V_C = IZ_{T_p} = (10\ \text{mA})(10\ \text{k}\Omega) = \mathbf{100\ V}$

e. $I_L = \dfrac{V_L}{X_L} = \dfrac{V_C}{2\pi f_p L} = \dfrac{100\ \text{V}}{2\pi(5.03\ \text{kHz})(1\ \text{mH})} = \dfrac{100\ \text{V}}{31.6\ \Omega} = \mathbf{3.16\ A}$

$$I_C = \frac{V_C}{X_C} = \frac{100\ \text{V}}{31.6\ \Omega} = \mathbf{3.16\ A}\ (= Q_p I)$$

Example 20.6 demonstrates the impact of R_s on the calculations associated with parallel resonance: The source impedance is the only factor limiting the input impedance and the level of V_C.

EXAMPLE 20.7 For the parallel resonant circuit in Fig. 20.33 with $R_s = \infty\ \Omega$:

a. Determine f_s, f_m, and f_p, and compare their levels.
b. Calculate the maximum impedance and the magnitude of the voltage V_C at f_m.
c. Determine the quality factor Q_p.
d. Calculate the bandwidth.
e. Compare the above results with those obtained using the equations associated with $Q_l \geq 10$.

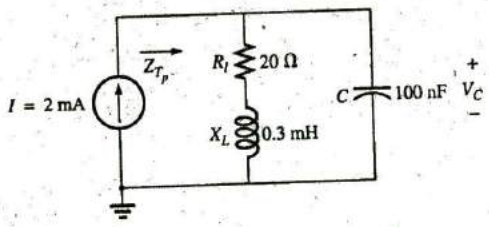


FIG. 20.33
Example 20.7.

Solutions:

a. $f_s = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.3\ \text{mH})(100\ \text{nF})}} = \mathbf{29.06\ kHz}$

$$f_m = f_s\sqrt{1 - \frac{1}{4}\left[\frac{R_l^2 C}{L}\right]}$$
$$= (29.06\ \text{kHz})\sqrt{1 - \frac{1}{4}\left[\frac{(20\ \Omega)^2(100\ \text{nF})}{0.3\ \text{mH}}\right]}$$
$$= \mathbf{25.58\ kHz}$$

$$f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}} = (29.06\text{ kHz})\sqrt{1 - \left[\frac{(20\ \Omega)^2(100\text{ nF})}{0.3\text{ mH}}\right]}$$
$$= \mathbf{27.06\ kHz}$$

Both f_m and f_p are less than f_s, as predicted. In addition, f_m is closer to f_s than f_p, as forecast. f_m is about 0.5 kHz less than f_s, whereas f_p is about 2 kHz less. The differences among f_s, f_m, and f_p suggest a low Q network.

b. $\mathbf{Z}_{T_m} = (R_l + jX_L) \| -jX_C$ at $f = f_m$

$$X_L = 2\pi f_m L = 2\pi(28.58\text{ kHz})(0.3\text{ mH}) = 53.87\ \Omega$$

$$X_C = \frac{1}{2\pi f_m C} = \frac{1}{2\pi(28.58\text{ kHz})(100\text{ nF})} = 55.69\ \Omega$$

$$R_l + jX_L = 20\ \Omega + j\,53.87\ \Omega = 57.46\ \Omega\ \angle 69.63°$$

$$\mathbf{Z}_{T_m} = \frac{(57.46\ \Omega\ \angle 69.63°)(55.69\ \Omega\ \angle -90°)}{20\ \Omega + j\,53.87\ \Omega - j\,55.69\ \Omega}$$
$$= \mathbf{159.34\ \Omega\ \angle -15.17°}$$

$$V_{C_{max}} = IZ_{T_m} = (2\text{ mA})(159.34\ \Omega) = \mathbf{318.68\ mV}$$

c. $R_s = \infty\ \Omega$; therefore,

$$Q_p = \frac{R_s \| R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = Q_l = \frac{X_L}{R_l}$$
$$= \frac{2\pi(27.06\text{ kHz})(0.3\text{ mH})}{20\ \Omega} = \frac{51\ \Omega}{20\ \Omega} = \mathbf{2.55}$$

The low Q confirms the conclusion of part (a). The differences among f_s, f_m, and f_p are significantly less for higher Q networks.

d. $$BW = \frac{f_p}{Q_p} = \frac{27.06\text{ kHz}}{2.55} = \mathbf{10.61\ kHz}$$

e. For $Q_l \geq 10$, $f_m \cong f_p = f_s = \mathbf{29.06\ kHz}$

$$Q_p = Q_l = \frac{2\pi f_s L}{R_l} = \frac{2\pi(29.06\text{ kHz})(0.3\text{ mH})}{20\ \Omega} = \mathbf{2.74}$$

(versus 2.55 above)

$$Z_{T_p} = Q_l^2 R_l = (2.74)^2 \cdot 20\ \Omega = \mathbf{150.15\ \Omega\ \angle 0°}$$

(versus 159.34 Ω ∠ −15.17° above)

$$V_{C_{max}} = IZ_{T_p} = (2\text{ mA})(150.15\ \Omega) = \mathbf{300.3\ mV}$$

(versus 318.68 mV above)

$$BW = \frac{f_p}{Q_p} = \frac{29.06\text{ kHz}}{2.74} = \mathbf{10.61\ kHz}$$

(versus 10.61 kHz above)

The results reveal that, even for a relatively low Q system, the approximate solutions are still close compared to those obtained using the full equations. The primary difference is between f_s and f_p (about 7% with the difference between f_s and f_m at less than 2%. For the future, using f_s to determine Q_l will certainly provide a measure of Q_l that can be used to determine whether the approximate approach is appropriate.

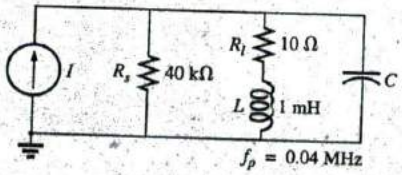


FIG. 20.34
Example 20.8.

EXAMPLE 20.8 For the network in Fig. 20.34 with f_p provided:

a. Determine Q_l.
b. Determine R_p.

c. Calculate Z_{T_p}.
d. Find C at resonance.
e. Find Q_p.
f. Calculate the BW and cutoff frequencies.

Solutions:

a. $$Q_l = \frac{X_L}{R_l} = \frac{2\pi f_p L}{R_l} = \frac{2\pi(0.04\text{ MHz})(1\text{ mH})}{10\ \Omega} = \mathbf{25.12}$$

b. $Q_l \geq 10$. Therefore,

$$R_p \cong Q_l^2 R_l = (25.12)^2(10\ \Omega) = \mathbf{6.31\ k\Omega}$$

c. $Z_{T_p} = R_s \| R_p = 40\text{ k}\Omega \| 6.31\text{ k}\Omega = \mathbf{5.45\ k\Omega}$

d. $Q_l \geq 10$. Therefore,

$$f_p \cong \frac{1}{2\pi\sqrt{LC}}$$

and $$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2(0.04\text{ MHz})^2(1\text{ mH})} = \mathbf{15.83\ nF}$$

e. $Q_l \geq 10$. Therefore,

$$Q_p = \frac{Z_{T_p}}{X_L} = \frac{R_s \| Q_l^2 R_l}{2\pi f_p L} \cong \frac{5.45\text{ k}\Omega}{2\pi(0.04\text{ MHz})(1\text{ mH})} = \mathbf{21.68}$$

f. $$BW = \frac{f_p}{Q_p} = \frac{0.04\text{ MHz}}{21.68} = \mathbf{1.85\ kHz}$$

$$f_1 = \frac{1}{4\pi C}\left[-\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

$$= \frac{1}{4\pi(15.9\text{ mF})}\left[-\frac{1}{5.45\text{ k}\Omega} + \sqrt{\frac{1}{(5.45\text{ k}\Omega)^2} + \frac{4(15.9\text{ mF})}{1\text{ mH}}}\right]$$

$$= 5.005 \times 10^6[-183.486 \times 10^{-6} + 7.977 \times 10^{-3}]$$
$$= 5.005 \times 10^6[7.794 \times 10^{-3}]$$
$$= \mathbf{39\ kHz}$$

$$f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

$$= 5.005 \times 10^6[183.486 \times 10^{-6} + 7.977 \times 10^{-3}]$$
$$= 5.005 \times 10^6[8.160 \times 10^{-3}]$$
$$= \mathbf{40.84\ kHz}$$

Note that $f_2 - f_1 = 40.84\text{ kHz} - 39\text{ kHz} = 1.84\text{ kHz}$, confirming our solution for the bandwidth above. Note also that the bandwidth is not symmetrical about the resonant frequency, with 1 kHz below and 840 Hz above.

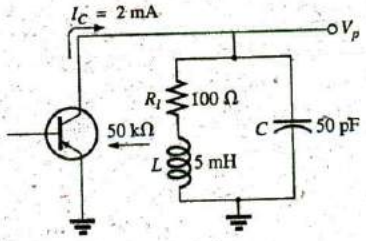


FIG. 20.35
Example 20.9.

EXAMPLE 20.9 The equivalent network for the transistor configuration in Fig. 20.35 is provided in Fig. 20.36.

a. Find f_p.
b. Determine Q_p.
c. Calculate the BW.
d. Determine V_p at resonance.
e. Sketch the curve of V_C versus frequency.

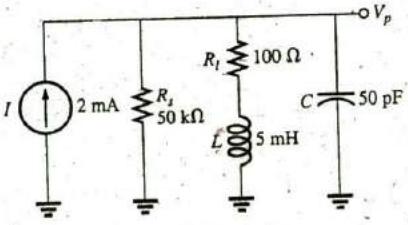


FIG. 20.36
Equivalent network for the transistor configuration in Fig. 20.35.

Solutions:

a. $f_s = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(5\text{ mH})(50\text{ pF})}} = 318.31\text{ kHz}$

$X_L = 2\pi f_s L = 2\pi(318.31\text{ kHz})(5\text{ mH}) = 10\text{ k}\Omega$

$Q_l = \dfrac{X_L}{R_l} = \dfrac{10\text{ k}\Omega}{100\text{ k}\Omega} = 100 > 10$

Therefore, $f_p = f_s = \mathbf{318.31\text{ kHz}}$. Using Eq. (20.31) results in $\cong 318.5$ kHz.

b. $Q_p = \dfrac{R_s \parallel R_p}{X_L}$

$R_p = Q_l^2 R_l = (100)^2\,100\,\Omega = 1\text{ M}\Omega$

$Q_p = \dfrac{50\text{ k}\Omega \parallel 1\text{ M}\Omega}{10\text{ k}\Omega} = \dfrac{47.62\text{ k}\Omega}{10\text{ k}\Omega} = \mathbf{4.76}$

Note the drop in Q from $Q_l = 100$ to $Q_p = 4.76$ due to R_s.

c. $BW = \dfrac{f_p}{Q_p} = \dfrac{318.31\text{ kHz}}{4.76} = \mathbf{66.87\text{ kHz}}$

On the other hand,

$$BW = \frac{1}{2\pi}\left(\frac{R_l}{L} + \frac{1}{R_s C}\right) = \frac{1}{2\pi}\left[\frac{100\,\Omega}{5\text{ mH}} + \frac{1}{(50\text{ k}\Omega)(50\text{ pF})}\right] = \mathbf{66.85\text{ kHz}}$$

compares very favorably with the above solution.

d. $V_p = IZ_{T_p} = (2\text{ mA})(R_s \parallel R_p) = (2\text{ mA})(47.62\text{ k}\Omega) = \mathbf{95.24\text{ V}}$

e. See Fig. 20.37.

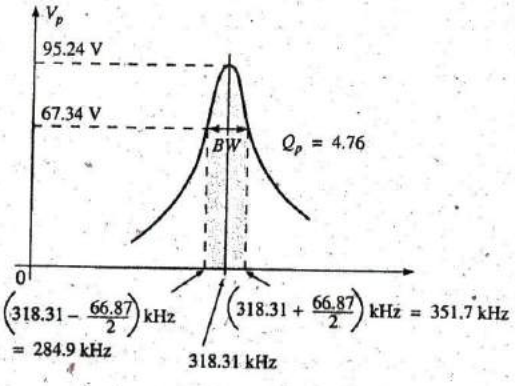


FIG. 20.37
Resonance curve for the network in Fig. 20.36.

EXAMPLE 20.10 Repeat Example 20.9, but ignore the effects of R_s, and compare results.

Solutions:

a. f_p is the same, **318.31 kHz.**

b. For $R_s = \infty\,\Omega$,

$$Q_p = Q_l = \mathbf{100} \qquad \text{(versus 4.76)}$$

c. $BW = \dfrac{f_p}{Q_p} = \dfrac{318.31\ \text{kHz}}{100} = \mathbf{3.18\ kHz}$ (versus 66.87 kHz)

d. $Z_{T_p} \cong R_p = 1\ \text{M}\Omega$ (versus 47.62 kΩ)

$V_p = IZ_{T_p} = (2\ \text{mA})(1\ \text{M}\Omega) = \mathbf{2000\ V}$ (versus 95.24 V)

The results obtained clearly reveal that the source resistance can have a significant impact on the response characteristics of a parallel resonant circuit.

EXAMPLE 20.11 Design a parallel resonant circuit to have the response curve in Fig. 20.38 using a 1 mH, 10 Ω inductor and a current source with an internal resistance of 40 kΩ.

Solution:

$$BW = \frac{f_p}{Q_p}$$

Therefore,

$$Q_p = \frac{f_p}{BW} = \frac{50{,}000\ \text{Hz}}{2500\ \text{Hz}} = \mathbf{20}$$

$$X_L = 2\pi f_p L = 2\pi(50\ \text{kHz})(1\ \text{mH}) = 314\ \Omega$$

and

$$Q_l = \frac{X_L}{R_l} = \frac{314\ \Omega}{10\ \Omega} = \mathbf{31.4}$$

$$R_p = Q_l^2 R = (31.4)^2(10\ \Omega) = \mathbf{9859.6\ \Omega}$$

$$Q_p = \frac{R}{X_L} = \frac{R_s \| 9859.6\ \Omega}{314\ \Omega} = 20 \quad \text{(from above)}$$

so that

$$\frac{(R_s)(9859.6)}{R_s + 9859.6} = 6280$$

resulting in

$$R_s = 17.298\ \text{k}\Omega$$

However, the source resistance was given as 40 kΩ. We must therefore add a parallel resistor (R') that will reduce the 40 kΩ to approximately 17.298 kΩ; that is,

$$\frac{(40\ \text{k}\Omega)(R')}{40\ \text{k}\Omega + R'} = 17.298\ \text{k}\Omega$$

Solving for R' gives

$$R' = \mathbf{30.48\ k\Omega}$$

The closest commercial value is **30 kΩ**. At resonance, $X_L = X_C$, and

$$X_C = \frac{1}{2\pi f_p C}$$

$$C = \frac{1}{2\pi f_p X_C} = \frac{1}{2\pi(50\ \text{kHz})(314\ \Omega)}$$

and

$$C \cong \mathbf{0.01\ \mu F} \quad \text{(commercially available)}$$

$$Z_{T_p} = R_s \| Q_l^2 R_l$$
$$= 17.298\ \text{k}\Omega \| 9859.6\ \Omega$$
$$= 6.28\ \text{k}\Omega$$

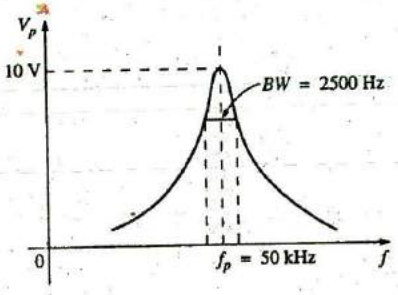


FIG. 20.38
Example 20.11.

with $$V_p = IZ_{T_p}$$

and $$I = \frac{V_p}{Z_{T_p}} = \frac{10\ \text{V}}{6.28\ \text{k}\Omega} \cong \mathbf{1.6\ mA}$$

The network appears in Fig. 20.39.

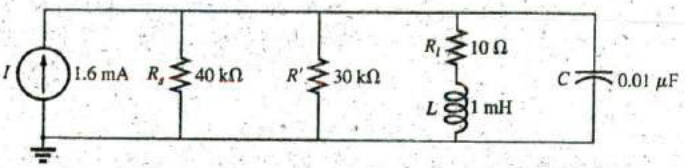


FIG. 20.39
Network designed to meet the criteria in Fig. 20.38.

20.13 APPLICATIONS

Stray Resonance

Stray resonance, like stray capacitance and inductance and unexpected resistance levels, can occur in totally unexpected situations and can severely affect the operation of a system. All that is required to produce stray resonance is, for example, a level of capacitance introduced by parallel wires or copper leads on a printed circuit board, or simply two parallel conductive surfaces with residual charge and inductance levels associated with any conductor or components such as tape recorder heads, transformers, and so on, that provide the elements necessary for a resonance effect. In fact, this resonance effect is a very common effect in a cassette tape recorder. The play/record head is a coil that can act like an inductor and an antenna. Combine this factor with the stray capacitance and real capacitance in the network to form the tuning network, and the tape recorder with the addition of a semiconductor diode can respond like an AM radio. As you plot the frequency response of any transformer, you normally find a region where the response has a peaking effect (look ahead at Fig. 25.21). This peaking is due solely to the inductance of the coils of the transformer and the stray capacitance between the wires.

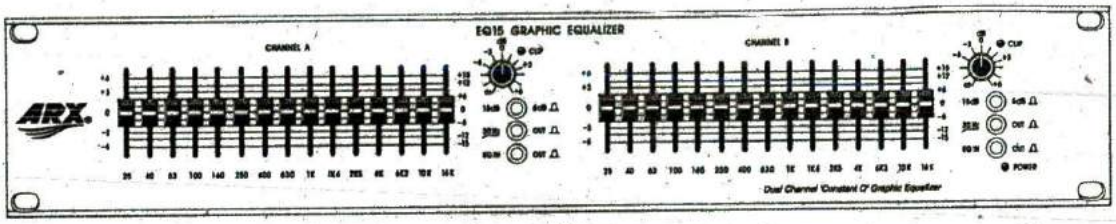
In general, any time you see an unexpected peaking in the frequency response of an element or a system, it is normally caused by a resonance condition. If the response has a detrimental effect on the overall operation of the system, a redesign may be in order, or a filter can be added that will block the frequencies that result in the resonance condition. Of course, when you add a filter composed of inductors and/or capacitors, you must be careful that you don't add another unexpected resonance condition. It is a problem that can be properly weighed only by constructing the system and exposing it to the full range of tests.

Graphic and Parametric Equalizers

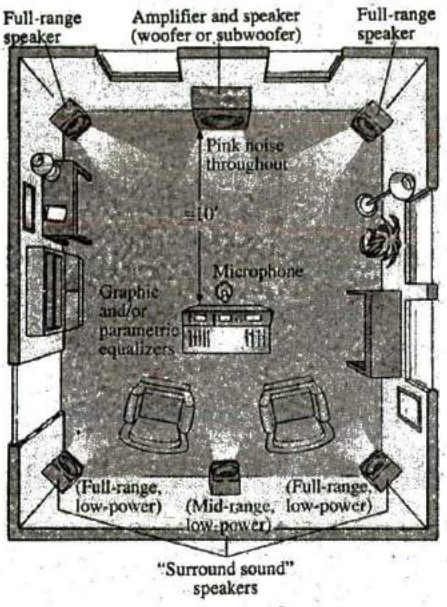
We have all noticed at one time or another that the music we hear in a concert hall doesn't quite sound the same when we play a recording of it on our home entertainment center. Even after we check the specifications of the speakers and amplifiers and find that both are nearly perfect (and the most expensive we can afford), the sound is still not what it should be. In general, we are experiencing the effects of the local environmental

characteristics on the sound waves. Some typical problems are hard walls or floors (stone, cement) that make high frequencies sound louder. Curtains and rugs, on the other hand, absorb high frequencies. The shape of the room and the placement of the speakers and furniture also affect the sound that reaches our ears. Another criterion is the echo or reflection of sound that occurs in the room. Concert halls are designed very carefully with vaulted ceilings and curved walls to allow a certain amount of echo. Even the temperature and humidity characteristics of the surrounding air affect the quality of the sound. It is certainly impossible, in most cases, to redesign your listening area to match a concert hall, but with the proper use of electronic systems you can develop a response that has all the qualities that you want from a home entertainment center.

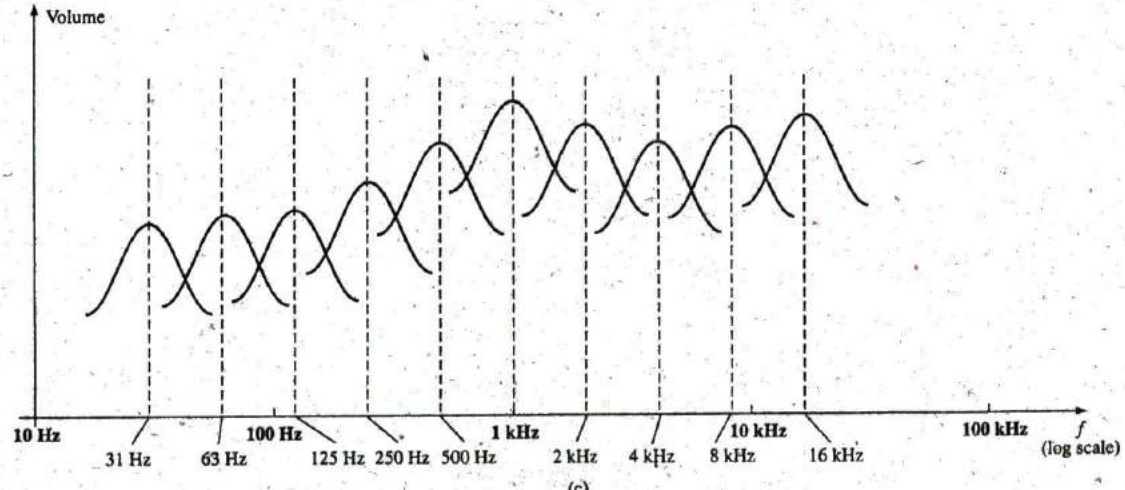
For a quality system, a number of steps can be taken: *characterization and digital delay (surround sound)* and *proper speaker and amplifier selection and placement.* Characterization is a process whereby a thorough sound absorption check of the room is performed and the frequency response determined. A *graphic equalizer* such as appearing in Fig. 20.40(a) is then used to make the response "flat" for the full range of frequencies. In other words, the room is made to appear as though all the frequencies receive equal amplification in the listening area. For instance, if the room is fully carpeted with full draping curtains, there is a considerable amount of high-frequency absorption, requiring that the high frequencies have additional amplification to match the sound levels of the mid and low frequencies. To *characterize* the typical rectangular-shaped room, a setup such as shown in Fig. 20.40(b) may be used. The amplifier and speakers are placed in the center of one wall; with additional speakers in the corners of the room facing the reception area. A mike is then placed in the reception area about 10 ft from the amplifier and centered between the two other speakers. A *pink noise* is then sent out from a spectrum analyzer (often an integral part of the graphic equalizer) to the amplifier and speakers. Pink noise is actually a square-wave signal whose amplitude and frequency can be controlled. A square-wave signal was chosen because a Fourier breakdown of a square-wave signal results in a broad range of frequencies for the system to check. You will find in Chapter 24 that a square wave can be constructed of an infinite series of sine waves of different frequencies. Once the proper volume of pink noise is established, the spectrum analyzer can be used to set the response of each slide band to establish the desired flat response. The center frequencies for the slides of the graphic equalizer in Fig. 20.40(a) are provided in Fig. 20.40(c), along with the frequency response for a number of adjoining frequencies evenly spaced on a logarithmic scale. Note that each center frequency is actually the resonant frequency for that slide. The design is such that each slide can control the volume associated with that frequency, but the bandwidth and frequency response stay fairly constant. A good spectrum analyzer has each slide set against a decibel (dB) scale (decibels are discussed in detail in Chapter 21). The decibel scale simply establishes a scale for the comparison of audio levels. At a normal listening level, usually a change of about 3 dB is necessary for the audio change to be detectable by the human ear. At low levels of sound, a 2 dB change may be detectable, but at loud sounds probably a 4 dB change would be necessary for the change to be noticed. These are not strict laws but guidelines commonly used by audio technicians. For the room in question, the mix of settings may be as shown in Fig. 20.40(c). Once set, the slides are not touched again. A flat response has been established for the room for the full audio range so that every sound or type of music is covered.



(a)



(b)



(c)

FIG. 20.40

(a) Dual-channel 15-band "Constant Q" graphic equalizer (Courtesy of ARX Systems.); *(b) setup; (c) frequency response.*

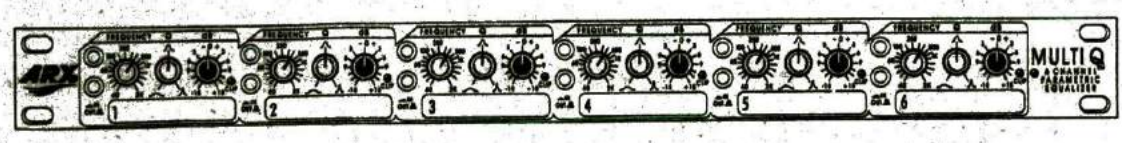


FIG. 20.41
Six-channel parametric equalizer.
(Courtesy of ARX Systems.)

A *parametric equalizer* such as appearing in Fig. 20.41 is similar to a *graphic equalizer,* but instead of separate controls for the individual frequency ranges, it uses three basic controls over three or four broader frequency ranges. The typical controls—the *gain, center frequency,* and *bandwidth*—are typically available for the *low-*, *mid-*, and *high-frequency* ranges. Each is fundamentally an independent control; that is, a change in one can be made without affecting the other two. For the parametric equalizer in Fig. 20.41, each of the six channels has a frequency control switch that, in conjunction with the $f \times 10$ switch, gives a range of center frequencies from 40 Hz through 16 kHz. It has controls for *BW* ("*Q*") from 3 octaves to 1/20 octave, and ±18 dB cut and boost. Some like to refer to the parametric equalizer as a *sophisticated tone control* and actually use it to enrich the sound after the flat response has been established by the graphic equalizer. The effect achieved with a standard tone control knob is sometimes referred to as "boring" compared to the effect established by a good parametric equalizer, primarily because the former can control only the volume and not the bandwidth or center frequency. In general, graphic equalizers establish the important *flat response,* while parametric equalizers are adjusted to provide the *type* and *quality* of sound you like to hear. You can "notch out" the frequencies that bother you and remove tape "hiss" and the "sharpness" often associated with CDs.

One characteristic of concert halls that is more difficult to fake is the fullness of sound that concert halls are able to provide. In the concert hall, you have the direct sound from the instruments and the reflection of sound off the walls and the vaulted ceilings, which were all carefully designed expressly for this purpose. Any reflection results in a delay in the sound waves reaching the ear, creating the fullness effect. Through digital delay, speakers can be placed to the back and side of a listener to establish the surround sound effect. In general, the delay speakers are much lower in wattage, with 20 W speakers typically used with a 100 W system. The echo response is one reason that people often like to play their stereos louder than they should for normal hearing. By playing the stereo louder, they create more echo and reflection off the walls, bringing into play some of the fullness heard at concert halls.

It is probably safe to say that any system composed of quality components, a graphic and parametric equalizer, and surround sound will have all the components necessary to have a quality reproduction of the concert hall effect.

20.14 COMPUTER ANALYSIS

PSpice

Series Resonance This chapter provides an excellent opportunity to demonstrate what computer software programs can do for us. Imagine having to plot a detailed resonance curve with all the calculations required

for each frequency. At every frequency, the reactance of the inductive and capacitive elements changes, and the phasor operations would have to be repeated—a long and arduous task. However, with PSpice, taking a few moments to enter the circuit and establish the desired simulation results in a detailed plot in a few seconds that can have plot points every microsecond!

For the first time, the horizontal axis is in the frequency domain rather than in the time domain as in all the previous plots. For the series resonant circuit in Fig. 20.42, the magnitude of the source was chosen to produce a maximum current of $I = 400\ \text{mV}/40\ \Omega = 10\ \text{mA}$ at resonance, and the reactive elements establish a resonant frequency of

$$f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(30\ \text{mH})(0.1\ \mu\text{F})}} \cong \mathbf{2.91\ kHz}$$

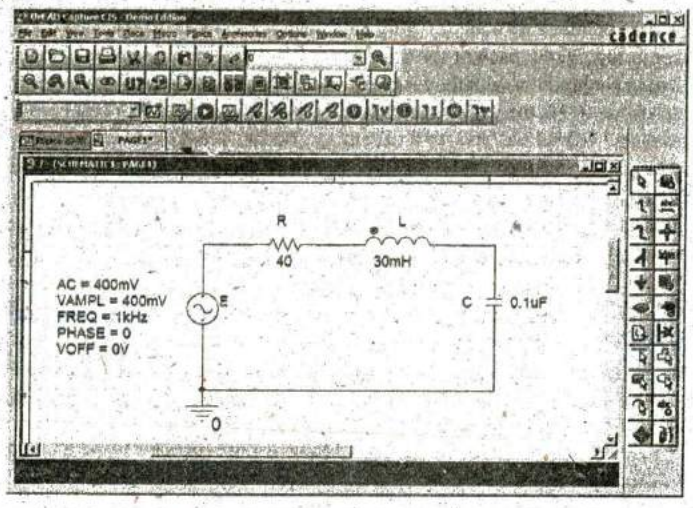


FIG. 20.42
Series resonant circuit to be analyzed using PSpice.

The quality factor is

$$Q_l = \frac{X_L}{R_l} = \frac{546.64\ \Omega}{40\ \Omega} \cong \mathbf{13.7}$$

which is relatively high and should give us a nice sharp response.

The bandwidth is

$$BW = \frac{f_s}{Q_l} = \frac{2.91\ \text{kHz}}{13.7} \cong \mathbf{212\ Hz}$$

which will be verified using our cursor options.

For the ac source, choose **VSIN**. All the parameters are set by double-clicking on the source symbol and entering the values in the **Property Editor** dialog box. For each, select **Name and Value** under **Display** followed by **Apply** before leaving the dialog box.

In the **Simulation Settings** dialog box, select **AC Sweep/Noise** and set the **Start Frequency** at 1 kHz, the **End Frequency** at 10 kHz, and the **Points/Decade** at 10,000. The **Logarithmic scale** and **Decade** settings remain at their default values. Choose 10,000 for **Points/Decade** to ensure a number of data points near the peak value. When the **SCHEMATIC1**

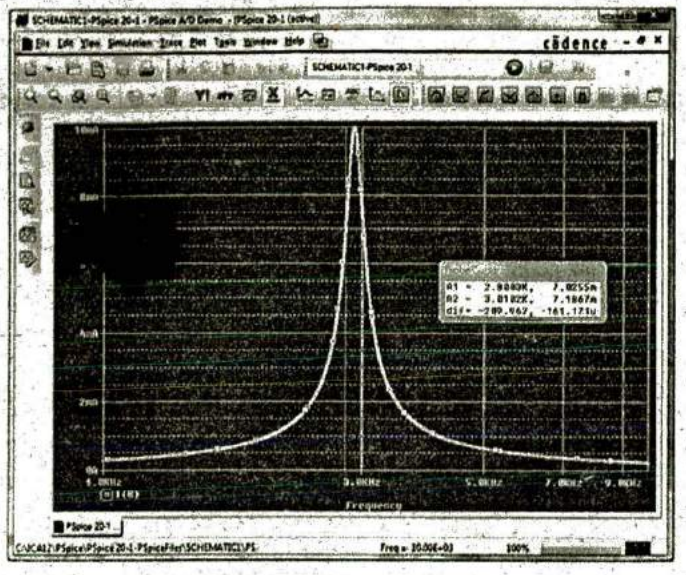


FIG. 20.43

Resonance curve for the current of the circuit in Fig. 20.42.

screen in Fig. 20.43 appears, **Trace-Add Trace-I(R)-OK** results in a logarithmic plot that peaks just to the left of 3 kHz. The spacing between grid lines on the **X-axis** should be increased, so select **Plot-Axis Settings-X Grid-unable Automatic-Spacing-Log-0.1-OK.** Next, select the **Toggle cursor** icon, and right-click to move the right cursor as close to 7.07 mA as possible (0.707 of the peak value to define the bandwidth) to obtain **A1** with a frequency of 2.80 kHz at a level of 7.03 mA—the best we can do with the 10,000 data points per decade. Now left-click, and place the left cursor as close to the same level as possible. The result is 3.01 kHz at a level of 7.19 mA for **A2.** The cursors were set in the order above to obtain a positive answer for the difference of the two as appearing in the third line of the **Probe Cursor** box. The resulting 209.96 Hz is an excellent match with the calculated value of 212 Hz.

Parallel Resonance Let us now investigate the parallel resonant circuit in Fig. 20.33 and compare the results with the longhand solution. The network appears in Fig. 20.44 using **ISRC** as the ac source voltage. Under the **Property Editor** heading, set the following values: **DC** = 0 A, **AC** = 2 mA, and **TRAN** = 0. Under **Display,** select **Do Not Display** for both **DC** and **TRAN** since they do not play a part in our analysis. In the **Simulation Settings** dialog box, select **AC Sweep/Noise,** and select the **Start Frequency** at 10 kHz since we know that it will resonate near 30 kHz. Choose the **End Frequency** as 100 kHz for a first run to see the results. Set the **Points/Decade** at 10,000 to ensure a good number of data points for the peaking region. After simulation, **Trace-Add Trace-V(C:1)-OK** results in the plot in Fig. 20.45 with a resonant frequency near 30 kHz. The selected range appears to be a good one, but the initial plot needed more grid lines on the *x*-axis, so use **Plot-Axis Settings-X-Grid-unenable Automatic-Spacing-Log-0.1-OK** to obtain a grid line at 10 kHz intervals. Next select the **Toggle cursor** pad and a left-click cursor is established on the screen. Choose the **Cursor Peak** pad to find

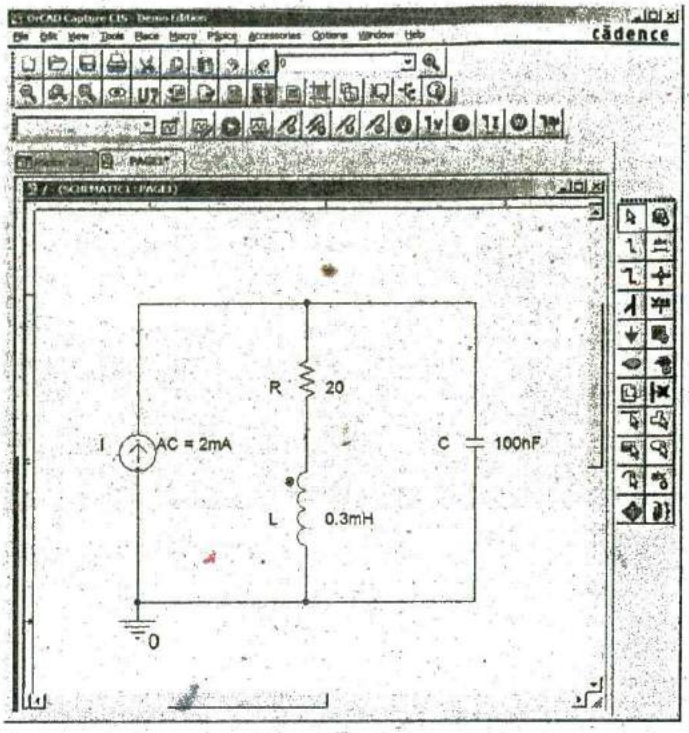


FIG. 20.44
Parallel resonant network to be analyzed using PSpice.

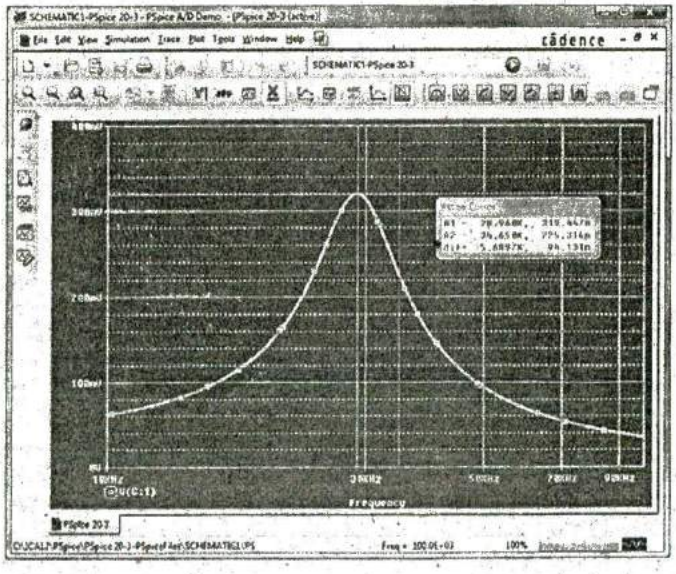


FIG. 20.45
Resonance curve for the voltage across the capacitor in Fig. 20.44.

the peak value of the curve. The result is **A1** = 319.45 mV at 28.96 kHz which is a very close match with the calculated value of 318.68 mV at 28.57 kHz for the maximum value of V_C. The bandwidth is defined at a level of 0.707(319.45 mV) = 225.85 mV. Using the right-click cursor, you find that the closest value is 225.32 mV for the 10,000 points of data per decade. The resulting frequency is 34.65 kHz as shown in the **Probe Cursor** box in Fig. 20.45.

