5

Kinetic Theory of Gases

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5.1. Introduction

A gas consists of an extremely large number of widely separated minute particles, called *molecules*, which are in a state of continuous motion. They rush about in all directions, and in all sorts of manner, and frequently collide with each other. Due to repeated collisions of the molecules, their velocities and directions get changed at random, as a result of which a molecular change prevails within the gas. The kinetic theory of gases covers the behaviour of gases under various physical conditions.

5.2. Assumptions in the Kinetic Theory of Gases

The kinetic theory of gases is based on the following assumptions :

1. The volume of a gas consists of a large number of minute particles called *molecules*. It has been experimentally found that there are about 26.8×10^{18} molecules in *1 ml. of gas at N.T.P.
2. The molecules are mere mass points. In other words, the size of a molecule is assumed to be negligible, as compared to the distance between the molecules.
3. The gas molecules are perfectly elastic spheres and exert negligible force of attraction or repulsion on one another, or on the walls of the containing vessel. Hence in a direct impact, the molecules rebound with the same velocity after each collision.
4. The molecules are continuously colliding against each other, and with the walls of the containing vessel. Between two collisions, a molecule moves in a straight line. This distance is called the free path of the molecule.
5. The time during which a collision takes place is negligible as compared to the time required to transverse the free path, *i.e.* collisions are instantaneous.

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* Since 1 ml = 10^{-6} m^3, therefore

1 m^3 of any gas at N.T.P. will contain $\frac{26.8 \times 10^{18}}{10^{-6}} = 26.8 \times 10^{24}$ molecules.

Since the volume of 1 kg molecule of any gas at N.T.P. is 22.4 m^3, therefore

1 kg molecule of any gas will contain $26.8 \times 10^{24} \times 22.4 = 6.02 \times 10^{26}$ molecules

6. The molecules are continuously in motion. They have velocities in all directions ranging from zero to infinity. The velocity of molecules increases with the increase in temperature. For simplicity, it is assumed that one-third of the molecules move parallel to the X-axis, one-third parallel to Y-axis and one-third parallel to Z-axis.

5.3. Velocity of a Molecule

We have already discussed that molecules of a gas move in an haphazard manner in all directions and have different velocities. The following three velocities are important from the subject point of view ;

1. Mean or average velocity, 2. Root mean square velocity, and 3. Most probable velocity.

These velocities are obtained as discussed below :

1. Mean or average velocity

Consider a gas containing n molecules, and having actual velocities of individual molecules as $C_1, C_2, C_3 \ldots\ldots C_n$.

$\therefore$ Mean or average velocity

$$= \frac{C_1 + C_2 + C_3 + \ldots + C_n}{n} = \sqrt{\frac{8kT}{\pi m}} = 1.595\sqrt{\frac{kT}{m}}$$

2. Root mean square (r.m.s.) velocity

The root mean square velocity,

$$C = \sqrt{\frac{C_1^2 + C_2^2 + C_3^2 + \ldots + C_n^2}{n}}$$

The value of r.m.s. velocity may also be obtained from the following relations :

(*i*) $$*C = \sqrt{\frac{3p}{\rho}} = 1.732\sqrt{\frac{p}{\rho}}$$

(*ii*) $$**C = \sqrt{\frac{3kT}{m}} = 1.732\sqrt{\frac{kT}{m}}$$

Since the velocity of a molecule changes with temperature, *the root mean square velocity of the molecules remains constant so long as the temperature remains constant.*

3. Most probable velocity

The Maxwell's distribution of molecular velocities and percentage of molecules is shown in Fig. 5.1.

We see that the maximum percentage of molecules (AB) at any temperature move with a velocity of OA. This velocity is known as *most probable velocity.*

The most probable velocity may, therefore, be defined as the *velocity with which the largest percentage of molecules in a gas are found to move at any given temperature.*

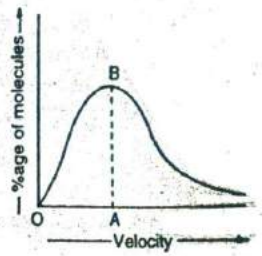


Fig. 5.1. Most probable velocity.

Notes : 1. The value of most probable velocity is given by :

$$\sqrt{\frac{2kT}{m}} = 1.414\sqrt{\frac{kT}{m}}$$

* See Art. 5.4.

** See Art. 5.6.

where k = Boltzmann's constant, Its value is same for all gases and may be taken as 1.38×10^{-26} kJ/K,

T = Absolute temperature, and

m = Mass of one molecule.

2. Due to the non-symmetrical shape of the curve, the mean velocity is larger than the most probable velocity, and the r.m.s. velocity is a little larger than either one of them.

3. From above, we see that

Mean or average velocity = 0.9209 × r.m.s. velocity

and most probable velocity = 0.8164 × r.m.s. velocity

5.4. Pressure Exerted by an Ideal Gas

The pressure exerted by a gas is due to the continuous bombardment on the walls of the containing vessel by the rapidly moving gas molecules.

Consider a hollow cubical vessel of each side equal to l, having perfectly elastic walls as shown in Fig. 5.2. Let it contain a perfect gas having a large number of molecules, say, n. Out of these, $n/3$ molecules are travelling back and forth parallel to each of the co-ordinate axis, *i.e.* X-axis, Y-axis and Z-axis with a velocity C (r.m.s. value).

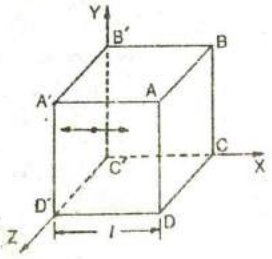


Fig. 5.2. Pressure exerted by an ideal gas.

Let us first consider the pressure exerted by one molecule, travelling back and forth parallel to X-axis. The molecule strikes the face $ABCD$ of the containing vessel and rebounds with the *same velocity* in the reverse direction, since the collision is perfectly elastic.

Let C = Velocity of molecule before collision, and

$-C$ = Velocity after collision.

∴ Momentum before collision = mC

and momentum after collision = $-mC$

Hence, change in momentum per collision per molecule

$$= mC - (-mC) = 2mC$$

The molecule now travels from face $ABCD$ to face $A'B'C'D'$ and travels back again to face $ABCD$. It is thus obvious that the distance travelled by the molecule between the two successive impacts on the face $ABCD$ is $2l$.

∴ Number of collisions per second

$$= \frac{C}{2l}$$

and change in momentum per collision per second

$$= 2mC \times \frac{C}{2l} = \frac{mC^2}{l}$$

and total change in momentum per second due to the collisions of $n/3$ molecules

$$= \frac{n}{3} \times \frac{mC^2}{l} = \frac{mn}{3l} \times C^2$$

Since the change in momentum per second is equal to the *impressed force, therefore,

Total force on the face *ABCD*

$$= \frac{mn}{3l} \times C^2$$

and** pressure exerted on the face *ABCD*

$$= \frac{\text{Force}}{\text{Area}} = \frac{mn}{3l \times l^2} \times C^2 \qquad \ldots (\because \text{Area of face} = l^2)$$

$$= \frac{mn}{3v} \times C^2 \qquad \ldots (\because \text{Volume, } v = l^3)$$

Now, $mn = M$, Molecular mass of the gas.

$\therefore$ Pressure exerted by an ideal gas,

$$p = \frac{1}{3} \times \frac{M}{v} \times C^2 = \frac{1}{3} \times \rho C^2 \qquad \ldots \left(\because \frac{M}{v} = \rho\right) \quad \ldots (i)$$

Note : The above equation may also be written as :

$$p = \frac{2}{3} \times \frac{1}{2} \rho C^2 = \frac{2}{3} \times \text{K.E. per molecule} = \frac{2}{3} E$$

Therefore, the pressure exerted by an ideal gas is equal to two-thirds of the kinetic energy of all the molecules contained in a unit volume of the gas.

Example 5.1. *Calculate the root mean square velocity of air at N.T.P. The density of air at N.T.P. is 1.29 kg/m³. Density of mercury = 13 600 kg/m³ and g = 9.81 m/s².*

Solution. Given : $p = 760$ mm of Hg ; $\rho = 1.29$ kg/m³ ; $\rho_m = 13\,600$ kg/m³

Let C = Root mean square velocity at N.T.P.

We know that $p = 760$ mm of Hg $= 0.76$ m of Hg

$$= 0.76 \times 13\,600 \times 9.81 = 101\,396 \text{ N/m}^2$$

We also know that

$$p = \frac{1}{3} \times \rho C^2 \text{ or } C = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3 \times 101\,396}{1.29}} = 485.6 \text{ m/s Ans.}$$

5.5. Kinetic Energy per kg Molecule of a Gas

We know that the pressure exerted by an ideal gas,

$$p = \frac{1}{3} \times \frac{M}{v} \times C^2 \text{ or } pv = \frac{1}{3} \times MC^2 \qquad \ldots (i)$$

Now, according to perfect gas equation, we have

$$pv = R_u T \qquad \ldots (ii)$$

where v = Volume of gas per kg molecule, and

R_u = Universal gas constant

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* Newton's second law of motion.

** The pressure exerted on any other face is same, *i.e.* $\frac{mn}{3v} \times C^2$

From equations (*i*) and (*ii*), we get

$$\frac{1}{3} \times MC^2 = R_u T \quad \text{or} \quad \frac{1}{2} \times MC^2 = \frac{3}{2} \times R_u T$$

Hence, *the kinetic energy per kg molecule of any gas at absolute temperature T is equal to* $\frac{3}{2} \times R_u T$.

5.6. Kinetic Interpretation of Temperature

We know that the pressure exerted by an ideal gas,

$$p = \frac{1}{3} \times \frac{M}{v} C^2 \quad \text{or} \quad pv = \frac{1}{3} \times MC^2$$

We also know that $pv = R_u T$

$$\therefore \quad \frac{1}{3} \times MC^2 = R_u T \quad \text{or} \quad \frac{1}{3} \times mNC^2 = R_u T$$

where m = Molecular mass of gas per molecule, and

N = Avogadro's number representing the number of molecules per kg molecule of a gas. Its value is same for all gases and may be taken as 6.02×10^{26}

From, above, we have

$$\frac{1}{3} \times mC^2 = \frac{R_u}{N} \times T \quad \text{or} \quad \frac{1}{2} \times mC^2 = \frac{3}{2} \times \frac{R_u}{N} \times T = \frac{3}{2} \times kT$$

where $k = \frac{R_u}{N}$ = Boltzmann's constant.

Hence, kinetic energy of translation of a molecule

$$= \frac{3}{2} \times kT$$

In other words, *the kinetic energy of translation of a molecule is proportional to the absolute temperature. This is known as kinetic interpretation of temperature.*

Notes : 1. The K.E. of translation of a molecule at a given absolute temperature is same for all the gases.

2. From equation $\frac{1}{3} \times mNC^2 = R_u T$,

$$C = \sqrt{\frac{3R_u T}{mN}} \quad \ldots (i)$$

$$= \sqrt{\frac{3kT}{m}} \quad \ldots \left(\because \frac{R_u}{N} = k \right) \ldots (ii)$$

$$= \sqrt{3RT} \quad \ldots \left(\because mN = M \text{ and } \frac{R_u}{M} = R \right) \ldots (iii)$$

where R = Characteristic gas constant.

In other words, $C \propto \sqrt{T}$

3. Since $C \propto \sqrt{T}$ or $C^2 \propto T$, therefore when $T = 0$, the velocity $C = 0$.

Hence, the absolute zero of temperature is that temperature at which the velocities of the molecules of a gas becomes zero.

Example 5.2. *Find the kinetic energy of the molecules of 1 kg of helium at 0° C if $R_u = 8.314$ kJ/kg K and molecular mass of helium is 4.*

Solution. Given : $T = 0°\ C = 0 + 273 = 273\ K$; $R_u = 8.314\ kJ/kg\ K$; $M = 4$

We know that number of molecules in 1 kg of helium

$$= \frac{N}{M} = \frac{N}{4}$$

and K.E. per molecule $= \frac{3}{2} kT$

∴ K.E. of the molecules in 1 kg of helium

= K.E. per molecule × No. of molecules

$$= \frac{3}{2} kT \times \frac{N}{4} = \frac{3}{8} R_u T \qquad \ldots (\because R_u = kN)$$

$$= \frac{3}{8} \times 8.314 \times 273 = 851\ kJ \text{ Ans.}$$

Example 5.3. *Avogadro's number is 6.02×10^{26} and Boltzmann's constant k is 1.38×10^{-23} J/K. What is the average velocity of a molecule of oxygen at 27° C ?*

Solution. Given : $N = 6.02 \times 10^{26}$; $k = 1.38 \times 10^{-23}$ J/K ; $T = 27°\ C = 27 + 273 = 300\ K$

We know that molecular mass of oxygen,

$$M = 16 \times 2 = 32$$

∴ Mass of one molecule,

$$m = \frac{M}{N} = \frac{32}{6.02 \times 10^{26}} = 5.316 \times 10^{-26}\ kg$$

We know that average velocity of a molecule,

$$C = \sqrt{\frac{8\,kT}{\pi m}} = 1.595 \sqrt{\frac{kT}{m}} = 1.595 \sqrt{\frac{1.38 \times 10^{-23} \times 300}{5.316 \times 10^{-26}}}\ m/s$$

$$= 445\ m/s \text{ Ans.}$$

Example 5.4. *Calculate the number of molecules in 1 m^3 of an ideal gas at 27° C and a pressure of 10 mm of Hg. The mean kinetic energy of a molecule at 27° C is 4×10^{-18} kJ and the density of mercury is 13 600 kg/m^3.*

Solution. Given : $T = 27°\ C$; $p = 10$ mm of Hg ; K.E. $= 4 \times 10^{-18}$ kJ ; $\rho_m = 13\,600\ kg/m^3$

We know that $p = 10$ mm of Hg $= 0.01$ m of Hg

$$= 0.01 \times 13\,600 \times 9.81 = 1334\ N/m^2$$

and kinetic energy per kg molecule of a gas

$$= \frac{3}{2} \times R_u T = \frac{3}{2} pv \qquad \ldots (\because pv = R_u T)$$

or kinetic energy per kg molecule per unit volume of a gas

$$= \frac{3}{2} p = \frac{3}{2} \times 1334 = 2001\ J = 2.001\ kJ$$

∴ Number of molecules in 1 m^3 of gas

$$= \frac{\text{K.E. per unit volume}}{\text{K.E. of a molecule}} = \frac{2.001}{4 \times 10^{-18}} = 5.0025 \times 10^{17} \text{ Ans.}$$

5.7. Application of Kinetic Theory to Laws of Perfect Gases

We have already discussed in Chapter 2, the various laws governing the perfect gases. Now we shall attempt to apply the kinetic theory to these laws.

1. *Boyle's law*

We know that pressure exerted by a perfect gas,

$$p = \frac{1}{3} \times \frac{M}{v} \times C^2 \quad \text{or} \quad pv = \frac{1}{3} \times MC^2$$

Since C depends on T ($\because C \propto \sqrt{T}$), therefore, when T is constant, the right hand side of the above expression is also constant.

$$\therefore \quad pv = \text{Constant} \qquad \ldots \text{(when } T \text{ is constant)}$$

2. *Charles' law*

We know that the pressure exerted by a perfect gas,

$$p = \frac{1}{3} \times \frac{M}{v} \times C^2 \quad \text{or} \quad pv = \frac{1}{3} \times MC^2$$

Since $C \propto \sqrt{T}$ or $C^2 \propto T$, therefore

$$pv \propto T$$

When p is kept constant, then

$$v \propto T \quad \text{or} \quad \frac{v}{T} = \text{Constant}$$

3. *Gay-Lussac law*

We have already seen in the above equation that

$$pv \propto T$$

When v is kept constant, then

$$p \propto T \quad \text{or} \quad \frac{p}{T} = \text{Constant}$$

4. *Perfect gas equation*

We have already seen in the above equation that

$$pv \propto T$$

or

$$\frac{pv}{T} = \text{Constant} \quad \text{or} \quad pv = R_u T \qquad \ldots \text{(General gas equation)}$$

5. *Avogadro's law*

It states, *"Equal volumes of all gases under the same conditions of temperature and pressure, contain the same number of molecules."*

Let m_1, n_1 and C_1 represent the mass per molecule, number of molecules per m^3 and root mean square velocity respectively for one gas and m_2, n_2 and C_2 the corresponding values for the other gas. If the two gases exert the same pressure p, then

$$p = \frac{1}{3} m_1 n_1 C_1^2 = \frac{1}{3} m_2 n_2 C_2^2 \qquad \ldots (i)$$

If the two gases are also at the same temperature, then the mean kinetic energy per molecule in the two gases is equal. In other words

$$\frac{1}{2} m_1 C_1^2 = \frac{1}{2} m_2 C_2^2$$

Dividing equation (*i*) by (*ii*),

$$n_1 = n_2$$

Example 5.5. *At what temperature will the velocity (root mean square velocity) of hydrogen be double of its value at N.T.P., if the pressure remains constant ?*

Solution. Given : $T_0 = 0^\circ C = 0 + 273 = 273$ K

Let C = Root mean square velocity at T_0, and

T = Absolute temperature at which the r.m.s. velocity is $2C$.

We know that $C \propto \sqrt{T}$ or $C^2 = T$

$$\therefore \quad \frac{C^2}{(2C)^2} = \frac{T_0}{T} \quad \text{or} \quad \frac{1}{4} = \frac{273}{T} \quad \text{or} \quad T = 1092 \text{ K} = 819^\circ \text{C Ans.}$$

Example 5.6. *Calculate the temperature at which root mean square velocity of a gas molecule is same as that of a molecule of another gas at 47° C. The molecular mass of first and second gases are 64 and 32 respectively.*

Solution. Given : $T_2 = 47^\circ C = 47 + 273 = 320$ K ; $M_1 = 64$; $M_2 = 32$

Let T_1 = Temperature of the first gas, and

C = Root mean square velocity.

We know that $p v = R_u T = \frac{1}{3} MC^2$

or $$\frac{MC^2}{T} = 3 R_u = \text{Constant}$$

$$\therefore \quad \frac{M_1 C_1^2}{T_1} = \frac{M_2 C_2^2}{T_2} \quad \text{or} \quad \frac{M_1}{T_1} = \frac{M_2}{T_2} \qquad \ldots (\because C_1 = C_2)$$

$$\therefore \quad T_1 = T_2 \times \frac{M_1}{M_2} = 320 \times \frac{64}{32} = 640 \text{ K} = 367^\circ \text{C Ans.}$$

Example 5.7. *The root mean square velocity of the molecules of a fixed mass of a gas is 895 m/s. Calculate the root mean square velocity of the molecules when the gas is compressed adiabatically to half of its original volume. The ratio of specific heats at constant pressure to that of constant volume is 1.4.*

Solution. Given : $C_1 = 895$ m/s ; $v_2 = 0.5\, v_1$; $\gamma = 1.4$

Let C_2 = Root mean square velocity of the molecules when the gas is compressed,

T_1 = Initial temperature of the gas, and

T_2 = Final temperature of the gas.

We know that for adiabatic compression,

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{\gamma - 1} = \left(\frac{0.5\, v_1}{v_1} \right)^{1.4 - 1} = (0.5)^{0.4} = 0.758 \qquad \ldots (i)$$

Since $C \propto \sqrt{T}$ or $C^2 \propto T$, therefore

$$\frac{C_1^2}{C_2^2} = \frac{T_1}{T_2} \text{ or } \frac{C_1^2}{C_2^2} = 0.758 \quad \ldots \text{[From equation (i)]}$$

$$\therefore \quad C_2 = \sqrt{\frac{C_1^2}{0.758}} = \sqrt{\frac{(895)^2}{0.758}} = 1028 \text{ m/s Ans.}$$

5.8. Degrees of Freedom

In order to describe completely the motion of a particle in one plane, only *two* quantities must be known, say its two rectangular components. Similarly, for a particle moving in space, *three* independent quantities must be known to describe its motion. A molecule in a rigid body can have three motions of vibration along any of the three co-ordinate axes in addition to its three motions of translation. It is thus obvious that in order to completely describe the state of motion of a particle, *six* independent quantities must be known.

In general, *the total number of independent quantities, which must be known for describing completely the state of motion of a body, are called its degrees of freedom.*

Now we shall consider the degrees of freedom for the following cases :

1. *Monoatomic gas.* A molecule of a monoatomic gas has only one atom. This molecule can rotate about its polar axis, and can also move bodily in the three perpendicular planes as shown in Fig. 5.3. Hence such a molecule will have *three translational degrees of freedom and one rotational.*

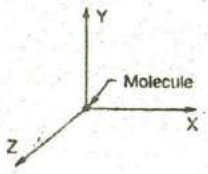


Fig. 5.3. Monoatomic gas molecule.

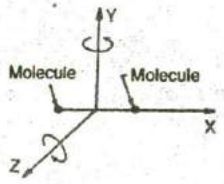


Fig. 5.4. Diatomic gas molecule.

2. *Diatomic gas.* A molecule of diatomic gas is like a dumb-bell. It has two atoms only. Such a molecule has appreciable moment of inertia and it is capable of rotational as well as translational motion. Its rotational motion can be resolved into two motions about two axes perpendicular to the line joining the atoms constituting a molecule as shown in Fig. 5.4. Hence *a diatomic gas possesses five degrees of freedom, i.e. three translational and two rotational.*

It can be observed that more the atoms in a molecule, the greater will be the number of degrees of freedom possessed by it.

5.9. Law of Equipartition of Energy*

This law states, *"The total energy of a molecule is shared equally by the various degrees of freedom possessed by it."*

In case of monoatomic molecules like argon and helium, the energy possessed by them is only that of translational type, the rotational energy being negligible. We have already discussed in Art. 5.4 that energy of translation per molecule

$$= \frac{3}{2} \times kT$$

* This law was devised by Maxwell in 1859 for translational degrees of freedom. But Boltzmann extended it further to rotational and vibrational energies.

This energy is equally distributed among the three translational degrees of freedom.

∴ Energy per molecule per degree of freedom

$$= \frac{1}{3} \times \frac{3}{2} kT = \frac{1}{2} kT$$

Hence, the law of equipartition of energy may be stated as follows :

"The total energy of any dynamical system in thermal equilibrium is divided equally among all its degrees of freedom and the energy associated with each degree of freedom is $\frac{1}{2} kT$."

5.10. Ratio of Molar Specific Heats

The ratio of molar specific heats for the following two cases is important from the subject point of view :

1. Monoatomic gas, and 2. Diatomic gas.

1. *Monoatomic gas.* We know that a monoatomic gas has *three* degrees of freedom of translation and energy per molecule of the gas per degree of freedom is $\frac{1}{2} kT$.

∴ Total energy per molecule for 3 degrees of freedom

$$= 3 \times \frac{1}{2} kT = \frac{3}{2} kT$$

and total energy per kg-molecule at temperature T K

$$= \frac{3}{2} kNT \qquad \ldots (N \text{ being number of molecules})$$

$$= \frac{3}{2} R_u T \qquad \ldots \left(\because k = \frac{R_u}{N} \right)$$

If the temperature of the gas is raised by 1 K, then total energy per kg-molecule at $(T + 1)$ K

$$= \frac{3}{2} R_u (T + 1)$$

It may be noted that the increase in total energy per kg-molecule per degree rise in temperature is equal to the heat required to raise the temperature of 1 kg molecule through 1°. This is known as *molar specific heat at constant volume, i.e.* c_{vm}.

∴ Molar specific heat at a constant volume,

$$c_{vm} = \frac{3}{2} R_u (T + 1) - \frac{3}{2} R_u T = \frac{3}{2} R_u$$

Since $c_{pm} - c_{vm} = R_u$, therefore

$$c_{pm} = c_{vm} + R_u = \frac{3}{2} R_u + R_u = \frac{5}{2} R_u$$

∴ Ratio of molar specific heats,

$$\gamma = \frac{c_{pm}}{c_{vm}} = \frac{\frac{5}{2} R_u}{\frac{3}{2} R_u} = \frac{5}{2} = 1.67$$

2. *Diatomic gas.* We know that the molecule of a diatomic gas has five degrees of freedom.

∴ Total energy per kg-molecule at temperature T K

$$= \frac{5}{2} R_u T$$

and total energy per kg-molecule at $(T + 1)$ K

$$= \frac{5}{2} R_u (T + 1)$$

Now, molar specific heat at a constant volume,

$$c_{vm} = \frac{5}{2} R_u (T + 1) - \frac{5}{2} R_u T = \frac{5}{2} R_u$$

Since $c_{pm} - c_{vm} = R_u$, therefore

$$c_{pm} = c_{vm} + R_u = \frac{5}{2} R_u + R_u = \frac{7}{2} R_u$$

∴ Ratio of molar specific heats,

$$\gamma = \frac{c_{pm}}{c_{vm}} = \frac{\frac{7}{2} R_u}{\frac{5}{2} R_u} = \frac{7}{5} = 1.4$$

5.11. Van der Waals' Equation of a Real Gas

Practically there is no *real gas* which obeys the gas laws perfectly. The deviation is small at ordinary temperatures and pressures, but it is large at high pressures and low temperatures. In deriving the perfect gas laws on the basis of kinetic theory of gases, the following assumptions were made which do not hold true for the real gases :

1. The molecules of a gas are mere mass-points occupying no space ; and
2. There is no attraction or repulsion between the molecules.

In actual practice, the molecules of all actual or real gases do occupy some space and do attract each other. Hence no real gas conforms to the perfect gas equation $p v = RT$. The Dutch physicist J.D. Van der Waals was the first scientist to correct this equation by applying corrections for the above two factors.

(*i*) *Correction for the size of molecules.* Consider some quantity of a gas contained in a vessel. We know that the molecules of a gas have a finite size, therefore the space available for the molecules to move about is less than the volume of the gas.

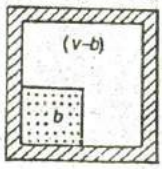


Fig. 5.5. Correction for the size of molecules.

Let v = Volume of the gas ; and

b = Volume occupied by all the molecules.

∴ Space actually available or the effective volume of the gas

$$= v - b$$

Note : In has been experimentally found that 'b' is four times the total volume of the molecules and not equal to the volume occupied by the molecules.

(*ii*) *Correction for the mutual attraction of molecules.* Consider a gas molecule A well within the body of the gas inside the vessel. It is attracted by other molecules in all directions with the same force and the net force on it is zero. But when it strikes the wall of the vessel, it is pulled back by other molecules. Its velocity, and hence the momentum, with which it strikes the wall would be less than the momentum with which it will strike in the absence of the force of attraction. It is obvious that when the pressure of the gas drops, the momentum of the molecules also decreases. It is due to this fact, that the decrease in pressure is proportional to :



Fig. 5.6. Correction for the mutual attraction of molecules.

1. The number of attracting molecules per unit volume, and
2. The number of attracting molecules striking a unit area of the walls of the containing vessel per unit time.

Each of these factors are proportional to the number of molecules per m^3 or the density of the gas.

$\therefore$ Decrease in pressure,

$$p' \propto (\text{density of the gas})^2 \propto \frac{1}{v^2} \quad \text{or} \quad p' = \frac{a}{v^2}$$

where a is constant of proportionality.

$$\therefore \text{Corrected pressure} = p + p' = p + \frac{a}{v^2}$$

Now with these corrections, the perfect gas equation for the real or actual gas becomes

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

This is known as *Van der Waals' equation for a real gas.*

EXERCISES

1. Calculate the r.m.s. velocity of nitrogen molecules at N.T.P. The density of nitrogen at N.T.P. is 1.25 kg/m^3 and density of mercury is 13 600 kg/m^3. Take $g = 9.81$ m/s^2 **[Ans. 493.3 m/s]**

2. Calculate the average kinetic energy of a gas molecule at a temperature of 300 K. The Boltzmann's constant is 1.38×10^{-26} kJ/K **[Ans. 621×10^{-26} kJ]**

3. Calculate the root mean square velocity and the mean kinetic energy of 1 kg molecule of hydrogen at N.T.P. The density of hydrogen is 0.09 kg/m^3. If the mass of a molecule of hydrogen is 3.34×10^{-27} kg, find Avogadro's number and Boltzmann's constant. **[Ans.** 1838.4 m/s ; 3380 kJ ; 6×10^{26} ; 1.386×10^{-25} kJ/K**]**

4. If the Avogadro's number is 6.02×10^{26} per kg mole, find the mass of a molecule of hydrogen. Calculate also the Boltzmann's constant, when $R_u = 8.314$ kJ/kg K.

[Ans. 0.33×10^{-26} kg ; 1.38×10^{-26} kJ/K**]**

5. The density of carbon dioxide gas at 0° C and at a pressure of 1 bar is 1.98 kg/m^3. Find the root mean square velocity of its molecules at 0° C and 30° C. The pressure is constant. **[Ans.** 389 m/s ; 411.0 m/s**]**

6. The root mean square velocity of the molecules of hydrogen at N.T.P. is 1840 m/s. Calculate the root mean square velocity of the oxygen molecules at N.T.P, if the molecular mass of hydrogen and oxygen are 2 and 32 respectively. **[Ans.** 460 m/s**]**

7. The root mean square velocity of oxygen at temperature 1092 K is 920 m/s. If the root mean square velocity of argon at a certain temperature is 450 m/s, find this temperature. The molecular mass of oxygen is 32 and that of argon is 40. **[Ans.** 327.6 K**]**

QUESTIONS

1. How many molecules are in one m^3 of air ?
2. What are the assumptions made in kinetic theory of gases ?
3. Define the following :
   (*a*) Mean velocity, (*b*) Root mean square velocity, and (*c*) Most probable velocity.
4. Derive an expression for the pressure exerted by a gas.
5. Show that the pressure of a gas in equal to two-thirds of the K.E. of translation per unit volume.
6. Write an expression for the r.m.s. velocity of a molecule in terms of :
   (*a*) Gas pressure and density,
   (*b*) Universal gas constant, absolute temperature and molecular mass of the gas,

(c) Boltzmann's constant, absolute temperature and mass of a gas molecule,

(d) Characteristic gas constant and absolute temperature of the gas.

7. Derive an expression for the average kinetic energy possessed by a gas molecule.

8. Derive the following gas laws as per kinetic theory of gases :

(a) Boyle's law, (b) Charles' law, (c) Gay-Lussac law, (d) Perfect gas equation, and (e) Avogadro's law.

9. What is meant by degrees of freedom possessed by a body ?

10. State the law of equipartition of energy. What is the amount of energy associated with each degree of freedom ?

OBJECTIVE TYPE QUESTIONS

1. The velocity of molecules

(a) increases with the increase of temperature

(b) increases with the decrease of temperature

(c) decreases with the increase of temperature

(d) remain constant at all temperatures

2. The root mean square velocity of the gas molecules (C) is given by

(a) $\sqrt{\frac{3m\,T}{k}}$ (b) $\sqrt{\frac{3\,T}{m\,k}}$ (c) $\sqrt{\frac{3k\,T}{m}}$ (d) $\sqrt{\frac{m\,k}{3T}}$

where m = Mass of one molecule of a gas,

k = Boltzmann's constant, and

T = Absolute temperature.

3. The ratio of root mean square velocity to average velocity of gas molecules at a particular temperature is

(a) 0.086 (b) 1.086 (c) 3.086 (d) 4.086

4. The pressure exerted by an ideal gas is ..... of the kinetic energy of all the molecules contained in a unit volume of gas.

(a) one-half (b) one-third (c) two-third (d) three-fourth

5. The kinetic energy per kg molecule of any gas at absolute temperature T is equal to

(a) $\frac{1}{2} R_u T$ (b) $\frac{3}{4} R_u T$ (c) $R_u T$ (d) $\frac{3}{2} R_u T$

ANSWERS

1. (a) 2. (c) 3. (b) 4. (c) 5. (d)

6

Thermodynamic Air Cycles

1. Introduction. 2. Assumptions in Thermodynamic Cycles. 3. Classifications of Thermodynamic Cycles. 4. Reversible Cycle. 5. Irreversible Cycle. 6. Reversibility and Irreversibility of Thermodynamic Processes. 7. Relation between Cycle and Engine. 8. Working of an Ideal Engine. 9. Important Terms used in Thermodynamic Cycles. 10. Efficiency of a Cycle. 11. Types of Thermodynamic Cycles. 12. Carnot Cycle. 13. Stirling Cycle. 14. Ericsson Cycle. 15. Joule's Cycle. 16. Otto Cycle. 17. Diesel Cycle. 18. Dual Combustion Cycle.