Now use the left-click cursor to find the same level to the left of the peak value so that you can determine the bandwidth. The closest that the left-click cursor can come to 225.85 mV is 224.96 mV at a frequency of 23.97 kHz. The bandwidth then appears as 10.68 kHz in the **Probe Cursor** box, matching the longhand solution of 10.68 kHz in Example 20.7.

You can now look at the phase angle of the voltage across the parallel network to find the frequency when the network appears resistive and the phase angle is 0°. First use **Trace-Delete All Traces,** and call up **P(V(C:1))** followed by **OK.** The result is the plot in Fig. 20.46, revealing that the phase angle is close to −90° at very high frequencies as the capacitive element with its decreasing reactance takes over the characteristics of the parallel network. At 10 kHz, the inductive element has a lower reactance than the capacitive element, and the network has a positive phase angle. Using the cursor option, move the left cursor along the horizontal axis until the phase angle is at its minimum value. As shown in Fig. 20.46, the smallest angle available with the determined data points is 125.15 mdegrees = 0.13°, which is certainly very close to 0°. The corresponding frequency is 27.07 kHz, which is essentially an exact match with the longhand solution of 27.05 kHz. Clearly, therefore, the frequency at which the phase angle is zero and the total impedance appears resistive is less than the frequency at which the output voltage is a maximum.

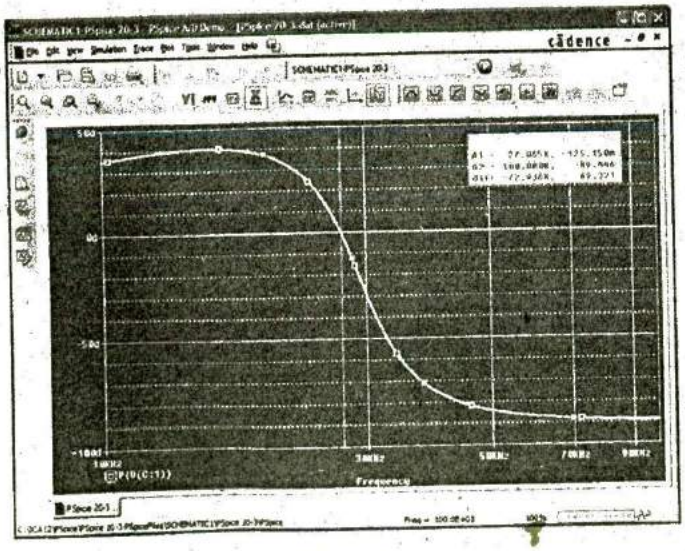


FIG. 20.46

Phase plot for the voltage v_C for the parallel resonant network in Fig. 20.44.

Multisim

The results of Example 20.9 are now confirmed using Multisim. The network in Fig. 20.36 appears as shown in Fig. 20.47 after all the elements have been placed as described in earlier chapters. The current source is selected in the **Select a Component** dialog box with the sequence **Family–SIGNAL_CURRENT_SOURCES,** followed by **Component–AC_CURRENT.** In particular, note that the frequency assigned to the 2 mA ac current source is 100 kHz. Since we have some idea that the resonant frequency is a few hundred kilohertz, it seems appropriate that the starting frequency for the plot begins at 100 kHz and extends to 1 MHz. Also, be sure that the **AC Magnitude** is set to 2 mA in the **Analysis Setup** within the **AC Current** dialog box. Keep in mind that the numbered nodal voltages are displayed on the network using a right-click of the mouse anywhere on the screen followed by **Properties-Sheet Properties-Net Names—Show all—OK.** The numbered nodes are very helpful when it comes to selecting traces to be seen on the screen.

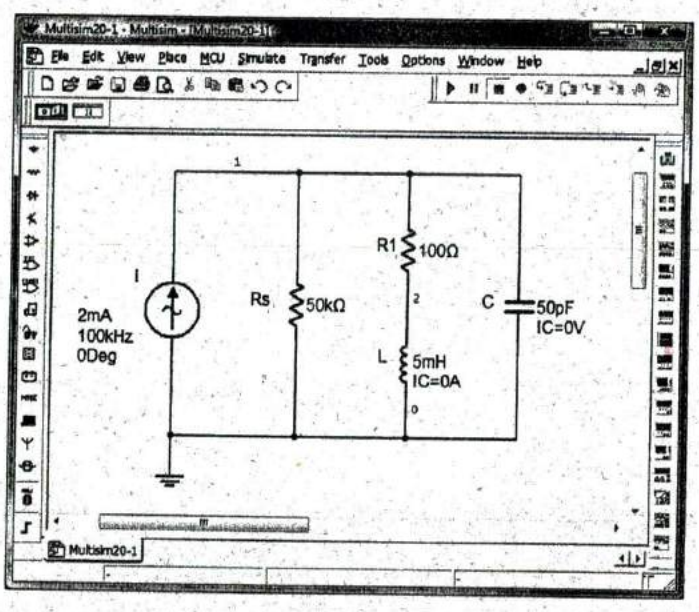


FIG. 20.47
Using Multisim to confirm the results of Example 20.9.

For simulation, first select the sequence **Simulate-Analyses-AC Analysis** to obtain the **AC Analysis** dialog box. Set the **Start frequency** at 100 kHz, the **Stop frequency** at 1 MHz, **Sweep type** at **Decade, Number of points per decade** at 1000, and the **Vertical scale** at **Linear.** Under **Output variables,** select **V(1)** as a **Variable for analysis** followed by **Simulate** to run the program. The results are the magnitude and phase plots in Fig. 20.48. Starting with the **Voltage** plot, select the **Show/Hide Grid** key, **Show/Hide Legend** key, and **Show/Hide Cursors** key. You will immediately note under the **AC Analysis** cursor box that the maximum value is 95.24 V and the minimum value

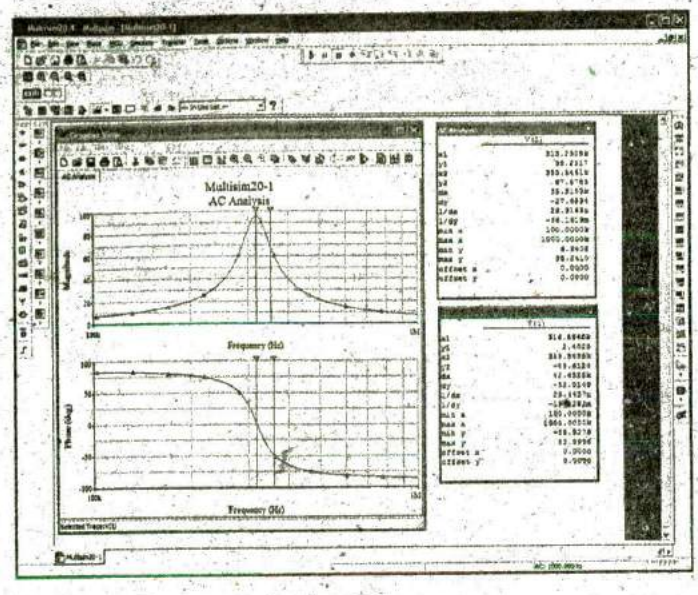


FIG. 20.48

Magnitude and phase plots for the voltage v_C of the network in Fig. 20.47.

is 6.94 V. By moving the cursor close to 95.24 V (**y1**), you can find the resonant frequency. As shown in the top cursor dialog box in Fig. 20.48, this is achieved at 318.23 kHz (**x1**). The other (blue) cursor can be used to define the high cutoff frequency for the bandwidth by first calculating the 0.707 level of the output voltage. The result is 0.707(95.24 V) = 67.33 V. The closest you can come to this level with the cursor is 67.57 V (**y2**) which defines a frequency of 353.55 kHz (**x2**). If you now use the red cursor to find the corresponding level below the resonant frequency, you find a level of 67.49 V (**y1**) at 287.08 kHz (**x1**). The resulting bandwidth is therefore 353.55 kHz − 287.08 kHz = 66.47 kHz.

You can now determine the resonant frequency if you define resonance as that frequency that results in a phase angle of 0° for the output voltage. By repeating the process described above for the phase plot, set the red cursor as close to 0° as possible. The result is 2.4°(**y1**) at 316.89 kHz (**x1**), clearly revealing that the resonant frequency defined by the phase angle is less than that defined by the peak voltage. However, with a Q_l of about 100, the difference of 1.99 kHz is not significant. Also note that when the second cursor was set on 359.55 kHz, the phase angle of −49.61° is close to the 45° expected at the cutoff frequency.

Again, the computer solution is a very close match with the longhand solution in Example 20.9 with a perfect match of 95.24 V for the peak value and only a small difference in bandwidth with 66.87 kHz in Example 20.9 and 66.63 kHz here. For the high cutoff frequency, the computer generated a result of 353.55 kHz, while the theoretical solution was 351.7 kHz. For the low cutoff frequency, the computer responded with 287.08 kHz compared to a theoretical solution of 284 kHz.

PROBLEMS

SECTIONS 20.2 THROUGH 20.7 Series Resonance

1. Find the resonant ω_s and f_s for the series circuit with the following parameters:
   a. $R = 10\ \Omega$, $L = 1$ H, $C = 16\ \mu$F
   b. $R = 300\ \Omega$, $L = 0.51$ H, $C = 0.16\ \mu$F
   c. $R = 20\ \Omega$, $L = 0.27$ mH, $C = 7.5\ \mu$F

2. For the series circuit in Fig. 20.49:
   a. Find the value of X_C for resonance.
   b. Determine the total impedance of the circuit at resonance.
   c. Find the magnitude of the current I.
   d. Calculate the voltages V_R, V_L, and V_C at resonance. How are V_L and V_C related? How does V_R compare to the applied voltage E?
   e. What is the quality factor of the circuit? Is it a high or low Q circuit?
   f. What is the power dissipated by the circuit at resonance?

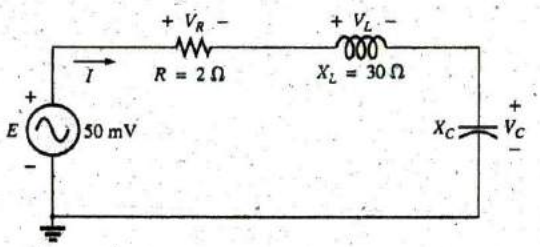


FIG. 20.49
Problem 2.

3. For the series circuit in Fig. 20.50:
   a. Find the value of X_L for resonance.
   b. Determine the magnitude of the current I at resonance.
   c. Find the voltages V_R, V_L, and V_C at resonance, and compare their magnitudes.
   d. Determine the quality factor of the circuit. Is it a high- or low-Q circuit?
   e. If the resonant frequency is 5 kHz, determine the value of L and C.
   f. Find the bandwidth of the response if the resonant frequency is 5 kHz.
   g. What are the low and high cutoff frequencies?

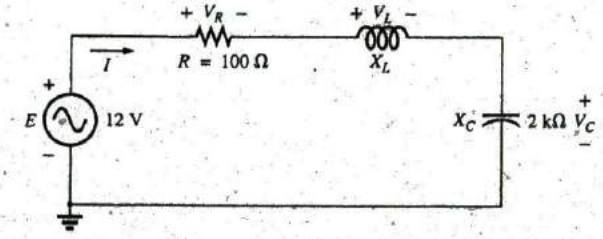


FIG. 20.50
Problem 3.

4. For the circuit in Fig. 20.51:
   a. Find the value of L in millihenries if the resonant frequency is 1800 Hz.
   b. Calculate X_L and X_C. How do they compare?
   c. Find the magnitude of the current I_{rms} at resonance.
   d. Find the power dissipated by the circuit at resonance.
   e. What is the apparent power delivered to the system at resonance?
   f. What is the power factor of the circuit at resonance?
   g. Calculate the Q of the circuit and the resulting bandwidth.
   h. Find the cutoff frequencies, and calculate the power dissipated by the circuit at these frequencies.

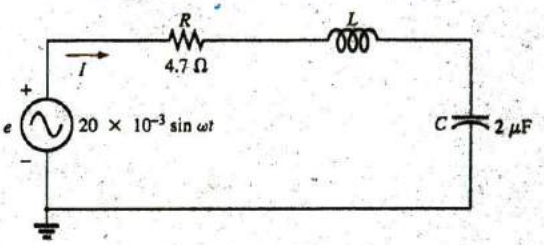


FIG. 20.51
Problem 4.

5. a. Find the bandwidth of a series resonant circuit having a resonant frequency of 6000 Hz and a Q_s of 15.
   b. Find the cutoff frequencies.
   c. If the resistance of the circuit at resonance is 3 Ω, what are the values of X_L and X_C in ohms?
   d. What is the power dissipated at the half-power frequencies if the maximum current flowing through the circuit is 0.5 A?

6. A series circuit has a resonant frequency of 10 kHz. The resistance of the circuit is 5 Ω, and X_C at resonance is 200 Ω.
   a. Find the bandwidth.
   b. Find the cutoff frequencies.
   c. Find Q_s.
   d. If the input voltage is 30 V ∠0°, find the voltage across the coil and capacitor in phasor form.
   e. Find the power dissipated at resonance.

7. a. The bandwidth of a series resonant circuit is 200 Hz. If the resonant frequency is 2000 Hz, what is the value of Q_s for the circuit?
   b. If $R = 2\ \Omega$, what is the value of X_L at resonance?
   c. Find the value of L and C at resonance.
   d. Find the cutoff frequencies.

8. The cutoff frequencies of a series resonant circuit are 5400 Hz and 6000 Hz.
   a. Find the bandwidth of the circuit.
   b. If Q_s is 9.5, find the resonant frequency of the circuit.
   c. If the resistance of the circuit is 2 Ω, find the value of X_L and X_C at resonance.
   d. Find the value of L and C at resonance.

*9. a. Design a series resonant circuit with an input voltage of 5 V ∠0° to have the following specifications:
   —Peak current of 500 mA at resonance
   —Bandwidth of 120 Hz
   —Resonant frequency of 8400 Hz
   b. Find the value of L and C and the cutoff frequencies.

*10. Design a series resonant circuit to have a bandwidth of 400 Hz using a coil with a Q_l of 20 and a resistance of 2 Ω. Find the values of L and C and the cutoff frequencies.

*11. A series resonant circuit is to resonate at $\omega_s = 2\pi \times 10^6$ rad/s and draw 20 W from a 120 V source at resonance. If the fractional bandwidth is 0.16:
   a. Determine the resonant frequency in hertz.
   b. Calculate the bandwidth in hertz.
   c. Determine the values of R, L, and C.
   d. Find the resistance of the coil if $Q_l = 80$.

*12. A series resonant circuit will resonate at a frequency of 1 MHz with a fractional bandwidth of 0.2. If the quality factor of the coil at resonance is 12.5 and its inductance is 100 μH, determine the following:
   a. The resistance of the coil.
   b. The additional resistance required to establish the indicated fractional bandwidth.
   c. The required value of capacitance.

SECTIONS 20.8 THROUGH 20.12 Parallel Resonance

13. For the "ideal" parallel resonant circuit in Fig. 20.52:
   a. Determine the resonant frequency (f_p).
   b. Find the voltage V_C at resonance.
   c. Determine the currents I_L and I_C at resonance.
   d. Find Q_p.

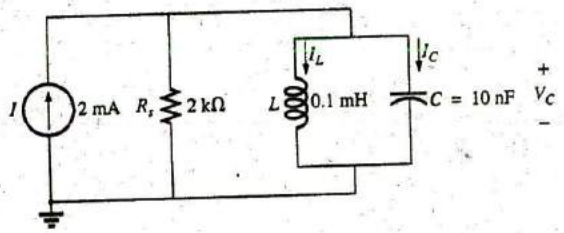


FIG. 20.52
Problem 13.

14. For the parallel resonant network in Fig. 20.53:
   a. Calculate f_s.
   b. Determine Q_l using $f = f_s$. Can the approximate approach be applied?
   c. Determine f_p and f_m.
   d. Calculate X_L and X_C using f_p. How do they compare?
   e. Find the total impedance at resonance (f_p).
   f. Calculate V_C at resonance (f_p).
   g. Determine Q_p and the BW using f_p.
   h. Calculate I_L and I_C at f_p.

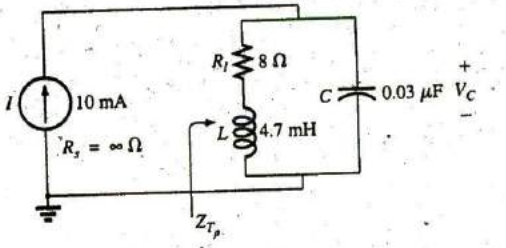


FIG. 20.53
Problem 14.

*15. The network of Fig. 20.54 has a supply with an internal resistance of 1 kΩ, an emf of 120 V, and a wide frequency range. The inductance of 200 μH has a Q_l of 15, which may be considered constant for the frequency range of interest. The tuning capacitor is fixed at 120 pF.
   a. Find the resonant frequency.
   b. Find the voltage across the tank circuit at resonance.
   c. Find the power delivered by the source at resonance.
   d. Calculate the power lost in the tank coil due to the internal resistance, and compare that to the power delivered in part (c).

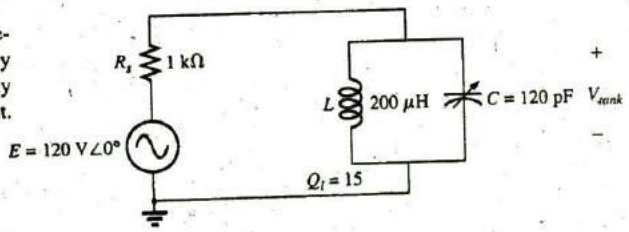


FIG. 20.54
Problem 15.

16. For the network in Fig. 20.55:
    a. Find the value of X_C at resonance (f_p).
    b. Find the total impedance Z_{T_p} at resonance (f_p).
    c. Find the currents I_L and I_C at resonance (f_p).
    d. If the resonant frequency is 20,000 Hz, find the value of L and C at resonance.
    e. Find Q_p and the BW.

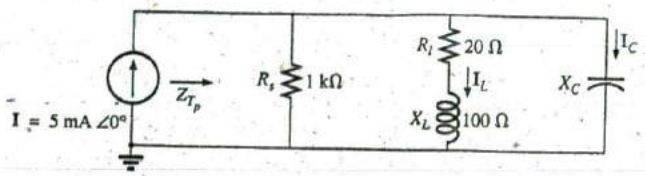


FIG. 20.55
Problem 16.

*17. The network shown in Fig. 20.56 is to resonate at $2 \times 10^6/2\pi$ Hz and have a bandwidth of $100{,}000/2\pi$ Hz. If the coil has a Q_l of 35 (constant for the frequency range of interest), calculate the values of R and C. Find the nearest commercial values available for R and C.

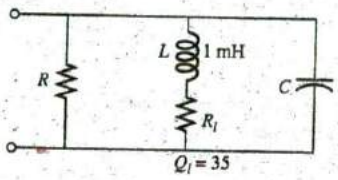


FIG. 20.56
Problem 17.

18. For the network in Fig. 20.57:
    a. Find the resonant frequencies f_s, f_p, and f_m. What do the results suggest about the Q_p of the network?
    b. Find the values of X_L and X_C at resonance (f_p). How do they compare?
    c. Find the impedance Z_{T_p} at resonance (fp).
    d. Calculate Q_p and the BW.
    e. Find the magnitude of currents I_L and I_C at resonance (f_p).
    f. Calculate the voltage V_C at resonance (f_p).

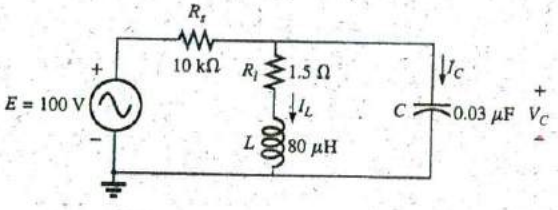


FIG. 20.57
Problem 18.

*19. Repeat Problem 18 for the network in Fig. 20.58.

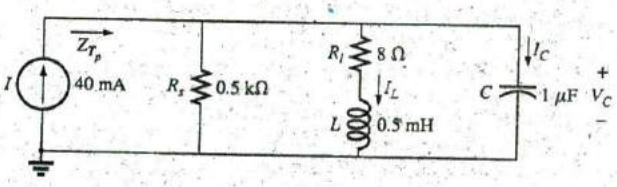


FIG. 20.58
Problems 19 and 29.

20. It is desired that the impedance $\mathbf{Z}_T$ of the high Q circuit in Fig. 20.59 be 50 kΩ ∠0° at resonance (f_p).
    a. Find the value of X_L.
    b. Compute X_C.
    c. Find the resonant frequency (f_p) if $L = 16$ mH.
    d. Find the value of C.

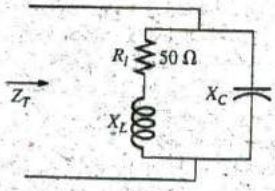


FIG. 20.59
Problem 20.

21. For the network in Fig. 20.60:
    a. Find f_p.
    b. Calculate the magnitude of V_C at resonance (f_p).
    c. Determine the power absorbed at resonance.
    d. Find the *BW*.

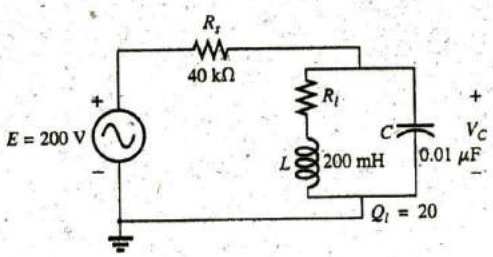


FIG. 20.60
Problem 21.

*22. For the network in Fig. 20.61:
    a. Find the value of X_L for resonance.
    b. Find Q_l.
    c. Find the resonant frequency (f_p) if the bandwidth is 1 kHz.
    d. Find the maximum value of the voltage V_C.
    e. Sketch the curve of V_C versus frequency. Indicate its peak value, resonant frequency, and band frequencies.

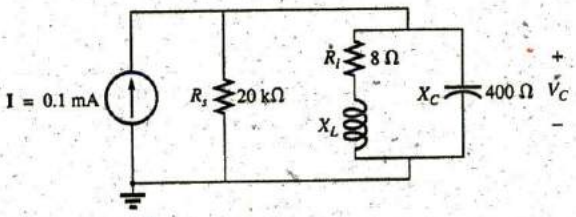


FIG. 20.61
Problem 22.

*23. Repeat Problem 22 for the network in Fig. 20.62.

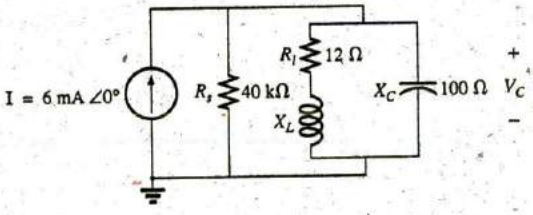


FIG. 20.62
Problem 23.

*24. For the network in Fig. 20.63:
    a. Find f_s, f_p, and f_m.
    b. Determine Q_l and Q_p at f_p after a source conversion is performed.
    c. Find the input impedance Z_{T_p}.
    d. Find the magnitude of the voltage V_C.
    e. Calculate the bandwidth using f_p.
    f. Determine the magnitude of the currents I_C and I_L.

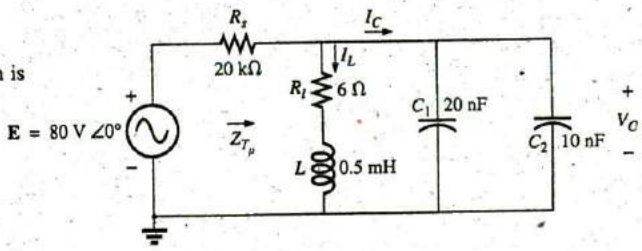


FIG. 20.63
Problem 24.

*25. For the network in Fig. 20.64, the following are specified:

$$f_p = 20 \text{ kHz}$$
$$BW = 1.8 \text{ kHz}$$
$$L = 2 \text{ mH}$$
$$Q_l = 80$$

Find R_s and C.

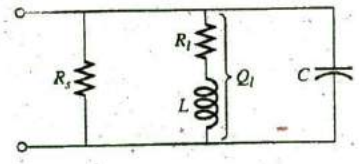


FIG. 20.64
Problem 25.

*26. Design the network in Fig. 20.65 to have the following characteristics:
   a. $BW = 500$ Hz
   b. $Q_p = 30$
   c. $V_{C_{max}} = 1.8$ V

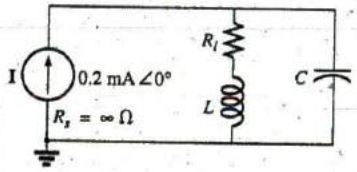


FIG. 20.65
Problem 26.

*27. For the parallel resonant circuit in Fig. 20.66:
   a. Determine the resonant frequency.
   b. Find the total impedance at resonance.
   c. Find Q_p.
   d. Calculate the BW.
   e. Repeat parts (a) through (d) for $L = 20\ \mu$H and $C = 20$ nF.
   f. Repeat parts (a) through (d) for $L = 0.4$ mH and $C = 1$ nF.
   g. For the network in Fig. 20.66 and the parameters of parts (e) and (f), determine the ratio L/C.
   h. Do your results confirm the conclusions in Fig. 20.28 for changes in the L/C ratio?

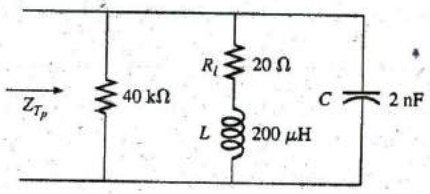


FIG. 20.66
Problem 27.

SECTION 20.14 Computer Analysis

PSpice or Multisim

28. Verify the results in Example 20.8. That is, show that the resonant frequency is 40 kHz, the cutoff frequencies are as calculated, and the bandwidth is 1.85 kHz.

29. Find f_p and f_m for the parallel resonant network in Fig. 20.58, and comment on the resulting bandwidth as it relates to the quality factor of the network.

GLOSSARY

Band (cutoff, half-power, corner) frequencies Frequencies that define the points on the resonance curve that are 0.707 of the peak current or voltage value. In addition, they define the frequencies at which the power transfer to the resonant circuit will be half the maximum power level.

Bandwidth (BW) The range of frequencies between the band, cutoff, or half-power frequencies.

Quality factor (Q) A ratio that provides an immediate indication of the sharpness of the peak of a resonance curve. The higher the Q, the sharper is the peak and the more quickly it drops off to the right and left of the resonant frequency.

Resonance A condition established by the application of a particular frequency (the resonant frequency) to a series or parallel R-L-C network. The transfer of power to the system is a maximum, and, for frequencies above and below, the power transfer drops off to significantly lower levels.

Selectivity A characteristic of resonant networks directly related to the bandwidth of the resonant system. High selectivity is associated with small bandwidth (high Q's), and low selectivity with larger bandwidths (low Q's).

Decibels, Filters, and Bode Plots



Objectives

- *Develop confidence in the use of logarithms and decibels in the description of power and voltage levels.*
- *Become familiar with the frequency response of high- and low-pass filters. Learn to calculate the cutoff frequency and describe the phase response.*
- *Be able to calculate the cutoff frequencies and sketch the frequency response of a pass-band, stop-band, or double-tuned filter.*
- *Develop skills in interpreting and establishing the Bode response of any filter.*
- *Become aware of the characteristics and operation of a crossover network.*

21.1 INTRODUCTION

The unit decibel (dB), defined by a logarithmic expression, is used throughout the industry to define levels of audio, voltage gain, energy, field strength, and so on. It will take some exposure to become totally comfortable with this unit of measure, but in time its usefulness will become quite apparent. For one thing, it removes the need to work with extremely large numbers that often lose their meaning when certain levels are reached. A good example of its use is in audio systems, where a 3 dB gain is one that can be detected by the human ear and normal listening levels. At low volumes, a 2 dB gain is often detectable, but at high levels it may take a 6 dB gain to detect the difference. Normal conversation occurs at about 60 dB, while damage to the ear drum can begin at 100 dB and higher. Additional examples of the use of decibels will be provided throughout the chapter to ensure some understanding of the impact of this unit of measurement. As noted above, it is used so extensively that it is of paramount importance that its use be completely and correctly understood.

The use of logarithms in industry is so extensive that a clear understanding of their purpose and use is an absolute necessity. At first exposure, logarithms often appear vague and mysterious due to the mathematical operations required to find the logarithm and antilogarithm using the longhand table approach that is typically taught in mathematics courses. However, almost all of today's scientific calculators have the common and natural log functions, eliminating the complexity of applying logarithms and allowing us to concentrate on the positive characteristics of using the function.

Logarithms

Basic Relationships Let us first examine the relationship between the variables of the logarithmic function. The mathematical expression

$$N = (b)^x$$

states that the number N is equal to the base b taken to the power x. A few examples:

$$100 = (10)^2$$
$$27 = (3)^3$$
$$54.6 = (e)^4 \qquad \text{where } e = 2.7183$$

To find the power x to satisfy the equation

$$1200 = (10)^x$$

you can determine the value of x using logarithms in the following manner:

$$x = \log_{10} 1200 = \mathbf{3.079}$$

revealing that

$$10^{3.079} = 1200$$

Note that the logarithm was taken to the base 10—the number to be taken to the power of x. There is no limitation to the numerical value of the base, except that tables and calculators are designed to handle either a base of 10 (common logarithm, LOG) or base $e = 2.7183$ (natural logarithm, LN). In review, therefore,

$$\boxed{\text{If } N = (b)^x, \text{ then } x = \log_b N.} \tag{21.1}$$

The base to be used is a function of the area of application. If a conversion from one base to the other is required, the following equation can be applied:

$$\boxed{\log_e x = 2.3 \log_{10} x} \tag{21.2}$$

In this chapter, we concentrate solely on the common logarithm. However, a number of the conclusions are also applicable to natural logarithms.

Some Areas of Application The following are some of the most common applications of the logarithmic function:

1. The response of a system can be plotted for a range of values that may otherwise be impossible or unwieldy with a linear scale.
2. Levels of power, voltage, and the like can be compared without dealing with very large or very small numbers that often cloud the true impact of the difference in magnitudes.
3. A number of systems respond to outside stimuli in a nonlinear logarithmic manner. The result is a mathematical model that permits a direct calculation of the response of the system to a particular input signal.
4. The response of a cascaded or compound system can be rapidly determined using logarithms if the gain of each stage is known on a logarithmic basis. This characteristic is demonstrated in an example to follow.

Graphs Graph paper is available in **semilog** and **log-log** varieties. Semilog paper has only one log scale, with the other a linear scale. Both scales of log-log paper are log scales. A section of semilog paper appears in Fig. 21.1. Note the linear (even-spaced-interval) vertical scaling and the repeating intervals of the log scale at multiples of 10.

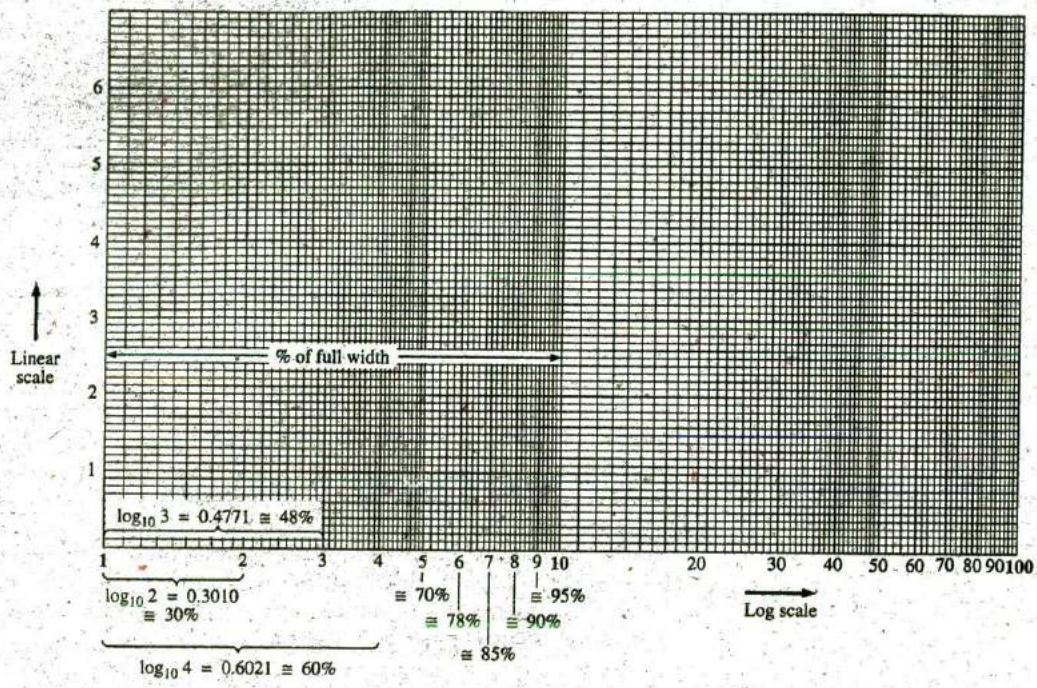


FIG. 21.1
Semilog graph paper.

The spacing of the log scale is determined by taking the common log (base 10) of the number. The scaling starts with 1, since $\log_{10} 1 = 0$. The distance between 1 and 2 is determined by $\log_{10} 2 = 0.3010$, or approximately 30% of the full distance of a log interval, as shown on the graph. The distance between 1 and 3 is determined by $\log_{10} 3 = 0.4771$, or about 48% of the full width. For future reference, keep in mind that almost 50% of the width of one log interval is represented by a 3 rather than by the 5 of a linear scale. In addition, note that the number 5 is about 70% of the full width, and 8 is about 90%. Remembering the percentage of full width of the lines 2, 3, 5, and 8 is particularly useful when the various lines of a log plot are left unnumbered.

Since

$$\log_{10} 1 = 0$$
$$\log_{10} 10 = 1$$
$$\log_{10} 100 = 2$$
$$\log_{10} 1000 = 3$$

the spacing between 1 and 10, 10 and 100, 100 and 1000, and so on, is the same as shown in Figs. 21.1 and 21.2.

It is important to realize that

a logarithmic scale cannot start at zero on semilog paper.

In Fig. 21.1, it starts at 1, and each interval is marked by a power of 10. Any number less than 1 is negative; for example,

$$\log_{10}(0.999) = -4.35 \times 10^{-4}$$

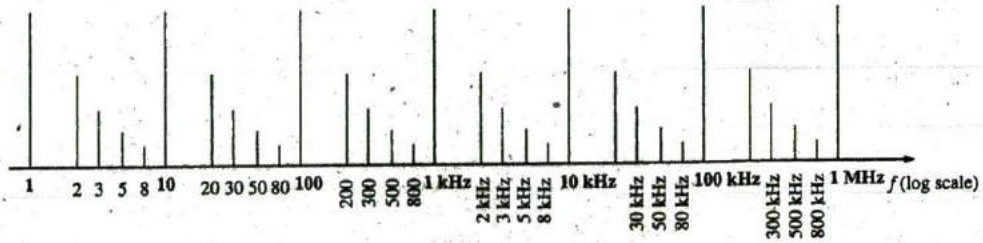


FIG. 21.2
Frequency log scale.

Note in Figs. 21.1 and 21.2 that the log scale becomes compressed at the high end of each interval. With increasing frequency levels assigned to each interval, a single graph can provide a frequency plot extending from 1 Hz to 1 MHz, as shown in Fig. 21.2, with particular reference to the 30%, 50%, 70%, and 90% levels of each interval.

On many log plots, the tick marks for most of the intermediate levels are left off because of space constraints. The following equation can be used to determine the logarithmic level at a particular point between known levels using a ruler or simply estimating the distances. The parameters are defined by Fig. 21.3. We have

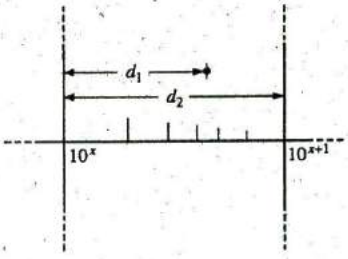


FIG. 21.3
Finding a value on a log plot.

$$\boxed{\text{Value} = 10^x \times 10^{d_1/d_2}} \tag{21.3}$$

The derivation of Eq. (21.3) is simply an extension of the details regarding distance appearing in Fig. 21.1.

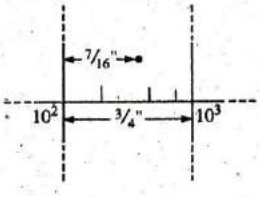


FIG. 21.4
Example 21.1.

EXAMPLE 21.1 Determine the value of the point appearing on the logarithmic plot in Fig. 21.4 using the measurements made by a ruler (linear).