6.1. Introduction

A thermodynamic cycle or a cyclic process consists of a series of thermodynamic operations (processes), which take place in a certain order, and the initial conditions are restored at the end of the processes. When the operations or processes of cycle are plotted on p-v diagram, they form a closed figure, each operation being represented by its own curve. Since the area under each curve gives the work done to some scale, during each operation, it therefore follows that the net work done during one cycle will be given by the enclosed area of the diagram as shown, shaded in Fig. 6.1.

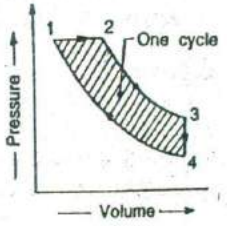


Fig. 6.1. A thermodynamic cycle.

The study of various thermodynamic cycles is very essential for the power developing systems (such as petrol engine diesel engine, gas turbine etc.). These engines use a mixture of fuel and air for their operations. Since the mass of fuel used, as compared to the mass of air is very small, therefore the mixture may be assumed to obey the properties of a perfect gas.

Notes : 1. A cycle, which requires four piston strokes and two complete revolutions of the crank is known as *four stroke cycle*. But a cycle, which requires only two piston strokes and one revolution of the crank, is known as *two stroke cycle*.

2. When air is assumed to be the working substance inside the engine cylinder, the cycle is called as an *air cycle*.

6.2. Assumptions in Thermodynamic Cycles

The analysis of all thermodynamic cycles (or air cycles) is based on the following assumptions :

1. The gas in the engine cylinder is a perfect gas, *i.e.* it obeys the gas laws and constant specific heats.
2. The physical constants of the gas in the engine cylinder are same as those of air at moderate temperatures.
3. All the compression and expansion processes are adiabatic and they take place without any internal friction.

4. Heat is supplied by bringing a hot body in contact with the cylinder at appropriate points during the process. Similarly heat is rejected by bringing a cold body in contact with the cylinder at these points.
5. The cycle is considered to be a closed one and the same air is used again and again to repeat the cycle.
6. No chemical reaction, whatsoever, takes place in the engine cylinder.

6.3. Classifications of Thermodynamic Cycles

The thermodynamic cycles, in general, may be classified into the following two types ;

1. Reversible or ideal cycle, and 2. Irreversible or natural or real cycle.

These cycles are discussed, in detail, in the following pages :

6.4. Reversible Cycle

A thermodynamically reversible cycle consists of reversible processes only. We have already discussed that a reversible process is one which is performed in such a way that at the end of the process, both the system and the surroundings may be restored to their initial states. For example, consider a process in which the system (gas) is expanded from state 1 to state 2 following the path 1-2 as shown in Fig. 6.2. Let during the thermodynamic process 1-2, the workdone by the system is W_{1-2} and the heat absorbed is Q_{1-2}. Now, if by doing the work (W_{1-2}) on the system (*i.e.* by compressing the gas) and extracting heat (Q_{1-2}) from the system, we can bring the system and the surroundings, back from state 2 to state 1 (*i.e.* initial state), following the same path 2-1, then the process is said to be a reversible process.

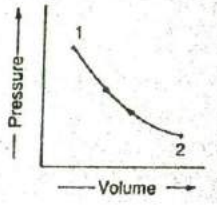


Fig. 6.2. Reversible process.

In a reversible process, there should not be any loss of heat due to friction, radiation or conduction, etc. A cycle will be reversible if all the processes constituting the cycle are reversible. Thus in a reversible cycle, the initial conditions are restored at the end of the cycle.

A little consideration will show, that when the operations are performed in the reversed order, the cycle draws heat from the cold body and rejects it to the hot body. This operation requires an external power to drive the mechanism according to second law of thermodynamics. A machine which operates on a reversed cycle is regarded as a "heat pump", such as a refrigerator, because it pumps heat from the cold body to the hot body. Following are the conditions for reversibility of a cycle :

1. The pressure and temperature of the working substance must not differ, appreciably, from those of the surroundings at any stage in the process.
2. All the processes, taking place in the cycle of operation, must be extremely slow.
3. The working parts of the engine must be friction free.
4. There should be no loss of energy during the cycle of operation.

Note : A reversible cycle should not be confused with a mechanically reversible engine. Steam engine cranks may be made to revolve in a reversed direction by mechanically altering the valve settings. But this does not reverse the cycle, on which it works. A two-stroke petrol engine may be made to revolve in reverse direction by altering the timing of ignition. But this also does not reverse the actual cycle.

6.5. Irreversible cycle

In the previous article, we have discussed that in a reversible process, the heat and work are completely restored back by reversing the process (*i.e.* by compressing the gas). But when the heat and work are not completely restored back by reversing the process, then the process is known as *irreversible process* (also called *natural* or *real process*). In an irreversible process, there is a loss of heat due to friction, radiation or conduction.

In actual practice most of the processes are irreversible to some degree. The main causes for the irreversiblity are:

1. mechanical and fluid friction, 2. unrestricted expansion, and 3. heat transfer with a finite temperature difference. Moreover, friction converts the mechanical work into heat. This heat cannot supply back the same amount of mechanical work, which was consumed for its production. Thus, if there is some friction involved in the process, it becomes irreversible. A cycle will be irreversible if any of the processes, constituting the cycle, is irreversible. Thus in an irreversible cycle, the initial conditions are not restored at the end of the cycle.

6.6. Reversibility and Irreversibility of Thermodynamic Processes

We have already discused the various thermodynamic processes in the last chapter. Now we shall discuss their conditions of reversibility and irreversibility.

1. *Isothermal and adiabatic processes.* It may be noted that a complete process or cycle is only an ideal case. But in actual practice, complete isothermal and adiabatic operations are not achieved. However, they can be approximated. The simple reason for the same is that it is impossible to transfer heat at a constant temperature in case of an isothermal operation. Moreover, it is also impossible to make an absolutely non-conducting cylinder in case of an adiabatic operation. In actual practice, however, an isothermal operation may be approached if the process is so slow that the heat is absorbed or rejected at such a rate that the temperature remains, practically, constant. Similarly, an adiabatic operation may be approached if the process takes place so *quickly* that no time is given to the heat to enter or leave the gas.

In view of the above, the isothermal and adiabatic processes are taken as reversible processes.

2. *Constant volume, constant pressure and constant $p\,v^n$ processes.* We know that when the temperature of the hot body, supplying the heat, remains constant during the process, the temperature of the working substance will vary as the operation proceeds. In view of this, the above three operations are irreversible. But, these can be made to approximate to reversibility by manipulating the temperature of the hot body to vary* so that at any stage the temperature of the working substance remains constant.

In this way, the constant volume, constant pressure and constant pv^n processes are regarded as reversible processes.

3. *Free expansion and throttling processes.* These processes are irreversible, as there is always a loss of heat due to friction when the working substance passes through an orifice.

6.7. Relation between Cycle and Engine

In the study of basic theory of a heat engine, it is assumed that the working fluid is used again and again in the cylinder. We say that the fluid has undergone a cycle, when it passes through different processes and returns back to its original state.

As a matter of fact, the working fluid in an actual engine does not go through a complete cycle, and operates on an open cycle. But for the purpose of simplicity in analysis, we study the closed cycle (*i.e.* ideal cycle), which closely approximates with the open cycle.

6.8. Working of an Ideal Engine

An ideal engine may be defined as a device, which develops work (*i.e.* power) continuously with the help of a working fluid, which undergoes some cyclic process. It is done with the help of a piston and cylinder as shown in Fig. 6.3.

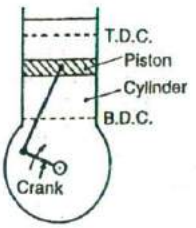


Fig. 6.3. Ideal engine.

In general, the arrangement of piston and cylinder, of an ideal engine, constitutes the cycle by the following processes :

* An apparatus known as regenerator is used for alternately storing and supplying heat in a reversible manner.

1. Air in the cylinder is heated with the help of some external source. It increases temperature and pressure of the enclosed air.
2. The air expands due to higher pressure and temperature. As a result of this, some work is done by the air.
3. The air then rejects some heat to the external source. Thus the air comes back to the original conditions.
4. The air is then compressed in the cylinder. As a result of this, some work is done on the air.

Note : The scientists, working on the research and development of the engines, have focussed their attention mainly on the process of heating the enclosed air. Thus the various engines are classified according to the process of heat addition.

6.9. Important Terms used in Thermodynamic Cycles

Though there are many terms used in thermodynamic cycles, yet the following are important from the subject point of view :

1. *Cylinder bore.* The inner diameter of the cylinder, in which the piston moves, is known as cylinder bore.

2. *Stroke length.* The piston moves in the cylinder due to rotation of the crank. Its extreme positions are known as top dead centre (TDC) and bottom dead centre (BDC) respectively as shown in Fig. 6.3. The distance between these two extreme positions is known as stroke length or stroke.

3. *Clearance volume.* The volume occupied by the working fluid, when piston reaches the top dead centre, is known as clearance volume. It is generally denoted by (v_c).

4. *Swept volume.* The volume swept by the piston, when it moves between the two extreme positions is known as swept volume or displacement volume or stroke volume. Mathematically, swept volume,

$$v_s = \text{Piston area} \times \text{Stroke length} = \frac{\pi}{4} \times d^2 \times l$$

where d = Cylinder bore or diameter of the piston.

5. *Total cylinder volume.* The volume occupied by the working fluid, when the piston is at the bottom dead centre, is known as total cylinder volume. Mathematically, total cylinder volume is equal to the sum of clearance volume (v_c) and swept volume (v_s).

6. *Compression ratio.* The ratio of total cylinder volume to the clearance volume is known as compression ratio. It is an important term used in an engine. Mathematically, compression ratio,

$$r = \frac{v_c + v_s}{v_c} = 1 + \frac{v_s}{v_c}$$

7. *Mean effective pressure.* As a matter of fact, pressure in the cylinder keeps on changing with the position of the piston. For all sorts of calculations, we need the mean effective pressure, which may be defined as the constant pressure acting on the piston during the working stroke. It will be able to do the same amount of work, as done by the actual varying pressure, produced during the cycle. It is ratio of work done to the stroke volume or piston displacement volume. Mathematically, mean effective pressure,

$$p_m = \frac{\text{Work done}}{\text{Stroke volume}}$$

6.10. Efficiency of a Cycle

It may be defined as the ratio of work done to the heat supplied during a cycle. Mathematically, efficiency of a cycle,

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}}$$

Since the work done during a cycle is equal to heat supplied *minus* the heat rejected, the efficiency of a cycle, therefore, may also be expressed as

$$\eta = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}}$$

Notes : 1. The efficiency, as given above, is the theoretical efficiency of the cycle. Therefore it is known as *theoretical thermal efficiency.*

2. It does not take into account the practical losses, which occur in the running of the engine.

3. In order to compare the efficiency of the thermodynamic cycles, air is assumed to be the working substance inside the engine cylinder. Moreover, air is assumed to behave as a perfect gas. The efficiency, thus, obtained is known as *air standard efficiency.* It is also called *ideal efficiency.*

6.11. Types of Thermodynamic Cycles

Though there are many types of thermodynamic cycles, yet the following are important from the subject point of view :

1. Carnot cycle, 2. Stirling cycle, 3. Ericsson cycle, 4. Joule cycle, 5. Otto cycle, 6. Diesel cycle, and 7. Dual combustion cycle.

The above mentioned cycles will be discussed, in detail, in the following pages.

6.12. Carnot Cycle

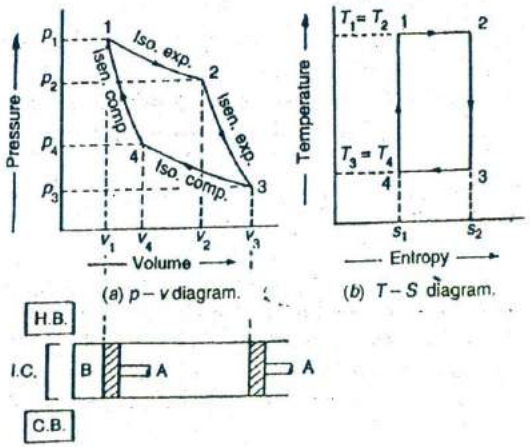


Fig. 6.4. Carnot cycle.

This cycle was devised by *Carnot, who was the first scientist to analyse the problem of the efficiency of a heat engine, disregarding its mechanical details. He focussed his attention on the basic features of a heat engine. In a Carnot cycle, the working substance is subjected to a cyclic operation consisting of two isothermal and two reversible adiabatic or isentropic operations. The *p-v* and *T-S* diagrams of this cycle are shown in Fig. 6.4 (*a*) and (*b*).

*. Nicolas Leonard Sadi Carnot was a French engineer. He devised this cycle in his early age.

The engine imagined by Carnot has air (which is supposed to behave like a perfect gas) as its working substance enclosed in a cylinder, in which a frictionless piston *A* moves. The walls of the cylinder and piston are perfect non-conductor of heat. However, the bottom *B* of the cylinder can be covered, at will, by an insulating cap (I.C.). The engine is assumed to work between two sources of infinite heat capacity, one at a higher temperature and the other at a lower temperature.

Now, let us consider the four stages of the Carnot's cycle. Let the engine cylinder contain m kg of air at its original condition represented by point 1 on the p-v and T-S diagrams. At this point, let p_1, T_1 and v_1 be the pressure, temperature and volume of the air, respectively.

1. *First stage (Isothermal expansion).* The source (hot body, H.B.) at a higher temperature is brought in contact with the bottom *B* of the cylinder. The air expands, practically at constant temperature T_1, from v_1 to v_2. It means that the temperature T_2 at point 2 is equal to the temperature T_1. This isothermal expansion is represented by curve 1-2 on p-v and T-S diagrams in Fig. 6.4 (*a*) and (*b*). It may be noted that the heat supplied by the hot body is fully absorbed by the air, and is utilised in doing external work.

∴ Heat supplied = *Work done by the air during isothermal expansion

or
$$Q_{1-2} = p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right) = m R T_1 \log_e \left(\frac{v_2}{v_1}\right) \quad \ldots (\because p_1 v_1 = m R T_1)$$
$$= 2.3\, m R T_1 \log r$$

where r = Expansion ratio = v_2/v_1.

2. *Second stage (Reversible adiabatic or isentropic expansion).* The hot body is removed from the bottom of the cylinder *B* and the insulating cap I.C. is brought in contact. The air is now allowed to expand reversibly and adiabatically. Thus the reversible adiabatic expansion is represented by the curve 2-3 on p-v and T-S diagrams. The temperature of the air falls from T_2 to T_3. Since no heat is absorbed or rejected by the air, therefore

Decrease in internal energy = Workdone by the air during adiabatic expansion

$$= \frac{p_2 v_2 - p_3 v_3}{\gamma - 1} = \frac{m R T_2 - m R T_3}{\gamma - 1} \quad \ldots (\because p v = m R T)$$

$$= \frac{m R (T_1 - T_3)}{\gamma - 1} \quad \ldots (\because T_1 = T_2)$$

3. *Third stage (Isothermal compression).* Now remove the insulating cap I.C. from the bottom of the cylinder and bring the cold body C.B. in its contact. The air is compressed practically at a constant temperature T_3 from v_3 to v_4. It means that the temperature T_4 (at point 4) is equal to the temperature T_3. This isothermal compression is represented by the curve 3-4 on p-v and T-S diagrams. It would be seen that during this process, the heat is rejected to the cold body and is equal to the work done on the air.

∴ Heat rejected = Work done on the air during isothermal compression

$$Q_{3-4} = p_3 v_3 \log_e \left(\frac{v_3}{v_4}\right) = m R T_3 \log_e \left(\frac{v_3}{v_4}\right) \quad \ldots (\because p v = m R T)$$
$$= 2.3\, m R T_3 \log r$$

---

* Since the temperature is constant, therefore there is no change in internal energy of the air, *i.e.* $dU = 0$. According to the first law of thermodynamics,

$$Q_{1-2} = dU + W_{1-2} \text{ or } Q_{1-2} = W_{1-2}$$

where r = *Compression ratio = v_3/v_4

4. *Fourth stage (Reversible adiabatic or isentropic compression).* Now again the insulated cap I.C. is brought in contact with the bottom of the cylinder B, and the air is allowed to be compressed reversibly and adiabatically. The reversible adiabatic compression is represented by the curve 4-1 on p-v and T-S diagrams. The temperature of the air increases from T_4 to T_1. Since no heat is absorbed or rejected by the air, therefore

Increase in internal energy = Work done on the air during adiabatic compression

$$= \frac{p_1 v_1 - p_4 v_4}{\gamma - 1} = \frac{m R T_1 - m R T_4}{\gamma - 1} \quad \ldots (\because p v = m R T)$$

$$(-) \quad = \frac{m R (T_1 - T_3)}{\gamma - 1} \quad \ldots (\because T_3 = T_4)$$

We see from the above discussion that the decrease in internal energy during reversible adiabatic expansion 2-3 is equal to the increase in internal energy during reversible adiabatic compression 4-1. Hence their net effect during the whole cycle is zero. We know that

Work done, W = Heat supplied – Heat rejected

$$= 2.3\, m R T_1 \log r - 2.3\, m R T_3 \log r = 2.3\, m R \log r (T_1 - T_3)$$

and efficiency $$**\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{2.3\, m R \log r (T_1 - T_3)}{2.3\, m R T_1 \log r}$$

$$= \frac{T_1 - T_3}{T_1} = 1 - \frac{T_3}{T_1}$$

---

* The expansion and compression ratios (r) must be equal, otherwise the cycle would not close.

We know that for reversible adiabatic or isentropic expansion process 2-3,

$$\frac{T_2}{T_3} = \left(\frac{v_3}{v_2}\right)^{\gamma - 1} \quad \text{or} \quad \frac{v_3}{v_2} = \left(\frac{T_2}{T_3}\right)^{\frac{1}{\gamma - 1}} \quad \ldots (i)$$

Similarly, for reversible adiabatic or isentropic compression process 4-1,

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1}\right)^{\gamma - 1} \quad \text{or} \quad \frac{v_4}{v_1} = \left(\frac{T_1}{T_4}\right)^{\frac{1}{\gamma - 1}} \quad \ldots (ii)$$

Since $T_1 = T_2$ and $T_3 = T_4$, therefore

$$\frac{v_3}{v_2} = \frac{v_4}{v_1} \quad \text{or} \quad r = \frac{v_2}{v_1} = \frac{v_3}{v_4}$$

Alternative Proof.

Heat supplied during isothermal expansion 1-2,

$$Q_{1\text{-}2} = T_1 (S_2 - S_1) \quad \ldots (\because \delta Q = T\, dS)$$

and heat rejected during isothermal compression 3-4,

$$Q_{3\text{-}4} = T_4 (S_2 - S_1) = T_3 (S_2 - S_1) \quad \ldots (\because T_4 = T_3)$$

We know that work done = Heat supplied – Heat rejected

$$= T_1 (S_2 - S_1) - T_3 (S_2 - S_1) = (T_1 - T_3)(S_2 - S_1)$$

∴ Efficiency, $$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{(T_1 - T_3)(S_2 - S_1)}{T_1 (S_2 - S_1)} = \frac{T_1 - T_3}{T_1} = 1 - \frac{T_3}{T_1}$$

The expression for the efficiency of a Carnot cycle may also be written as discussed below :

We know that for reversible adiabatic or isentropic expansion 2-3,

$$\frac{T_2}{T_3} = \left(\frac{v_3}{v_2}\right)^{\gamma-1} \quad \text{or} \quad \frac{T_1}{T_3} = \left(\frac{v_3}{v_2}\right)^{\gamma-1} \qquad \ldots(\because T_2 = T_1) \ldots (i)$$

Similarly, for reversible adiabatic or isentropic compression process 4-1,

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1}\right)^{\gamma-1} \quad \text{or} \quad \frac{T_1}{T_3} = \left(\frac{v_4}{v_1}\right)^{\gamma-1} \qquad \ldots(\because T_4 = T_3) \ldots (ii)$$

From equations (*i*) and (*ii*),

$$\frac{v_3}{v_2} = \frac{v_4}{v_1} \quad \text{or} \quad \frac{v_2}{v_1} = \frac{v_3}{v_4} = r$$

where r = Ratio of expansion or compression.

$$\therefore \quad \frac{T_1}{T_3} = (r)^{\gamma-1}$$

We know that efficiency,

$$\eta = 1 - \frac{T_3}{T_1} = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - \frac{1}{r^{\gamma-1}}$$

Notes : 1. From the above equation, we see that the efficiency of Carnot's cycle increases as T_1 is increased or T_3 is decreased. In other words, the heat should be taken in at as high a temperature as possible, and rejected at as low a temperature as possible. It may be noted that 100% efficiency can be achieved, only, if T_3 reaches absolute zero, though it is impossible to achieve in practice.

2. In the above theory, we have taken temperature at points 1, 2, 3 and 4 as T_1, T_2, T_3 and T_4 respectively in order to keep similarity between Carnot cycle and other cycles. But some authors take it T_1 (for points 1 and 2) and T_2 (for points 3 and 4). In that case, they obtain the relation for efficiency as,

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

3. It may be noted that it is impossible to make an engine working on Carnot's cycle. The simple reason for the same is that the isothermal expansion 1-2 will have to be carried out extremely slow to ensure that the air is always at temperature T_1. Similarly, the isothermal compression 3-4 will have to be carried out extremely slow. But reversible adiabatic expansion 2-3 and reversible adiabatic compression 4-1 should be carried out as quickly as possible, in order to approach ideal adiabatic conditions. We know that sudden changes in the speed of an engine are not possible in actual practice. Moreover, it is impossible to completely eliminate friction between the various moving parts of the engine, and also heat losses due to conduction, radiation, etc. It is thus obvious, that it is impossible to realise Carnot's engine in actual practice. However, such an imaginary engine is used as the ultimate standard of comparison of all heat engines.

Example 6.1. *A Carnot engine, working between 650 K and 310 K, produces 150 kJ of work. Find thermal efficiency and heat added during the process.*

Solution. $T_1 = 650$ K ; $T_3 = 310$ K ; $W = 150$ kJ

Thermal efficiency

We know that thermal efficiency,

$$\eta = \frac{T_1 - T_3}{T_1} = \frac{650 - 310}{650} = 0.523 \quad \text{or} \quad 52.3\% \text{ Ans.}$$

Heat added during the process

We know that heat added during the process,

$$Q_{1-2} = \frac{W}{\eta} = \frac{150}{0.523} = 286.8 \text{ kJ Ans.}$$

Example 6.2. *A Carnot engine operates between two reservoirs at temperatures T_1 and T_3. The work output of the engine is 0.6 times the heat rejected. The difference in temperatures between the source and the sink is 200° C. Calculate the thermal efficiency, source temperature and the sink temperature.*

Solution. Given : $W = 0.6 \times$ Heat rejected $= 0.6\, Q_{3-4}$; $T_1 - T_3 = 200°$ C

Thermal efficiency

We know that the thermal efficiency,

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{\text{Work done}}{\text{Work done} + \text{Heat rejected}}$$

$$= \frac{0.6\, Q_{3-4}}{0.6\, Q_{3-4} + Q_{3-4}} = \frac{0.6}{1.6} = 0.375 \quad \text{or} \quad 37.5\% \text{ Ans.}$$

Source and sink temperatures

Let T_1 = Source temperature, and

T_3 = Sink temperature.

We know that thermal efficiency (η),

$$0.375 = \frac{T_1 - T_3}{T_1} = \frac{200}{T_1}$$

$$\therefore \quad T_1 = 200 / 0.375 = 533.3 \text{ K} = 260.3° \text{ C Ans.}$$

and $$T_3 = T_1 - 200 = 260.3 - 200 = 60.3° \text{ C Ans.}$$

Example 6.3. *An engineer claims his engine to develop 3.75 kW. On testing, the engine consumes 0.44 kg of fuel per hour having a calorific value of 42 000 kJ/kg. The maximum temperature recorded in the cycle is 1400° C and minimum is 350° C. Find whether the engineer is justified in his claim.*

Solution. Give : $P = 3.75$ kW; Fuel consumed = 0.44 kg/h; Calorific value = 42 000 kJ/kg

$T_1 = 1400°$ C $= 1400 + 273 = 1673$ K ; $T_3 = 350°$ C $= 350 + 273 = 623$ K

We know that the maximum efficiency, between two specified temperatures, is that of Carnot cycle.

$$\therefore \quad \eta_{carnot} = \frac{T_1 - T_3}{T_1} = \frac{1673 - 623}{1673} = 0.627 \quad \text{or} \quad 62.7\%$$

We also know that the heat supplied to the engine by the fuel

$$= \text{Fuel consumed} \times \text{Calorific value of fuel}$$

$$= 0.44 \times 42\,000 = 18\,480 \text{ kJ/h} = 5.13 \text{ kJ/s}$$

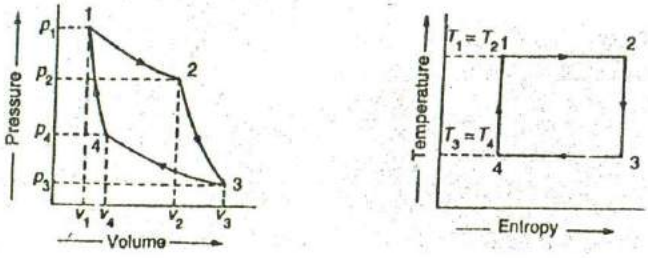
and workdone by the engine $= 3.75$ kW $= 3.75$ kJ/s

$$\therefore \text{ Efficiency claimed} = \frac{\text{Workdone}}{\text{Heat supplied}} = \frac{3.75}{5.13} = 0.731 \quad \text{or} \quad 73.1\ \%$$

Since efficiency of the engine claimed (73.1%) is more than the maximum possible efficiency (62.7%), therefore the engineer is not justified in his claim. **Ans.**

Example 6.4. *A Carnot cycle works with isentropic compression ratio of 5 and isothermal expansion ratio of 2. The volume of air at the beginning of the isothermal expansion is 0.3 m³. If the maximum temperature and pressure is limited to 550 K and 21 bar, determine : 1. minimum temperature in the cycle ; 2. thermal efficiency of the cycle ; 3. pressure at all salient points ; 4. change of entropy during the isothermal expansion, 5. work done per cycle, and 6. mean effective pressure. Take ratio of specific heats as 1.4.*

Solution. Given : $v_4 / v_1 = 5$; $v_2 / v_1 = 2$; $v_1 = 0.3\ m^3$; $T_1 = 550$ K ; $p_1 = 21$ bar $= 2.1 \times 10^6$ N/m² ; $\gamma = 1.4$



(*a*) *p-v* diagram. (*b*) *T-S* diagram.

Fig. 6.5

1. *Minimum temperature in the cycle*

Let T_4 (or T_3) = Minimum temperature in the cycle.

The cycle on *p-v* and *T-S* diagrams is shown in Fig. 6.5 (*a*) and (*b*) respectively. First of all, let us consider the isentropic compression process 4-1. We know that

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1}\right)^{\gamma - 1} = (5)^{1.4-1} = (5)^{0.4} = 1.9036$$

$$\therefore \quad T_4 = T_1 / 1.9036 = 550 / 1.9036 = 289\ \text{K} = 16^\circ\text{C}\ \text{Ans.}$$

2. *Thermal efficiency of the cycle*

We know that thermal efficiency of the cycle,

$$\eta = \frac{T_1 - T_3}{T_1} = \frac{550 - 289}{550} = 0.4745 \text{ or } 47.45\ \%\ \text{Ans.}$$

$\ldots (\because T_3 = T_4)$

3. *Pressure at all the salient points*

Let p_2, p_3, p_4 = Pressures at points 2, 3 and 4 respectively.

First of all, let us consider the isothermal expansion process 1-2. We know that in an isothermal expansion,

$$p_1 v_1 = p_2 v_2 \quad \text{or} \quad p_2 = p_1 \times \frac{v_1}{v_2} = 21 \times \frac{1}{2} = 10.5 \text{ bar Ans.}$$

$\ldots (\because v_2 / v_1 = 2)$

Now consider the isentropic expansion process 2-3, we know that

$$p_2 v_2^{\gamma} = p_3 v_3^{\gamma} \text{ or } p_3 = p_2 \left(\frac{v_2}{v_3}\right)^{\gamma} = 10.5 \left(\frac{1}{5}\right)^{1.4} = 10.5\,(0.2)^{1.4}$$

$$= 1.103 \text{ bar } \textbf{Ans.} \qquad \ldots \left(\because \frac{v_4}{v_1} = \frac{v_3}{v_2}\right)$$

Now consider the isentropic compression process 4-1. We know that

$$p_4 v_4^{\gamma} = p_1 v_1^{\gamma} \text{ or } p_4 = p_1 \left(\frac{v_1}{v_4}\right)^{\gamma} = 21 \left(\frac{1}{5}\right)^{1.4} = 21\,(0.2)^{1.4}$$

$$= 2.206 \text{ bar } \textbf{Ans.}$$

4. *Change of entropy during the isothermal expansion*

We know that change of entropy during the isothermal expansion,

$$S_2 - S_1 = 2.3\, m R \log\left(\frac{v_2}{v_1}\right) = 2.3 \times \frac{p_1 v_1}{T_1} \log\left(\frac{v_2}{v_1}\right) \quad \ldots (\because p_1 v_1 = m R T_1)$$

$$= 2.3 \times \frac{2.1 \times 10^6 \times 0.3}{550} \times \log 2 = 2.636 \times 10^3 \times 0.301 = 793 \text{ J/K}$$

$$= 0.793 \text{ kJ/K } \textbf{Ans.}$$

5. *Workdone per cycle*

We know that heat supplied during the cycle,

$$Q_{1\text{-}2} = T_1 (S_2 - S_1) = 550 \times 0.793 = 436 \text{ kJ}$$

and heat rejected during the cycle,

$$Q_{3\text{-}4} = T_3 (S_2 - S_1) = 289 \times 0.793 = 229 \text{ kJ}$$

∴ Workdone per cycle,

$$W = \text{Heat supplied} - \text{Heat rejected} = 436 - 229 = 207 \text{ kJ } \textbf{Ans.}$$

Note: The heat supplied and heat rejected may also be obtained as discussed below :
We know that heat supplied,

$$Q_{1\text{-}2} = 2.3\, p_1 v_1 \log\left(\frac{v_2}{v_1}\right) = 2.3 \times 2.1 \times 10^6 \times 0.3 \log 2$$

$$= 1.449 \times 10^6 \times 0.301 = 436 \times 10^3 \text{ J} = 436 \text{ kJ}$$

and heat rejected,

$$Q_{3\text{-}4} = 2.3\, p_3 v_3 \log\left(\frac{v_3}{v_4}\right) = 2.3 \times 0.1103 \times 10^6 \times 3 \log 2$$

$$= 0.761 \times 10^6 \times 0.301 = 229 \times 10^3 \text{ J} = 229 \text{ kJ}$$

$$\ldots \left[\begin{array}{l} \because \dfrac{v_3}{v_4} = \dfrac{v_2}{v_1} = 2 \text{ and } \dfrac{v_4}{v_1} = 5 \\ \text{or } \dfrac{v_3}{v_4} \times \dfrac{v_4}{v_1} = \dfrac{v_3}{v_1} = 2 \times 5 = 10 \\ \therefore v_3 = 10\, v_1 = 10 \times 0.3 = 3 \text{ m}^3 \end{array}\right]$$

6. *Mean effective pressure*

We know that the stroke volume in a Carnot cycle

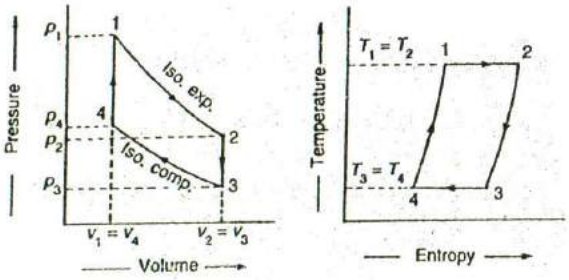
$$= v_3 - v_1 = 3.0 - 0.3 = 2.7 \text{ m}^3$$

∴ Mean effective pressure

$$= \frac{\text{Work done}}{\text{Stroke volume}} = \frac{207}{2.7} = 76.7 \text{ kN/m}^2 \text{ Ans.}$$

6.13. Stirling Cycle

This cycle was devised by *Stirling, which consists of two isothermal processes and two constant volume processes. The last two processes are performed with the help of a regenerator to make this cycle reversible. The p-v and T-S diagrams of this cycle are shown in Fig. 5.6.