Solution:

$$\frac{d_1}{d_2} = \frac{7/16''}{3/4''} = \frac{0.438''}{0.750''} = 0.584$$

Using a calculator gives

$$10^{d_1/d_2} = 10^{0.584} = 3.837$$

Applying Eq. (21.3) gives

$$\text{Value} = 10^x \times 10^{d_1/d_2} = 10^2 \times 3.837$$
$$= \mathbf{383.7}$$

21.2 PROPERTIES OF LOGARITHMS

There are a few characteristics of logarithms that should be emphasized:

1. ***The common or natural logarithm of the number 1 is 0:***

$$\boxed{\log_{10} 1 = 0} \tag{21.4}$$

just as $10^x = 1$ requires that $x = 0$.

2. ***The log of any number less than 1 is a negative number:***

$$\log_{10} \tfrac{1}{2} = \log_{10} 0.5 = -0.3$$
$$\log_{10} \tfrac{1}{10} = \log_{10} 0.1 = -1$$

3. ***The log of the product of two numbers is the sum of the logs of the numbers:***

$$\boxed{\log_{10} ab = \log_{10} a + \log_{10} b} \qquad (21.5)$$

4. ***The log of the quotient of two numbers is the log of the numerator minus the log of the denominator:***

$$\boxed{\log_{10} \frac{a}{b} = \log_{10} a - \log_{10} b} \qquad (21.6)$$

5. ***The log of a number taken to a power is equal to the product of the power and the log of the number:***

$$\boxed{\log_{10} a^n = n \log_{10} a} \qquad (21.7)$$

Calculator Functions

Using the TI-89 calculator, the common logarithm of a number is determined by first selecting the CATALOG key and then scrolling to find the common logarithm function. The time involved in scrolling through the options can be reduced by first selecting the key with the first letter of the desired function—in this case, *L*, as shown below, to find the common logarithm of the number 80.



1.90

For the reverse process, to find N, or the antilogarithm, use the function 10. In this case, the function 10 is found after the letter *Z* in the catalog. It is easier if you first select *A* to start at the top of the listing and then go backwards to the power of ten function by using the upward moving scroll option. The antilogarithm of the number 0.6 is found as follows:



3.98

EXAMPLE 21.2 Evaluate each of the following logarithmic expressions:

a. $\log_{10} 0.004$
b. $\log_{10} 250{,}000$
c. $\log_{10}(0.08)(240)$
d. $\log_{10} \dfrac{1 \times 10^4}{1 \times 10^{-4}}$
e. $\log_{10}(10)^4$

Solutions:

a. **−2.40**
b. **+5.40**
c. $\log_{10}(0.08)(240) = \log_{10} 0.08 + \log_{10} 240 = -1.097 + 2.380$
$= 1.28$

d. $\log_{10}\frac{1\times10^{4}}{1\times10^{-4}} = \log_{10}1\times10^{4} - \log_{10}1\times10^{-4} = 4-(-4)$
$= \mathbf{8}$

e. $\log_{10}10^{4} = 4\log_{10}10 = 4(1) = \mathbf{4}$

21.3 DECIBELS

Power Gain

Two levels of power can be compared using a unit of measure called the *bel*, which is defined by the following equation:

$$\boxed{B = \log_{10}\frac{P_2}{P_1}} \quad \text{(bels)} \tag{21.8}$$

However, to provide a unit of measure of *less* magnitude, a **decibel** is defined, where

$$\boxed{1 \text{ bel} = 10 \text{ decibels (dB)}} \tag{21.9}$$

The result is the following important equation, which compares power levels P_2 and P_1 in decibels:

$$\boxed{\text{dB} = 10\log_{10}\frac{P_2}{P_1}} \quad \text{(decibels, dB)} \tag{21.10}$$

If the power levels are equal ($P_2 = P_1$), there is no change in power level, and dB = 0. If there is an increase in power level ($P_2 > P_1$), the resulting decibel level is positive. If there is a decrease in power level ($P_2 < P_1$), the resulting decibel level will be negative.

For the special case of $P_2 = 2P_1$, the gain in decibels is

$$\text{dB} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}2 = \mathbf{3\ dB}$$

Therefore, for a speaker system, a 3 dB increase in output requires that the power level be doubled. In the audio industry, it is a generally accepted rule that an increase in sound level is accomplished with 3 dB increments in the output level. In other words, a 1 dB increase is barely detectable, and a 2 dB increase just discernible. A 3 dB increase normally results in a readily detectable increase in sound level. An additional increase in the sound level is normally accomplished by simply increasing the output level another 3 dB. If an 8 W system were in use, a 3 dB increase would require a 16 W output, whereas an additional increase of 3 dB (a total of 6 dB) would require a 32 W system, as demonstrated by the calculations

$$\text{dB} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{16}{8} = 10\log_{10}2 = \mathbf{3\ dB}$$

$$\text{dB} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{32}{8} = 10\log_{10}4 = \mathbf{6\ dB}$$

For $P_2 = 10P_1$,

$$\text{dB} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}10 = 10(1) = \mathbf{10\ dB}$$

resulting in the unique situation where the power gain has the same magnitude as the decibel level.

For some applications, a reference level is established to permit a comparison of decibel levels from one situation to another. For communication systems, a commonly applied reference level is

$$P_{ref} = 1\text{ mW} \qquad \text{(across a 600 } \Omega \text{ load)}$$

Eq. (21.10) is then typically written as

$$\boxed{dB_m = 10\log_{10}\frac{P}{1\text{ mW}}\bigg|_{600\,\Omega}} \tag{21.11}$$

Note the subscript m to denote that the decibel level is determined with a reference level of 1 mW.

In particular, for $P = 40$ mW,

$$\text{dB}_m = 10\log_{10}\frac{40\text{ mW}}{1\text{ mW}} = 10\log_{10}40 = 10(1.6) = \mathbf{16\ dB}_m$$

whereas for $P = 4$ W,

$$\text{dB}_m = 10\log_{10}\frac{4000\text{ mW}}{1\text{ mW}} = 10\log_{10}4000 = 10(3.6) = \mathbf{36\ dB}_m$$

Even though the power level has increased by a factor of 4000 mW/40 mW = 100, the dB_m increase is limited to 20 dB_m. In time, the significance of dB_m levels of 16 dB_m and 36 dB_m will generate an immediate appreciation regarding the power levels involved. An increase of 20 dB_m is also associated with a significant gain in power levels.

Voltage Gain

Decibels are also used to provide a comparison between voltage levels. Substituting the basic power equations $P_2 = V_2^2/R_2$ and $P_1 = V_1^2/R_1$ into Eq. (21.10) results in

$$\begin{aligned}\text{dB} &= 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{V_2^2/R_2}{V_1^2/R_1}\\ &= 10\log_{10}\frac{V_2^2/V_1^2}{R_2/R_1} = 10\log_{10}\left(\frac{V_2}{V_1}\right)^2 - 10\log_{10}\left(\frac{R_2}{R_1}\right)\end{aligned}$$

and

$$\text{dB} = 20\log_{10}\frac{V_2}{V_1} - 10\log_{10}\frac{R_2}{R_1}$$

For the situation where $R_2 = R_1$, a condition normally assumed when comparing voltage levels on a decibel basis, the second term of the preceding equation drops out ($\log_{10} 1 = 0$), and

$$\boxed{\text{dB}_v = 20\log_{10}\frac{V_2}{V_1}} \quad \text{(dB)} \tag{21.12}$$

Note the subscript v to define the decibel level obtained.

EXAMPLE 21.3 Find the voltage gain in dB of a system where the applied signal is 2 mV and the output voltage is 1.2 V.

Solution:

$$dB_v = 20 \log_{10} \frac{V_o}{V_i} = 20 \log_{10} \frac{1.2 \text{ V}}{2 \text{ mV}} = 20 \log_{10} 600 = \mathbf{55.56\ dB}$$

for a voltage gain $A_v = V_o/V_i$ of 600.

EXAMPLE 21.4 If a system has a voltage gain of 36 dB, find the applied voltage if the output voltage is 6.8 V.

Solution:

$$dB_v = 20 \log_{10} \frac{V_o}{V_i}$$

$$36 = 20 \log_{10} \frac{V_o}{V_i}$$

$$1.8 = \log_{10} \frac{V_o}{V_i}$$

From the antilogarithm,

$$\frac{V_o}{V_i} = 63.096$$

and

$$V_i = \frac{V_o}{63.096} = \frac{6.8 \text{ V}}{63.096} = \mathbf{107.77\ mV}$$

TABLE 21.1

V_o/V_i	$dB = 20 \log_{10} (V_o/V_i)$
1	0 dB
2	6 dB
10	20 dB
20	26 dB
100	40 dB
1,000	60 dB
100,000	100 dB

Table 21.1 compares the magnitude of specific gains to the resulting decibel level. In particular, note that when voltage levels are compared, a doubling of the level results in a change of 6 dB rather than 3 dB as obtained for power levels.

In addition, note that an increase in gain from 1 to 100,000 results in a change in decibels that can easily be plotted on a single graph. Also note that doubling the gain (from 1 to 2 and 10 to 20) results in a 6 dB increase in the decibel level, while a change of 10 to 1 (from 1 to 10, 10 to 100, and so on) always results in a 20 dB decrease in the decibel level.

Human Auditory Response

One of the most frequent applications of the decibel scale is in the communication and entertainment industries. The human ear does not respond in a linear fashion to changes in source power level; that is, a doubling of the audio power level from 1/2 W to 1 W does not result in a doubling of the loudness level for the human ear. In addition, a change from 5 W to 10 W is received by the ear as the same change in sound intensity as experienced from 1/2 W to 1 W. In other words, the ratio between levels is the same in each case (1 W/0.5 W = 10 W/5 W = 2), resulting in the same decibel or logarithmic change defined by Eq. (21.7). The ear, therefore, responds in a logarithmic fashion to changes in audio power levels.

To establish a basis for comparison between audio levels, a reference level of 0.0002 **microbar** (μbar) was chosen, where 1 μbar is equal to the sound pressure of 1 dyne per square centimeter, or about 1 millionth of the normal atmospheric pressure at sea level. The 0.0002 μbar level is

the threshold level of hearing. Using this reference level, the sound pressure level in decibels is defined by the following equation:

$$\boxed{\mathrm{dB}_s = 20 \log_{10} \frac{P}{0.0002\ \mu\mathrm{bar}}} \qquad (21.13)$$

where P is the sound pressure in microbars.

The decibel levels in Fig. 21.5 are defined by Eq. (21.13). Meters designed to measure audio levels are calibrated to the levels defined by Eq. (21.13) and shown in Table 21.2.

TABLE 21.2

Typical sound levels and their decibel levels.

Output Power, Average value in watts.	dB_s	
	160	Jet engine
	150	
	140	Community siren
	130	Jackhammer
Threshold of pain	120	Live music concert, iPods and MP3 players at full volume
300	110	Health club, movie theater
100		Chain saw
30	100	Very loud music, motorcycle
10		
3	90	Loud music, heavy truck, subway train
1		
0.3	80	Orchestra, highway traffic, alarm clock
0.1		
0.03	70	Average conversation
0.01		Quiet music
0.003	60	
0.001		
0.0003	50	Average residence, computer system
	40	Background music
	30	Quiet office, computer hard drive
	20	Whispering
	10	Faint sounds, paper rustling
0.0002 μbar of pressure	0	Threshold of hearing

Dynamic range ≅ 120 dB_s

In particular take note of the sound level for iPods and MP3 players, for which it is suggested, based on research, that they should not be used for more than 1 hour a day at 60% volume to prevent permanent hearing damage. Always remember that hearing damage is usually not reversible, so that any loss is for the long term.

A common question regarding audio levels is how much the power level of an acoustical source must be increased to double the sound level received by the human ear. The question is not as simple as it first seems due to considerations such as the frequency content of the sound, the acoustical conditions of the surrounding area, the physical characteristics of the surrounding medium, and—of course—the unique characteristics of the human ear. However, a general conclusion can be formulated that has practical value if we note the power levels of an acoustical source

appearing to the left in Table 21.2. Each power level is associated with a particular decibel level, and a change of 10 dB in the scale corresponds to an increase or a decrease in power by a factor of 10. For instance, a change from 90 dB to 100 dB is associated with a change in wattage from 3 W to 30 W. Through experimentation, it has been found that on an average basis the loudness level doubles for every 10 dB change in audio level—a conclusion somewhat verified by the examples to the right in Table 21.2.

To double the sound level received by the human ear, the power rating of the acoustical source (in watts) must be increased by a factor of 10.

In other words, doubling the sound level available from a 1 W acoustical source requires moving up to a 10 W source.

Further:

At normal hearing levels, it would take a change of about 3 dB (twice the power level) for the change to be noticeable to the human ear.

At low levels of sound, a change of 2 dB may be noticeable, but it may take a 6 dB (four times the power level) change for much higher levels of sound.

One final example of the use of dB as a unit of measurement is the LRAD (Long Range Acoustic Device) appearing in Fig. 21.5. It emits a tone between 2100 Hz and 3100 Hz at 145 dB that is effective at up to 500 m, or almost two football fields. The sound at its peak is thousands of times louder than a smoke alarm. It can be used to transmit critical information and instructions and is capable of strong deterrent tones against intruders.



FIG. 21.5
LRAD (Long Range Acoustic Device) 1000X. (Courtesy of the American Technology Corporation.)

Instrumentation

A number of modern VOMs and DMMs have a dB scale designed to provide an indication of power ratios referenced to a standard level of 1 mW at 600 Ω. Since the reading is accurate only if the load has a characteristic impedance of 600 Ω, the 1 mW, 600 Ω reference level is normally printed somewhere on the face of the meter, as shown in Fig. 21.6. The dB scale is usually calibrated to the lowest ac scale of the meter. In other words, when making the dB measurement, choose the lowest ac voltage scale, but read the dB scale. If a higher voltage scale is chosen, a

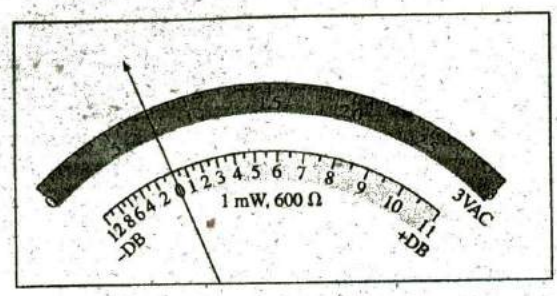


FIG. 21.6
Defining the relationship between a dB scale referenced to 1 mW, 600 Ω and a 3 V rms voltage scale.

correction factor must be used, which is sometimes printed on the face of the meter but is always available in the meter manual. If the impedance is other than 600 Ω or not purely resistive, other correction factors must be used that are normally included in the meter manual. Using the basic power equation $P = V^2/R$ reveals that 1 mW across a 600 Ω load is the same as applying 0.775 V rms across a 600 Ω load; that is, $V = \sqrt{PR} = \sqrt{(1\text{ mW})(600\Omega)} = 0.775\text{V}$. The result is that an analog display will have 0 dB [defining the reference point of 1 mW, dB = $10 \log_{10} P_2/P_1 = 10 \log_{10} (1\text{ mW}/1\text{ mW(ref)} = 0\text{ dB}$] and 0.775 V rms on the same pointer projection, as shown in Fig. 21.6. A voltage of 2.5 V across a 600 Ω load results in a dB level of dB = $20 \log_{10} V_2/V_1 = 20 \log_{10}$ 2.5 V/0.775 = 10.17 dB, resulting in 2.5 V and 10.17 dB appearing along the same pointer projection. A voltage of less than 0.775 V, such as 0.5 V, results in a dB level of dB = $20 \log_{10} V_2/V_1 = 20 \log_{10}$ 0.5 V/0.775 V = −3.8 dB, also shown on the scale in Fig. 21.6. Although a reading of 10 dB reveals that the power level is 10 times the reference, don't assume that a reading of 5 dB means that the output level is 5 mW. The 10 : 1 ratio is a special one in logarithmic use. For the 5 dB level, the power level must be found using the antilogarithm (3.126), which reveals that the power level associated with 5 dB is about 3.1 times the reference or 3.1 mW. A conversion table is usually provided in the manual for such conversions.

21.4 FILTERS

Any combination of passive (R, L, and C) and/or active (transistors or operational amplifiers) elements designed to select or reject a band of frequencies is called a **filter.** In communication systems, filters are used to pass those frequencies containing the desired information and to reject the remaining frequencies. In stereo systems, filters can isolate particular bands of frequencies for increased or decreased emphasis by the output acoustical system (amplifier, speaker, and so on). Filters are used to filter out any unwanted frequencies, commonly called *noise,* due to the nonlinear characteristics of some electronic devices or signals picked up from the surrounding medium. In general, there are two classifications of filters:

1. **Passive filters** are those filters composed of series or parallel combinations of R, L, and C elements.
2. **Active filters** are filters that employ active devices such as transistors and operational amplifiers in combination with R, L, and C elements.

Since this text is limited to passive devices, the analysis of this chapter is limited to passive filters. In addition, only the most fundamental forms are examined in the next few sections. The subject of filters is a very broad one that continues to receive extensive research support from industry and the government as new communication systems are developed to meet the demands of increased volume and speed. There are courses and texts devoted solely to the analysis and design of filter systems, which can become quite complex and sophisticated. In general, however, all filters belong to the four broad categories of **low-pass, high-pass, pass-band,** and **stop-band,** as depicted in Fig. 21.7. For each form, there are critical frequencies that define the regions of pass-bands and stop-bands (often called *reject* bands). Any frequency in the pass-band will pass through to the next stage with at least 70.7% of the maximum output voltage. Recall the use of the 0.707 level to define the

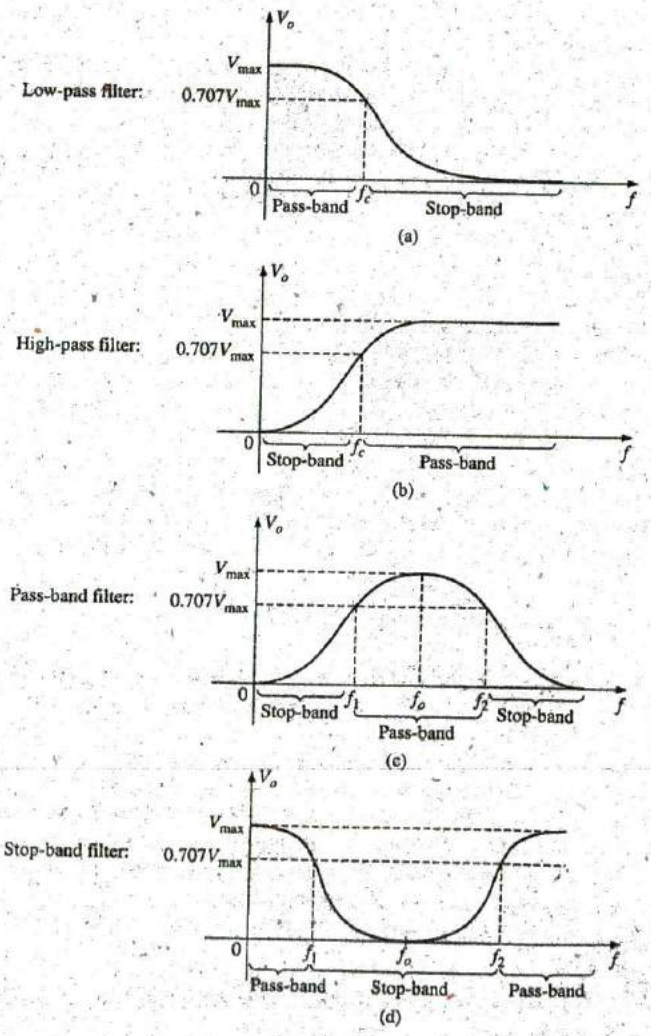


FIG. 21.7
Defining the four broad categories of filters.

bandwidth of a series or parallel resonant circuit (both with the general shape of the pass-band filter).

For some stop-band filters, the stop-band is defined by conditions other than the 0.707 level. In fact, for many stop-band filters, the condition that $V_o = 1/1000V_{max}$ (corresponding to -60 dB in the discussion to follow) is used to define the stop-band region, with the pass-band continuing to be defined by the 0.707 V level. The resulting frequencies between the two regions are then called the *transition frequencies* and establish the *transition region.*

At least one example of each filter in Fig. 21.7 is discussed in some detail in the sections to follow. Take particular note of the relative simplicity of some of the designs.

21.5 *R-C* LOW-PASS FILTER

The *R-C* filter, incredibly simple in design, can be used as a low-pass or a high-pass filter. If the output is taken off the capacitor, as shown in Fig. 21.8, it responds as a low-pass filter. If the positions of the resistor and capacitor are interchanged and the output is taken off the resistor, the response is that of a high-pass filter.

A glance at Fig. 21.7(a) reveals that the circuit should behave in a manner that results in a high-level output for low frequencies and a declining level for frequencies above the critical value. Let us first examine the network at the frequency extremes of $f = 0$ Hz and very high frequencies to test the response of the circuit.

At $f = 0$ Hz,

$$X_C = \frac{1}{2\pi fC} = \infty\,\Omega$$

and the open-circuit equivalent can be substituted for the capacitor, as shown in Fig. 21.9, resulting in $\mathbf{V}_o = \mathbf{V}_i$.

At very high frequencies, the reactance is

$$X_C = \frac{1}{2\pi fC} \cong 0\,\Omega$$

and the short-circuit equivalent can be substituted for the capacitor, as shown in Fig. 21.10, resulting in $\mathbf{V}_o = 0$ V.

A plot of the magnitude of V_o versus frequency results in the curve in Fig. 21.11. Our next goal is now clearly defined: Find the frequency at which the transition takes place from a pass-band to a stop-band.

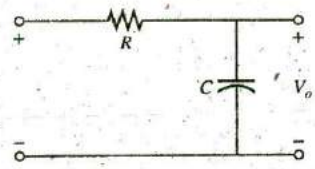


FIG. 21.8
Low-pass filter.

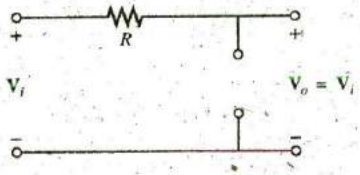


FIG. 21.9
R-C low-pass filter at low frequencies.

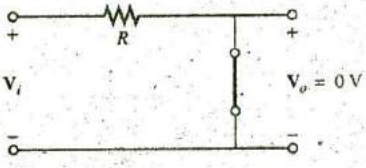


FIG. 21.10
R-C low-pass filter at high frequencies.

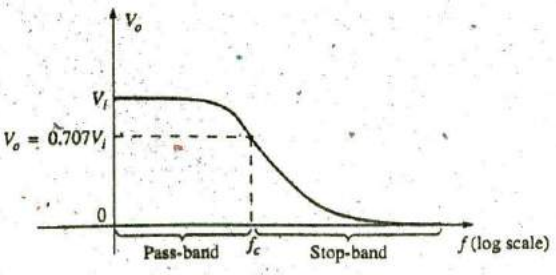


FIG. 21.11
V_o versus frequency for a low-pass R-C filter.

For filters, a normalized plot is used more often than the plot of V_o versus frequency in Fig. 21.11.

Normalization is a process whereby a quantity such as voltage, current, or impedance is divided by a quantity of the same unit of measure to establish a dimensionless level of a specific value or range.

A normalized plot in the filter domain can be obtained by dividing the plotted quantity such as V_o in Fig. 21.11 with the applied voltage V_i for the frequency range of interest. Since the maximum value of V_o for the low-pass filter in Fig. 21.8 is V_i, each level of V_o in Fig. 21.11 is divided by the level of V_i. The result is the plot of $A_v = V_o/V_i$ in Fig. 21.12. Note that the maximum value is 1 and the cutoff frequency is defined at the 0.707 level.

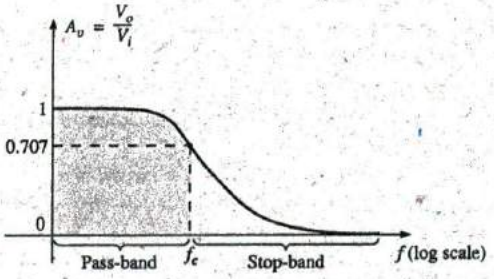


FIG. 21.12
Normalized plot of Fig. 21.11.

At any intermediate frequency, the output voltage $\mathbf{V}_o$ in Fig. 21.8 can be determined using the voltage divider rule:

$$\mathbf{V}_o = \frac{X_C \angle -90° \mathbf{V}_i}{R - jX_C}$$

or

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{X_C \angle -90°}{R - jX_C} = \frac{X_C \angle -90°}{\sqrt{R^2 + X_C^2} \angle -\tan^{-1}(X_C/R)}$$

and

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{X_C}{\sqrt{R^2 + X_C^2}} \angle -90° + \tan^{-1}\left(\frac{X_C}{R}\right)$$

The magnitude of the ratio V_o/V_i is therefore determined by

$$A_v = \frac{V_o}{V_i} = \frac{X_C}{\sqrt{R^2 + X_C^2}} = \frac{1}{\sqrt{\left(\frac{R}{X_C}\right)^2 + 1}} \tag{21.14}$$

and the phase angle is determined by

$$\theta = -90° + \tan^{-1}\frac{X_C}{R} = -\tan^{-1}\frac{R}{X_C} \tag{21.15}$$

For the special frequency at which $X_C = R$, the magnitude becomes

$$A_v = \frac{V_o}{V_i} = \frac{1}{\sqrt{\left(\frac{R}{X_C}\right)^2 + 1}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

which defines the critical or cutoff frequency in Fig. 21.12.

The frequency at which $X_C = R$ is determined by

$$\frac{1}{2\pi f_c C} = R$$

and

$$\boxed{f_c = \frac{1}{2\pi RC}} \qquad (21.16)$$

The impact of Eq. (21.16) extends beyond its relative simplicity. For any low-pass filter, the application of any frequency less than f_c results in an output voltage V_o that is at least 70.7% of the maximum. For any frequency above f_c, the output is less than 70.7% of the applied signal.

Solving for $\mathbf{V}_o$ and substituting $\mathbf{V}_i = V_i \angle 0°$ gives

$$\mathbf{V}_o = \left[\frac{X_C}{\sqrt{R^2 + X_C^2}} \angle\theta\right] \mathbf{V}_i = \left[\frac{X_C}{\sqrt{R^2 + X_C^2}} \angle\theta\right] V_i \angle 0°$$

and

$$\mathbf{V}_o = \frac{X_C V_i}{\sqrt{R^2 + X_C^2}} \angle\theta$$

The angle θ is, therefore, the angle by which $\mathbf{V}_o$ leads $\mathbf{V}_i$. Since $\theta = -\tan^{-1} R/X_C$ is always negative (except at $f = 0$ Hz), it is clear that $\mathbf{V}_o$ will always lag $\mathbf{V}_i$, leading to the label *lagging network* for the network in Fig. 21.8.

At high frequencies, X_C is very small and R/X_C is quite large, resulting in $\theta = -\tan^{-1} R/X_C$ approaching $-90°$.

At low frequencies, X_C is quite large and R/X_C is very small, resulting in $\theta = -\tan^{-1} R/X_C$ approaching $90°$.

At $X_C = R$, or $f = f_c$, $-\tan^{-1} R/X_C = -\tan^{-1} 1 = -45°$.

A plot of θ versus frequency results in the phase plot in Fig. 21.13.

The plot is of $\mathbf{V}_o$ leading $\mathbf{V}_i$, but since the phase angle is always negative, the phase plot in Fig. 21.14 ($\mathbf{V}_o$ lagging $\mathbf{V}_i$) is more appropriate. Note that a change in sign requires that the vertical axis be changed to the angle by which $\mathbf{V}_o$ lags $\mathbf{V}_i$. In particular, note that the phase angle between $\mathbf{V}_o$ and $\mathbf{V}_i$ is less than 45° in the pass-band and approaches 0° at lower frequencies.

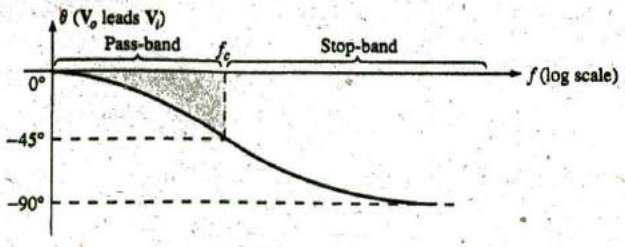


FIG. 21.13
Angle by which V_o leads V_i.

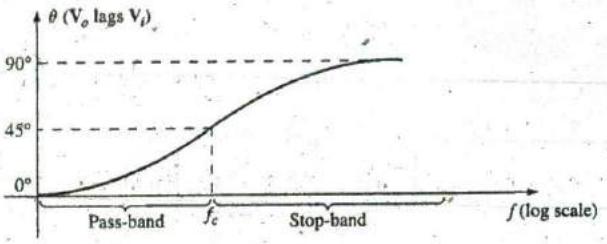


FIG. 21.14
Angle by which V_o lags V_i.

In summary, for the low-pass *R-C* filter in Fig. 21.8:

$$f_c = \frac{1}{2\pi RC}$$

For $f < f_c$, $V_o > 0.707V_i$

whereas for $f > f_c$, $V_o < 0.707V_i$

At f_c, V_o lags V_i by 45°

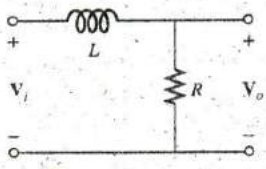


FIG. 21.15
Low-pass R-L filter.

The low-pass filter response in Fig. 21.7(a) can also be obtained using the *R-L* combination in Fig. 21.15 with

$$f_c = \frac{R}{2\pi L} \tag{21.17}$$

In general, however, the *R-C* combination is more popular due to the smaller size of capacitive elements and the nonlinearities associated with inductive elements. The details of the analysis of the low-pass *R-L* can be an exercise for independent study.

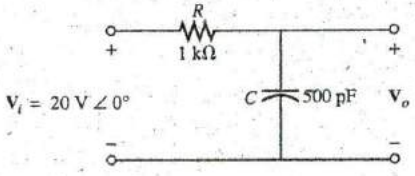


FIG. 21.16
Example 21.5.

EXAMPLE 21.5

a. Sketch the output voltage V_o versus frequency for the low-pass *R-C* filter in Fig. 21.16.
b. Determine the voltage V_o at $f = 100$ kHz and 1 MHz, and compare the results to the results obtained from the curve in part (a).
c. Sketch the normalized gain $A_v = V_o/V_i$.

Solutions:

a. Eq. (21.16):

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(1\text{ k}\Omega)(500\text{ pF})} = \mathbf{318.31\text{ kHz}}$$

At f_c, $V_o = 0.707(20\text{ V}) = 14.14\text{ V}$. See Fig. 21.17.

b. Eq. (21.14):

$$V_o = \frac{V_i}{\sqrt{\left(\frac{R}{X_C}\right)^2 + 1}}$$

At $f = 100$ kHz,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100\text{ kHz})(500\text{ pF})} = 3.18\text{ k}\Omega$$

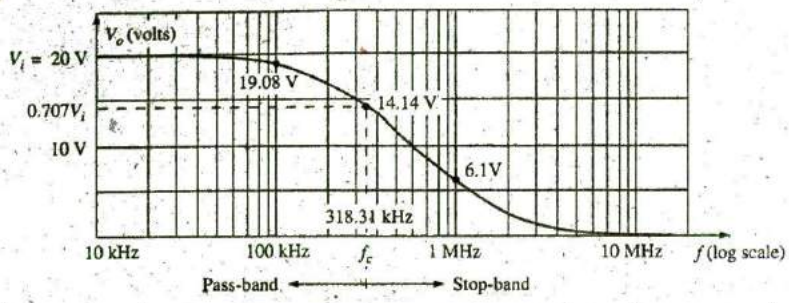


FIG. 21.17
Frequency response for the low-pass R-C network in Fig. 21.16.

$$\text{and } V_o = \frac{20\text{ V}}{\sqrt{\left(\frac{1\text{ k}\Omega}{3.18\text{ k}\Omega}\right)^2 + 1}} = \mathbf{19.08\text{ V}}$$

At $f = 1$ MHz,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1\text{ MHz})(500\text{ pF})} = 0.32\text{ k}\Omega$$

$$\text{and } V_o = \frac{20\text{ V}}{\sqrt{\left(\frac{1\text{ k}\Omega}{0.32\text{ k}\Omega}\right)^2 + 1}} = \mathbf{6.1\text{ V}}$$

Both levels are verified by Fig. 21.17.

c. Dividing every level in Fig. 21.17 by $V_i = 20$ V results in the normalized plot in Fig. 21.18.

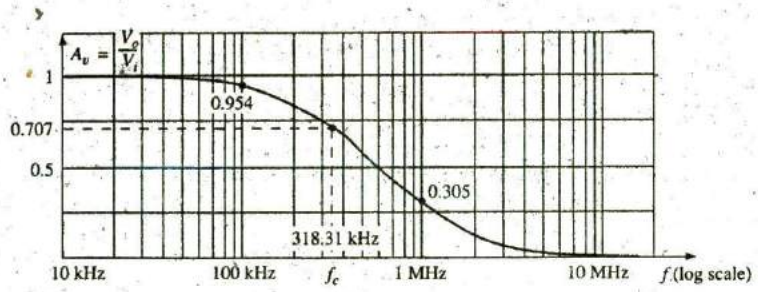


FIG. 21.18
Normalized plot of Fig. 21.17.

21.6 *R-C* HIGH-PASS FILTER

As noted in Section 21.5, a high-pass *R-C* filter can be constructed by simply reversing the positions of the capacitor and resistor, as shown in Fig. 21.19.

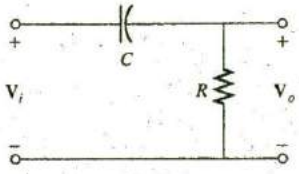


FIG. 21.19
High-pass filter.

At very high frequencies, the reactance of the capacitor is very small, and the short-circuit equivalent can be substituted, as shown in Fig. 21.20. The result is that $\mathbf{V_o = V_i}$.

At $f = 0$ Hz, the reactance of the capacitor is quite high, and the open-circuit equivalent can be substituted, as shown in Fig. 21.21. In this case, $\mathbf{V_o = 0\text{ V}}$.

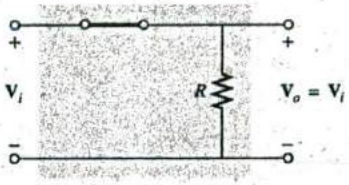


FIG. 21.20
R-C high-pass filter at very high frequencies.

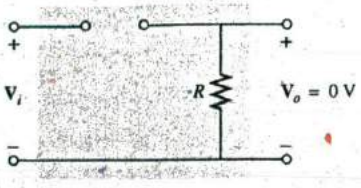


FIG. 21.21
R-C high-pass filter at $f = 0$ Hz.

A plot of the magnitude versus frequency is provided in Fig. 21.22, with the normalized plot in Fig. 21.23.

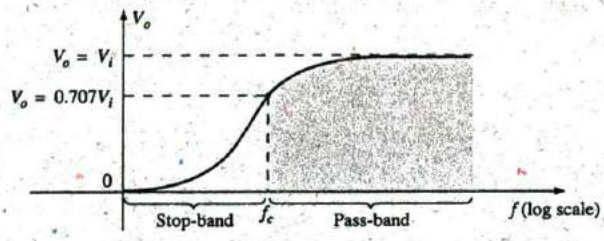


FIG. 21.22
V_o versus frequency for a high-pass R-C filter.

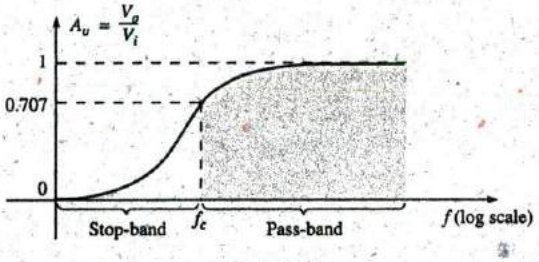


FIG. 21.23
Normalized plot of Fig. 21.22.

At any intermediate frequency, the output voltage can be determined using the voltage divider rule:

$$\mathbf{V}_o = \frac{R\angle 0^\circ\,\mathbf{V}_i}{R - jX_C}$$

or

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R\angle 0^\circ}{R - jX_C} = \frac{R\angle 0^\circ}{\sqrt{R^2 + X_C^2}\angle -\tan^{-1}(X_C/R)}$$

and

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{\sqrt{R^2 + X_C^2}}\angle \tan^{-1}(X_C/R)$$

The magnitude of the ratio $\mathbf{V}_o/\mathbf{V}_i$ is therefore determined by

$$A_v = \frac{V_o}{V_i} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{1}{\sqrt{1 + \left(\frac{X_C}{R}\right)^2}} \qquad (21.18)$$

and the phase angle θ by

$$\boxed{\theta = \tan^{-1}\frac{X_C}{R}} \tag{21.19}$$

For the frequency at which $X_C = R$, the magnitude becomes

$$\frac{V_o}{V_i} = \frac{1}{\sqrt{1 + \left(\frac{X_C}{R}\right)^2}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

as shown in Fig. 21.23.

The frequency at which $X_C = R$ is determined by

$$X_C = \frac{1}{2\pi f_c C} = R$$

and

$$\boxed{f_c = \frac{1}{2\pi RC}} \tag{21.20}$$

For the high-pass *R-C* filter, the application of any frequency greater than f_c results in an output voltage V_o that is at least 70.7% of the magnitude of the input signal. For any frequency below f_c, the output is less than 70.7% of the applied signal.

For the phase angle, high frequencies result in small values of X_C, and the ratio X_C/R approaches zero with $\tan^{-1}(X_C/R)$ approaching 0°, as shown in Fig. 21.24. At low frequencies, the ratio X_C/R becomes quite large, and $\tan^{-1}(X_C/R)$ approaches 90°. For the case $X_C = R$, $\tan^{-1}(X_C/R) = \tan^{-1} 1 = 45°$. Assigning a phase angle of 0° to $\mathbf{V}_i$ such that $\mathbf{V}_i = V_i \angle 0°$, we obtain the phase angle associated with $\mathbf{V}_o$ as θ, resulting in $\mathbf{V}_o = V_o \angle \theta$ and revealing that θ is the angle by which $\mathbf{V}_o$ leads $\mathbf{V}_i$. Since the angle θ is the angle by which $\mathbf{V}_o$ leads $\mathbf{V}_i$ throughout the frequency range in Fig. 21.24, the high-pass *R-C* filter is referred to as a *leading network*.

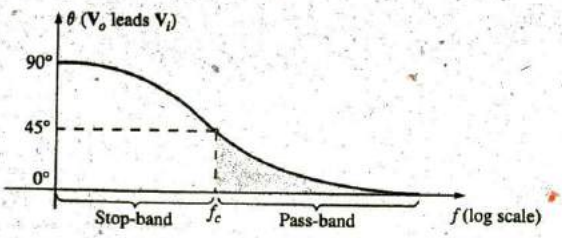


FIG. 21.24

Phase-angle response for the high-pass R-C filter.

In summary, for the high-pass *R-C* filter:

$$f_c = \frac{1}{2\pi RC}$$

For $f < f_c$, $V_o < 0.707V_i$

whereas for $f > f_c$, $V_o > 0.707V_i$

At f_c, $\mathbf{V}_o$ leads $\mathbf{V}_i$ by 45°

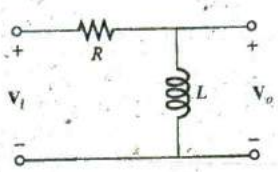


FIG. 21.25
High-pass R-L filter.

The high-pass filter response in Fig. 21.23 can also be obtained using the same elements in Fig. 21.15 but interchanging their positions, as shown in Fig. 21.25.

EXAMPLE 21.6 Given $R = 20\ \text{k}\Omega$ and $C = 1200\ \text{pF}$:

a. Sketch the normalized plot if the filter is used as both a high-pass and a low-pass filter.
b. Sketch the phase plot for both filters in part (a).
c. Determine the magnitude and phase of $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_i$ at $f = \frac{1}{2}f_c$ for the high-pass filter.

Solutions:

a. $$f_c = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(20\ \text{k}\Omega)(1200\ \text{pF})}$$
$$= \mathbf{6631.46\ Hz}$$

The normalized plots appear in Fig. 21.26.

b. The phase plots appear in Fig. 21.27.

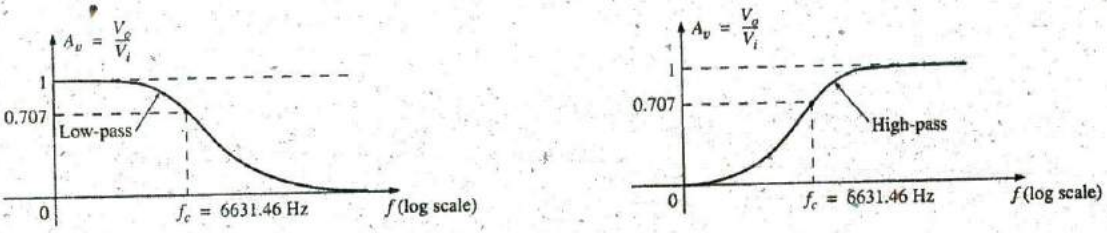


FIG. 21.26
Normalized plots for a low-pass and a high-pass filter using the same elements.

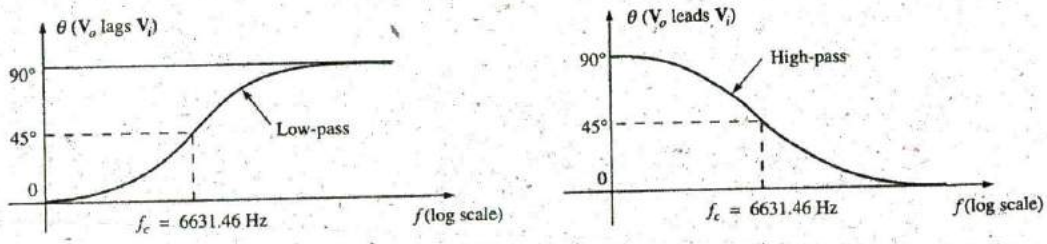


FIG. 21.27
Phase plots for a low-pass and a high-pass filter using the same elements.

c. $$f = \frac{1}{2}f_c = \frac{1}{2}(6631.46\ \text{Hz}) = 3315.73\ \text{Hz}$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{(2\pi)(3315.73\ \text{Hz})(1200\ \text{pF})}$$
$$\cong 40\ \text{k}\Omega$$

$$A_v = \frac{V_o}{V_i} = \frac{1}{\sqrt{1 + \left(\frac{X_C}{R}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{40\,\text{k}\Omega}{20\,\text{k}\Omega}\right)^2}} = \frac{1}{\sqrt{1 + (2)^2}}$$

$$= \frac{1}{\sqrt{5}} = 0.4472$$

$$\theta = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\frac{40\,\text{k}\Omega}{20\,\text{k}\Omega} = \tan^{-1}2 = 63.43°$$

and $$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = 0.447\angle 63.43°$$

21.7 PASS-BAND FILTERS

The most direct way to obtain the pass-band characteristics of Fig. 21.7(c) is to use a series or parallel resonant circuit as described in Chapter 20. In each case, however, V_o will not be equal to V_i in the pass-band, but a frequency range in which V_o will be equal to or greater than $0.707V_{max}$ can be defined.

For the series resonant circuit in Fig. 21.28, $X_L = X_C$ at resonance, and

$$\boxed{V_{o_{max}} = \frac{R}{R + R_l}V_i}_{f = f_s} \qquad (21.21)$$

and

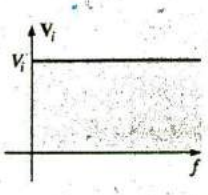
$$\boxed{f_s = \frac{1}{2\pi\sqrt{LC}}} \qquad (21.22)$$

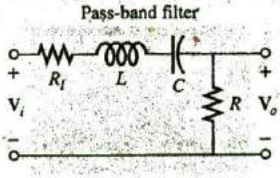
with

$$\boxed{Q_s = \frac{X_L}{R + R_l}} \qquad (21.23)$$

and

$$\boxed{BW = \frac{f_s}{Q_s}} \qquad (21.24)$$





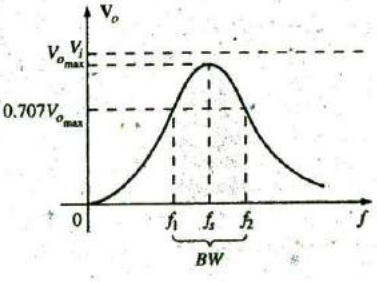


FIG. 21.28
Series resonant pass-band filter.

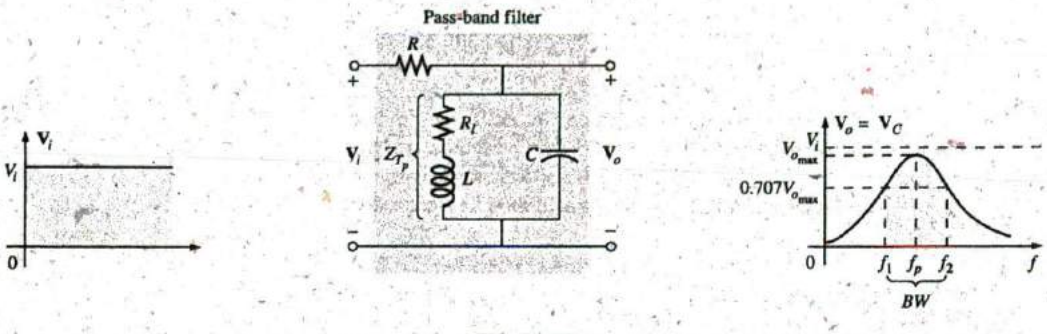


FIG. 21.29
Parallel resonant pass-band filter.

For the parallel resonant circuit in Fig. 21.29, Z_{T_p} is a maximum value at resonance, and

$$V_{o_{max}} = \frac{Z_{T_p} V_i}{Z_{T_p} + R} \quad f = f_p \tag{21.25}$$

with

$$Z_{T_p} = Q_l^2 R_l \quad Q_l \geq 10 \tag{21.26}$$

and

$$f_p = \frac{1}{2\pi\sqrt{LC}} \quad Q_l \geq 10 \tag{21.27}$$

For the parallel resonant circuit

$$Q_p = \frac{X_L}{R_l} \tag{21.28}$$

and

$$BW = \frac{f_p}{Q_p} \tag{21.29}$$

As a first approximation that is acceptable for most practical applications, it can be assumed that the resonant frequency bisects the bandwidth.

EXAMPLE 21.7

a. Determine the frequency response for the voltage V_o for the series circuit in Fig. 21.30.
b. Plot the normalized response $A_v = V_o/V_i$.
c. Plot a normalized response defined by $A'_v = A_v/A_{v_{max}}$.

Solutions:

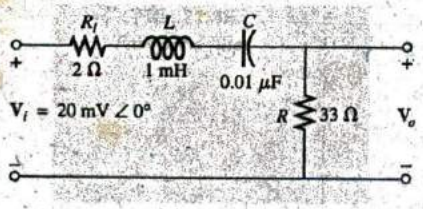


FIG. 21.30
Series resonant pass-band filter for Example 21.7.

a. $$f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1\text{ mH})(0.01\ \mu\text{F})}} = \mathbf{50{,}329.21\ Hz}$$

$$Q_s = \frac{X_L}{R + R_l} = \frac{2\pi(50{,}329.21\text{ Hz})(1\text{ mH})}{33\ \Omega + 2\ \Omega} = \mathbf{9.04}$$

$$BW = \frac{f_s}{Q_s} = \frac{50{,}329.21\text{ Hz}}{9.04} = \mathbf{5.57\ kHz}$$

At resonance:

$$V_{o_{max}} = \frac{RV_i}{R + R_l} = \frac{33\ \Omega(V_i)}{33\ \Omega + 2\ \Omega} = 0.943V_i = 0.943(20\ \text{mV})$$
$$= \mathbf{18.86\ mV}$$

At the cutoff frequencies:

$$V_o = (0.707)(0.943V_i) = 0.667V_i = 0.667(20\ \text{mV})$$
$$= \mathbf{13.34\ mV}$$

Note Fig. 21.31.

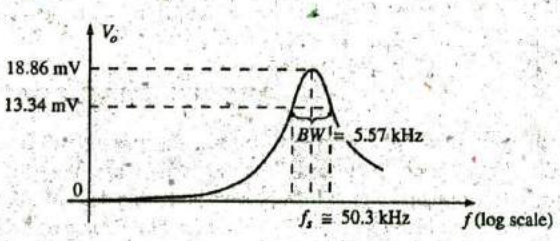


FIG. 21.31
Pass-band response for the network.

b. Dividing all levels in Fig. 21.31 by $V_i = 20$ mV results in the normalized plot in Fig. 21.32(a).
c. Dividing all levels in Fig. 21.32(a) by $A_{v_{max}} = 0.943$ results in the normalized plot in Fig. 21.32(b).

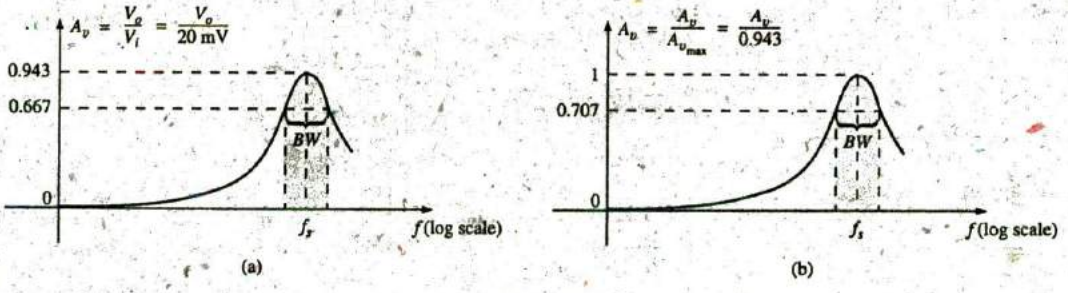


FIG. 21.32
Normalized plots for the pass-band filter in Fig. 21.30.

Another way to produce a band-pass filter is to use a low-pass and a high-pass filter in cascade, as shown in Fig. 21.33.

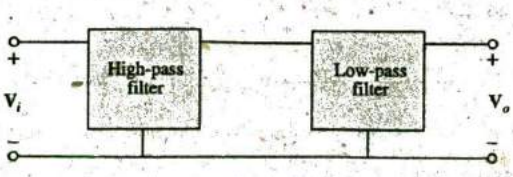


FIG. 21.33
Pass-band filter.

The components are chosen to establish a cutoff frequency for the high-pass filter that is lower than the critical frequency of the low-pass filter, as shown in Fig. 21.34. A frequency f_1 may pass through the low-pass filter but have little effect on V_o due to the reject characteristics of the high-pass filter. A frequency f_2 may pass through the high-pass filter unmolested but be prohibited from reaching the high-pass filter by the low-pass characteristics. A frequency f_o near the center of the pass-band passes through both filters with very little degeneration.

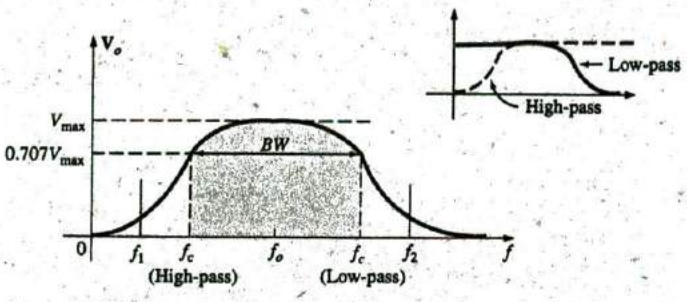


FIG. 21.34
Pass-band characteristics.

The network in Example 21.8 generates the characteristics of Fig. 21.34. However, for a circuit such as the one shown in Fig. 21.35, there is a loading between stages at each frequency that affects the level of $\mathbf{V}_o$. Through proper design, the level of $\mathbf{V}_o$ may be very near the level of $\mathbf{V}_i$ in the pass-band, but it will never equal it exactly. In addition, as the critical frequencies of each filter get closer and closer together to increase the quality factor of the response curve, the peak values within the pass-band continue to drop. For cases where $V_{o_{max}} \neq V_{i_{max}}$, the bandwidth is defined at 0.707 of the resulting $V_{o_{max}}$.

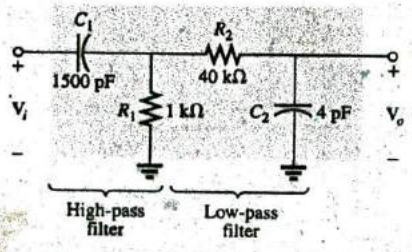


FIG. 21.35
Pass-band filter.

EXAMPLE 21.8 For the pass-band filter in Fig. 21.35:

a. Determine the critical frequencies for the low- and high-pass filters.
b. Using only the critical frequencies, sketch the response characteristics.
c. Determine the actual value of V_o at the high-pass critical frequency calculated in part (a), and compare it to the level that defines the upper frequency for the pass-band.

Solutions:

a. High-pass filter:

$$f_c = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi(1\,\text{k}\Omega)(1500\,\text{pF})} = \mathbf{106.1\ kHz}$$

Low-pass filter:

$$f_c = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi(40\,\text{k}\Omega)(4\,\text{pF})} = \mathbf{994.72\ kHz}$$

b. In the mid-region of the pass-band at about 500 kHz, an analysis of the network reveals that $V_o \cong 0.9V_i$, as shown in Fig. 21.36. The bandwidth is therefore defined at a level of $0.707(0.9V_i) = 0.636V_i$, as also shown in Fig. 21.36.

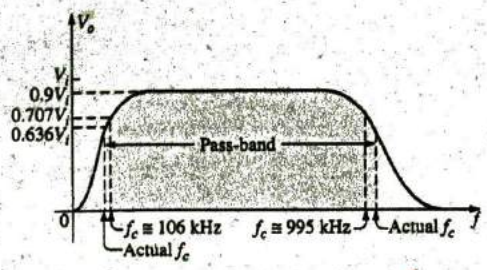


FIG. 21.36

Pass-band characteristics for the filter in Fig. 21.35.

c. At $f = 994.72$ kHz,

$$X_{C_1} = \frac{1}{2\pi f C_1} \cong 107\ \Omega$$

and

$$X_{C_2} = \frac{1}{2\pi f C_2} = R_2 = 40\ \text{k}\Omega$$

resulting in the network in Fig. 21.37.

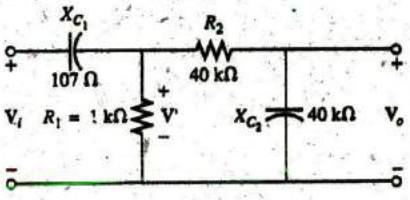


FIG. 21.37

Network of Fig. 21.35 at $f = 994.72$ kHz.

The parallel combination $R_1 \| (R_2 - jX_{C_2})$ is essentially 0.976 kΩ ∠0° because the $R_2 - X_{C_2}$ combination is so large compared to the parallel resistor R_1.

Then

$$\mathbf{V}' = \frac{0.976\ \text{k}\Omega\ \angle 0°(\mathbf{V}_i)}{0.976\ \text{k}\Omega - j\,0.107\ \text{k}\Omega} \cong 0.994\mathbf{V}_i\ \angle 6.26°$$

with

$$\mathbf{V}_o = \frac{(40\ \text{k}\Omega\ \angle -90°)(0.994\mathbf{V}_i\ \angle 6.26°)}{40\ \text{k}\Omega - j\,40\ \text{k}\Omega}$$

$$\mathbf{V}_o \cong 0.703\mathbf{V}_i\ \angle -39°$$

so that

$$V_o \cong 0.703V_i \quad \text{at} \quad f = 994.72\ \text{kHz}$$

Since the bandwidth is defined at $0.636V_i$ the upper cutoff frequency will be higher than 994.72 kHz, as shown in Fig. 21.36.

21.8 BAND-REJECT FILTERS

Since the characteristics of a band-reject filter (also called stop-band or notch filter) are the inverse of the pattern obtained for the band-pass filter, a band-reject filter can be designed by simply applying Kirchhoff's voltage law to each circuit. That is, since the sum of the voltages around a closed loop must equal zero at any instant, if we take the output off the series resistor in a series resonant circuit and the series resistive load in a parallel resonant circuit, we will obtain the desired band-reject response. The above is demonstrated by the waveforms of Fig. 21.38.

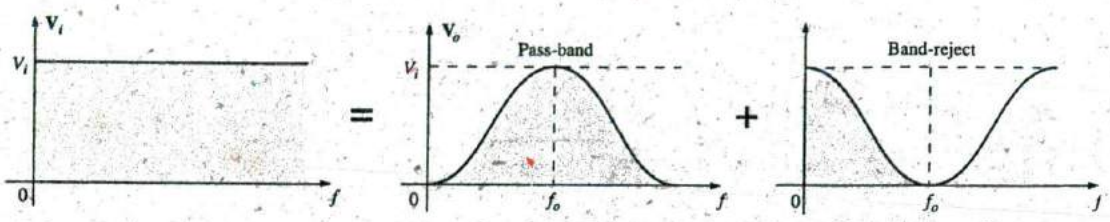


FIG. 21.38

Demonstrating how an applied signal of fixed magnitude can be broken down into a pass-band and band-reject response curve.

For the series resonant circuit in Fig. 21.39, Eqs. (21.22) through (21.24) still apply, but now, at resonance,

$$V_{o_{\min}} = \frac{R_l V_i}{R_l + R} \tag{21.30}$$

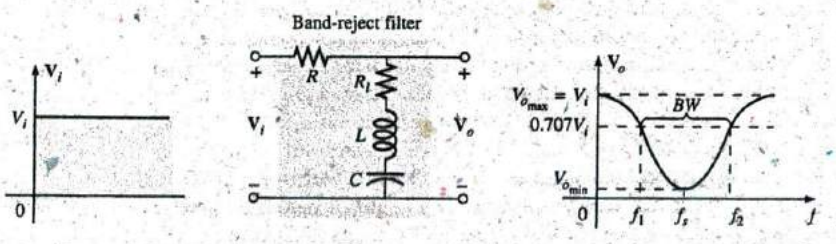


FIG. 21.39

Band-reject filter using a series resonant circuit.

For the parallel resonant circuit in Fig. 21.40, Eqs. (21.26) through (21.29) are still applicable, but now, at resonance,

$$V_{o_{\min}} = \frac{RV_i}{R + Z_{T_p}} \tag{21.31}$$

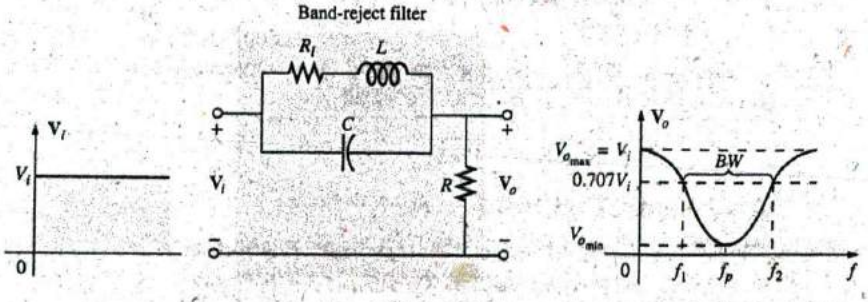


FIG. 21.40

Band-reject filter using a parallel resonant network.

The maximum value of V_o for the series resonant circuit is V_i at the low end due to the open-circuit equivalent for the capacitor and V_i at the high end due to the high impedance of the inductive element.

For the parallel resonant circuit, at $f = 0$ Hz, the inductor can be replaced by a short-circuit equivalent, and the capacitor can be replaced by its open circuit, and $V_o = RV_i/(R + R_l)$. At the high-frequency end, the capacitor approaches a short-circuit equivalent, and V_o increases toward V_i.

Band-reject filters can also be constructed using a low-pass and a high-pass filter. However, rather than the cascaded configuration used for the pass-band filter, a parallel arrangement is required, as shown in Fig. 21.41. A low-frequency f_1 can pass through the low-pass filter, and a higher-frequency f_2 can use the parallel path, as shown in Figs. 21.41 and 21.42. However, a frequency such as f_o in the reject-band is higher than the low-pass critical frequency and lower than the high-pass critical frequency, and it is therefore prevented from contributing to the levels of V_o above $0.707V_{max}$.

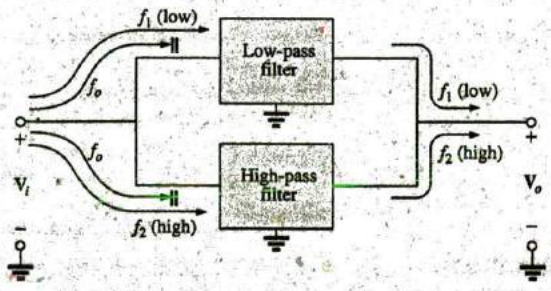


FIG. 21.41
Band-reject filter.

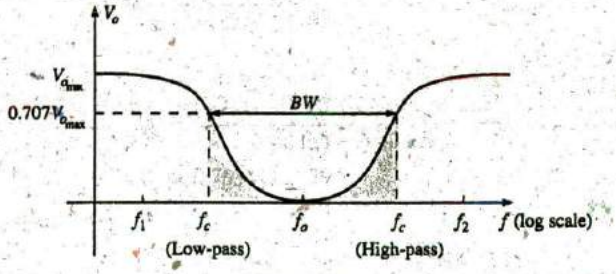


FIG. 21.42
Band-reject characteristics.

21.9 DOUBLE-TUNED FILTER

Some network configurations display both a pass-band and a stop-band characteristic, such as shown in Fig. 21.43. Such networks are called **double-tuned filters.** For the network in Fig. 21.43(a), the parallel resonant circuit establishes a stop-band for the range of frequencies not permitted to establish a significant V_L. The greater part of the applied voltage appears across the parallel resonant circuit for this frequency range due to its very high impedance compared with R_L. For the pass-band, the parallel resonant circuit is designed to be capacitive (inductive if L_s is replaced by C_s). The inductance L_s is chosen to cancel the effects of the resulting net capacitive reactance at the resonant pass-band

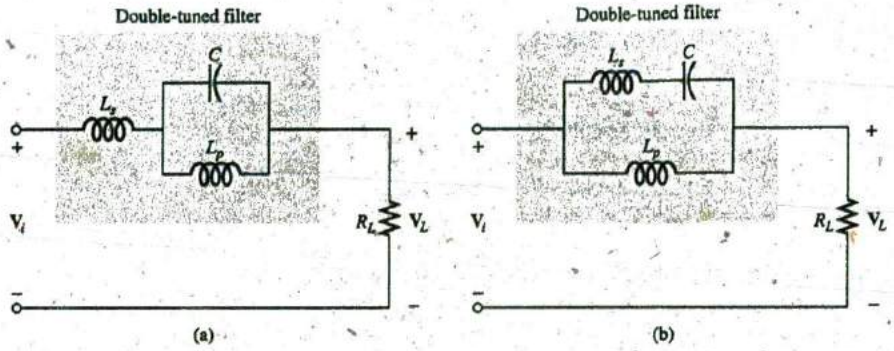


FIG. 21.43
Double-tuned networks.

frequency of the tank circuit, thereby acting as a series resonant circuit. The applied voltage then appears across R_L at this frequency.

For the network in Fig. 21.43(b), the series resonant circuit still determines the pass-band, acting as a very low impedance across the parallel inductor at resonance. At the desired stop-band resonant frequency, the series resonant circuit is capacitive. The inductance L_p is chosen to establish parallel resonance at the resonant stop-band frequency. The high impedance of the parallel resonant circuit results in a very low load voltage V_L.

For rejected frequencies below the pass-band, the networks should appear as shown in Fig. 21.43. For the reverse situation, L_s in Fig. 21.43(a) and L_p in Fig. 21.43(b) are replaced by capacitors.

EXAMPLE 21.9 For the network in Fig. 21.43(b), determine L_s and L_p for a capacitance C of 500 pF if a frequency of 200 kHz is to be rejected and a frequency of 600 kHz accepted.

Solution: For series resonance, we have

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

and
$$L_s = \frac{1}{4\pi^2 f_s^2 C} = \frac{1}{4\pi^2(600\text{ kHz})^2(500\text{ pF})} = \mathbf{140.7\ \mu H}$$

At 200 kHz,

$$X_{L_s} = \omega L = 2\pi f_s L_s = (2\pi)(200\text{ kHz})(140.7\ \mu\text{H}) = 176.8\ \Omega$$

and
$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi)(200\text{ kHz})(500\text{ pF})} = 1591.5\ \Omega$$

For the series elements,

$$j(X_{L_s} - X_C) = j(176.8\ \Omega - 1591.5\ \Omega) = -j\,1414.7\ \Omega = -jX'_C$$

At parallel resonance ($Q_l \geq 10$ assumed),

$$X_{L_p} = X'_C$$

and
$$L_p = \frac{X_{L_p}}{\omega} = \frac{1414.7\ \Omega}{(2\pi)(200\text{ kHz})} = \mathbf{1.13\text{ mH}}$$

The frequency response for the preceding network appears as one of the examples of PSpice in the last section of the chapter.

21.10 BODE PLOTS

There is a technique for sketching the frequency response of such factors as filters, amplifiers, and systems on a decibel scale that can save a great deal of time and effort and provide an excellent way to compare decibel levels at different frequencies.

The curves obtained for the magnitude and/or phase angle versus frequency are called Bode plots (Fig. 21.44). Through the use of straight-line segments called idealized Bode plots, the frequency response of a system can be found efficiently and accurately.

To ensure that the derivation of the method is correctly and clearly understood, the first network to be analyzed is examined in some detail. The second network is treated in a shorthand manner, and finally a method for quickly determining the response is introduced.



FIG. 21.44
Hendrik Wade Bode.
Courtesy of AT&T Archives.

American (Madison, WI; Summit, NJ; Cambridge, MA)
(1905–81)
V.P. at Bell Laboratories
Professor of Systems Engineering, Harvard University

In his early years at Bell Laboratories, Hendrik Bode was involved with *electric filter* and *equalizer design*. He then transferred to the Mathematics Research Group, where he specialized in research pertaining to electrical networks theory and its application to long-distance communication facilities. In 1948 he was awarded the Presidential Certificate of Merit for his work in electronic fire control devices. In addition to the publication of the book *Network Analysis and Feedback Amplifier Design* in 1945, which is considered a classic in its field, he was granted 25 patents in electrical engineering and systems design. Upon retirement, Bode was elected Gordon McKay Professor of Systems Engineering at Harvard University. He was a fellow of the IEEE and American Academy of Arts and Sciences.

High-Pass *R-C* Filter

Let us start by reexamining the high-pass filter in Fig. 21.45. The high-pass filter was chosen as our starting point because the frequencies of primary interest are at the low end of the frequency spectrum.

The voltage gain of the system is given by

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R - jX_C} = \frac{1}{1 - j\dfrac{X_C}{R}} = \frac{1}{1 - j\dfrac{1}{2\pi fCR}}$$

$$= \frac{1}{1 - j\left(\dfrac{1}{2\pi RC}\right)\dfrac{1}{f}}$$

If we substitute

$$\boxed{f_c = \frac{1}{2\pi RC}} \tag{21.32}$$

which we recognize as the cutoff frequency of earlier sections, we obtain

$$\boxed{\mathbf{A}_v = \frac{1}{1 - j(f_c/f)}} \tag{21.33}$$

We will find in the analysis to follow that the ability to reformat the gain to one having the general characteristics of Eq. (21.33) is critical to the application of the Bode technique. Different configurations result in variations of the format of Eq. (21.33), but the desired similarities become obvious as we progress through the material.

In magnitude and phase form,

$$\boxed{\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = A_v \angle\theta = \frac{1}{\sqrt{1 + (f_c/f)^2}} \angle \tan^{-1}(f_c/f)} \tag{21.34}$$

providing an equation for the magnitude and phase of the high-pass filter in terms of the frequency levels.

Using Eq. (21.12) gives

$$A_{v_{dB}} = 20 \log_{10} A_v$$

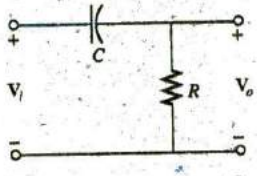


FIG. 21.45
High-pass filter.

and, substituting the magnitude component of Eq. (21.34) gives

$$A_{v_{dB}} = 20\log_{10}\frac{1}{\sqrt{1+(f_c/f)^2}} = \underbrace{20\log_{10}1}_{0} - 20\log_{10}\sqrt{1+(f_c/f)^2}$$

and

$$A_{v_{dB}} = -20\log_{10}\sqrt{1+\left(\frac{f_c}{f}\right)^2}$$

Recognizing that $\log_{10}\sqrt{x} = \log_{10}x^{1/2} = \frac{1}{2}\log_{10}x$, we have

$$A_{v_{dB}} = -\frac{1}{2}(20)\log_{10}\left[1+\left(\frac{f_c}{f}\right)^2\right]$$

$$= -10\log_{10}\left[1+\left(\frac{f_c}{f}\right)^2\right]$$

For frequencies where $f \ll f_c$ or $(f_c/f)^2 \gg 1$,

$$1+\left(\frac{f_c}{f}\right)^2 \cong \left(\frac{f_c}{f}\right)^2$$

and

$$A_{v_{dB}} = -10\log_{10}\left(\frac{f_c}{f}\right)^2$$

but

$$\log_{10}x^2 = 2\log_{10}x$$

resulting in

$$A_{v_{dB}} = -20\log_{10}\frac{f_c}{f}$$

However, logarithms are such that

$$-\log_{10}b = +\log_{10}\frac{1}{b}$$

and substituting $b = f_c/f$, we have

$$\boxed{A_{v_{dB}} = +20\log_{10}\frac{f}{f_c}}_{f \ll f_c} \tag{21.35}$$

First note the similarities between Eq. (21.35) and the basic equation for gain in decibels: $G_{dB} = 20\log_{10}V_o/V_i$. The comments regarding changes in decibel levels due to changes in V_o/V_i can therefore be applied here also, except now a change in frequency by a 2 : 1 ratio results in a 6 dB change in gain. A change in frequency by a 10 : 1 ratio results in a 20 dB change in gain.

Two frequencies separated by a 2 : 1 ratio are said to be an octave apart. For Bode plots, a change in frequency by one octave will result in a 6 dB change in gain.

Two frequencies separated by a 10 : 1 ratio are said to be a decade apart. For Bode plots, a change in frequency by one decade will result in a 20 dB change in gain.

One may wonder about all the mathematical development to obtain an equation that initially appears confusing and of limited value. As specified, Eq. (21.35) is accurate only for frequency levels much less than f_c.

First, realize that the mathematical development of Eq. (21.35) does not have to be repeated for each configuration encountered. Second, the equation itself is seldom applied but simply used to define a straight line

on a log plot that permits a sketch of the frequency response of a system with a minimum of effort and a high degree of accuracy.

To plot Eq. (21.35), consider the following levels of increasing frequency:

For $f = f_c/10$, $f/f_c = 0.1$ and $+20 \log_{10} 0.1 = -20$ dB
For $f = f_c/4$, $f/f_c = 0.25$ and $+20 \log_{10} 0.25 = -12$ dB
For $f = f_c/2$, $f/f_c = 0.51$ and $+20 \log_{10} 0.5 = -6$ dB
For $f = f_c$, $f/f_c = 1$ and $+20 \log_{10} 1 = 0$ dB

Note from the above equations that as the frequency of interest approaches f_c, the dB gain becomes less negative and approaches the final normalized value of 0 dB. The positive sign in front of Eq. (21.35) can therefore be interpreted as an indication that the dB gain will have a positive slope with an increase in frequency. A plot of these points on a log scale results in the straight-line segment in Fig. 21.46 to the left of f_c.

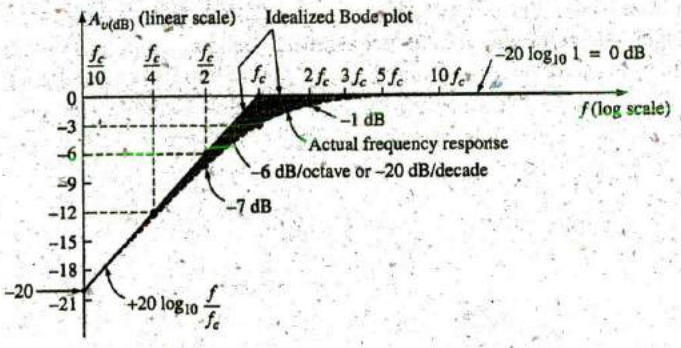


FIG. 21.46
Idealized Bode plot for the low-frequency region.

For the future, note that the resulting plot is a straight line intersecting the 0 dB line at f_c. It increases to the right at a rate of +6 dB per octave or +20 dB per decade. In other words, once f_c is determined, find $f_c/2$, and a plot point exists at −6 dB (or find $f_c/10$, and a plot point exists at −20 dB).

Bode plots are straight-line segments because the dB change per decade or octave is constant.

The actual response approaches an asymptote (straight-line segment) defined by $A_{v_{dB}} = 0$ dB since at high frequencies

$$f >> f_c \quad \text{and} \quad f_c/f \cong 0$$

with
$$A_{v_{dB}} = 20 \log_{10} \frac{1}{\sqrt{1 + (f_c/f)^2}} = 20 \log_{10} \frac{1}{\sqrt{1 + 0}}$$
$$= 20 \log_{10} 1 = 0 \text{ dB}$$

The two asymptotes defined above intersect at f_c, as shown in Fig. 21.46, forming an envelope for the actual frequency response.

At $f = f_c$, the cutoff frequency,

$$A_{v_{dB}} = 20 \log_{10} \frac{1}{\sqrt{1 + (f_c/f)^2}} = 20 \log_{10} \frac{1}{\sqrt{1 + 1}} = 20 \log_{10} \frac{1}{\sqrt{2}}$$
$$= \mathbf{-3 \text{ dB}}$$

At $f = 2f_c$,

$$A_{v_{dB}} = -20\log_{10}\sqrt{1 + \left(\frac{f_c}{2f_c}\right)^2} = -20\log_{10}\sqrt{1 + \left(\frac{1}{2}\right)^2}$$
$$= -20\log_{10}\sqrt{1.25} = \mathbf{-1\ dB}$$

as shown in Fig. 21.46.