(*a*) p-v diagram. (*b*) T-S diagram.

Fig. 6.6. Stirling cycle.

Let us now consider the four stages of the Striling cycle. Let the engine cylinder contain m kg of air at its original position represented by point 1. At this point, let p_1, T_1 and v_1 be the pressure, temperature and volume of the air respectiveiy.

1. *First stage (Isothermal expansion).* The air expands isothermally, practically, at constant temperature (T_1 or T_2) from v_1 to v_2. It means that the temperature T_2 (at point 2) is equal to temperature T_1. This isothermal expansion is represented by the curve 1-2 in Fig. 6.6 (*a*) and (*b*). The heat supplied by the external source is absorbed during this process.

∴ Heat supplied = Work done during isothermal expansion

$$Q_{1\text{-}2} = p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right) = m R T_1 \log_e \left(\frac{v_2}{v_1}\right) \quad \ldots (\because p_1 v_1 = m R T_1)$$

$$= 2.3\, m R T_1 \log r$$

where r = Expansion ratio $= v_2 / v_1$

2. *Second stage (Constant volume cooling).* The air now passes through the regenerator, and is cooled at constant volume to a temperature T_3. This process is represented by a curve 2-3 on p-v and T-S diagrams. In this process, heat is rejected to the regenerator.

∴ Heat rejected to the regenerator,

$$Q_{2\text{-}3} = m c_v (T_2 - T_3)$$

* Robert Stirling devised this cycle in 1845.

3. *Third stage (Isothermal compression).* The air is now compressed isothermally, practically, at a constant temperature (T_3) in the engine cylinder from v_3 to v_4. This process is represented by the curve 3-4 on p-v and T-S diagrams. Again, heat is rejected by the air during this process.

∴ Heat rejected by the air,

$$Q_{3\text{-}4} = p_3 v_3 \log_e \left(\frac{v_3}{v_4}\right) = m R T_3 \log_e \left(\frac{v_3}{v_4}\right) \quad \ldots (\because p_3 v_3 = m R T_3)$$

$$= 2.3\, m R T_3 \log r$$

where r = Compression ratio $= v_3 / v_4$

4. *Fourth stage (Constant volume heating).* Lastly, the air is heated at a constant volume to a temperature T_1, by passing it though the regenerator in the reverse direction to that of process 2-3. It may be noted that the air restores initial conditions at the end of this process, and completes the cycle. It is thus obvious, that the heat is absorbed by the air from the regenerator during this process, *i.e.* 4-1.

∴ Heat absorbed by the air,

$$Q_{4\text{-}1} = \text{Heat taken in from the regenerator}$$

$$= m c_v (T_1 - T_4) = m c_v (T_1 - T_3) \quad \ldots (\because T_3 = T_4)$$

We see that the heat rejected to the regenerator during process 2-3 is equal to the heat taken in from the regenerator during process 4-1. Hence there is no interchange of heat to the system during these processes. The only interchange of heat is during the two isothermal processes.

∴ Work done = Heat supplied – Heat rejected

$$= 2.3\, m R T_1 \log r - 2.3\, m R T_3 \log r = 2.3\, m R \log r (T_1 - T_3)$$

and efficiency,

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{2.3\, m R \log r (T_1 - T_3)}{2.3\, m R T_1 \log r}$$

$$= \frac{T_1 - T_3}{T_1} = 1 - \frac{T_3}{T_1}$$

Notes : 1. The efficiency of the Stirling cycle is same as that of Carnot cycle. This is due to the fact that the cycle is reversible, and all reversible cycles have the same efficiency.

2. If the efficiency of regenerator is η_R, then heat taken in from the regenerator during process 4-1 will be $m c_v (T_1 - T_3)(1 - \eta_R)$. In that case,

$$\eta = \frac{2.3\, m R \log r (T_1 - T_3)}{2.3\, m R T_1 \log r + m c_v (T_1 - T_3)(1 - \eta_R)}$$

$$= \frac{2.3\, R \log r (T_1 - T_3)}{2.3\, R T_1 \log r + c_v (T_1 - T_3)(1 - \eta_R)}$$

Example 6.5. *An air engine, working on Stirling cycle, has lower limit of temperature as 400° C. The maximum and minimum pressure limits are 12 bar and 2 bar. If the expansion ratio of the cycle is 3, find its ideal efficiency.*

Solution. Given : $T_3 = T_4 = 400°\text{ C} = 400 + 273 = 673\text{ K}$; $p_1 = 12$ bar ; $p_3 = 2$ bar ; $v_2 / v_1 = v_3 / v_4 = 3$

We know that in isothermal expansion process 1-2 (Refer Fig. 6.6),

$$p_1 v_1 = p_2 v_2 \quad \text{or} \quad p_2 = p_1 \times \frac{v_1}{v_2} = 12 \times \frac{1}{3} = 4\text{ bar}$$

Similarly, in isothermal compression process 3-4,

$$p_3 v_3 = p_4 v_4 \quad \text{or} \quad p_4 = p_3 \times \frac{v_3}{v_4} = 2 \times 3 = 6 \text{ bar}$$

Since the air is heated at constant volume in process 4-1, therefore

$$T_1 = T_4 \times \frac{p_1}{p_4} = 673 \times \frac{12}{6} = 1346 \text{ K} \qquad \ldots \left(\because \frac{p_1}{T_1} = \frac{p_4}{T_4} \right)$$

$\therefore$ Efficiency, $$\eta = 1 - \frac{T_3}{T_1} = 1 - \frac{673}{1346} = 0.5 \text{ or } 50\% \text{ Ans.}$$

Example 6.6. *A Stirling air engine is fitted with a regenerator of efficiency 90%. It operates between the temperature limits of 350° C and 50° C. Determine the efficiency of the engine, assuming isothermal expansion ratio as 2. Take $c_p = 1.005$ kJ/kg K and $c_v = 0.712$ kJ/kg K.*

Solution. Given : $\eta_R = 90\% = 0.9$; $T_1 = 350° \text{ C} = 350 + 273 = 623$ K ; $T_3 = 50° \text{ C} = 50 + 273 = 323$ K ; $r = v_2 / v_1 = 2$

We know that gas constant,

$$R = c_p - c_v = 1.005 - 0.712 = 0.293 \text{ kJ/kg K}$$

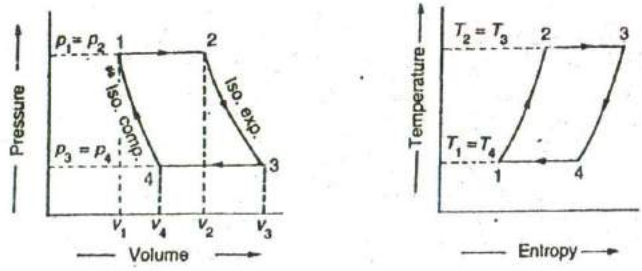
$\therefore$ Efficiency of the engine,

$$\eta = \frac{2.3 R \log r (T_1 - T_3)}{2.3 R T_1 \log r + c_v (T_1 - T_3) (1 - \eta_R)}$$

$$= \frac{2.3 \times 0.293 \log 2 (623 - 323)}{2.3 \times 0.293 \times 623 \log 2 + 0.712 (623 - 323) (1 - 0.9)}$$

$$= \frac{60.85}{126.37 + 21.36} = 0.412 \text{ or } 41.2\% \text{ Ans.}$$

6.14. Ericsson Cycle



(*a*) *p-v* diagram. (*b*) *T-S* diagram.

Fig. 6.7. Ericsson cycle.

This cycle was invented by *Ericsson, which consists of two isothermal and two constant pressure processes. It is made thermodynamically reversible by the action of a regenerator. The *p-v* and *T-S* diagrams of the cycle are shown in Fig. 6.7 (*a*) and (*b*). This cycle is used these days in the manufacture of closed-cycle type gas turbines.

* J. Ericsson was an American engineer, who invented this engine in 1840. He used a hot air engine, working on this cycle, for running a ship (known as Ericsson) in 1853.

Now, let us consider the four stages of the Ericsson cycle. Let the engine contain m kg of air at its original position represented by point 1 on p-v and T-S diagrams. At this point, let p_1, T_1 and v_1 be the pressure, temperature and volume of the air.

1. *First stage (Constant pressure heating).* The air is heated at constant pressure from initial temperature T_1 to a temperature T_2 represented by the curve 1-2 in Fig. 6.7 (*a*) and (*b*).

∴ Heat supplied to the air,

$$Q_{1\text{-}2} = m\, c_p\, (T_2 - T_1)$$

2. *Second stage (Isothermal expansion).* The air is allowed to expand isothermally (*i.e.* at constant temperature $T_2 = T_3$) from initial volume v_2 to v_3 represented by the curve 2-3 in Fig. 6.7 (*a*) and (*b*). We know that a part of the heat supplied in the first stage is utilised for doing work in isothermal expansion.

∴ Heat utilised during isothermal expansion,

$$Q_{2\text{-}3} = p_2 v_2 \log_e \left(\frac{v_3}{v_2}\right) = m R T_2 \log_e \left(\frac{v_3}{v_2}\right) \qquad \ldots (\because p v = m R T)$$

$$= 2.3\, m R T_2 \log r$$

where r = Expansion ratio = v_3 / v_2

3. *Third stage (Constant pressure cooling).* The air is now cooled at a constant pressure from initial temperature T_3 to a temperature T_4 represented by the curve 3-4 in Fig. 6.7 (*a*) and (*b*).

∴ Heat rejected by the air,

$$Q_{3\text{-}4} = m\, c_p\, (T_3 - T_4)$$

4. *Fourth stage (Isothermal compression).* Finally, the air is compressed isothermally (*i.e.* at constant temperature $T_4 = T_1$) from initial volume v_3 to v_4 represented by the curve 4-1 in Fig. 6.7 (*a*) and (*b*). We know that some heat is rejected by the air for doing work on the air.

∴ Heat rejected during isothermal compression,

$$Q_{4\text{-}1} = p_4 v_4 \log_e \left(\frac{v_4}{v_1}\right) = m R T_4 \log_e \left(\frac{v_4}{v_1}\right) \qquad \ldots (\because p v = m R T)$$

$$= 2.3\, m R T_4 \log r$$

where r = Compression ratio = v_4 / v_1

We see from the above, that the heat supplied during the process 1-2 is equal to the heat rejected during the process 3-4 (because $T_2 - T_1 = T_3 - T_4$).

∴ Work done = Heat supplied – Heat rejected

$$= 2.3\, m R T_2 \log r - 2.3\, m R T_4 \log r = 2.3\, m R \log r\, (T_2 - T_4)$$

and efficiency,

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{2.3\, m R \log r\, (T_2 - T_4)}{2.3\, m R T_2 \log r}$$

$$= \frac{T_2 - T_4}{T_2} = 1 - \frac{T_4}{T_2} = 1 - \frac{T_1}{T_3}$$

Notes : 1. The efficiency of the Ericsson cycle is same as that of Carnot efficiency *i.e.*

$$\eta = 1 - \frac{\text{Lowest temperature}}{\text{Highest temperature}}$$

2. If the regenerator efficiency is η_R, then heat taken in from the regenerator during process 4-1 will be $mc_p (T_2 - T_4)(1 - \eta_R)$. In that case,

$$\eta = \frac{2.3\, m\, R \log r\, (T_2 - T_4)}{2.3\, m\, R\, T_2 \log r + m\, c_p (T_2 - T_4)(1 - \eta_R)}$$

$$= \frac{2.3\, R \log r\, (T_2 - T_4)}{2.3\, R\, T_2 \log r + c_p (T_2 - T_4)(1 - \eta_R)}$$

Example 6.7. *An Ericsson regenerative engine works between the temperature limits of 45° C and 230° C. If the ratio of expansion is 2, determine : 1. Work done per kg of air, and 2. Efficiency of the cycle.*

Assume R = 0.287 kJ/kg K.

Solution. Given : $T_1 = T_4 = 45°\,C = 45 + 273 = 318$ K ; $T_2 = T_3 = 230°\,C = 230 + 273 = 503$ K ; $r = 2$; $R = 0.287$ kJ/kg K

1. *Workdone per kg of air*

We know that workdone per kg of air,

$$= 2.3\, m\, R \log r\, (T_2 - T_4)$$

$$= 2.3 \times 1 \times 0.287 \log 2\, (503 - 318) = 36.76 \text{ kJ/kg}$$ **Ans.**

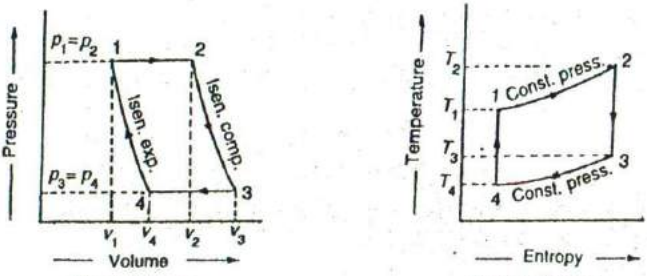
2. *Efficiency of the cycle*

We know that efficiency of the cycle,

$$\eta = 1 - \frac{T_1}{T_3} = 1 - \frac{318}{503} = 0.368 \text{ or } 36.8\,\%$$ **Ans.**

6.15. Joule's Cycle

It consists of two constant pressure and two reversible adiabatic or isentropic processes as shown on *p-v* and *T-S* diagrams in Fig. 6.8 (*a*) and (*b*).



(*a*) *p-v* diagram. (*b*) *T-S* diagram.

Fig. 6.8. Joule's cycle.

Now, let us consider the four stages of the Joule's cycle. Let the engine cylinder contain *m* kg of air at its original condition represented by point 1 on *p-v* and *T-S* diagram. At this point, let p_1, T_1 and v_1 be the pressure, temperature and volume of the air.

1. *First stage (Constant pressure heating).* The air is heated at a constant pressure from initial temperature T_1 to a temperature T_2 represented by the curve 1-2 in Fig. 6.8 (*a*) and (*b*).

∴ Heat supplied to the air,

$$Q_{1\text{-}2} = m\, c_p (T_2 - T_1)$$

2. *Second stage (Reversible adiabatic or isentropic expansion)* The air is allowed to expand reversibly and adiabatically from v_2 to v_3. The reversible adiabatic expansion is represented by the curve 2-3 in Fig. 6.8 (*a*) and (*b*). The temperature of the air falls from T_2 to T_3. In this process, no heat is absorbed or rejected by the air.

3. *Third stage (Constant pressure cooling)* The air is now cooled at constant pressure from temperature T_3 to a temperature T_4 represented by 3-4 in Fig. 6.8 (*a*) and (*b*).

$\therefore$ Heat rejected by the air,

$$Q_{3\text{-}4} = m\, c_p\, (T_3 - T_4)$$

4. *Fourth stage (Reversible adiabatic or isentropic compression)*. The air is now compressed reversibly and adiabatically from v_4 to v_1. The reversible adiabatic compression is represented by the curve 4-1 in Fig. 6.8 (*a*) and (*b*). The temperature of the air increases from T_4 to T_1. Again, no heat is absorbed or rejected by the air.

We see, from above, that there is no interchange of heat during the two reversible adiabatic processes. The only interchange of heat takes place during constant pressure processes.

$\therefore$ Work done = Heat supplied – Heat rejected

$$= m\, c_p\, (T_2 - T_1) - m\, c_p\, (T_3 - T_4)$$

and efficiency,

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{m\, c_p\, (T_2 - T_1) - m\, c_p\, (T_3 - T_4)}{m\, c_p\, (T_2 - T_1)}$$

$$= 1 - \frac{T_3 - T_4}{T_2 - T_1} = 1 - \frac{T_3\left(1 - \dfrac{T_4}{T_3}\right)}{T_2\left(1 - \dfrac{T_1}{T_2}\right)} \qquad \ldots (i)$$

We know that for reversible adiabatic or isentropic expansion 2-3,

$$\frac{T_3}{T_2} = \left(\frac{v_2}{v_3}\right)^{\gamma - 1} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma - 1}{\gamma}} \qquad \ldots (ii)$$

and for reversible adiabatic or isentropic compression 4-1,

$$\frac{T_4}{T_1} = \left(\frac{v_1}{v_4}\right)^{\gamma - 1} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} \qquad \ldots (iii)$$

From equations (*ii*) and (*iii*), we find that

$$\frac{T_3}{T_2} = \frac{T_4}{T_1} \quad \text{or} \quad \frac{T_4}{T_3} = \frac{T_1}{T_2} \qquad \ldots (\because p_1 = p_2 \text{ and } p_3 = p_4)$$

Substituting the value of T_4 / T_3 in equation (*i*),

$$\eta = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_1} \qquad \ldots \left(\because \frac{T_3}{T_2} = \frac{T_4}{T_1}\right)$$

From equations (*ii*) and (*iii*), we find that

$$\left(\frac{v_2}{v_3}\right)^{\gamma - 1} = \left(\frac{v_1}{v_4}\right)^{\gamma - 1} \qquad \ldots \left(\because \frac{p_3}{p_2} = \frac{p_4}{p_1}\right)$$

or $$\frac{v_2}{v_3} = \frac{v_1}{v_4} = \frac{1}{r} \quad \ldots\left(\because r = \frac{v_3}{v_2} = \frac{v_4}{v_1}\right)$$

$$\therefore \quad \frac{T_3}{T_2} = \frac{T_4}{T_1} = \left(\frac{1}{r}\right)^{\gamma-1} = \frac{1}{(r)^{\gamma-1}}$$

and $$\eta = 1 - \frac{1}{(r)^{\gamma-1}}$$

Notes : 1. The efficiency of the Joule's cycle is lower than Carnot efficiency. The reason is that all the heat is not taken in at the highest temperature and rejected at the lowest temperature.

2. The cycle is not thermodynamically reversible, because there is no regenerator to provide a constant temperature during heating and cooling at constant pressure.

3. The reversed Joule cycle is known as *Bell-coleman cycle or Brayton cycle and is applied to refrigerators, where air is used as a refrigerant.

Example 6.8. *An air motor works on Joule's cycle between 5 bar and 1 bar. The temperature at the beginning of isentropic expansion is 773 K and at the beginning of isentropic compression is 293 K. Determine the work done per kg of air and ideal efficiency. Assume c_p = 1 kJ/kg K and γ = 1.4.*

Solution. Given : $p_1 = p_2 = 5$ bar ; $p_3 = p_4 = 1$ bar ; $T_2 = 773$ K ; $T_4 = 293$ K ; $c_p = 1$ kJ/kg K ; $\gamma = 1.4$

Work done per kg of air

We know that for isentropic expansion process 2-3 (Refer Fig. 6.8),

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5}\right)^{\frac{1.4-1}{1.4}} = (0.2)^{0.286} = 0.6311$$

$$\therefore \quad T_3 = T_2 \times 0.6311 = 773 \times 0.6311 = 488 \text{ K}$$

Similarly, for isentropic compression process 4-1,

$$\frac{T_4}{T_1} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5}\right)^{\frac{1.4-1}{1.4}} = (0.2)^{0.286} = 0.6311$$

$$\therefore \quad T_1 = T_4 / 0.6311 = 293 / 0.6311 = 464 \text{ K}$$

We know that heat supplied

$$= m\, c_p (T_2 - T_1) = 1 \times 1\,(773 - 464) = 309 \text{ kJ}$$

and heat rejected $$= m\, c_p (T_3 - T_4) = 1 \times 1\,(488 - 293) = 195 \text{ kJ}$$

$\therefore$ Workdone = Heat supplied – Heat rejected = 309 – 195 = 114 kJ **Ans.**

Ideal efficiency

We know that ideal efficiency,

$$\eta = \frac{\text{Workdone}}{\text{Heat supplied}} = \frac{114}{309} = 0.369 \text{ or } 36.9\% \text{ Ans.}$$

Example 6.9. *A hot air engine works on Brayton cycle with initial and final pressures of air as 3 bar and 1 bar respectively. If the temperature before isentropic compression and isentropic*

* Please refer Chapter 35 (Art. 35.10).

expansion are 298 K and 923 K, determine : 1. heat supplied per kg of air ; 2. heat rejected per kg of air ; 3. work done per kg of air ; and 4. efficiency of the engine. Take c_p as 1 kJ/kg K, and c_v = 0.715 kJ/kg K.

Solution. Given : $p_1 = p_2 = 3$ bar ; $p_3 = p_4 = 1$ bar ; $T_4 = 298$ K ; $T_2 = 923$ K ; $c_p = 1$ kJ/kg K ; $c_v = 0.715$ kJ/kg K

We know that the ratio of specific heats or isentropic index,

$$\gamma = c_p / c_v = 1 / 0.715 = 1.4$$

Heat supplied per kg of air

We know that in isentropic expansion process 4-1 (Refer Fig. 6.8),

$$\frac{T_1}{T_4} = \left(\frac{p_1}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3}{1}\right)^{\frac{1.4-1}{1.4}} = (3)^{0.286} = 1.35$$

$$\therefore \quad T_1 = T_4 \times 1.35 = 298 \times 1.35 = 402 \text{ K}$$

and heat supplied per kg of air during constant pressure process 1-2,

$$Q_{1\text{-}2} = m\, c_p (T_2 - T_1) = 1 \times 1 (923 - 402) = 521 \text{ kJ} \quad \text{Ans.}$$

Heat rejected per kg of air

We know that in isentropic compression process 2-3,

$$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3}{1}\right)^{\frac{1.4-1}{1.4}} = (3)^{0.286} = 1.35$$

$$\therefore \quad T_3 = T_2 / 1.35 = 923 / 1.35 = 684 \text{ K}$$

and heat rejected per kg of air during constant pressure process 3-4,

$$Q_{3\text{-}4} = m\, c_p (T_3 - T_4) = 1 \times 1 (684 - 298) = 386 \text{ kJ} \quad \text{Ans.}$$

Work done per kg of air

We know that work done per kg of air,

$$W = Q_{1\text{-}2} - Q_{3\text{-}4} = 521 - 386 = 135 \text{ kJ} \quad \text{Ans.}$$

Efficiency of the engine

We know that efficiency of the engine,

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{135}{521} = 0.259 \text{ or } 25.9\ \% \quad \text{Ans.}$$

6.16. Otto Cycle

The first successful engine working on this *cycle was built by A. Otto. These days, many gas, petrol and many of the oil engines run on this cycle. It is the also known as *constant volume cycle*, as the heat is received and rejected at a constant volume.

This cycle is taken as a standard of comparison for internal combustion engines. For the purpose of comparison with other cycles, the air is assumed to be the working substance.

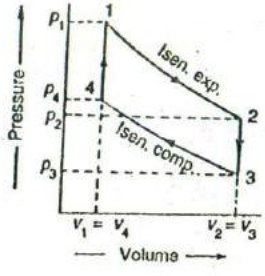
The engine conceived by Otto has air enclosed in a cylinder, whose walls are perfectly non-conductor of heat, but the bottom is a perfect conductor of heat. There is also a hot body and cold

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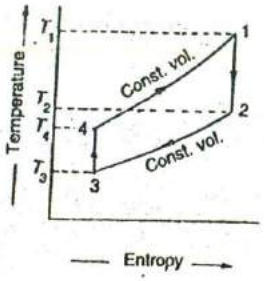
* This cycle was originally devised by a Frenchman Beau-de-Rochas in 1862. The first successful engine, working on this cycle, was built by a German engineer Nicholas A. Otto in 1876.

body and an insulating cap, which are alternately brought in contact with the bottom of the cylinder (*i.e.* a cylinder similar to that of Carnot).

The ideal Otto cycle consists of two constant volume and two reversible adiabatic or isentropic processes as shown on p-v and T-S diagrams in Fig. 6.9 (*a*) and (*b*).



(*a*) p-v diagram.



(*b*) T-S diagram.

Fig. 6.9. Otto cycle.

Let the engine cylinder contain m kg of air at point 1. At this point, let p_1, T_1 and v_1 be the pressure, temperature and volume of the air. Following are the four stages of the ideal cycle :

1. *First stage (Reversible adiabatic or isentropic expansion).* The air is expanded reversibly and adiabatically from initial temperature T_1 to a temperature T_2 as shown by the curve 1-2 in Fig. 6.9 (*a*) and (*b*). In this process, no heat is absorbed or rejected by the air.

2. *Second stage (Constant volume cooling).* The air is cooled at constant volume from temperature T_2 to a temperature T_3 as shown by the curve 2-3 in Fig. 6.9 (*a*) and (*b*). We know that heat rejected by the air during this process.

$$Q_{2-3} = m\, c_v\, (T_2 - T_3)$$

3. *Third stage (Reversible adiabatic or isentropic compression).* The air is compressed reversibly and adiabatically from temperature T_3 to a temperature T_4 as shown in by the curve 3-4 in Fig. 6.9 (*a*) and (*b*). In this process, no heat is absorbed or rejected by the air.

4. *Fourth stage (Constant volume heating).* The air is now heated at constant volume from temperature T_4 to a temperature T_1 as shown by the curve 4-1 in Fig. 6.9 (*a*) and (*b*). We know that heat absorbed by the air during this process,

$$Q_{4-1} = m\, c_v\, (T_1 - T_4)$$

We see that the air has been brought back to its original conditions of pressure, volume and temperature, thus completing the cycle.

We know that work done = Heat absorbed – Heat rejected

$$= m\, c_v\, (T_1 - T_4) - m\, c_v\, (T_2 - T_3)$$

∴ Ideal efficiency or air standard efficiency,

$$\eta = \frac{\text{Work done}}{\text{Heat absorbed}}$$

$$= \frac{m\, c_v\, (T_1 - T_4) - m\, c_v\, (T_2 - T_3)}{m\, c_v\, (T_1 - T_4)}$$

$$= 1 - \frac{T_2 - T_3}{T_1 - T_4} = 1 - \frac{T_3\left(\frac{T_2}{T_3} - 1\right)}{T_4\left(\frac{T_1}{T_4} - 1\right)} \quad \ldots (i)$$

We know that for reversible adiabatic or isentropic expansion process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = \left(\frac{1}{r}\right)^{\gamma - 1} \quad \ldots (ii)$$

where r = Expansion ratio = v_2 / v_1

Similarly, for reversible adiabatic or isentropic compression process 3-4,

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma - 1} = \left(\frac{1}{r}\right)^{\gamma - 1} \quad \ldots (iii)$$

where r = Compression ratio = $v_3 / v_4 = v_2 / v_1$

From equations (*ii*) and (*iii*), we find that

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{1}{r}\right)^{\gamma - 1} = \frac{1}{(r)^{\gamma - 1}} \quad \text{or} \quad \frac{T_1}{T_4} = \frac{T_2}{T_3}$$

Substituting the value of T_1 / T_4 in equation (*i*),

$$\eta = 1 - \frac{T_3}{T_4} = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{(r)^{\gamma - 1}} \quad \ldots \left(\because \frac{T_3}{T_4} = \frac{T_2}{T_1}\right) \ldots (iv)$$

Notes : 1. We see from equation (*iv*) that the efficiency of Otto cycle depends on compression ratio (r) only.

2. The efficiency increases with the increase in compression ratio (r). In actual practice, r can not be increased beyond a value of 7 or so.

3. Compression ratio, $r = \dfrac{\text{Total cylinder volume}}{\text{Clearance volume}}$

$$= \frac{\text{Clearance volume + Stroke volume}}{\text{Clearance volume}} = \frac{v_c + v_s}{v_c}$$

$$\therefore \text{Clearance volume, } v_c = \frac{\text{Stroke volume}}{r - 1} = \frac{v_s}{r - 1}$$

4. The relations between pressure and temperature or pressure and volume may be obtained from the usual reversible adiabatic or isentropic processes, *i.e.*

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma - 1}{\gamma}} \quad \text{and} \quad \frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^{\gamma} \quad \text{or} \quad \frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

Example 6.10. *In an Otto cycle, the temperature at the beginning and end of the isentropic compression are 316 K and 596 K respectively. Determine the air standard efficiency and the compression ratio. Take $\gamma = 1.4$.*

Solution. Given : $T_3 = 316$ K ; $T_4 = 596$ K ; $\gamma = 1.4$

Compression ratio

Let r = Compression ratio = v_3 / v_4

We know that for isentropic compression 3-4 (Refer Fig. 6.9),

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{1.4-1} = \frac{1}{(r)^{0.4}}$$

$$\therefore \quad (r)^{0.4} = \frac{T_4}{T_3} \text{ or } r = \left(\frac{T_4}{T_3}\right)^{\frac{1}{0.4}} = \left(\frac{596}{316}\right)^{2.5} = 4.885 \text{ Ans.}$$

Air standard efficiency

We know that air standard efficiency,

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(4.885)^{1.4-1}} = 1 - \frac{1}{1.886}$$

$$= 1 - 0.53 = 0.47 \text{ or } 47\% \text{ Ans.}$$

Example 6.11. *An engine, working on the Otto cycle, has a cylinder diameter of 150 mm and a stroke of 225 mm. The clearance volume is 1.25×10^{-3} m³. Find the air standard efficiency of this engine. Take $\gamma = 1.4$.*

Solution. Given : $d = 150$ mm $= 0.15$ m ; $l = 225$ mm $= 0.225$ m ; $v_c = 1.25 \times 10^{-3}$ m³ ; $\gamma = 1.4$

We know that swept volume

$$v_s = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4} (0.15)^2 (0.225) = 3.976 \times 10^{-3} \text{ m}^3$$

$$\therefore \text{ Compression ratio, } r = \frac{v_c + v_s}{v_c} = \frac{1.25 \times 10^{-3} + 3.976 \times 10^{-3}}{1.25 \times 10^{-3}} = 4.18$$

We know that air standard efficiency

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(4.18)^{1.4-1}} = 1 - \frac{1}{1.772}$$

$$= 1 - 0.564 = 0.436 \text{ or } 43.6\% \text{ Ans.}$$

Example 6.12. *A certain quantity of air at a pressure of 1 bar and temperature 70° C is compressed reversibly and adiabatically until the pressure is 7 bar in an Otto cycle engine. 460 kJ of heat per kg of air is now added at constant volume. Determine : 1. compression ratio of the engine, 2. temperature at the end of compression, and 3. temperature at the end of heat addition.*

Take for air, $c_p = 1$ kJ/kg K and $c_v = 0.707$ kJ/kg K.

Solution. Given : $p_3 = 1$ bar ; $T_3 = 70°$ C $= 70 + 273 = 343$ K ; $p_4 = 7$ bar ; $Q_{4-1} = 460$ kJ ; $m = 1$ kg ; $c_p = 1$ kJ/kg K ; $c_v = 0.707$ kJ/kg K

We know that ratio of specific heats,

$$\gamma = c_p / c_v = 1 / 0.707 = 1.41$$

1. *Compression ratio of the engine*

Let r = Compression ratio of the engine $= v_3 / v_4$.

We know that $p_3 v_3^{\gamma} = p_4 v_4^{\gamma}$

$$\therefore \quad \frac{v_3}{v_4} = \left(\frac{p_4}{p_3}\right)^{\frac{1}{\gamma}} \text{ or } r = \left(\frac{7}{1}\right)^{\frac{1}{1.41}} = (7)^{0.709} = 3.97 \text{ Ans.}$$

2. *Temperature at the end of compression*

Let T_4 = Temperature at the end of compression.

We know that $\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1} = \left(\frac{1}{3.97}\right)^{1.41-1} = (0.252)^{0.41} = 0.568$

$\therefore$ $T_4 = T_3 / 0.568 = 343 / 0.568 = 604$ K $= 331°$ C **Ans.**

3. *Temperature at the end of heat addition*

Let T_1 = Temperature at the end of heat addition.