At $f = f_c/2$,

$$A_{v_{dB}} = -20\log_{10}\sqrt{1 + \left(\frac{f_c}{f_c/2}\right)^2} = -20\log_{10}\sqrt{1 + (2)^2}$$
$$= -20\log_{10}\sqrt{5}$$
$$= \mathbf{-7\ dB}$$

separating the idealized Bode plot from the actual response by 7 dB − 6 dB = 1 dB, as shown in Fig. 21.46.

Reviewing the above,

at $f = f_c$, the actual response curve is 3 dB down from the idealized Bode plot, whereas at $f = 2f_c$ and $f_c/2$, the actual response curve is 1 dB down from the asymptotic response.

The phase response can also be sketched using straight-line asymptotes by considering a few critical points in the frequency spectrum.

Eq. (21.34) specifies the phase response (the angle by which $\mathbf{V}_o$ leads $\mathbf{V}_i$) by

$$\boxed{\theta = \tan^{-1}\frac{f_c}{f}} \qquad \textbf{(21.36)}$$

For frequencies well below f_c ($f << f_c$), $\theta = \tan^{-1}(f_c/f)$ approaches 90°, and for frequencies well above f_c ($f >> f_c$), $\theta = \tan^{-1}(f_c/f)$ will approach 0°, as discovered in earlier sections of the chapter. At $f = f_c$, $\theta = \tan^{-1}(f_c/f) = \tan^{-1} 1 = 45°$.

Defining $f << f_c$ for $f = f_c/10$ (and less) and $f >> f_c$ for $f = 10f_c$ (and more), we can define

an asymptote at $\theta = 90°$ for $f << f_c/10$, an asymptote at $\theta = 0°$ for $f >> 10f_c$, and an asymptote from $f_c/10$ to $10f_c$ that passes through $\theta = 45°$ at $f = f_c$.

The asymptotes defined above all appear in Fig. 21.47. Again, the Bode plot for Eq. (21.36) is a straight line because the change in phase angle will be 45° for every tenfold change in frequency.

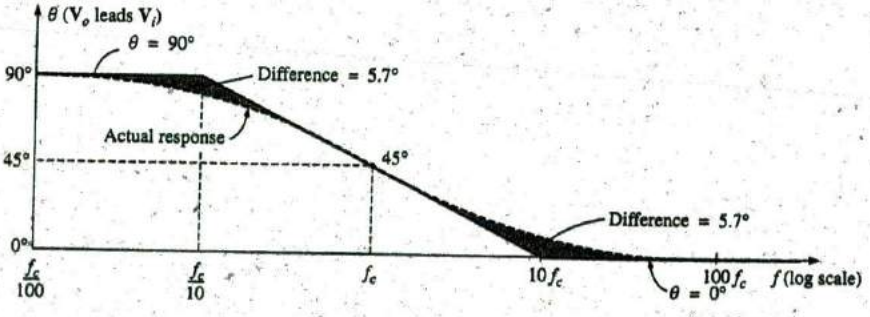


FIG. 21.47
Phase response for a high-pass R-C filter.

Substituting $f = f_c/10$ into Eq. (21.36) gives

$$\theta = \tan^{-1}\left(\frac{f_c}{f_c/10}\right) = \tan^{-1} 10 = \mathbf{84.29°}$$

for a difference of $90° - 84.29° \cong 5.7°$ from the idealized response.

Substituting $f = 10f_c$ gives

$$\theta = \tan^{-1}\left(\frac{f_c}{10f_c}\right) = \tan^{-1}\frac{1}{10} \cong \mathbf{5.7°}$$

In summary, therefore,

at $f = f_c$, $\theta = 45°$, whereas at $f = f_c/10$ and $10f_c$, the difference between the actual phase response and the asymptotic plot is 5.7°.

EXAMPLE 21.10

a. Sketch $A_{v_{dB}}$ versus frequency for the high-pass *R-C* filter in Fig. 21.48.

b. Determine the decibel level at $f = 1$ kHz.

c. Sketch the phase response versus frequency on a log scale.

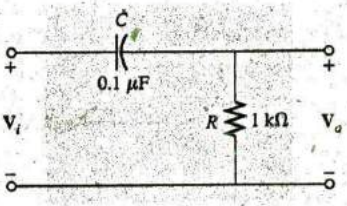


FIG. 21.48
Example 21.10.

Solutions:

a. $$f_c = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(1\text{ k}\Omega)(0.1\mu\text{F})} = 1591.55\text{ Hz}$$

The frequency f_c is identified on the log scale as shown in Fig. 21.49. A straight line is then drawn from f_c with a slope that will intersect -20 dB at $f_c/10 = 159.15$ Hz or -6 dB at $f_c/2 = 795.77$ Hz. A second asymptote is drawn from f_c to higher frequencies at 0 dB. The actual response curve can then be drawn through the -3 dB level at f_c approaching the two asymptotes of Fig. 21.49. Note the 1 dB difference between the actual response and the idealized Bode plot at $f = 2f_c$ and $0.5f_c$.

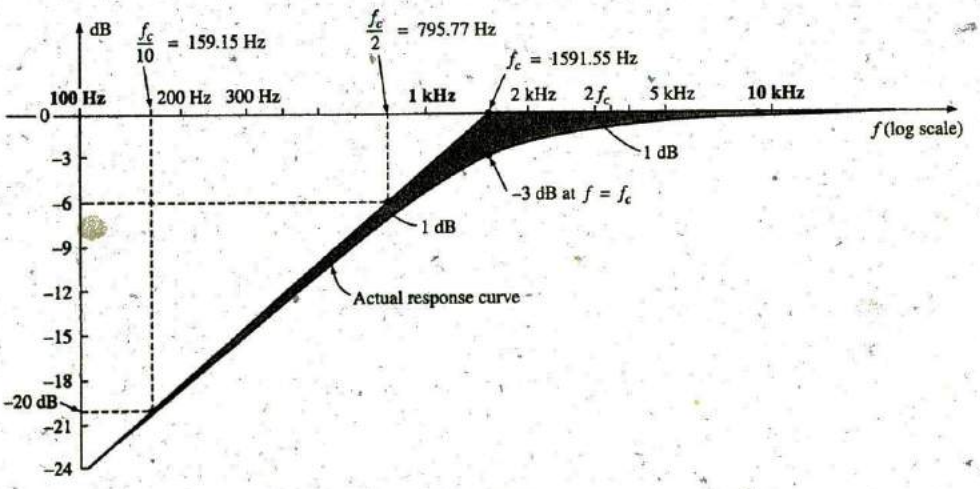


FIG. 21.49
Frequency response for the high-pass filter in Fig. 21.48.

Note that in the solution to part (a), there is no need to use Eq. (21.35) or to perform any extensive mathematical manipulations.

b. Eq. (21.33):

$$|A_{v_{dB}}| = 20\log_{10}\frac{1}{\sqrt{1+\left(\frac{f_c}{f}\right)^2}} = 20\log_{10}\frac{1}{\sqrt{1+\left(\frac{1591.55\text{ Hz}}{1000}\right)^2}}$$

$$= 20\log_{10}\frac{1}{\sqrt{1+(1.592)^2}} = 20\log_{10}0.5318 = \mathbf{-5.49\text{ dB}}$$

as verified by Fig. 21.49.

c. See Fig. 21.50. Note that $\theta = 45°$ at $f = f_c = 1591.55$ Hz, and the difference between the straight-line segment and the actual response is 5.7° at $f = f_c/10 = 159.2$ Hz and $f = 10f_c = 15{,}923.6$ Hz.

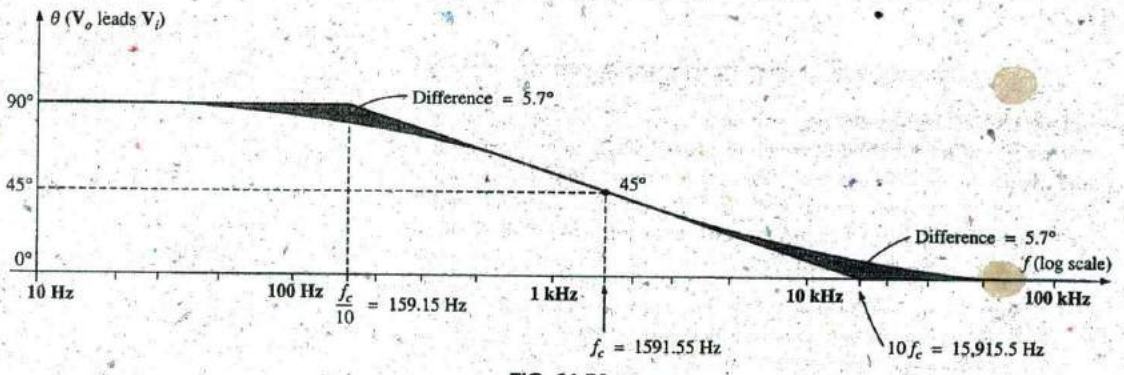


FIG. 21.50
Phase plot for the high-pass R-C filter.

Low-Pass *R-C* Filter

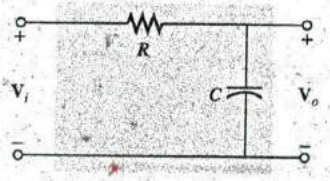


FIG. 21.51
Low-pass filter.

For the low-pass filter in Fig. 21.51,

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-jX_C}{R-jX_C} = \frac{1}{\frac{R}{-jX_C}+1}$$

$$= \frac{1}{1+j\frac{R}{X_C}} = \frac{1}{1+j\frac{R}{\frac{1}{2\pi fC}}} = \frac{1}{1+j\frac{f}{\frac{1}{2\pi RC}}}$$

and

$$\boxed{\mathbf{A}_v = \frac{1}{1+j(f/f_c)}} \qquad \textbf{(21.37)}$$

with

$$\boxed{f_c = \frac{1}{2\pi RC}} \qquad \textbf{(21.38)}$$

as defined earlier.

Note that now the sign of the imaginary component in the denominator is positive and f_c appears in the denominator of the frequency ratio rather than in the numerator, as in the case of f_c for the high-pass filter.

In terms of magnitude and phase,

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \mathbf{A}_v \angle\theta = \frac{1}{\sqrt{1 + (f/f_c)^2}} \angle -\tan^{-1}(f/f_c) \quad (21.39)$$

An analysis similar to that performed for the high-pass filter results in

$$A_{v_{dB}} \cong -20 \log_{10} \frac{f}{f_c} \quad f >> f_c \quad (21.40)$$

Note in particular that the equation is exact only for frequencies much greater than f_c, but a plot of Eq. (21.40) does provide an asymptote that performs the same function as the asymptote derived for the high-pass filter. In addition, note that it is exactly the same as Eq. (21.35), except for the minus sign, which suggests that the resulting Bode plot will have a negative slope [recall the positive slope for Eq. (21.35)] for increasing frequencies beyond f_c.

A plot of Eq. (21.40) appears in Fig. 21.52 for $f_c = 1$ kHz. Note the 6 dB drop at $f = 2f_c$ and the 20 dB drop at $f = 10f_c$.

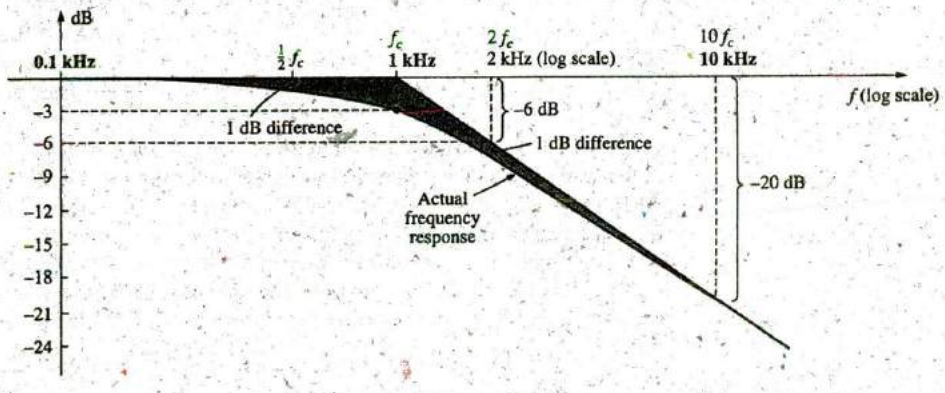


FIG. 21.52

Bode plot for the high-frequency region of a low-pass R-C filter.

At $f >> f_c$, the phase angle $\theta = -\tan^{-1}(f/f_c)$ approaches $-90°$, whereas at $f << f_c$, $\theta = -\tan^{-1}(f/f_c)$ approaches $0°$. At $f = f_c$, $\theta \cong -\tan^{-1} 1 = -45°$, establishing the plot in Fig. 21.53. Note again the 45° change in phase angle for each tenfold increase in frequency.

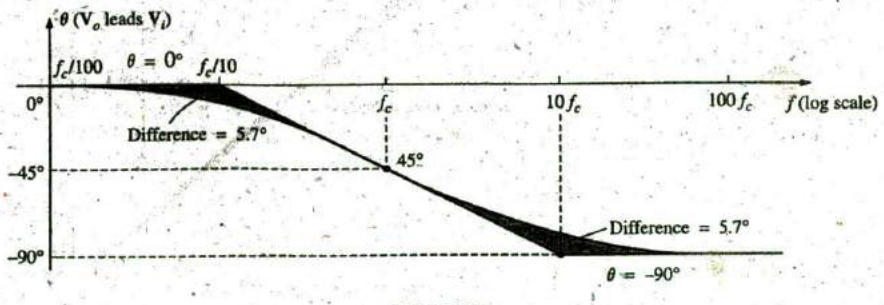


FIG. 21.53

Phase plot for a low-pass R-C filter.

Even though the preceding analysis has been limited solely to the *R-C* combination, the results obtained will have an impact on networks that are a great deal more complicated. One good example is the high- and low-frequency response of a standard transistor configuration. Some capacitive elements in a practical transistor network affect the low-frequency response, and others affect the high-frequency response. In the absence of the capacitive elements, the frequency response of a transistor ideally stays level at the midband value. However, the coupling capacitors at low frequencies and the bypass and parasitic capacitors at high frequencies define a bandwidth for numerous transistor configurations. In the low-frequency region, specific capacitors and resistors form an *R-C* combination that defines a low cutoff frequency. There are then other elements and capacitors forming a second *R-C* combination that define a high cutoff frequency. Once the cutoff frequencies are known, the −3 dB points are set, and the bandwidth of the system can be determined.

21.11 SKETCHING THE BODE RESPONSE

In the previous section, we found that normalized functions of the form appearing in Fig. 21.54 had the Bode envelope and the dB response indicated in the same figure. In this section, we introduce additional functions and their responses that can be used in conjunction with those in Fig. 21.54 to determine the dB response of more sophisticated systems in a systematic, time-saving, and accurate manner.

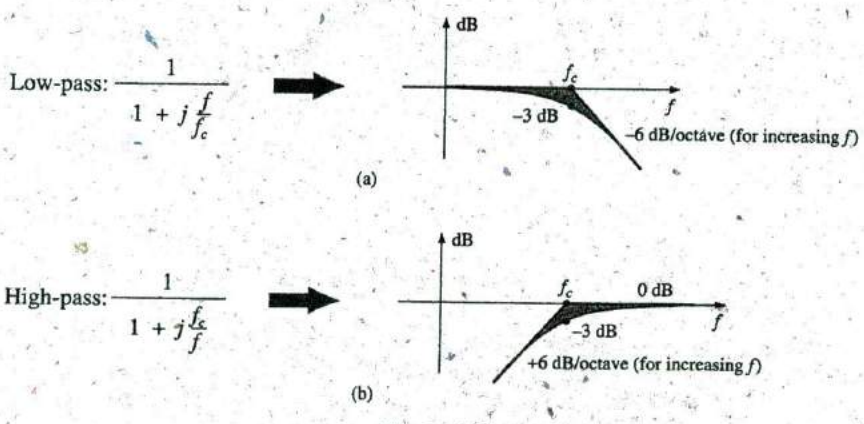


FIG. 21.54
dB response of (a) low-pass filter and (b) high-pass filter.

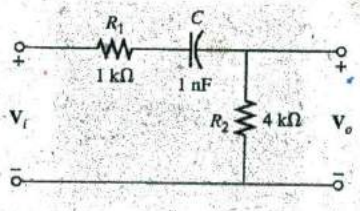


FIG. 21.55
High-pass filter with attenuated output.

As an avenue toward introducing an additional function that appears quite frequently, let us examine the high-pass filter in Fig. 21.55, which has a high-frequency output less than the full applied voltage.

Before developing a mathematical expression for $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_i$, let us first make a rough sketch of the expected response.

At $f = 0$ Hz, the capacitor assumes its open-circuit equivalence, and $V_o = 0$ V. At very high frequencies, the capacitor can assume its short-circuit equivalence, and

$$V_o = \frac{R_2}{R_1 + R_2} V_i = \frac{4\,\text{k}\Omega}{1\,\text{k}\Omega + 4\,\text{k}\Omega} V_i = 0.8\,V_i$$

The resistance to be used in the equation for cutoff frequency can be determined by determining the Thévenin resistance "seen" by the capacitor. Setting $V_i = 0$ V and solving for R_{Th} (for the capacitor C) results in the network in Fig. 21.56, where it is quite clear that

$$R_{Th} = R_1 + R_2 = 1\ \text{k}\Omega + 4\ \text{k}\Omega = 5\ \text{k}\Omega$$

Therefore,

$$f_c = \frac{1}{2\pi R_{Th}C} = \frac{1}{2\pi(5\ \text{k}\Omega)(1\ \text{nF})} = 31.83\ \text{kHz}$$

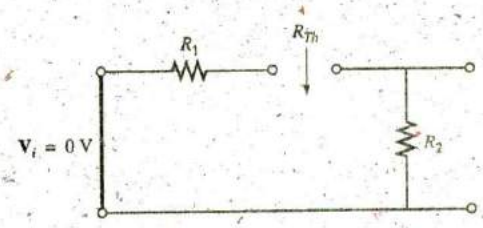


FIG. 21.56
Determining R_{Th} for the equation for cutoff frequency.

A sketch of V_o versus frequency is provided in Fig. 21.57(a). A normalized plot using V_i as the normalizing quantity results in the response in Fig. 21.57(b). If the maximum value of A_v is used in the normalization process, the response in Fig. 21.57(c) is obtained. For all the plots obtained in the previous section, V_i was the maximum value, and the ratio V_o/V_i had a maximum value of 1. For many situations, this will not be the case, and we must be aware of which ratio is being plotted versus frequency. The dB response curves for the plots in Figs. 21.57(b) and 21.57(c) can both be obtained quite directly using the foundation established by the conclusions depicted in Fig. 21.54, but we must be aware of what to expect and how they will differ. In Fig. 21.57(b), we are comparing the output level to the input voltage. In Fig. 21.57(c), we are plotting A_v versus the maximum value of A_v. On most data sheets and for the majority of the investigative techniques commonly used, the normalized plot in Fig. 21.57(c) is used because it establishes 0 dB as an asymptote for the dB plot. To ensure that the impact of using either Fig. 21.57(b) or Fig. 21.57(c) in a frequency plot is understood, the analysis of the filter in Fig. 21.55 includes the resulting dB plot for both normalized curves.

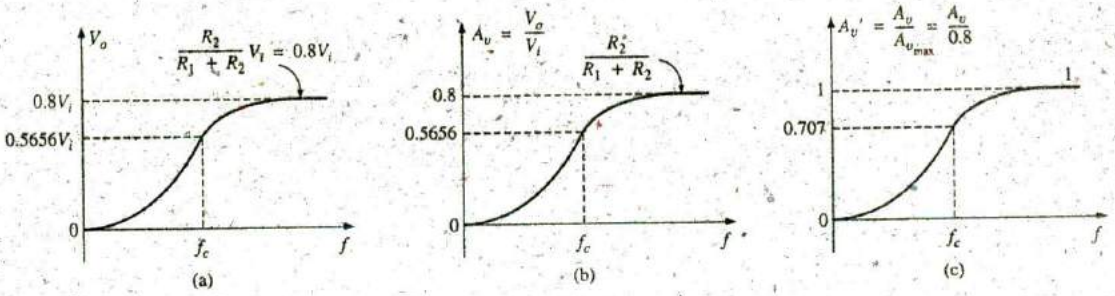


FIG. 21.57
Finding the normalized plot for the gain of the high-pass filter in Fig. 21.55 with attenuated output.

For the network in Fig. 21.55,

$$\mathbf{V}_o = \frac{R_2\mathbf{V}_i}{R_1 + R_2 - jX_C} = R_2\left[\frac{1}{R_1 + R_2 - jX_C}\right]\mathbf{V}_i$$

Dividing the top and bottom of the equation by $R_1 + R_2$ results in

$$\mathbf{V}_o = \frac{R_2}{R_1 + R_2}\left[\frac{1}{1 - j\dfrac{X_C}{R_1 + R_2}}\right]$$

but $$-j\frac{X_C}{R_1+R_2} = -j\frac{1}{\omega(R_1+R_2)C} = -j\frac{1}{2\pi f(R_1+R_2)C}$$
$$= -j\frac{f_c}{f} \text{ with } f_c = \frac{1}{2\pi R_{Th}C} \text{ and } R_{Th} = R_1 + R_2$$

so that $$\mathbf{V}_o = \frac{R_2}{R_1+R_2}\left[\frac{1}{1-j(f_c/f)}\right]\mathbf{V}_i$$

If we divide both sides by $\mathbf{V}_i$, we obtain

$$\boxed{\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_2}{R_1+R_2}\left[\frac{1}{1-j(f_c/f)}\right]} \tag{21.41}$$

from which the magnitude plot in Fig. 21.57(b) can be obtained. If we divide both sides by $\mathbf{A}_{v_{max}} = R_2/(R_1 + R_2)$, we have

$$\boxed{\mathbf{A}'_v = \frac{\mathbf{A}_v}{\mathbf{A}_{v_{max}}} = \frac{1}{1-j(f_c/f)}} \tag{21.42}$$

from which the magnitude plot in Fig. 21.57(c) can be obtained.

Based on the previous section, a dB plot of the magnitude of $A'_v = A_v/A_{v_{max}}$ is now quite direct using Fig. 21.54(b). The plot appears in Fig. 21.58.

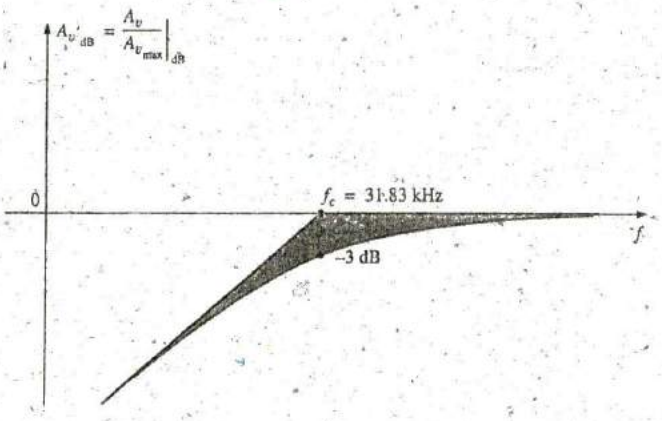


FIG. 21.58
dB plot for A'_v for the high-pass filter in Fig. 21.55.

For the gain $A_v = V_o/V_i$, we can apply Eq. (21.5):

$$20\log_{10} ab = 20\log_{10} a + 20\log_{10} b$$

where

$$20\log_{10}\left\{\frac{R_2}{R_1+R_2}\left[\frac{1}{1-j(f_c/f)}\right]\right\}$$
$$= 20\log_{10}\frac{R_2}{R_1+R_2} + 20\log_{10}\frac{1}{\sqrt{1+(f_c/f)^2}}$$

The second term results in the same plot in Fig. 21.58, but the first term must be added to the second to obtain the total dB response.

Since $R_2/(R_1 + R_2)$ must always be less than 1, we can rewrite the first term as

$$20 \log_{10} \frac{R_2}{R_1 + R_2} = 20 \log_{10} \frac{1}{\frac{R_1 + R_2}{R_2}} = \underbrace{20 \log_{10} 1}_{0} - 20 \log_{10} \frac{R_1 + R_2}{R_2}$$

and

$$\boxed{20 \log_{10} \frac{R_2}{R_1 + R_2} = -20 \log_{10} \frac{R_1 + R_2}{R_2}} \qquad (21.43)$$

providing the drop in dB from the 0 dB level for the plot. Adding one log plot to the other *at each frequency*, as permitted by Eq. (21.5), results in the plot in Fig. 21.59.

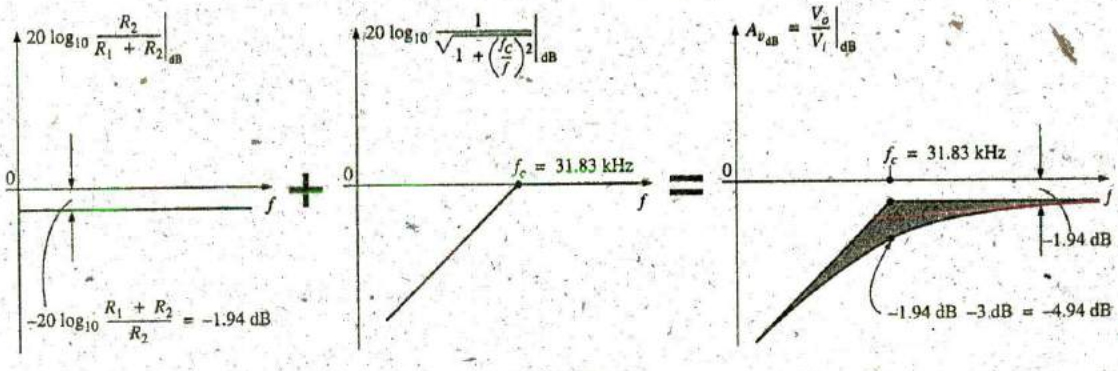


FIG. 21.59

Obtaining a dB plot of $A_{v_{dB}} = \left.\frac{V_o}{V_i}\right|_{dB}$

For the network in Fig. 21.55, the gain $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_i$ can also be found in the following manner:

$$\mathbf{V}_o = \frac{R_2 \mathbf{V}_i}{R_1 + R_2 - jX_C}$$

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_2}{R_1 + R_2 - jX_C} = \frac{jR_2}{j(R_1 + R_2) + X_C}$$

$$= \frac{jR_2/X_C}{j(R_1 + R_2)/X_C + 1} = \frac{j\,\omega R_2 C}{1 + j\,\omega(R_1 + R_2)C}$$

$$= \frac{j\,2\pi f R_2 C}{1 + j\,2\pi f(R_1 + R_2)C}$$

and

$$\boxed{\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(f/f_1)}{1 + j(f/f_c)}} \qquad (21.44)$$

with $f_1 = \dfrac{1}{2\pi R_2 C}$ and $f_c = \dfrac{1}{2\pi(R_1 + R_2)C}$

The denominator of Eq. (21.44) is a match of the denominator of the low-pass function in Fig. 21.54(a). The numerator, however, is a new

function that defines a unique Bode asymptote that will prove useful for a variety of network configurations.

Applying Eq. (21.5) gives

$$20\log_{10}\frac{V_o}{V_i} = 20\log_{10}\left[\frac{f}{f_1}\right]\left[\frac{1}{\sqrt{1+(f/f_c)^2}}\right]$$

$$= 20\log_{10}(f/f_1) + 20\log_{10}\frac{1}{\sqrt{1+(f/f_c)^2}}$$

Let us now consider specific frequencies for the first term.

At $f = f_1$:

$$20\log_{10}\frac{f}{f_1} = 20\log_{10}1 = 0\text{ dB}$$

At $f = 2f_1$:

$$20\log_{10}\frac{f}{f_1} = 20\log_{10}2 = +6\text{ dB}$$

At $f = \frac{1}{2}f_1$:

$$20\log_{10}\frac{f}{f_1} = 20\log_{10}0.5 = -6\text{ dB}$$

A dB plot of $20\log_{10}(f/f_1)$ is provided in Fig. 21.60. Note that the asymptote passes through the 0 dB line at $f = f_1$ and has a positive slope of +6 dB/octave (or 20 dB/decade) for frequencies above and below f_1 for increasing values of f.

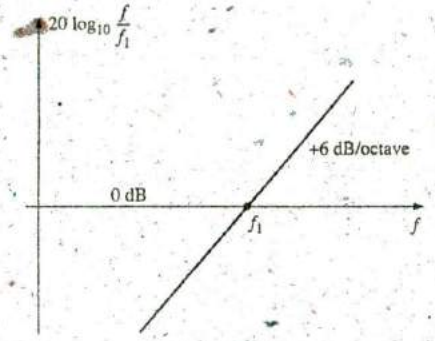


FIG. 21.60
dB plot of f/f_1.

If we examine the original function A_v, we find that the phase angle associated with $j\,f/f_1 = f/f_1\,\angle 90°$ is fixed at 90°, resulting in a phase angle for A_v of $90° - \tan^{-1}(f/f_c) = +\tan^{-1}(f_c/f)$.

Now that we have a plot of the dB response for the magnitude of the function f/f_1, we can plot the dB response of the magnitude of A_v using a procedure outlined by Fig. 21.61.

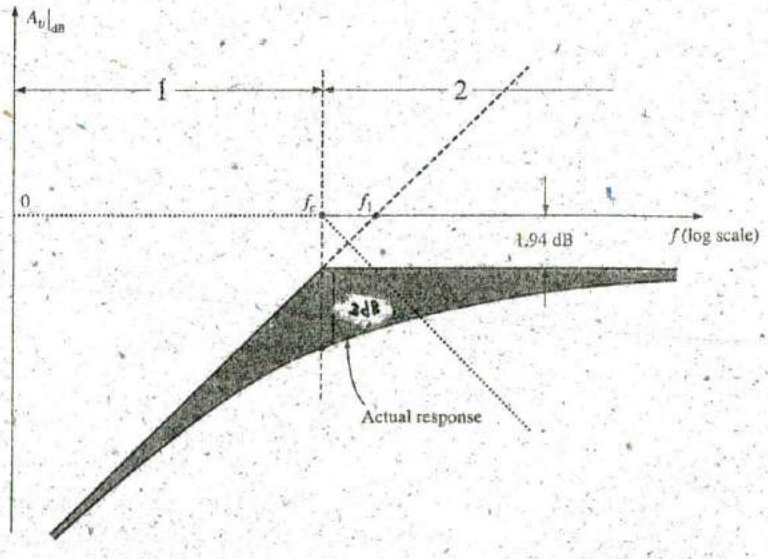


FIG. 21.61
Plot of $A_v|_{dB}$ for the network in Fig. 21.55.

Solving for f_1 and f_c gives

$$f_1 = \frac{1}{2\pi R_2 C} = \frac{1}{2\pi(4\text{ k}\Omega)(1\text{ nF})} = 39.79\text{ kHz}$$

with $$f_c = \frac{1}{2\pi(R_1 + R_2)C} = \frac{1}{2\pi(5\text{ k}\Omega)(1\text{ nF})} = 31.83\text{ kHz}$$

For this development, the straight-line asymptotes for each term resulting from the application of Eq. (21.5) are drawn on the same frequency axis to permit an examination of the impact of one line section on the other. For clarity, the frequency spectrum in Fig. 21.61 has been divided into two regions.

In region 1, we have a 0 dB asymptote and one increasing at 6 dB/octave for increasing frequencies. The sum of the two as defined by Eq. (21.5) is simply the 6 dB/octave asymptote shown in the figure.

In region 2, one asymptote is increasing at 6 dB, and the other is decreasing at −6 dB/octave for increasing frequencies. The net effect is that one cancels the other for the region greater than $f = f_c$, leaving a horizontal asymptote beginning at $f = f_c$. A careful sketch of the asymptotes on a log scale reveals that the horizontal asymptote is at −1.94 dB, as obtained earlier for the same function. The horizontal level can also be determined by simply plugging $f = f_c$ into the Bode plot defined by f/f_1; that is,

$$20\log\frac{f_c}{f_1} = 20\log_{10}\frac{f_c}{f_1} = 20\log_{10}\frac{31.83\text{ kHz}}{39.79\text{ kHz}}$$
$$= 20\log_{10}0.799 = -1.94\text{ dB}$$

The actual response can then be drawn using the asymptotes and the known differences at $f = f_c$ (−3 dB) and at $f = 0.5f_c$ or $2f_c$ (−1 dB).

In summary, therefore, the same dB response for $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_i$ can be obtained by isolating the maximum value or defining the gain in a different form. The latter approach permitted the introduction of a new function for our catalog of idealized Bode plots that will prove useful in the future.

21.12 LOW-PASS FILTER WITH LIMITED ATTENUATION

Our analysis now continues with the low-pass filter in Fig. 21.62, which has limited attenuation at the high-frequency end. That is, the output will not drop to zero as the frequency becomes relatively high. The filter is similar in construction to Fig. 21.55, but note that now $\mathbf{V}_o$ includes the capacitive element.

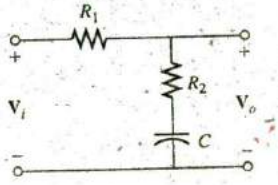


FIG. 21.62
Low-pass filter with limited attenuation.

At $f = 0$ Hz, the capacitor can assume its open-circuit equivalence, and $\mathbf{V}_o = \mathbf{V}_i$. At high frequencies, the capacitor can be approximated by a short-circuit equivalence, and

$$\mathbf{V}_o = \frac{R_2}{R_1 + R_2}\mathbf{V}_i$$

A plot of V_o versus frequency is provided in Fig. 21.63(a). A sketch of $A_v = V_o/V_i$ appears as shown in Fig. 21.63(b).

An equation for $\mathbf{V}_o$ in terms of $\mathbf{V}_i$ can be derived by first applying the voltage divider rule:

$$\mathbf{V}_o = \frac{(R_2 - jX_C)\mathbf{V}_i}{R_1 + R_2 - jX_C}$$

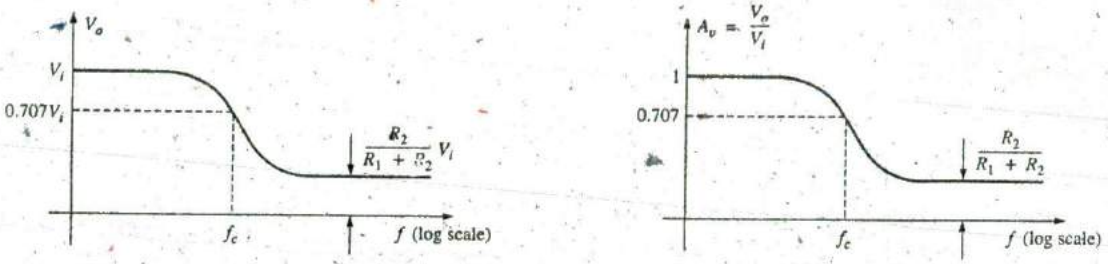


FIG. 21.63
Low-pass filter with limited attenuation.

$$\text{and} \quad \mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_2 - jX_C}{R_1 + R_2 - jX_C} = \frac{R_2/X_C - j}{(R_1 + R_2)/X_C - j}$$
$$= \frac{(j)(R_2X_C - j)}{(j)((R_1 + R_2)/X_C - j)}$$
$$= \frac{j(R_2/X_C) + 1}{j((R_1 + R_2)/X_C) + 1} = \frac{1 + j\,2\pi f R_2 C}{1 + j\,2\pi f(R_1 + R_2)C}$$

$$\text{so that} \quad \boxed{\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1 + j(f/f_1)}{1 + j(f/f_c)}} \tag{21.45}$$

$$\text{with} \quad f_1 = \frac{1}{2\pi R_2 C} \quad \text{and} \quad f_c = \frac{1}{2\pi(R_1 + R_2)C}$$

The denominator of Eq. (21.45) is simply the denominator of the low-pass function in Fig. 21.54(a). The numerator, however, is new and must be investigated.

Applying Eq. (21.5) gives

$$A_{v_{dB}} = 20 \log_{10} \frac{V_o}{V_i} = 20 \log_{10} \sqrt{1 + (f/f_1)^2} + 20 \log_{10} \frac{1}{\sqrt{1 + (f/f_c)^2}}$$

For $f >> f_1$, $(f/f_1)^2 >> 1$, and the first term becomes

$$20 \log_{10} \sqrt{(f/f_1)^2} = 20 \log_{10}((f/f_1)^2)^{1/2} = 20 \log_{10}(f/f_1)|_{f >> f_1}$$

which defines the idealized Bode asymptote for the numerator of Eq. (21.45).

At $f = f_1$, $20 \log_{10} 1 = 0$ dB, and at $f = 2f_1$, $20 \log_{10} 2 = 6$ dB. For frequencies much less than f_1, $(f/f_1)^2 << 1$, and the first term of the Eq. (21.5) expansion becomes $20 \log_{10} \sqrt{1} = 20 \log_{10} 1 = 0$ dB, which establishes the low-frequency asymptote.

The full idealized Bode response for the numerator of Eq. (21.45) is provided in Fig. 21.64.

We are now in a position to determine $A_v|_{dB}$ by plotting the asymptote for each function of Eq. (21.45) on the same frequency axis, as shown in Fig. 21.65. Note that f_c must be less than f_1 since the denominator of f_1 includes only R_2, whereas the denominator of f_c includes both R_2 and R_1.

Since $R_2/(R_1 + R_2)$ will always be less than 1, we can use an earlier development to obtain an equation for the drop in dB below the 0 dB axis at high frequencies. That is,

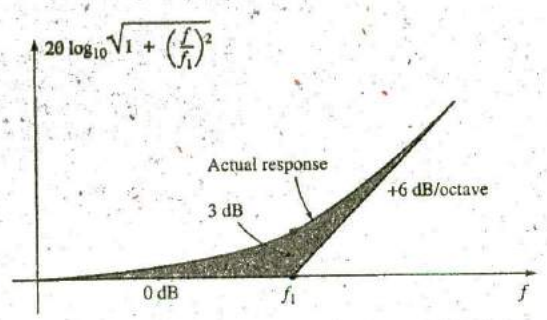


FIG. 21.64

Idealized and actual Bode response for the magnitude of $(1 + j\,(f/f_1))$.

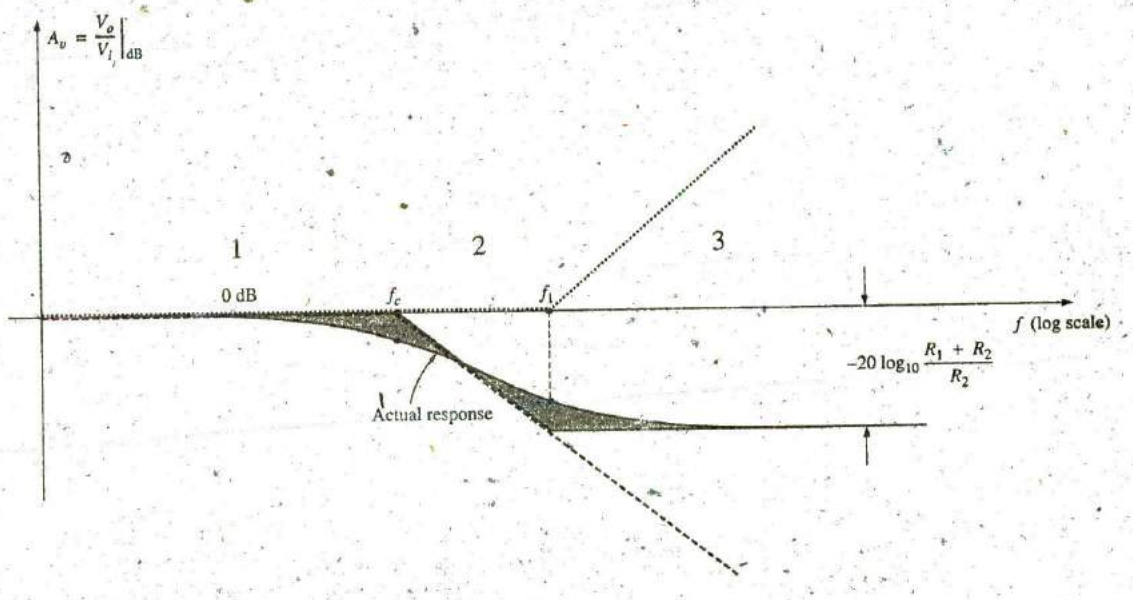


FIG. 21.65

$A_{v_{dB}}$ versus frequency for the low-pass filter with limited attenuation of Fig. 21.62.

$$20 \log_{10} R_2/(R_1 + R_2) = 20 \log_{10} 1/((R_1 + R_2)/R_2)$$
$$= \underbrace{20 \log_{10} 1}_{0} - 20 \log_{10}((R_1 + R_2)/R_2)$$

and

$$\boxed{20 \log_{10} \frac{R_2}{R_1 + R_2} = -20 \log_{10} \frac{R_1 + R_2}{R_2}} \qquad (21.46)$$

as shown in Fig. 21.65.

In region 1 in Fig. 21.65, both asymptotes are at 0 dB, resulting in a net Bode asymptote at 0 dB for the region. At $f = f_c$, one asymptote maintains its 0 dB level, whereas the other is dropping by 6 dB/octave. The sum of the two is the 6 dB drop per octave shown for the region. In region 3, the -6 dB/octave asymptote is balanced by the $+6$ dB/octave asymptote, establishing a level asymptote at the negative dB level attained by the f_c asymptote at $f = f_1$. The dB level of the horizontal asymptote in

region 3 can be determined by using Eq. (21.46) or by substituting $f = f_1$ into the asymptotic expression defined by f_c.

The full idealized Bode envelope is now defined, permitting a sketch of the actual response by shifting 3 dB in the right direction at each corner frequency, as shown in Fig. 21.65.

The phase angle associated with $\mathbf{A}_v$ can be determined directly from Eq. (21.45). That is,

$$\theta = \tan^{-1} f/f_1 - \tan^{-1} f/f_c \tag{21.47}$$

A full plot of θ versus frequency can be obtained by substituting various key frequencies into Eq. (21.47) and plotting the result on a log scale.

The first term of Eq. (21.47) defines the phase angle established by the numerator of Eq. (21.45). The asymptotic plot established by the numerator is provided in Fig. 21.66. Note the phase angle of 45° at $f = f_1$ and the straight-line asymptote between $f_1/10$ and $10f_1$.

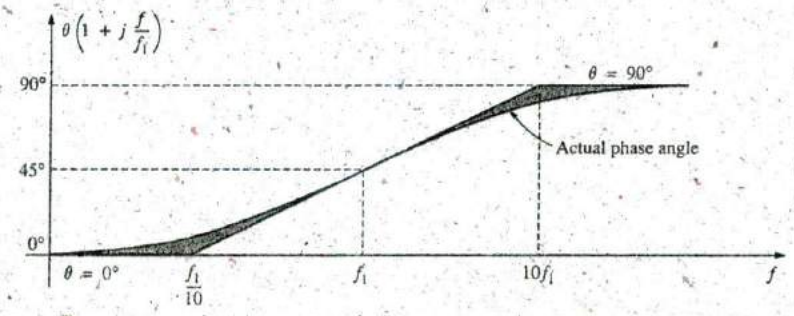


FIG. 21.66
Phase angle for $(1 + j\,(f/f_1))$.

Now that we have an asymptotic plot for the phase angle of the numerator, we can plot the full phase response by sketching the asymptotes for both functions of Eq. (21.45) on the same graph, as shown in Fig. 21.67.

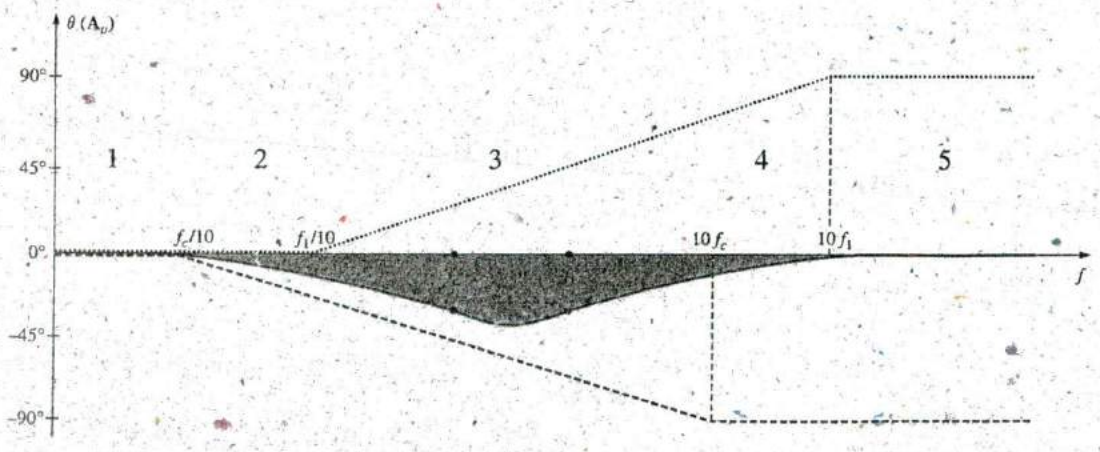


FIG. 21.67
Phase angle for the low-pass filter in Fig. 21.62.

The asymptotes in Fig. 21.67 clearly indicate that the phase angle will be 0° in the low-frequency range and 0° (90° − 90° = 0°) in the high-frequency range. In region 2, the phase plot drops below 0° due to the impact of the f_c asymptote. In region 4, the phase angle increases since the asymptote due to f_c remains fixed at −90°, whereas that due to f_1 is increasing. In the midrange, the plot due to f_1 is balancing the continued negative drop due to the f_c asymptote, resulting in the leveling response indicated. Due to the equal and opposite slopes of the asymptotes in the midregion, the angles of f_1 and f_c will be the same, but note that they are less than 45°. The maximum negative angle will occur between f_1 and f_c. The remaining points on the curve of Fig. 21.67 can be determined by simply substituting specific frequencies into Eq. (21.45). However, it is also useful to know that the most dramatic (the quickest) changes in the phase angle occur when the dB plot of the magnitude also goes through its greatest changes (such as at f_1 and f_c).

21.13 HIGH-PASS FILTER WITH LIMITED ATTENUATION

The filter in Fig. 21.68 is designed to limit the low-frequency attenuation in much the same manner as described for the low-pass filter of the previous section.

At $f = 0$ Hz, the capacitor can assume its open-circuit equivalence, and $\mathbf{V}_o = [R_2/(R_1 + R_2)]\mathbf{V}_i$. At high frequencies, the capacitor can be approximated by a short-circuit equivalence, and $\mathbf{V}_o = \mathbf{V}_i$.

The resistance to be used when determining f_c can be found by finding the Thévenin resistance for the capacitor C, as shown in Fig. 21.69. A careful examination of the resulting configuration reveals that $R_{Th} = R_1 \parallel R_2$ and $f_c = 1/2\pi(R_1 \parallel R_2)C$.

A plot of V_o versus frequency is provided in Fig. 21.70(a), and a sketch of $A_v = V_o/V_i$ appears in Fig. 21.70(b).

An equation for $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_i$ can be derived by first applying the voltage divider rule:

$$\mathbf{V}_o = \frac{R_2\mathbf{V}_i}{R_2 + R_1 \parallel -jX_C}$$

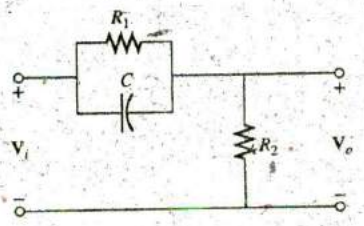


FIG. 21.68
High-pass filter with limited attenuation.

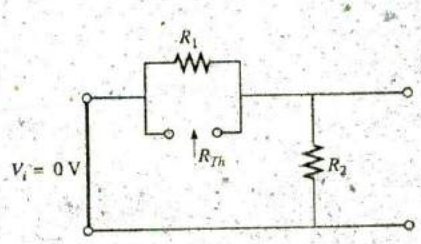


FIG. 21.69
Determining R for the f_c calculation for the filter in Fig. 21.68.

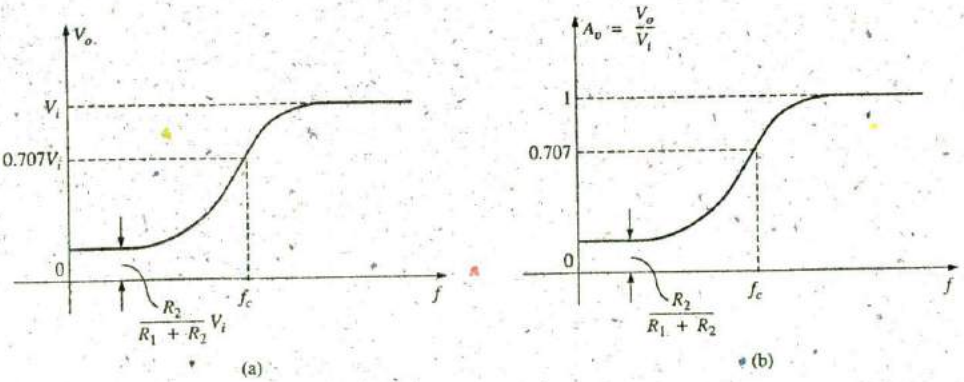


FIG. 21.70
High-pass filter with limited attenuation.

and
$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_2}{R_2 + R_1 \| -jX_C} = \frac{R_2}{R_2 + \dfrac{R_1(-jX_C)}{R_1 - jX_C}}$$
$$= \frac{R_2(R_1 - jX_C)}{R_2(R_1 - jX_C) - jR_1X_C} = \frac{R_1R_2 - jR_2X_C}{R_1R_2 - jR_2X_C - jR_1X_C}$$
$$= \frac{R_1R_2 - jR_2X_C}{R_1R_2 - j(R_1 + R_2)X_C} = \frac{1 - j\dfrac{R_2X_C}{R_1R_2}}{1 - j\dfrac{(R_1 + R_2)}{R_1R_2}X_C}$$
$$= \frac{1 - j\dfrac{X_C}{R_1}}{1 - j\dfrac{X_C}{\dfrac{R_1R_2}{R_1 + R_2}}} = \frac{1 - j\dfrac{X_C}{R_1}}{1 - j\dfrac{X_C}{R_1 \| R_2}} = \frac{1 - j\dfrac{1}{2\pi fR_1C}}{1 - j\dfrac{1}{2\pi f(R_1 \| R_2)C}}$$

so that
$$\boxed{\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1 - j(f_1/f)}{1 - j(f_c/f)}} \tag{21.48}$$

with
$$f_1 = \frac{1}{2\pi R_1C} \quad \text{and} \quad f_c = \frac{1}{2\pi(R_1 \| R_2)C}$$

The denominator of Eq. (21.48) is simply the denominator of the high-pass function in Fig. 21.54(b). The numerator, however, is new and must be investigated.

Applying Eq. (21.5) gives
$$A_{v_{dB}} = 20\log_{10}\frac{V_o}{V_i} = 20\log_{10}\sqrt{1 + (f_1/f)^2} + 20\log_{10}\frac{1}{\sqrt{1 + (f_c/f)^2}}$$

For $f << f_1$, $(f_1/f)^2 >> 1$, and the first term becomes
$$20\log_{10}\sqrt{(f_1/f)^2} = 20\log_{10}(f_1/f)|_{f<<f_1}$$
which defines the idealized Bode asymptote for the numerator of Eq. (21.48).

At $f = f_1$, $20\log_{10} 1 = 0$ dB
At $f = 0.5f_1$, $20\log_{10} 2 = 6$ dB
At $f = 0.1f_1$, $20\log_{10} 10 = 20$ dB

For frequencies greater than f_1, $f_1/f << 1$ and $20\log_{10} 1 = 0$ dB, which establishes the high-frequency asymptote. The full idealized Bode plot for the numerator of Eq. (21.48) is provided in Fig. 21.71.

We are now in a position to determine $A_{v_{dB}}$ by plotting the asymptotes for each function of Eq. (21.48) on the same frequency axis, as shown in Fig. 21.72. Note that f_c must be more than f_1 since $R_1 \| R_2$ must be less than R_1.

When determining the linearized Bode response, let us first examine region 2, where one function is 0 dB and the other is dropping at 6 dB/octave for decreasing frequencies. The result is a decreasing asymptote from f_c to f_1. At the intersection of the resultant of region 2 with f_1, we enter region 1, where the asymptotes have opposite slopes and cancel the effect of each other. The resulting level at f_1 is determined by $-20\log_{10}(R_1 + R_2)/R_2$, as found in earlier sections. The drop can also be determined by substituting $f = f_1$ into the asymptotic equation defined

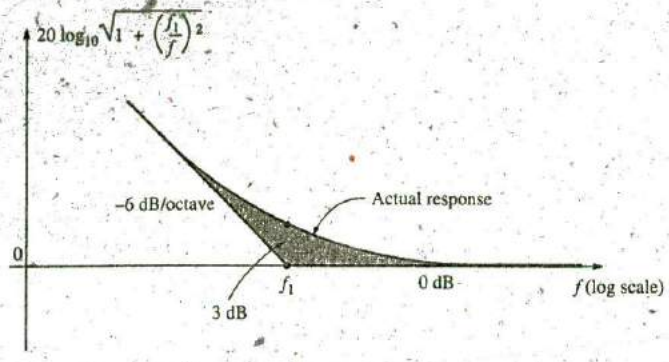


FIG. 21.71

Idealized and actual Bode response for the magnitude of $(1 - j(f_1/f))$.

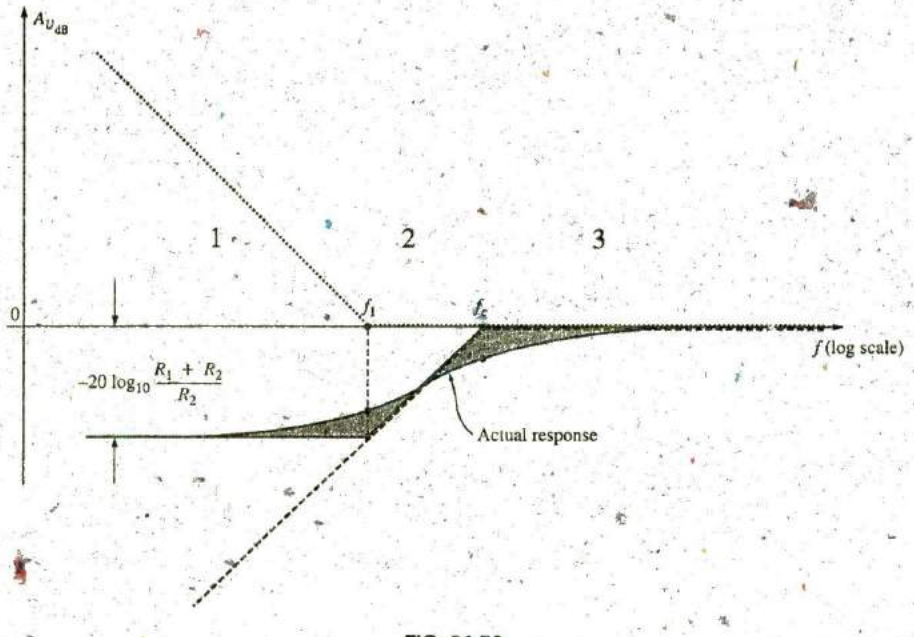


FIG. 21.72

$A_{v_{dB}}$ versus frequency for the high-pass filter with limited attenuation in Fig. 21.68.

for f_c. In region 3, both are at 0 dB, resulting in a 0 dB asymptote for the region. The resulting asymptotic and actual responses both appear in Fig. 21.72.

The phase angle associated with A_v can be determined directly from Eq. (21.48); that is,

$$\theta = -\tan^{-1}\frac{f_1}{f} + \tan^{-1}\frac{f_c}{f} \qquad (21.49)$$

A full plot of θ versus frequency can be obtained by substituting various key frequencies into Eq. (21.49) and plotting the result on a log scale.

The first term of Eq. (21.49) defines the phase angle established by the numerator of Eq. (21.48). The asymptotic plot resulting from the

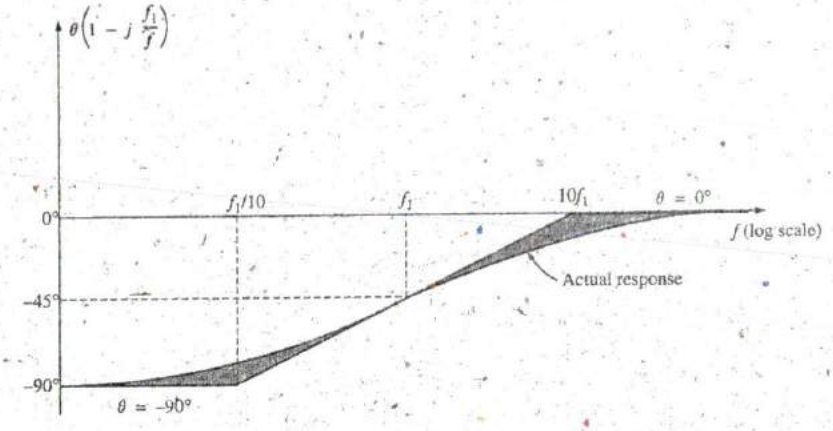


FIG. 21.73
Phase angle for $(1 - j(f_1/f))$.

numerator is provided in Fig. 21.73. Note the leading phase angle of 45° at $f = f_1$ and the straight-line asymptote from $f_1/10$ to $10f_1$.

Now that we have an asymptotic plot for the phase angle of the numerator, we can plot the full phase response by sketching the asymptotes for both functions of Eq. (21.48) on the same graph, as shown in Fig. 21.74.

The asymptotes in Fig. 21.74 clearly indicate that the phase angle will be 90° in the low-frequency range and 0° (90° − 90° = 0°) in the high-frequency range. In region 2, the phase angle is increasing above 0° because one angle is fixed at 90° and the other is becoming less negative. In region 4, one is 0° and the other is decreasing, resulting in a decreasing θ for this region. In region 3, the positive angle is always greater than the negative angle, resulting in a positive angle for the entire region. Since the slopes of the asymptotes in region 3 are equal but opposite, the angles at f_c and f_1 are the same. Fig. 21.74 reveals that the angle at f_c and f_1 will be less than 45°. The maximum angle occurs

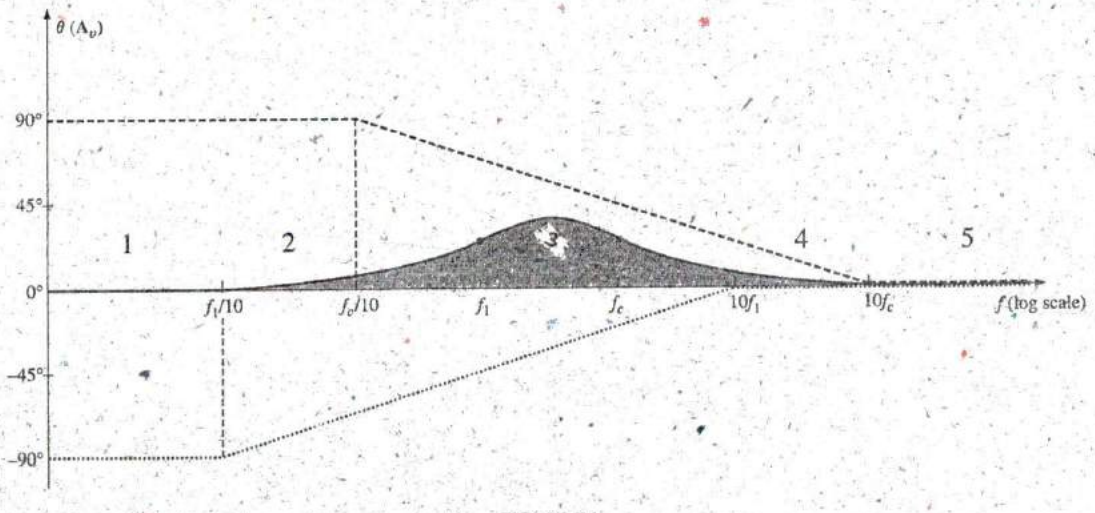


FIG. 21.74
Phase response for the high-pass filter in Fig. 21.68.

between f_c and f_1, as shown in the figure. Note again that the greatest change in θ occurs at the corner frequencies, matching the regions of greatest change in the dB plot.

EXAMPLE 21.11 For the filter in Fig. 21.75:

a. Sketch the curve of $A_{v_{dB}}$ versus frequency using a log scale.
b. Sketch the curve of θ versus frequency using a log scale.

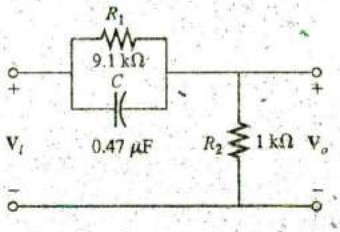


FIG. 21.75
Example 21.11.

Solutions:

a. For the break frequencies,

$$f_1 = \frac{1}{2\pi R_1 C} = \frac{1}{2\pi(9.1\text{ k}\Omega)(0.47\ \mu\text{F})} = \mathbf{37.2\text{ Hz}}$$

$$f_c = \frac{1}{2\pi\left(\dfrac{R_1R_2}{R_1 + R_2}\right)C} = \frac{1}{2\pi(0.9\text{ k}\Omega)(0.47\ \mu\text{F})} = \mathbf{376.25\text{ Hz}}$$

The maximum low-level attenuation is

$$-20\log_{10}\frac{R_1 + R_2}{R_2} = -20\log_{10}\frac{9.1\text{ k}\Omega + 1\text{ k}\Omega}{1\text{ k}\Omega}$$
$$= -20\log_{10} 10.1 = \mathbf{-20.09\text{ dB}}$$

The resulting plot appears in Fig. 21.76.

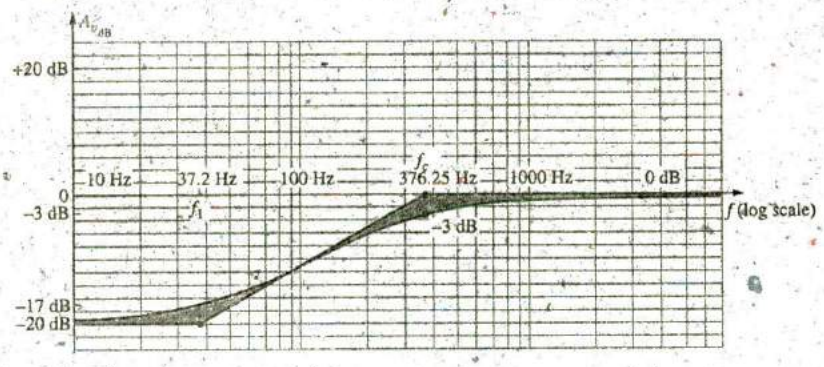


FIG. 21.76
$A_{v_{dB}}$ versus frequency for the filter in Fig. 21.75.

b. For the break frequencies:

At $f = f_1 = 37.2$ Hz,

$$\theta = -\tan^{-1}\frac{f_1}{f} + \tan^{-1}\frac{f_c}{f}$$
$$= -\tan^{-1} 1 + \tan^{-1}\frac{376.25\text{ Hz}}{37.2\text{ Hz}}$$
$$= -45° + 84.35°$$
$$= \mathbf{39.35°}$$

At $f = f_c = 376.26$ Hz,

$$\theta = -\tan^{-1}\frac{37.2\text{ Hz}}{376.26\text{ Hz}} + \tan^{-1}1$$
$$= -5.65° + 45°$$
$$= \mathbf{39.35°}$$

At a frequency midway between f_c and f_1 on a log scale, for example, 120 Hz,

$$\theta = -\tan^{-1}\frac{37.2\text{ Hz}}{120\text{ Hz}} + \tan^{-1}\frac{376.26\text{ Hz}}{120\text{ Hz}}$$
$$= -17.22° + 72.31°$$
$$= \mathbf{55.09°}$$

The resulting phase plot appears in Fig. 21.77.

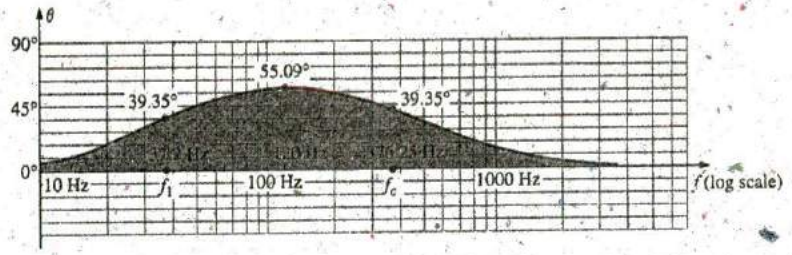


FIG. 21.77

θ (the phase angle associated with A_v) versus frequency for the filter in Fig. 21.75.

21.14 ADDITIONAL PROPERTIES OF BODE PLOTS

Bode plots are not limited to filters but can be applied to any system for which a dB-versus-frequency plot is desired. Although the previous sections did not cover all the functions that lend themselves to the idealized linear asymptotes, many of those most commonly encountered have been introduced.

We now examine some of the special situations that can develop that further demonstrate the adaptability and usefulness of the linear Bode approach to frequency analysis.

In all the situations described in this chapter, there was only one term in the numerator or denominator. For situations where there is more than one term, there will be an interaction between functions that must be examined and understood. In many cases, the use of Eq. (21.5) will prove useful. For example, if $\mathbf{A}_v$ has the format

$$\boxed{\mathbf{A}_v = \frac{200(1 - jf_2/f)(jf/f_1)}{(1 - jf_1/f)(1 + jf/f_2)} = \frac{(a)(b)(c)}{(d)(e)}} \qquad (21.50)$$

we can expand the function in the following manner:

$$A_{v_{dB}} = 20\log_{10}\frac{(a)(b)(c)}{(d)(e)}$$
$$= 20\log_{10}a + 20\log_{10}b + 20\log_{10}c - 20\log_{10}d - 20\log_{10}e$$

revealing that the net or resultant dB level is equal to the algebraic sum of the contributions from all the terms of the original function. We can, therefore, add algebraically the linearized Bode plots of all the terms in each frequency interval to determine the idealized Bode plot for the full function.

If two terms happen to have the same format and corner frequency, as in the function

$$A_v = \frac{1}{(1 - jf_1/f)(1 - jf_1/f)}$$

the function can be rewritten as

$$A_v = \frac{1}{(1 - jf_1/f)^2}$$

so that

$$A_{v_{dB}} = 20 \log_{10} \frac{1}{(\sqrt{1 + (f_1/f)^2})^2}$$
$$= -20 \log_{10}(1 + (f_1/f)^2)$$

for $f \ll f_1$, $(f_1/f)^2 \gg 1$, and

$$A_{v_{dB}} = -20 \log_{10}(f_1/f)^2 = -40 \log_{10} f_1/f$$

versus the $-20 \log_{10}(f_1/f)$ obtained for a single term in the denominator. The resulting dB asymptote will drop, therefore, at a rate of -12 dB/octave (-40 dB/decade) for decreasing frequencies rather than -6 dB/octave. The corner frequency is the same, and the high-frequency asymptote is still at 0 dB. The idealized Bode plot for the above function is provided in Fig. 21.78.

Note the steeper slope of the asymptote and the fact that the actual curve now passes -6 dB below the corner frequency rather than -3 dB, as for a single term.

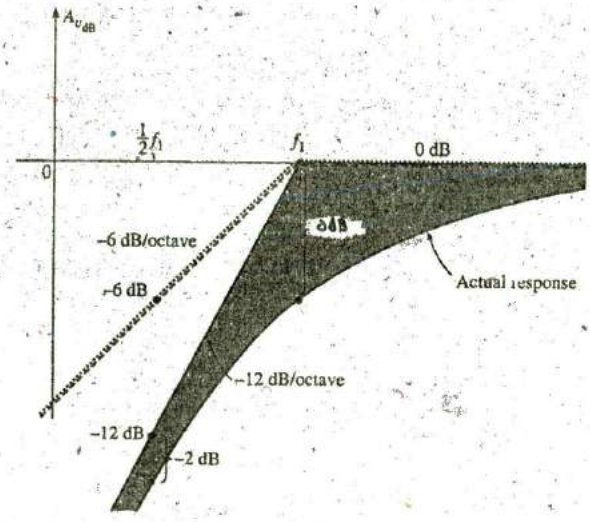


FIG. 21.78

Plotting the linearized Bode plot of $\frac{1}{(1 - j(f_1/f))^2}$

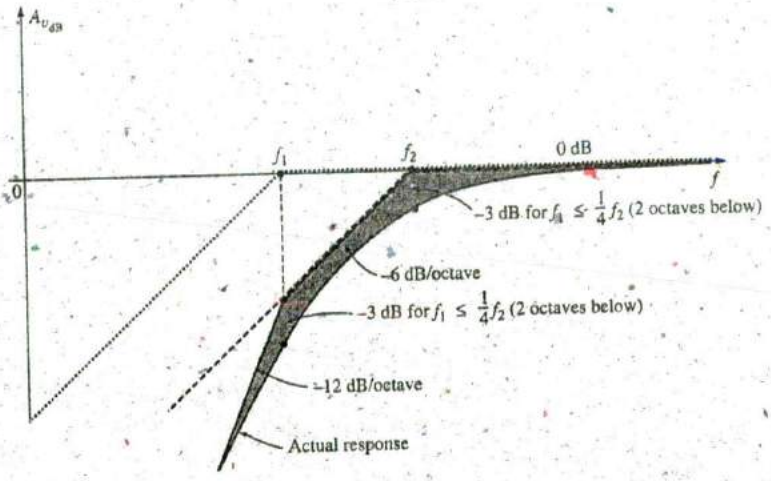


FIG. 21.79

Plot of $A_{v_{dB}}$ *for* $\frac{1}{(1 - j(f_1/f))(1 - j(f_2/f))}$ *with* $f_1 < f_2$.

Keep in mind that if the corner frequencies of the two terms in the numerator or denominator are close but not exactly equal, the total dB drop is the algebraic sum of the contributing terms of the expansion. For instance, consider the linearized Bode plot in Fig. 21.79 with corner frequencies f_1 and f_2.

In region 3, both asymptotes are 0 dB, resulting in an asymptote at 0 dB for frequencies greater than f_2. For region 2, one asymptote is at 0 dB, whereas the other drops at −6 dB/octave for decreasing frequencies. The net result for this region is an asymptote dropping at −6 dB, as shown in the same figure. At f_1, we find two asymptotes dropping off at −6 dB for decreasing frequencies. The result is an asymptote dropping off at −12 dB/octave for this region.

If f_1 and f_2 are at least two octaves apart, the effect of one on the plotting of the actual response for the other can almost be ignored. In other words, for this example, if $f_1 < \frac{1}{4}f_2$, the actual response is down −3 dB at $f = f_2$ and f_1.

The above discussion can be expanded for any number of terms at the same frequency or in the same region. For three equal terms in the denominator, the asymptote will drop at −18 dB/octave, and so on. Eventually, the procedure will become self-evident and relatively straightforward to apply. In many cases, the hardest part of finding a solution is to put the original function in the desired form.

EXAMPLE 21.12 A transistor amplifier has the following gain:

$$A_v = \frac{100}{\left(1 - j\frac{50\text{ Hz}}{f}\right)\left(1 - j\frac{200\text{ Hz}}{f}\right)\left(1 + j\frac{f}{10\text{ kHz}}\right)\left(1 + j\frac{f}{20\text{ kHz}}\right)}$$

a. Sketch the normalized response $A'_v = A_v/A_{v_{max}}$, and determine the bandwidth of the amplifier.

b. Sketch the phase response, and determine a frequency where the phase angle is close to 0°.

Solutions:

a. $A'_v = \dfrac{A_v}{A_{v_{max}}} = \dfrac{A_v}{100}$

$$= \frac{1}{\left(1 - j\frac{50\text{ Hz}}{f}\right)\left(1 - j\frac{200\text{ Hz}}{f}\right)\left(1 + j\frac{f}{10\text{ kHz}}\right)\left(1 + j\frac{f}{20\text{ kHz}}\right)}$$

$$= \frac{1}{(a)(b)(c)(d)} = \left(\frac{1}{a}\right)\left(\frac{1}{b}\right)\left(\frac{1}{c}\right)\left(\frac{1}{d}\right)$$

and

$$A'_{v_{dB}} = -20\log_{10}a - 20\log_{10}b - 20\log_{10}c - 20\log_{10}d$$

clearly substantiating the fact that the total number of decibels is equal to the algebraic sum of the contributing terms.

A careful examination of the original function reveals that the first two terms in the denominator are high-pass filter functions, whereas the last two are low-pass functions. Fig. 21.80 demonstrates how the combination of the two types of functions defines a bandwidth for the amplifier. The high-frequency filter functions have defined the low cutoff frequency, and the low-frequency filter functions have defined the high cutoff frequency.

Plotting all the idealized Bode plots on the same axis results in the plot in Fig. 21.81. Note for frequencies less than 50 Hz that the

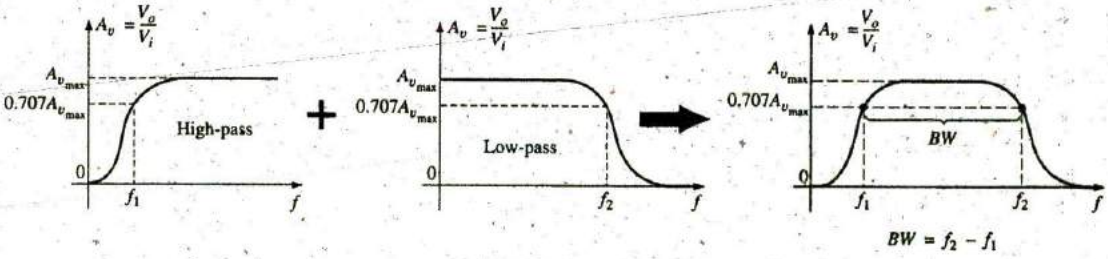


FIG. 21.80

Finding the overall gain versus frequency for Example 21.12.