We know that heat added at constant volume ($Q_{4\text{-}1}$),

$$460 = m\, c_v (T_1 - T_4) = 1 \times 0.707\,(T_1 - 604)\ \text{kJ}$$

$\therefore$ $T_1 - 604 = 460 / 0.707 = 651$ or $T_1 = 1255$ K $= 982°$ C **Ans.**

Example 6.13. *In an Otto cycle, air at 1 bar and 290 K is compressed isentropically until the pressure is 15 bar. The heat is added at constant volume until the pressure rises to 40 bar. Calculate the air standard efficiency and the mean effective pressure for the cycle. Take $c_v = 0.717$ kJ/kg K and $R_u = 8.314$ kJ/kg mole K.*

Solution. Given : $p_3 = 1$ bar ; $T_3 = 290$ K ; $p_4 = 15$ bar ; $p_1 = 40$ bar ; $c_v = 0.717$ kJ/kg K ; $R_u = 8.314$ kJ/kg mole K

We know that characteristic gas constant,

$$R = \frac{\text{Universal gas constant } (R_u)}{\text{Molecular mass } (M)} = \frac{8.314}{28.97} = 0.287 \text{ kJ/kg K}$$

... ($\because$ M for air = 28.97 kg)

and $c_p = R + c_v = 0.287 + 0.717 = 1.004$ kJ/kg K

$\therefore$ $\gamma = c_p / c_v = 1.004 / 0.717 = 1.4$

Air standard efficiency

We know that for isentropic compression process 3-4 (Refer Fig. 6.9).

$$p_3 v_3^{\gamma} = p_4 v_4^{\gamma} \quad \text{or} \quad \frac{v_3}{v_4} = \left(\frac{p_4}{p_3}\right)^{\frac{1}{\gamma}}$$

$\therefore$ Compression ratio, $r = \frac{v_3}{v_4} = \left(\frac{p_4}{p_3}\right)^{\frac{1}{\gamma}} = \left(\frac{15}{1}\right)^{\frac{1}{1.4}} = (15)^{0.714} = 6.914$

We know that air standard efficiency,

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(6.914)^{1.4-1}} = 1 - \frac{1}{2.167}$$

$$= 1 - 0.4615 = 0.5385 \text{ or } 53.85\ \%\ \text{Ans.}$$

Mean effective pressure

First of all, let us find the workdone during the cycle per kg of air.

Let T_4 = Temperature at the end of isentropic compression, and

T_1 = Temperature at the end of constant volume heating.

We know that $\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1} = \left(\frac{1}{6.914}\right)^{1.4-1} = \frac{1}{2.167}$

$\therefore$ $T_4 = T_3 \times 2.167 = 290 \times 2.167 = 628.5$ K

Now for constant volume heating process 4-1,

$$\frac{p_4}{T_4} = \frac{p_1}{T_1} \quad \text{or} \quad T_1 = T_4 \times \frac{p_1}{p_4} = 628.5 \times \frac{40}{15} = 1676 \text{ K}$$

We know that heat supplied,

$$Q_{4-1} = m\, c_v\, (T_1 - T_4) = 1 \times 0.717\, (1676 - 628.5) = 751 \text{ kJ}$$

$\therefore$ Workdone during the cycle

$$= \eta \times \text{Heat supplied} = 0.5385 \times 751 = 404.4 \text{ kJ}$$

Now let us find the stroke volume (*i.e.* $v_3 - v_4$ or $v_2 - v_1$). We know that

$$p_3 v_3 = m R T_3 \quad \text{or} \quad v_3 = \frac{m R T_3}{p_3} = \frac{1 \times 287 \times 290}{0.1 \times 10^6} = 0.8323 \text{ m}^3$$

... (R is taken in J/kg K and p_3 in N/m^2)

$\therefore$ $v_4 = v_3 / 6.914 = 0.8323 / 6.914 = 0.1204$ m^3

... ($\because r = v_3/v_4 = 6.914$)

and stroke volume $= v_3 - v_4 = 0.8323 - 0.1204 = 0.7119$ m^3

We know that mean effective pressure

$$= \frac{\text{Work done}}{\text{Stroke volume}} = \frac{404.4}{0.7119} = 568 \text{ kN/m}^2 = 5.68 \text{ bar Ans.}$$

Example 6.14. *In an air standard Otto cycle, the compression ratio is 7 and the compression begins at 1 bar and 313 K. The heat added is 2510 kJ/kg. Find : 1. maximum temperature and pressure of the cycle ; 2. workdone per kg of air ; 3. cycle efficiency ; and 4. mean effective pressure.*

Take for air, $c_v = 0.713$ kJ/kg K and $R = 287$ J/kg K.

Solution. Given : $r = v_3 / v_4 = v_2 / v_1 = 7$; $p_3 = 1$ bar $= 0.1 \times 10^6$ N/m^2 ; $T_3 = 313$ K ; $Q_{4-1} = 2510$ kJ/kg ; $c_v = 0.713$ kJ/kg K ; $R = 287$ J/kg K $= 0.287$ kJ/kg K

We know that $c_p - c_v = R$ or $c_p = R + c_v = 0.287 + 0.713 = 1$ kJ/kg K

and $\gamma = c_p / c_v = 1 / 0.713 = 1.4$

1. Maximum temperature and pressure of the cycle

Let T_1 = Maximum temperature, and

p_1 = Maximum pressure.

We know that for isentropic compression process 3-4 (Refer Fig. 6.9),

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1} = \left(\frac{1}{7}\right)^{1.4-1} = \frac{1}{7^{0.4}} = \frac{1}{2.178}$$

$\therefore \quad T_4 = T_3 \times 2.178 = 313 \times 2.178 = 682\ \text{K}$

and $p_3 v_3^{\gamma} = p_4 v_4^{\gamma}$ or $p_4 = p_3 \left(\frac{v_3}{v_4}\right)^{\gamma} = 1 \times 7^{1.4} = 15.24\ \text{bar}$

We know that heat supplied during constant volume process 4-1 (Q_{4-1}),

$$2510 = m c_v (T_1 - T_4) = 1 \times 0.713 (T_1 - 682) \qquad \ldots (\because m = 1\ \text{kg})$$

$\therefore \quad T_1 - 682 = 2510 / 0.713 = 3520$ or $T_1 = 4202$ K **Ans.**

We also know that $\frac{p_4}{T_4} = \frac{p_1}{T_1}$ or $p_1 = p_4 \times \frac{T_1}{T_4} = 15.24 \times \frac{4202}{682} = 93.9\ \text{bar}$ **Ans.**

2. *Workdone per kg of air*

We know that for isentropic expansion process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = \left(\frac{1}{r}\right)^{\gamma - 1} = \left(\frac{1}{7}\right)^{1.4 - 1} = \frac{1}{7^{0.4}} = \frac{1}{2.178}$$

$\therefore \quad T_2 = T_1 / 2.178 = 4202 / 2.178 = 1929\ \text{K}$

and heat rejected per kg of air,

$$Q_{2-3} = m c_v (T_2 - T_3) = 1 \times 0.713 (1929 - 313) = 1152\ \text{kJ}$$

$\therefore$ Workdone per kg of air,

$$w = \text{Heat supplied} - \text{Heat rejected} = Q_{4-1} - Q_{2-3}$$
$$= 2510 - 1152 = 1358\ \text{kJ}\ \textbf{Ans.}$$

3. *Cycle efficiency*

We know that cycle efficiency,

$$\eta = \frac{\text{Workdone}}{\text{Heat supplied}} = \frac{1358}{2510} = 0.541 \text{ or } 54.1\%\ \textbf{Ans.}$$

Note. The cycle efficiency may also be calculated as follows :

We know that $\eta = 1 - \frac{1}{(r)^{\gamma - 1}} = 1 - \frac{1}{(7)^{1.4 - 1}} = 1 - \frac{1}{2.178} = 0.541$ or 54.1% **Ans.**

4. *Mean effective pressure*

First of all, let us find the stroke volume (*i.e.* $v_3 - v_4$ or $v_2 - v_1$).

We know that

$$p_3 v_3 = m R T_3 \text{ or } v_3 = \frac{m R T_3}{p_3} = \frac{1 \times 287 \times 313}{0.1 \times 10^6} = 0.9\ \text{m}^3$$

and $v_4 = v_3 / r = 0.9 / 7 = 0.1286\ \text{m}^3 \qquad \ldots (\because v_3 / v_4 = r)$

We know that stroke volume,

$$v_3 - v_4 = 0.9 - 0.1286 = 0.7714\ \text{m}^3$$

$\therefore$ Mean effective pressure

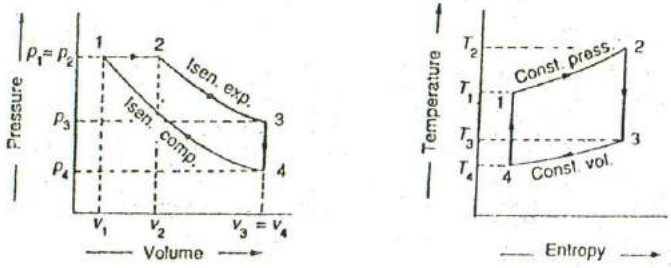
$$= \frac{\text{Workdone}}{\text{Stroke volume}} = \frac{1358}{0.7714} = 1760\ \text{kN/m}^2 = 17.6\ \text{bar}\ \textbf{Ans.}$$

6.17. Diesel Cycle

This cycle was devised by Dr. Rudolph Diesel in 1893, with an idea to attain a higher thermal efficiency, with a high compression ratio. This is an important cycle on which all the diesel engines work. It is also known as *constant pressure cycle* as heat is received at a constant pressure.

The engine imagined by Diesel has air enclosed in the cylinder, whose walls are perfectly non-conductor of heat, but bottom is a perfect conductor of *heat. Again, there is a hot body, cold body and an insulating cap, which are alternately brought in contact with the cylinder.

The ideal diesel cycle consists of two reversible adiabatic or isentropic, a constant pressure and a constant volume processes. These processes are represented on p-v and T-S diagrams as shown in Fig. 6.10 (*a*) and (*b*).



(*a*) p-v diagram. (*b*) T-S diagram.

Fig. 6.10. Diesel cycle.

Let the engine cylinder contain m kg of air at point 1. At this point let, p_1, T_1 and v_1 be the pressure, temperature and volume of the air. Following are four stages of an ideal diesel cycle.

1. *First stage (Constant pressure heating).* The air is heated at constant pressure from initial temperature T_1 to a temperature T_2 represented by the curve 1-2 in Fig. 6.10 (*a*) and (*b*).

∴ Heat supplied to the air,

$$Q_{1\text{-}2} = m\, c_p\, (T_2 - T_1)$$

Note : Since the supply of heat is cut off at point 2, therefore it is known as cut-off point.

2. *Second stage (Reversible adiabatic or isentropic expansion).* The air is expanded reversibly and adiabatically from temperature T_2 to a temperature T_3 as shown by the curve 2-3 in Fig. 6.10 (*a*) and (*b*). In this process, no heat is absorbed or rejected by the air.

3. *Third stage (Constant volume cooling).* The air is now cooled at constant volume from temperature T_3 to a temperature T_4 as shown by the curve 3-4 in Fig. 6.10 (*a*) and (*b*).

∴ Heat rejected by the air,

$$Q_{3\text{-}4} = m\, c_v\, (T_3 - T_4)$$

4. *Fourth stage (Reversible adiabatic or isentropic compression).* The air is compressed reversibly and adiabatically from temperature T_4 to a temperature T_1 represented by the curve 4-1 in Fig. 6.10 (*a*) and (*b*). In this process, no heat is absorbed or rejected by the air.

We see that the air has been brought back to its original conditions of pressure, volume and temperature, thus completing the cycle. We know that

---

* A similar type of cylinder was designed by Otto.

$$\text{Work done} = \text{Heat absorbed} - \text{Heat rejected}$$
$$= m c_p (T_2 - T_1) - m c_v (T_3 - T_4)$$

∴ Air standard efficiency,

$$\eta = \frac{\text{Work done}}{\text{Heat absorbed}} = \frac{m c_p (T_2 - T_1) - m c_v (T_3 - T_4)}{m c_p (T_2 - T_1)}$$
$$= 1 - \frac{c_v}{c_p}\left(\frac{T_3 - T_4}{T_2 - T_1}\right) = 1 - \frac{1}{\gamma}\left(\frac{T_3 - T_4}{T_2 - T_1}\right) \quad \ldots (i)$$

Now let compression ratio,

$$r = \frac{v_4}{v_1}$$

Cut-off ratio, $\rho = \frac{v_2}{v_1}$

Expansion ratio, $r_1 = \frac{v_3}{v_2} = \frac{v_4}{v_2}$ $\ldots (\because v_3 = v_4)$

$$= \frac{v_4}{v_1} \times \frac{v_1}{v_2} = r \times \frac{1}{\rho} = \frac{r}{\rho}$$

We know for constant pressure heating process 1-2,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \quad \ldots \text{(Charles's law)}$$

$$\therefore \quad T_2 = T_1 \times \frac{v_2}{v_1} = T_1 \times \rho \quad \ldots (ii)$$

Similarly, for reversible adiabatic or isentropic expansion process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{v_2}{v_3}\right)^{\gamma - 1} = \left(\frac{1}{r_1}\right)^{\gamma - 1} = \left(\frac{\rho}{r}\right)^{\gamma - 1}$$

$$\therefore \quad T_3 = T_2 \left(\frac{\rho}{r}\right)^{\gamma - 1} = T_1 \times \rho \left(\frac{\rho}{r}\right)^{\gamma - 1} \quad \ldots (iii)$$

and for reversible adiabatic or isentropic compression process 4-1,

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1}\right)^{\gamma - 1} = (r)^{\gamma - 1} \quad \text{or} \quad T_1 = T_4 (r)^{\gamma - 1} \quad \ldots (iv)$$

Substituting the value of T_1 in equations (*ii*) and (*iii*),

$$T_2 = T_4 (r)^{\gamma - 1} \times \rho \quad \ldots (v)$$

and

$$T_3 = T_4 (r)^{\gamma - 1} \times \rho \left(\frac{\rho}{r}\right)^{\gamma - 1} = T_4 \rho^{\gamma} \quad \ldots (vi)$$

Now substituting the values of T_1, T_2 and T_3 in equation (*i*),

$$\eta = 1 - \frac{1}{\gamma}\left[\frac{(T_4 \rho^{\gamma}) - T_4}{T_4 (r)^{\gamma-1} \rho - T_4 (r)^{\gamma-1}}\right]$$

$$= 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}\right] \quad \ldots (vii)$$

Notes : 1. The efficiency of the ideal Diesel cycle is lower than that of Otto cycle, for the same compression ratio. This is due to the fact that the cut-off ratio (ρ) is always greater than unity and hence the term within the bracket of equation (*vii*) increases with the increase of cut-off ratio. Thus the negative term increases and the efficiency is reduced.

2. The Diesel cycle efficiency increases with decrease in cut-off ratio and approaches maximum (equal to Otto cycle efficiency) when the term within the bracket is unity.

Example 6.15. *In a diesel engine, the compression ratio is 13 : 1 and the fuel is cut-off at 8% of the stroke. Find the air standard efficiency of the engine. Take γ for air as 1.4.*

Solution. Given : $r = v_4 / v_1 = 13$; $\gamma = 1.4$

Since the cut-off takes place at 8% of the stroke, therefore volume at cut-off,

$$v_2 = v_1 + 8\% \text{ of stroke volume} = v_1 + 0.08 (v_4 - v_1)$$

Let us assume that the clearance volume (v_1) = 1 m^3.

$$\therefore \quad v_4 = 13 \text{ m}^3 \quad \ldots (\because v_4 / v_1 = 13)$$

and stroke volume, $v_4 - v_1 = 13 - 1 = 12$ m^3

∴ Volume at cut-off

$$v_2 = v_1 + 0.08 (v_4 - v_1) = 1 + 0.08 \times 12 = 1.96 \text{ m}^3$$

We know that cut-off ratio,

$$\rho = v_2 / v_1 = 1.96 / 1 = 1.96$$

∴ Air standard efficiency,

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}\right] = 1 - \frac{1}{(13)^{1.4-1}}\left[\frac{(1.96)^{1.4} - 1}{1.4 (1.96 - 1)}\right]$$

$$= 1 - 0.417 = 0.583 \quad \text{or} \quad 58.3\% \text{ Ans.}$$

Example 6.16. *In an ideal Diesel cycle, the temperatures at the beginning and end of compression are 57° C and 603° C respectively. The temperatures at the beginning and end of expansion are 1950° C and 870° C respectively. Determine the ideal efficiency of the cycle. $\gamma = 1.4$.*

If the compression ratio is 14 and the pressure at the beginning of the compression is 1 bar, calculate the maximum pressure in the cycle.

Solution. Given : $T_4 = 57° \text{C} = 57 + 273 = 330$ K ; $T_1 = 603° \text{C} = 603 + 273 = 876$ K ; $T_2 = 1950° \text{C} = 1950 + 273 = 2223$ K ; $T_3 = 870° \text{C} = 870 + 273 = 1143$ K ; $\gamma = 1.4$; $r = v_4 / v_1 = 14$; $p_4 = 1$ bar

Ideal efficiency of the cycle

We know that ideal efficiency of the cycle,

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{T_3 - T_4}{T_2 - T_1}\right) = 1 - \frac{1}{1.4}\left(\frac{1143 - 330}{2223 - 876}\right)$$

$$= 1 - 0.431 = 0.569 \quad \text{or} \quad 56.9\% \text{ Ans.}$$

Maximum pressure in the cycle

Let p_1 = Maximum pressure in the cycle.

We know that for reversible adiabatic compression,

$$p_1 v_1^{\gamma} = p_4 v_4^{\gamma} \text{ or } p_1 = p_4 \left(\frac{v_4}{v_1}\right)^{\gamma} = 1\,(14)^{1.4} = 40.23 \text{ bar } \textbf{Ans.}$$

Example 6.17. *An ideal Diesel engine has a diameter 150 mm and stroke 200 mm. The clearance volume is 10 per cent of the swept volume. Determine the compression ratio and the air standard efficiency of the engine if the cut-off takes place at 6 per cent of the stroke.*

Solution. Given : $d = 150$ mm $= 0.15$ m ; $l = 200$ mm $= 0.2$ m ; $v_c = 10\%$ of $v_s = 0.1\, v_s$

Compression ratio

We know that stroke volume,

$$v_s = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4} (0.15)^2\, 0.2 = 3.53 \times 10^{-3} \text{ m}^3$$

$$\therefore \quad v_c = 0.1\, v_s = 0.1 \times 3.53 \times 10^{-3} = 0.353 \times 10^{-3} \text{ m}^3$$

We know that compression ratio,

$$r = \frac{\text{Total volume}}{\text{Clearance volume}} = \frac{v_c + v_s}{v_c} = \frac{0.353 \times 10^{-3} + 3.53 \times 10^{-3}}{0.353 \times 10^{-3}}$$

$$= 11 \textbf{ Ans.}$$

Air standard efficiency

Since the cut-off takes place at 6% of the stroke, therefore volume at cut-off,

$$v_2 = v_1 + 0.06\, v_s = v_c + 0.06\, v_s \qquad \ldots (\because v_1 = v_c)$$

$$= 0.353 \times 10^{-3} + 0.06 \times 3.53 \times 10^{-3} = 0.565 \times 10^{-3} \text{ m}^3$$

$$\therefore \text{ Cut-off ratio, } \quad \rho = \frac{v_2}{v_1} = \frac{v_2}{v_c} = \frac{0.565 \times 10^{-3}}{0.353 \times 10^{-3}} = 1.6$$

We know that air standard efficiency,

$$\eta = 1 - \frac{1}{(r)^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}\right] = 1 - \frac{1}{(11)^{1.4-1}} \left[\frac{(1.6)^{1.4} - 1}{1.4\,(1.6 - 1)}\right]$$

$$= 1 - \frac{1}{2.61} \times 1.11 = 1 - 0.4246 = 0.5753 \text{ or } 57.53\% \textbf{ Ans.}$$

Example 6.18. *The compression ratio of an ideal air standard Diesel cycle is 15. The heat transfer is 1465 kJ/kg of air. Find the pressure and temperature at the end of each process and determine the cycle efficiency.*

What is the mean effective pressure of the cycle, if the inlet conditions are 300 K and 1 bar.

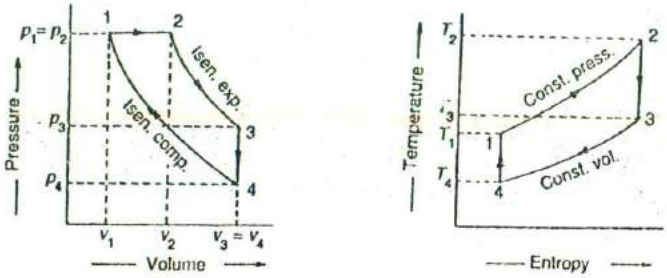
Solution. Given : $r = v_4 / v_1 = 15$; $Q_{1-2} = 1465$ kJ/kg ; $T_4 = 300$ K ; $p_4 = 1$ bar $= 0.1 \times 10^6$ N/m^2

Pressure and temperature at the end of each process

The p-v and T-S diagram for the Diesel cycle is shown in Fig. 6.11.

Let p_1, p_2 and p_3 = Pressures at points 1, 2 and 3 respectively.

T_1, T_2 and T_3 = Temperature at points 1, 2 and 3 respectively.



(a) p–v diagram. (b) T–S diagram.

Fig. 6.11

First of all, consider the isentropic compression process 4-1. We know that

$$p_4 v_4^{\gamma} = p_1 v_1^{\gamma} \text{ or } p_1 = p_4 \left(\frac{v_4}{v_1} \right)^{\gamma} = 1 (15)^{1.4} = 44.3 \text{ bar Ans.}$$

and

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1} \right)^{\gamma - 1} = (15)^{1.4-1} = (15)^{0.4} = 2.954$$

$\therefore$ $$T_1 = T_4 \times 2.954 = 300 \times 2.954 = 886.2 \text{ K Ans.}$$

Now consider the constant pressure process 1-2. We know that

$$p_2 = p_1 = 44.3 \text{ bar Ans.}$$

and heat supplied per kg of air during constant pressure process 1-2 (Q_{1-2}),

$$1465 = m c_p (T_2 - T_1) = 1 \times 1 (T_2 - 886.2) = T_2 - 886.2$$

... (Taking c_p for air = 1 kJ/kg K)

$\therefore$ $$T_2 = 1465 + 886.2 = 2351.2 \text{ K Ans.}$$

Now consider the isentropic expansion process 2-3. First of all, let us find the volumes at points 2 and 3.

Let v_2 = Volume at point 2, and

v_3 = Volume at point 3.

We know that $$p_4 v_4 = m R T_4 \text{ or } v_4 = \frac{m R T_4}{p_4} = \frac{1 \times 287 \times 300}{0.1 \times 10^6} = 0.861 \text{ m}^3$$

... ($\because$ R for air = 287 J/kg K)

$\therefore$ $$v_3 = 0.861 \text{ m}^3 \quad \ldots (\because v_3 = v_4)$$

and $$v_1 = v_4 / 15 = 0.861 / 15 = 0.0574 \text{ m}^3 \quad \ldots (\because v_4 / v_1 = 15)$$

We also know that for the constant pressure process 1-2,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \quad \text{or} \quad v_2 = v_1 \times \frac{T_2}{T_1} = 0.0574 \times \frac{2351.2}{886.2} = 0.1523 \text{ m}^3$$

Now $$\frac{T_2}{T_3} = \left(\frac{v_3}{v_2}\right)^{\gamma - 1} = \left(\frac{0.861}{0.1523}\right)^{1.4-1} = (5.65)^{0.4} = 2$$

$\therefore$ $$T_3 = T_2 / 2 = 2351.2 / 2 = 1175.6 \text{ K}$$ **Ans.**

We know that $$p_2 v_2^{\gamma} = p_3 v_3^{\gamma} \quad \text{or} \quad p_3 = p_2 \left(\frac{v_2}{v_3}\right)^{\gamma} = 44.3 \left(\frac{0.1523}{0.861}\right)^{1.4} = 3.92 \text{ bar}$$ **Ans.**

Cycle efficiency

We know that heat rejected per kg of air during the constant volume process 3-4,

$$Q_{3-4} = m\, c_v (T_3 - T_4) = 1 \times 0.712\,(1175.6 - 300) = 623.4 \text{ kJ}$$

... (Taking c_v for air = 0.712 kJ/kg K)

$\therefore$ Cycle efficiency, $$\eta = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}} = \frac{1465 - 623.4}{1465}$$

$$= 0.5745 \quad \text{or} \quad 57.45\% \text{ Ans.}$$

Mean effective pressure

We know that workdone per kg of air during the cycle

$$= \text{Heat supplied} - \text{Heat rejected} = 1465 - 623.4 = 841.6 \text{ kJ}$$

and stroke volume $$= v_4 - v_1 = 0.861 - 0.0574 = 0.8036 \text{ m}^3$$

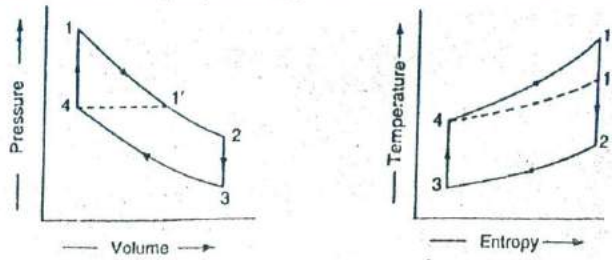
$\therefore$ Mean effective pressure

$$= \frac{\text{Workdone}}{\text{Stroke volume}} = \frac{841.6}{0.8036} = 1047.3 \text{ kN/m}^2$$

$$= 10.473 \text{ bar}$$ **Ans.** ... ($\because$ 1 bar = 100 kN/m^2)

Example 6.19. *Find the air standard efficiencies for the Otto and Diesel cycles on the basis of equal compression ratio of 10 and equal heat rejection of 840 kJ/kg. The suction conditions are 1 bar and 328 K.*

Solution. Given : $r = v_3 / v_4 = 10$; $Q_{2-3} = 840$ kJ/kg ; $p_3 = 1$ bar ; $T_3 = 328$ K



(*a*) *p-v* diagram. (*b*) *T-S* diagram.

Fig. 6.12

In the *p-v* and *T-S* diagrams as shown in Fig. 6.12 (*a*) and (*b*), the cycle 1-2-3-4-1 represents an Otto cycle and the cycle 4-1'-2-3-4 represents a Diesel cycle.

First of all, let us consider isentropic compression process 3-4, which is common for both the cycles.

Let T_4 and p_4 = Temperature and pressure at the end of isentropic compression.

We know that $\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = (10)^{1.4-1} = (10)^{0.4} = 2.512$

$\therefore$ $T_4 = T_3 \times 2.512 = 328 \times 2.512 = 824$ K

We also know that

$$p_3 v_3^{\gamma} = p_4 v_4^{\gamma} \text{ or } p_4 = p_3\left(\frac{v_3}{v_4}\right)^{\gamma} = 1\,(10)^{1.4} = 25.12 \text{ bar}$$

Now consider the constant volume process 2-3 which is also common for both the cycles.

We know that heat rejected per kg of gas during this process (Q_{2-3}),

$$840 = m\,c_v\,(T_2 - T_3) = 1 \times 0.712\,(T_2 - 328)$$

... (Taking c_v for air = 0.712 kJ/kg K)

$\therefore$ $T_2 - 328 = 840 / 0.712 = 1180$ or $T_2 = 1180 + 328 = 1508$ K

and pressure at point 2, $p_2 = p_3 \times \frac{T_2}{T_3} = 1 \times \frac{1508}{328} = 4.6$ bar ... $\left(\because \frac{p_2}{T_2} = \frac{p_3}{T_3}\right)$

Otto cycle

Consider the isentropic expansion process 1-2.

Let T_1 = Temperature at the beginning of isentropic expansion.

We know that $\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = (10)^{1.4-1} = 10^{0.4} = 2.512$

... ($\because v_1 = v_4$ and $v_2 = v_3$)

$\therefore$ $T_1 = T_2 \times 2.512 = 1508 \times 2.512 = 3788$ K

and heat supplied per kg of air during constant volume process 4-1,

$$Q_{4-1} = m\,c_v\,(T_1 - T_4) = 1 \times 0.712\,(3788 - 824) = 2110 \text{ kJ/kg}$$

$\therefore$ Efficiency of Otto cycle,

$$\eta_O = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}} = \frac{2110 - 840}{2110} = 0.60 \text{ or } 60\% \text{ Ans.}$$

Diesel cycle

Consider the isentropic expansion process 1′–2.

Let T_1' = Temperature at the beginning of isentropic expansion.

We know that $\frac{T_1'}{T_2} = \left(\frac{p_1'}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{25.12}{4.58}\right)^{\frac{1.4-1}{1.4}} = (5.485)^{0.286} = 1.627$

... ($\because p_1' = p_4 = 25.12$ bar)

$\therefore$ $T_1' = T_2 \times 1.627 = 1508 \times 1.627 = 2453$ K

and heat supplied per kg of air during constant pressure process 4-1′,

$$Q_{4\text{-}1}' = m\, c_p\, (T_1' - T_4) = 1 \times 1\,(2453 - 824) = 1629 \text{ kJ/kg}$$

... (Taking c_p for air = 1 kg/kg K)

∴ Efficiency of the Diesel cycle,

$$\eta_D = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}} = \frac{1629 - 840}{1629}$$

$$= 0.484 \text{ or } 48.4\ \% \text{ Ans.}$$

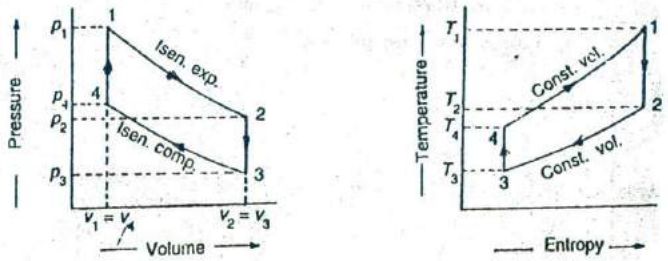
Example 6.20. *Two engines are to operate on Otto and Diesel cycles with the following data :*

Maximum temperature = 1500 K ; Exhaust temperature = 700 K ; Ambient conditions = 1 bar and 300 K.

Compare the compression ratios, maximum pressures, and efficiencies of the two engines.

Solution. Given : $T_1 = 1500$ K ; $T_2 = 700$ K ; $p_3 = 1$ bar ; $T_3 = 300$ K

First of all let us consider an Otto cycle as shown in Fig. 6.13.



(*a*) *p-v* diagram. (*b*) *T-S* diagram.

Fig. 6.13. Otto cycle.

Let r = Compression ratio = $v_3 / v_4 = v_2 / v_1$, and

p_1 = Maximum pressure.

We know that for isentropic expansion process 1-2,

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma - 1} = r^{\gamma - 1}$$

or

$$r = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma - 1}} = \left(\frac{1500}{700}\right)^{\frac{1}{1.4 - 1}} = 6.72$$

Now for isentropic compression process 3-4,

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (6.72)^{1.4 - 1} = 2.143$$

$$T_4 = T_3 \times 2.143 = 300 \times 2.143 = 643 \text{ K}$$

and $$p_4 v_4^{\gamma} = p_3 v_3^{\gamma} \quad \text{or} \quad p_4 = p_3 \left(\frac{v_3}{v_4}\right)^{\gamma} = p_3 \times r^{\gamma} = 1\,(6.72)^{1.4} = 14.4 \text{ bar}$$

For constant volume process 4-1,

$$\frac{p_1}{T_1} = \frac{p_4}{T_4} \quad \text{or} \quad p_1 = p_4 \times \frac{T_1}{T_4} = 14.4 \times \frac{1500}{643} = 33.6 \text{ bar}$$

We know that efficiency of Otto cycle,

$$\eta_O = 1 - \frac{1}{(r)^{\gamma - 1}} = 1 - \frac{1}{(6.72)^{1.4-1}} = 1 - \frac{1}{2.143} = 1 - 0.467$$

$$= 0.533 \text{ or } 53.3\% \quad \textbf{Ans.}$$

Note. The efficiency may also be calculated as follows :

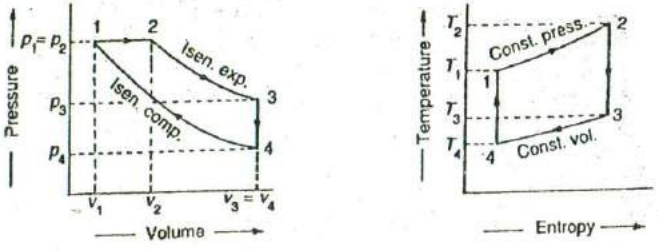
We know that $$\eta_O = 1 - \frac{T_2 - T_3}{T_1 - T_4} = 1 - \frac{700 - 300}{1500 - 643} = 1 - 0.467 = 0.533 \text{ or } 53.3\%$$

Now let us consider the Diesel cycle as shown in Fig. 6.14. In this case

$T_2 = 1500$ K ; $T_3 = 700$ K ; $T_4 = 300$ K ; $p_4 = 1$ bar

Let r = Compression ratio = v_4 / v_1, and

$p_1 = p_2$ = Maximum pressure.



(*a*) *p-v* diagram. (*b*) *T-S* diagram.

Fig. 6.14. Diesel cycle.