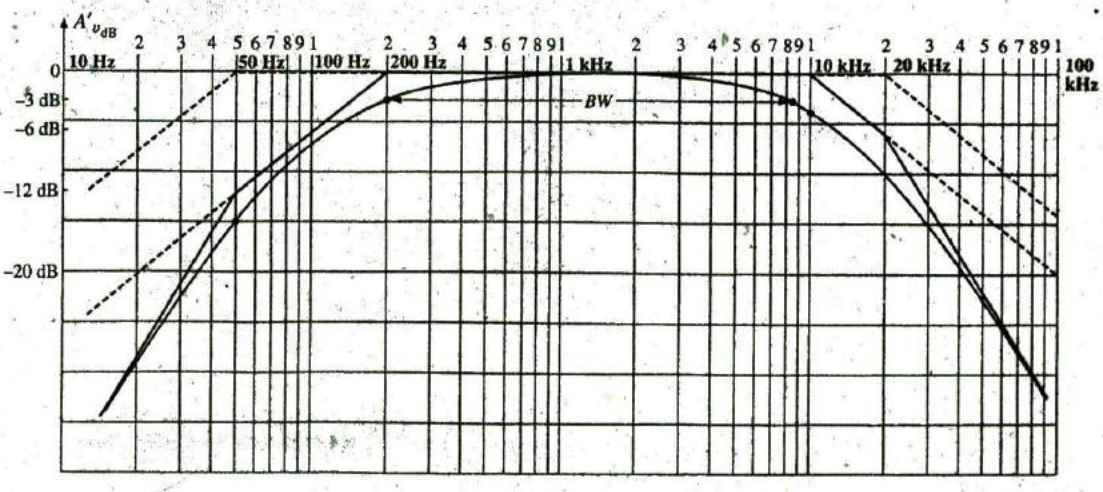


FIG. 21.81

$A'_{v_{dB}}$ versus frequency for Example 21.12.

resulting asymptote drops off at −12 dB/octave. In addition, since 50 Hz and 200 Hz are separated by two octaves, the actual response will be down by only about −3 dB at the corner frequencies of 50 Hz and 200 Hz.

For the high-frequency region, the corner frequencies are not separated by two octaves, and the difference between the idealized plot and the actual Bode response must be examined more carefully. Since 10 kHz is one octave below 20 kHz, we can use the fact that the difference between the idealized response and the actual response for a single corner frequency is 1 dB. If we add an additional −1 dB drop due to the 20 kHz corner frequency to the −3 dB drop at $f = 10$ kHz, we can conclude that the drop at 10 kHz will be −4 dB, as shown on the plot. To check the conclusion, let us write the full expression for the dB level at 10 kHz and find the actual level for comparison purposes.

$$A'_{v_{dB}} = -20\log_{10}\sqrt{1+\left(\frac{50\text{ Hz}}{10\text{ kHz}}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{200\text{ Hz}}{10\text{ kHz}}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{10\text{ kHz}}{10\text{ kHz}}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{10\text{ kHz}}{20\text{ kHz}}\right)^2}$$
$$= -0.00011\text{ dB} - 0.0017\text{ dB} - 3.01\text{ dB} - 0.969\text{ dB}$$
$$= -3.98\text{dB} \cong \mathbf{-4\text{ dB}} \quad \text{as before}$$

An examination of the above calculations reveals that the last two terms predominate in the high-frequency region and essentially eliminate the need to consider the first two terms in that region. For the low-frequency region, examining the first two terms is sufficient.

Proceeding in a similar fashion, we find a −4 dB difference at $f =$ 20 kHz, resulting in the actual response appearing in Fig. 21.81. Since the bandwidth is defined at the −3 dB level, a judgment must be made as to where the actual response crosses the −3 dB level in the high-frequency region. A rough sketch suggests that it is near 8.5 kHz. Plugging this frequency into the high-frequency terms results in

$$A'_{v_{dB}} = -20\log_{10}\sqrt{1+\left(\frac{8.5\text{ kHz}}{10\text{ kHz}}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{8.5\text{ kHz}}{20\text{ kHz}}\right)^2}$$
$$= -2.148\text{ dB} - 0.645\text{ dB} \cong \mathbf{-2.8\text{ dB}}$$

which is relatively close to the −3 dB level, and

$$BW = f_{high} - f_{low} = 8.5\text{ kHz} - 200\text{ Hz} = \mathbf{8.3\text{ kHz}}$$

In the midrange of the bandwidth, $A'_{v_{dB}}$ approaches 0 dB. At $f =$ 1 kHz:

$$A'_{v_{dB}} = -20\log_{10}\sqrt{1+\left(\frac{50\text{ Hz}}{1\text{ kHz}}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{200\text{ Hz}}{1\text{ kHz}}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{1\text{ kHz}}{10\text{ kHz}}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{1\text{ kHz}}{20\text{ kHz}}\right)^2}$$
$$= -0.0108\text{ dB} - 0.1703\text{ dB} - 0.0432\text{ dB} - 0.0108\text{ dB}$$
$$= \mathbf{-0.235\text{ dB}} \cong \mathbf{-\frac{1}{5}\text{ dB}}$$

which is certainly close to the 0 dB level, as shown on the plot.

b. The phase response can be determined by substituting a number of key frequencies into the following equation, derived directly from the original function $\mathbf{A}_v$:

$$\theta = \tan^{-1}\frac{50\text{ Hz}}{f} + \tan^{-1}\frac{200\text{ Hz}}{f} - \tan^{-1}\frac{f}{10\text{ kHz}} - \tan^{-1}\frac{f}{20\text{ kHz}}$$

However, let us make full use of the asymptotes defined by each term of $\mathbf{A}_v$ and sketch the response by finding the resulting phase angle at critical points on the frequency axis. The resulting asymptotes and phase plot are provided in Fig. 21.82. Note that at $f = 50$ Hz, the sum of the two angles determined by the straight-line asymptotes is $45° + 75° = 120°$ (actual = 121°). At $f = 1$ kHz, if we subtract 5.7° for one corner frequency, we obtain a net angle of $14° - 5.7° \cong 8.3°$ (actual = 5.6°).

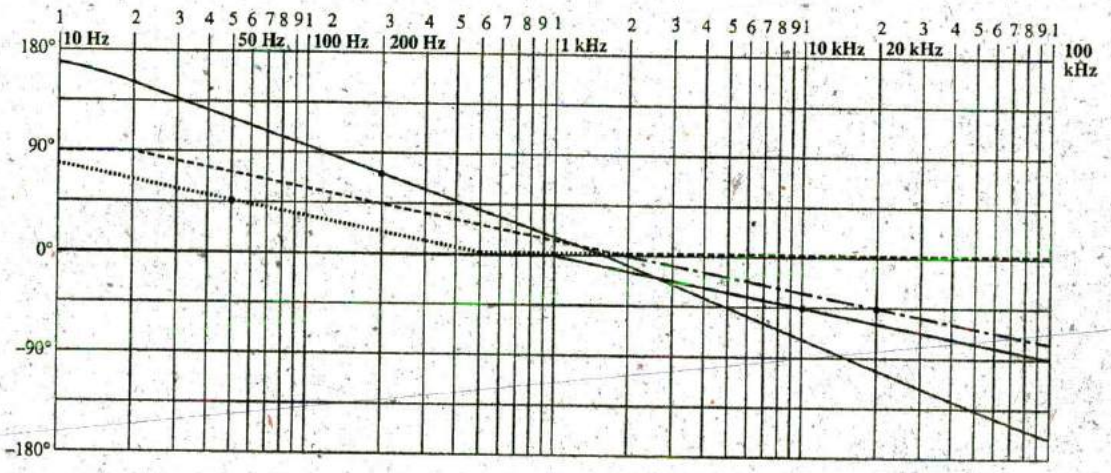


FIG. 21.82
Phase response for Example 21.12.

At 10 kHz, the asymptotes leave us with $\theta \cong -45° - 32° = -77°$ (actual = −71.56°). The net phase plot appears to be close to 0° at about 1300 Hz. To check on our assumptions and the use of the asymptotic approach, plug $f = 1300$ Hz into the equation for θ:

$$\theta = \tan^{-1}\frac{50\text{ Hz}}{1300\text{ Hz}} + \tan^{-1}\frac{200\text{ Hz}}{1300\text{ Hz}} - \tan^{-1}\frac{1300\text{ Hz}}{10\text{ kHz}} - \tan^{-1}\frac{1300\text{ Hz}}{20\text{ kHz}}$$
$$= 2.2° + 8.75° - 7.41° - 3.72°$$
$$= -0.18° \cong 0° \quad \text{as predicted}$$

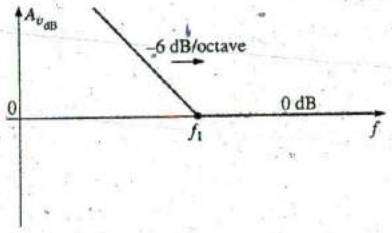
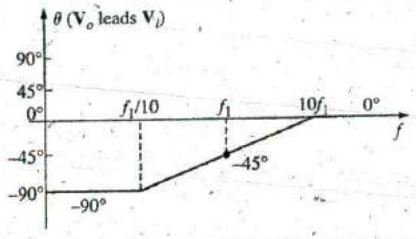
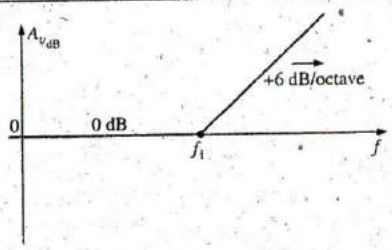
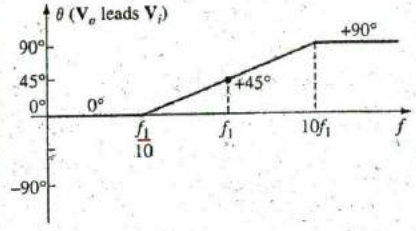
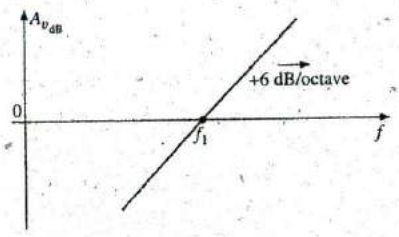
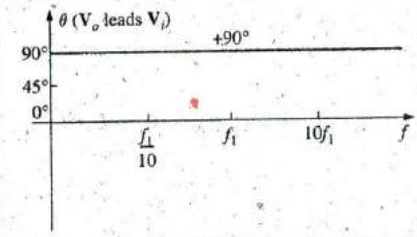
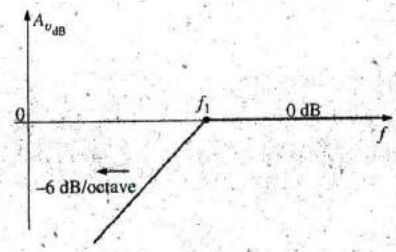
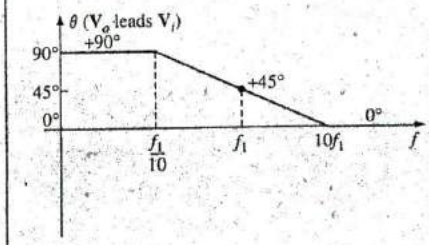
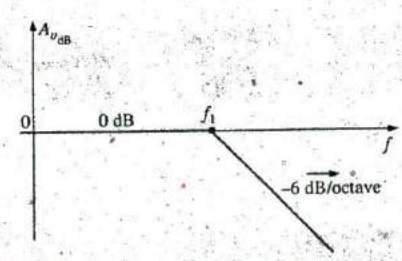
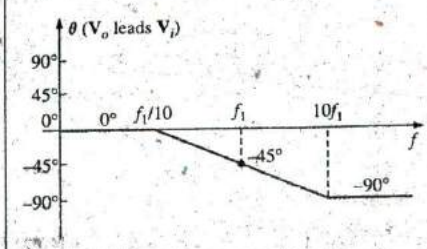
In total, the phase plot appears to shift from a positive angle of 180° ($\mathbf{V}_o$ leading $\mathbf{V}_i$) to a negative angle of 180° as the frequency spectrum extends from very low frequencies to high frequencies. In the midregion, the phase plot is close to 0° ($\mathbf{V}_o$ in phase with $\mathbf{V}_i$), much like the response to a common-base transistor amplifier.

Table 21.3 consolidates some of the material introduced in this chapter and provides a reference for future investigations. It includes the linearized dB and phase plots for the functions appearing in the first column. There are many other functions, but these provide a foundation onto which others can be added.

Reviewing the development of the filters in Sections 21.12 and 21.13 shows that establishing the function $\mathbf{A}_v$ in the proper form is the most

TABLE 21.3

Idealized Bode plots for various functions.

Function	dB Plot	Phase Plot
$\mathbf{A}_v = 1 - j\frac{f_1}{f}$		
$\mathbf{A}_v = 1 + j\frac{f}{f_1}$		
$\mathbf{A}_v = j\frac{f}{f_1}$		
$\mathbf{A}_v = \frac{1}{1 - j\frac{f_1}{f}}$		
$\mathbf{A}_v = \frac{1}{1 + j\frac{f}{f_1}}$		

difficult part of the analysis. However, with practice and an awareness of the desired format, you will discover methods that will significantly reduce the effort involved.

21.15 CROSSOVER NETWORKS

The topic of *crossover networks* is included primarily to present an excellent demonstration of filter operation without a high level of complexity. Crossover networks are used in audio systems to ensure that the proper frequencies are channeled to the appropriate speaker. Although less expensive audio systems have only one speaker to cover the full audio range from about 20 Hz to 20 kHz, better systems have at least three speakers to cover the low range (20 Hz to about 500 Hz), the midrange (500 Hz to about 5 kHz), and the high range (5 kHz and up). The term *crossover* comes from the fact that the system is designed to have a crossover of frequency spectrums for adjacent speakers at the -3 dB level, as shown in Fig. 21.83. Depending on the design, each filter can drop off at 6 dB, 12 dB, or 18 dB, with complexity increasing with the desired dB drop-off rate. The three-way crossover network in Fig. 21.83 is quite simple in design, with a low-pass *R-L* filter for the *woofer*, an *R-L-C* pass-band filter for the midrange, and a high-pass *R-C* filter for the *tweeter*. The basic equations for the components are provided below. Note the similarity between the equations, with the only difference for each type of element being the cutoff frequency. We have

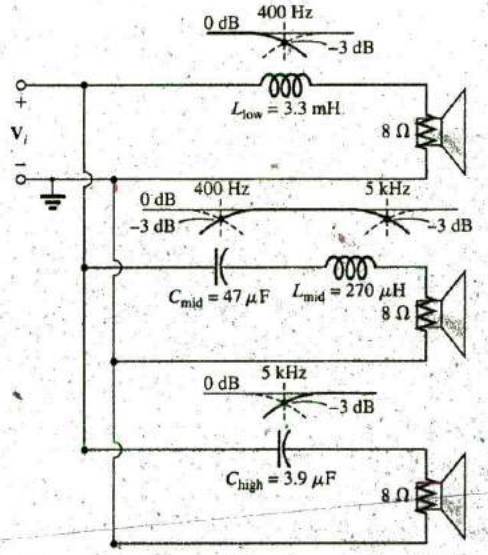


FIG. 21.83
Three-way, crossover network with 6 dB per octave.

$$\boxed{L_{low} = \frac{R}{2\pi f_1} \qquad L_{mid} = \frac{R}{2\pi f_2}} \tag{21.51}$$

$$\boxed{C_{mid} = \frac{1}{2\pi f_1 R} \qquad C_{high} = \frac{1}{2\pi f_2 R}} \tag{21.52}$$

For the crossover network in Fig. 21.83 with three 8 Ω speakers, the resulting values are

$$L_{low} = \frac{R}{2\pi f_1} = \frac{8\ \Omega}{2\pi(400\ \text{Hz})} = 3.183\ \text{mH} \rightarrow 3.3\ \text{mH}$$

(commercial value)

$$L_{mid} = \frac{R}{2\pi f_2} = \frac{8\ \Omega}{2\pi(5\ \text{kHz})} = 254.65\ \mu\text{H} \rightarrow 270\ \mu\text{H}$$

(commercial value)

$$C_{mid} = \frac{1}{2\pi f_1 R} = \frac{1}{2\pi(400\ \text{Hz})(8\ \Omega)} = 49.736\ \mu\text{F} \rightarrow 47\ \mu\text{F}$$

(commercial value)

$$C_{high} = \frac{1}{2\pi f_2 R} = \frac{1}{2\pi(5\ \text{kHz})(8\ \Omega)} = 3.979\ \mu\text{F} \rightarrow 3.9\ \mu\text{F}$$

(commercial value)

as shown in Fig. 21.83.

For each filter, a rough sketch of the frequency response is included to show the crossover at the specific frequencies of interest. Because all three speakers are in parallel, the source voltage and impedance for each are the same. The total loading on the source is obviously a function of

the frequency applied, but the total delivered is determined solely by the speakers since they are essentially resistive in nature.

To test the system, let us apply a 4 V signal at a frequency of 1 kHz (a predominant frequency of the typical human auditory response curve) and see which speaker has the highest power level.

At $f = 1$ kHz,

$$X_{L_{low}} = 2\pi f L_{low} = 2\pi(1\text{ kHz})(3.3\text{ mH}) = 20.74\ \Omega$$

$$\mathbf{V}_o = \frac{(\mathbf{Z}_R \angle 0°)(V_i \angle 0°)}{\mathbf{Z}_T} = \frac{(8\ \Omega \angle 0°)(4\text{ V} \angle 0°)}{8\ \Omega + j\,20.74\ \Omega}$$

$$= 1.44\text{ V} \angle -68.90°$$

$$X_{L_{mid}} = 2\pi f L_{mid} = 2\pi(1\text{ kHz})(270\ \mu\text{H}) = 1.696\ \Omega$$

$$X_{C_{mid}} = \frac{1}{2\pi f C_{mid}} = \frac{1}{2\pi(1\text{ kHz})(47\ \mu\text{F})} = 3.386\ \Omega$$

$$\mathbf{V}_o = \frac{(\mathbf{Z}_R \angle 0°)(V_i \angle 0°)}{\mathbf{Z}_T} = \frac{(8\ \Omega \angle 0°)(4\text{ V} \angle 0°)}{8\ \Omega + j\,1.696\ \Omega - j\,3.386\ \Omega}$$

$$= 3.94\text{ V} \angle 11.93°$$

$$X_{C_{high}} = \frac{1}{2\pi f C_{high}} = \frac{1}{2\pi(1\text{ kHz})(3.9\ \mu\text{F})} = 40.81\ \Omega$$

$$\mathbf{V}_o = \frac{(\mathbf{Z}_R \angle 0°)(V_i \angle 0°)}{\mathbf{Z}_T} = \frac{(8\ \Omega \angle 0°)(4\text{ V} \angle 0°)}{8\ \Omega - j\,40.81\ \Omega}$$

$$= 0.77\text{ V} \angle 78.91°$$

Using the basic power equation $P = V^2/R$, we find the power to the woofer,

$$P_{low} = \frac{V^2}{R} = \frac{(1.44\text{V})^2}{8\ \Omega} = \mathbf{0.259\ W}$$

to the midrange speaker,

$$P_{mid} = \frac{V^2}{R} = \frac{(3.94\text{ V})^2}{8\ \Omega} = \mathbf{1.94\ W}$$

and to the tweeter,

$$P_{high} = \frac{V^2}{R} = \frac{(0.77\text{ V})^2}{8\ \Omega} = \mathbf{0.074\ W}$$

resulting in a power ratio of 7.5 : 1 between the midrange and the woofer and 26 : 1 between the midrange and the tweeter. Obviously, the response of the midrange speaker totally overshadows that of the other two.

21.16 APPLICATIONS

Attenuators

Attenuators are, by definition, any device or system that can reduce the power or voltage level of a signal while introducing little or no distortion. There are two general types: passive and active. The passive type uses only resistors, while the active type uses electronic devices such as transistors and integrated circuits. Since electronics is a subject for the courses to follow, only the resistive type is covered here. Attenuators are commonly used in audio equipment (such as the graphic and parametric equalizers introduced in Chapter 20), antenna systems, AM or FM systems where attenuation may be required before the signals are mixed, and any other application where a reduction in signal strength is required.

The unit in Fig. 21.84 has coaxial input and output terminals and switches to set the level of dB reduction. It has a flat response from dc to about 6 GHz, which essentially means that its introduction into the network will not affect the frequency response for this band of frequencies. The design is rather simple, with resistors connected in either a *tee* (T) or a *wye* (Y) configuration, as shown in Figs. 21.85 and 21.86, respectively, for a 50 Ω coaxial system. In each case, the resistors are chosen to ensure that the input impedance and output impedance match the line. That is, the input and output impedances of each configuration will be 50 Ω. For a number of dB attenuations, the resistor values for the T and Y are provided in Figs. 21.85 and 21.86. Note in each design that two of the resistors are the same, while the third is a much smaller or larger value.

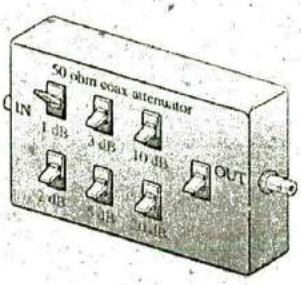
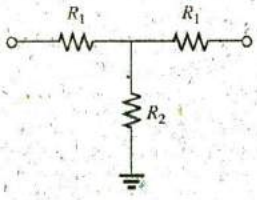
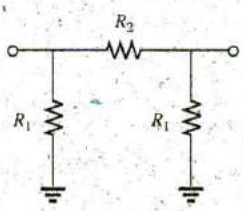


FIG. 21.84
Passive coax attenuator.



Attenuation	R_1	R_2
1 dB	2.9 Ω	433.3 Ω
2 dB	5.7 Ω	215.2 Ω
3 dB	8.5 Ω	141.9 Ω
5 dB	14.0 Ω	82.2 Ω
10 dB	26.0 Ω	35.0 Ω
20 dB	41.0 Ω	10.0 Ω

FIG. 21.85
Tee (T) configuration.

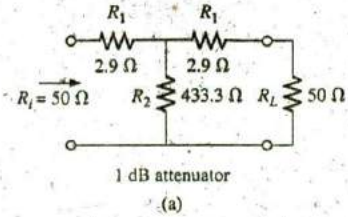


Attenuation	R_1	R_2
1 dB	870.0 Ω	5.8 Ω
2 dB	436.0 Ω	11.6 Ω
3 dB	292.0 Ω	17.6 Ω
5 dB	178.6 Ω	30.4 Ω
10 dB	96.2 Ω	71.2 Ω
20 dB	61.0 Ω	247.5 Ω

FIG. 21.86
Wye (Y) configuration.

For the 1 dB attenuation, the resistor values were inserted for the T configuration in Fig. 21.87(a). Terminating the configuration with a 50 Ω load, we find through the following calculations that the input impedance is, in fact, 50 Ω:

$$R_i = R_1 + R_2 \| (R_1 + R_L) = 2.9\ \Omega + 433.3\ \Omega \| (2.9\ \Omega + 50\ \Omega)$$
$$= 2.9\ \Omega + 47.14\ \Omega$$
$$= \mathbf{50.04\ \Omega}$$



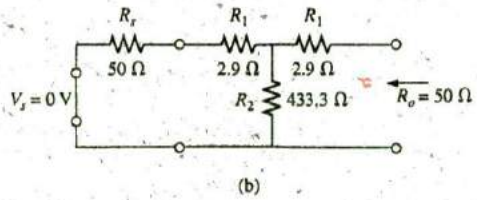


FIG. 21.87
1 dB attenuator: (a) loaded; (b) finding R_o.

Looking back from the load as shown in Fig. 21.87(b) with the source set to zero volts, we find through the following calculations that the output impedance is also 50 Ω:

$$R_o = R_1 + R_2 \parallel (R_1 + R_s) = 2.9\,\Omega + 433.3\,\Omega \parallel (2.9\,\Omega + 50\,\Omega)$$
$$= 2.9\,\Omega + 47.14\,\Omega$$
$$= \mathbf{50.04\,\Omega}$$

In Fig. 21.88, a 50 Ω load has been applied, and the output voltage is determined as follows:

$$R' = R_2 \parallel (R_1 + R_L) = 47.14\,\Omega \quad \text{from above}$$

and
$$V_{R_2} = \frac{R'V_s}{R' + R_1} = \frac{47.14\,\Omega V_s}{47.14\,\Omega + 2.9\,\Omega} = 0.942V_s$$

with
$$V_L = \frac{R_L V_{R_2}}{R_L + R_1} = \frac{50\,\Omega(0.942V_s)}{50\,\Omega + 2.9\,\Omega} = 0.890V_s$$

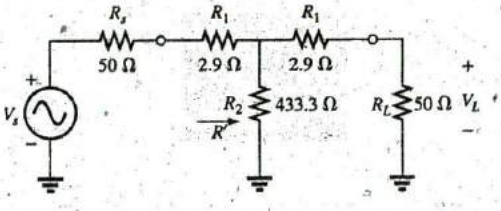


FIG. 21.88
Determining the voltage levels for the 1 dB attenuator in Fig. 21.87(a).

Calculating the drop in dB results in the following:

$$A_{v_{dB}} = 20\log_{10}\frac{V_L}{V_s} = 20\log_{10}\frac{0.890V_s}{V_s}$$
$$= 20\log_{10}0.890 = \mathbf{-1.01\ dB}$$

substantiating the fact that there is a 1 dB attenuation.

As mentioned earlier, there are other methods for attenuation that are more sophisticated in design and beyond the scope of the coverage of this text. However, the above designs are quite effective, relatively inexpensive, and perform quite well.

Noise Filters

Noise is a problem that can occur in any electronic system. In general, the presence of any unwanted signal can affect the overall operation of a system. It can come from a power source (60 Hz hum), from feedback networks, from mechanical systems connected to electrical systems, from stray capacitive and inductive effects, or possibly from a local signal source that is not properly shielded—the list is endless. To solve a noise problem, an analyst needs a broad practical background, a sense for the origin for the unwanted noise, and the ability to remove it in the simplest, most direct way. Generally noise problems arise during the testing phase, not during the original design phase. Although sophisticated methods may be needed, most situations are resolved simply by rearranging an element or two of a value sensitive to the problem.

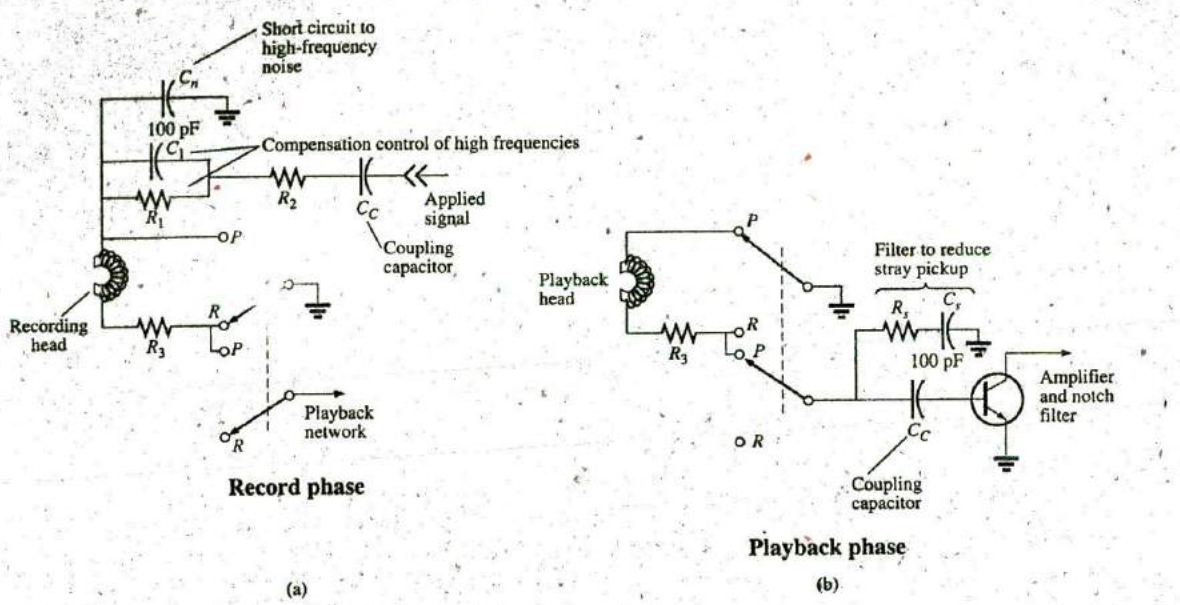


FIG. 21.89
Noise reduction in a tape recorder.

In Fig. 21.89, two capacitors have been strategically placed in the tape recording and playback sections of a tape recorder to remove the undesirable high-frequency noise (rushing sound) that can result from unexpected, randomly placed particles on a magnetic tape, noise coming down the line, or noise introduced from the local environment. During the record mode, with the switches in the positions shown (R), the 100 pF capacitor at the top of the schematic acts as a short circuit to the high-frequency noise. The capacitor C_1 is included to compensate for the fact that recording on a tape is not a linear process versus frequency. In other words, certain frequencies are recorded at higher amplitudes than others.

Fig. 21.90 is a sketch of recording level versus frequency, clearly indicating that the human audio range of about 40 Hz to 20 kHz is very poor for the tape recording process, starting to rise only after 20 kHz. Thus, tape recorders must include a fixed biasing frequency that, when added to the actual audio signal, brings the frequency range to be amplified to the region of high-amplitude recording. On some tapes, the *actual*

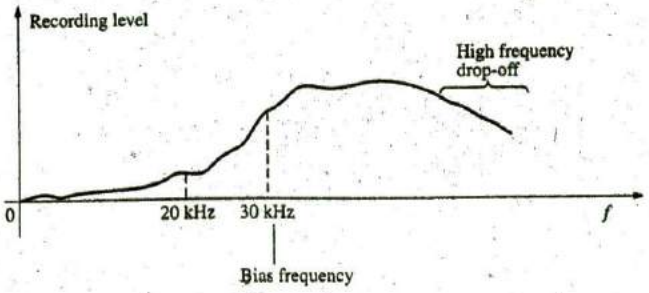


FIG. 21.90
Noise reduction in a tape recorder.

bias frequency is provided, while on others, the phrase *normal bias* is used. Even after you pass the bias frequency, there is a frequency range that follows that drops off considerably. Compensation for this drop-off is provided by the parallel combination of the resistor R_1 and the capacitor C_1 mentioned above. At frequencies near the bias frequency, the capacitor is designed to act essentially like an open circuit (high reactance), and the head current and voltage are limited by the resistors R_1 and R_2. At frequencies in the region where the tape gain drops off with frequency, the capacitor begins to take on a lower reactance level and reduce the net impedance across the parallel branch of R_1 and C_1. The result is an increase in head current and voltage due to the lower net impedance in the line, resulting in a leveling in the tape gain following the bias frequency. Eventually, the capacitor begins to take on the characteristics of a short circuit, effectively shorting out the resistance R_1, and the head current and voltage will be a maximum. During playback, this bias frequency is eliminated by a notch filter so that the original sound is not distorted by the high-frequency signal.

During playback (*P*), the upper circuit in Fig. 21.89 is set to ground by the upper switch, and the lower network comes into play. Again note the second 100 pF capacitor connected to the base of the transistor to short to ground any undesirable high-frequency noise. The resistor is there to absorb any power associated with the noise signal when the capacitor takes on its short-circuit equivalence. Keep in mind that the capacitor was chosen to act as a short-circuit equivalent for a particular frequency range and not for the audio range, where it is essentially an open circuit.

Alternators in a car are notorious for developing high-frequency noise down the line to the radio, as shown in Fig. 21.91(a). This problem is usually alleviated by placing a high-frequency filter in the line as shown. The inductor of 1 H offers a high impedance for the range of noise frequencies, while the capacitor (1000 μF to 47,000 μF) acts as a short-circuit equivalent to any noise that happens to get through. For the speaker system in Fig. 21.91(b), the push-pull power arrangement of transistors in the output section can often develop a short period of time between pulses where the strong signal voltage is zero volts. During this

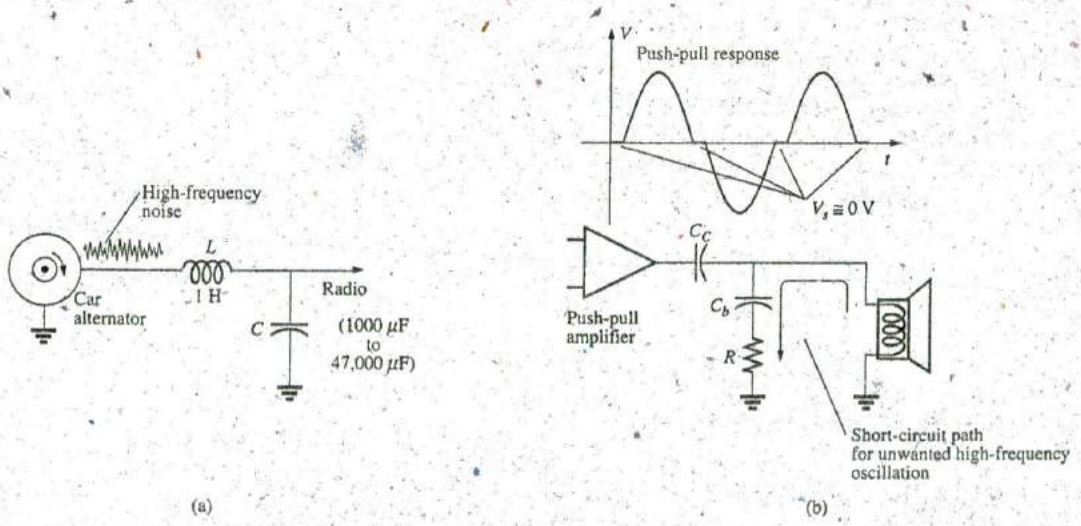


FIG. 21.91

Noise generation: (a) due to a car alternator; (b) from a push-pull amplifier.

short period, the coil of the speaker rears its inductive effects, sees an unexpected path to ground like a switch opening, and quickly cuts off the speaker current. Through the familiar relationship $v_L = L(di_L/dt)$, an unexpected voltage develops across the coil and sets a high-frequency oscillation on the line that finds its way back to the amplifier and causes further distortion. This effect can be subdued by placing an *R-C* path to ground that offers a low-resistance path from the speaker to ground for a range of frequencies typically generated by this signal distortion. Since the capacitor assumes a short-circuit equivalence for the range of noise disturbance, the resistor was added to limit the current and absorb the energy associated with the signal noise.

In regulators, such as the 5 V regulator in Fig. 21.92(a), when a spike in current comes down the line for any number of reasons, there is a voltage drop along the line, and the input voltage to the regulator drops. The regulator, performing its primary function, senses this drop in input voltage and increases its amplification level through a feedback loop to maintain a constant output. However, the spike is of such short duration that the output voltage has a spike of its own because the input voltage has quickly returned to its normal level, and with the increased amplification the output jumps to a higher level. Then the regulator senses its error and quickly cuts its gain. The sensitivity to changes in the input level has caused the output level to go through a number of quick oscillations that can be a real problem for the equipment to which the dc voltage is applied: A high-frequency noise signal has been developed. One way to subdue this reaction and, in fact, slow the system response down so that very short interval spikes have less impact is to add a capacitor across the output as shown in Fig. 21.92(b). Since the regulator is providing a fixed dc level, a large capacitor of 1 μF can be used to short-circuit a wide range of high-frequency disturbances. However, you don't want to make the capacitor too large or you'll get too much *damping*, and large overshoots and undershoots can develop. To maximize the input of the added capacitor, you must place it physically closer to the regulator to ensure that noise is not picked up between the regulator and capacitor and to avoid developing any delay time between output signal and capacitive reaction.

In general, as you examine the schematic of working systems and see elements that don't appear to be part of any standard design procedure, you can assume that they are either protective devices or due to noise on the line that is affecting the operation of the system. Noting their type, value, and location often reveals their purpose and modus operandi.

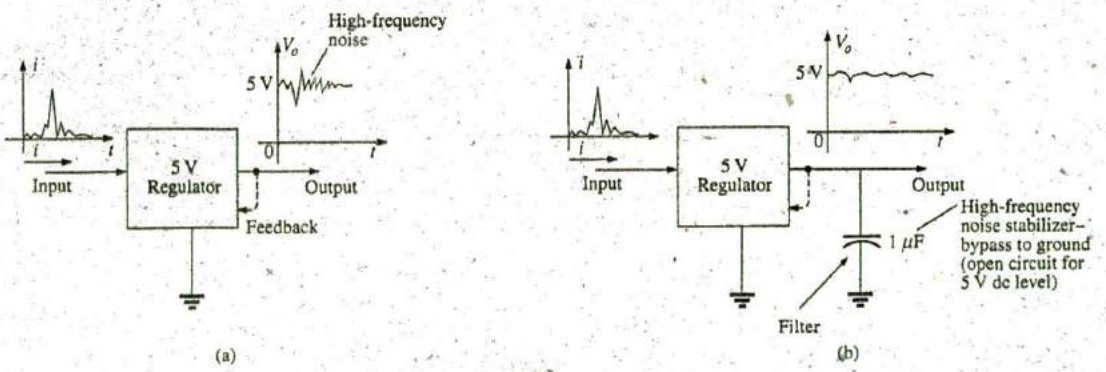


FIG. 21.92

Regulator: (a) effect of spike in current on the input side; (b) noise reduction.