We know that for isentropic expansion process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{v_2}{v_3}\right)^{\gamma - 1} \quad \text{or} \quad \frac{v_2}{v_3} = \left(\frac{T_3}{T_2}\right)^{\frac{1}{\gamma - 1}} = \left(\frac{700}{1500}\right)^{\frac{1}{1.4-1}} = 0.1488 \quad \ldots (i)$$

Now for isentropic compression process 4-1,

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1}\right)^{\gamma - 1}$$

or $$T_1 = T_4 \left(\frac{v_4}{v_1}\right)^{\gamma - 1} = 300 \left(\frac{v_4}{v_1}\right)^{1.4-1} = 300 \left(\frac{v_4}{v_1}\right)^{0.4} \quad \ldots (ii)$$

and for constant pressure process 1-2,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \text{ or } T_1 = T_2 \times \frac{v_1}{v_2} = 1500 \times \frac{v_1}{v_2} \quad \ldots (iii)$$

From equations (*ii*) and (*iii*),

$$300\left(\frac{v_4}{v_1}\right)^{0.4} = 1500 \times \frac{v_1}{v_2} \text{ or } \left(\frac{v_4}{v_1}\right)^{0.4} \times \frac{v_2}{v_1} = \frac{1500}{300} \quad \ldots (iv)$$

We know that $$\frac{v_2}{v_1} = \frac{v_4}{v_1} \times \frac{v_2}{v_4} = \frac{v_4}{v_1} \times \frac{v_2}{v_3} \quad \ldots (\because v_4 = v_3)$$

$$= \frac{v_4}{v_1} \times 0.1488 \quad \ldots \text{[From equation } (i)\text{]}$$

Substituting the value of v_2/v_1 in equation (*iv*), we have

$$\left(\frac{v_4}{v_1}\right)^{0.4} \times \frac{v_4}{v_1} \times 0.1488 = \frac{1500}{300}$$

or $$\left(\frac{v_4}{v_1}\right)^{1.4} = \frac{1500}{300} \times \frac{1}{0.1488} = 33.6$$

$$\therefore \quad \frac{v_4}{v_1} = (33.6)^{\frac{1}{1.4}} = 12.3 \text{ or } r = 12.3$$

Now for isentropic compression process 4-1,

$$p_4 v_4^{\gamma} = p_1 v_1^{\gamma} \text{ or } p_1 = p_4\left(\frac{v_4}{v_1}\right)^{\gamma} = 1\,(12.3)^{1.4} = 33.6 \text{ bar}$$

and from equation (*ii*), for isentropic compression process,

$$T_1 = 300\left(\frac{v_4}{v_1}\right)^{0.4} = 300\,(12.3)^{0.4} = 818.6 \text{ K}$$

We know that efficiency of Diesel cycle,

$$\eta_D = 1 - \frac{1}{\gamma}\left(\frac{T_3 - T_4}{T_3 - T_1}\right) = 1 - \frac{1}{1.4}\left(\frac{700 - 300}{1500 - 818.6}\right)$$

$$= 1 - 0.419 = 0.581 \text{ or } 58.1\ \%$$

∴ Ratio of compression ratios,

$$\frac{r \text{ for Otto cycle}}{r \text{ for Diesel cycle}} = \frac{6.72}{12.3} = 0.546 \text{ Ans.}$$

Ratio of maximum pressures,

$$\frac{p_1 \text{ for Otto cycle}}{p_1 \text{ for Diesel cycle}} = \frac{33.6}{33.6} = 1 \text{ Ans.}$$

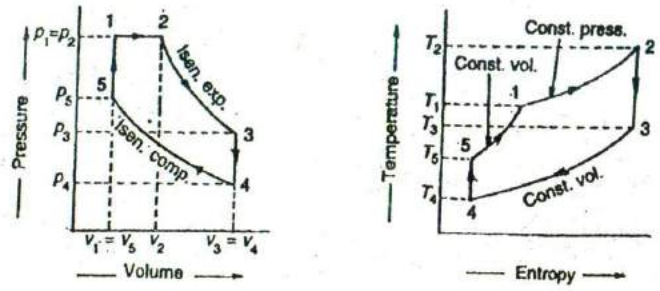
Ratio of efficiencies,

$$\frac{\eta \text{ for Otto cycle}}{\eta \text{ for Diesel cycle}} = \frac{0.533}{0.581} = 0.917 \text{ Ans.}$$

6.18. Dual Combustion Cycle

This cycle is a combination of Otto and Diesel cycles. It is sometimes called *semi-diesel cycle*, because semi-diesel engines work on this cycle. In this cycle, heat is absorbed partly at a constant volume and partly at a constant pressure.

The ideal dual combustion cycle consists of *two reversible adiabatic or isentropic , two constant volume* and *a constant pressure processes*. These processes are represented on p-v and T-S diagram as shown in Fig. 6.15 (*a*) and (*b*).



(*a*) p-v diagram. (*b*) T-S diagram.

Fig. 6.15. Dual combustion cycle.

Let the engine cylinder* contain m kg of air at point 1. At this point, let p_1, T_1 and v_1 be the pressure, temperature and volume of the air. Following are the five stages of an ideal dual combustion cycle.

1. *First stage (Constant pressure heating).* The air is heated at constant pressure from initial temperature T_1 to a temperature T_2 represented by the curve 1-2 in Fig. 6.15 (*a*) and (*b*).

$\therefore$ Heat absorbed by the air, $Q_{1\text{-}2} = m\, c_p\, (T_2 - T_1)$

2. *Second stage (Reversible adiabatic or isentropic expansion).* The air is expanded reversibly and adiabatically from temperature T_2 to a temperature T_3 as shown by the curve 2-3 in Fig. 6.15 (*a*) and (*b*). In this process, no heat is absorbed or rejected by the air .

3. *Third stage (Constant volume cooling).* The air is now cooled at constant volume from temperature T_3 to temperature T_4 as shown by the curve 3-4 in Fig. 6.15 (*a*) and (*b*).

$\therefore$ Heat rejected by the air, $Q_{3\text{-}4} = m\, c_v\, (T_3 - T_4)$

4. *Fourth stage (Reversible adiabatic or isentropic compression).* The air is compressed reversibly and adiabatically from temperature T_4 to a temperature T_5 as shown by the curve 4-5 in Fig. 6.15 (*a*) and (*b*). In this process, no heat is absorbed or rejected by the air.

5. *Fifth stage (Constant volume heating).* The air is finally heated at constant volume from temperature T_5 to a temperature T_1 as shown by the curve 5-1 in Fig. 6.15 (*a*) and (*b*).

$\therefore$ Heat absorbed by the air, $Q_{5\text{-}1} = m\, c_v\, (T_1 - T_5)$

We see that the air has been brought back to its original conditions of pressure, volume and temperature, thus completing the cycle. We know that

$$\text{Work done} = \text{Heat absorbed} - \text{Heat rejected}$$
$$= [m\, c_p\, (T_2 - T_1) + m\, c_v\, (T_1 - T_5)] - m\, c_v\, (T_3 - T_4)$$

* The cylinder is of similar type as discussed in Otto and Diesel cycles.

and air standard efficiency, $\eta = \dfrac{\text{Workdone}}{\text{Heat absorbed}}$

$$= \frac{m c_p (T_2 - T_1) + m c_v (T_1 - T_5) - m c_v (T_3 - T_4)}{m c_p (T_2 - T_1) + m c_v (T_1 - T_5)}$$

$$= 1 - \frac{c_v (T_3 - T_4)}{c_p (T_2 - T_1) + c_v (T_1 - T_5)}$$

$$= 1 - \frac{T_3 - T_4}{\gamma (T_2 - T_1) + (T_1 - T_5)} \quad \ldots (i)$$

Now, let compression ratio,

$$r = \frac{v_4}{v_5} = \frac{v_3}{v_1} \quad \ldots (\because v_3 = v_4 \text{ and } v_5 = v_1)$$

Cut-off ratio, $\rho = \dfrac{v_2}{v_1} = \dfrac{v_2}{v_5} \quad \ldots (\because v_1 = v_5)$

and pressure ratio,

$$\alpha = \frac{p_1}{p_5}$$

We know that in constant pressure heating process 1-2,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \quad \ldots \text{(Charles' law)}$$

$$\therefore \quad T_2 = T_1 \times \frac{v_2}{v_1} = T_1 \rho \quad \ldots (ii)$$

Similarly, in reversible adiabatic or isentropic expansion process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{v_2}{v_3}\right)^{\gamma - 1} = \left(\frac{v_2}{v_1} \times \frac{v_1}{v_3}\right)^{\gamma - 1} = \left(\frac{\rho}{r}\right)^{\gamma - 1}$$

$$\therefore \quad T_3 = T_2 \left(\frac{\rho}{r}\right)^{\gamma - 1} = T_1 \rho \left(\frac{\rho}{r}\right)^{\gamma - 1} \quad \ldots (iii)$$

and in reversible adiabatic or isentropic compression process 4-5,

$$\frac{T_5}{T_4} = \left(\frac{v_4}{v_5}\right)^{\gamma - 1} = (r)^{\gamma - 1}$$

$$\therefore \quad T_5 = T_4 (r)^{\gamma - 1} \quad \ldots (iv)$$

Now in constant volume heating process 5-1,

$$\frac{p_5}{T_5} = \frac{p_1}{T_1} \quad \ldots \text{(Gay-Lussac law)}$$

$$\therefore \quad T_1 = T_5 \times \frac{p_1}{p_5} = T_5 \alpha = T_4 (r)^{\gamma - 1} \alpha \quad \ldots (v)$$

Substituting the value of T_1 in equations (*ii*) and (*iii*),

$$T_2 = T_4 (r)^{\gamma-1} \alpha \rho$$

and

$$T_3 = T_4 (r)^{\gamma-1} \alpha \rho \left(\frac{\rho}{r}\right)^{\gamma-1} = T_4 \alpha \rho^{\gamma}$$

Now substituting the values of T_1, T_2, T_3 and T_5 in equation (*i*),

$$\eta = 1 - \frac{T_4 \alpha \rho^{\gamma} - T_4}{\gamma [T_4 (r)^{\gamma-1} \alpha \rho) - T_4 (r)^{\gamma-1} \alpha] + [T_4 (r)^{\gamma-1} \alpha - T_4 (r)^{\gamma-1}]}$$

$$= 1 - \frac{T_4 (\alpha \rho^{\gamma} - 1)}{T_4 (r)^{\gamma-1} [\gamma (\alpha \rho - \alpha) + (\alpha - 1)]}$$

$$= 1 - \frac{(\alpha \rho^{\gamma} - 1)}{(r)^{\gamma-1} [\gamma \alpha (\rho - 1) + (\alpha - 1)]}$$

$$= 1 - \frac{1}{(r)^{\gamma-1}} \left[\frac{\alpha \rho^{\gamma} - 1}{(\alpha - 1) + \gamma \alpha (\rho - 1)}\right] \quad \ldots (vi)$$

Notes : 1. For Otto cycle, $\rho = 1$. Substituting this value in equation (*vi*),

$$\eta_{otto} = 1 - \frac{1}{(r)^{\gamma-1}} \quad \ldots \text{(Same as before)}$$

2. For Diesel cycle, $\alpha = 1$. Substituting this value in equation (*vi*),

$$\eta_{diesel} = 1 - \frac{1}{(r)^{\gamma-1}} \left(\frac{\rho^{\gamma} - 1}{\gamma (\rho - 1)}\right) \quad \ldots \text{(Same as before)}$$

3. The efficiency of dual combustion cycle is greater than Diesel-cycle and less than Otto cycle, for the same compression ratio.

Example 6.21. *An oil engine, working on the dual combustion cycle, has a compression ratio 10 and cut-off takes place at 1/10 of the stroke. If the pressure at the beginning of compression is 1 bar and maximum pressure 40 bar, determine the air standard efficiency of the cycle. Take $\gamma = 1.4$.*

Solution. Given : $r = v_4 / v_5 = 10$; Cut-off = 1/10 of stroke ; $p_4 = 1$ bar ; $p_1 = p_2 = 40$ bar ; $\gamma = 1.4$

Since the cut-off takes place at 1/10 of the stroke, therefore volume at cut-off,

$$v_2 = v_1 + 1/10 \text{ of stroke volume} = v_1 + \frac{1}{10}(v_4 - v_1)$$

$$= v_5 + 0.1 (v_4 - v_5) \quad \ldots (\because v_1 = v_5)$$

$$\therefore \text{ Cut-off ratio, } \rho = \frac{v_2}{v_5} = 1 + 0.1 \left(\frac{v_4}{v_5} - 1\right) \quad \ldots \text{(Dividing both sides by } v_5)$$

$$= 1 + 0.1 (10 - 1) = 1.9$$

We know that for isentropic compression process 4-5 (Refer Fig. 6.15),

$$p_4 v_4^{\gamma} = p_5 v_5^{\gamma} \text{ or } p_5 = p_4 \left(\frac{v_4}{v_5}\right)^{\gamma} = 1 (10)^{1.4} = 25.12 \text{ bar}$$

Pressure ratio, $\alpha = p_1 / p_5 = 40 / 25.12 = 1.59$

We know that air standard efficiency,

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{\alpha\rho^{\gamma}-1}{(\alpha-1)+\gamma\alpha(\rho-1)}\right]$$

$$= 1 - \frac{1}{(10)^{1.4-1}}\left[\frac{1.59\,(1.9)^{1.4}-1}{(1.59-1)+1.4\times1.59\,(1.9-1)}\right]$$

$$= 1 - \frac{1}{2.512}\left[\frac{3.9-1}{0.59+2}\right] = 1 - 0.446 = 0.554 \text{ or } 55.4\ \%\ \textbf{Ans.}$$

Example 6.22. *In a *compression ignition engine, working on a dual combustion cycle, pressure and temperature at the start of compression are 1 bar and 300 K respectively. At the end of compression, pressure reaches a value of 25 bar. 420 kJ of heat is supplied per kg of air during constant volume heating and pressure becomes 2.8 bar at the end of isentropic expansion. Estimate the ideal thermal efficiency. Take $c_p = 1.005$ kJ/kg K and $c_v = 0.712$ kJ/kg K.*

Solution. Given : $p_4 = 1$ bar ; $T_4 = 300$ K ; $p_5 = 25$ bar ; $Q_{5-1} = 420$ kJ/kg ; $p_3 = 2.8$ bar ; $c_p = 1.005$ kJ/kg K ; $c_v = 0.712$ kJ/kg K

We know that ratio of specific heats or isentropic index,

$$\gamma = c_p / c_v = 1.005 / 0.712 = 1.4$$

First of all, consider the isentropic compression process 4-5 (Refer Fig. 6.15).

We know that

$$p_4 v_4^{\gamma} = p_5 v_5^{\gamma}$$

∴ Compression ratio,

$$r = \frac{v_4}{v_5} = \left(\frac{p_5}{p_4}\right)^{\frac{1}{\gamma}} = \left(\frac{25}{1}\right)^{\frac{1}{1.4}} = (25)^{0.714} = 9.96$$

We also know that $\dfrac{T_5}{T_4} = \left(\dfrac{v_4}{v_5}\right)^{\gamma-1} = (9.96)^{1.4-1} = 2.5$

∴ $T_5 = T_4 \times 2.5 = 300 \times 2.5 = 750$ K

Now consider the constant volume heating process 5-1. We know that heat supplied per kg of air (Q_{5-1}),

$$420 = m\,c_v\,(T_1 - T_5) = 1 \times 0.712\,(T_1 - 750)$$

∴ $T_1 - 750 = 420 / 0.712 = 590$ or $T_1 = 590 + 750 = 1340$ K

and pressure ratio, $\alpha = p_1 / p_5 = T_1 / T_5 = 1340 / 750 = 1.787$ ...($\because p_1 / T_1 = p_5 / T_5$)

∴ $p_1 = p_5 \times 1.787 = 25 \times 1.787 = 44.7$ bar

Now consider the isentropic expansion 2-3. We know that

$$p_2 v_2^{\gamma} = p_3 v_3^{\gamma} \text{ or } \frac{v_2}{v_3} = \left(\frac{p_3}{p_2}\right)^{\frac{1}{\gamma}} = \left(\frac{2.8}{44.7}\right)^{\frac{1}{1.4}} = 0.138 \quad \ldots(\because p_2 = p_1)$$

* A Diesel engine is also called compression ignition engine.

and cut-off ratio, $\rho = \frac{v_2}{v_1} = \frac{v_2}{v_5} = \frac{v_2}{v_3} \times \frac{v_3}{v_5} = \frac{v_2}{v_3} \times \frac{v_4}{v_5}$ ... ($\because v_1 = v_5$ and $v_3 = v_4$)

$= 0.138 \times 9.96 = 1.37$

We know that ideal thermal efficiency,

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}} \left[\frac{\alpha \rho^{\gamma} - 1}{(\alpha - 1) + \gamma \alpha (\rho - 1)} \right]$$

$$= 1 - \frac{1}{(9.96)^{1.4-1}} \left[\frac{1.787 (1.37)^{1.4} - 1}{(1.787 - 1) + 1.4 \times 1.787 (1.37 - 1)} \right]$$

$$= 1 - \frac{1}{2.51} \left[\frac{2.78 - 1}{0.787 + 0.926} \right] = 1 - 0.414 = 0.586 \text{ or } 58.6\% \text{ Ans.}$$

Example 6.23. *An engine working on a dual combustion cycle has a pressure of 1 bar and 50° C before compression. The air is then compressed isentropically to 1/15th of its original volume. The maximum pressure is twice the pressure at the end of isentropic compression. If the cut-off ratio is 2, determine the temperature at the end of each process and the ideal efficiency of the cycle, $\gamma = 1.4$.*

Solution. Given : $p_4 = 1$ bar ; $T_4 = 50° C = 50 + 273 = 323$ K ; $v_5 = v_4 / 15$; $p_1 = 2 p_5$; $\rho = v_2 / v_1 = 2$; $\gamma = 1.4$

We know that $v_5 = v_4 / 15$

∴ Compression ratio,

$$r = v_4 / v_5 = 15$$

Temperature at the end of isentropic compression

Let T_5 = Temperature at the end of isentropic compression.

We know that for isentropic compression process 4-5 (Refer Fig. 6.15),

$$\frac{T_5}{T_4} = \left(\frac{v_4}{v_5} \right)^{\gamma - 1} = (15)^{1.4-1} = 2.954$$

∴ $T_5 = T_4 \times 2.954 = 323 \times 2.954 = 954$ K $= 681° C$ **Ans.**

Temperature at the end of constant volume heating

Let T_1 = Temperature at the end of constant volume heating.

We know that for a constant pressure process 5-1,

$$\frac{p_5}{T_5} = \frac{p_1}{T_1} \text{ or } T_1 = T_5 \times \frac{p_1}{p_5} = 954 \times 2 = 1908 \text{ K} = 1635° C \text{ Ans.}$$

... ($\because p_1 = 2 p_5$)

Temperature at the end of constant pressure heating

Let T_2 = Temperature at the end of constant pressure heating.

We know that for a constant volume process 1-2,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \text{ or } T_2 = T_1 \times \frac{v_2}{v_1} = 1908 \times 2 = 3816 \text{ K} = 3543° C \text{ Ans.}$$

... ($\because v_2 / v_1 = 2$)

Temperature at the end of isentropic expansion 2-3,

Let T_3 = Temperature at the end of isentropic expansion.

We know that for isentropic expansion process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{v_2}{v_3}\right)^{\gamma-1} = \left(\frac{v_2}{v_1} \times \frac{v_1}{v_3}\right)^{1.4-1} = \left(2 \times \frac{1}{15}\right)^{0.4} = 0.447$$

$$\ldots\left(\because \frac{v_1}{v_3} = \frac{v_5}{v_4} = \frac{1}{15}\right)$$

$$\therefore \quad T_3 = T_2 \times 0.447 = 3816 \times 0.447 = 1706\ \text{K} = 1433^\circ\text{C}\ \textbf{Ans.}$$

Ideal efficiency of the cycle

We know that ideal efficiency of the cycle,

$$\eta = 1 - \frac{T_3 - T_4}{\gamma(T_2 - T_1) + (T_1 - T_5)}$$

$$= 1 - \frac{1706 - 323}{1.4(3816 - 1908) + (1908 - 954)} = 1 - \frac{1383}{2671 + 954}$$

$$= 1 - 0.3815 = 0.6185 \text{ or } 61.85\ \%\ \textbf{Ans.}$$

Example 6.24. *An ideal dual cycle engine works with a stroke volume of 10 litres of air with a compression ratio of 16. The pressure and temperature of air before isentropic compression is 1 bar and 300 K respectively. If the heat is added at a constant pressure of 70 bar and for 5% of the stroke, determine : 1. Pressure ratio ; 2. Cut-off ratio ; 3. Mass of air contained in the cylinder ; 4. Heat added per cycle ; 5. Heat rejected per cycle ; 6. Work done per cycle ; 7. Thermal efficiency of the cycle ; and 8. Mean effective pressure. Take c_p = 1 kJ/kg K and c_v = 0.714 kJ/kg K.*

Solution. Given : $v_s = v_4 - v_5 = v_3 - v_1 = 10$ litres $= 10 \times 10^{-3} = 0.01\ \text{m}^3$; $r = v_4 / v_5 = 16$; $p_4 = 1$ bar $= 0.1 \times 10^6\ \text{N/m}^2$; $T_4 = 300$ K ; $p_1 = p_2 = 70$ bar ; Cut-off = 5% of stroke ; $c_p = 1$ kJ/kg K ; $c_v = 0.714$ kJ/kg K

We know that $\gamma = c_p / c_v = 1 / 0.714 = 1.4$

1. *Pressure ratio*

Let p_5 = Pressure at point 5, and

α = Pressure ratio $= p_1 / p_5$.

We know that in isentropic compression process 4-5 (Refer Fig. 6.15),

$$p_4 v_4^\gamma = p_5 v_5^\gamma \quad \text{or} \quad p_5 = p_4 \left(\frac{v_4}{v_5}\right)^\gamma = 1\,(16)^{1.4} = 48.5\ \text{bar}$$

$\therefore$ Pressure ratio,

$$\alpha = p_1 / p_5 = 70 / 48.5 = 1.44\ \textbf{Ans.}$$

2. *Cut-off ratio*

We know that compression ratio,

$$r = v_4 / v_5 = 16 \quad \text{or} \quad v_4 = 16\, v_5 \qquad \ldots(i)$$

and

$$v_4 - v_5 = 10\ \text{litres} = 0.01\ \text{m}^3 \qquad \ldots(ii)$$

From equations (*i*) and (*ii*),

$$v_5 = 0.67 \times 10^{-3}\ \text{m}^3 \quad \text{and} \quad v_4 = 10.67 \times 10^{-3}\ \text{m}^3$$

Since the cut-off takes place at 5% of the stroke, therefore volume at cut-off,

$$v_2 = v_1 + 5\%\, v_s = v_1 + 0.05\,(v_4 - v_5)$$

$$= 0.67 \times 10^{-3} + 0.05 \times 0.01 = 1.17 \times 10^{-3}\ \text{m}^3 \qquad \ldots(\because v_1 = v_5)$$

$\therefore$ Cut-off ratio, $\rho = v_2 / v_1 = 1.17 \times 10^{-3} / 0.67 \times 10^{-3} = 1.746$ **Ans.**

3. *Mass of air contained in the cylinder*

Let m = Mass of air contained in the cylinder.

We know that $p_4 v_4 = m R T_4$

$\therefore$ $$m = \frac{p_4 v_4}{R T_4} = \frac{0.1 \times 10^6 \times 10.67 \times 10^{-3}}{287 \times 300} = 0.012 \text{ kg}$$ **Ans.**

... (Taking $R = 287$ J/kg K)

4. *Heat added per cycle*

First of all, let us find the temperature at points 1 and 2 *i.e.* T_1 and T_2 respectively.

We know that for isentropic compression process 4-5,

$$\frac{T_5}{T_4} = \left(\frac{v_4}{v_5}\right)^{\gamma - 1} = (16)^{1.4-1} = 3.03$$

$\therefore$ $$T_5 = T_4 \times 3.03 = 300 \times 3.03 = 909 \text{ K}$$

Similarly for constant volume process 5-1,

$$\frac{p_5}{T_5} = \frac{p_1}{T_1} \quad \text{or} \quad T_1 = T_5 \times \frac{p_1}{p_5} = 909 \times \frac{70}{48.5} = 1312 \text{ K}$$

and for constant pressure process 1-2,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \quad \text{or} \quad T_2 = T_1 \times \frac{v_2}{v_1} = 1312 \times \frac{1.17 \times 10^{-3}}{0.67 \times 10^{-3}} = 2291 \text{ K}$$

$\ldots(\because v_1 = v_5)$

We know that heat added during constant pressure process 1-2,

$$Q_{1-2} = m c_p (T_2 - T_1) = 0.012 \times 1\,(2291 - 1312) = 11.75 \text{ kJ}$$

and heat added during constant volume process 5-1,

$$Q_{5-1} = m c_v (T_1 - T_5) = 0.012 \times 0.714\,(1312 - 909) = 3.45 \text{ kJ}$$

$\therefore$ Total heat added per cycle,

$$Q = Q_{1-2} + Q_{5-1} = 11.75 + 3.45 = 15.20 \text{ kJ}$$ **Ans.**

5. *Heat rejected per cycle*

First of all, let us find the temperature (T_3) at point 3.

We know that for isentropic expansion process 2-3,

$$\frac{T_2}{T_3} = \left(\frac{v_3}{v_2}\right)^{\gamma - 1} = \left(\frac{10.67 \times 10^{-3}}{1.17 \times 10^{-3}}\right)^{1.4-1} = 2.42 \qquad \ldots(\because v_3 = v_4)$$

$$\therefore \quad T_3 = T_2 / 2.42 = 2291 / 2.42 = 947 \text{ K}$$

and heat rejected per cycle (during constant volume process 3-4),

$$= m c_v (T_3 - T_4) = 0.012 \times 0.714 (947 - 300) = 5.54 \text{ kJ} \quad \textbf{Ans.}$$

6. *Work done per cycle*

We also know that work done per cycle

$$= \text{Heat added} - \text{Heat rejected} = 15.20 - 5.54 = 9.66 \text{ kJ} \quad \textbf{Ans.}$$

7. *Thermal efficiency of the cycle*

We know that thermal efficiency of the cycle,

$$\eta = \frac{\text{Work done per cycle}}{\text{Heat added per cycle}} = \frac{9.66}{15.20} = 0.636 \text{ or } 63.6\% \quad \textbf{Ans.}$$

8. *Mean effective pressure.*

We know that mean effective pressure,

$$= \frac{\text{Work done}}{\text{Stroke volume}} = \frac{9.66}{0.01} = 966 \text{ kN/m}^2 = 9.66 \text{ bar} \quad \textbf{Ans.}$$

EXERCISES

1. An ideal engine is imagined to be working on Carnot's cycle. The working fluid receives heat at a temperature of 590 K and rejects at a temperature of 295 K. Find the theoretical efficiency of the cycle.

If the engine working on this cycle absorbs 35 kJ/s from the hot body, calculate the net work-done per second. **[Ans. 50% ; 17.5 kJ/s]**

2. In a carnot cycle engine, the temperature of the source and sink are 700° C and 50° C. The heat supply is 84 kJ/s. Find the power developed by the engine.

If the temperature of source is 500° C and the heat supply and workdone are 420 kJ and 200 kJ respectively, find the sink temperature. **[Ans. 56.112 kW ; 132° C]**

3. An inventor claims that his petrol engine operating between temperatures 2000° C and 600° C will produce 0.735 kW consuming 0.12 kg per hour of petrol having 46 000 kJ/kg calorific value. Check the validity of the claim. **[Ans. Inventor's claim is valid]**

4. Determine the efficiency of the regenerative Stirling cycle working between the temperature limits of 700 K and 350 K. The isothermal expansion or compression ratio is 4. **[Ans. 50%]**

5. A regenerative Stirling air engine works between the temperatures of 400° C and 15° C. The ratio of isothermal expansion is 3. Find ideal efficiency of the cycle, if regenerator efficiency is 80%. Assume c_p = 0.996 kJ/kg K, and c_v = 0.707 kJ/kg K. **[Ans. 45.6%]**

6. In a Stirling engine working with a perfect regenerator, the lower limit of temperature is 538° C. The maximum and minimum pressures are 12 bar and 2 bar. Determine the ideal efficiency, if the expansion ratio is 3. **[Ans. 50%]**

7. An Ericsson engine having an expansion ratio 2 works between the temperature ranges of 700 K and 298 K. Calculate the ideal efficiency when :

1. the engine is fitted with a perfect regenerator, and 2. the efficiency of regenerator is 0.85.

Assume c_p = 0.996 kJ/kg K and c_v = 0.712 kJ/kg K **[Ans. 57.4% ; 40%]**

8. A gas turbine working on Joule's cycle receives air at 1 bar and 15° C. It is compressed isentropically to 5 bar and then heated at a constant pressure till its temperature reaches 800° C. The hot air is then expanded isentropically in the turbine back to its original pressure of 1 bar. Determine the work done kg of air supplied and thermal efficiency of the turbine. Assume γ = 1.4 and c_p = 1 kJ/kg K. **[Ans. 228 kJ ; 37%]**

9. The efficiency of an Otto cycle is 50% and γ = 1.5. Find the compression ratio. **[Ans. 4]**

10. An engine working on ideal Otto cycle has temperature and pressure, at the beginning of isentropic compression as 25° C and 1.5 bar respectively. Find the compression ratio, if $\gamma = 1.4$ and thermal efficiency of the engine = 48%. Also find temperature and pressure at the end of compression.

[**Ans.** 5.13 ; 300° C ; 14.8 bar]

11. An engine working on constant volume cycle has a clearance volume of 1 litre and a stroke volume of 6 litres. The suction pressure and temperature are 1 bar and 20° C respectively. The pressure at the end of heat addition is 25 bar. Determine ; 1. pressure and volume at salient points of the cycle ; 2. thermal efficiency, and 3. workdone per cycle.

Take c_v during heat addition = 0.807 kJ/kg K ; c_v during heat rejection = 0.737 kJ/kg K and $\gamma = 1.4$. [Ans. $p_2 = 1.64$ bar = $p_4 = 15.24$ bar, $v_1 = v_4 = 1$ litre ; $v_2 = v_3 = 7$ litre ; 54.1% ; 192 kJ/kg]

12. A four-cylinder petrol engine has a swept volume of 2×10^{-3} m^3 and the clearance volume in each cylinder is 60×10^{-6} m^3. If the pressure and temperature at the beginning of compression are 1.02 bar and 24° C and the maximum cycle temperature is 1400° C, calculate 1. the air standard efficiency, and 2. the mean effective pressure. Assume for air, $c_p = 1.005$ kJ/kg K and $c_v = 0.716$ kJ/kg K. [**Ans.** 59.1% ; 5.3 bar]

13. An air standard Diesel cycle has a compression ratio of 14. The pressure at the beginning of compression stroke is 1 bar and the temperature is 27° C. The maximum temperature of the cycle is 2500°C. Determine the efficiency of this engine. [**Ans.** 53.6 %]

14. A diesel engine has a bore of 250 mm and a stroke of 400 mm. The cut-off takes place at 5 per cent of the stroke. Estimate : 1. Air standard efficiency, and 2. Mean effective pressure if the clearance volume and the pressure at the end of suction stroke are 1.2 litres and 1 bar respectively. [**Ans.** 64.2% ; 5.11 bar]

15. Estimate the loss in air standard efficiency for an engine of compression ratio of 15 and the cut-off changes from 5 to 15% of the stroke. [**Ans.** 6.6%]

16. An ideal Diesel cycle operates on 1 kg of standard air with initial pressure of 1 bar and a temperature of 35° C. The pressure at the end of compression is 33 bar and the cut-off 6% of the stroke. Determine : 1. the compression ratio, 2. the percentage clearance, 3. the heat supplied, and 4. the heat rejected.

Take $\gamma = 1.4$ and $c_p = 1$ kJ/kg K. [**Ans.** 12.14 ; 8.98% of stroke ; 559 kJ ; 230 kJ]

17. An ideal Diesel cycle has compression ratio of 16 with lowest temperature and pressure as 27°C and 1 bar respectively. If heat added during the cycle is 500 kJ/kg of air, determine 1. pressure and temperature at each salient point, 2. work done in the cycle/kg, 3. thermal efficiency of the cycle, and 4. mean effective pressure. [**Ans.** $p_1 = p_2 = 48.5$ bar, $T_1 = 636°$ C, $T_2 = 1136°$ C, $p_3 = 1.87$ bar, $T_3 = 175°$ C ; 394.3 kJ/kg ; 78.9% ; 4.9 bar]

18. Two engines are to operate on Otto and Diesel cycles with the following data :

Maximum temperature = 1500 K ; Exhaust temperature = 700 K ; Ambient condition = 1 bar and 300 K.

Find the compression ratios, maximum pressures and efficiencies.

[**Ans.** Otto cycle : 6.72 ; 33.6 bar ; 53.3 % ; Diesel cycle: 12.3 ; 33.6 bar ; 58.1%]

19. An engine working on the dual combustion cycle has pressure ratio of 1.41. The diameter of engine cylinder is 200 mm and stroke 420 mm. If the compression ratio is 13.5 and cut-off occurs at 5% of the stroke, determine the air standard efficiency of the engine. [**Ans.** 61.8%]

20. An oil engine operates on ideal dual combustion cycle as follows :

Pressure at the beginning of compression = 0.97 bar ; Temperature at the beginning of compression = 55° C ; Pressure ratio = 1.5 ; Cut-off ratio = 1.6 ; Compression ratio = 11 ; Ratio of specific heats, $\gamma = 1.4$.

Determine the temperature at the end of each operation and the efficiency of the engine.

[Ans. $T_5 = 583°$ C ; $T_1 = 1011°$ C ; $T_2 = 1782°$ C ; $T_3 = 676°$ C ; 58.8%]

QUESTIONS

1. What is a thermodynamic cycle ? Distinguish between a four-stroke cycle and a two-stroke cycle.

2. Explain reversible and irreversible processes. Is it possible to realise these processes ?

3. What are the conditions of reversibility ? Prove that all reversible engines are equally efficient between the same temperature limits.

4. Do you agree with the statement that the conception of thermodynamic reversibility is purely hypothetical ? Explain fully.

5. Prove that the efficiency of the Carnot engine working between the temperature limits T_1 and T_2 is equal to $\frac{T_1 - T_2}{T_1}$, and show that no engine can be more efficient than this when working over the same temperature range.

6. Derive an expression for the efficiency of a Stirling air engine with the aid of p-v and T-S diagrams.

7. Deduce an expression for thermal efficiency of Joule's air engine and show that it is less than Carnot efficiency.

8. Explain briefly Otto cycle with the help of p-v and T-S diagram, and derive an expression for the ideal efficiency of Otto cycle.

9. Show that the efficiency of Otto cycle is a function of compression ratio only.

10. Explain briefly the Diesel cycle with the help of p-v and T-S diagrams and derive an expression for the ideal efficiency of a Diesel cycle.

11. For a given compression ratio ; the air standard Diesel cycle is less efficient than air standard Otto cycle. Explain.

12. Derive an expression for the ideal efficiency of dual combustion cycle, using ideal air as the working fluid.

OBJECTIVE TYPE QUESTIONS

1. Carnot cycle consists of

(*a*) two constant volume and two reversible adiabatic processes

(*b*) two isothermal and two reversible adiabatic processes

(*c*) two constant pressure and two reversible adiabatic processes

(*d*) one constant volume, one constant pressure and two reversible adiabatic processes

2. The efficiency of Carnot cycle may be increased by

(*a*) increasing the highest temperature (*b*) decreasing the highest temperature

(*c*) increasing the lowest temperature (*d*) decreasing the lowest temperature

3. A cycle consisting of two constant volume and two isothermal processes is known as

(*a*) Carnot cycle (*b*) Joule cycle (*c*) Diesel cycle (*d*) Stirling cycle

4. The efficiency of Ericsson cycle is .....Carnot cycle.

(*a*) greater than (*b*) less than (*c*) equal to

5. Otto cycle is also known as

(*a*) constant pressure cycle (*b*) constant volume cycle

(*c*) constant temperature cycle (*d*) constant entropy cycle

6. For the same compression ratio, the efficiency of Diesel cycle is ..... Otto cycle.

(*a*) greater than (*b*) less than (*c*) equal to

7. The efficiency of Diesel cycle approaches to Otto cycle efficiency when

(*a*) cut-off is increased (*b*) cut-off is decreased

(*c*) cut-off is zero (*d*) cut-off is constant

8. The air standard efficiency of an Otto cycle is given by

(*a*) $1-\frac{1}{r^{\gamma-1}}$ (*b*) $1+\frac{1}{r^{\gamma-1}}$ (*c*) $1-r^{\gamma-1}$ (*d*) $1+r^{\gamma-1}$

9. The efficiency of the dual combustion cycle, for the same compression ratio, is .... Diesel cycle.

(*a*) greater than (*b*) less than (*c*) equal to

10. For the same maximum pressure and temperature of Otto, Diesel and dual combustion air standard cycles,

(*a*) the compression ratios will be same

(*b*) the heat supplied to the cycles will be same

(*c*) the air standard efficiency will have the same value

(*d*) the heat rejected by the engine will be the same

ANSWERS

1. (*b*)	2. (*b*)	3. (*d*)	4. (*c*)	5. (*b*)
6. (*b*)	7. (*b*)	8. (*a*)	9. (*a*)	10. (*d*)

7

Formation and Properties of Steam

1. Introduction. 2. Formation of Steam at a Constant Pressure from Water. 3. Temperature v_s. Total Heat Graph during Steam Formation. 4. Important Terms for Steam. 5. Steam Tables and their Uses. 6. Superheated Steam. 7. Advantages of Superheating the Steam. 8. External Work Done During Evaporation. 9. Internal Energy of Steam. 10. Measurement of Dryness Fraction of Steam. 11. Barrel Calorimeter. 12. Separating Calorimeter. 13. Throttling Calorimeter. 14. Combined Separating and Throttling Calorimeter.

7.1. Introduction

Steam is a vapour of water, and is invisible when pure and dry. It is used as the working substance in the operation of steam engines and steam turbines. Steam does not obey laws of perfect gases, until it is perfectly dry. It has already been discussed that when the dry vapour is heated further, it becomes superheated vapour which behaves, more or less, like a perfect gas.

7.2. Formation of Steam at a Constant Pressure from Water

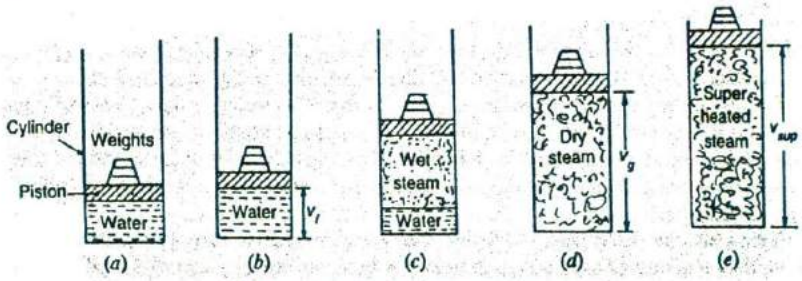


Fig. 7.1. Formation of steam at constant pressure.

Consider 1 kg of water at 0° C contained in the piston-cylinder arrangement as shown in Fig. 7.1 (*a*). The piston and weights maintain a constant pressure in the cylinder. If we heat the water contained in the cylinder, it will be converted into steam as discussed below :

1. The volume of water will increase slightly with the increase in temperature as shown in Fig. 7.1 (*b*). It will cause the piston to move slightly upwards and hence work is obtained. This increase in volume of water (or work) is generally, neglected for all types of calculations.

2. On further heating, temperature reaches boiling point. The boiling point of water, at normal atmospheric pressure of 1.013 bar is 100° C, but it increases with the increase in pressure. When the boiling point is reached, the temperature remains constant and the water evaporates, thus pushing the piston up against the constant pressure. Consequently, the specific volume of steam increases as shown in Fig. 7.1 (*c*). At this stage, the steam will have some particles of water in suspension, and is termed as *wet steam*. This process will continue till the whole water is converted into wet steam.

3. On further heating, the water particles in suspension will be converted into steam. The entire steam, in such a state, is termed as *dry steam* or *saturated steam* as shown in Fig. 7.1 (*d*). Practically, the dry steam behaves like a perfect gas.

4. On further heating, the temperature of the steam starts rising. The steam, in such a state, is termed as *superheated steam* as shown in Fig. 7.1 (*e*).

7.3. Temperature vs. Total Heat Graph during Steam Formation

The process of steam formation, as discussed above, may also be represented on a graph, whose abscissa represents the total heat and the vertical ordinate represents the temperature. The point *A* represents the initial condition of water at 0° C and pressure *p* (in bar) as shown in Fig. 7.2. Line *ABCD* shows the relation between temperature and heat at a specific pressure of *p* (in bar).

During the formation of the superheated steam, from water at freezing point, the heat is absorbed in the following three stages :

1. The heating of water up to boiling temperature or saturation temperature (*t*) is shown by *AB* in Fig. 7.2. The heat absorbed by the water is *AP*, known as *sensible heat* or *liquid heat* or *total heat of water.*

2. The change of state from liquid to steam is shown by *BC*. The heat absorbed during this stage is *PQ*, known as *latent heat of vaporisation.*

3. The superheating process is shown by *CD*. The heat absorbed during this stage is *QR*, known as *heat of superheat.* Line *AR* represents the *total heat of the superheated steam.*

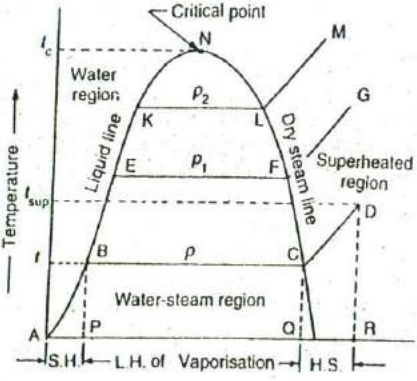


Fig. 7.2. Temperature-total heat graph during steam formation.

If the pressure is increased (say p_1 bar), the boiling temperature also increases. The point *E* represents the boiling temperature or saturation temperature at pressure p_1 and *F* is the point of dry saturated steam. Line *FG* shows the constant pressure process, in which the steam is superheated. Similarly, a family of curves may be drawn for different pressures as shown in the figure.

The line passing through the points *A, B, E, K* is known as *saturated liquid line* which forms boundary line between water and steam. Similarly, a line passing through dry steam points *L, F, C* is known as *dry saturated steam line* which forms boundary line between wet and superheated steam. Sometimes, these terms are briefly written as *liquid line* and *dry steam line,* but the word "saturated" is always understood.

It may also be noted from the figure, that when the pressure and saturation temperature increases, the latent heat of vaporisation decreases. it becomes zero at a point (*N*) where liquid and dry steam lines meet. This point *N* is known as the *critical point* and at this point, the liquid and vapour phases merge, and become identical in every respect. The temperature corresponding to critical point *N* is known as *critical temperature* and the pressure is known as *critical pressure.* For steam, the critical temperature is 374.15° C and critical pressure is 221.2 bar.

7.4. Important Terms for Steam

Though there are many terms applied to steam, yet the following are important from the subject point of view :

1. *Wet steam.* When the steam contains moisture or particles of water in suspension, it is said to be *wet steam.* It means that the evaporation of water is not complete, and the whole of the latent heat has not been absorbed.

2. *Dry saturated steam.* When the wet steam is further heated, and it does not contain any suspended particles of water, it is known as *dry saturated steam.* The dry saturated steam has absorbed its full latent heat and behaves practically, in the same way as a perfect gas.

3. *Superheated steam.* When the dry steam is further heated at a constant pressure, thus raising its temperature, it is said to be *superheated steam*. Since the pressure is constant, therefore the volume of superheated steam increases. It may be noted that the volume of one kg of superheated steam is considerably greater than the volume of one kg of dry saturated steam at the same pressure.

In actual practice, the superheated steam is produced in a separate apparatus known as superheater, so that it is out of contact with water from which it was formed.

4. *Dryness fraction or quality of wet steam.* It is the ratio of the mass of actual dry steam, to the mass of same quantity of wet steam, and is generally denoted by 'x'. Mathematically,

$$x = \frac{m_g}{m_g + m_f} = \frac{m_g}{m}$$

where m_g = Mass of actual dry steam,

m_f = Mass of water in suspension, and

m = Mass of wet steam = $m_g + m_f$

Note: The value of dryness fraction, in case of dry steam, is unity. At this stage, the mass of water in suspension (m_f) is zero.

5. *Sensible heat of water.* It is the amount of heat absorbed by 1 kg of water, when heated at a constant pressure, from the freezing point (0° C) to the temperature of formation of steam, *i.e.* saturation temperature (t). The sensible heat is also known as *liquid heat.*

The specific heat of water at constant pressure is usually taken as 4.2 kJ/kg K. Therefore heat absorbed by 1 kg of water from 0° C to t °C or sensible heat

= Mass × Sp. heat × Rise in temperature

$= 1 \times 4.2\,[(t + 273) - (0 + 273)] = 4.2\,t$ kJ/kg

Thus the sensible heat of water in kJ/kg may be obtained directly by multiplying the specific heat of water and the saturation temperature (t) in °C.

It may be noted that the sensible heat of water is taken equal to the * specific enthalpy (*i.e.* total heat) of water. It is, usually, denoted by h_f in steam tables and its value may be determined for any given pressure in kJ/kg.

---

* We know that specific enthalpy (*i.e.* enthalpy per kg) of water = Specific internal energy + Flow work *i.e.* work required in changing the volume against pressure p.

or $h = u + pdv$

Let v_f = Specific volume of water at 0° C (*i.e.* volume of 1 kg of water at 0° C).

When this water is heated from 0° C to t° C, there is a negligible change in volume. In other words, the specific volume of water at t° C is nearly the same *i.e.* v_f.

∴ Specific enthalpy of water at 0° C,

$$h_0 = u_0 + p\,v_f \quad \ldots (i)$$

Similarly, specific enthalpy of water at t° C,

$$h_f = v_f + p\,v_f \quad \ldots (ii)$$

∴ Change in specific enthalpy of water (*i.e.* enthalpy of 1 kg of water when heated from 0°C to t°C),

$$h_f - h_0 = u_f - u_0 \quad \ldots (iii)$$

Since the specific internal energy of water at 0° C is zero and the specific volume (v_f) is negligible, therefore $h_0 = 0$. Now from equation (*iii*),

$h_f = u_f$ *i.e.* specific internal energy at t° C or the heat absorbed by 1 kg of water in raising its temperature from 0° C to t° C

= Sensible heat

6. **Latent heat of vaporisation.* It is the amount of heat absorbed to evaporate 1 kg of water, at its boiling point or saturation temperature without change of temperature. It is denoted by h_{fg} and its value depends upon the pressure. The heat of vaporisation of water or latent heat of steam is 2257 kJ/kg at atmospheric pressure.

It has been experimentally found that the value of h_{fg} decreases as the pressure increases and it is zero at critical pressure. If the steam is wet with a dryness fraction x, then the heat absorbed by it during evaporation is $x\, h_{fg}$.

7. *Enthalpy or total heat of steam.* It is amount of heat absorbed by water from freezing point to saturation temperature *plus* the heat absorbed during evaporation.

∴ Enthalpy or total heat of steam

= Sensible heat + Latent heat

It is denoted by h_g and its value for the dry saturated steam may be read directly from the steam tables. The expressions for the enthalpy of wet steam, dry steam and superheated steam are as follows :

(*i*) *Wet steam.* The enthalpy of wet steam is given by :

$$h = h_f + x\, h_{fg} \qquad \ldots (i)$$

where x is the dryness fraction of steam.

(*ii*) *Dry steam.* We know that in case of dry steam, $x = 1$.

$$\therefore \qquad h = h_g = h_f + h_{fg} \qquad \ldots (ii)$$

(*iii*) *Superheated steam.* If we further add heat to the dry steam, its temperature increases while pressure remaining constant. This increase in temperature shows the superheat stage of the steam. Thus, the total heat required for the steam to be superheated is :

h_{sup} = Total heat for dry steam + Heat for superheated steam

$$= h_f + h_{fg} + c_p\,(t_{sup} - t) = h_g + c_p\,(t_{sup} - t) \quad **$$

where c_p = Mean specific heat at constant pressure for superheated steam,

t_{sup} = Temperature of the superheated steam, and

t = Saturation temperature at the given constant pressure.

Notes : 1. The difference $(t_{sup} - t)$ is known as *degree of superheat.*

2. The value of c_p for steam lies between 1.67 kJ/kg K to 2.5 kJ/kg K.

8. *Specific volume of steam.* It is the volume occupied by the steam per unit mass at a given temperature and pressure, and is expressed in m^3/kg. It is the reciprocal of density of steam in kg/m^3. It may be noted that the value of specific volume decreases with the increase in pressure. The expressions for the volumes occupied by steam are as follows :

(*i*) *Wet steam.* Consider 1 kg of wet steam of dryness fraction x. We know that this steam will have x kg of dry steam and $(1 - x)$ kg of water. Let v_f be the volume of 1 kg of water, then

Volume of one kg of wet steam

$$= x\, v_g + (1 - x)\, v_f$$

---

* See also Art. 7.8

** It makes no difference whether the individual temperatures (t_{sup} and t) are in Kelvin or °C because of difference in temperatures.

Since v_f is very small as compared to v_g, therefore the expression $(1-x)\,v_f$ may be neglected.

$\therefore$ Volume of 1 kg of wet steam $= x\,v_g$ m^3

or specific volume of wet steam, $v = x\,v_g$ m^3/kg ... (*i*)

(*ii*) *Dry steam.* We know that in case of dry steam, the mass of water in suspension is zero and dryness fraction is unity. Therefore specific volume of dry steam

$= v_g$ m^3/kg ... (*ii*)

(*iii*) *Superheated steam.* We have already discussed that when the dry saturated steam is further heated under a constant pressure, there is an increase in volume with the rise in temperature. The superheated steam behaves more or less like a perfect gas. Therefore, according to Charles' law,

$$\frac{v_{sup}}{T_{sup}} = \frac{v_g}{T} \text{ or } v_{sup} = \frac{v_g\,T_{sup}}{T} \quad \ldots (\because p \text{ is constant}) \ldots (iii)$$

where v_{sup} = Specific volume of superheated steam,

v_g = Specific volume of dry steam at the pressure of steam formation,

T_{sup} = Absolute temperature of superheated steam, and

T = Absolute saturation temperature at the pressure of steam formation.

7.5. Steam Tables and their Uses

The properties of dry saturated steam like its temperature of formation (saturation temperature), sensible heat, latent heat of vaporisation, enthalpy or total heat, specific volume, entropy etc., vary with pressure, and can be found by experiments only. These properties have been carefully determined, and made available in a tabular form known as *steam tables.* It may be noted, that there is a slight difference in figures, quoted by various authors, and even in their various editions of a book. But in this book we shall use the standard properties which are internationally recognised.

There are two important steam tables, one in terms of absolute pressure and other in terms of temperature. An extract of the properties of dry saturated steam is shown in the following two tables :

Table 7.1 (Pressure)

Absolute Pressure (p) in bar	*Temperature (t) in °C*	*Specific volume in m³/kg*		*Specific enthalpy in kJ/kg*			*Specific entropy in kJ/kg K*		
		Water (v_f)	Steam (v_g)	Water (h_f)	Evaporation (h_{fg})	Steam (h_g)	Water (s_f)	Evaporations (s_{fg})	Steam (s_g)
0.010	6.983	0.001 000	129.21	29.3	2485.1	2514.4	0.106	8.871	8.977
0.015	13.04	0.001 001	87.982	54.7	2470.8	2525.5	0.196	8.634	8.830
0.20	60.09	0.001 017	7.649	251.5	2358.4	2609.9	0.832	7.077	7.909

Table 7.2 (Temperature)

Temperature in (t) in °C	*Absolute pressure (p) in bar*	*Specific volume in m³/kg*		*Specific enthalpy in kJ/kg*			*Specific entropy in kJ/kg K*		
		Water (v_f)	Steam (v_g)	Water (h_f)	Evaporation (h_{fg})	Steam (h_g)	Water (s_f)	Evaporations (s_{fg})	Steam (s_g)
0	0.006 11	0.001 000	206.31	0.0	2501.6	2501.6	0.000	9.158	9.158
5	0.008 72	0.001 000	147.16	21.0	2489.7	2510.7	0.076	8.951	9.027
10	0.012 27	0.001 000	106.43	42.0	2477.9	2519.9	0.151	8.751	8.902

Example 7.1. *Calculate the enthalpy of 1 kg of steam at a pressure of 8 bar and dryness fraction of 0.8. How much heat would be required to raise 2 kg of this steam from water at 20° C?*

Solution. Given : $p = 8$ bar ; $x = 0.8$

Enthalpy of 1 kg of steam

From steam tables, corresponding to a pressure of 8 bar, we find that

$$h_f = 720.9 \text{ kJ/kg} \text{ and } h_{fg} = 2046.5 \text{ kJ/kg}$$

We know that enthalpy of 1 kg of wet steam,

$$h = h_f + x\, h_{fg} = 720.9 + 0.8 \times 2046.5 = 2358.1 \text{ kJ}$$ **Ans.**

Heat required to raise 2 kg of this steam from water at 20° C

We have calculated above the enthalpy or total heat required to raise 1 kg of steam from water at 0° C. Since the water, in this case, is already at 20° C, therefore

Heat already in water $= 4.2 \times 20 = 84$ kJ

∴ Heat required per kg of steam

$$= 2358.1 - 84 = 2274.1 \text{ kJ}$$

and heat required for 2 kg of steam

$$= 2 \times 2274.1 = 4548.2 \text{ kJ}$$ **Ans.**

Example 7.2. *Determine the quantity of heat required to produce 1 kg of steam at a pressure of 6 bar at a temperature of 25° C, under the following conditions :*

1. when the steam is wet having a dryness fraction 0.9 ;

2. when the steam is dry saturated ; and

3. when it is superheated at a constant pressure at 250° C assuming the mean specific heat of superheated steam to be 2.3 kJ/kg K.

Solution. Given : $p = 6$ bar ; $t_w = 25°$ C ; $x = 0.9$; $t_{sup} = 250°$ C ; $c_p = 2.3$ kJ/kg K

From steam tables, corresponding to a pressure of 6 bar, we find that

$$h_f = 670.4 \text{ kJ/kg} ; \; h_{fg} = 2085 \text{ kJ/kg} \text{ and } t = 158.8° \text{ C}$$

1. *When the steam is wet*

We know that enthapy or total heat of 1 kg of wet steam,

$$h = h_f + x\, h_{fg} = 670.4 + 0.9 \times 2085 = 2546.9 \text{ kJ}$$

Since the water is at a temperature of 25° C, therefore

Heat already in water $= 4.2 \times 25 = 105$ kJ

∴ Heat actually required $= 2546.9 - 105 = 2441.9$ kJ **Ans.**

2. *When the steam is dry saturated*

We know that enthalpy or total heat of 1 kg of dry saturated steam,

$$h_g = h_f + h_{fg} = 670.4 + 2085 = 2755.4 \text{ kJ}$$

∴ Heat actually required $= 2755.4 - 105 = 2650.4$ kJ **Ans.**

3. *When the steam is superheated*

We know that enthalpy or total heat of 1 kg of superheated steam,

$$h_{sup} = h_g + c_p\,(t_{sup} - t) = 2755.4 + 2.3\,(250 - 158.8) = 2965.16 \text{ kJ}$$

∴ Heat actually required $= 2965.16 - 105 = 2860.16$ kJ **Ans.**

Example 7.3. *Determine the condition of steam in the following cases :*

1. At a pressure of 10 bar and temperature 200° C.

2. At a pressure of 10 bar and volume 0.175 m³/kg.

Solution. Given : $p = 10$ bar. ; $t = 200°$ C ; $v = 0.175$ m³/kg

1. *Condition of steam at temperature of 200° C*

From steam tables, corresponding to a pressure of 10 bar, we find that

$$v_g = 0.194 \text{ m}^3/\text{kg} ; \; h_f = 762.6 \text{ kJ/kg} ; \text{ and } t = 179.9° \text{ C}$$

Since the saturation temperature at 10 bar is (179.9° C) or lower than the given temperature of the steam (200° C), therefore the given steam is superheated. The degree of superheat

$$= 200 - 179.9 = 20.1° \text{ C} \quad \textbf{Ans.}$$

2. *Condition of steam at a volume of 0.175 m³/kg*

Since the volume of given steam (0.175 m³/kg) is less than the specific volume of the dry saturated steam (0.194 m³/kg), therefore the given steam is wet. The dryness fraction of steam,

$$x = \frac{0.175}{0.194} = 0.902 \quad \textbf{Ans.}$$

Example 7.4. *Steam enters an engine at a pressure of 12 bar with a 67° C of superheat. It is exhausted at a pressure of 0.15 bar and 0.95 dry. Find the drop in enthalpy of the steam.*

Solution. Given : $p_1 = 12$ bar ; $t_{sup} - t = 67°$ C ; $p_2 = 0.15$ bar ; $x = 0.95$

From steam tables, corresponding to a pressure of 12 bar, we find that

$$h_f = 798.4 \text{ kJ/kg} ; \; h_{fg} = 1984.3 \text{ kJ/kg}$$

We know that enthalpy or total heat of 1 kg of superheated steam,

$$h_{sup} = h_f + h_{fg} + c_p (t_{sup} - t) = 798.4 + 1984.3 + 2 \times 67$$

$$= 2916.7 \text{ kJ/kg} \quad \textbf{Ans.} \qquad \ldots (\text{Taking } c_p = 2 \text{ kJ/kg K})$$

Similarly, from steam tables, corresponding to a pressure of 0.15 bar, we find that

$$h_f = 226 \text{ kJ/kg} ; \; h_{fg} = 2373.2 \text{ kJ/kg}$$

We know that enthalpy or total heat of 1 kg of wet steam,

$$h = h_f + x h_{fg} = 226 + 0.95 \times 1984.3 = 2111 \text{ kJ/kg}$$

∴ Drop in enthalpy of the steam

$$= h_{sup} - h = 2916.7 - 2111 = 805.7 \text{ kJ/kg} \quad \textbf{Ans.}$$

Example 7.5. *A steam engine obtains steam from a boiler at a pressure of 15 bar and 0.98 dry. It was observed that the steam loses 21 kJ of heat per kg as it flows through the pipe line, pressure remaining constant. Calculate dryness fraction of the steam, at the engine end of the pipeline.*

Solution. Given : $p = 15$ bar ; $x = 0.98$; Heat loss = 21 kJ/kg

From steam tables, corresponding to a pressure of 15 bar, we find that

$$h_f = 844.6 \text{ kJ/kg} ; \; h_{fg} = 1945.3 \text{ kJ/kg}$$

We know that enthalpy of wet steam at the boiler end,

$$h_1 = h_f + x h_{fg} = 844.6 + 0.98 \times 1945.3 = 2751 \text{ kJ/kg}$$

Since the steam loses 21 kJ/kg of steam, therefore enthalpy of wet steam at the engine end,

$$h_2 = 2751 - 21 = 2730 \text{ kJ}$$

Let x_2 = Dryness fraction of steam at the engine end.

Since the pressure remains constant, therefore h_f and h_{fg} is same. We know that

$$h_2 = h_f + x_2 h_{fg}$$

$$2730 = 844.6 + x_2 \times 1945.3 \text{ or } x_2 = 0.97 \text{ Ans.}$$

7.6. Superheated Steam

We have already discussed that whenever dry steam or saturated steam is further heated, then the steam is termed as superheated steam. The process of superheating is assumed to follow constant pressure process (or Charles' law).

The values of saturation temperature, specific volume, specific enthalpy and specific entropy at a given pressure of the superheated steam are also given in the tabular form known as steam tables of superheated steam.

7.7. Advantages of Superheating the Steam

The following advantages of superheating the steam are important from the subject point of view :

1. The superheated steam contains more heat contents, and hence its capacity to do work is also increased without increasing its pressure.

2. The superheating is done in a superheater, which obtains heat from waste furnace gases. These gases would have otherwise passed, uselessly, through the chimney.

3. The high temperature of the superheated steam results in an increase of thermal efficiency.

4. Since the superheated steam is at a higher temperature than that corresponding to its pressure, therefore it can be considerably cooled during expansion in an engine cylinder. This is done before the temperature of superheated steam falls below that at which it condenses and, thereby, becomes wet. It is thus obvious, that heat losses due to condensation steam on cylinder walls, etc., are avoided to a great extent.

Example 7.6. *Determine the volume of 1 kg of superheated steam at a pressure of 20 bar and a temperature of 300° C.*

Solution. Given : $p = 20$ bar ; $T_{sup} = 300°\text{ C} = 300 + 273 = 573$ K

Let v_{sup} = Volume of 1 kg of superheated steam.

From steam tables, corresponding to a pressure of 20 bar, we find that

$$t = 212.4°\text{ C and } v_g = 0.1 \text{ m}^3/\text{kg}$$

$$\therefore \quad T = 212.4 + 273 = 485.4 \text{ K}$$

We know that the superheated steam obeys Charles's law.

$$\therefore \quad \frac{v_g}{T} = \frac{v_{sup}}{T_{sup}} \qquad \ldots (\because p \text{ is constant})$$

or $$v_{sup} = \frac{v_g}{T} \times T_{sup} = \frac{0.1}{485.4} \times 573 = 0.118 \text{ m}^3 \text{ Ans.}$$

Example 7.7. *A boiler is supplied with feed water at a temperature of 45° C. The water is converted into steam at a pressure of 5.5 bar and a temperature of 188° C. Determine the quantity of heat supplied per kg of steam. Assume suitable data.*

Solution. Given : $t_w = 45°\,C$; $p = 5.5$ bar ; $t_{sup} = 188°\,C$

From steam tables, corresponding to a pressure of 5.5 bar, we find that

$$h_f = 655.8 \text{ kJ/kg} ; \; h_{fg} = 2095.9 \text{ kJ/kg} ; \; t = 155.5°\,C$$

Since the temperature of the given steam (188° C) is higher than the saturation temperature (155.5° C), therefore the steam produced is superheated. Now let us assume the value of c_p for superheated steam as 2.1 kJ/kg K. We know that enthalpy or total heat of superheated steam,

$$h_{sup} = h_f + h_{fg} + c_p (t_{sup} - t)$$
$$= 655.8 + 2095.9 + 2.1 (188 - 155.5) = 2820 \text{ kJ/kg}$$

Since the water is supplied at 45° C, therefore heat already present in the feed water

$$= 4.2 \times 45 = 189 \text{ kJ}$$

$\therefore$ Heat actually required = 2820 – 189 = 2631 kJ/kg **Ans.**

7.8. External Work Done during Evaporation

We have already discussed that whenever water at boiling temperature is heated at a constant pressure, it gets converted into steam after absorbing the latent heat. This latent heat is utilised in the following two ways :

1. In overcoming the internal molecular resistance of water in changing its state from the saturated water to dry saturated steam.
2. In overcoming the external resistance to the movement of the piston due to increase in volume during evaporation.

The first effect is called *internal work* or *internal latent heat*, as the change takes place within the body of the steam, and represents the energy stored in the steam. The second effect is called *external work of evaporation*, and represents the energy which has been taken out of the steam.

Let p = Pressure on the piston in bar = $p \times 10^5$ N/m^2,

v_f = Volume of water in m^3 at pressure p, and

v_g = Volume of steam in m^3 at pressure p.