21.17 COMPUTER ANALYSIS

PSpice

Double-Tuned Filter Our analysis now turns to a fairly complex-looking filter for which an enormous amount of time would be required to generate a detailed plot of gain versus frequency using a handheld calculator. It is the same filter examined in Example 21.9, so we have a chance to test our theoretical solution. The schematic appears in Fig. 21.93 with **VAC** again chosen since the frequency range of interest is set by the **Simulation Profile.** Again, the attributes for the source are set in the **Property Editor** box rather than by selecting the components from the screen. Note the need for the two resistors in series with the inductors since inductors cannot be considered ideal elements. The small value of the resistive elements, however, has no effect on the results obtained.

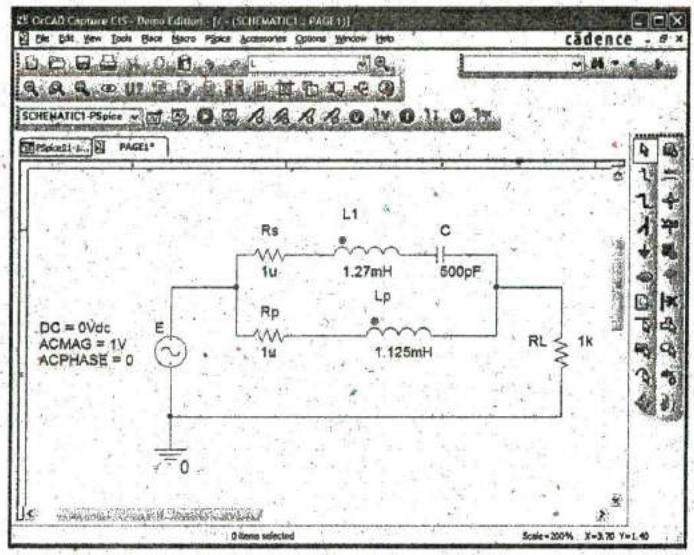


FIG. 21.93
Using PSpice to analyze a double-tuned filter.

In the **Simulation Settings** dialog box, select **AC Sweep** again with a **Start Frequency** of 100 Hz and an **End Frequency** of 10 MHz (be sure to enter this value as **10MEGHZ**) to ensure that the full-range effect is provided. Then use the axis controls to close in on the desired plot. The **Points/Decade** remains at 10k, although with this range of frequencies it may take a few seconds to simulate. Once the **SCHEMATIC1** appears, **Trace-Add Trace-V(RL:1)-OK** results in the plot in Fig. 21.94. Quite obviously, there is a reject-band around 200 kHz and a pass-band around 600 kHz. Note that up to 10 kHz, there is another pass-band as the inductor L_p provides an almost direct path of low impedance from input to output. At frequencies approaching 10 MHz, there is a continous stop-band due to the open-circuit equivalence of the L_p inductor. Using the cursor option, place the left-click cursor on the minimum point of the graph by using the **Cursor Trough** keypad (the second pad to the right of the **Toggle cursor** pad). Right-click to identify the frequency of the maximum point on the curve near 600 kHz. The results appearing in the **Probe Cursor** box clearly support our theoretical calculations of 200 kHz for the band-stop minimum (**A1** = 201.05 kHz with a magnitude

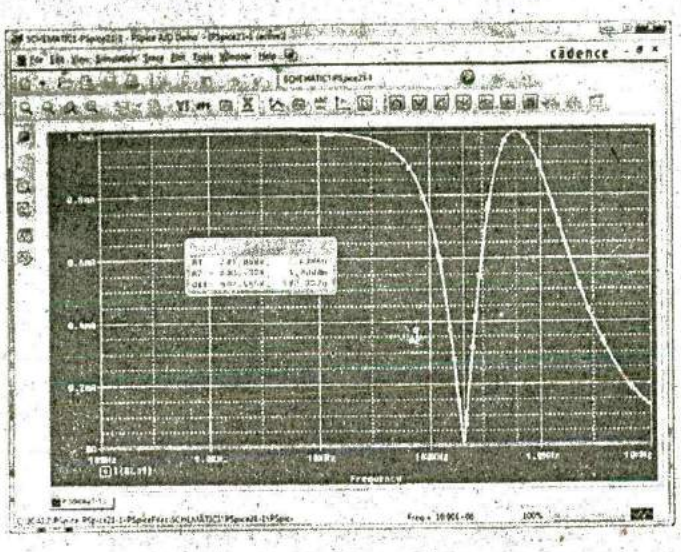


FIG. 21.94

Magnitude plot versus frequency for the voltage across R_L of the network in Fig. 21.93.

of essentially 0 V) and 600 kHz for the pass-band maximum (**A2** = 603.53 kHz with a magnitude of 1 V).

Let us now concentrate on the range from 10 kHz to 1 MHz, where most of the filtering action is taking place. That was the advantage of choosing such a wide range of frequencies when the **Simulation Settings** were set up. The data have been established for the broad range of frequencies, and you can simply select a band of interest once the region of most activity is defined. If the frequency range were too narrow in the original simulation, another simulation would have to be defined. Select **Plot-Axis Settings-X Axis-User Defined-10kHz to 1MEGHz-OK** to obtain the plot at the bottom of Fig. 21.95. A dB plot of the results can

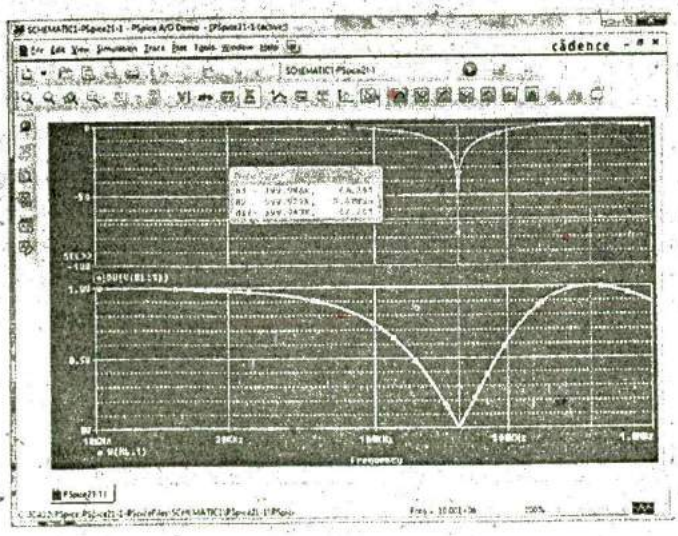


FIG. 21.95

dB and magnitude plot for the voltage across R_L of the network in Fig. 21.93.

also be displayed in the same figure by selecting **Plot-Add Plot to Window-Trace-Add Trace-DB(V(RL:1))-OK,** resulting in the plot at the top of the figure. Using the left-click cursor option and the **Cursor Trough** key, you find that the minimum is at −67.26 dB at a frequency of essentially 200 kHz, which is an excellent characteristic for a band-stop filter. Using the right-click cursor and setting it on 600 kHz, you find that the drop is −3.61 μdB, or essentially 0 dB, which is excellent for the pass-band region.

Multisim

High-Pass Filter The Multisim computer analysis is an investigation of the high-pass filter in Fig. 21.96. The cutoff frequency is determined by $f = 1/2\pi RC = 1.592$ kHz, with the voltage across the resistor approaching 1 V at high frequencies at a phase angle of 0°.

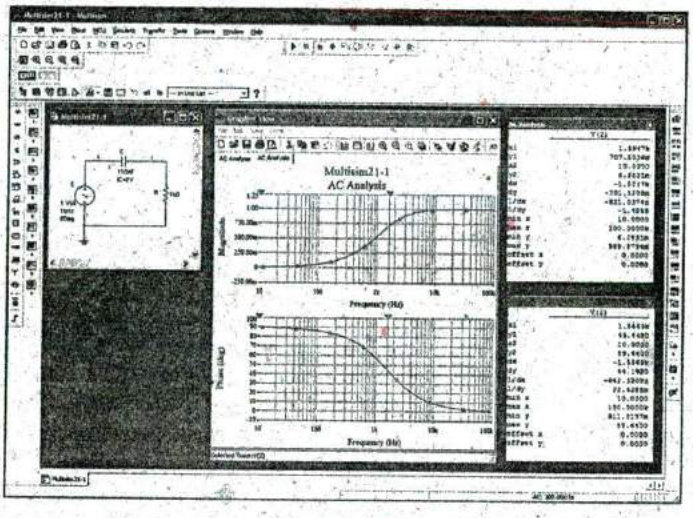


FIG. 21.96
High-pass R-C filter to be investigated using PSpice.

For this analysis, the **Component: AC_POWER** under **POWER_SOURCES** was chosen. The components of the source were set in the **AC_POWER** dialog box as shown in Fig. 21.96. Use the sequence **Simulate-Analyses-AC Analysis** to obtain the **AC Analysis** dialog box. Select the following settings: **Start frequency:** 10 Hz, **Stop frequency:** 100 kHz, **Sweep type:** Decade (logarithmic), **Number of points per decade:** 1000, **Vertical scale:** Linear. Under the **Output** option, move $2 to **Selected variables for analysis** using the **Add** option and remove the $1 option using the **Remove** option. Select **Simulate** to obtain the response in Fig. 21.96. Add the grid option to each, and then select **Show/Hide Cursors** to permit a determination of the magnitude and phase angle at the cutoff frequency. As shown in Fig. 21.96, the magnitude is 0.707 at 1.59 kHz and the phase angle is 45.45° at 1.57 kHz—very close to the expected results.

PROBLEMS

SECTION 21.1 Logarithms

1. a. Determine the frequencies (in kHz) at the points indicated on the plot in Fig. 21.97(a).
   b. Determine the voltages (in mV) at the points indicated on the plot in Fig. 21.97(b).

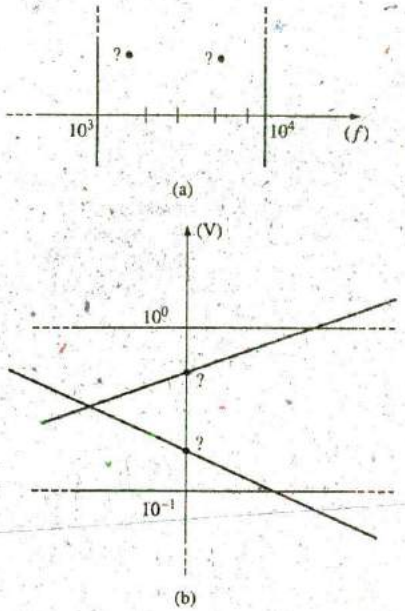


FIG. 21.97
Problem 1.

SECTION 21.2 Properties of Logarithms

2. Determine $\log_{10} x$ for each value of x.
   a. 100,000 b. 0.0001
   c. 10^8 d. 10^{-6}
   e. 20 f. 8643.4
   g. 56,000 h. 0.318
3. Given $N = \log_{10} x$, determine x for each value of N.
   a. 3 b. 12
   c. 0.2 d. 0.04
   e. 10 f. 3.18
   g. 1.001 h. 6.1
4. Determine $\log_e x$ for each value of x.
   a. 100,000 b. 0.0001
   c. 20 d. 8643.4

   Compare with the solutions to Problem 2.
5. Determine $\log_{10} 48 = \log_{10}(8)(6)$, and compare to $\log_{10} 8 + \log_{10} 6$.
6. Determine $\log_{10} 0.2 = \log_{10} 18/90$, and compare to $\log_{10} 18 - \log_{10} 90$.
7. Verify that $\log_{10} 0.5$ is equal to $-\log_{10} 1/0.5 = -\log_{10} 2$.
8. Find $\log_{10}(3)^3$, and compare with $3 \log_{10} 3$.

SECTION 21.3 Decibels

9. a. Determine the number of bels that relate power levels of $P_2 = 280$ mW and $P_1 = 4$ mW.
   b. Determine the number of decibels for the power levels of part (a), and compare results.
10. A power level of 100 W is 6 dB above what power level?
11. If a 2 W speaker is replaced by one with a 40 W output, what is the increase in decibel level?
12. Determine the dB_m level for an output power of 120 mW.
13. Find the dB_v gain of an amplifier that raises the voltage level from 0.1 mV to 8.4 mV.
14. Find the output voltage of an amplifier if the applied voltage is 20 mV and a dB_v gain of 22 dB is attained.
15. If the sound pressure level is increased from 0.001 μbar to 0.016 μbar, what is the increase in dB_s level?
16. What is the required increase in acoustical power to raise a sound level from that of quiet music to very loud music? Use Fig. 21.5.
17. a. Using semilog paper, plot X_L versus frequency for a 10 mH coil and a frequency range of 100 Hz to 1 MHz. Choose the best vertical scaling for the range of X_L.
    b. Repeat part (a) using log-log graph paper. Compare to the results of part (a). Which plot is more informative?
    c. Using semilog paper, plot X_C versus frequency for a 1 μF capacitor and a frequency range of 10 Hz to 100 kHz. Again choose the best vertical scaling for the range of X_C.
    d. Repeat part (a) using log-log graph paper. Compare to the results of part (c). Which plot is more informative?
18. a. For the meter of Fig. 21.6, find the power delivered to a load for an 8 dB reading.
    b. Repeat part (a) for a -5 dB reading.

SECTION 21.5 *R-C* Low-Pass Filter

19. For the *R-C* low-pass filter in Fig. 21.98:
    a. Sketch $A_v = V_o/V_i$ versus frequency using a log scale for the frequency axis. Determine $A_v = V_o/V_i$ at $0.1 f_c$, $0.5f_c$, f_c, $2f_c$, and $10f_c$.
    b. Sketch the phase plot of θ versus frequency, where θ is the angle by which $\mathbf{V}_o$ leads $\mathbf{V}_i$. Determine θ at $f = 0.1f_c$, $0.5f_c$, f_c, $2f_c$, and $10f_c$.

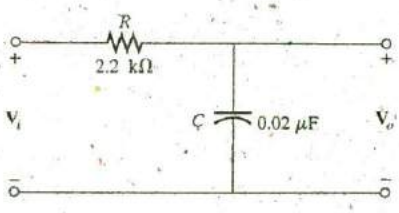


FIG. 21.98
Problem 19.

*20. For the network in Fig. 21.99:
   a. Determine V_o at a frequency one octave above the critical frequency.
   b. Determine V_o at a frequency one decade below the critical frequency.
   c. Do the levels of parts (a) and (b) verify the expected frequency plot of V_o versus frequency for the filter?

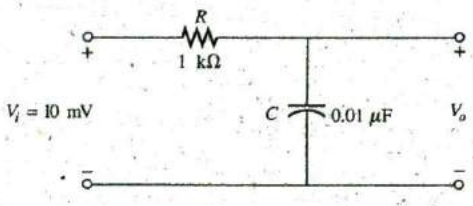


FIG. 21.99
Problem 20.

21. Design an *R-C* low-pass filter to have a cutoff frequency of 500 Hz using a resistor of 1.2 kΩ. Then sketch the resulting magnitude and phase plot for a frequency range of $0.1f_c$ to $10f_c$.

22. For the low-pass filter in Fig. 21.100:

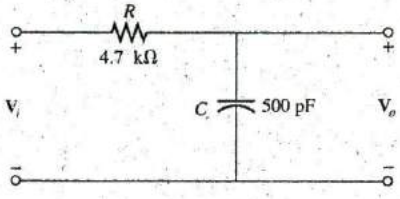


FIG. 21.100
Problem 22.

   a. Determine f_c.
   b. Find $A_v = V_o/V_i$ at $f = 0.1f_c$, and compare to the maximum value of 1 for the low-frequency range.
   c. Find $A_v = V_o/V_i$ at $f = 10f_c$, and compare to the minimum value of 0 for the high-frequency range.
   d. Determine the frequency at which $A_v = 0.01$ or $V_o = \frac{1}{100}V_i$.

SECTION 21.6 *R-C* High-Pass Filter

23. For the *R-C* high-pass filter in Fig. 21.101:
   a. Sketch $A_v = V_o/V_i$ versus frequency using a log scale for the frequency axis. Determine $A_v = V_o/V_i$ at f_c, one octave above and below f_c, and one decade above and below f_c.
   b. Sketch the phase plot of θ versus frequency, where θ is the angle by which $\mathbf{V}_o$ leads $\mathbf{V}_i$. Determine θ at the same frequencies noted in part (a).

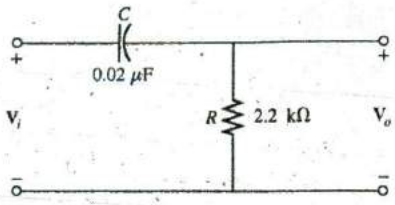


FIG. 21.101
Problem 23.

24. For the network in Fig. 21.102:
   a. Determine $A_v = V_o/V_i$ at $f = f_c$ for the high-pass filter.
   b. Determine $A_v = V_o/V_i$ at two octaves above f_c. Is the rise in V_o significant from the $f = f_c$ level?
   c. Determine $A_v = V_o/V_i$ at two decades above f_c. Is the rise in V_o significant from the $f = f_c$ level?
   d. If V_i = 10 mV, what is the power delivered to R at the critical frequency?

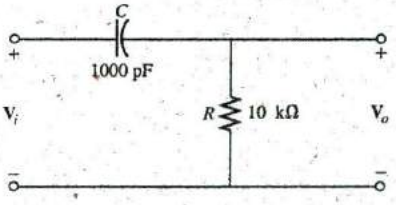


FIG. 21.102
Problem 24.

25. Design a high-pass *R-C* filter to have a cutoff or corner frequency of 2 kHz, given a capacitor of 0.1 μF. Choose the closest commercial value for R, and then recalculate the resulting corner frequency. Sketch the normalized gain $A_v = V_o/V_i$ for a frequency range of $0.1f_c$ to $10f_c$.

26. For the high-pass filter in Fig. 21.103:
   a. Determine f_c.
   b. Find $A_v = V_o/V_i$ at $f = 0.01f_c$, and compare to the minimum level of 0 for the low-frequency region.
   c. Find $A_v = V_o/V_i$ at $f = 100f_c$, and compare to the maximum level of 1 for the high-frequency region.
   d. Determine the frequency at which $V_o = \frac{1}{2}V_i$.

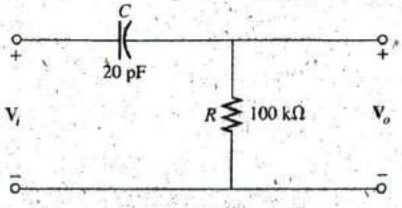


FIG. 21.103
Problems 26 and 54.

SECTION 21.7 Pass-Band Filters

27. For the pass-band filter in Fig. 21.104:
    a. Sketch the frequency response of $A_v = V_o/V_i$ against a log scale extending from 10 Hz to 10 kHz.
    b. What are the bandwidth and the center frequency?

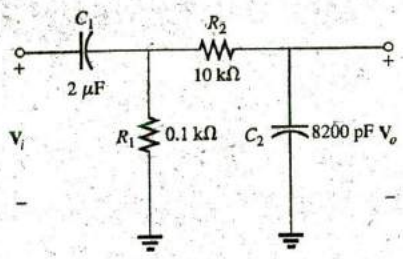


FIG. 21.104
Problems 27 and 28.

*28. Design a pass-band filter such as the one appearing in Fig. 21.104 to have a low cutoff frequency of 4 kHz and a high cutoff frequency of 80 kHz.

29. For the pass-band filter in Fig. 21.105:
    a. Determine f_s.
    b. Calculate Q_s and the BW for V_o.
    c. Sketch $A_v = V_o/V_i$ for a frequency range of 1 kHz to 1 MHz.
    d. Find the magnitude of V_o at $f = f_s$ and the cutoff frequencies.

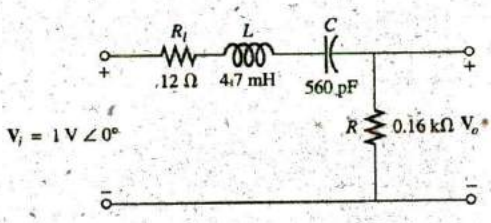


FIG. 21.105
Problem 29.

30. For the pass-band filter in Fig. 21.106:

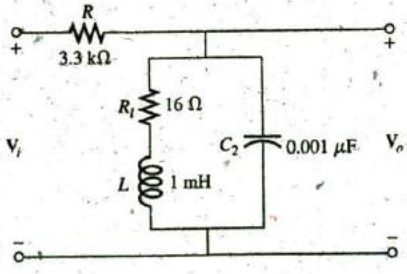


FIG. 21.106
Problems 30 and 55.

    a. Determine the frequency response of $A_v = V_o/V_i$ for a frequency range of 100 Hz to 1 MHz.
    b. Find the quality factor Q_p and the BW of the response.

SECTION 21.8 Stop-Band Filters

*31. For the stop-band filter in Fig. 21.107:
    a. Determine Q_s.
    b. Find the bandwidth and the half-power frequencies.
    c. Sketch the frequency characteristics of $A_v = V_o/V_i$.
    d. What is the effect on the curve of part (c) if a load of 2 kΩ is applied?

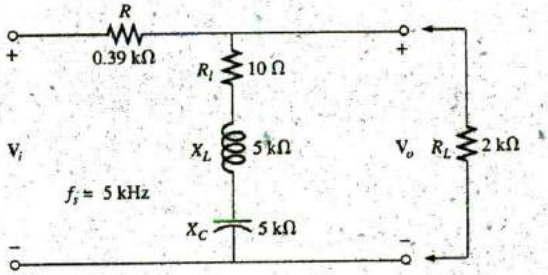


FIG. 21.107
Problem 31.

*32. For the pass-band filter in Fig. 21.108:
    a. Determine Q_p ($R_L = \infty\ \Omega$, an open circuit).
    b. Sketch the frequency characteristics of $A_v = V_o/V_i$.
    c. Find Q_p (loaded) for $R_L = 100$ kΩ, and indicate the effect of R_L on the characteristics of part (b).
    d. Repeat part (c) for $R_L = 20$ kΩ.

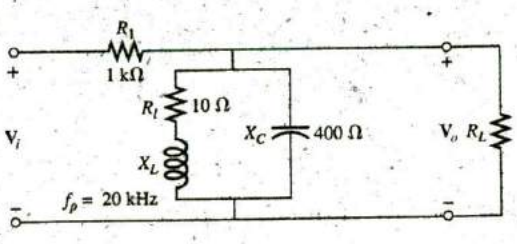


FIG. 21.108
Problem 32.

SECTION 21.9 Double-Tuned Filter

33. a. For the network in Fig. 21.43(a), if $L_p = 400\ \mu$H ($Q > 10$), $L_s = 60\ \mu$H, and $C = 120$ pF, determine the rejected and accepted frequencies.
    b. Sketch the response curve for part (a).

34. a. For the network in Fig. 21.43(b), if the rejected frequency is 30 kHz and the accepted is 100 kHz, determine the values of L_s and L_p ($Q > 10$) for a capacitance of 200 pF.
    b. Sketch the response curve for part (a).

SECTION 21.10 Bode Plots

35. a. Sketch the idealized Bode plot for $A_v = V_o/V_i$ for the high-pass filter in Fig. 21.109.
    b. Using the results of part (a), sketch the actual frequency response for the same frequency range.
    c. Determine the decibel level at f_c, $\frac{1}{2}f_c$, $2f_c$, $\frac{1}{10}f_c$, and $10f_c$.
    d. Determine the gain $A_v = V_o/V_i$ at $f = f_c$, $\frac{1}{2}f_c$, and $2f_c$.
    e. Sketch the phase response for the same frequency range.

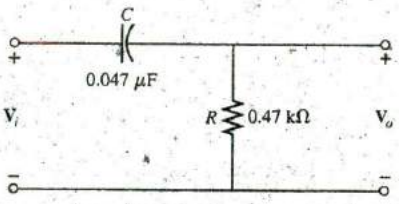


FIG. 21.109
Problem 35.

*36. a. Sketch the response of the magnitude of V_o (in terms of V_i) versus frequency for the high-pass filter in Fig. 21.110.
    b. Using the results of part (a), sketch the response $A_v = V_o/V_i$ for the same frequency range.
    c. Sketch the idealized Bode plot.
    d. Sketch the actual response, indicating the dB difference between the idealized and the actual response at $f = f_c$, $0.5f_c$, and $2f_c$.
    e. Determine $A_{v_{dB}}$ at $f = 1.5f_c$ from the plot of part (d), and then determine the corresponding magnitude of $A_v = V_o/V_i$.
    f. Sketch the phase response for the same frequency range (the angle by which $\mathbf{V}_o$ leads $\mathbf{V}_i$).

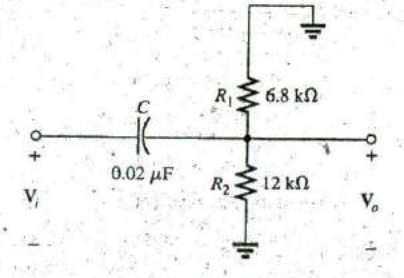


FIG. 21.110
Problem 36.

37. a. Sketch the idealized Bode plot for $A_v = V_o/V_i$ for the low-pass filter in Fig. 21.111.
    b. Using the results of part (a), sketch the actual frequency response for the same frequency range.
    c. Determine the decibel level at f_c, $\frac{1}{2}f_c$, $2f_c$, $\frac{1}{10}f_c$, and $10f_c$.
    d. Determine the gain $A_v = V_o/V_i$ at $f = f_c$, $\frac{1}{2}f_c$, and $2f_c$.
    e. Sketch the phase response for the same frequency range.

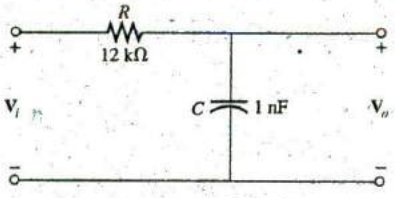


FIG. 21.111
Problem 37.

*38. a. Sketch the response of the magnitude of V_o (in terms of V_i) versus frequency for the low-pass filter in Fig. 21.112.
    b. Using the results of part (a), sketch the response $A_v = V_o/V_i$ for the same frequency range.
    c. Sketch the idealized Bode plot.
    d. Sketch the actual response indicating the dB difference between the idealized and the actual response at $f = f_c$, $0.5f_c$, and $2f_c$.
    e. Determine $A_{v_{dB}}$ at $f = 0.25f_c$ from the plot of part (d), and then determine the corresponding magnitude of $A_v = V_o/V_i$.
    f. Sketch the phase response for the same frequency range (the angle by which $\mathbf{V}_o$ leads $\mathbf{V}_i$).

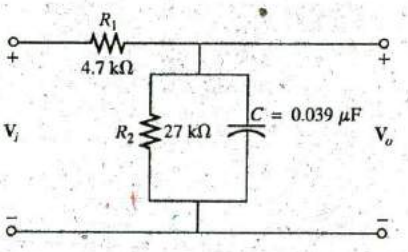


FIG. 21.112
Problem 38.

SECTION 21.11 Sketching the Bode Response

39. For the filter in Fig. 21.113:
    a. Sketch the curve of $A_{v_{dB}}$ versus frequency using a log scale.
    b. Sketch the curve of θ versus frequency for the same frequency range as in part (a).

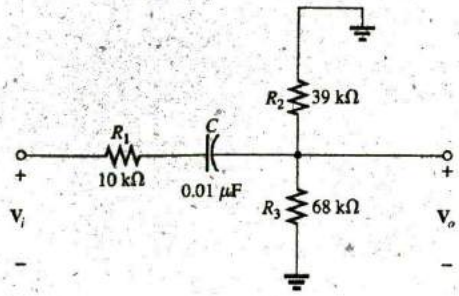


FIG. 21.113
Problem 39.

*40. For the filter in Fig. 21.114:
   a. Sketch the curve of $A_{v_{dB}}$ versus frequency using a log scale.
   b. Sketch the curve of θ versus frequency for the same frequency range as in part (a).

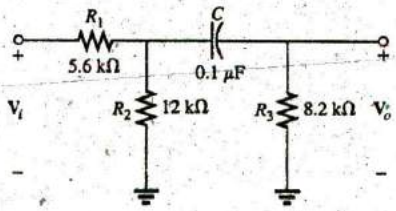


FIG. 21.114
Problem 40.

SECTION 21.12 Low-Pass Filter with Limited Attenuation

41. For the filter in Fig. 21.115:
   a. Sketch the curve of $A_{v_{dB}}$ versus frequency using the idealized Bode plots as a guide.
   b. Sketch the curve of θ versus frequency.

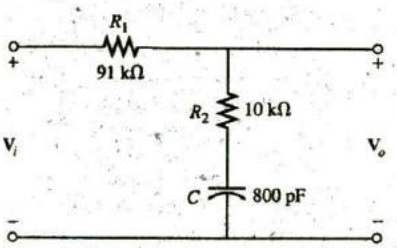


FIG. 21.115
Problem 41.

*42. For the filter in Fig. 21.116:
   a. Sketch the curve of $A_{v_{dB}}$ versus frequency using the idealized Bode plots as a guide.
   b. Sketch the curve of θ versus frequency.

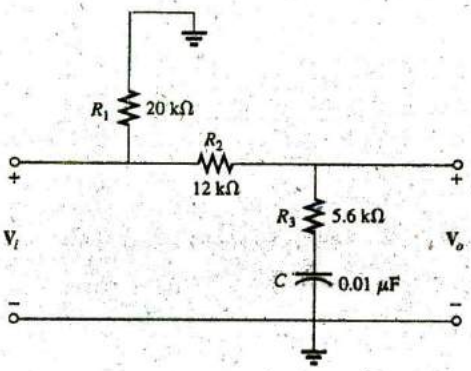


FIG. 21.116
Problem 42.

SECTION 21.13 High-Pass Filter with Limited Attenuation

43. For the filter in Fig. 21.117:
   a. Sketch the curve of $A_{v_{dB}}$ versus frequency using the idealized Bode plots as an envelope for the actual response.
   b. Sketch the curve of θ (the angle by which $\mathbf{V}_o$ leads $\mathbf{V}_i$) versus frequency.

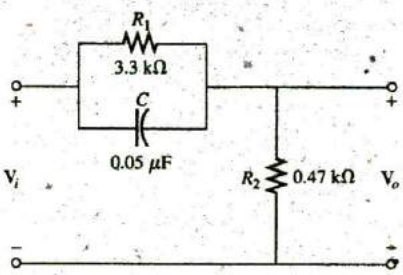


FIG. 21.117
Problem 43.

*44. For the filter in Fig. 21.118:
   a. Sketch the curve of $A_{v_{dB}}$ versus frequency using the idealized Bode plots as an envelope for the actual response.
   b. Sketch the curve of θ (the angle by which $\mathbf{V}_o$ leads $\mathbf{V}_i$) versus frequency.

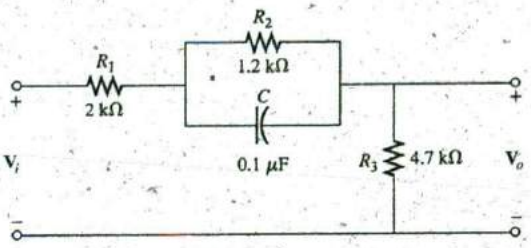


FIG. 21.118
Problem 44.

SECTION 21.14 Additional Properties of Bode Plots

45. A bipolar transistor amplifier has the following gain:

$$\mathbf{A}_v = \frac{160}{\left(1 - j\frac{100\text{ Hz}}{f}\right)\left(1 - j\frac{130\text{ Hz}}{f}\right)\left(1 + j\frac{f}{20\text{ kHz}}\right)\left(1 + j\frac{f}{50\text{ kHz}}\right)}$$

a. Sketch the normalized Bode response $A'_{v_{dB}} = (A_v/A_{v_{max}})|_{dB}$, and determine the bandwidth of the amplifier. Be sure to note the corner frequencies.

b. Sketch the phase response, and determine a frequency where the phase angle is relatively close to 45°.

46. A JFET transistor amplifier has the following gain:

$$\mathbf{A}_v = \frac{-5.6}{\left(1 - j\frac{10\text{ Hz}}{f}\right)\left(1 - j\frac{45\text{ Hz}}{f}\right)\left(1 - j\frac{68\text{ Hz}}{f}\right)\left(1 + j\frac{f}{23\text{ kHz}}\right)\left(1 + j\frac{f}{50\text{ kHz}}\right)}$$

a. Sketch the normalized Bode response $A'_{v_{dB}} = (A_v/A_{v_{max}}|_{dB})$, and determine the bandwidth of the amplifier. When you normalize, be sure that the maximum value of A'_v is +1. Clearly indicate the cutoff frequencies on the plot.

b. Sketch the phase response, and note the regions of greatest change in phase angle. How do the regions correspond to the frequencies appearing in the function $\mathbf{A}_v$?

47. A transistor amplifier has a midband gain of −120, a high cutoff frequency of 36 kHz, and a bandwidth of 35.8 kHz. In addition, the actual response is also about −15 dB at $f =$ 50 Hz. Write the transfer function $\mathbf{A}_v$ for the amplifier.

48. Sketch the Bode plot of the following function:

$$\mathbf{A}_v = \frac{0.05}{0.05 - j\,100/f}$$

49. Sketch the Bode plot of the following function:

$$\mathbf{A}_v = \frac{200}{200 + j\,0.1f}$$

50. Sketch the Bode plot of the following function:

$$\mathbf{A}_v = \frac{jf/1000}{(1 + jf/1000)(1 + jf/10{,}000)}$$

***51.** Sketch the Bode plot of the following function:

$$\mathbf{A}_v = \frac{(1 + jf/1000)(1 + jf/2000)}{(1 + jf/3000)^2}$$

***52.** Sketch the Bode plot of the following function (note the presence of ω rather than f):

$$\mathbf{A}_v = \frac{40(1 + j0.001\,\omega)}{(j0.001\,\omega)(1 + j0.0002\,\omega)}$$

SECTION 21.15 Crossover Networks

***53.** The three-way crossover network in Fig. 21.119 has a 12 dB rolloff at the cutoff frequencies.

a. Determine the ratio V_o/V_i for the woofer and tweeter at the cutoff frequencies of 400 Hz and 5 kHz, respectively, and compare to the desired level of 0.707.

b. Calculate the ratio V_o/V_i for the woofer and tweeter at a frequency of 3 kHz, where the midrange speaker is designed to predominate.

c. Determine the ratio V_o/V_i for the midrange speaker at a frequency of 3 kHz, and compare to the desired level of 1.

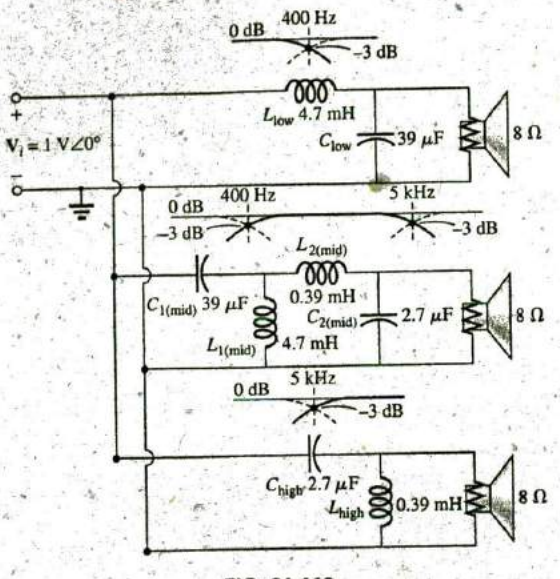


FIG. 21.119
Problems 53 and 57.

SECTION 21.17 Computer Analysis

PSpice or Multisim

54. Using schematics, obtain the magnitude and phase response versus frequency for the network in Fig. 21.103.

55. Using schematics, obtain the magnitude and phase response versus frequency for the network in Fig. 21.106.

***56.** Obtain the dB and phase plots for the network in Fig. 21.75, and compare with the plots in Figs. 21.76 and 21.77.

***57.** Using schematics, obtain the magnitude and dB plot versus frequency for each filter in Fig. 21.119, and verify that the curves drop off at 12 dB per octave.

GLOSSARY

Active filter A filter that uses active devices such as transistors or operational amplifiers in combination with *R*, *L*, and *C* elements.

Bode plot A plot of the frequency response of a system using straight-line segments called *asymptotes*.

Decibel A unit of measurement used to compare power levels.

Double-tuned filter A network having both a pass-band and a stop-band region.

Filter Networks designed to either pass or reject the transfer of signals at certain frequencies to a load.

High-pass filter A filter designed to pass high frequencies and reject low frequencies.

Log-log paper Graph paper with vertical and horizontal log scales.

Low-pass filter A filter designed to pass low frequencies and reject high frequencies.

Microbar (μbar) A unit of measurement for sound pressure levels that permits comparing audio levels on a dB scale.

Pass-band (band-pass) filter A network designed to pass signals within a particular frequency range.

Passive filter A filter constructed of series, parallel, or series-parallel *R*, *L*, and *C* elements.

Semilog paper Graph paper with one log scale and one linear scale.

Stop-band filter A network designed to reject (block) signals within a particular frequency range.