We know that work done during evaporation,

$$W = \text{Pressure} \times \text{Change in volume}$$
$$= p \times 10^5 (v_g - v_f) \text{ J} = 100 p (v_g - v_f) \text{ kJ}$$

At low pressures, the volume of water (v_f) is very small, as compared to the volume of steam (v_g). Therefore neglecting the value of v_f in the above equation,

$$W = 100 p v_g \text{ kJ}$$

If the steam is not completely dry, and has dryness fraction of x, then work done,

$$W = 100 p x v_g \text{ kJ}$$

If the steam is superheated, then *work done,

$$W = 100 p v_{sup} \text{ kJ}$$

---

* The work done by the superheated steam may also be found out as discussed below :

Work done during superheating = $100 p v_{sup}$ kJ

$\therefore$ Total work done = Work done during evaporation + Work done during superheating

$= 100 p v_g + 100 p (v_{sup} - v_g) = 100 p v_{sup}$ kJ

Example 7.8. *Find the external work done during evaporation per kg of steam at a pressure of 15 bar when the steam is (a) 90% dry and (b) dry saturated.*

Solution. Given : $p = 15$ bar

From steam tables, corresponding to a pressure of 15 bar, we find that

$$v_g = 0.1317 \text{ m}^3/\text{kg}$$

(a) Workdone when the steam is 90% dry (i.e. $x = 0.9$)

We know that workdone,

$$w = 100\, p\, x\, v_g = 100 \times 15 \times 0.9 \times 0.1317 = 177.8 \text{ kJ/kg}$$ **Ans.**

(b) Workdone when the steam is dry saturated

We know that workdone,

$$w = 100\, p\, v_g = 100 \times 15 \times 0.1317 = 197.5 \text{ kJ/kg}$$ **Ans.**

7.9. Internal Energy of Steam

It is the actual heat energy stored in steam, above the freezing point of water. The internal energy may be calculated by subtracting the external work done during evaporation from the enthalpy or total heat of steam. Mathematically,

Internal energy of steam

= Enthallpy or total heat – External workdone during evaporation

The expressions of internal energy per kg of steam (u) for wet steam, dry saturated steam and superheated steam are given below :

(a) For wet steam

$$u = h - 100\, p\, x\, v_g = h_f + x\, h_{fg} - 100\, p\, x\, v_g \text{ kJ/kg}$$

(b) For dry saturated steam

$$u = h_g - 100\, p\, v_g = h_f + h_{fg} - 100\, p\, v_g \text{ kJ/kg}$$

(c) For superheated steam,

$$u = h_{sup} - 100\, p\, v_{sup} = [h_g + c_p\,(t_{sup} - t)] - 100\, p\, v_{sup} \text{ kJ/kg}$$

Note: If the steam of initial internal energy (u_1) is heated in a cylinder to a final internal energy (u_2), then change in internal energy,

$$u = u_2 - u_1$$

Example 7.9. *Calculate the internal energy of 1 kg of steam at a pressure of 10 bar when the steam is : (a) 0.9 dry ; and (b) dry saturated. The volume of water may be neglected.*

Solution. Given : $p = 10$ bar

From steam tables, corresponding to a pressure of 10 bar, we find that

$$h_f = 762.6 \text{ kJ/kg} ; \; h_{fg} = 2013.6 \text{ kJ/kg and } v_g = 0.1943 \text{ m}^3/\text{kg}$$

(a) Internal energy of 1 kg of steam when it is 0.9 dry (i.e. $x = 0.9$)

We know that internal energy for wet steam

$$u = h_f + x\, h_{fg} - 100\, p\, x\, v_g$$

$$= 762.6 + 0.9 \times 2013.6 - 100 \times 10 \times 0.9 \times 0.1943 \text{ kJ}$$

$$= 2574.8 - 174.8 = 2400 \text{ kJ}$$ **Ans.**

(b) Internal energy of 1 kg of steam when it is dry saturated

We know that internal energy for dry saturated steam,

$$u = h_f + h_{fg} - 100\, p\, v_g$$
$$= 762.6 + 2013.6 - 100 \times 10 \times 0.1943$$
$$= 2776.2 - 194.3 = 2581.9 \text{ kJ Ans.}$$

Example 7.10. *Find the internal energy of 1 kg of superheated steam at a pressure of 10 bar and 280° C. If this steam be expanded to a pressure of 1.6 bar and 0.8 dry, determine the change in internal energy. Assume specific heat of superheated steam as 2.1 kJ/kg K.*

Solution. Given : $p_1 = 10$ bar ; $t_{sup} = 280°$ C ; $p_2 = 1.6$ bar ; $x = 0.8$; $c_p = 2.1$ kJ/kg K

From steam tables, corresponding to a pressure of 10 bar, we find that

$$h_f = 762.6 \text{ kJ/kg} ; \quad h_{fg} = 2013.6 \text{ kJ/kg} ; \quad v_g = 0.1943 \text{ m}^3/\text{kg} ; \quad \text{and} \quad t = 179.9° \text{ C}$$

We know that enthalpy of 1 kg of superheated steam,

$$h_{sup} = h_f + h_{fg} + c_p (t_{sup} - t)$$
$$= 762.6 + 2013.6 + 2.1\,(280 - 179.9) = 2986.4 \text{ kJ/kg}$$

and
$$v_{sup} = v_g \times \frac{T_{sup}}{T} = 0.1943 \times \frac{(280 + 273)}{(179.9 + 273)} = 0.237 \text{ m}^3/\text{kg}$$

∴ Internal energy of superheated steam,

$$u_{sup} = h_{sup} - 100\, p_1\, v_{sup}$$
$$= 2986.4 - 100 \times 10 \times 0.237 = 2749.4 \text{ kJ/kg}$$

The superheated steam now expands to a pressure of 1.6 bar. From steam tables corresponding to a pressure of 1.6 bar, we find that

$$h_f = 475.4 \text{ kJ/kg} ; \quad h_{fg} = 2220.9 \text{ kJ/kg} ; \text{ and } v_g = 1.091 \text{ m}^3/\text{kg}$$

We know that internal energy of the expanded steam,

$$u_e = h_f + x\, h_{fg} - 100\, p_2\, x\, v$$
$$= 475.4 + 0.8 \times 2220.9 - 100 \times 1.6 \times 0.8 \times 1.091$$
$$= 2252.1 - 139.7 = 2112.4 \text{ kJ/kg}$$

∴ Change in internal energy

$$= u_{sup} - u_e = 2749.4 - 2112.4 = 637 \text{ kJ/kg Ans.}$$

Example 7.11. *A vessel contains 2 kg of steam at a pressure of 8 bar. Find the amount of heat, which must be rejected, so as to reduce the quality of steam in the vessel to be 70%.*

Solution. Given : $m = 2$ kg ; $p_1 = 8$ bar ; $x_2 = 70\% = 0.7$

From steam tables, corresponding to a pressure of 8 bar, we find that

$$h_{g1} = 2767.5 \text{ kJ/kg} ; \quad \text{and} \quad v_{g1} = 0.24 \text{ m}^3/\text{kg}$$

∴ Volume of vessel $= 2 \times 0.24 = 0.48 \text{ m}^3$

and initial internal energy per kg of steam,

$$u_1 = h_{g1} - 100\, p_1\, v_{g1} = 2767.5 - 100 \times 8 \times 0.24 = 2575.5 \text{ kJ/kg}$$

Now neglecting the volume of water in the wet steam (with dryness fraction of 0.7), the volume occupied by the steam will be equal to the volume of vessel.

$\therefore \quad 0.48 = 2 \times 0.7 \times v_{g2} = 1.4\, v_{g2}$ or $v_{g2} = 0.343\ m^3/kg$

From steam tables, corresponding to a specific volume of 0.343 m^3/kg, we find that

$$p_2 = 5.5\ bar\ ;\ h_f = 655.8\ kJ/kg\ ;\ \text{and}\ h_{fg} = 2095.9\ kJ/kg$$

$\therefore$ Final internal energy per kg of steam,

$$u_2 = h_f + x_2 h_{fg} - 100\, p_2 x_2 v_{g2}$$
$$= 655.8 + 0.7 \times 2095.9 - 100 \times 5.5 \times 0.7 \times 0.343$$
$$= 2123 - 132 = 1991\ kJ/kg$$

and heat rejected $= u_1 - u_2 = 2575.5 - 1991 = 584.5\ kJ/kg$

$\therefore$ Heat rejected by 2 kg of steam

$$= 2 \times 584.5 = 1169\ kJ\ \textbf{Ans.}$$

Example 7.12. *Steam at 18 bar and dryness 0.9 is heated at constant pressure until dry and saturated. Find the increase in volume, heat supplied and work done per kg of steam.*

If the volume is now kept constant, find how much heat must be extracted to reduce the pressure to 14 bar.

Solution. Given : $p = 18\ bar$; $x = 0.9$; $p_1 = 14\ bar$

From steam tables, corresponding to a pressure of 18 bar, we find that

$$h_f = 884.5\ kJ/kg\ ;\ h_{fg} = 1910.3\ kJ/kg\ ;\ h_g = 2794.8\ kJ/kg\ ;\ \text{and}\ v_g = 0.1103\ m^3/kg$$

Increase in volume

We know that volume of wet steam,

$$v_f = x\, v_g = 0.9 \times 0.1103 = 0.0993\ m^3/kg$$

$\therefore$ Increase in volume

$$= v_g - v_f = 0.1103 - 0.0993 = 0.011\ m^3/kg\ \textbf{Ans.}$$

Heat supplied

We know that enthalpy or total heat of wet steam,

$$h = h_f + x\, h_{fg} = 884.5 + 0.9 \times 1910.3 = 2603.8\ kJ/kg$$

$\therefore$ Heat supplied $= h_g - h = 2794.8 - 2603.8 = 191\ kJ/kg$ **Ans.**

Workdone

We know that workdone,

$$w = 100\, p\, (v_g - v_f) = 100 \times 18 \times 0.011 = 19.8\ kJ/kg\ \textbf{Ans.}$$

Heat extracted to reduce the pressure to 14 bar

We know that during constant volume process, no work is done and extraction of heat will result in the reduction of internal energy of the steam. In other words, the steam is cooled, which results in the reduction of dryness fraction.

From steam tables, corresponding to a pressure of 14 bar, we find that

$$h_{f1} = 830.1\ kJ/kg\ ;\ h_{fg1} = 1957.7\ kJ/kg\ ;\ \text{and}\ v_{g1} = 0.1407\ m^3/kg$$

Let x_1 = Dryness fraction of steam.

Since the volume is constant, therefore

$$v_g = x_1 v_{g1} \quad \text{or} \quad x_1 = v_g / v_{g1} = 0.1103 / 0.1407 = 0.78$$

Now internal energy of steam at a pressure of 18 bar,

$$u = h_g - 100\,p\,v_g = 2794.8 - 100 \times 18 \times 0.1103 = 2596.3 \text{ kJ/kg}$$

and internal energy of steam at a pressure of 14 bar,

$$u_1 = h_{f1} + x_1\,h_{fg1} - 100\,p_1\,x_1\,v_{g1}$$
$$= 830.1 + 0.78 \times 1957.7 - 100 \times 14 \times 0.78 \times 0.1407$$
$$= 2357 - 154 = 2203 \text{ kJ/kg}$$

$\therefore$ Heat extracted to reduce the pressure

$$= u - u_1 = 2596.3 - 2203 = 393.3 \text{ kJ/kg}$$ **Ans.**

7.10. Measurement of Dryness Fraction of Steam

Though there are a number of steam calorimeters to determine the dryness fraction of wet steam, yet the following are important from the subject point of view :

1. Barrel calorimeter ; 2. Separating calorimeter, 3. Throttling calorimeter, and 4. Combined separating and throttling calorimeter.

Now we shall discuss these calorimeters, in detail, in the following pages.

7.11. Barrel Calorimeter

In barrel calorimeter, as shown in Fig. 7.3, the known mass of steam sample and of known pressure is condensed by mixing it with a known mass of cold water. The steam condenses into water, thereby increasing the mass and temperature of the water in the calorimeter. The heat supplied by the condensing steam is determined, with the help of the initial and final temperatures of the water and calorimeter.

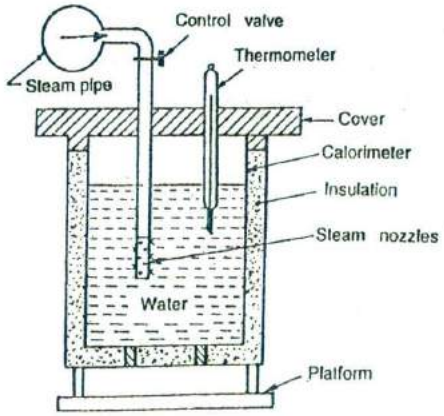


Fig. 7.3. Barrel calorimeter.

Let p = Pressure of the steam in bar,

t = Temperature of steam formation at pressure p (from steam tables),

h_{fg} = Latent heat of steam at pressure p (from steam tables),

m_c = Mass of the calorimeter,

c_c = Specific heat of the calorimeter,

m_e = Water equivalent of the calorimeter = $m_c \times c_c$,

m_s = Mass of the steam condensed,

m_w = Mass of cold water in the calorimeter,

t_1 = Initial temperature of water and calorimeter,

t_2 = Final temperature of water and calorimeter,

c_w = Specific heat of water (4.2 kJ / kg K),

x = Dryness fraction of steam sample.

According to the law of conservation, the heat lost by steam is equal to the heat gained by water and calorimeter, provided there is no addition or loss of heat.

Heat lost by steam $= m_s [(x\, h_{fg}) + c_w (t - t_2)]$ ... (*i*)

Heat gained by water and calorimeter,

$= (m_w c_w + m_c c_c)(t_2 - t_1)$ ... (*ii*)

Equating equations (*i*) and (*ii*),

$$m_s [(x\, h_{fg}) + c_w (t - t_2)] = (m_w c_w + m_c c_c)(t_2 - t_1)$$

From this expression, the dryness fraction of steam (x) may be determined.

Example 7.13. *In a laboratory experiment on wet steam by a barrel calorimeter, the following observations were recorded :*

Mass of copper calorimeter = *1 kg*
Mass of calorimeter + water = *3.8 kg*
Mass of calorimeter + water + steam = *4 kg*
Initial temperature of water = *10° C*
Final temperature of water = *50° C*
Steam pressure = *5.5 bar*

If the specific heat of copper is 0.406 kJ/kg K, determine the dryness fraction of steam.

Solution. Given : $m_c = 1$ kg ; $m_c + m_w = 3.8$ kg or $m_w = 3.8 - m_c = 3.8 - 1 = 2.8$ kg ; $m_c + m_w + m_s = 4$ kg or $m_s = 4 - (m_c + m_w) = 4 - 3.8 = 0.2$ kg ; $t_1 = 10°$ C ; $t_2 = 50°$ C ; $p = 5.5$ bar ; $c_c = 0.406$ kJ/kg K

From steam tables, corresponding to a pressure of 5.5 bar, we find that

$t = 155.5°$ C ; and $h_{fg} = 2095.5$ kJ/kg

Let x = Dryness fraction of steam.

We know that heat lost by steam

$= m_s [(x\, h_{fg}) + c_w (t - t_2)]$

$= 0.2 [x \times 2095.5 + 4.2 (155.5 - 50)]$ ... ($\because$ c_w for water = 4.2 kJ/kg K)

$= 419.1\, x + 88.6$ kJ ... (*i*)

and heat gained by water and calorimeter

$= (m_w c_w + m_c c_c)(t_2 - t_1)$

$= (2.8 \times 4.2 + 1 \times 0.406)(50 - 10) = 486.6$ kJ ... (*ii*)

Equating equations (*i*) and (*ii*),

$$419.1\,x + 88.6 = 486.6 \text{ or } x = 0.95 \textbf{ Ans.}$$

7.12. Separating Calorimeter

The separating calorimeter is used to determine the dryness fraction of steam by mechanically separating the water particles from the wet steam.

The wet steam enters at the top of the calorimeter through a control valve as shown in Fig. 7.4. It strikes the perforated cup and thereby undergoes a quick reversal of directions of motion. The water particles, due to their greater moment of inertia, tend to move on, and consequently get separated from the mixture. The separated water collects at the bottom of the inner chamber, and its amount may be determined by the graduated water gauge. The amount of dry steam leaving the outer chamber of the calorimeter may be measured by condensing it in a weighed quantity of cold water or alternatively by reading the pressure gauge which is calibrated in terms of rate of steam flow for this purpose.

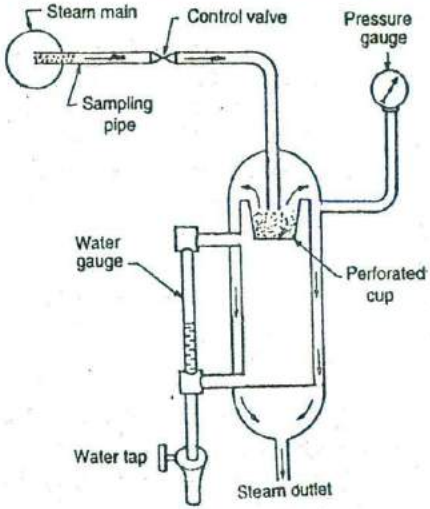


Fig. 7.4. Separating calorimeter.

Let m = Mass of water collected in a certain time,

M = Mass of dry steam passing in the same time, and

x = Dryness fraction of wet steam.

We know that dryness fraction,

$$x = \frac{\text{Mass of dry steam}}{\text{Mass of wet steam}} = \frac{M}{M+m}$$

Example 7.14. *In a laboratory experiment, a sample of wet steam is allowed to pass through a separating calorimeter. At some instant, the water collected in the chamber was 0.1 kg whereas the condensed steam was found to be 1.25 kg. Determine the dryness fraction of the steam entering the calorimeter.*

Solution. Given : $m = 0.1$ kg ; $M = 1.25$ kg

We know that dryness fraction of steam,

$$x = \frac{M}{M+m} = \frac{1.25}{1.25+0.1} = 0.926 \textbf{ Ans.}$$

7.13. Throttling Calorimeter

A throttling calorimeter used to determine the dryness fraction of steam is shown in Fig. 7.5. It consists of a separator A into which steam is admitted through a control valve from the steam main. The pressure and temperature are measured by the pressure gauge and the thermometer T_1 provided in this section. It may be noted that temperature recorded by T_1 is same as the saturation temperature corresponding to the pressure of steam in calorimeter B. This steam is then throttled through a narrow aperture of restricted valve openings, its total heat remaining constant. The steam is in the superheated

state after throttling at a lower pressure than previous. The temperature and pressure of steam leaving the calorimeter B is noted by the thermometer T_2 and manometer respectively.

Let p_1 = Pressure of steam before throttling, *i.e.* pressure of steam main,

p_2 = Pressure of steam after throttling,

h_{f1} = Sensible heat of water at pressure p_1, ... (From steam tables)

h_{fg1} = Latent heat of steam at pressure p_1, ... (From steam tables)

h_{g2} = Total heat of dry steam at pressure p_2, ... (From steam tables)

t_{sup} = Temperature of superheated steam after throttling,

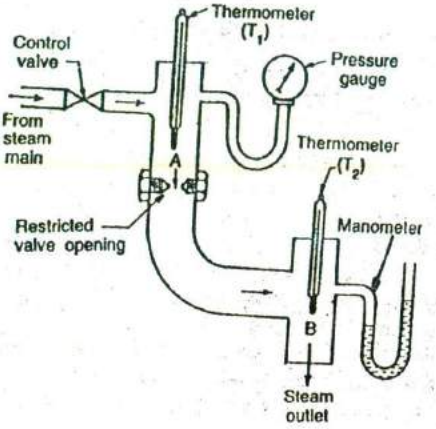


Fig. 7.5. Throttling calorimeter.

t_2 = Saturation temperature at pressure p_2, ... (From steam tables)

c_p = Specific heat of superheated steam, and

x = Dryness fraction of steam before throttling.

Since the steam has undergone a throttling process, therefore

Total heat before throttling = Total heat after throttling

$$\therefore \quad h_{f1} + x\, h_{fg1} = h_{g2} + c_p\,(t_{sup} - t_2)$$

Example 7.15. *In a throttling calorimeter, the steam is admitted at a pressure of 10 bar. If it is discharged at atmospheric pressure and 110° C after throttling, determine the dryness fraction of steam. Assume specific heat of steam as 2.2 kJ/kg K.*

Solution. Given : $p_1 = 10$ bar ; $p_2 = 1.013$ bar ; $t_{sup} = 110°$ C ; $c_p = 2.2$ kJ/kg K

Let x = Dryness fraction of steam.

From steam tables, corresponding to a pressure of 10 bar, we find that

$$h_{f1} = 762.6 \text{ kJ/kg} ; \quad \text{and} \quad h_{fg1} = 2013.6 \text{ kJ/kg}$$

and corresponding to a pressure of 1.013 bar,

$$h_{g2} = 2676 \text{ kJ/kg} ; \text{ and } t_2 = 100° \text{ C}$$

We know that $h_{f1} + x\, h_{fg1} = h_{g2} + c_p\,(t_{sup} - t_2)$

$$762.6 + x \times 2013.6 = 2676 + 2.2\,(110 - 100) = 2698$$

$$\therefore \quad x = \frac{2698 - 762.6}{2013.6} = 0.961 \text{ Ans.}$$

7.14. Combined Separating and Throttling Calorimeter

A very successful method of measuring the dryness fraction of steam is by a combined separating and throttling calorimeter as shown in Fig. 7.6.

In this calorimeter, the wet steam is first collected in a perforated collecting pipe and then passed through a separating calorimeter. A part of water is removed by the separating calorimeter owing to quick change of direction of flow. The resulting semi-dry steam is throttled into a throttling

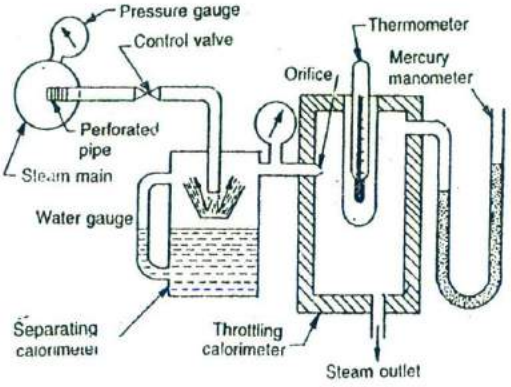


Fig. 7.6. Combined separating and throttling calorimeter.

calorimeter. This method ensures that the steam will be superheated after throttling. This instrument is well insulated to prevent any loss of heat.

Let x_1 = Dryness fraction of steam considering separating calorimeter, and

x_2 = Dryness fraction of steam entering the throttling calorimeter.

Now the actual dryness fraction of steam in the steam main,

$$x = x_1 \times x_2$$

Note : It is not possible to obtain results with this instrument if the final condition of steam is wet. The final condition of steam must be just dry or superheated.

Example 7.16. *In a laboratory experiment, the following observations were recorded to find the dryness fraction of steam by combined separating and throttling calorimeter :*

Total quantity of steam passed = *36 kg*
Water drained from separator = *1.8 kg*
Steam pressure before throttling = *12 bar*
Temperature of steam after throttling = *110° C*
Pressure after throttling = *1.013 bar*
Specific heat of steam = *2.1 kJ/kg K*
Determine the dryness fraction of steam before inlet to the calorimeter.

Solution. Given : $m_s + m_w = 36$ kg ; $m_w = 1.8$ kg ; $p_1 = 12$ bar ; $t_{sup} = 110°$ C ; $p_2 = 1.013$ bar ; $c_p = 2.1$ kJ/kg K

Let x_1 = Dryness fraction for separating calorimeter,

x_2 = Dryness fraction for throttling calorimeter,

x = Actual dryness fraction entering the combined separating and throttling calorimeter.

We know that mass of dry steam,

$$m_s = (m_s + m_w) - m_w = 36 - 1.8 = 34.2 \text{ kg}$$

∴ Dryness fraction for separating calorimeter,

$$x_1 = \frac{m_s}{m_s + m_w} = \frac{34.2}{36} = 0.95$$

From steam tables, corresponding to a pressure of 12 bar, we find that

$$h_{f1} = 798.4 \text{ kJ/kg} ; \text{ and } h_{fg1} = 1984.3 \text{ kJ/kg}$$

and corresponding to a pressure of 1.013 bar,

$$h_{g2} = 2676 \text{ kJ/kg} ; \text{ and } t_2 = 100^\circ \text{C}$$

We know that $h_{f1} + x_2 h_{fg1} = h_{g2} + c_p (t_{sup} - t_2)$

$$798.4 + x_2 \times 1984.3 = 2676 + 2.1 (110 - 100) = 2697$$

$$\therefore \quad x_2 = \frac{2697 - 798.4}{1984.3} = 0.957$$

We also know that the actual dryness fraction of the steam entering the combined separating and throttling calorimeter,

$$x = x_1 \times x_2 = 0.95 \times 0.957 = 0.909 \text{ Ans.}$$

Note : The value of x may also be obtained as follows :

First of all, the value fo x_2 is obtained as before, *i.e.* $x_2 = 0.957$

Now mass of dry steam entering the throttling calorimeter

= Mass of dry steam leaving the separating calorimeter

$= 34.2 \times 0.957 = 32.73$ kg

∴ Dryness fraction of steam entering the separating calorimeter,

$$x = \frac{32.73}{36} = 0.909 \text{ Ans.}$$

EXERCISES

1. 5 kg of steam at a pressure of 5 bar is produced from water at 20° C. Determine the amount of heat supplied, if the steam is 0.9 dry. **[Ans.** 12 265 kJ**]**

2. Find the amount of heat required to convert 1 kg of steam at a pressure of 0.5 bar and dryness fraction 0.9 to dry saturated steam. **[Ans.** 230.5 kJ**]**

3. Find the mass of 0.50 m^3 of wet steam at a pressure of 4 bar and dryness fraction 0.8. Also determine the enthalpy of 1 m^3 of steam. **[Ans.** 1.35 kg ; 6240 kJ**]**

4. One kg of water at 47.8° C is heated under a constant pressure of 14 bar until it is converted into steam with 110° C of superheat. Determine the quantity of heat supplied during superheating and the total heat of superheated steam. Assume $c_p = 2.1$ kJ/kg K. **[Ans.** 201 kJ ; 2818 kJ**]**

5. A certain amount of steam is produced at a pressure of 8 bar and a dryness fraction 0.8. Determine : 1. external workdone during evaporation ; and 2. internal latent heat of steam. **[Ans.** 153.6 kJ ; 1483.6 kJ**]**

6. Find the internal energy of 1 kg of steam at a pressure of 7 bar under the following conditions :

1. when it is wet having dryness fraction 0.8 ; and 2. when it is dry saturated.

[Ans. 2196 kJ/kg ; 2571 kJ/kg**]**

7. Calculate the internal energy per kg of superheated steam at a pressure of 10 bar and a temperature of 300° C. If the steam is expanded to 1.4 bar and dryness fraction 0.8, find the change of internal energy. **[Ans.** 701 kJ/kg**]**

8. Find the internal energy of 1 kg of steam at 20 bar in the following cases :

1. when the stem is wet having dryness fraction 0.9, and 2. when the steam is superheated when its temperature is 400° C.

Take specific heat for superheated steam as 2.3 kJ/kg K. [Ans. 2429.3 kJ/kg ; 2952.7 kJ/kg]

9. Two boilers, one with a superheater and the other without a superheater are delivering equal quantities of steam into a common mains. The pressure in the boiler and the mains is 14.7 bar. The temperature of steam from the boiler with superheater is 325° C and that of steam mains 250° C. Estimate the quality of steam supplied by the other boiler. **[Ans. 0.97]**

10. Steam for a small boiler is discharged through a pipe into a barrel containing water and after a few minutes, observations were taken of the mass and temperature of the water. From the collected data given below, calculate the dryness fraction of the steam.

Mass of empty barrel = 25 kg ; Mass of barrel + cold water = 135 kg ; Mass of barrel + cold water + steam = 140 kg ; Initial temperature of water = 15° C ; Final temperature of water = 42° C ; Steam pressure = 8 bar ; Specific heat of the material of barrel = 0.4 kJ/kg K **[Ans. 0.96]**

11. In an experiment, it was found that the steam enters a throttling calorimeter at a pressure of 12.25 bar. After throttling, the pressure and temperature was measured as 1.013 bar and 115° C respectively. Estimate the dryness fraction of steam. **[Ans. 0.96]**

12. In a combined separating and throttling calorimeter, the following observations were made :

Total quantity of steam = 23.4 kg ; Water drained from separator = 1.2 kg ; Steam pressure before throttling = 8.25 bar ; Temperature of steam on leaving = 111.4° C ; Steam pressure on leaving = 1.013 bar.

Find the dryness fraction of steam on entry. Specific heat of superheated steam is 2 kJ/kg K. **[Ans. 0.92]**

QUESTIONS

1. What is meant by saturation temperature and saturation pressure ?
2. Explain how the wet steam, dry saturated steam and superheated steam is produced.
3. Define the following terms :
   (*i*) Sensible heat of water ; (*ii*) Latent heat of vaporisation ;
   (*iii*) Total heat of steam ; (*iv*) Dryness fraction of steam ; and
   (*v*) Volume of superheated steam.
4. List out the advantages of superheated steam.
5. Derive an expression for the external work done during evaporation :
   (*i*) when the steam is wet ; (*ii*) when the steam is dry ; and
   (*iii*) when the steam is superheated.
6. What is meant by the internal energy of steam ?

OBJECTIVE TYPE QUESTIONS

1. Water at standard atmospheric conditions
   (*a*) behaves as an ideal gas (*b*) is mostly liquid
   (*c*) is far above its critical state (*d*) is far below its critical state
2. The locus of saturated liquid line and saturated vapour line meets at
   (*a*) boiling point (*b*) critical point (*c*) ice point (*d*) triple point
3. For steam
   (*a*) the critical temperature is 221.2° C and critical pressure is 374.15 bar
   (*b*) the critical temperature is 374.15° C and critical pressure is 221.2 bar
   (*c*) the critical temperature is 221.2° C and critical pressure is 221.2 bar
   (*d*) the critical temperature is 374.15° C and critical pressure is 374.15 bar
4. The latent heat of steam at atmospheric pressure is
   (*a*) 1535 kJ/kg (*b*) 1875 kJ/kg (*c*) 2257 kJ/kg (*d*) 2685 kJ/kg
5. With the increase in pressure
   (*a*) the boiling point of water decreases and enthalpy of evaporation increases

(*b*) the boiling point of water increases and enthalpy of evaporation decreases

(*c*) both the boiling point of water and the enthalpy of evaporation decreases

(*d*) both the boiling point of water and the enthalpy of evaporation increases.

6. The dryness fraction of steam is equal to

(*a*) $\frac{m_g}{m_g + m_f}$ (*b*) $\frac{m_f}{m_g + m_f}$ (*c*) $\frac{m_g}{m_f}$ (*d*) $\frac{m_f}{m_g}$

where m_g = Mass of dry steam, and

m_f = Mass of water in suspension.

7. The specific volume of water when heated from 0° C

(*a*) increases steadily (*b*) decreases steadily

(*c*) first decreases and then increases (*d*) first increases and then decreases

8. The enthalpy of dry saturated steam. . . . . .with the increase in pressure.

(*a*) decreases (*b*) increases (*c*) remains constant

9. The amount of heat absorbed to evaporate 1 kg of water from its saturation temperature, without change of temperature, is called

(*a*) sensible heat of water (*b*) latent heat of vaporisation

(*c*) enthalpy of steam (*d*) entropy of steam

10. The internal energy (*u*) of a dry steam is given by

(*a*) $u = h_g + 100\, p\, v_g$ (*b*) $u = h_g - 100\, p\, v_g$

(*c*) $u = h_g / 100\, p\, v_g$ (*d*) $u = 100\, p\, v_g / h_g$

where h_g = Enthalpy of 1 kg of dry steam in kJ/kg,

p = Pressure of steam in bar, and

v_g = Specific volume of 1 kg of dry steam in m^3/kg.

ANSWERS

1. (*d*)	2. (*b*)	3. (*b*)	4. (*c*)	5. (*b*)
6. (*a*)	7. (*c*)	8. (*a*)	9. (*b*)	10. (*b*)

8

Entropy of Steam

1. Introduction. 2. Entropy of Water. 3. Entropy Increase during Evaporation. 4. Entropy of Wet and Dry Steam. 5. Entropy of Superheated Steam. 6. Temperature-Entropy (T-s) Diagram for Water and Steam. 7. Isothermal Lines on T-s diagram. 8. Isentropic Lines on T-s diagram. 9. Enthalpy-Entropy (h-s) Diagram for Water and Steam or Mollier Chart. 10. Dryness Fraction Lines on h-s Diagram. 11. Constant Volume Lines on h-s Diagram. 12. Constant Pressure Lines on h-s Diagram. 13. Isothermal Lines on h-s Diagram. 14. Isentropic Lines on h-s Diagram. 15. Throttling Lines on h-s Diagram.

8.1. Introduction

We have already discussed, in detail, the entropy of perfect gases in chapter 4. The entropy of steam is also an important property, which increases with the addition of heat, and decreases with its removal.

The entropy of steam consists of :

1. Increase in entropy of water during heating from freezing point to boiling point corresponding to the pressure at which the water is being heated,
2. Increase in entropy during evaporation, and
3. Increase in entropy during superheating.

Notes : (*a*) Since the enthalpy or total heat of steam is reckoned from freezing point of water *i.e.* 0° C, therefore entropy is also reckoned from 0° C.

(*b*) The entropy of water at 0° C is regarded as zero.

(*c*) The absolute temperatures are used in entropy calculations.

8.2. Entropy of Water

Consider 1 kg of water being heated at a constant pressure from freezing temperature (*i.e.* 0° C or 273 K) to the boiling temperature. Now consider an instant, when the absolute temperature of water is T K.

Let, for a small rise in temperature of dT, the heat absorbed by 1 kg of water is δq. Then

$$\delta q = \text{Mass} \times \text{Sp. heat of water} \times \text{Rise in temperature.}$$

$$= 1 \times c_w \times dT = c_w\, dT$$

where c_w = Specific heat of water.

The value of c_w may be taken as 4.2 kJ/kg K.

We know that increase in entropy for rise in temperature dT,

$$ds = \frac{\delta q}{T} = \frac{c_w\, dT}{T} \qquad \ldots (\because \delta q = c_w\, dT)$$

The total increase in entropy of water from freezing point to boiling point, may be obtained by integrating the above expression within the limits 273 K and T K.

$$\int_0^s ds = \int_{273}^{T} \frac{c_w\, dT}{T}$$

$$\therefore \qquad s_f = c_w \log_e \left(\frac{T}{273}\right) = 2.3\, c_w \log \left(\frac{T}{273}\right)$$

Note : The value of s_f may be directly seen from the steam tables.

8.3. Entropy Increase during Evaporation

When the water is completely evaporated into steam, it absorbs full latent heat (h_{fg}) at constant temperature T, corresponding to the given pressure. We know that

$$\text{Entropy} = \frac{\text{Heat absorbed}}{\text{Absolute temperature}}$$

$\therefore$ Increase of entropy during evaporation,

$$s_{fg} = \frac{h_{fg}}{T}$$

If the steam is wet with dryness fraction x, the evaporation will be partial. In such a case, heat absorbed

$$= x\, h_{fg}$$

$\therefore$ Increase in entropy, $s_{fg} = \dfrac{x\, h_{fg}}{T}$

8.4. Entropy of Wet and Dry Steam

The entropy of wet and dry steam, above the freezing point of water, is the entropy of water *plus* the entropy during evaporation. Mathematically, increase in entropy

$$= s_f + \frac{x\, h_{fg}}{T} = s_f + x\, s_{fg} \qquad \text{... (For wet steam)}$$

$$= s_f + \frac{h_{fg}}{T} = s_f + s_{fg} = s_g \qquad \text{... (For dry steam)}$$

Note : The entropy of dry steam is denoted by s_g. Its value may also be directly read from the steam tables.

8.5. Entropy of Superheated Steam

We know that during superheating, the heat is supplied at a constant pressure, and the temperature of dry steam (T) increases to the temperature of superheated steam (T_{sup}). For a small rise in temperature dT, the heat absorbed,

$$\delta q = c_p\, dT \text{ per kg of steam}$$

or $$\frac{\delta q}{T} = \frac{c_p\, dT}{T} \qquad \text{... (Dividing both sides by } T\text{)}$$

$\therefore$ $$ds = \frac{c_p\, dT}{T} \qquad \left(\because \frac{\delta q}{T} = ds\right)$$

where c_p = Specific heat of superheated steam at constant pressure. Its value may be taken between 1.67 kJ/kg K to 2.5 kJ/kg K.

The total increase in entropy from T K to T_{sup} K may be obtained by integrating the above expression between the proper limits.

$$\int_{s_g}^{s_{sup}} ds = c_p \int_{T}^{T_{sup}} \frac{dT}{T}$$

or $$s_{sup} - s_g = c_p \log_e \left(\frac{T_{sup}}{T} \right) = 2.3\, c_p \log \left(\frac{T_{sup}}{T} \right)$$

where $(s_{sup} - s_g)$ is the increase in entropy.

Note : The entropy of 1 kg of superheated steam is given by :

$$s_{sup} = s_g + 2.3\, c_p \log \left(\frac{T_{sup}}{T} \right) = \left[s_f + \frac{h_{fg}}{T} \right] + 2.3\, c_p \log \left(\frac{T_{sup}}{T} \right)$$

Example 8.1. *Find the entropy of 1 kg of dry saturated steam at a pressure of 5.2 bar. The boiling point of water at this pressure is given as 152.6° C and its total heat at this temperature is 2110 kJ/kg.*

Solution. Given : $p = 5.2$ bar ; $T = 152.6°$ C $= 152.6 + 273 = 425.6$ K ; $h_{fg} = 2110$ kJ/kg

We know that entropy of 1 kg of dry saturated steam,

$$s_g = s_f + \frac{h_{fg}}{T} = 2.3\, c_w \log \left(\frac{T}{273} \right) + \frac{h_{fg}}{T}$$

$$= 2.3 \times 4.2 \log \left(\frac{425.6}{273} \right) + \frac{2110}{425.6}$$

$$= 1.86 + 4.96 = 6.82 \text{ kJ/kg K} \quad \textbf{Ans.}$$

Note : The value of entropy of dry saturated steam (s_g) may be directly read from steam tables, corresponding to a pressure of 5.2 bar.

Example 8.2. *Calculate the entropy of 1 kg of wet steam with dryness fraction of 0.9 at a pressure of 8.4 bar.*

Solution. Given ; $x = 0.9$; $p = 8.4$ bar

From steam tables, corresponding to a pressure of 8.4 bar, we find that

$T = 172.4°$ C $= 172.4 + 273 = 445.4$ K ; $h_{fg} = 2039.6$ kJ/kg ; $s_f = 2.066$ kJ/kg K

We know that entropy of 1 kg of wet steam,

$$s = s_f + \frac{x\, h_{fg}}{T} = 2.066 + \frac{0.9 \times 2039.6}{445.4}$$

$$= 2.066 + 4.12 = 6.186 \text{ kJ/kg K} \quad \textbf{Ans.}$$

Note : The entropy of 1 kg of wet steam may also be determined as follows :

From steam tables, corresponding to a pressure of 8.4 bar, we find that

$$s_f = 2.066 \text{ kJ/kg K} \text{ and } s_{fg} = 4.577 \text{ kJ/kg K}$$

We know that entropy of 1 kg of wet steam,

$$s = s_f + x\, s_{fg} = 2.066 + 0.9 \times 4.577 = 2.066 + 4.12 = 6.186 \text{ kJ/kg K} \quad \textbf{Ans.}$$

Example 8.3. *Determine the entropy per kg of superheated steam at a pressure of 20 bar and a temperature of 250° C. Assume c_p for superheated steam as 2.2 kJ/kg K.*

Solution. Given : $p = 20$ bar ; $T_{sup} = 250°$C $= 250 + 273 = 523$ K ; $c_p = 2.2$ kJ/kg K

From steam tables, corresponding to a pressure of 20 bar, we find that

$$T = 212.4°\,C = 212.4 + 273 = 485.4\ K\ ;\ s_g = 6.337\ kJ/kg\ K$$

We know that entropy per kg of superheated steam,

$$s_{sup} = s_g + 2.3\, c_p \log\left(\frac{T_{sup}}{T}\right) = 6.337 + 2.3 \times 2.2 \log\left(\frac{523}{485.4}\right)$$

$$= 6.5\ kJ/kg\ K\ \textbf{Ans.}$$

8.6. Temperature-Entropy (*T-s*) Diagram for Water and Steam

The temperature-entropy (briefly written as *T-s*) diagram is very useful in solving the problems on adiabatic expansion and compression of steam. The abscissa of the diagram represents the entropy of 1 kg of water and steam above the freezing point of water. The vertical ordinate shows the values of temperature as shown in Fig. 8.1.

Now consider 1 kg of water being heated at a constant pressure (p). We know that when heat is added to the water, its entropy will increase. If we plot a graph between temperature and entropy, we shall find that the entropy increases along logarithmic curve with the increase in temperature, till the boiling temperature (T) corresponding to the pressure (p) is reached. It is shown by the graph AB. The corresponding increase in entropy (s_f) is given by the line Ab as shown in Fig. 8.1.

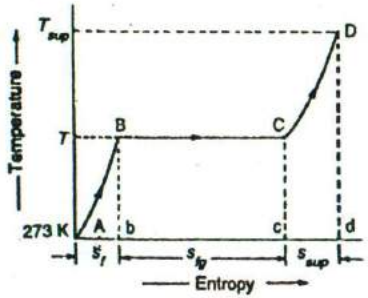


Fig. 8.1. *T-s* diagram.

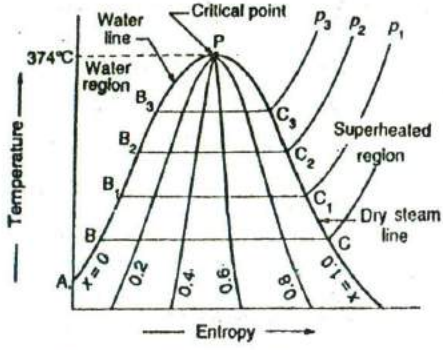


Fig. 8.2. *T-s* graph for water and steam.

On further heating, we find that the water starts evaporating and receives heat at constant temperature (T). The entropy goes on increasing, till the entire latent heat required to evaporate 1 kg of water has been supplied. The increase in entropy (s_{fg}) during evaporation is given by the line bc in Fig. 8.1.

If we draw a family of similar curves, at different pressures, and plot the points B_1, B_2 and C_1, C_2 etc., then the line joining the points A, B, B_1, B_2 etc. is called *water line*. Similarly, the line joining the points C, C_1, C_2 etc. is called *dry steam line*. The point, where the water line and dry steam line meets (*i.e.* P) is called *critical point*, and it represents 374.15° C as shown in Fig. 8.2.

Now consider the line BC, which represents increase in entropy at constant temperature. We see that dryness fraction increases with entropy from $x = 0$ at B to $x = 1$ at C. The line BC is divided into 10 parts, each representing dryness fraction from 0 to 1.0. Now draw similar lines $B_1\,C_1$, $B_2\,C_2$ etc. and mark the values of dryness fraction. The line joining the same values of dryness fraction for different pressures are known as constant dryness fraction lines. In Fig. 8.2, dryness fraction lines at intervals of 0.2 have been drawn.

Notes : 1. In actual practice, the *T-s* diagram in not carried up to the critical point.

2. The advantage of *T-s* diagram over *p-v* diagram is that area under the *T-s* diagram represents heat absorbed or rejected during the process whereas area under the *p-v* diagram represents the workdone.

8.7. Isothermal Lines on *T-s* Diagram

We know that there is no change of temperature during an isothermal process. It is thus obvious, that an isothermal process may be represented by a horizontal line on the *T-s* diagram, as shown in Fig. 8.3.

Consider a sample of 1 kg of wet steam at a temperature of 250° C and dryness fraction of 0.2, which may be shown by the point *A** on the *T-s* diagram. Let this sample be expanded isothermally, till its dryness fraction is 0.8. Now mark this point *B*. This process may be represented by the line *AB* on the *T-s* diagram. The change in entropy during this process may be found out by the difference of readings (s_A and s_B) obtained by drawing vertical lines through *A* and *B*. In this case, the reading at *A* is 3.45 and that at *B* is 5.42. Thus increase in entropy during this process is 5.42 – 3.45 = 1.97 kJ/kg K.

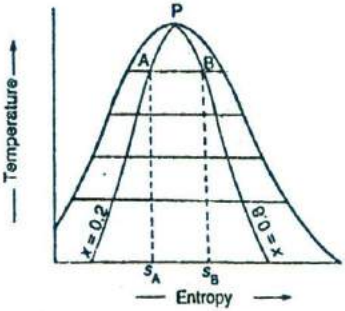


Fig. 8.3. Isothermal lines on *T-s* diagram.

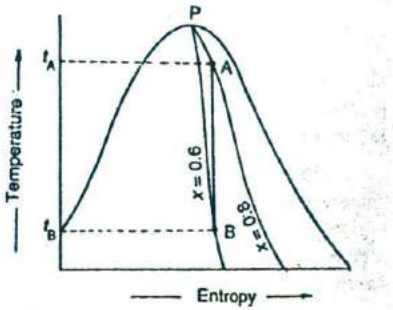


Fig. 8.4. Isentropic lines on *T-s* diagram.

8.8. Isentropic Lines on *T-s* Diagram

We know that there is no increase or decrease in the enthalpy or total heat during an isentropic process. It is thus obvious, that there is no change in entropy also during the process. Therefore, the isentropic process may be represented by a vertical line on the *T-s* diagram, as shown in Fig. 8.4.

Consider a sample of 1 kg of wet steam at a temperature of 310° C and dryness fraction of 0.8, which may be shown by the point *A* on the *T-s* diagram. Let this sample be expanded isentropically till its dryness fraction is 0.6. Now mark this point *B*. This process may be represented by the line *AB* on the *T-s* diagram. The change of temperature during the process may be found out by the difference between the initial temperature of 310° C and the final temperature obtained by drawing horizontal line through *B*. In this case, the final temperature is 40° C. Thus fall in temperature during this process is 310 – 40 = 270° C.

8.9. Enthalpy-Entropy (*h-s*) Diagram for Water and Steam or Mollier Chart

It is a graphical representation of the steam tables, in which the enthalpy (*h*) is plotted along the ordinate and the entropy (*s*) along abscissa. First of all, enthalpy and entropy of water and dry saturated steam, for any particular pressure, are obtained from the steam tables. These values of enthalpies and entropies are plotted and then liquid line and dry saturated line is obtained. Both these lines meet at *C*, *i.e.* the critical point as shown in Fig. 8.5. The critical point corresponds to the enthalpy of liquid and dry saturated steam at 221.2 bar.

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* The location of point *A* may be marked where the horizontal line through 250° C and logarithmic line of dryness fraction of 0.2 meets. Similarly, the location of point *B* may be marked where the horizontal line through 250° C and logarithmic line of dryness fraction 0.8 meets

The enthalpy-entropy chart, like temperature-entropy chart, is also very useful in solving the problems on isentropic expansion and compression of steam. In actual diagram, abscissa of the

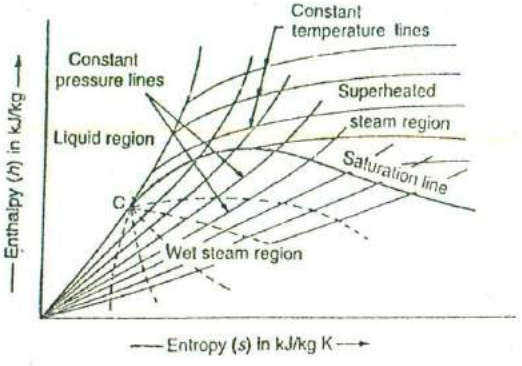


Fig. 8.5 *h-s* diagram for water and steam

diagram represents the entropy of 1 kg of water and steam (*i.e.* specific entropy) above the freezing point of water. The vertical ordinate shows the values of specific enthalpy (*i.e.* total heat) as shown in Fig. 8.6.

The diagram is divided into two portions, by a line termed as *saturation line*. In the lower (*i.e.* wet) region, the temperature of steam remains constant at a given pressure. In the upper (*i.e.* super heat) region the temperature of steam increases at the given pressure.

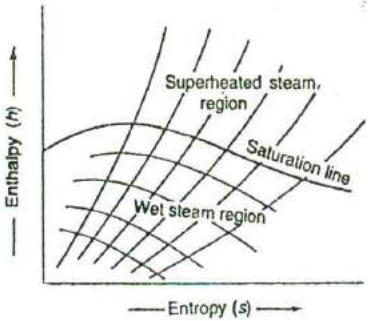


Fig. 8.6 *h-s* diagram or Mollier chart.

The Mollier diagram has the following lines :

1. Dryness fraction lines,
2. Constant volume lines,
3. Constant pressure lines,
4. Isothermal lines,
5. Isentropic lines, and
6. Throttling lines.

These lines are discussed, in detail, in the following pages.

8.10. Dryness Fraction Lines on *h-s* Diagram

The dryness fraction lines are drawn only below the saturation line (which represents dryness fraction equal to 1.0). These lines represent the condition of wet steam between various values of enthalpy and entropy as shown in Fig. 8.7.

Now consider a sample of 1 kg of wet steam of dryness fraction 0.9. In the initial stage, let the sample have enthalpy of 2500 kJ/kg which may be shown by the point *A* on the *h-s* diagram, Let this sample be cooled at constant dryness fraction to an enthalpy of 2400 kJ/kg which may be shown by point *B*. Now the change of entropy during this process may be found out by the difference of initial and final entropies obtained by drawing vertical lines through *A* and *B*. In this case, the entropies at

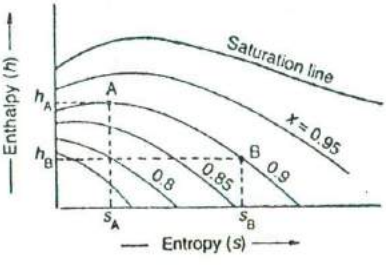


Fig. 8.7. Dryness fraction on *h-s* diagram

A and *B* are 6.5 and 7.02 respectively. Thus increase in entropy during the process is 7.02 – 6.5 = 0.52 kJ/kg K.

8.11. Constant Volume Lines on *h-s* Diagram

The constant volume lines are drawn in both the wet steam region and superheated steam region. These lines are straight in the wet steam region, but curved upwards above the saturation curve *i.e.* superheated region as shown in Fig. 8.8.

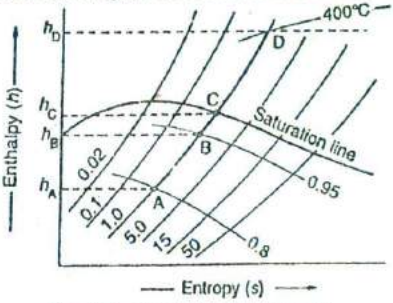


Fig. 8.8. Constant volume lines on *h-s* diagram.

Now consider a sample of wet steam of dryness faction 0.8 having enthalpy (total heat) of 2250 kJ/kg, which may be shown by point *A* on the *h-s* diagram. From the Mollier diagram, we find that the volume of this steam is 1.0 m^3/kg. Let this sample be heated at constant volume of 1.0 m^3/kg till its dryness fraction is 0.95. Now mark this point *B*. This process may be represented by the line *AB* on the Mollier diagram. Now the change of enthalpy (total heat) during this process may be found out by the difference between the initial enthalpy of 2250 kJ/kg and the final enthalpy obtained by drawing a horizontal line through *B*. In this case, the final value of enthalpy is 2590 kJ/kg. Thus increase in enthalpy during this process is 2590-2250 = 340 kJ/kg.

If this sample of steam is further heated at constant volume of 1.0 m^3/kg first into dry saturated steam (shown by point *C*) and then into superheated steam at 400° C (shown by point *D*), we find that enthalpy at *C* and *D* is 2700 kJ/kg and 3275 kJ/kg respectively. Thus increase in enthalpy from *A* to *C* is 2700 – 2250 = 450 kJ/kg. Similarly, increase in enthalpy from *A* to *D* is 3275 – 2250 = 1025 kJ/kg.

Note : Increase in entropy during the process from *A* to *B*, *C* or *D* may be found out by obtaining the corresponding values of entropy.

8.12. Constant Pressure Lines on *h-s* Diagram

The constant pressure lines are drawn in both the wet steam region and superheated steam region. These lines are straight in the wet steam region, because during vaporisation the increase of enthalpy (total heat) is directly proportional to the increase in quality, and hence to the increase in entropy. The constant pressure lines are curved slightly upwards above the saturation curve *i.e.* in the superheated region as shown in Fig. 8.9.

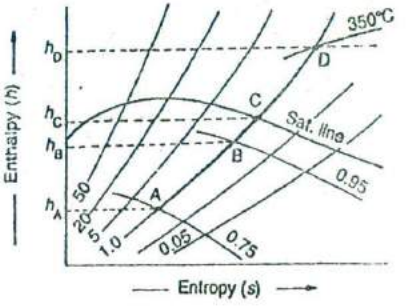


Fig. 8.9. Constant pressure lines on *h-s* diagram.

Now consider a sample of wet steam of dryness fraction 0.75 having enthalpy (total heat) of 2110 kJ/kg which may be shown by the point *A* on the *h-s* diagram. From the *h-s* diagram, we find that the pressure of this steam is 1.0 bar. Let this sample be heated at constant pressure of 1.0 bar till its dryness fraction is 0.95. Now mark this point *B*. This process may be represented by the line *AB* on the Mollier diagram. Now the change of enthalpy during the process may be found out by the difference between the initial enthalpy of 2110 kJ/kg and final enthalpy obtained by drawing a horizontal line through *B*. In this case, the final value of enthalpy is 2570 kJ/kg. Thus increase in enthalpy during this process is 2570 – 2110 = 460 kJ/kg.

If this sample of steam is further heated at constant pressure of 1.0 bar first into dry saturated steam (shown by point *C*) and then into superheated steam at 350° C (shown by point *D*), we find that the enthalpy at *C* and *D* is 2680 kJ/kg and 3170 kJ/kg respectively. Thus increase in enthalpy from *A* to *C* is 2680 – 2110 = 570 kJ/kg. Similarly, increase in enthalpy from *A* to *D* is 3170 – 2110 = 1060 kJ/kg.

Note : Increase of entropy during the process from *A* to *B*, *C* or *D* may be found out by obtaining the corresponding values of entropy.

8.13. Isothermal Lines on *h-s* Diagram

The isothermal (or constant temperature) lines are drawn only above the saturation line. These lines represent the condition of superheated steam between various values of enthalpy and entropy as shown in Fig. 8.10.

Now consider a sample of 1 kg of superheated steam at a temperature of 400° C. In the initial stage, let the sample has enthalpy of 2900 kJ/kg, which may be shown by the point *A* on the *h-s* diagram. Let this sample be heated isothermally (*i.e.* at the constant temperature of 400° C) till the enthalpy is 3270 kJ/kg which may be shown by the point *B*. Now the change of entropy during this process may be found out by the difference of initial and final entropies obtained by drawing vertical lines through *A* and *B*. In this case, the entropies at *A* and *B* are 5.7 and 7.6 respectively. Thus increase in entropy during the process is 7.6 – 5.7 = 1.9 kJ/kg K.

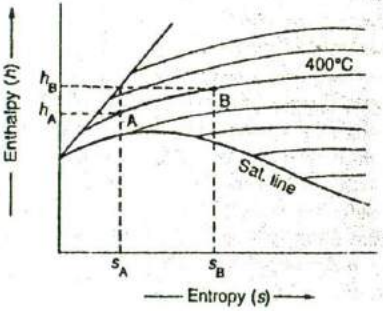


Fig. 8.10. Isothermal lines on *h-s* diagram.

8.14. Isentropic Lines on *h-s* Diagram

We know that during an *isentropic process, there is no change of entropy. It is thus obvious, that the isentropic process is marked by a vertical line *AB* on the *h-s* diagram as shown in Fig. 8.11.

Now consider a sample of superheated steam at a temperature of 300° C and pressure 10 bar which may be shown by the point *A* on the *h-s* diagram. From the Mollier diagram, we find that the enthalpy of this steam is 3050 kJ/kg. Let this sample be expanded to a pressure of 0.5 bar. Now draw a vertical line, through *A* to meet the pressure line of 0.5 bar, which may be shown by point *B*. From the diagram, we find that the enthalpy of steam at 0.5 bar is 2480 kJ/kg. Thus decrease in enthalpy from *A* to *B* is 3050 – 2480 = 570 kJ/kg.

Note: From the Mollier diagram, we find that the specific volume of steam at *A* and *B* is 0.18 and 3.3 m^3/kg respectively. Dryness fraction of steam at *B* is 0.928.

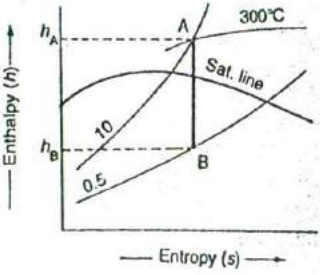


Fig. 8.11. Isentropic lines on *h-s* diagram

8.15. Throttling Lines on *h-s* Diagram

We know that during a **throttling process, there is no change of enthalpy. It is thus obvious, that the throttling process is marked by a horizontal line *AB* on the *h*-s diagram as shown in Fig. 8.12.

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* For details, please refer Art. 3.10.

** For details, please refer Art. 3.21.

Now consider a sample of wet steam at a pressure of 14 bar and dryness fraction 0.88 which may be shown by the point *A* on the *h-s* diagram. From the Mollier diagram, we find that the entropy of this steam is 5.94 kJ/kg K. Let this sample be expanded by throttling to a pressure of 1.0 bar. Now draw a horizontal line through *A* to meet the pressure line of 1.0 bar, which may be shown by the point *B*. From the Mollier diagram, we find that the entropy at *B* is 7.04 kJ/kg K. Thus increase in entropy from *A* to *B* is 7.04 – 5.94 = 1.1 kJ/kg K.

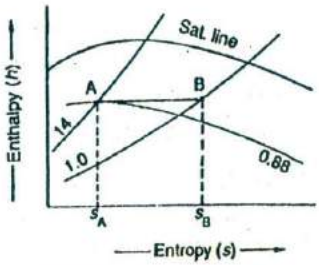


Fig. 8.12. Throttling lines on *h-s* diagram.

Note : From the Mollier diagram, we also find that specific volume of steam at *A* and *B* is 0.14 and 1.7 m^3/kg respectively. Dryness fraction of steam at *B* is 0.95.

Example 8.4. *Using Mollier chart, find enthalpy drop and final condition of steam when it is expanded isentropically from an initial pressure of 30 bar and 350° C to a pressure of 1 bar.*

Solution. Given : $p_1 = 30$ bar ; $t_1 = 350°$ C ; $p_2 = 1$ bar

1. First of all, on the Mollier chart mark a point *A*, where the pressure line through p_1 (*i.e.* 30 bar) and temperature line through t_1 (*i.e.* 350° C) meets as shown in Fig. 8.13.

2. Since the steam is expanded isentropically (*i.e.* at a constant entropy), therefore draw a vertical line through *A* to meet the pressure line p_2 (*i.e.* 1 bar) at point *B* as shown in Fig. 8.13.

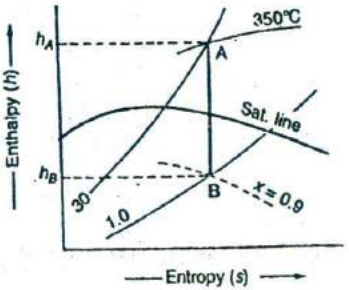


Fig. 8.13

Enthalpy drop during the process

From the Mollier chart, we find that enthalpy at *A*,

$$h_A = 3120 \text{ kJ/kg}$$

and enthalpy at *B*, $h_B = 2450$ kJ/kg

∴ Enthalpy drop during the process,

$$h = h_A - h_B = 3120 - 2450 = 670 \text{ kJ/kg}$$ **Ans.**

Final condition of the steam

From the Mollier chart, we also find that final condition (*i.e.* dryness fraction) of the steam at *B* is 0.9 **Ans.**

Example 8.5. *A sample of steam at 200° C and a pressure of 7 bar is first expanded at a constant enthalpy to 3 bar, and then at constant entropy to 0.5 bar. With the help of Mollier diagram, determine the change of entropy and enthalpy during the processes.*

Solution. Given : $t_1 = 200°$ C ; $p_1 = 7$ bar ; $p_2 = 3$ bar ; $p_3 = 0.5$ bar

1. First of all, on the Mollier diagram mark point *A* where the pressure line through p_1 (*i.e.* 7 bar) and temperature line through t_1 (*i.e.* 200° C) meets as shown in Fig. 8.14.

2. Since the steam is first expanded at constant enthalpy, therefore draw a horizontal line through *A* to meet the pressure line through p_2 (*i.e.* 3 bar) at point *B* as shown in Fig. 8.14.

3. Now as the steam is expanded at constant entropy, therefore draw a vertical line through B to meet the pressure line p_3 (*i.e.* 0.5 bar) at point C as shown in Fig. 8.14.

From the Mollier diagram, we find that

$h_A = h_B = 2850$ kJ/kg ; $s_A = 6.84$ kJ/kg K

$h_C = 2535$ kJ/kg ; $s_B = s_C = 7.24$ kJ/kg K

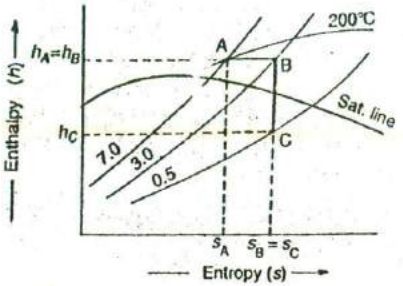


Fig. 8.14

Change of entropy and enthalpy during constant enthalpy process

We know that change of entropy

$= s_B - s_A = 7.24 - 6.84$

$= 0.4$ kJ/kg K Ans.

and change of enthalpy = 0 Ans.

Change of entropy and enthalpy during constant entropy process

We know that change of entropy

= 0 Ans.

and change of enthalpy $= h_B - h_C = 2850 - 2535 = 315$ kJ/kg Ans.

EXERCISES

1. Calculate the entropy of 1 kg of water at 90° C. [Ans. 1.195 kJ/kg K]
2. Calculate the entropy increase during evaporation of 1 kg of dry saturated steam at a pressure of 7 bar. [Ans. 4.714 kJ/kg K]
3. Determine the entropy of 1 kg of wet steam at a pressure of 10.5 bar and of dryness fraction 0.9. [Ans. 6.127 kJ/kg K]
4. Determine the entropy of 1 kg of superheated steam at 14 bar and 300° C. Assume $c_p = 2.1$ kJ/kg K. [Ans. 6.89 kJ/kg K]
5. Calculate the entropy of 1 kg of superheated steam at a pressure of 15 bar and 100° C of superheat. Assume $c_p = 2.3$ kJ/kg K. [Ans. 6.883 kJ/kg K]
6. Dry saturated steam at a pressure of 10 bar is expanded isentropically to a pressure of 0.7 bar. Using Mollier diagram, determine the enthalpy drop. [Ans. 430 kJ/kg]

QUESTIONS

1. What is the arbitrary temperature of zero entropy chosen for water ?
2. Is the temperature used in calculations for entropy °C or K ?
3. Write the equation for the entropy increase during superheating.
4. Show the following processes on T-s diagram :
   (*i*) heating of water from 0° C to boiling temperature ;
   (*ii*) evaporation of water ; and
   (*iii*) superheating of steam.
5. What is meant by constant dryness fraction lines ? How these are plotted on T-s diagram ?
6. Write a short note on the Mollier chart and its construction.
7. Name the various uses of Mollier chart.
8. Show the throttling and isentropic expansion of steam on Mollier chart.

OBJECTIVE TYPE QUESTIONS

1. The entropy of water at 0° C is

(*a*) 0 (*b*) 1 (*c*) 2.3 (*d*) 4.2

2. When water is heated from the freezing point to the boiling point, then the increase in entropy is given by

(*a*) $c_w \log (273)$ (*b*) $c_w \log (T)$ (*c*) $c_w \log_e \left(\frac{T}{273}\right)$ (*d*) $c_w \log_e \left(\frac{273}{T}\right)$

where c_w = Specific heat of water, and

T = Boiling temperature in K.

3. The entropy of dry steam (s_g) is given by

(*a*) $s_f + \frac{T}{h_{fg}}$ (*b*) $s_f + \frac{h_{fg}}{T}$ (*c*) $s_f - \frac{T}{h_{fg}}$ (*d*) $s_f - \frac{h_{fg}}{T}$

where s_f = Entropy of water,

T = Absolute temperature, and

h_{fg} = Latent heat of vaporisation.

4. When dry steam at a temperature T is heated upto a temperature of superheated steam (T_{sup}), then the increase in entropy ($s_{sup} - s_g$) is given by

(*a*) $c_p \log_e \left(\frac{T_{sup}}{T}\right)$ (*b*) $2.3\, c_p \log_e \left(\frac{T_{sup}}{T}\right)$

(*c*) $c_p \log_e \left(\frac{T}{T_{sup}}\right)$ (*d*) $2.3\, c_p \log_e \left(\frac{T}{T_{sup}}\right)$

where c_p = Specific heat of superheatedsteam.

5. The isentropic process, on the Mollier diagram, is represented by a

(*a*) horizontal line (*b*) vertical line (*c*) inclined line (*d*) curve

6. The throttling process, on the *h-s* diagram, will be a

(*a*) horizontal line (*b*) vertical line (*c*) inclined line (*d*) curve

ANSWERS

1. (*a*) 2. (*c*) 3. (*b*) 4. (*a*) 5. (*b*)
6. (*a*)